Factorizing one-loop contributions to two-loop Bhabha scattering and automatization of Feynman diagram calculations

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Abstract

In higher order calculations a number of new technical problems arise: one needs diagrams in arbitrary dimension in order to obtain their needed ε**-expansion, zero Gram determinants** appear, renormalization produces diagrams with 'dots' on the lines, i.e. **higher order powers of scalar propagators**. All these problems cannot be accessed by the 'standard' Passarino-Veltman approach: there is not available what is needed for higher loops. We demonstrate our method of how to solve these problems.

We are moving in the direction of **two-loop** Bhabha scattering, which is extremely important, in particular at higher energies, for the luminosity determination of the coming accellerators.

Factorizing one-loop contributions are obtained directly by 'squaring', e.g.:

$$\text{1-loop} \times \text{1-loop} = \text{2-loop contribution}$$

and via **renormalization.** As an example mass-renormalization: in any Feynman diagram we replace

$$\frac{1}{k^2-(m^2+\delta(m^2))} \to \frac{1}{k^2-m^2}(1+\frac{\delta(m^2)}{k^2-m^2}),$$

thus obtaining 'dotted' diagrams.

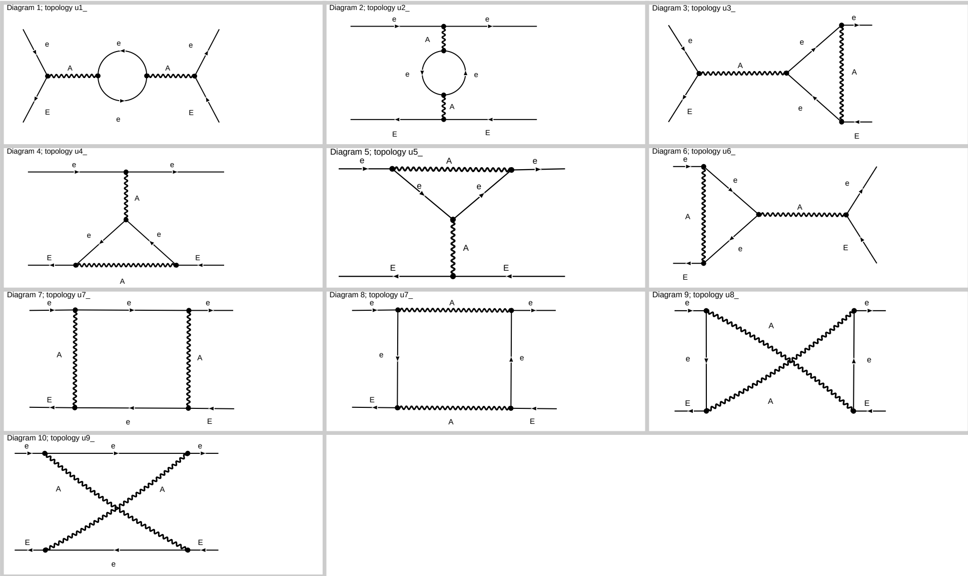
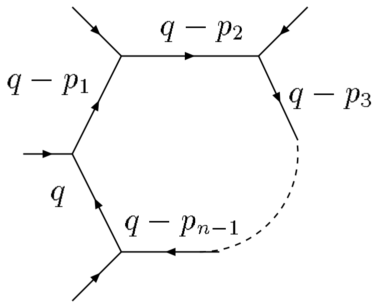


Diagram 1; topology u1_
Diagram 2; topology u2_
Diagram 3; topology u3_
Diagram 4; topology u4_
Diagram 5; topology u5_
Diagram 6; topology u6_
Diagram 7; topology u7_
Diagram 8; topology u7_
Diagram 9; topology u8_
Diagram 10; topology u9_

Calculation of 1-loop integrals

Ref.
J.F. , F. Jegerlehner and O.V. Tarasov, Nucl.Phys. B566 (2000) p.423



One-loop diagram with n external legs.

Tensor integrals

$$I_{n,r}^{(d)} = \int \frac{d^dq}{\pi^{d/2}} \prod_{j=1}^{n} \frac{q_{\mu_1} \dots q_{\mu_r}}{c_j^{\nu_j}},$$
$$= T_{\mu_1 \dots \mu_r}(\{p_s\}, \{\partial_j\}, \mathbf{d}) I_n^{(d)}$$

where $\nu_j =$ 'index'

$$c_j = (q-p_j)^2 - m_j^2 + i\epsilon \quad \text{for} \quad j < n \quad \text{and} \quad c_n = q^2 - m_n^2 + i\epsilon.$$

Tensor operator:

$$T_{\mu_1 \dots \mu_r}(\{p_s\}, \{\partial_j\}, \mathbf{d}^+) = \frac{1}{i^r} \prod_{j=1}^{r} \frac{\partial}{\partial a_{\mu_j}} \exp\left[i \left(\sum_{k=1}^{n-1} (ap_k)\alpha_k - \frac{1}{4}a^2 \right) \rho \right] \Bigg|_{\substack{a_j=0 \\ \alpha_j = i\partial_j \\ \rho = i\mathbf{d}^+}} .$$

with

$$\partial_j = \frac{\partial}{\partial m_j^2}, \qquad \mathbf{d}^+ I^{(d)} = I^{(d+2)},$$

Only scalar integrals remain to be evaluated !

Integrals with non-zero Gram determinants

$$\Delta_n = \begin{vmatrix} Y_{11} & Y_{12} & \ldots & Y_{1n} \\ Y_{12} & Y_{22} & \ldots & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{1n} & Y_{2n} & \ldots & Y_{nn} \end{vmatrix}.$$

where

$$Y_{ij} = -(p_i - p_j)^2 + m_i^2 + m_j^2,$$

"Modified Cayley determinant" of the diagram with internal lines $1 \ldots n$

$$()_n \equiv \begin{vmatrix} 0 & 1 & 1 & \ldots & 1 \\ 1 & Y_{11} & Y_{12} & \ldots & Y_{1n} \\ 1 & Y_{12} & Y_{22} & \ldots & Y_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & Y_{1n} & Y_{2n} & \ldots & Y_{nn} \end{vmatrix},$$

"Signed minors"

$$\begin{pmatrix} j_1 j_2 \cdots \\ k_1 k_2 \ldots \end{pmatrix}_n$$

will be labeled by the rows $j_1, j_2, \ldots$ and columns $k_1, k_2, \ldots$ excluded from $()_n$. E.g. we have

$$\Delta_n = \begin{pmatrix} 0 \\ 0 \end{pmatrix}_n$$

Recursion Relations:

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}_n \nu_j \mathbf{j}^+ I_n^{(d)} = \sum_{k=1}^{n} \begin{pmatrix} 0j \\ 0k \end{pmatrix}_n \left[d - \sum_{i=1}^{n} \nu_i (\mathbf{k}^- \mathbf{i}^+ + 1) \right] I_n^{(d)}.$$

The operators $\mathbf{j}^{\pm}$ etc. shift the indices $\nu_j \to \nu_j \pm 1$.

This relation reduces the indices and leaves the dimension.

The following one reduces simultaneously indices and dimension:

$$()_n \, \nu_j \mathbf{j}^+ I_n^{(d+2)} = \left[-\begin{pmatrix} j \\ 0 \end{pmatrix}_n + \sum_{k=1}^{n} \begin{pmatrix} j \\ k \end{pmatrix}_n \mathbf{k}^- \right] I_n^{(d)},$$

A relation reducing the space time dimension is given by:

$$(d-\sum_{i=1}^{n}\nu_i+1)\,()_n\,I_n^{(d+2)}=\left[\binom{0}{0}_n-\sum_{k=1}^{n}\binom{0}{k}_n\mathbf{k}^-\right]I_n^{(d)}.$$

Integrals with zero kinematic determinants

$$(d-\sum_{i=1}^{n}\nu_i+1)\,()_n\,I_n^{(d+2)}=-\sum_{k=1}^{n}\binom{0}{k}_n\mathbf{k}^-I_n^{(d)}.$$

To increase again the dimension d one uses relation

$$\sum_{j=1}^{n}\nu_j\mathbf{j}^+I_n^{(d+2)}\;=-I_n^{(d)}\;.$$

As an example we show the relation for the three-point integral C_0 with zero Gram determinant:

$$(s-4m_e^2)C_0(m_e,0,m_e,m_e^2,m_e^2,s)=-\frac{2(d-3)}{(d-4)}B_0(m_e^2,m_e^2,s)-\frac{(d-2)}{m_e^2(d-4)}A_0(m_e^2)$$

Decomposition of the diagrams into Amplitudes (diagram 7)

$$\begin{array}{llllllll}
O_1 = & \overline{U}(-p_2) & I & U(p_1) & \cdot & \overline{V}(p_4) & I & V(-p_3) \\
O_2 = & \overline{U}(-p_2) & \hat{p}_4 & U(p_1) & \cdot & \overline{V}(p_4) & I & V(-p_3) \\
O_3 = & \overline{U}(-p_2) & I & U(p_1) & \cdot & \overline{V}(p_4) & \hat{p}_2 & V(-p_3) \\
O_4 = & \overline{U}(-p_2) & \hat{p}_4 & U(p_1) & \cdot & \overline{V}(p_4) & \hat{p}_2 & V(-p_3) \\
O_5 = & \overline{U}(-p_2) & \gamma_\mu & U(p_1) & \cdot & \overline{V}(p_4) & \gamma_\mu & V(-p_3) \\
O_6 = & \overline{U}(-p_2) & \gamma_\mu \hat{p}_4 & U(p_1) & \cdot & \overline{V}(p_4) & \gamma_\mu & V(-p_3) \\
O_7 = & \overline{U}(-p_2) & \gamma_\mu & U(p_1) & \cdot & \overline{V}(p_4) & \gamma_\mu \hat{p}_2 & V(-p_3) \\
O_8 = & \overline{U}(-p_2) & \gamma_\mu \gamma_\nu & U(p_1) & \cdot & \overline{V}(p_4) & \gamma_\nu \gamma_\mu & V(-p_3) \\
O_9 = & \overline{U}(-p_2) & \gamma_\mu \gamma_\nu \hat{p}_4 & U(p_1) & \cdot & \overline{V}(p_4) & \gamma_\nu \gamma_\mu & V(-p_3) \\
O_{10} = & \overline{U}(-p_2) & \gamma_\mu \gamma_\nu & U(p_1) & \cdot & \overline{V}(p_4) & \gamma_\nu \gamma_\mu \hat{p}_2 & V(-p_3) \\
O_{11} = & \overline{U}(-p_2) & \gamma_\mu \gamma_\nu \hat{p}_4 & U(p_1) & \cdot & \overline{V}(p_4) & \gamma_\nu \gamma_\mu \hat{p}_2 & V(-p_3) \\
O_{12} = & \overline{U}(-p_2) & \gamma_\mu \gamma_\nu \gamma_\rho & U(p_1) & \cdot & \overline{V}(p_4) & \gamma_\rho \gamma_\nu \gamma_\mu & V(-p_3)
\end{array}$$

For diagram 8:

$$\overline{U}(-p_2) \quad \cdot \quad V(-p_3) \qquad \overline{V}(p_4) \quad \cdot \quad U(p_1)$$

The on-shell diagram reads

$$\mathbf{Diagram} = \sum_{\mathbf{j=1}}^{\mathbf{12}} \mathbf{A_j O_j}.$$

with **amplitudes** $\mathbf{A_j}$.

Crossing relations (e.g. diagram 8 ◁▷ diagram 9)

$$\begin{aligned}
A_1^{(9)} &= A_1^{(8)} - 2m_e A_7^{(8)} - 4m_e A_9^{(8)} + 2dA_8^{(8)} \\
A_2^{(9)} &= -A_2^{(8)} + 4A_7^{(8)} - 2(d-4)A_9^{(8)} - 8m_e A_{11}^{(8)} \\
A_3^{(9)} &= -A_3^{(8)} + 2A_6^{(8)} - 2(d-2)A_{10}^{(8)} - 4m_e A_{11}^{(8)} \\
A_4^{(9)} &= A_4^{(8)} - 2(d-2)A_{11}^{(8)} \\
A_5^{(9)} &= -A_5^{(8)} + 2m_e A_6^{(8)} + 4m_e A_7^{(8)} + 8m_e A_9^{(8)} + 4m_e A_{10}^{(8)} - 12m_e^2 A_{11}^{(8)} - (6d-4)A_{12}^{(8)} \\
A_6^{(9)} &= -4m_e A_{11}^{(8)} + A_6^{(8)} \\
A_7^{(9)} &= -4m_e A_{11}^{(8)} + A_7^{(8)} \\
A_8^{(9)} &= -A_8^{(8)} \\
A_9^{(9)} &= A_9^{(8)} \\
A_{10}^{(9)} &= A_{10}^{(8)} \\
A_{11}^{(9)} &= -A_{11}^{(8)} \\
A_{12}^{(9)} &= A_{12}^{(8)}
\end{aligned}$$

and **exchanging t ◁▷ u**. These relations also hold for crossing diagram 7 ◁▷ diagram 10 (s ◁▷ u). For the crossing 7 ◁▷ 8 (s ◁▷ t) only a change of sign occurs in the amplitudes. **Thus:** we need to calculate **only one** SE, vertex and box.

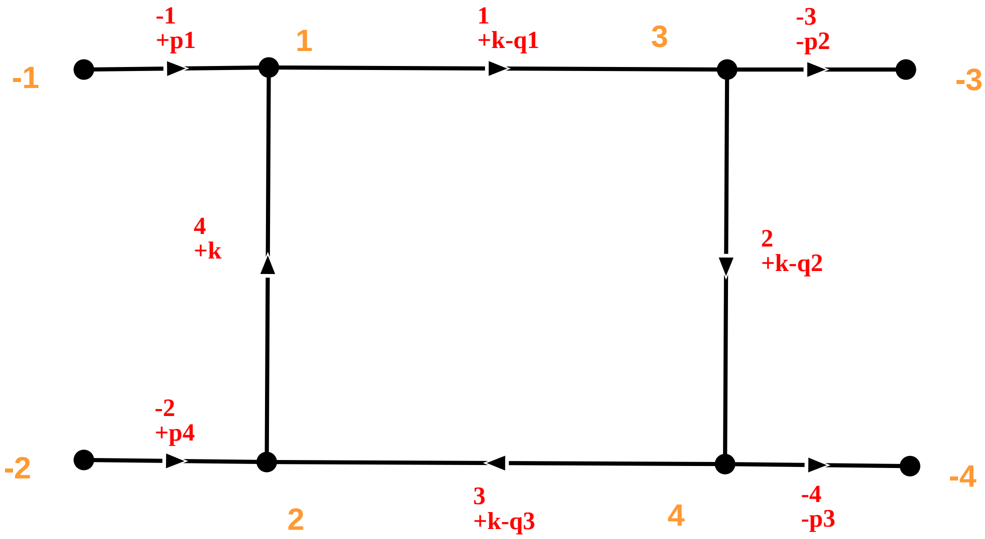
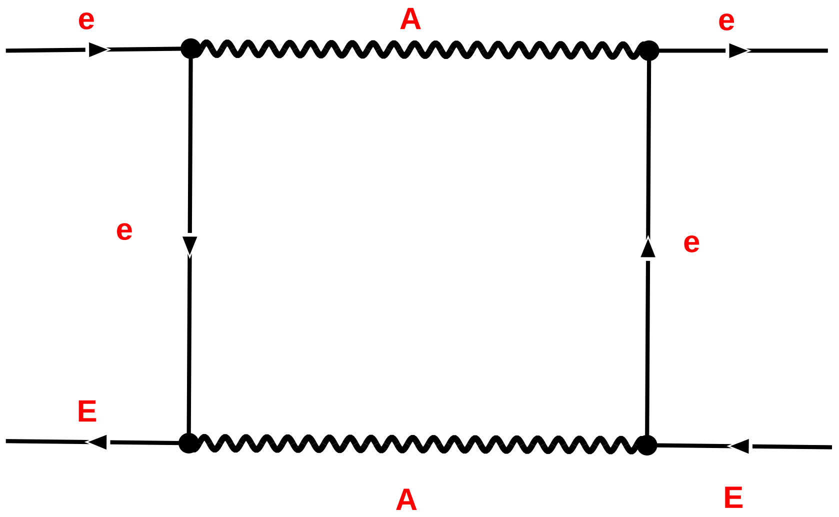


Diagram 8 topology u7_ (unique u7_) momentaset 1 (of 1)



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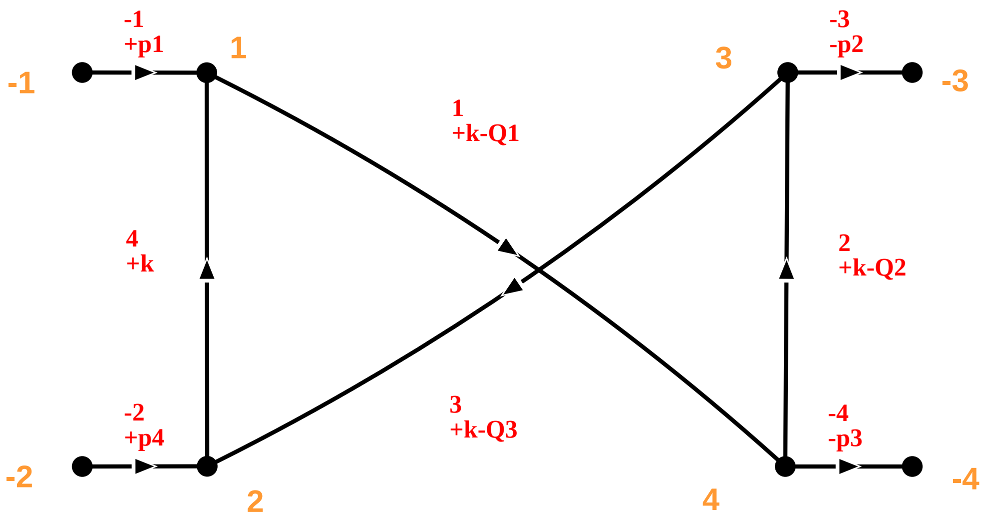
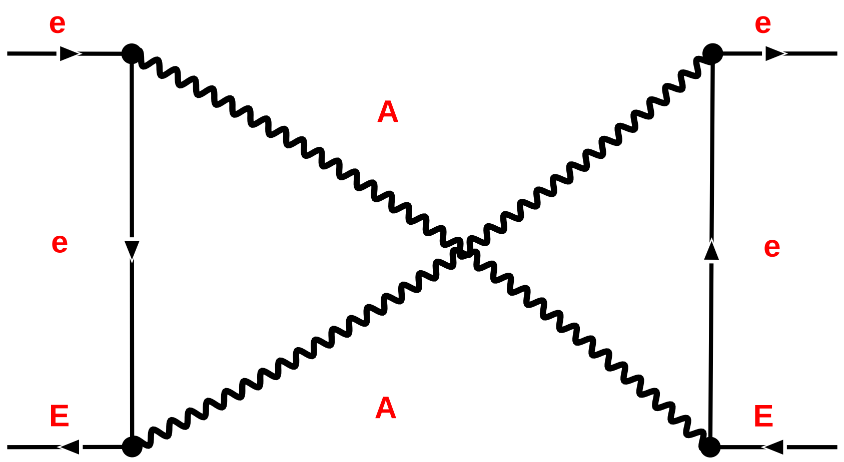


Diagram 9 topology u8_ (unique u8_) momentaset 1 (of 1)



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Further relations between amplitudes of **one** diagram (obtained by solving a system of equations):

$$
\begin{aligned}
A_3 &= A_2 \\
A_7 &= A_6 \\
A_{10} &= A_9 \\
A_8 &= \frac{1}{4}A_1 - \frac{m_e}{2}A_2 + \;(\;A_{12}\;) \\
A_9 &= \frac{1}{4m_e}A_1 - \frac{1}{2}A_2 - \frac{1}{2}A_6 \\
A_{11} &= \frac{1}{4m_e^2}A_1 - \frac{1}{2m_e}A_2
\end{aligned}
$$

Thus: there are only **6 indpendent amplitudes**.

Contribution to the differential cross section

$$\frac{d\sigma}{dcos\theta} = \frac{\pi\alpha^2}{2s}\sum_{j=1}^{12}\left[\frac{B_j(s,t)}{4}A_j(s,t) - \frac{B_j(t,s)}{4}A_j(t,s).\right]$$

$$B_1 = -\frac{4}{t}s^2 + (2d - 8 + \frac{24}{t}m_e^2)s - \frac{32}{t}m_e^4 - 8dm_e^2 - 32\frac{m_e^2}{s}(t - 2m_e^2),$$

$$B_2 = \frac{4}{t}m_e(s^2 + 8m_e^4) + (2(4-d)m_e - \frac{24}{t}m_e^3)s - 4(d+2)m_e t + 8(d-2)m_e^3 - 16\frac{m_e}{s}(t - 2m_e^2)^2,$$

$$B_3 = B_2,$$

$$B_4 = -\frac{4}{t}(m_e^2 + t)s^2 + ((-4 - 2d)t + 2dm_e^2 + \frac{24}{t}m_e^4)s - 2(d+4)t^2 + 8(1+d)m_e^2 t - 8(d-4)m_e^4$$

$$-\frac{32}{t}m_e^6 - \frac{8}{s}(t - 2m_e^2)^3,$$

$$B_5 = -\frac{1}{m_e}B_6 + 4td + \frac{8(st + 2t^2 + 2m_e^2 s - 4m_e^2 t)}{s},$$

$$\frac{B_6}{m_e} = -4\frac{(d-2)}{t}(s^2 - 2tm_e^2) + (2(d-2)(d-10) + 8(d-4)\frac{m_e^2}{t})s + 32\frac{m_e^4}{t} + 32\frac{m_e^2}{s}(t - 2m_e^2),$$

$$B_7 = B_6,$$

$$B_8 = -4\frac{(d-2)^2}{t}s^2 + (2(d-2)(d-4)(d-10) + 8(3d^2 - 8d + 8)\frac{m_e^2}{t})s - 8(d-2)(d-4)t$$

$$-8(d-2)(d^2 - 10d + 20)m_e^2 - 32(d^2 - 2d + 2)\frac{m_e^4}{t} - 32d\frac{m_e^2}{s}(t - 2m_e^2),$$

$$\frac{B_9}{m_e} = 4(d-2)^2\frac{s^2}{t} + (2(2-d)(d-4)(d-10) - 8(3d^2 - 8d + 8)\frac{m_e^2}{t})s - 4(d^3 + 34d$$

$$-10d^2 - 28)t + 8(d^3 - 14d^2 + 52d - 60)m_e^2 + 32(d^2 - 2d + 2)\frac{m_e^4}{t} - 16d\frac{(t - 2m_e^2)^2}{s},$$

$$B_{10} = B_9,$$

$$B_{11} = (-4(d-4)^2 t - 4(d-2)^2 m_e^2)s^3 + (2(4-d)(d^2 - 4d - 2)t^2 + 2(d^3 + 20d - 12d^2 + 32)m_e^2 t$$

$$+8(3d^2 - 8d + 8)m_e^4)s^2 + ((20d^2 - 68d + 64 - 2d^3)t^3 + 8(d-4)(d^2 - 8d + 10)m_e^2 t^2 - 8(d-2)$$

$$(d^2 - 14d + 28)m_e^4 t - 32(d^2 - 2d + 2)m_e^6)s - 8(d-2)t^3(t - 6m_e^2) - 32(3d-4)m_e^4 t^2 + 64dm_e^6 t,$$

$$B_{12} = 4(d-2)^3 s^3 + (-2(d-2)(d^3 - 20d^2 + 110d - 208)t - 8(24d - 16 + d^3 - 12d^2)m_e^2)s^2$$

$$+((12d^3 - 120d^2 - 352 + 400d)t^2 - 8(3d^3 - 42d^2 + 148d - 160)m_e^2 t - 32(4 - 6d + 3d^2)m_e^4)s$$

$$+16(3d-2)t(t^2 - 4m_e^2 t + 4m_e^4).$$

Contribution to the differential cross section

$$\frac{d\sigma}{dcos\theta} = \frac{\pi\alpha^2}{2s}\sum_{j=1}^{12}\left[\frac{B_j(s,t)}{4}A_j(s,t) - \frac{B_j(t,s)}{4}A_j(t,s).\right]$$

$$B_1 = -\frac{4}{t}(s^2 - 6m_e^2 s + 8m_e^4) - 32m_e^2 - 32\frac{m_e^2}{s}(t - 2m_e^2),$$
$$B_2 = 4\frac{m_e}{t}s^2 - 24\frac{m_e^3}{t}s - 24m_e t + 16m_e^3 + 32\frac{m_e^5}{t} - 16\frac{m_e}{s}(t - 2m_e^2)^2,$$
$$B_3 = B_2,$$
$$B_4 = -4(\frac{m_e^2}{t} + 1)s^2 + (-12t + 8m_e^2 + 24\frac{m_e^4}{t})s - 16t^2 + 40m_e^2 t - 32\frac{m_e^6}{t} - \frac{8}{s}(t - 2m_e^2)^3,$$
$$B_5 = 8\frac{s^2}{t} + 24s + 24t - 32\frac{m_e^4}{t} + 16\frac{(t - 2m_e^2)^2}{s},$$
$$B_6 = -8\frac{m_e s^2}{t} - 24m_e s + 16m_e^3 + 32\frac{m_e^5}{t} + 32\frac{m_e^3}{s}(t - m_e^2),$$
$$B_7 = B_6,$$
$$B_8 = -16\frac{s^2}{t} + 192\frac{m_e^2}{t}s + 64m_e^2 - 320\frac{m_e^4}{t} - 128\frac{m_e^2}{s}(t - 2m_e^2),$$
$$B_9 = 16\frac{m_e s^2}{t} - 192\frac{m_e^3 s}{t} - 48m_e t - 96m_e^3 + 320\frac{m_e^5}{t} - 64\frac{m_e}{s}(t - 2m_e^2)^2,$$
$$B_{10} = B_9,$$
$$B_{11} = -16m_e^2 s^3 - 32m_e^2(t - 8m_e^2)s^2 - 16(t^3 - 12m_e^4 t + 0m_e^6)s - 16t^4 + 96m_e^2 t^3 - 256m_e^4 t(t - m_e^2),$$

$$B_{12} = 32s^3 + 96(t + 4m_e^2)s^2 + (96t^2 + 384m_e^2 t - 896m_e^4)s + 160t(t - 2m_e^2)^2.$$

'Undotted' Diagrams; common factor $\frac{e^2}{(4\pi)^{d/2}}$ (normalized to Born term)

Selfenergy - diagrams:

$$A_5 = A_0 \frac{4}{s^2}[\frac{1}{d-1} - 1] + B_e \frac{2}{s}[\frac{1}{d-1}(1-z^2) - 1]$$

Vertex - diagrams:

$$A_2 = A_0 \frac{2}{m_e s^2} x^2 [2\frac{1}{d-3} - (d-4)] + B_e \frac{4m_e}{s^2} x^2 [1 - (d-4)]$$

$$A_5 = 2(-A_0 \frac{1}{m_e^2 s}[2\frac{1}{d-3} - x^2 z^2] + B_e \frac{1}{s}[x^2(1+z^2) + d - 4] + F_s \frac{2}{(d-4)s}[1+x^2])$$

'Dotted' Diagrams; common factor $\delta(m_e)/m_e \; e^4$

Selfenergy - diagrams:

$$A_5 = A_0 \frac{4}{s^2}(d-2)x^2 z^2 - B_e \frac{2}{s} z^2 [1 - (d-3)x^2]$$

Vertex - diagrams:

$$A_2 = -A_0 \frac{4}{m_e s^2} x^2 [\frac{1}{d-3} x^2 + (d-4)(1 + (d-4)(1 - x^2/2)) - 3 + 2x^2]$$
$$-B_e \frac{4m_e}{s^2} x^2 [(d-4)(x^2 - (d-4)x^2 z^2) + 2x^2 z^2] + F_s \frac{8m_e}{(d-4)s^2} x^2 [1 + x^2]$$

$$A_5 = 2(A_0 \frac{2}{s^2} x^2 [(\frac{1}{d-3} + 6\frac{1}{d-5}) - (d-4)(2x^2 - 1 + (d-4)) + 7 - 4x^2]$$
$$+A_0 \frac{2}{m_e^2 s}[1 + 3\frac{1}{d-5}] - B_e \frac{1}{s} x^2 z^2 [(d-4)(2x^2 + (d-4)) + 2x^2] - F_s \frac{2}{(d-4)s} x^2 z^2)$$

Notation:

$A_0 = A_0(m_e), B_t = B_0(m_e, m_e, t), B_0 = B_0(0, 0, s), B_e = B_0(m_e, m_e, s)$

$F_s = A_0(m_e)/{m_e}^2 + B_0(m_e, m_e, s), F_t = A_0(m_e)/{m_e}^2 + B_0(m_e, m_e, t),$

$C_0 = C_0(0, m_e, 0, m_e^2, m_e^2, s), D_0 = D_0(m_e, 0, m_e, 0, m_e^2, m_e^2, m_e^2, m_e^2, t, s).$

F_s, F_t: only with factor $\frac{1}{d-4}$!

Further abbreviations:

$r_t = t/s, \quad r_u = u/s$

and $x^2 = 1/(1 - 4m_e^2/s), \quad y^2 = 1/(1 - 4m_e^2/t), \quad z^2 = \frac{4m_e^2}{s}$ $(x^2 - 1 = x^2 z^2)$.

‘Undotted’ Box-Diagrams; common factor $\frac{e^2}{(4\pi)^{d/2}} r_t^{-2} r_u^{-2}$

$$
\begin{aligned}
&A_1 = \\
&A_0\frac{2r_t}{s^2}(-\frac{1}{(d-3)}(1+r_u+2r_tr_ux^2+r_uy^2-z^2)-(r_u+2r_t+2r_tr_ux^2+(1+5r_u)y^2))- \\
&B_tr_t\frac{z^2}{s}(2r_t+(1+5r_u)y^2)-B_0r_t\frac{z^2}{s}(1+2r_tx^2)r_u+F_tr_t\frac{z^2}{s(d-4)}(1-2r_t-(1+4r_u)y^2-z^2) \\
&-C_0r_tz^2(\frac{1}{(d-3)}(r_t+r_u-z^2(1+r_t+r_u-z^2))-(r_t+r_u+2r_t^2+6r_tr_u-2z^2r_t(1-r_ux^2)))/2 \\
&+D_0r_tsz^2(\frac{1}{(d-3)}(r_t+z^2(r_u-z^2(1+r_t+r_u-z^2))) \\
&+(r_t(1+2r_t+2r_u+2r_t^2+6r_tr_u+4r_u^2)-2z^2r_t(1+2r_t+3r_u-z^2)))/4,
\end{aligned}
$$

$$
\begin{aligned}
&A_2 = \\
&A_0\frac{2r_t}{m_es^2}(-\frac{1}{(d-3)}(1+r_t+r_u+r_tr_ux^2-z^2)-(r_t+r_u+r_tr_ux^2+(1+2r_u)y^2)) \\
&-B_t\frac{4m_er_t}{s^2}(r_t+(1+2r_u)y^2)-B_0\frac{4m_er_t}{s^2}(r_u+r_tr_ux^2)+F_t\frac{4m_er_t}{s^2(d-4)}(1-(1+2r_u)y^2-z^2) \\
&-C_0\frac{2m_er_t}{s}(\frac{1}{(d-3)}((1+r_t)(r_u+r_t)-z^2(1+r_u+2r_t-z^2))-(r_t+r_t^2 \\
&+(1+3r_t)r_u-z^2r_t(1-x^2r_u)))+D_0m_er_t(\frac{1}{(d-3)}(r_t+r_t^2+z^2(r_t^2+(1+r_t)r_u \\
&-z^2(1+r_u+2r_t-z^2)))+r_t(1+3r_t+2r_t^2+2r_u(2+3r_t+2r_u)-z^2(3+4r_t+6r_u-2z^2))),
\end{aligned}
$$

$$
\begin{aligned}
&A_4 = \\
&A_0\frac{2r_t}{m_e^2s^2}(-\frac{1}{(d-3)}(1-2r_t(1+r_t)+r_u(1-2r_t)+2r_tr_ux^2-r_uy^2-z^2(1-2r_t)) \\
&-(r_u+2r_t+2r_tr_ux^2+(1-r_u)y^2))-B_t\frac{4r_t}{s^2}(2r_t+(1-r_u)y^2)-B_0\frac{4r_t}{s^2}(r_u+2r_tr_ux^2) \\
&+F_t\frac{4r_t}{s^2(d-4)}(1-2r_tr_u-4r_t-2r_t^2-y^2-z^2(1-2r_t))-C_0\frac{2r_t}{s}(\frac{1}{(d-3)}(r_u+r_t(1-2r_t \\
&-2r_t^2-2r_u(1+r_t))-z^2(1+r_u(1-2r_t)-r_t(1+4r_t)-z^2(1-2r_t)))-(r_u+r_t+2r_t^2) \\
&+2z^2r_t(1-r_ux^2))+D_0r_t(\frac{1}{(d-3)}(r_t(1-2r_t-2r_t^2)+z^2(r_u(1-2r_t-2r_t^2)-2r_t^3 \\
&-z^2(1-r_t-4r_t^2+r_u(1-2r_t)-z^2(1-2r_t))))+r_t(1+2r_u+2r_t+2r_t^2) \\
&-2z^2r_t(1+2r_t-z^2)),
\end{aligned}
$$

$$
\begin{aligned}
&A_5 = \\
&A_0\frac{r_t}{m_e^2 s}(-\frac{1}{(d-3)}(2r_t(1-r_tr_u-r_t^2+r_tr_ux^2)+z^2(r_tr_u-r_t+3r_t^2-r_uy^2-z^2r_t)) \\
&-(2r_t(1+r_t+2r_u+r_tr_ux^2)+2z^2(1+r_u)y^2))-B_t\frac{4r_t}{s}(r_t(1+r_t+r_u)+z^2(1+r_u)y^2) \\
&-B_0\frac{4r_t}{s}(r_tr_u(1+r_tx^2))-F_t\frac{2r_t}{s(d-4)}(2r_t(r_t+r_t^2+(2+r_t)r_u)+z^2(r_t(1-r_u-3r_t) \\
&+(2+3r_u)y^2+z^2r_t))-C_0r_t^2(\frac{1}{(d-3)}(2(r_t-r_t^3+(1-r_t^2)r_u)-z^2(2+r_t+r_u \\
&-3r_tr_u-5r_t^2-z^2(1-r_u-4r_t+z^2)))-2(r_t+r_t^2+(1+2r_t)r_u-z^2r_t(1-r_ux^2))) \\
&+D_0r_t^2s(\frac{1}{(d-3)}(r_t-r_t^3+z^2(r_t+3r_t^2-2r_t^3+2(1-r_t^2)r_u-z^2(2(1+r_t)-5r_t^2 \\
&+(1-3r_t)r_u-z^2(1-r_u-4r_t+z^2))))/2)+r_t(1+r_t)(1+r_t+2r_u) \\
&-z^2(4r_t(1+r_t)-(r_u-4r_t)r_u-2z^2r_t)/2),
\end{aligned}
$$

$$
\begin{aligned}
&A_6 = \\
&A_0\frac{r_t}{m_e s^2}(\frac{1}{(d-3)}(2(r_t-r_tr_ux^2-r_uy^2))-2(r_t+r_tr_ux^2+2r_uy^2)) \\
&-B_t\frac{4m_er_t}{s^2}(r_t+2r_uy^2)-B_0\frac{4m_er_t^2}{s^2}r_ux^2-F_t\frac{2m_er_t}{s^2(d-4)}(4r_t+2r_uy^2) \\
&+C_0\frac{2m_er_t^2}{s}(\frac{1}{(d-3)}(r_t+r_u-z^2)+r_t+2r_u-z^2(1-r_ux^2)) \\
&-D_0m_er_t^2(\frac{1}{(d-3)}(r_t+z^2(r_t+r_u-z^2))+(r_t+r_u-z^2)),
\end{aligned}
$$

$$
\begin{aligned}
&A_{12} = \\
&-A_0\frac{r_t^2}{m_e^2 s(d-3)}r_u/2+F_t\frac{r_t^2r_u}{s(d-4)}-C_0r_t^2(\frac{1}{(d-3)}((r_t+r_u)r_u-z^2r_u))/2 \\
&+D_0r_t^2s(\frac{1}{(d-3)}(r_tr_u+z^2((r_t+r_u)r_u-z^2r_u)))/4.
\end{aligned}
$$

'Dotted' Box-Diagrams; common factor $\frac{\delta(\mathbf{m_e})}{\mathbf{m_e}}\frac{\mathbf{e^2}}{(4\pi)^{\mathbf{d/2}}}\mathbf{r_t^{-2}r_u^{-2}}$

$$A_1 =$$
$$A_0\frac{1}{s^2}((d-4)(-2r_tr_u(1+2x^2r_t+y^2)-2z^2y^4(1-3r_u))-4r_tr_u(1+2x^2r_t+y^2)$$
$$+2r_t(1-y^2)-2z^2(r_t(1-2y^2)+y^4(2-5r_u)))+B_t\frac{z^2}{s}((d-4)(-z^2y^4(1-3r_u))$$
$$+r_t(1-y^2)-z^2((1-2y^2)r_t+y^4(1-2r_u)))+B_0\frac{z^2}{s}r_t((d-4)((r_t+y^2)(1-2r_u)$$
$$-r_u(1+2x^2r_t))+((r_t+y^2)(1-2r_u)-r_u(1+2x^2r_t)))+F_t\frac{z^2}{s(d-4)}(r_t(1-y^2)$$
$$-z^2(r_t(1-2y^2)+y^4r_u))+C_0z^22r_t((d-4)(-(r_t+y^2)(1-2r_u)+r_u(1+2r_t)$$
$$+z^2(2x^2r_tr_u+y^2(1-2r_u)))++z^2(2x^2r_tr_u+1-r_t+r_u-z^2))/4$$
$$+D_0z^2sr_t(-(d-4)z^2y^2(1-2r_u)+4r_tr_u^2+z^2((y^2(1-2r_u)-(1-2r_t(1-r_u)))+z^2))/4,$$

$$A_2 =$$
$$A_0\frac{1}{m_es^2}((d-4)(-2r_t(r_t+r_u+x^2r_tr_u+y^2(1+r_u))+2z^2y^4r_u)-\frac{6}{(d-5)}r_t(y^2(1+r_u)$$
$$+r_t)+2r_t(1-3r_t-2r_u-2x^2r_tr_u-y^2(4+3r_u))-2z^2(r_t-2y^4r_u))$$
$$+B_t\frac{4m_e}{s^2}(r_t(1-y^2)+z^2((d-4)y^4r_u-r_t+y^4r_u))+B_0\frac{4m_e}{s^2}r_t(d-3)(r_t-r_u+y^2$$
$$-2r_tr_ux^2+z^2r_tr_ux^2)+F_t\frac{m_e}{s^2(d-4)}(4r_t((1-y^2)-z^2))+C_0\frac{2r_tm_e}{s}((d-4)(2r_tr_u$$
$$-(r_t-r_u)-y^2+z^2(r_t+y^2+x^2r_tr_u))+z^2(1+r_t+(1+x^2r_t)r_u-z^2))$$
$$-D_02m_er_t((d-4)z^2(r_t+y^2)/2+r_t(1+r_t)r_u+z^2(1-y^2-r_t(1+r_t+2r_u)$$
$$-z^2(1-r_t))/2),$$

$$A_4 =$$
$$A_0\frac{1}{m_e^2s^2}((d-4)(2r_t(2r_t(3+r_t+2r_u-x^2r_u)+4y^2-r_u(1-5y^2))-2z^2(2r_t(r_t+y^2)$$
$$+(3+5r_u)y^4))+\frac{12}{(d-5)}r_t(r_t(3+r_t+r_u)+2(1+r_u)y^2-z^2r_t)+2r_t(6r_t(3+r_t)$$
$$-(3-10r_t+4r_tx^2-15y^2)r_u+12y^2)-2z^2(4r_t^2-2r_tr_uy^2+(6+9r_u)y^4))$$
$$-B_t\frac{4}{s^2}((d-4)z^2(2r_ty^2+(3+5r_u)y^4)+r_tr_u(1-y^2)-z^2(2r_t(r_t+y^2(1+r_u))$$
$$-(3+4r_u)y^4))+B_0\frac{4}{s^2}r_t((d-4)(1-y^2-z^2(1+2x^2r_tr_u))+(1-y^2)$$
$$-z^2(1+2x^2r_tr_u))-F_t\frac{1}{(d-4)s^2}(4r_tr_u(1-y^2)-4z^2(r_t(2r_t+4y^2+2y^2r_u)+r_uy^4))$$
$$-C_0\frac{2r_t}{s}((d-4)(1-y^2-z^2(1-2r_t(2(1+r_u)+r_t-x^2r_u-z^2)-y^2))+2z^2r_t(2+r_t$$
$$-x^2r_u-z^2))+D_0r_tz^2((d-4)(2r_t(2+r_t+2r_u-z^2)+y^2)-(1+y^2)+z^2),$$

$A_5 =$

$$A_0\frac{1}{m_e^2 s}((d-4)(2r_t^2(r_t^2+3r_t+2+r_u(2+2r_t+2r_u-x^2r_t))$$
$$-z^2(r_t(3r_t^2+5r_t+2r_tr_u-y^2(1+2r_u)r_u)-z^2(r_t(r_t-y^2)-(1+r_u)y^4)))$$
$$+\frac{3}{(d-5)}(2r_t^2(r_t^2+3r_t+2+r_u(2+2r_u+r_t))-z^2r_t(3r_t^2+3r_t+r_tr_u$$
$$-y^2 2(1+r_u+r_u^2)-z^2r_t))+2r_t^2(3r_t^2+9r_t+5r_tr_u+6(1+r_u+r_u^2)-$$
$$2x^2r_tr_u)+z^2(r_t(1+r_u-22r_t-14r_t^2-6r_tr_u+y^2(3+17r_u+12r_u^2))/2$$
$$+z^2(r_t(8r_t-2-4y^2(1-r_u))-4y^4(2+r_u))/4))-B_t\frac{z^2}{s}(2(d-4)(2r_t(r_t$$
$$+y^2(1+r_u))+z^2y^2(r_t+y^2(1+r_u)))-r_t(1+4r_t^2+(1+4r_t)r_u-5y^2(1-r_u))$$
$$+z^2(r_t(1+2r_t)-2y^2(r_tr_u-y^2)))-B_0\frac{1}{s}(d-3)(8r_t^2(1+r_t+r_u)r_u+z^2r_t(1+3r_t$$
$$+r_u(1-4r_t)+2x^2r_t(1+2r_t)r_u+2y^2(1+2r_u+2r_u^2)-z^2(1+2x^2r_tr_u)))+$$
$$F_t\frac{z^2}{(d-4)s}(r_t(1+r_u+4r_t(1+r_u+r_t)-y^2(1-9r_u))-z^2(r_t(1+2r_t-2y^2(1+r_u))$$
$$-2y^4r_u))+C_0r_t((d-4)(4r_t(1+r_t+r_u)r_u+z^2((1+r_u-r_t(1+10r_u+8r_u^2$$
$$+r_t(8+8r_u+4r_t))+4x^2r_t^2r_u+2y^2(1+2r_u+2r_u^2))/2-z^2(1-4r_t(1+r_u)-$$
$$6r_t^2+y^2(2+4r_u+4r_u^2)+2z^2r_t)/2))-z^2(r_t(r_u+2r_t(1+r_t)-2x^2r_tr_u)-z^2(1+r_u$$
$$+3r_t+6r_t^2-z^2(1+2r_t))/2))+D_0r_tz^2s((d-4)(r_t(1+2(r_t+(1+r_t+r_u)r_u)+r_t^2)$$
$$-z^2(r_t(2+2r_u+3r_t)-y^2(1+2r_u+2r_u^2)-z^2r_t)/2)$$
$$-r_t(2+2r_t+2r_tr_u+r_u^2+3/2r_u)+z^2(r_t(3+2r_u)-y^2(1+2r_u+2r_u^2))/2),$$

$A_6 =$

$$A_0\frac{1}{m_e s^2}((d-4)(2r_t(r_t-x^2r_tr_u+y^2)-2z^2y^4(1-r_u))+\frac{6}{(d-5)}(r_t(r_t$$
$$+y^2(1+r_u)))+2r_t(3r_t-2x^2r_tr_u+3y^2)+2z^2y^2(2r_t-y^2(2-r_u)))-$$
$$B_t\frac{4m_e}{s^2}((d-4)z^2y^4(1-r_u)+y^2r_tr_u-z^2y^2(2r_t-y^2))-$$
$$B_0\frac{4m_e}{s^2}(d-3)(r_t(r_t+y^2)r_u+z^2x^2r_t^2r_u)-F_t\frac{4m_e}{(d-4)s^2}(r_tr_uy^2-z^2y^2(2r_t-r_uy^2))+$$
$$C_0\frac{2m_e}{s}r_t((d-4)((r_t+y^2)r_u-z^2(r_t-r_u(r_tx^2-y^2)))+r_tr_u-z^2(2r_t-x^2r_tr_u))+$$
$$D_0r_tm_e((d-4)(z^2(r_t+y^2r_u))+r_t(1+2r_u+2r_t)r_u+z^2(r_t(1-r_t-3r_u)-y^2r_u+z^2r_t)),$$

$A_{12} =$

$$-A_0\frac{2}{s^2}r_tr_uy^2-B_t\frac{z^2}{s}r_tr_uy^2-F_t\frac{z^2}{(d-4)s}r_tr_uy^2+C_0 2z^2r_t^2r_u/4-D_0z^2sr_t^2r_u/4,$$

One loop integrals occurring in Bhabha scattering

1. Two - point integrals

$$I_G^{(d)} = I_2^{(d)}\Big|_{m_1=0,m_2=0,p^2} = \frac{\sqrt{\pi}}{(-p^2)^{(2-\frac{d}{2})}} \frac{\Gamma\left(2-\frac{d}{2}\right)\Gamma\left(\frac{d}{2}-1\right)}{2^{d-3}\,\Gamma\left(\frac{d-1}{2}\right)}$$

$$I_F^{(d)} = I_2^{(d)}\Big|_{m_1=m,m_2=m,p^2=m^2} = (m^2)^{-(2-\frac{d}{2})}\,\Gamma\left(2-\frac{d}{2}\right)\,{}_2F_1\left[\begin{matrix}1, 2-\frac{d}{2}\,; \\ \frac{3}{2}\,;\end{matrix}\;\frac{p^2}{4m^2}\right].$$

2. Three - point integrals

For **two zero masses** and final on-shell momenta we have (from **J. F., F. Jegerlehner and O.V Tarasov, unpublished**)

$$\frac{J_G}{\Gamma\left(2-\frac{d}{2}\right)} = \frac{(m^2)^{\frac{d}{2}-3}}{2(d-3)}\,{}_2F_1\left[\begin{matrix}1, 1\,; \\ \frac{d-1}{2}\,;\end{matrix}\;1-\frac{p^2}{4m^2}\right] - \frac{\sqrt{\pi}\Gamma\left(\frac{d-2}{2}\right)(-p^2)^{\frac{d-4}{2}}}{2^{d-2}\Gamma\left(\frac{d-1}{2}\right)m^2}\,{}_2F_1\left[\begin{matrix}1, \frac{d-2}{2}\,; \\ \frac{d-1}{2}\,;\end{matrix}\;1-\frac{p^2}{4m^2}\right],$$

where

$$J_G = -C_0(0, me, 0, m_e^2, m_e^2, p^2).$$

For the other master three-point integral with **two equal masses** and one zero mass and final on-shell momenta the **Gram determinant is zero**. This integral can be expressed in terms of a propagator integral as shown before. A compact result also is

$$J_F = \frac{\Gamma\left(2-\frac{d}{2}\right)}{2\,m^{6-d}}\,{}_2F_1\left[\begin{matrix}1, 3-\frac{d}{2}\,; \\ \frac{3}{2}\,;\end{matrix}\;\frac{p^2}{4m^2}\right],$$

where

$$J_F = -C_0(m_e, 0, m_e, m_e^2, m_e^2, p^2).$$