26 September 1996

PHYSICS LETTERS B

ELSEVIER

Physics Letters B 385 (1996) 411–414

On the asymptotic behaviour of $F_2(x, Q^2)$

A. DeRoeck [a], M. Klein [b], Th. Naumann [b,1]

[a] *DESY, Notkestr. 85, D-22607 Hamburg Germany*

[b] *DESY Zeuthen, Platanenallee 6, D-15738 Zeuthen Germany*

Received 6 May 1996; revised manuscript received 25 June 1996
Editor: K. Winter

Abstract

We discuss how the proton structure function $F_2(x, Q^2)$ is described in the HERA kinematic range by double asymptotic expressions for low x and large Q^2.

Previous fixed target lepton-nucleon scattering experiments [1,2] measured the proton structure function $F_2(x, Q^2)$ at Bjorken-$x \gtrsim 0.01$ and were sensitive to the valence quark content of the nucleon. HERA experiments collect data at values of x low enough to neglect valence quarks and at values of Q^2 large enough to apply perturbation theory. Here Q^2 is the four-momentum transfer squared from the incoming lepton to the proton. The proton structure function F_2 strongly rises with decreasing x over the whole kinematic range accessible to the HERA experiments H1 and ZEUS [3,4]. Such a rise was predicted more than twenty years ago [5] from the leading order renormalization group equations of perturbative QCD. Ball and Forte pursued these ideas [6] and proposed a way to demonstrate that the low x data at HERA exhibit scaling properties dominantly generated by QCD radiation. They gave an expression for $F_2(x, Q^2)$ in the double asymptotic limit of low x and large Q^2. The recent $F_2(x, Q^2)$ measurement of H1 is in accord with such scaling behaviour. Hence these data are expected to be sensitive to the fundamental QCD evolution dynamics, and not to depend on unknown starting distributions. The formalism of the double asymptotic analysis was extended to two loops in [7].

We define the logarithmic QCD evolution variables

$$t = \ln(Q^2/\Lambda^2), \quad \xi = \ln(1/x)$$

where Λ is the QCD mass scale, and the evolution length T of $\alpha_S(Q^2)$ from a starting point Q_0^2 to Q^2

$$T = \ln(\alpha_S(Q_0^2)/\alpha_S(Q^2)).$$

To leading order T is simply

$$T = 4\pi/b_0 \int_{t_0}^{t} dt \alpha_S(t) = \ln(t/t_0).$$

The T dependence of F_2 allows to determine α_S. Here $b_0 = 11 - 2n_f/3$ is the leading coefficient of the renormalization group equations, with n_f the number of flavours.

For large t and ξ and an $F_2(x, Q^2)$ which at Q_0^2 is not too singular in x the NLO double asymptotic expression for $F_2(x, Q^2)$ is [7]

$$F_2 \sim N_F\ (1 - f_{\text{NLO}}) \times \exp(2\gamma\sqrt{\xi T} - \delta T + \tfrac{1}{4}\ln T - \tfrac{3}{4}\ln\xi). \qquad (1)$$

[1] Fax: +49-33762-77330; e-mail: naumann@ifh.de.

PII S0370-2693(96)00976-8

with

$$\gamma^2 = 12/b_0, \quad \delta = (11 + 2n_f/27)/b_0.$$

The normalization coefficient is

$$N_F = \sqrt{\gamma/\pi}\, 5n_f/324.$$

For $n_f = 4$ this gives $\gamma = 6/5$, $\delta = 4/3$ and $N_F = 0.038$. The NLO correction term $f_{\rm NLO}$ is

$$f_{\rm NLO} = \frac{\sqrt{\xi/T}}{2\pi\gamma}[\epsilon(\alpha_S(Q_0^2) - \alpha_S(Q^2)) - 13\alpha_S(Q^2)]$$

with

$$\epsilon = (206n_f/27 + 6b_1/b_0)/b_0, \quad b_1 = 102 - 38n_f/3.$$

The leading order formula is recovered setting $f_{\rm NLO} = 0$ and $b_1 = 0$. Defining the leading exponent as $\lambda\xi = 2\gamma\sqrt{\xi T}$ and the subleading term as

$$\alpha = -\delta T + \tfrac{1}{4}\ln T - \tfrac{3}{4}\ln\xi$$

we rewrite F_2 as

$$F_2 \sim x^{-\lambda}e^{\alpha}. \tag{2}$$

The leading term in the double asymptotic formula for F_2 corresponds to the double leading log approximation DLL [8] of the DGLAP equations [9]. It generates the growth of the structure function with falling x proportional to $x^{-\lambda}$. The subleading term falls with x but slower than the leading term grows.

Expression (1) for the structure function is a prediction of QCD in the limit of small x and large Q^2. It does not depend on the shape of the parton distribution functions at Q_0^2 if these are sufficiently soft. The starting point Q_0^2 of the QCD evolution is a free parameter. Conventional QCD analyses of structure functions [10–13] fit input parton distributions at some Q_0^2 and evolve them to larger Q^2 assuming a singular behaviour $F_2 \sim x^{-\lambda}$ of the input distributions. On the contrary, the approach of GRV [14] starts with non-singular, valence-like input distributions at some low Q_0^2 and generates the steep rise of the F_2 from QCD dynamics.

We now test the double scaling limit of QCD fitting expression (1) for $F_2(x, Q^2)$ to the latest measurement of the proton structure function by the H1 experiment [3] at HERA. Since T depends on Λ this allows to determine Λ and α_S in this limit. When passing the heavy flavour thresholds we adjust Λ according to [15]. In order to reach the asymptotic region in T and ξ we demand $Q^2 \geq Q^2_{\rm min} = 5$ GeV$^2 > Q_0^2$ and $x < 0.1$ which corresponds to $\xi T \gtrsim 2$ approximately. The fit using the total errors gives the central value $\Lambda = 248$ MeV with a $\chi^2/{\rm ndf} = 122/145$ and $Q_0^2 = 1.12$ GeV^2. The Λ symbol is taken as an abbreviation for $\Lambda^{(4)}_{\overline{\rm MS}}$ [15,16]. The result is shown as a full line in Fig. 1 and describes the data for $Q^2 \geq 5$ GeV^2 reasonably well. A fit of the LO expression of Eq. (1) to the F_2 data is found to drop too fast with x. The $\chi^2/{\rm ndf}$ increases by a factor of two.

The statistical error is $\delta\Lambda_{\rm stat} = 23$ MeV from a fit with the statistical errors only. Using the full systematic error matrix of H1 we then vary each of the measured quantities entering the F_2 analysis. From the variations of the Λ determination with the varied F_2 we get the full systematic error $\delta\Lambda_{\rm syst} = 83$ MeV. In order to estimate the systematic error coming from the kinematic cuts we also varied $Q^2_{\rm min}$ from 5 to 60 GeV^2 and asked for $x < 0.05$ or $x > 0.001$. We also replaced the upper x limit by demanding that $\xi T > 2$. The resulting Λ variation is 46 MeV. The Λ determination translates into $\alpha_S(M_Z^2) = 0.113 \pm 0.002({\rm stat}) \pm 0.007({\rm syst})$. Clearly, the ultimate determination of Λ should be performed within a full QCD analysis. Within the precision of the present data, however, a fit to an analytic expression is much simpler and more transparent than solving the DGLAP equations and fitting a large number of parameters which are often highly correlated, in particular with Λ.

The heavy flavour treatment causes a theoretical uncertainty. Since we work at Q^2 above the charm threshold only the bottom quark mass m_b enters the uncertainty. We chose an effective $m_b = 4.74$ GeV according to [17]. Taking $m_b = 4.50$ GeV instead changes Λ by 13 MeV. Using a different prescription [18] for the transition between $\Lambda^{(n_f)}$ and $\Lambda^{(n_f-1)}$ changes Λ by 7 MeV. Changing as an extreme assumption the mass scale from Q^2 to W^2 changes Λ by 15 MeV. The heavy flavour treatment thus yields a systematic error on α_S of 0.002. In order to check the influence of the asymptotic expression (1) on the α_S determination we also fitted it to the H1 QCD fit [3] and to a set of GRV parametrizations with varying Λ [19] for $x > 0.001$. For the Q_0^2 obtained above the Λ val-

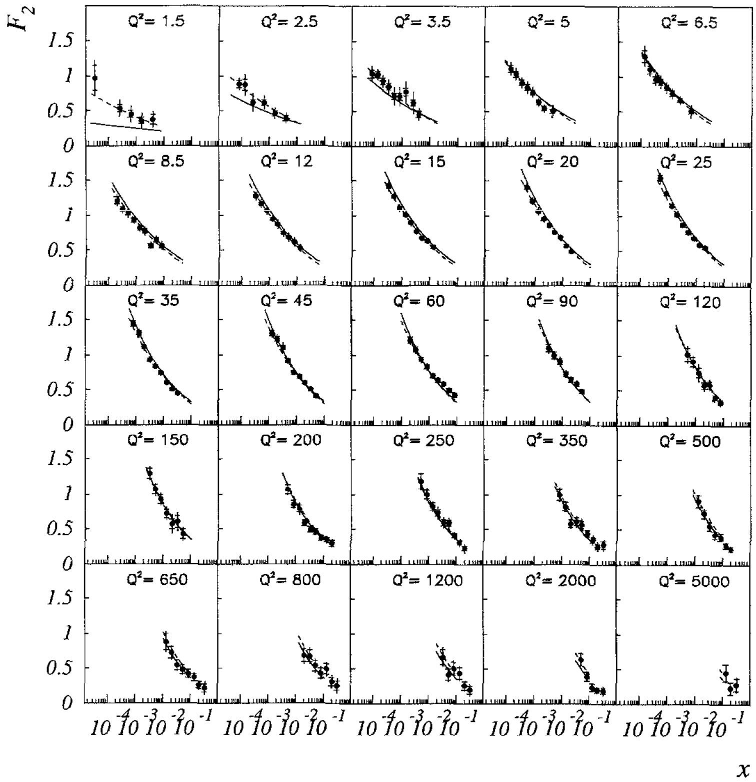

Fig. 1. The proton structure function $F_2(x, Q^2)$ as measured by the H1 experiment at HERA together with a fit to the NLO double asymptotic expression (1) (full line) for $Q^2 > 5$ GeV^2 and with a fit to the modified DLL expression $F_2 = N_f e^{-\gamma\sqrt{T/\xi}}$ (dashed line) in the full Q^2 range.

ues from these fits follow the variations of the values used in the QCD evolutions of these parton densities and differ by at most 34 MeV from the input values. This corresponds to an α_S error of 0.003. The largest theoretical error, however, is anticipated to come from higher order perturbative corrections and subleading terms and is also present in full QCD analyses. This error is quoted to be in the range of 0.004 [20] to 0.007 [21]. We conservatively quote our result with a theoretical error of 0.007.

In order to include the data at $Q^2 < 5$ GeV^2 we also fitted the leading expression of Eq. (1) $F_2 = aN_f x^{-c2\gamma\sqrt{T/\xi}}$ for four flavours keeping the factors a and c free. Since we got $a = 1.02$ and $c = 0.46$ we set $a = 1$ and $c = 0.5$ and fitted $F_2 = N_f x^{-\gamma\sqrt{T/\xi}}$. The result is shown in Fig. 1 as a dashed line. The values of the only two fit parameters are $Q_0^2 = 0.365 \pm 0.026(\text{stat}) \pm 0.048(\text{syst})$ and $\Lambda = (243 \pm 13 \pm 23)$ MeV. The two parameters resemble the GRV Q_0^2 scale and the QCD parameter Λ. The χ^2/ndf of the fit using full errors is 109/167. Leaving the flavour number n_f free gives very similar Q_0^2 and Λ values, yielding $n_f = 3.83 \pm 0.18 \pm 0.34$ with almost the same $\chi^2/\text{ndf} = 108/166$. For $x < 0.1$ this fit describes the F_2 data surprisingly well with the two parameters Q_0^2 and Λ only down to the lowest Q^2 values available.

Summarizing, for $Q^2 \geq 5$ GeV^2 a NLO double asymptotic expression for $F_2(x, Q^2)$ describes the HERA data well. We use its Q^2 dependence to extract $\alpha_S(M_Z^2) = 0.113 \pm 0.002(\text{stat}) \pm 0.007(\text{syst})$. We quote a theory error of 0.007 which is subject to uncertainties. We found a simple parametrization $F_2 = N_f e^{-\gamma\sqrt{T/\xi}}$ for the proton structure function which is valid over more than three orders of magnitude in x and Q^2 including the region $Q^2 < 5$ GeV^2 and depends on two fitted scales Q_0^2 and Λ only.

We thank R. Ball, J. Blümlein, S. Forte and A. Vogt for helpful discussions.

References

[1] NMC Collaboration, M. Arneodo et al., Phys. Lett. B 364 (1995) 107.
[2] BCDMS Collaboration, A.C. Benvenuti et al., Phys. Lett. B 223 (1989) 485.
[3] H1 Collaboration, T. Ahmed et al., DESY preprint 96-039, submitted to Nucl. Phys. B.
[4] ZEUS Collaboration, M. Derrick et al., Z. Phys. C 65 (1995) 379; C 69 (1996) 607.
[5] A. De Rujula et al., Phys. Rev. D 10 (1974) 1649.
[6] R.D. Ball and S. Forte, Phys. Lett. B 335 (1994) 77.
[7] S. Forte and R.D. Ball, XXXV Cracow School of Theoretical Physics, Zakopane, June 1995, and CERN-TH/95-323. To be published in: Acta Phys. Pol.
[8] L.V. Gribov, E.M. Levin and M.G. Ryskin, Phys. Rep. 100 (1983) 1.
[9] Yu.L. Dokshitzer, Sov. Phys. JETP 46 (1977) 641;
V.N. Gribov and L.N. Lipatov, Sov. J. Nucl. Phys. 15 (1972) 438 and 675;
G. Altarelli and G. Parisi, Nucl. Phys. B 126 (1977) 297.
[10] H1 Collaboration, T. Ahmed et al., Phys. Lett. B 354 (1995) 494.
[11] ZEUS Collaboration, M. Derrick et al., Phys. Lett. B 345 (1995) 576.
[12] A.D. Martin, W.J. Stirling and R.G. Roberts, Phys. Lett. B 354 (1995) 155.
[13] H.L. Lai et al., CTEQ Collaboration, Phys. Rev. D 51 (1995) 4763.
[14] M. Glück, E. Hoffmann and E. Reya, Z. Phys. C 13 (1982) 119;
M. Glück, E. Reya and M. Stratmann, Nucl. Phys. B 422 (1994) 37.
[15] W.J. Marciano, Phys. Rev. D 29 (1984) 580.
[16] W. Bernreuther and W. Wetzel, Nucl. Phys. B 197 (1982) 228.
[17] Particle Data Group, Phys. Rev. D 50 (1994) 1435.
[18] W. Bernreuther, Ann. Phys. 151 (1983) 127.
[19] A. Vogt, Phys. Lett. B 354 (1995) 145.
[20] M. Virchaux and A. Milsztajn, Phys. Lett. B 274 (1992) 221;
A.D. Martin, W.J. Stirling and R.G. Roberts, Phys. Lett. B 266 (1991) 173.
[21] R.D. Ball and S. Forte, Phys. Lett. B 358 (1995) 365, and contribution to the Workshop on Deep Inelastic Scattering, Rome, April 1996, to appear in the Proceedings.