Elementary Particles

and

Interactions

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DESY Hamburg
Protons 920 GeV
HERA
Electrons 27.6 GeV
PETRA


DESY Zeuthen

CERN

European Organization for Nuclear Research

Organisation Européenne pour la Recherche Nucléaire

DESY International Summer Student Program

July 17 to September 6, 2018

DESY is one of the world's leading accelerator centers for investigating the structure of matter. DESY develops and builds large particle accelerators and conducts research in the fields of photon science and particle physics. The research facilities of DESY are used by a large international community of scientists. Each summer DESY offers students in physics or related natural science disciplines the opportunity to participate in its research activities. About 100 students from all over the world take part in DESY's research and attend the lecture program.

www.desy.de/summerstudents

>100 students
from ~30 countries

Photon Science

Summer students join experiments which are carried out with soft and hard X-rays using a variety of spectroscopic and scattering techniques for research in the fields of physics, chemistry, biology etc. Activities range from preparation, realization and evaluation of measurements to improvements of instrumentation.

Elementary Particle Physics, Astroparticle Physics and Accelerators

Summer students will work in the analysis, software or detector related fields of experiments in elementary particle physics (LHC, ILC, HERA, BELLE II, ALPS-II) and astroparticle physics (CTA), development of particle accelerators, theory of elementary particles or computing.

Application Deadline: 31 January, 2018

Qualified applicants should have completed three years of full time studies on a university level by summer 2017.
All participating students will obtain financial support.

1. Introduction

1. Leptons
2. Hadrons

2. The Quark Model

1. Isospin
2. Multiplets and SU(N)
3. Spectroscopy of Light Quarks
4. Spectroscopy of Heavy Quarks
5. Measurement of Quark Charges
6. *Particle Mixing, CKM matrix and GIM mechanism*

3. Symmetries and Conservation Laws

1. Charge Conjugation C
2. Parity P
3. *CP Violation*

4. Phenomenology of Weak Interactions

1. Neutral and Charged Currents
2. Neutrino Physics
3. Cross section and Fermi Constant
4. W- and Z-Bosons

5. Gauge Theory of Weak Interactions

1. Gauge Theories
2. Higgs Mechanism
3. Lagrangian of Weak Interactions
4. Experimental Tests of the Standard Model
5. Higgs Search

6. Gauge Theory of Strong Interactions

1. Quantum Chromodynamics
2. Hadron Structure

7. Grand Unification and Cosmology

1. Unified Gauge Theories
2. Proton Decay and Baryon Asymmetry of the Universe
3. Supersymmetry and Superstrings
4. Cosmology and Particle Physics

Literature

Full list in: www-zeuthen.desy.de/~naumann/lectures/literature.pdf

General

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Gauge Theories and Electroweak Interactions

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Quantum Chromodynamics and Hadron Structure

F.E. Close, An Introduction to Quarks and Partons, Academic Press, 1979.
R.G. Roberts, The Structure of the Proton, Cambridge University Press, 1993.
G. Dissertori, I.G.Knowles, M.Schmelling, Quantum Chromodynamics: High Energy Experiments and Theory, Oxford University Press, 2003.
G. Sterman et al., Handbook of Perturbative QCD, www.phys.psu.edu/~cteq/#Handbook
W. Tung, Perturbative QCD and the Parton Structure of the Nucleon, www.physics.smu.edu/~olness/cteqpp/tung2003/IntroPqcd.pdf

Detectors

H. Kolanoski, N. Wermes, Teilchendetektoren, Springer Spektrum, Berlin 2016.
K. Kleinknecht, Detektoren für Teilchenstrahlung, Teubner, 2005.
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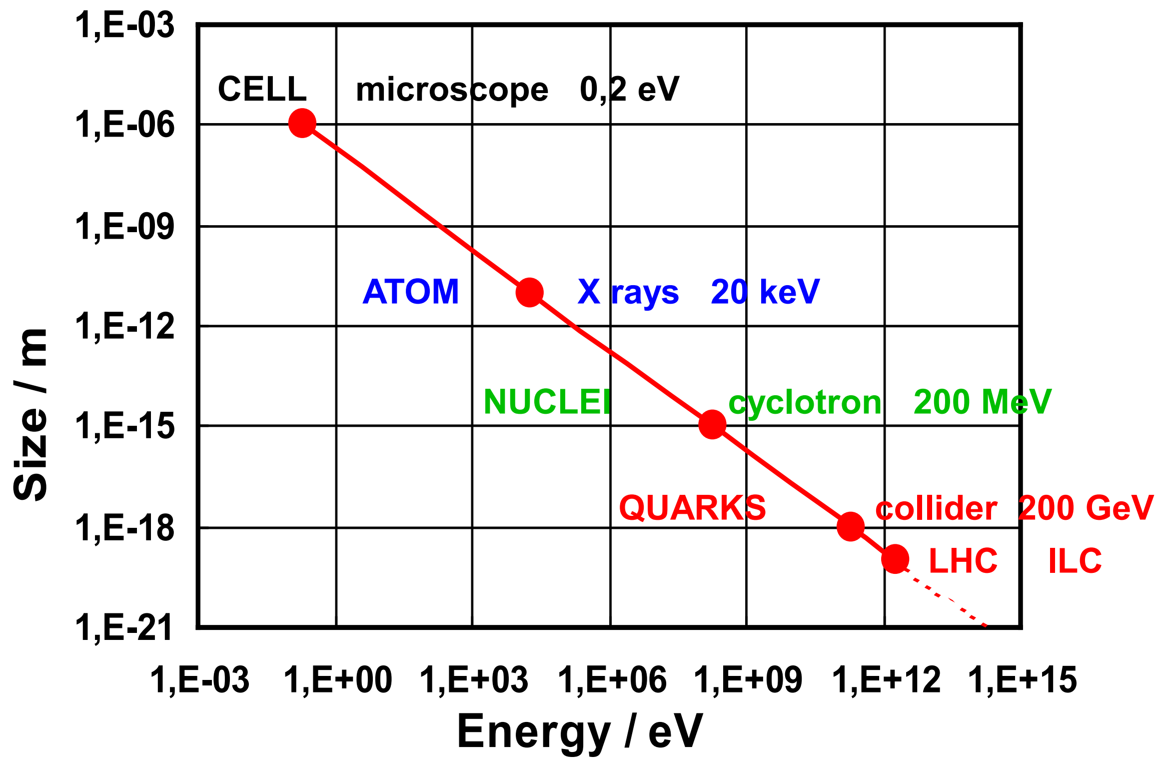
Structure of Matter

subatomic units:
electron-volt

Energy	Size	Device	Object	Year
0.2 eV	10^{-6} m	microscope	cell	1600
20 keV	10^{-11} m	X rays	atom	1910
200 MeV	10^{-15} m	cyclotron	nuclei	1946
200 GeV	10^{-18} m	collider	quarks	1998

uncertainty relation

$$1 \text{ eV} = k \cdot 11\,604 \text{ K}$$

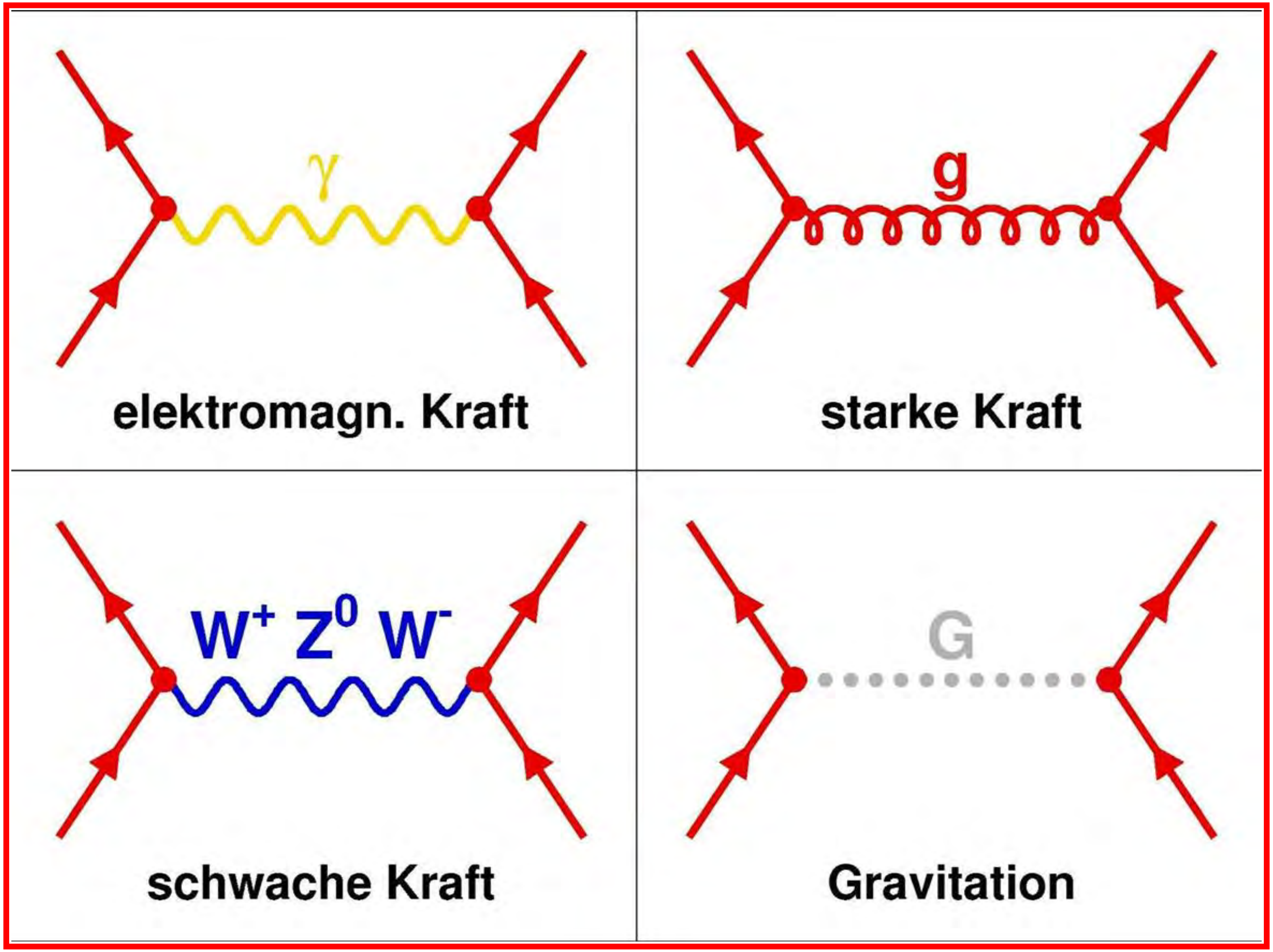


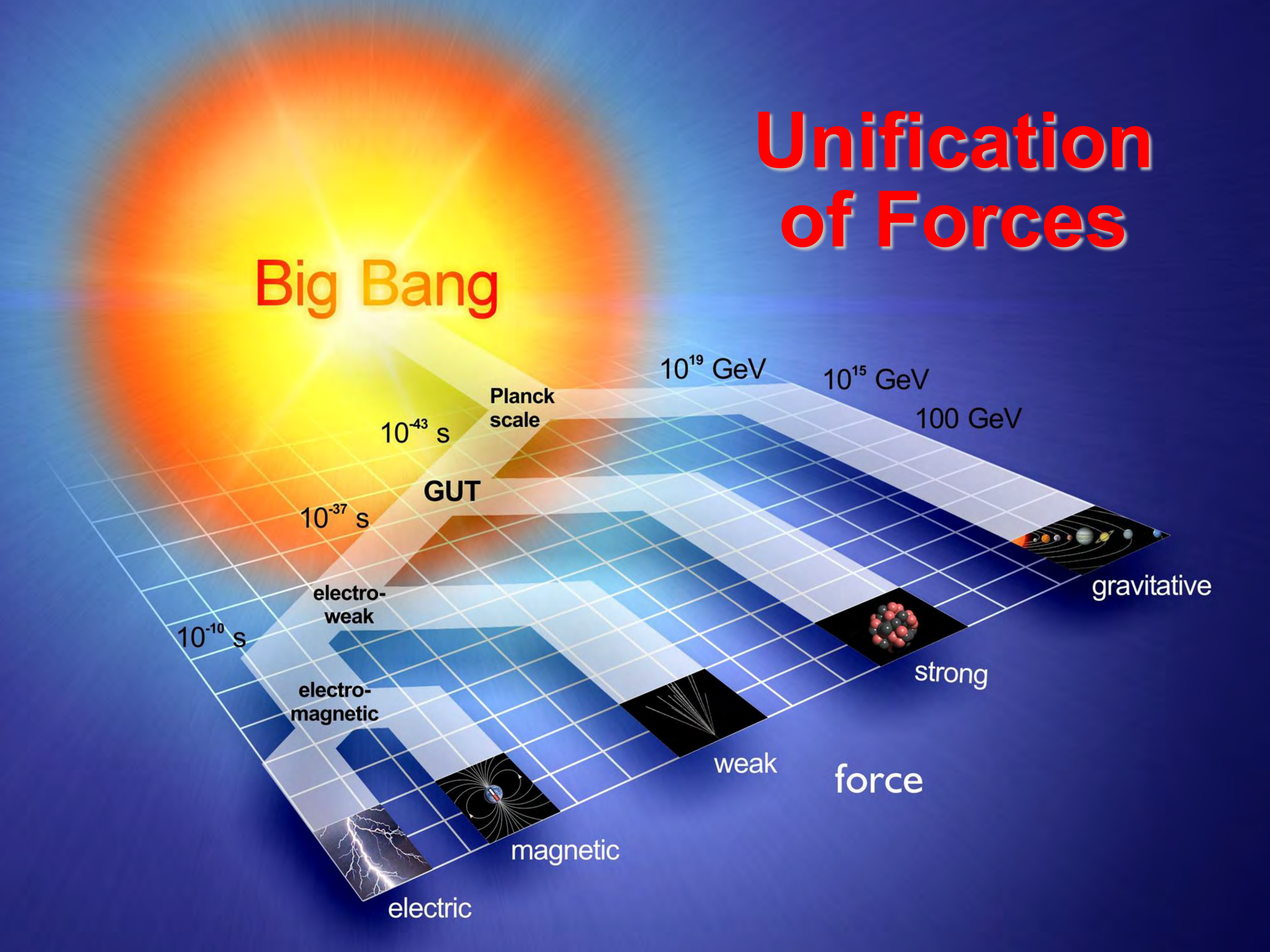
$$\Delta p \, \Delta x = \hbar$$

≈

200 MeV/c fm

The Forces

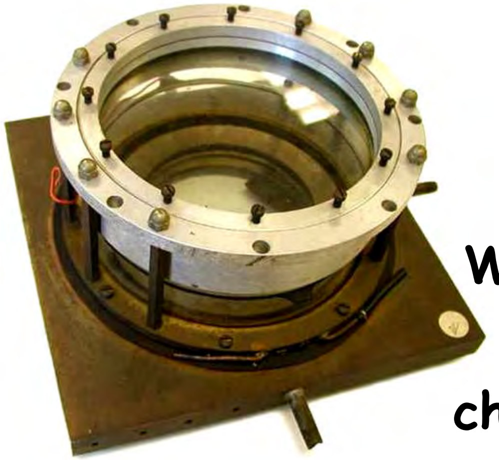

γ
elektromagn. Kraft
g
starke Kraft
W+ Z0 W-
schwache Kraft
G
Gravitation



Unification of Forces
Big Bang
Planck scale
10^{19} GeV
10^{15} GeV
100 GeV
10^{-43} s
GUT
10^{-37} s
electro-weak
10^{-10} s
electro-magnetic
gravitative
strong
weak
force
magnetic
electric

The Positron

- **curvature** R in magnetic field B: $R \sim p \sim 1/B$
- **ionization** I ~ velocity $\beta = v/c$: $I \sim 1/\beta \sim 1/(p/m)$



Wilson's cloud chamber

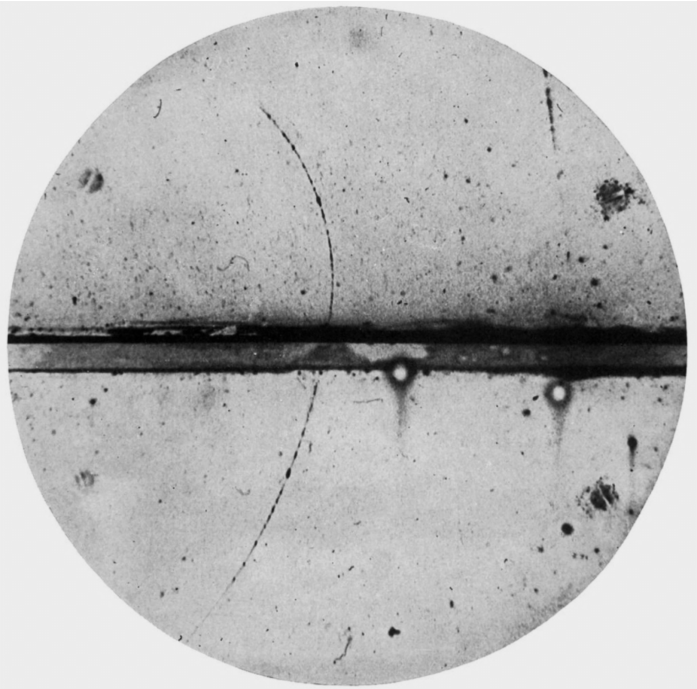
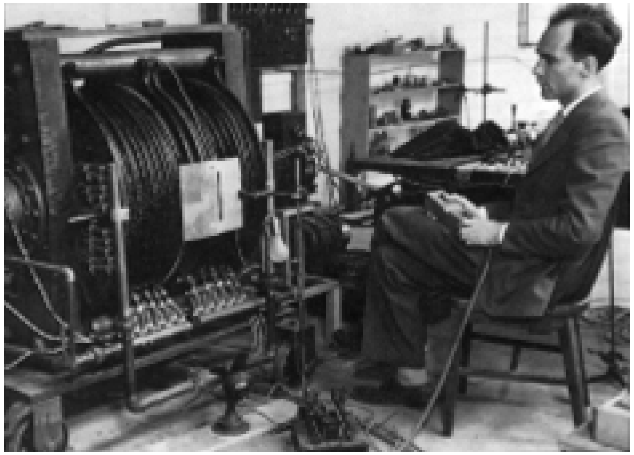


FIG. 1. A 63 million volt positron ($H\rho = 2.1\times10^5$ gauss-cm) passing through a 6 mm lead plate and emerging as a 23 million volt positron ($H\rho = 7.5\times10^4$ gauss-cm). The length of this latter path is at least ten times greater than the possible length of a proton path of this curvature.

C.Anderson,

The Positive Electron

Phys.Rev. 43, 491 (1933)



Nobel prize 1936 C. Anderson for the discovery of the positron

Nobel prize 1927 Ch. Wilson for the cloud chamber

Antimatter

Dirac, 1928:

relativistic theory of electrons

The only equation in Westminster Abbey:

$$(i\hbar c\,\not{\partial} - mc^2)\psi = 0$$

kinetic energy

non-relativistic: $E = p^2 / 2m$

relativistic: $E^2 = p^2 + m^2$

$E = \pm\sqrt{p^2 + m^2}$

electrons + ... ?

holes = antimatter = positron

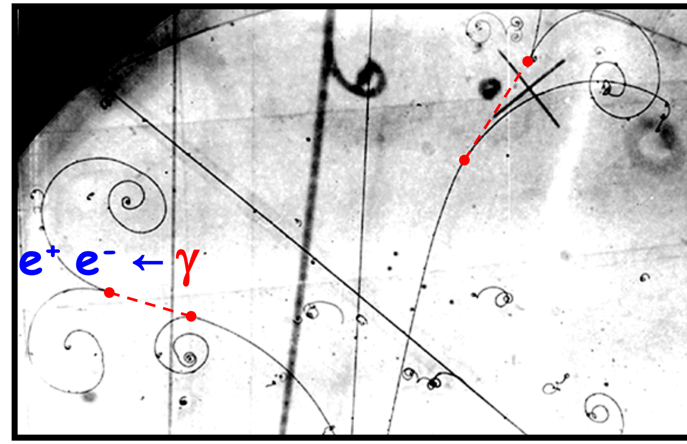
another mirror world: supersymmetry?

Nobel prize  **1933**

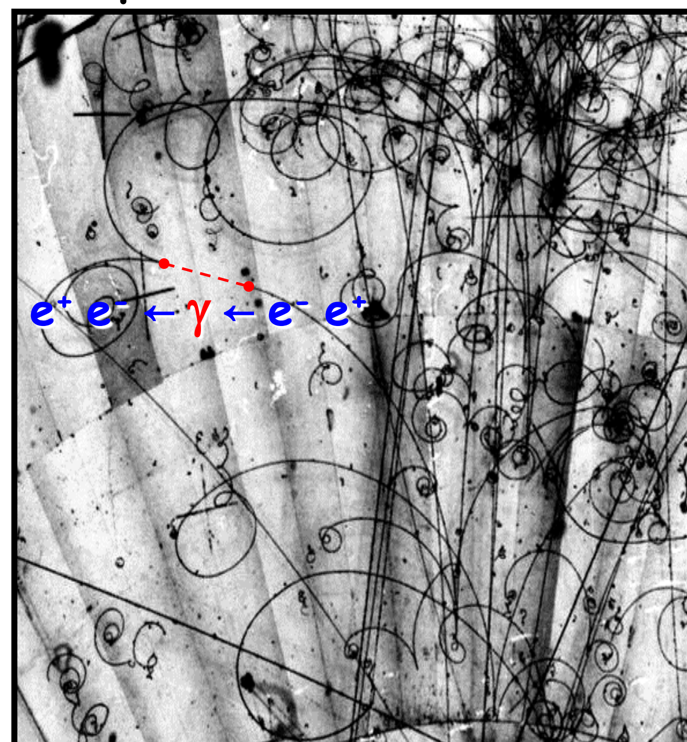
Dirac: „The equation was smarter than I was."

P.A.M. Dirac, Proc. Royal Soc. A117 610 (1928), A126 360 (1930).

bremsstrahlung



positron annihilation



The Muon

1935: H. Yukawa:
carrier of nuclear force
mass ~200 MeV (between e + p)

Nobel prize 1949

J.C. Street, E.C. Stevenson,

New Evidence for the Existence of a Particle of Mass Intermediate between the Proton and the Electron, Phys. Rev. 52, 1003 (1937).

Too penetrating - NOT the Yukawa particle !

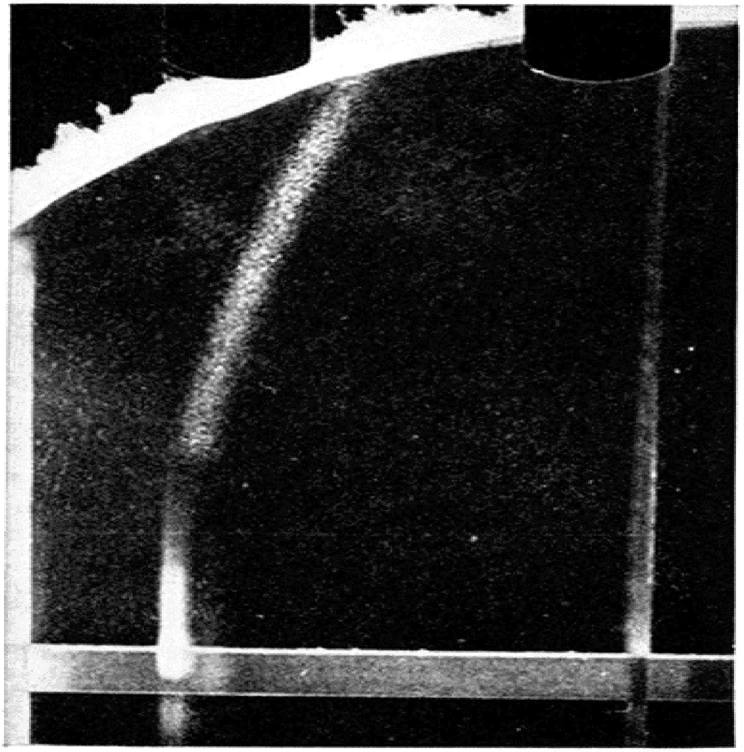
I.I. Rabi

Nobel 1944

Who ordered that ?

curvature R in magnetic field B:
$R \sim p \sim 1/B$

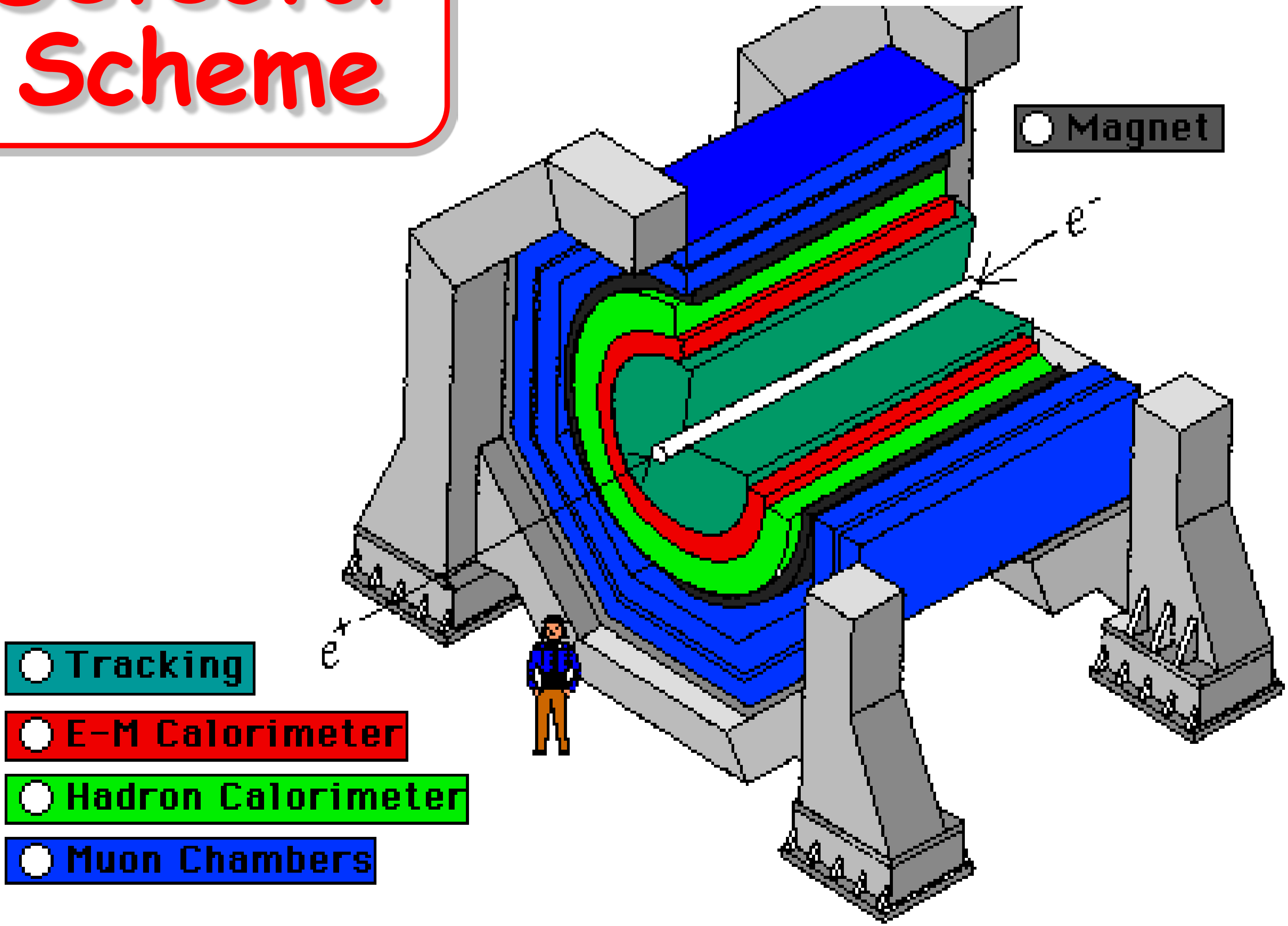
ionization I ~ velocity $\beta = v/c$:
$I \sim 1/\beta \sim 1/(p/m)$



I got what I wanted,
but it wasn't what I expected.

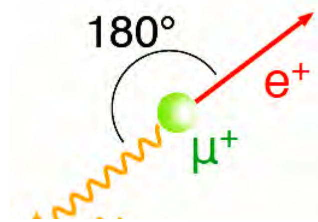
William Hamilton, New Yorker Cartoon, October 23, 1978

Detector Scheme



Lepton Number Conservation

$\mu \rightarrow e\,\gamma$



MEG, PSI, Villingen, CH:

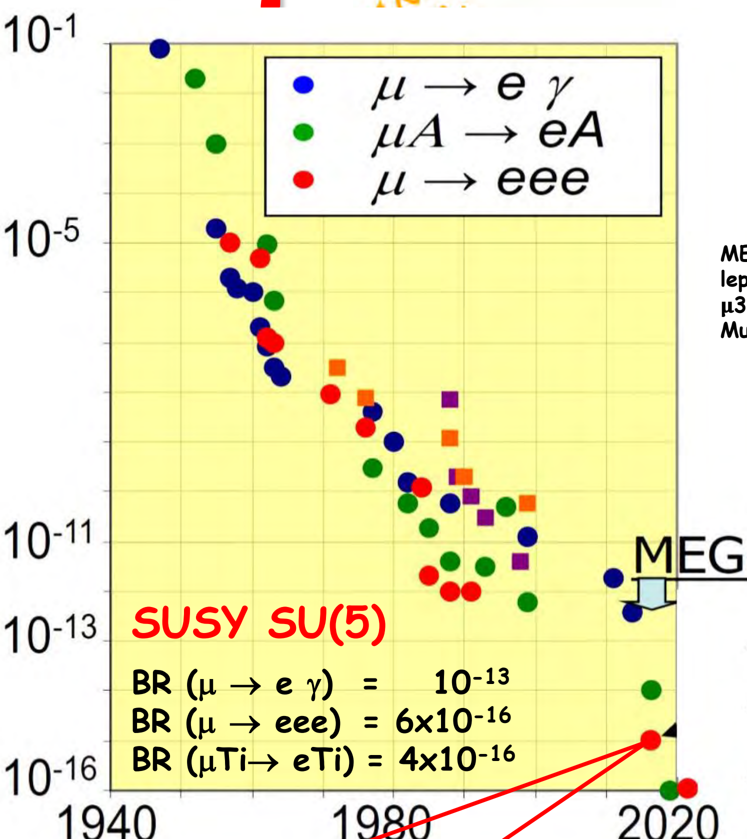
$7.5 \cdot 10^{14}$ ($3 \cdot 10^{7}$ /s) μ decays:

find lepton nr violating decays:

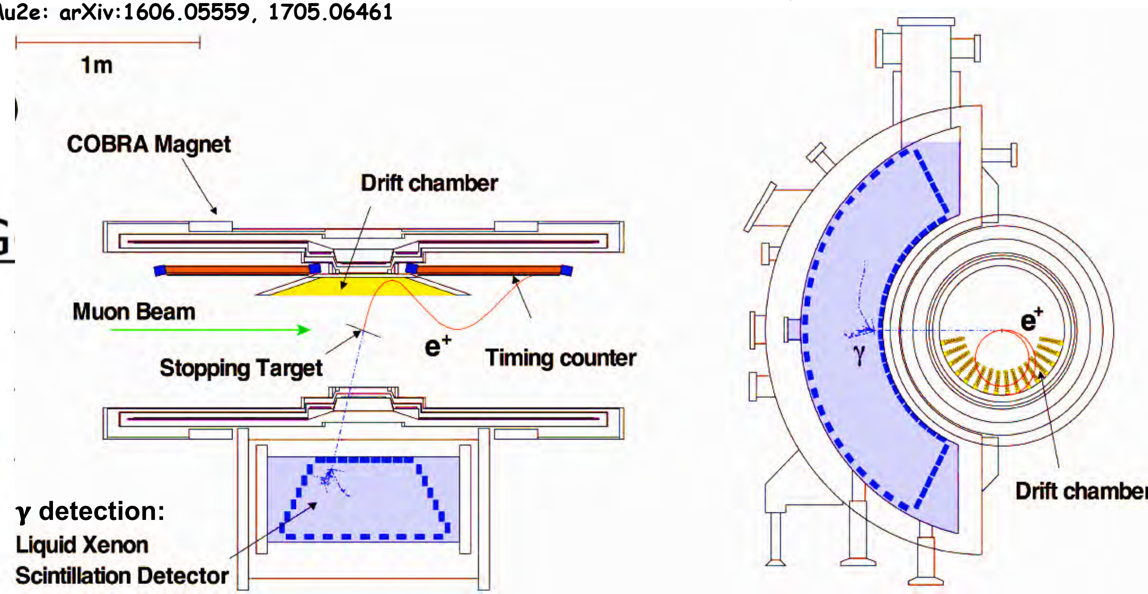
$(\mu \rightarrow e\,\gamma) / (\mu \rightarrow \text{all}) < 4.2 \cdot 10^{-13}$

Mu3e PSI: 10^{16} (10^{9}/s) decays $\mu \rightarrow eee$, $10^{-15\ldots16}$ sensitivity

MEG PSI: arXiv:1605.05081, 1606.08168 . Eur.Phys.J. C76 (2016) 434. Phys.Rev.Lett. 110, 201801 (2013)
lepton expts: arXiv:1307.5787 meg.web.psi.ch
μ3e PSI: arXiv:1301.6113 arXiv:1605.02906 1802.09851 Eur.Phys.J. C 57 (2008) 13
Mu2e: arXiv:1606.05559, 1705.06461

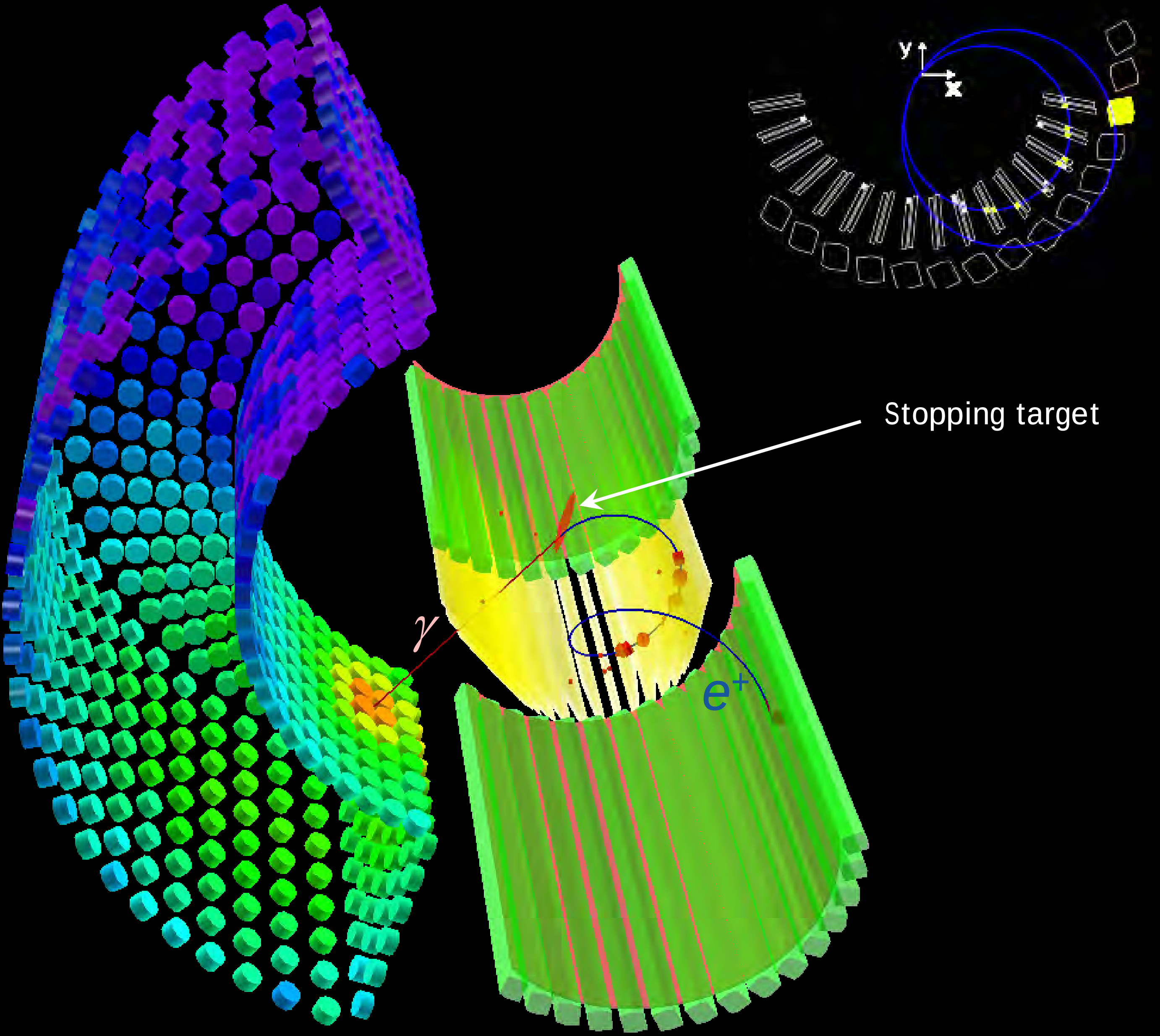


MEG PSI:	2016:	BR < 4 10^{-13}
MEG II:	2018-20:	BR < 5 10^{-14}
Mu3e PSI:	>2020:	BR < $10^{-15\ldots16}$
Mu2e FNAL	>2020:	BR < 6 10^{-17}

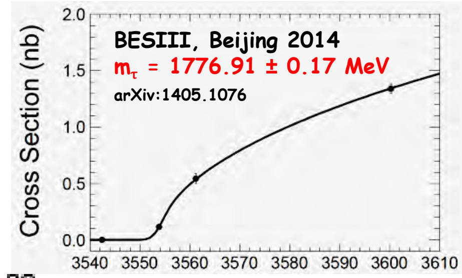


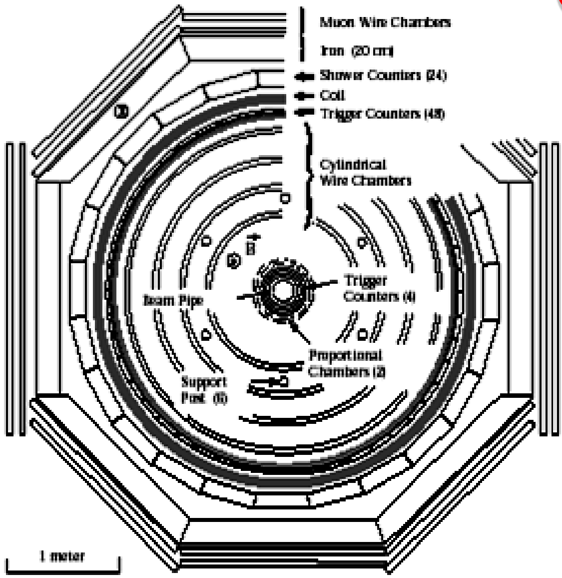
$\mu^+ \to e^+ \gamma$ candidate

MEG

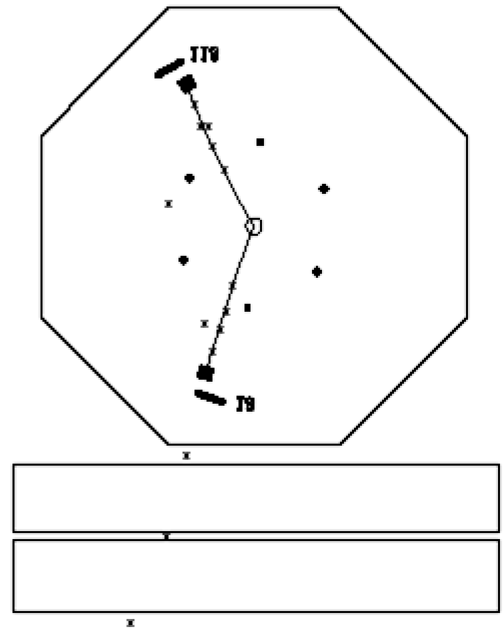


τ lepton





MARK-I detector
at SPEAR $e^+ e^-$ ring
SLAC, USA.



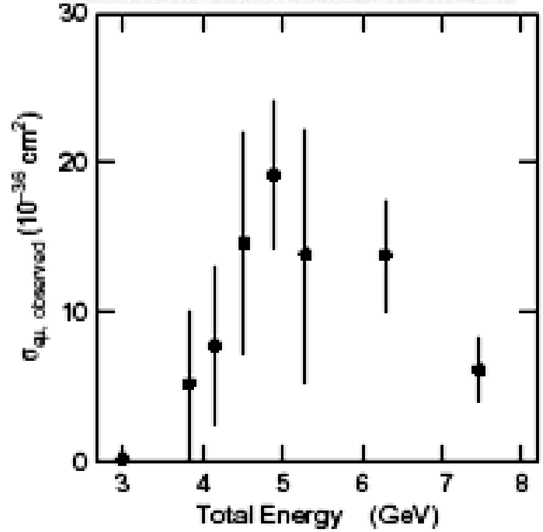


FIG. 2. The *observed* cross section for the signature *e-μ* events.

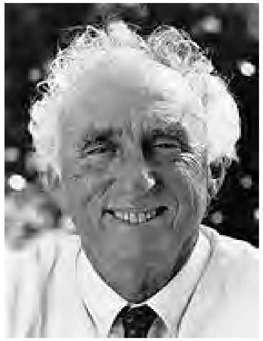
M. Perl et al., 1975-77:

$$e^+ e^- \rightarrow L^+ L^-$$

above 3.5 GeV threshold: unlike lepton pair production:

$L^+ \rightarrow e^+$ + unseen ν's carrying off energy + L_e
$L^- \rightarrow \mu^-$ + unseen ν's carrying off energy + L_μ

confirmed 1997 in Pluto+DASP @ DESY
Belle at KEKB, Japan, 2010: 719 million τ pairs



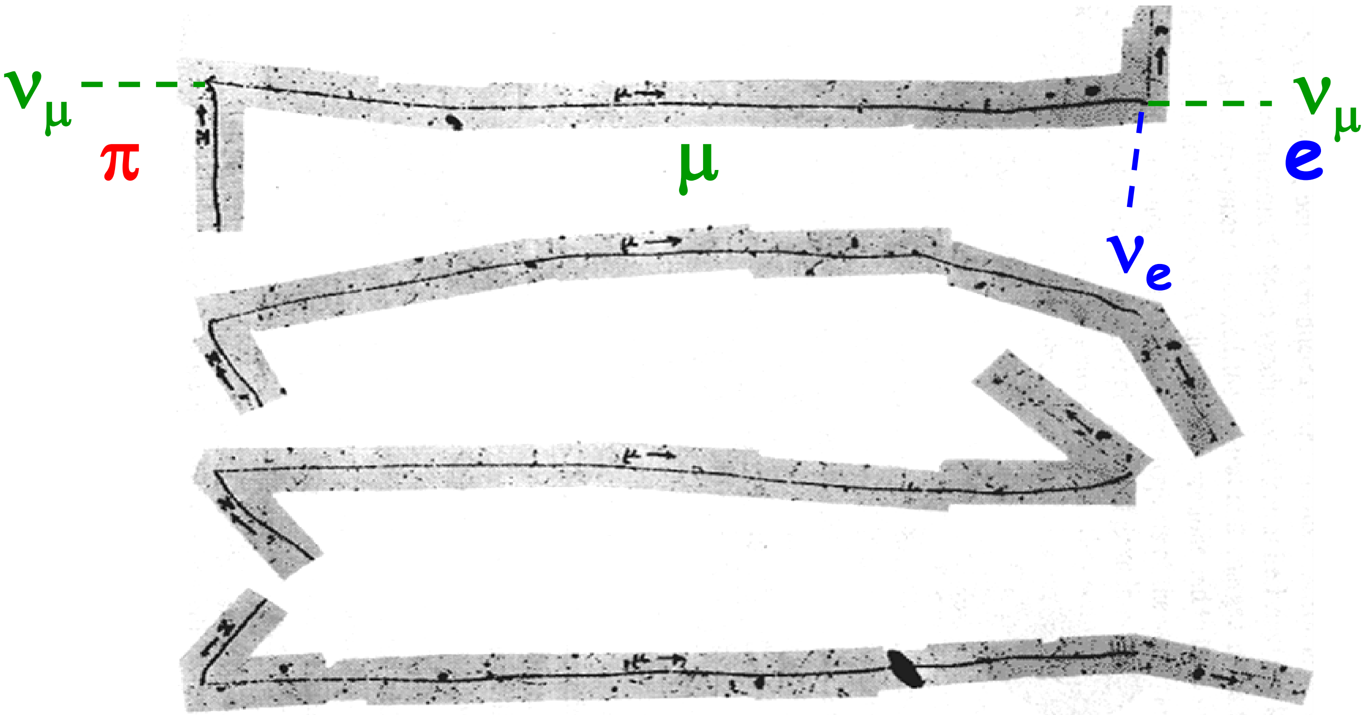
Martin Perl
1927-2014

Nobel prize

1995

The Pion

- stopping track + typical decay cascade
- decay product fixed range in nuclear emulsion = always same E
- two body decay to muon + neutrino

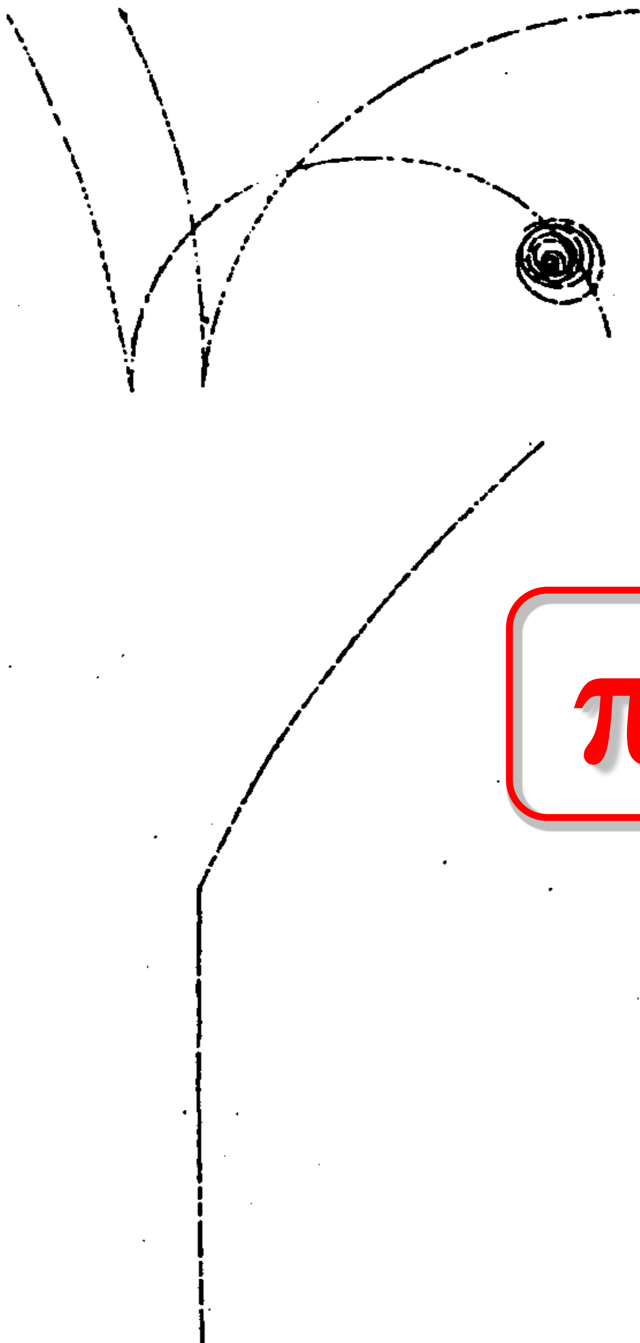


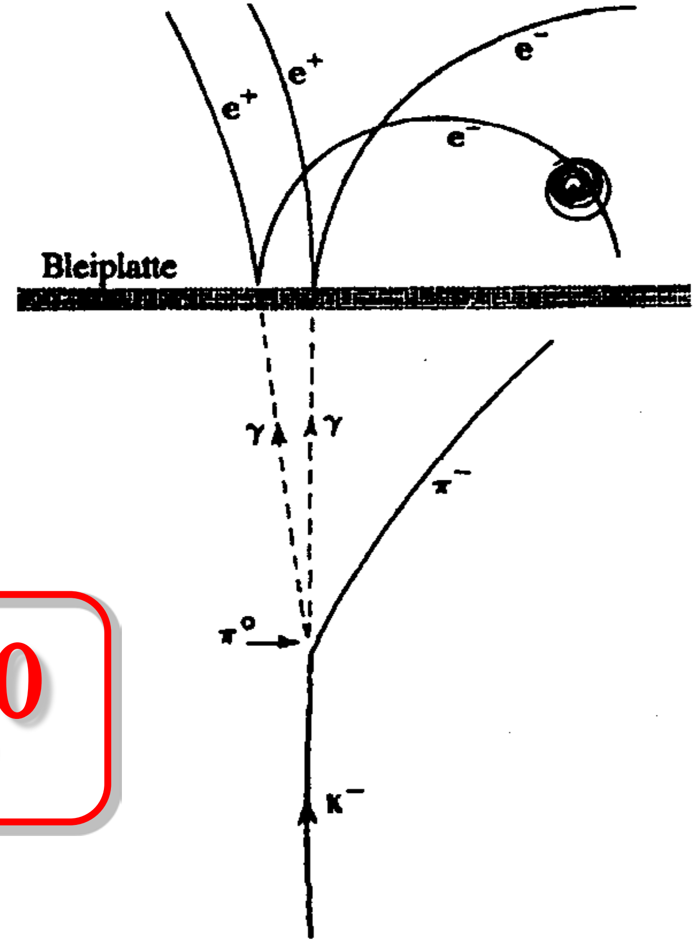
Nobel prize 1948: P. Blackett, Use of cloud chambers in cosmic radiation

Nobel prize 1950: C.F. Powell, Discoveries on mesons with emulsions

(still used, τ in OPERA, Gran Sasso)

π^0



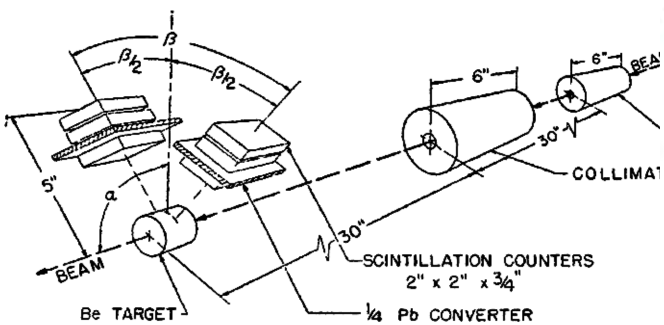


Die beiden Bilder zeigen eine Fotografie (bei der alle für den Vorgang unwichtigen Linien gelöscht wurde) und eine Zeichnung zur Erklärung des Zerfalls eines negativen Kaons in ein negatives und ein neutrales Pion. Die enge Spirale rechts oben stammt von einem Elektron, das aus einem Atom der Blasenkammerflüssigkeit herausgeschlagen wurde. Die Aufnahme stammt aus der Blasenkammer des LBL.

π^0 mass

J.Steinberger,
W.Panofsky, J.Steller, 1950 :

Phys.Rev. 78 (1950) 802.



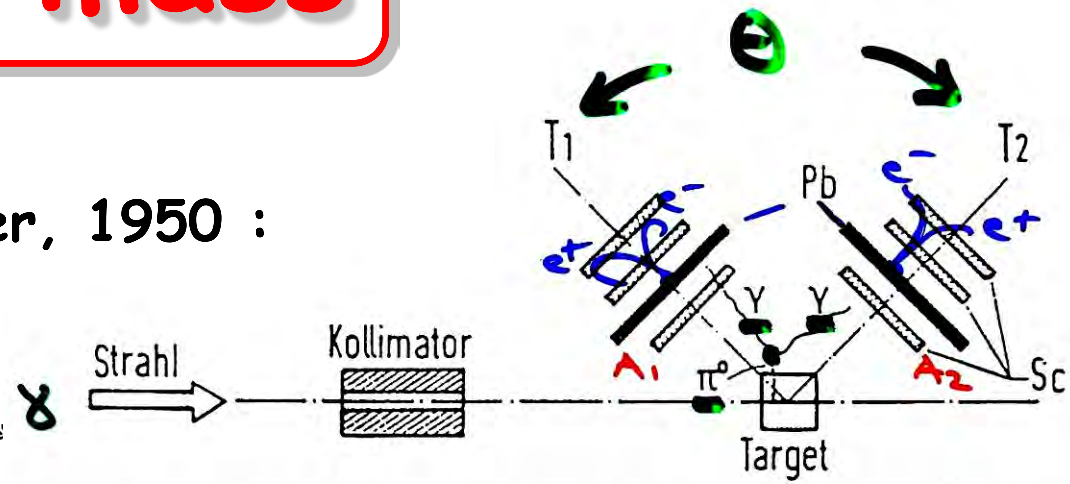
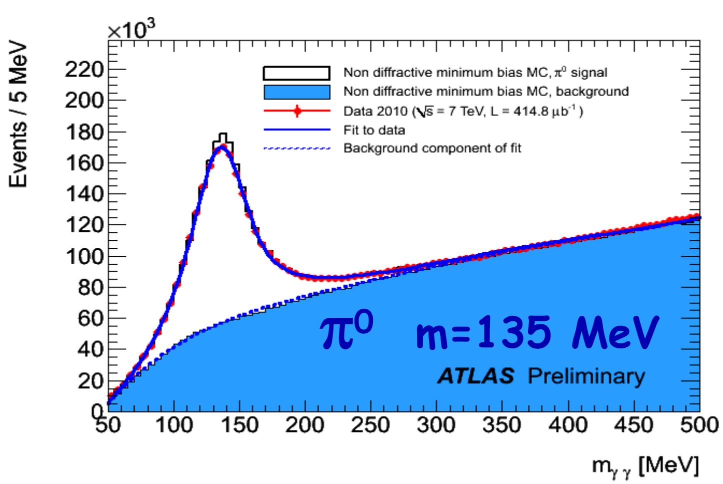


Fig. 1.9 Prinzipskizze zur Entdeckung des π^0-Mesons: Ein γ-Strahl aus einem Synchrotron mit einer Maximalenergie von 330 MeV erzeugt nach Kollimation in einem Target aus Be (39 mm lang) Kernreaktionen. Dabei entstehen u. a. π^0-Mesonen. Diese zerfallen nach dem Schema $\pi^0 \to \gamma + \gamma$ in zwei γ-Quanten. Die zwei γ-Quanten werden simultan in zwei Zählerteleskopen T1 und T2 nachgewiesen: Ein erster Szintillationszähler ist in Antikoinzidenz geschaltet und gewährleistet, daß das Hodoskop nicht auf ein geladenes einfallendes Teilchen anspricht. Es folgt eine Bleischicht, um das einfallende Photon in ein e^+e^--Paar zu konventieren. Zwei weitere Szintillationszähler weisen das erzeugte Elektron-Positronpaar nach. Die Kinematik des $\pi^0 \to \gamma + \gamma$-Zerfalls erzwingt einen Mindestwinkel zwischen den beiden γ-Quanten. Macht man den Winkel zwischen T1 und T2 kleiner als diesen Mindestwinkel, so verschwindet die Koinzidenzrate. Dies beweist direkt den Ursprung der beiden γ-Quanten aus dem 2γ-Zerfall eines massiven Teilchens



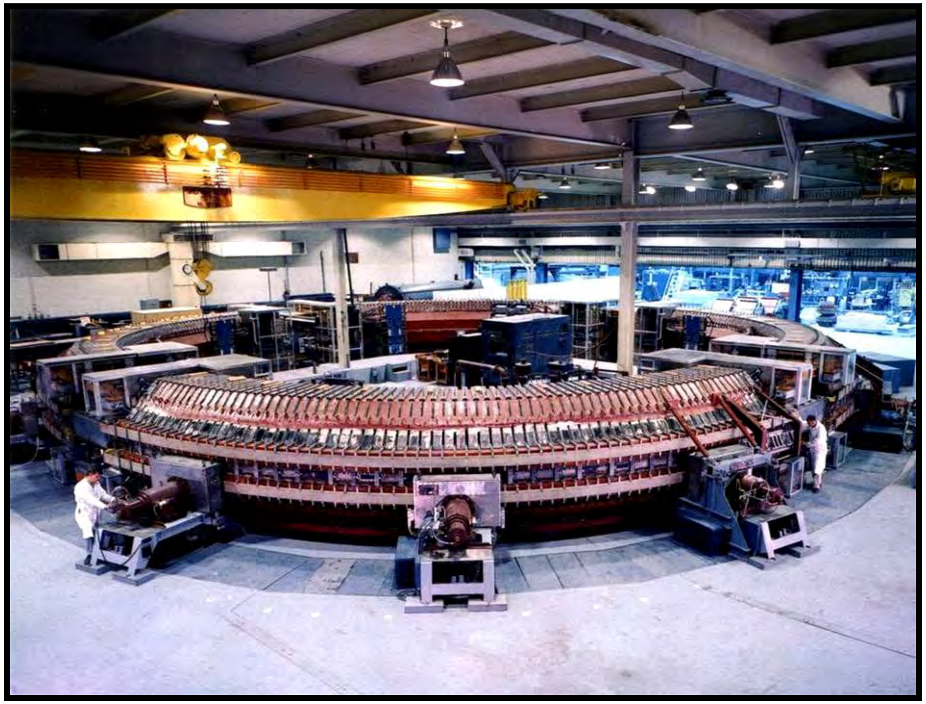
$$\theta < \theta_{min} \Rightarrow N(T_1 \cdot T_2) \to 0$$

$$\theta_{min} = ?$$

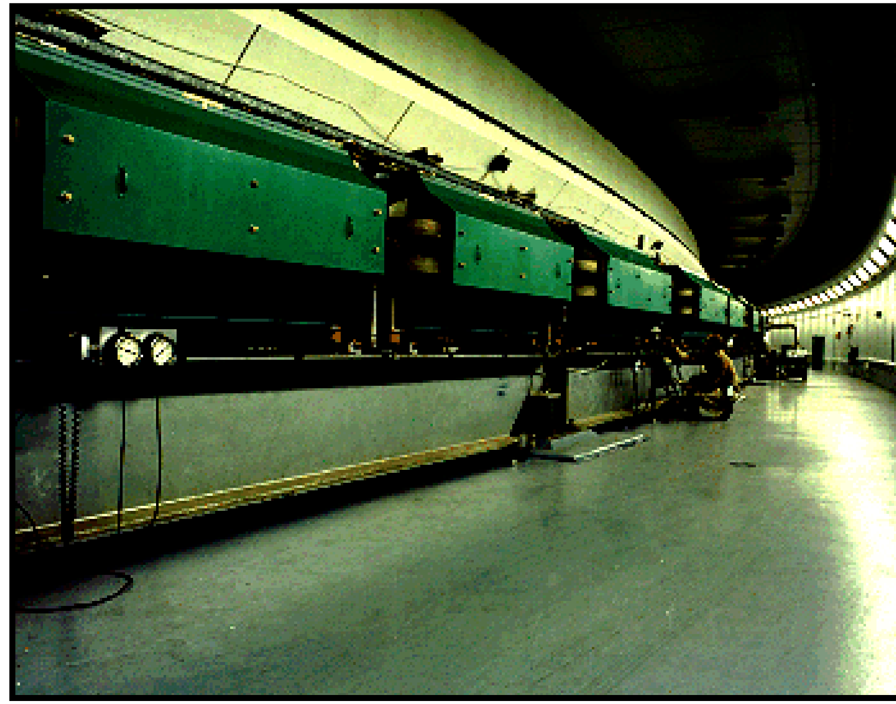
Accelerators

find particle zoo:

BNL Brookhaven National Lab., USA

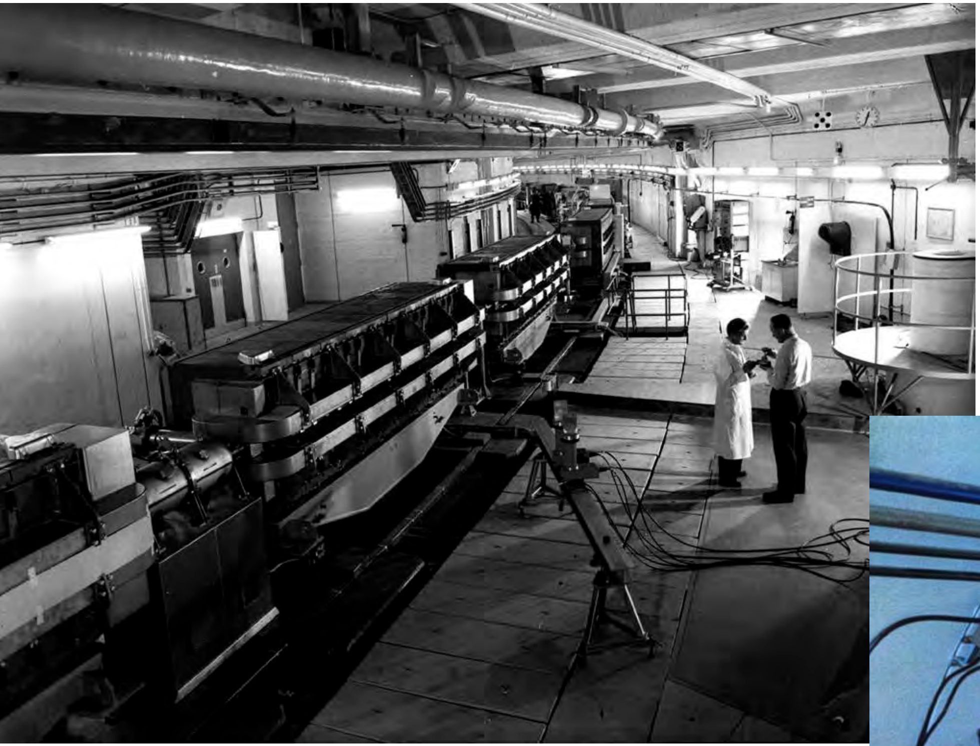


Cosmotron: 1st proton synchrotron.
1953: 3.3 GeV



AGS: Alternating Gradient Synchrotron
1960: 33 GeV

CERN Accelerators



CERN 1959:

26 GeV

Proton Synchrotron

CERN 1976:

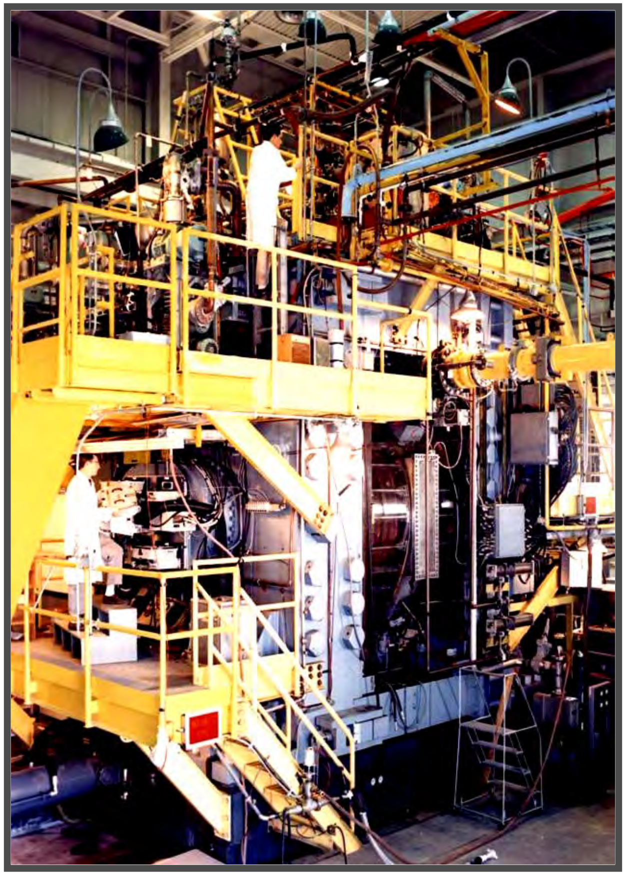
400 GeV   7 km

Super Proton Synchrotron

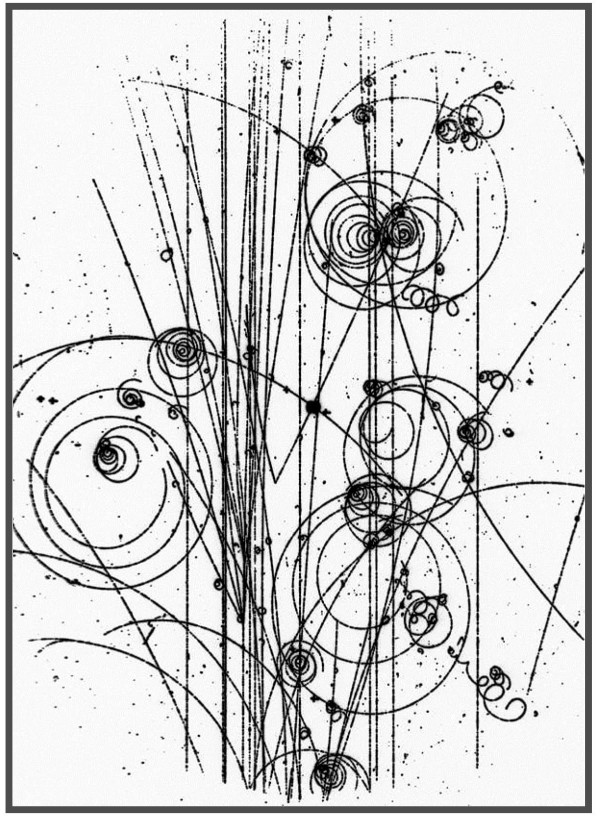
injector chain PS → SPS → LHC

Bubble Chambers

Particle



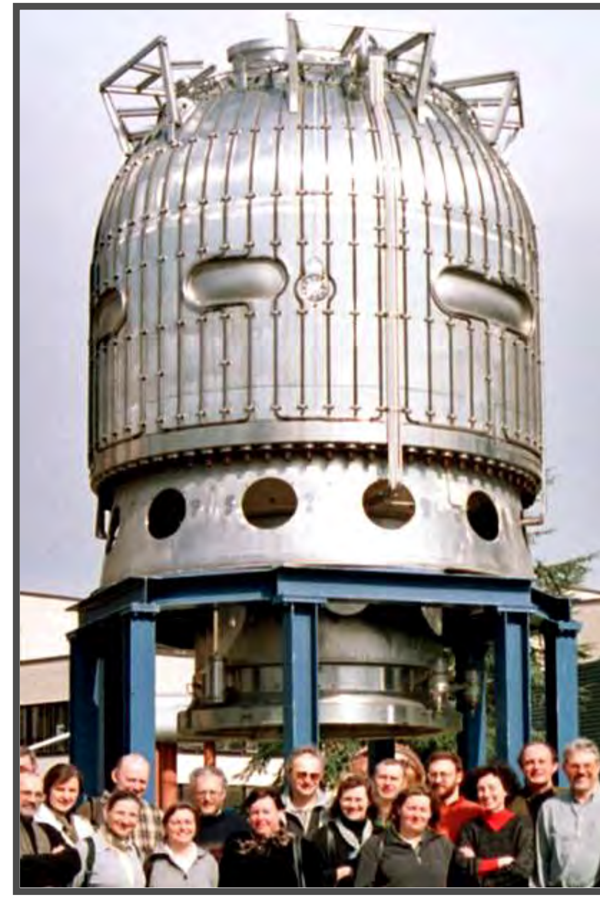
BNL 80" Bubble Chamber
1963-74
1964 Ω^-
Nobel prize



Zoo

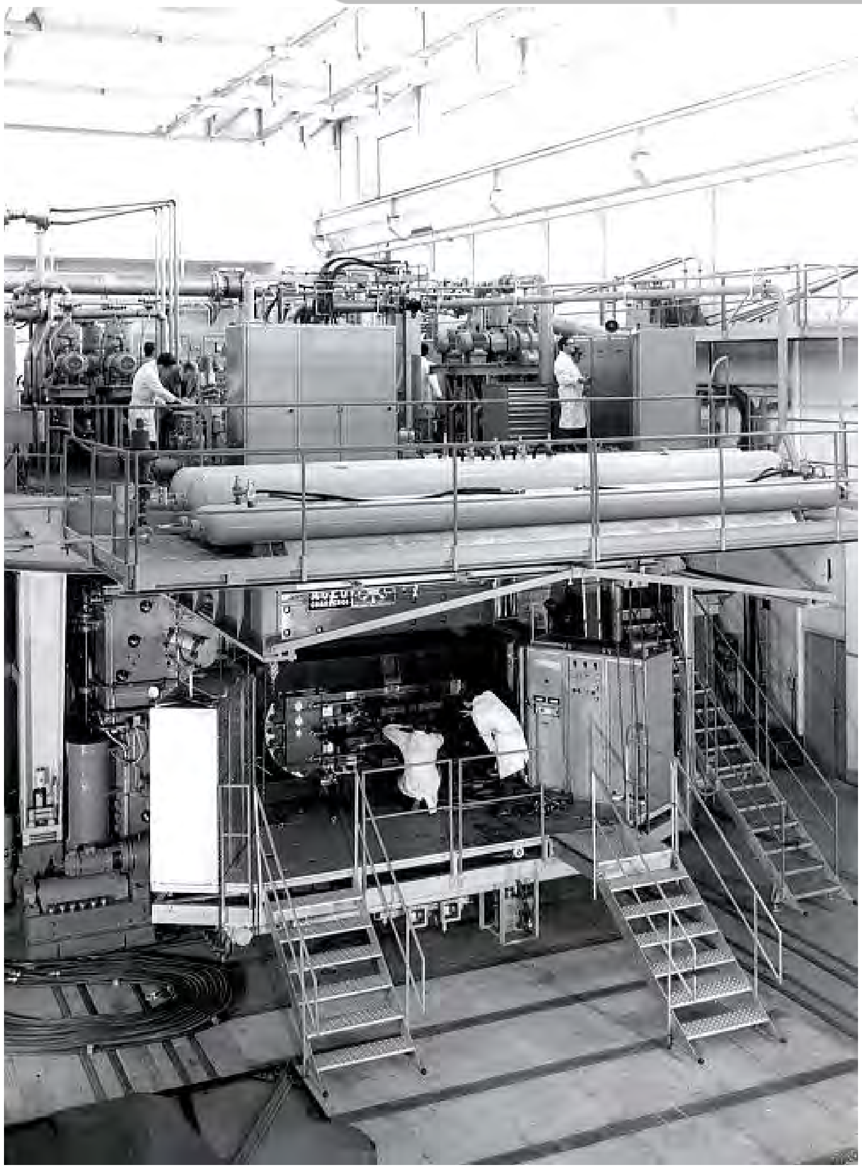
Quark model !

~100 million photos
US + EU

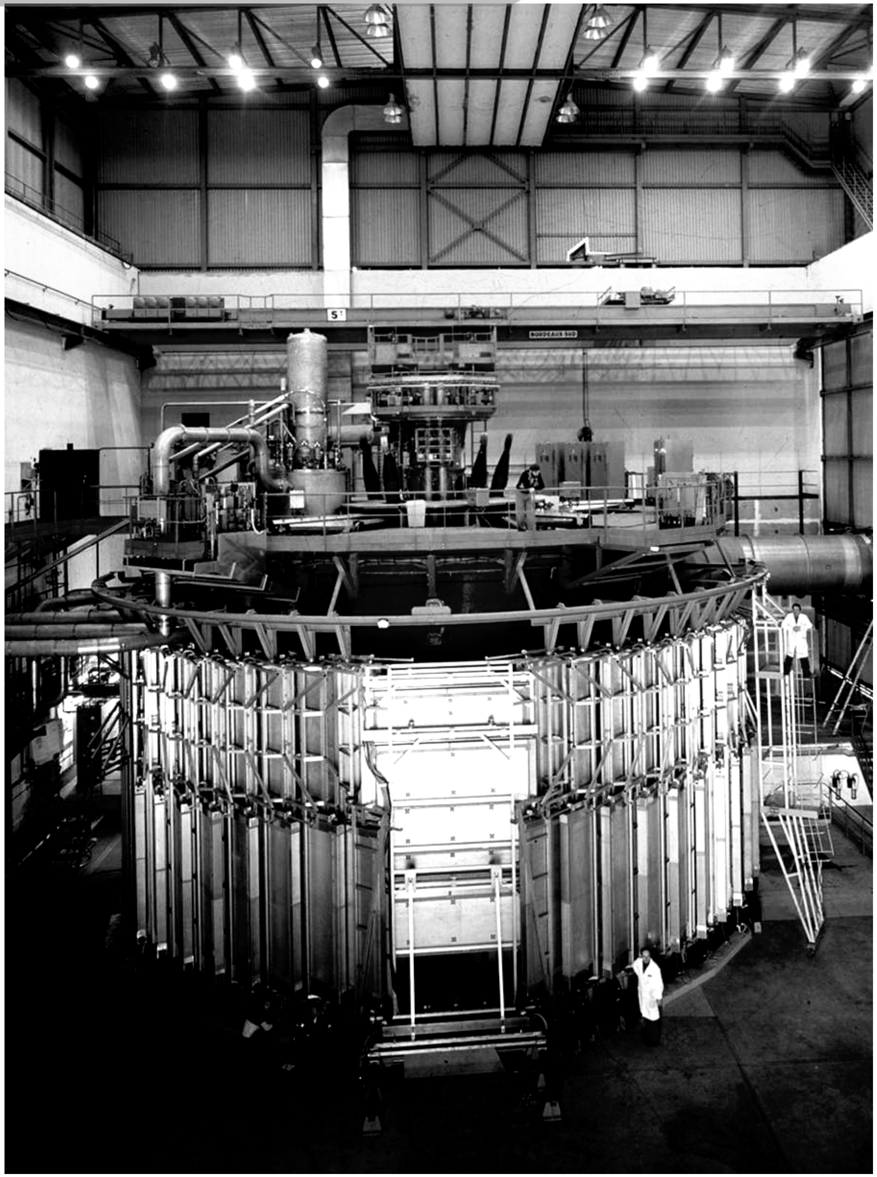


CERN: BEBC
Big European Bubble Chamber
3.7 m, 35 m^3 H, D, Ne.
6 million photos 1973-84.
piston 2 t. magnet 3.5 T, 0.8 GJ

CERN Bubble Chambers



2m Hydrogen Bubble Chamber



Big European Bubble Chamber: 20 m^3

Strangeness

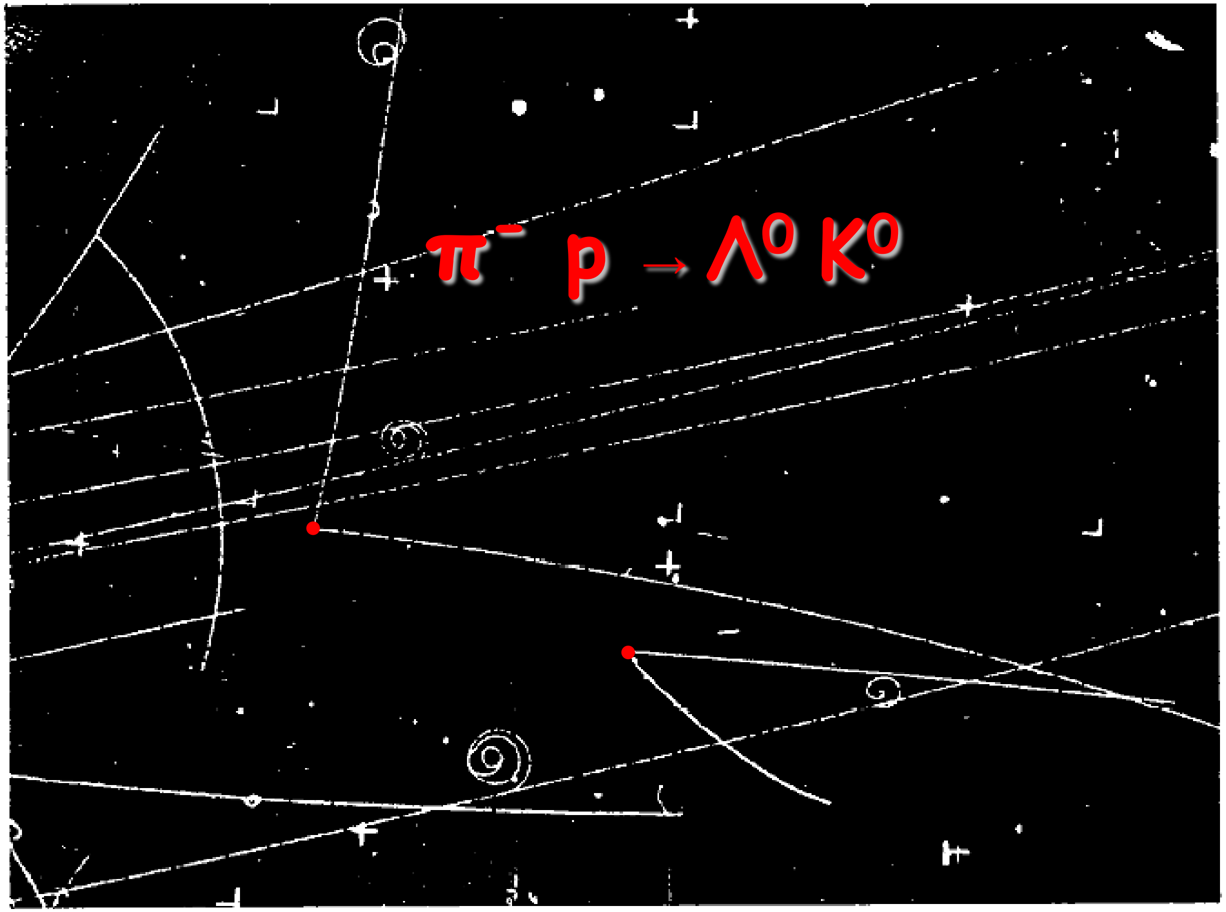


Fig. 12. Associated production, $\pi^- + p \to \Lambda^\circ + K^\circ$ at about 1 GeV with subsequent decays in Alvarez's hydrogen bubble chamber.

Gell-Mann 1953:

Strange particles:

always
pair produced
in strong interactions:

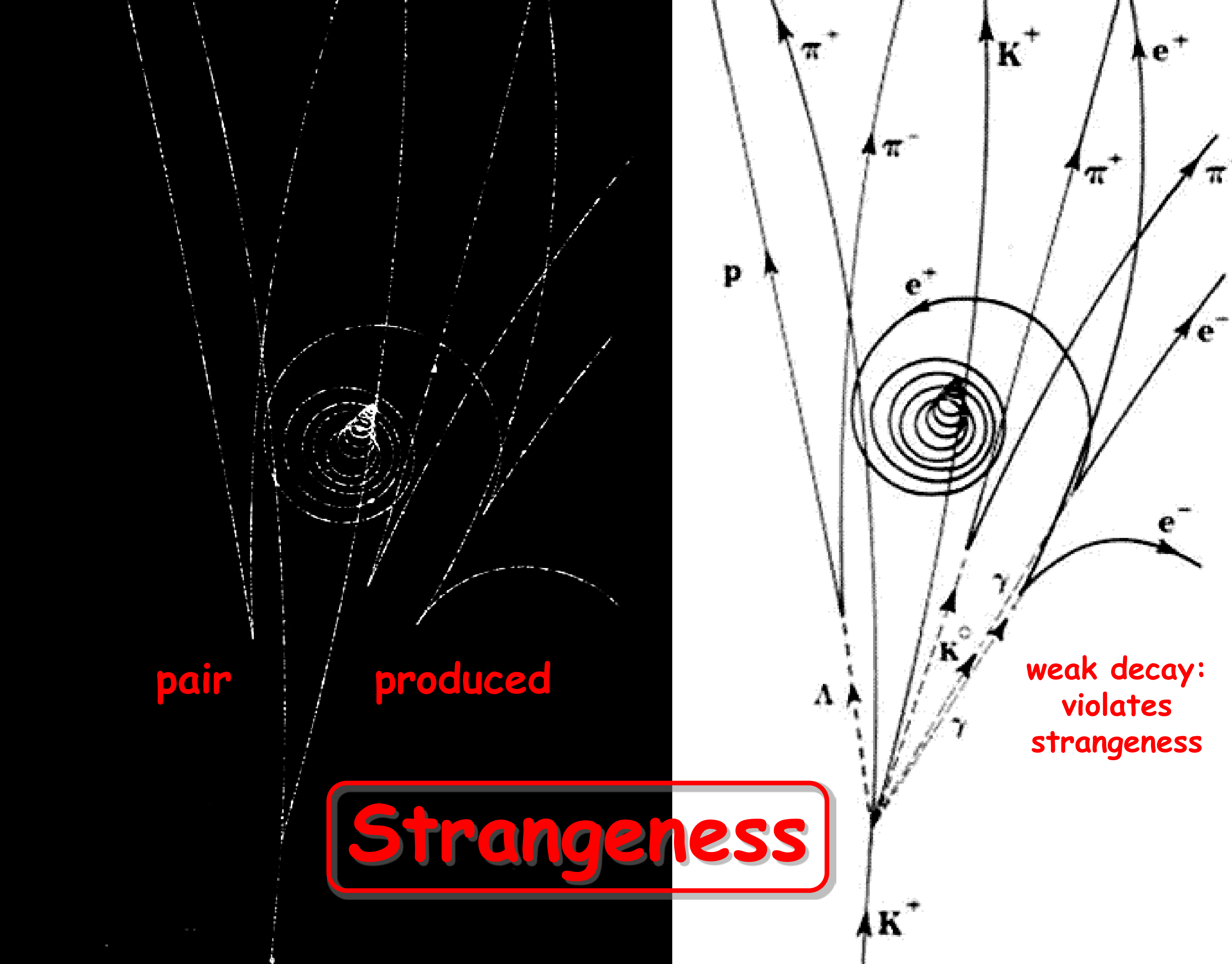
new
conservation law !

life time $\tau \sim 10^{-8 \ldots -10}$ s
decay length $c\tau \sim$ cm…m

D. Glaser, Nobel prize 1960
L. Alvarez, 1968

invention of bubble chambers
use of bubble chambers

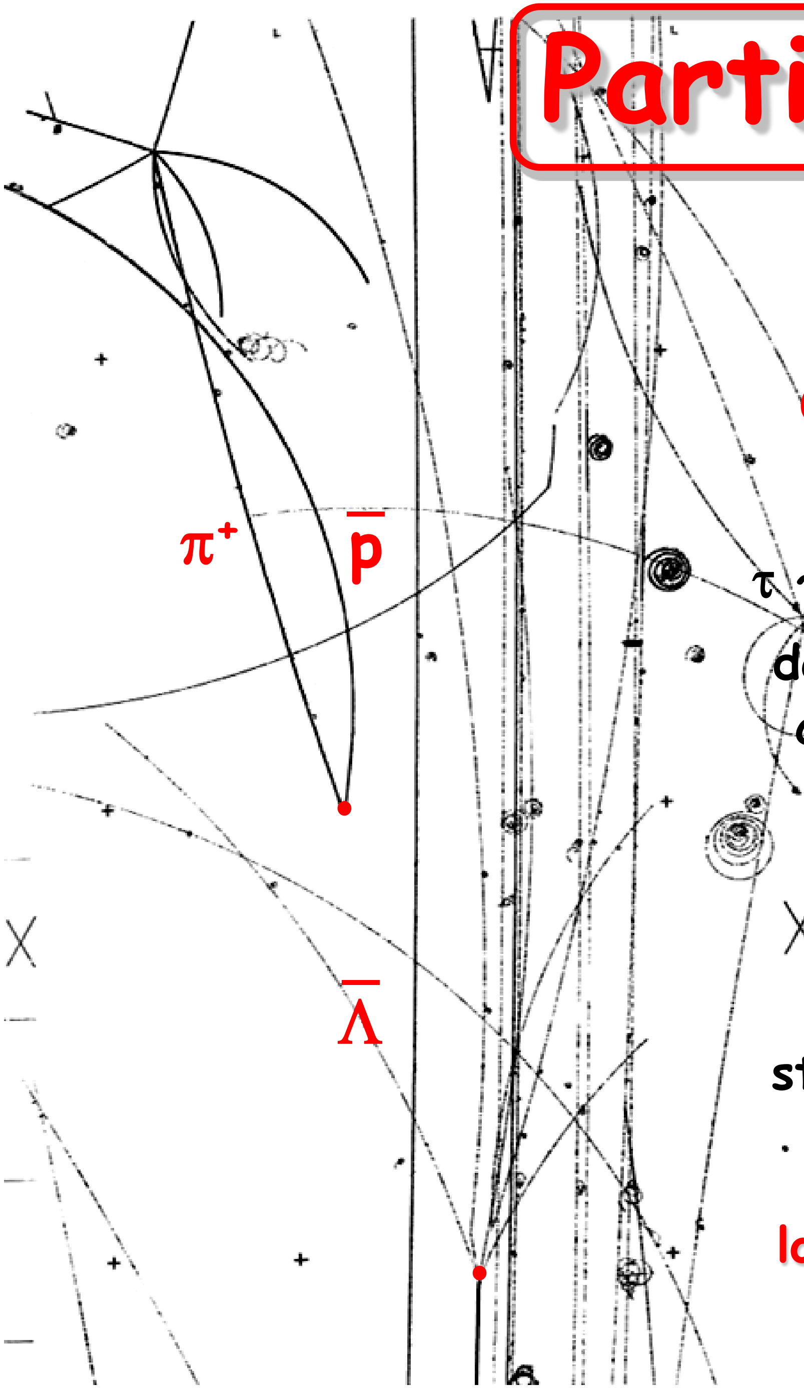
Strangeness



pair produced

weak decay: violates strangeness

Particle Zoo



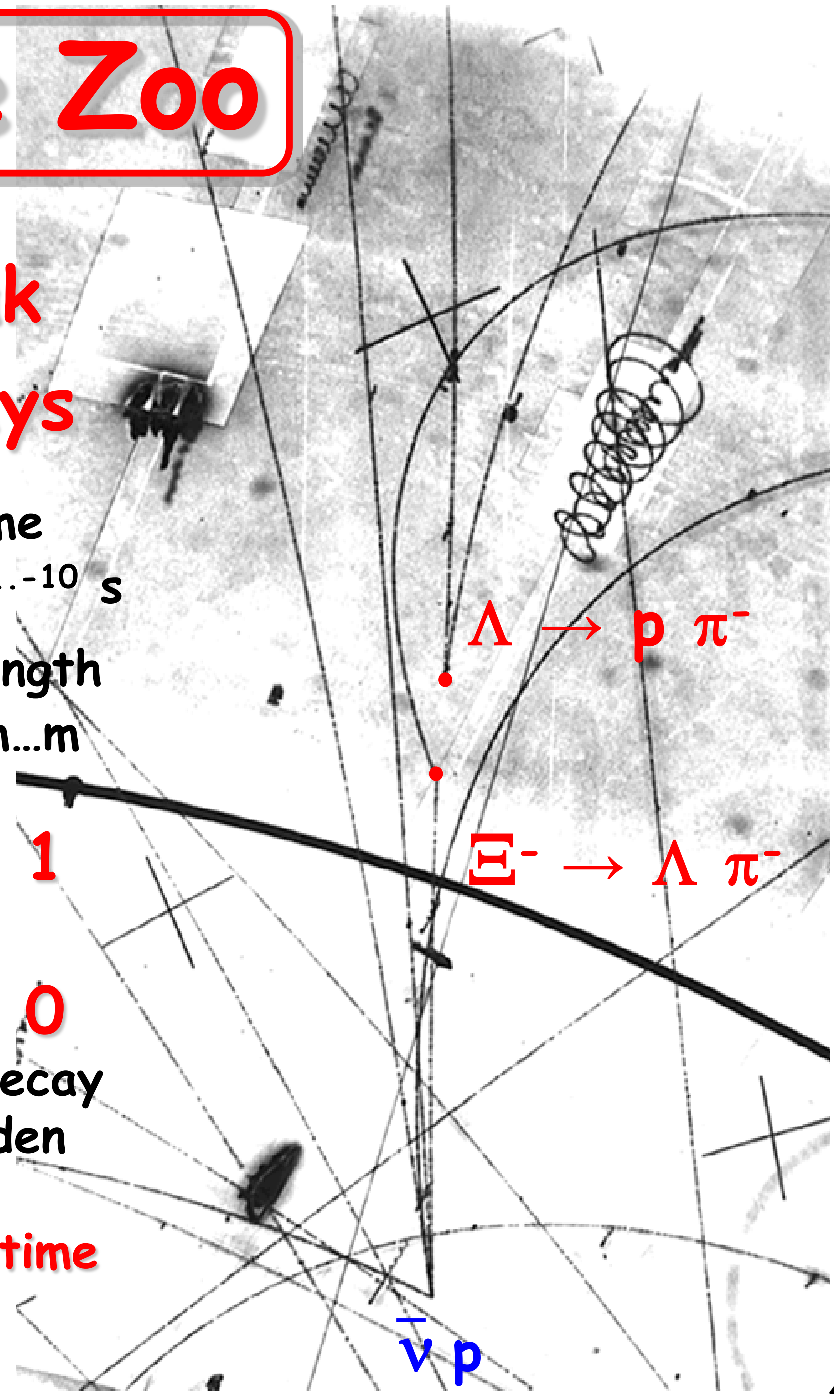
weak decays

lifetime
$\tau \sim 10^{-8...-10}$ s

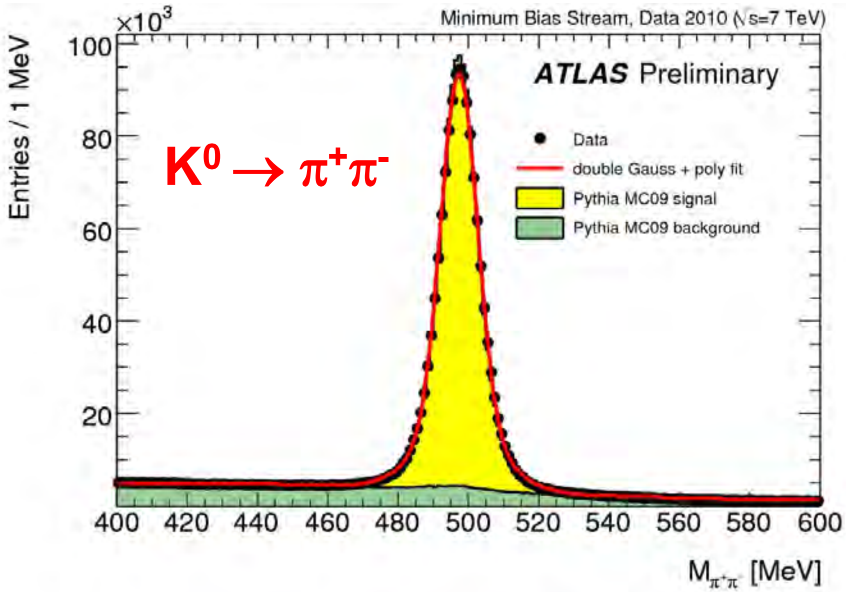
decay length
$c\tau \sim$ cm...m

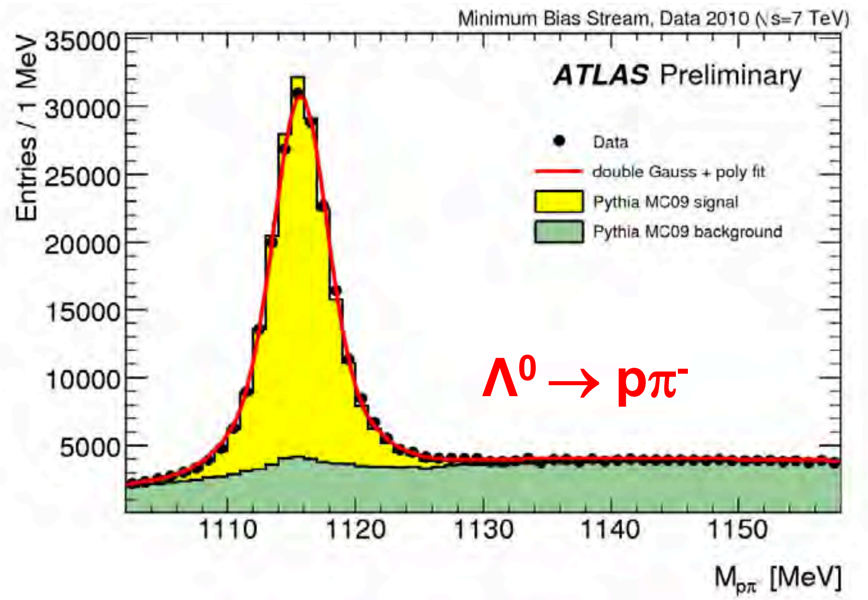
$\Delta S = 1$

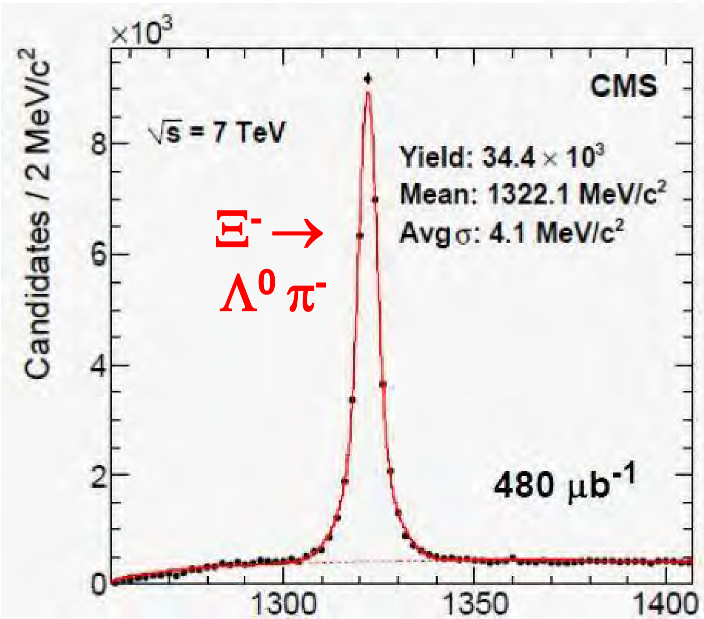
$\Delta S \neq 0$
strong decay forbidden
=>
long lifetime

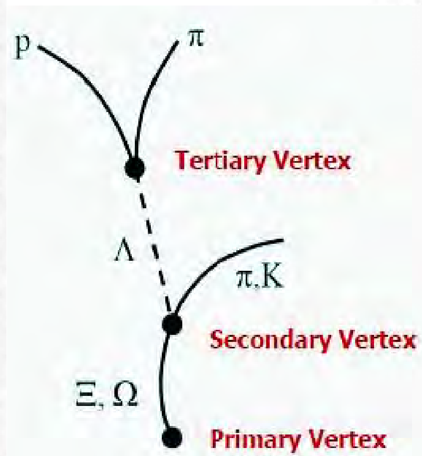


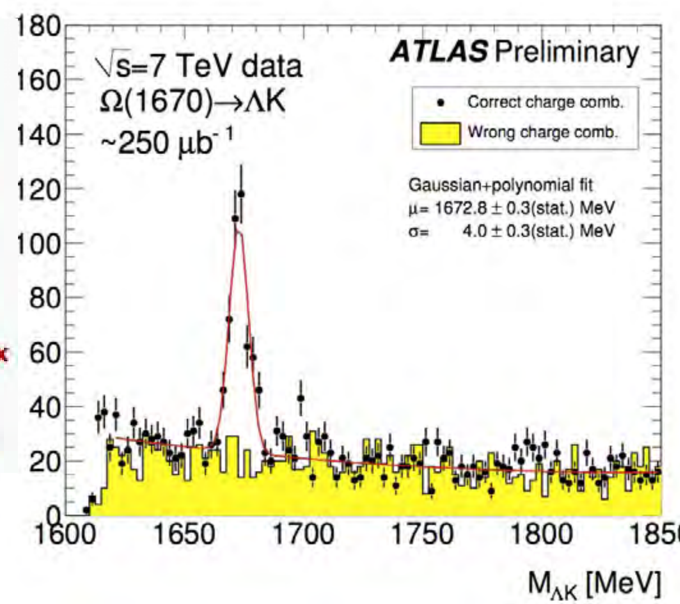
strange particles



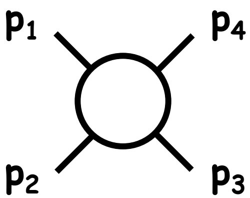








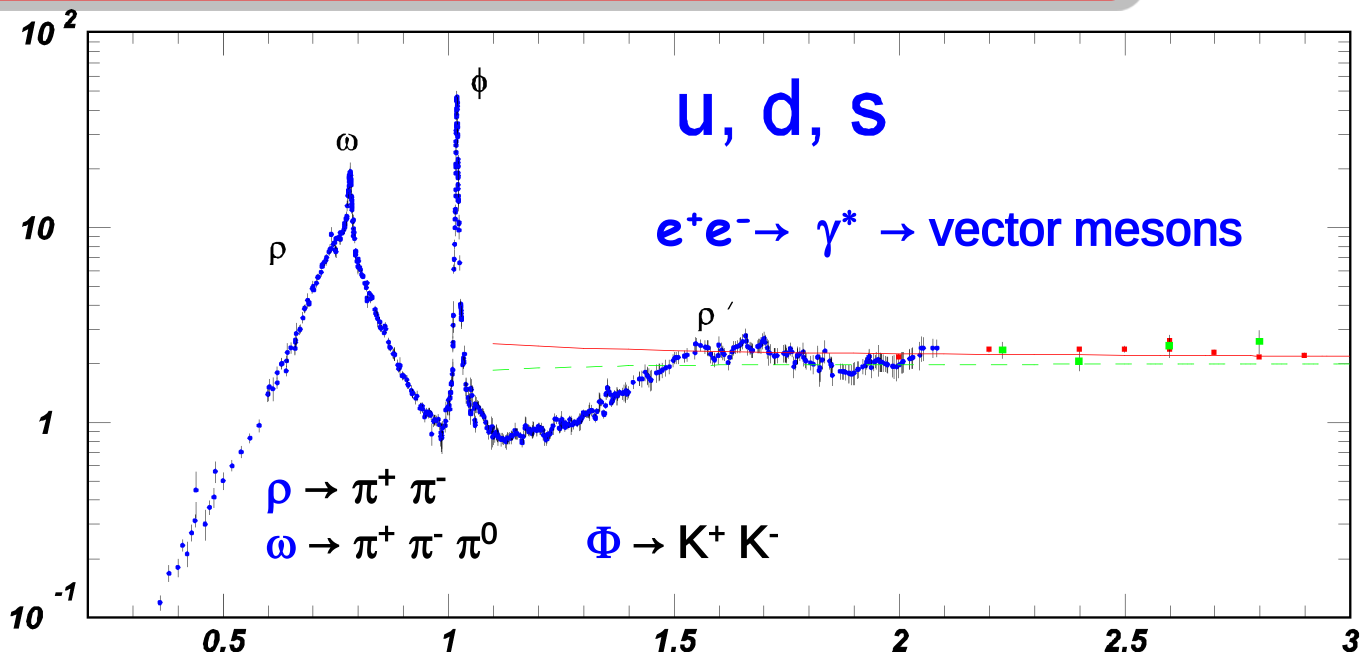
Strongly bound hadrons



four-momentum
$p_i = (E_i, \vec{p_i})$

invariant mass
of a particle:

$m^2 = p^2 = s = (p_1+p_2)^2 = (p_3+p_4)^2$



decay

width:

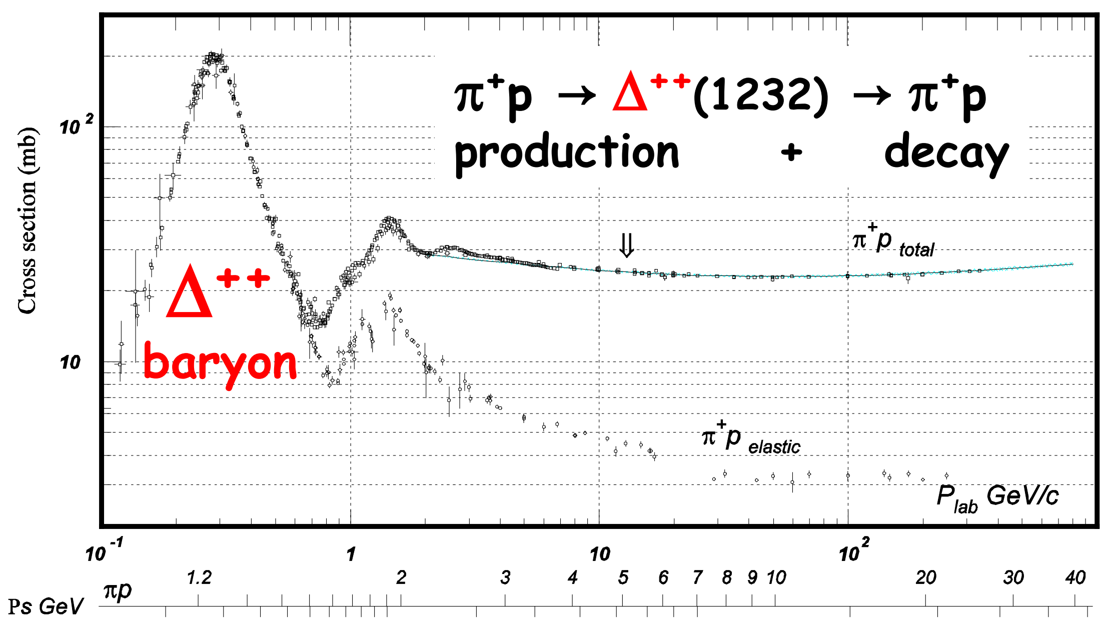
$\Gamma \sim 120$ MeV

lifetime:

$\tau \sim 10^{-23}$ s

decay length:

$c\tau \sim \text{fm} \sim r_{proton}$



production

The Particle Zoo

Light Mesons

$n\,^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $-\bar{u}s$	I = 0 f'	I = 0 f	$\theta_{\rm quad}$ [°]	$\theta_{\rm lin}$ [°]
$1\,^1S_0$	0^{-+}	π	K	η	$\eta'(958)$	−11.5	−24.6
$1\,^3S_1$	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$	38.7	36.0
$1\,^1P_1$	1^{+-}	$b_1(1235)$	K_{1B}†	$h_1(1380)$	$h_1(1170)$		
$1\,^3P_0$	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$		
$1\,^3P_1$	1^{++}	$a_1(1260)$	K_{1A}†	$f_1(1420)$	$f_1(1285)$		
$1\,^3P_2$	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$	29.6	28.0
$1\,^1D_2$	2^{-+}	$\pi_2(1670)$	$K_2(1770)$†	$\eta_2(1870)$	$\eta_2(1645)$		
$1\,^3D_1$	1^{--}	$\rho(1700)$	$K^*(1680)$‡		$\omega(1650)$		
$1\,^3D_2$	2^{--}		$K_2(1820)$‡				
$1\,^3D_3$	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$	32.0	31.0
$1\,^3F_4$	4^{++}	$a_4(2040)$	$K_4^*(2045)$		$f_4(2050)$		
$1\,^3G_5$	5^{--}	$\rho_5(2350)$					
$1\,^3H_6$	6^{++}	$a_6(2450)$			$f_6(2510)$		
$2\,^1S_0$	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$	$\eta(1295)$	−22.4	−22.6
$2\,^3S_1$	1^{--}	$\rho(1450)$	$K^*(1410)$‡	$\phi(1680)$	$\omega(1420)$		

LIGHT UNFLAVORED ($S = C = B = 0$)

	$I^G(J^{PC})$		$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$	• $\pi_2(1670)$	$1^-(2^{-+})$
• π^0	$1^-(0^{-+})$	• $\phi(1680)$	$0^-(1^{--})$
• η	$0^+(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$
• $f_0(600)$	$0^+(0^{++})$	• $\rho(1700)$	$1^+(1^{--})$
• $\rho(770)$	$1^+(1^{--})$	$a_2(1700)$	$1^-(2^{++})$
• $\omega(782)$	$0^-(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$
• $\eta'(958)$	$0^+(0^{-+})$	$\eta(1760)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$	• $\pi(1800)$	$1^-(0^{-+})$
• $a_0(980)$	$1^-(0^{++})$	$f_2(1810)$	$0^+(2^{++})$
• $\phi(1020)$	$0^-(1^{--})$	• $\phi_3(1850)$	$0^-(3^{--})$
• $h_1(1170)$	$0^-(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$
• $b_1(1235)$	$1^+(1^{+-})$	$\rho(1900)$	$1^+(1^{--})$
• $a_1(1260)$	$1^-(1^{++})$	$f_2(1910)$	$0^+(2^{++})$
• $f_2(1270)$	$0^+(2^{++})$	• $f_2(1950)$	$0^+(2^{++})$
• $f_1(1285)$	$0^+(1^{++})$	$\rho_3(1990)$	$1^+(3^{--})$
• $\eta(1295)$	$0^+(0^{-+})$	• $f_2(2010)$	$0^+(2^{++})$
• $\pi(1300)$	$1^-(0^{-+})$	$f_0(2020)$	$0^+(0^{++})$
• $a_2(1320)$	$1^-(2^{++})$	• $a_4(2040)$	$1^-(4^{++})$
• $f_0(1370)$	$0^+(0^{++})$	• $f_4(2050)$	$0^+(4^{++})$
$h_1(1380)$	$?^-(1^{+-})$	$\pi_2(2100)$	$1^-(2^{-+})$
• $\pi_1(1400)$	$1^-(1^{-+})$	$f_0(2100)$	$0^+(0^{++})$
• $\eta(1405)$	$0^+(0^{-+})$	$f_2(2150)$	$0^+(2^{++})$
• $f_1(1420)$	$0^+(1^{++})$	$\rho(2150)$	$1^+(1^{--})$
• $\omega(1420)$	$0^-(1^{--})$	$f_0(2200)$	$0^+(0^{++})$
$f_2(1430)$	$0^+(2^{++})$	$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
• $a_0(1450)$	$1^-(0^{++})$		
• $\rho(1450)$	$1^+(1^{--})$	$\eta(2225)$	$0^+(0^{-+})$
• $\eta(1475)$	$0^+(0^{-+})$	$\rho_3(2250)$	$1^+(3^{--})$
• $f_0(1500)$	$0^+(0^{++})$	• $f_2(2300)$	$0^+(2^{++})$
$f_1(1510)$	$0^+(1^{++})$	$f_4(2300)$	$0^+(4^{++})$
• $f_2'(1525)$	$0^+(2^{++})$	• $f_2(2340)$	$0^+(2^{++})$
$f_2(1565)$	$0^+(2^{++})$	$\rho_5(2350)$	$1^+(5^{--})$
$h_1(1595)$	$0^-(1^{+-})$	$a_6(2450)$	$1^-(6^{++})$
• $\pi_1(1600)$	$1^-(1^{-+})$	$f_6(2510)$	$0^+(6^{++})$
$a_1(1640)$	$1^-(1^{++})$		
$f_2(1640)$	$0^+(2^{++})$		
• $\eta_2(1645)$	$0^+(2^{-+})$		
• $\omega(1650)$	$0^-(1^{--})$		
• $\omega_3(1670)$	$0^-(3^{--})$		

OTHER LIGHT

Further States

STRANGE ($S = \pm1$, $C = B = 0$)

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K_S^0	$1/2(0^-)$
• K_L^0	$1/2(0^-)$
$K_0^*(800)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
$K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED ($C = \pm1$)

	$I(J^P)$
• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
• $D_2^*(2460)^0$	$1/2(2^+)$
• $D_2^*(2460)^\pm$	$1/2(2^+)$
$D^*(2640)^\pm$	$1/2(?^?)$

CHARMED, STRANGE ($C = S = \pm1$)

	$I(J^P)$
• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{sJ}^*(2317)^\pm$	$0(0^+)$
• $D_{sJ}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}(2573)^\pm$	$0(?^?)$

BOTTOM ($B = \pm1$)

	$I^G(J^{PC})$
• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B_s^0/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
$B_J^*(5732)$	$?(?^?)$

BOTTOM, STRANGE ($B = \pm1$, $S = \mp1$)

• B_s^0	$0(0^-)$
B_s^*	$0(1^-)$
$B_{sJ}^*(5850)$	$?(?^?)$

BOTTOM, CHARMED ($B = C = \pm1$)

• $B_c^\pm$	$0(0^-)$

$c\bar{c}$

• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
$h_c(1P)$	$?^?(?^{??})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
$\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
$\psi(3836)$	$0^-(2^{--})$
$X(3872)$	$?^?(?^?)$
• $\psi(4040)$	$0^-(1^{--})$
• $\psi(4160)$	$0^-(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$

$b\bar{b}$

$\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\Upsilon(4S)$	$0^-(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

NON-$q\bar{q}$ CANDIDATES

NON-$q\bar{q}$ CANDIDATES

Particle Types

LEPTONS
B=0 L=1

fundamental fermions
no strong interaction
no substructure
spin = 1/2

ELECTRONS
Q=1

e m = 511 keV
μ 106 MeV
τ 1.8 GeV

NEUTRINOS
Q=0

ν_e m $\lesssim$ 1 eV
ν_μ
ν_τ

HADRONS
L=0

B baryon nr
L lepton nr
S strangeness

BARYONS
B=1

NUCLEONS
S=0

N = (n, p)

m_p = 938 MeV
$m_n - m_p$ = 1 MeV

p stable
$n \rightarrow p\, e^- \bar{\nu}_e$ τ = 886 s

HYPERONS
S≠0

Σ^+ (1189) $\rightarrow$ N π
Λ^o (1116) $\rightarrow$ p π^-
Ξ^- (1321) $\rightarrow$ $\Lambda\, \pi^-$

$\tau \approx 10^{-10}$ s $c\tau \approx$ cm

MESONS
B=0

Pions = ($\pi^+\ \pi^0\ \pi^-$) **S=0**
$m(\pi^\pm)$ = 140 MeV
$m(\pi^0)$ = 135 MeV
$\pi^+ \rightarrow \mu^+ \nu_\mu$ τ = 26 ns $c\tau$ = 8 m

Kaons = ($K^+\ K^0$) , ($K^-\ \bar{K}^0$) **S=1**

$m(K^\pm)$ = 494 MeV
$m(K^0)$ = 498 MeV
$K^+ \rightarrow \mu^+ \nu_\mu$ τ = 12 ns $c\tau$ = 3 m

2. Quark Model

Iso-Spin

- **observation:** level scheme of **mirror nuclei** similar:

  $^{15}_{7}N \equiv {}^{15}_{8}O$ $\quad$ $^{14}_{6}C \; {}^{14}_{7}N \; {}^{14}_{8}O$ $\quad$ $^{13}_{7}N \equiv {}^{13}_{6}C$

- **strong interactions symmetric** w.r.t. exchange $n \leftrightarrow p$?

  can we say: $n \equiv p$???

- **but:** $^{3}_{1}H = npn \neq pnp = {}^{3}_{2}He$

  nn $\neq$ np = $^{2}_{1}H$ $\neq$ pp
  not bound deuterium repulsive

  p e = bound H atom $\neq$ **n e** = not bound

- **symmetry broken** by electromagnetic interaction:

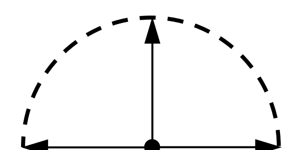
  $(m_n - m_p) / m_p = 1$ MeV / 1 GeV = 1 ‰ small

- and by weak interaction: $n \rightarrow p \, e^- \, \nu_e$

- **Heisenberg 1932:** isotopic spin:

  rotations in 3D real vector space **SO(3)** (spin algebra) isomorphic to
  rotations in 2D complex vector space **SU(2)** (isospin algebra)

- **nucleons (n, p) I = 1/2 (2I+1) = 2 iso-doublet**
- **deuterium (n p) I = 0 (2I+1) = 1 iso-singlet**
- **pions (π^+ π^o π^-) I = 1 (2I+1) = 3 iso-triplet**



I_3 = Q π^- π^o π^+

I_3 = Q-B/2 incl. nucleon baryon nr

- same **algebra** as for spin:

  Clebsch-Gordon coefficients give
  cross section and decay branching ratios. See exercises !

Clebsch-Gordon coefficients

- Quantum Mechanics: vector addition of angular momenta $j_1 \oplus j_2$:
- triangular condition: $|j_1 - j_2| < J < j_1 + j_2$ $M = m_1 + m_2$

$$|JM\rangle = \sum_{m_1, m_2} |j_1\, m_1\, j_2\, m_2\rangle \langle j_1\, m_1\, j_2\, m_2\, |JM\rangle$$

- multiplicity: $\sum_{j=|j_1-j_2|}^{j_1+j_2} (2j+1) = (2j_1+1)(2j_2+1)$
- normalization: $\sum_{m_1, m_2} |\langle j_1\, m_1\, j_2\, m_2\, |JM\rangle|^2 = 1$ =>

$$\langle j_1 m_1 j_2 m_2 | jm \rangle = \delta_{m_1+m_2,m} \sqrt{\frac{(j_1+j_2-j)!(j+j_1-j_2)!(j+j_2-j_1)!(2j+1)}{(j+j_1+j_2+1)!}}$$
$$\times \sum_k \frac{(-1)^k \sqrt{(j_1+m_1)!(j_1-m_1)!(j_2+m_2)!(j_2-m_2)!(j+m)!(j-m)!}}{k!(j_1+j_2-j-k)!(j_1-m_1-k)!(j_2+m_2-k)!(j-j_2+m_1+k)!(j-j_1-m_2+k)!}$$

Wigner 1931
Racah 1942

- Abramowitz, M. and Stegun, I.A. (Eds.). "Vector-Addition Coefficients." § 27.9 in "Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables", 9th printing. New York: Dover, pp. 1006-1010, 1972.
- Condon, E.U. and Shortley, G. § 3.6-3.14 in "The Theory of Atomic Spectra." Cambridge, England: Cambridge University Press, pp. 56-78, 1951.
- Messiah, A. "Clebsch-Gordan Coefficients and 3j Symbols." Appendix C.I in "Quantum Mechanics", Vol. 2. Amsterdam, Netherlands: North-Holland, pp. 1054-1060, 1962.

Clebsch-Gordan coefficients

$J_1 x J_2$

$J = J_1 \otimes J_2$

$M = m_1 + m_2$

Notation:

		J J … / M M …
m_1 m_2		
m_1 m_2		Coefficients
⋮ ⋮		

1/2 × 1/2

m_1	m_2	J=1, M=+1
+1/2	+1/2	1

m_1	m_2	J=1, M=0	J=0, M=0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	J=1, M=−1
−1/2	−1/2	1

1 × 1/2

m_1	m_2	J=3/2, M=+3/2
+1	+1/2	1

m_1	m_2	J=3/2, M=+1/2	J=1/2, M=+1/2
+1	−1/2	1/3	2/3
0	+1/2	2/3	−1/3

m_1	m_2	J=3/2, M=−1/2	J=1/2, M=−1/2
0	−1/2	2/3	1/3
−1	+1/2	1/3	−2/3

m_1	m_2	J=3/2, M=−3/2
−1	−1/2	1

2 × 1

m_1	m_2	J=3, M=+3
+2	+1	1

m_1	m_2	J=3, M=+2	J=2, M=+2
+2	0	1/3	2/3
+1	+1	2/3	−1/3

m_1	m_2	J=3, M=+1	J=2, M=+1	J=1, M=+1
+2	−1	1/15	1/3	3/5
+1	0	8/15	1/6	−3/10
0	+1	6/15	−1/2	1/10

m_1	m_2	J=3, M=0	J=2, M=0	J=1, M=0
+1	−1	1/5	1/2	3/10
0	0	3/5	0	−2/5
−1	+1	1/5	−1/2	3/10

m_1	m_2	J=3, M=−1	J=2, M=−1	J=1, M=−1
0	−1	6/15	1/2	1/10
−1	0	8/15	−1/6	−3/10
−2	+1	1/15	−1/3	3/5

m_1	m_2	J=3, M=−2	J=2, M=−2
−1	−1	2/3	1/3
−2	0	1/3	−2/3

m_1	m_2	J=3, M=−3
−2	−1	1

1 × 1

m_1	m_2	J=2, M=+2
+1	+1	1

m_1	m_2	J=2, M=+1	J=1, M=+1
+1	0	1/2	1/2
0	+1	1/2	−1/2

m_1	m_2	J=2, M=0	J=1, M=0	J=0, M=0
+1	−1	1/6	1/2	1/3
0	0	2/3	0	−1/3
−1	+1	1/6	−1/2	1/3

m_1	m_2	J=2, M=−1	J=1, M=−1
0	−1	1/2	1/2
−1	0	1/2	−1/2

m_1	m_2	J=2, M=−2
−1	−1	1

2 × 1/2

m_1	m_2	J=5/2, M=+5/2
+2	1/2	1

m_1	m_2	J=5/2, M=3/2	J=3/2, M=+3/2
+2	−1/2	1/5	4/5
+1	+1/2	4/5	−1/5

$j_1 \times j_2$ 2 / +1/2 +1/2

m_1	m_2	J=5/2, M=−1/2	J=3/2, M=−1/2
+1	−1/2	2/5	3/5
0	+1/2	3/5	−2/5

m_1	m_2	J=5/2, M=−3/2	J=3/2, M=−3/2
0	−1/2	3/5	2/5
−1	+1/2	2/5	−3/5

m_1	m_2	J=5/2, M=−5/2
−1	−1/2	4/5
−2	+1/2	1/5

m_1	m_2	(J=3/2 column)
−1	−1/2	1/5
−2	+1/2	−4/5

m_1	m_2	
−2	−1/2	1

3/2 × 1/2

m_1	m_2	J=2, M=+2
+3/2	+1/2	1

m_1	m_2	J=2, M=+1	J=1, M=+1
+3/2	−1/2	1/4	3/4
+1/2	+1/2	3/4	−1/4

m_1	m_2	J=2, M=0	J=1, M=0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	J=2, M=−1	J=1, M=−1
−1/2	−1/2	3/4	1/4
−3/2	+1/2	1/4	−3/4

m_1	m_2	J=2, M=−2
−3/2	−1/2	1

3/2 × 1

m_1	m_2	J=5/2, M=+5/2
+3/2	+1	1

m_1	m_2	J=5/2, M=+3/2	J=3/2, M=+3/2
+3/2	0	2/5	3/5
+1/2	+1	3/5	−2/5

m_1	m_2	J=5/2, M=+1/2	J=3/2, M=+1/2	J=1/2, M=+1/2
+3/2	−1	1/10	2/5	1/2
+1/2	0	3/5	1/15	−1/3
−1/2	+1	3/10	−8/15	1/6

m_1	m_2	J=5/2, M=−1/2	J=3/2, M=−1/2	J=1/2, M=−1/2
+1/2	−1	3/10	8/15	1/6
−1/2	0	3/5	−1/15	−1/3
−3/2	+1	1/10	−2/5	1/2

m_1	m_2	J=5/2, M=−3/2	J=3/2, M=−3/2
−1/2	−1	3/5	2/5
−3/2	0	2/5	−3/5

m_1	m_2	J=5/2, M=−5/2
−3/2	−1	1

3/2 × 3/2

m_1	m_2	J=3, M=+3
+3/2	+3/2	1

m_1	m_2	J=3, M=+2	J=2, M=+2
+3/2	+1/2	1/2	1/2
+1/2	+3/2	1/2	−1/2

m_1	m_2	J=3, M=+1	J=2, M=+1	J=1, M=+1
+3/2	−1/2	1/5	1/2	3/10
+1/2	+1/2	3/5	0	−2/5

J: 3 2 1 0

2 × 3/2

m_1	m_2	J=7/2, M=+7/2
+2	+3/2	1

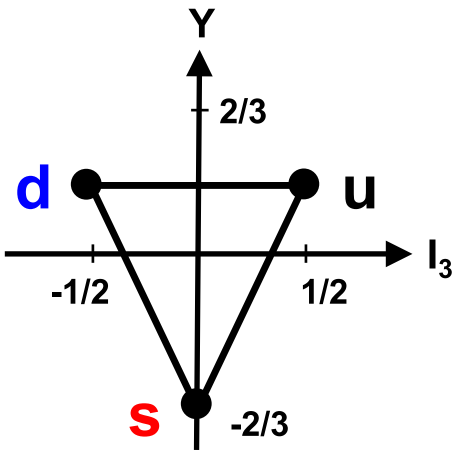
m_1	m_2	J=7/2, M=+5/2	J=5/2, M=+5/2
+2	+1/2	3/7	4/7

J: 7/2 5/2 3/2

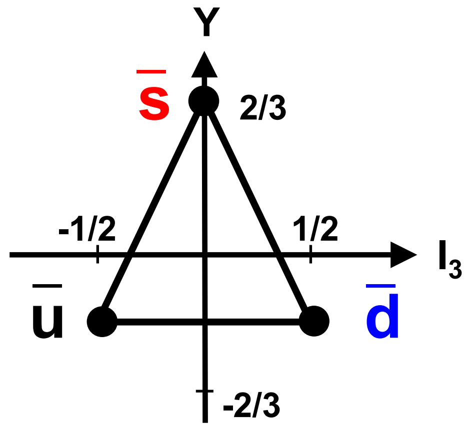
Quarks

Quark	Q	I_3	B	S	Y
u	+2/3	+1/2	1/3	0	+1/3
d	-1/3	- 1/2	1/3	0	+1/3
s	-1/3	0	1/3	-1	-2/3

spin
J = 1/2



Quark



Antiquark

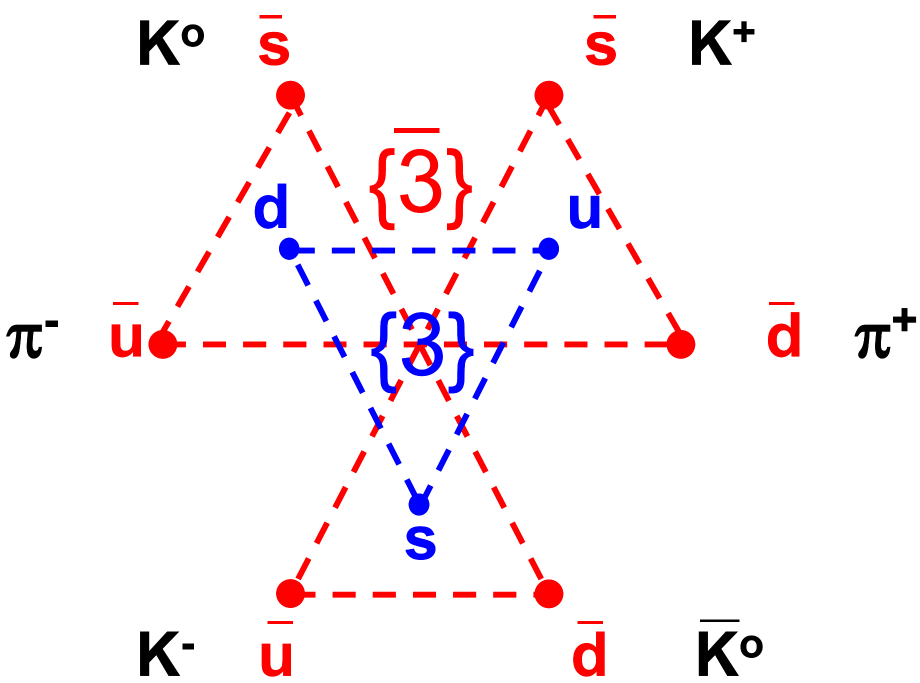
$Y = B + S$ hypercharge

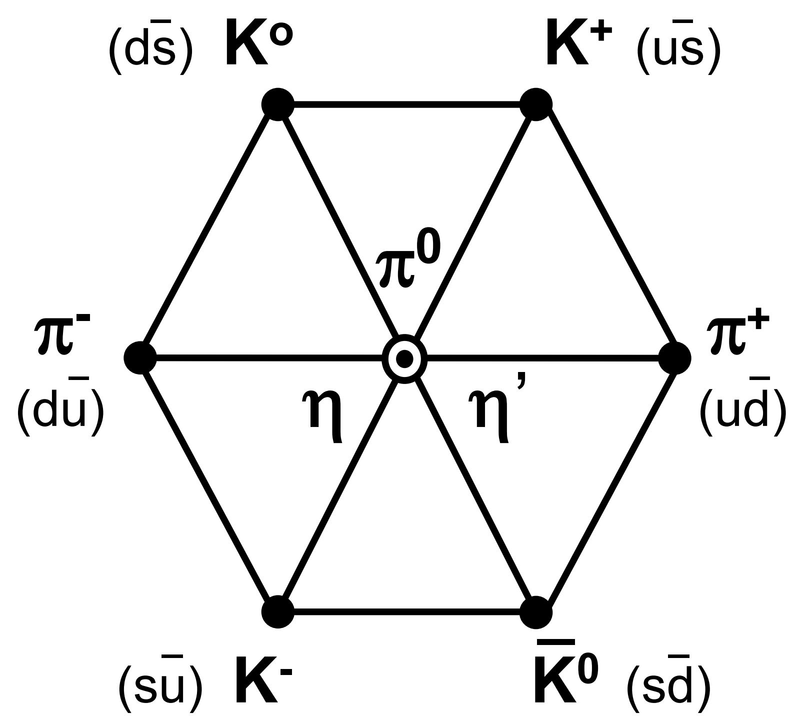
$I_3 = Q - Y/2$ Gell-Mann - Nishijima rule

Meson Octet

$$\{q\} \otimes \{\bar{q}\} = \nabla \otimes \Delta = \{2\} \otimes \{\bar{2}\} = \{1\} \oplus \{3\} \quad \text{in SU(2)}$$

$$= \{3\} \otimes \{\bar{3}\} = \{1\} \oplus \{8\} \quad \text{in SU(3)}$$



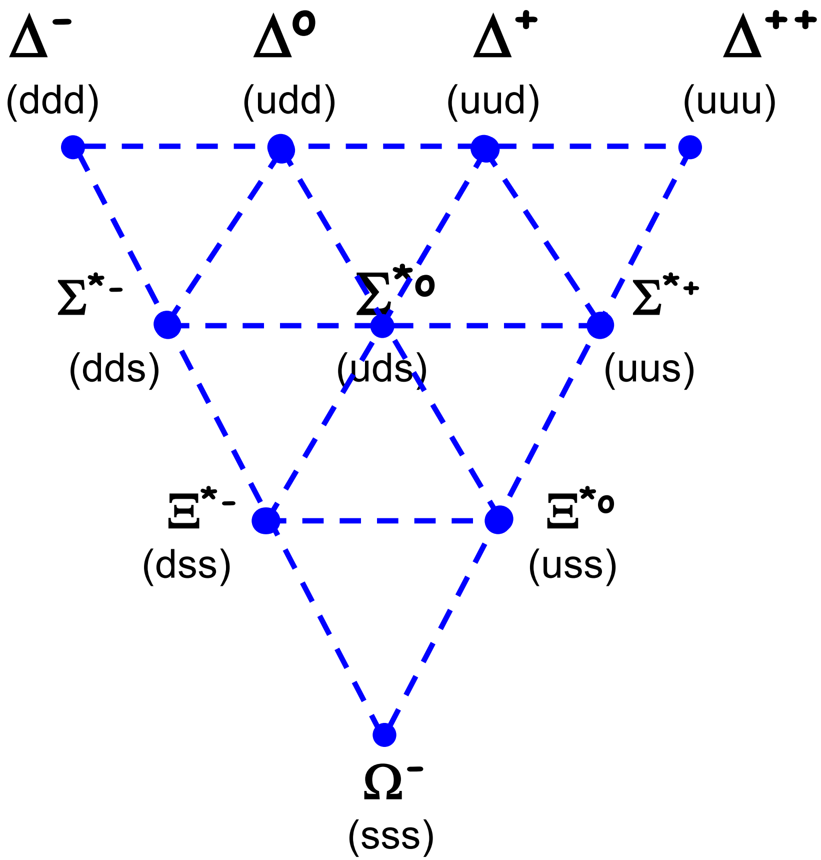


Baryon Decuplet

$= \{q\} \otimes \{q\} \otimes \{q\} = \nabla \otimes \nabla \otimes \nabla$

$= \{2\} \otimes \{2\} \otimes \{2\} = \{2\} \oplus \{2\} \oplus \{4\}$ in $SU(2)$

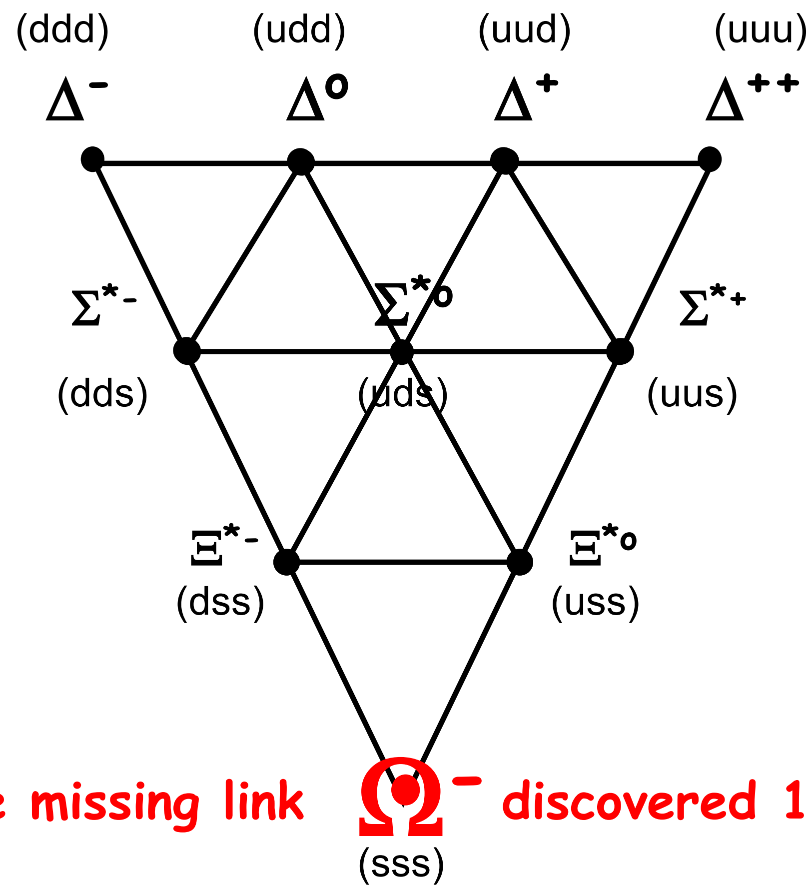
$= \{3\} \otimes \{3\} \otimes \{3\} = \{1\} \oplus \{8\} \oplus \{8\} \oplus \{10\}$ in $SU(3)$

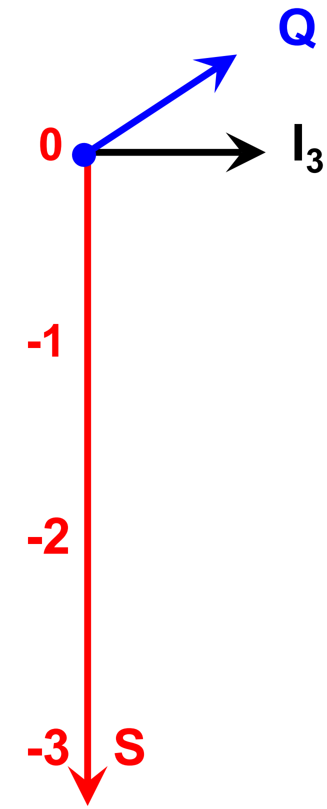


{4} in SU(2)

{10} in SU(3)

Baryon Decuplet





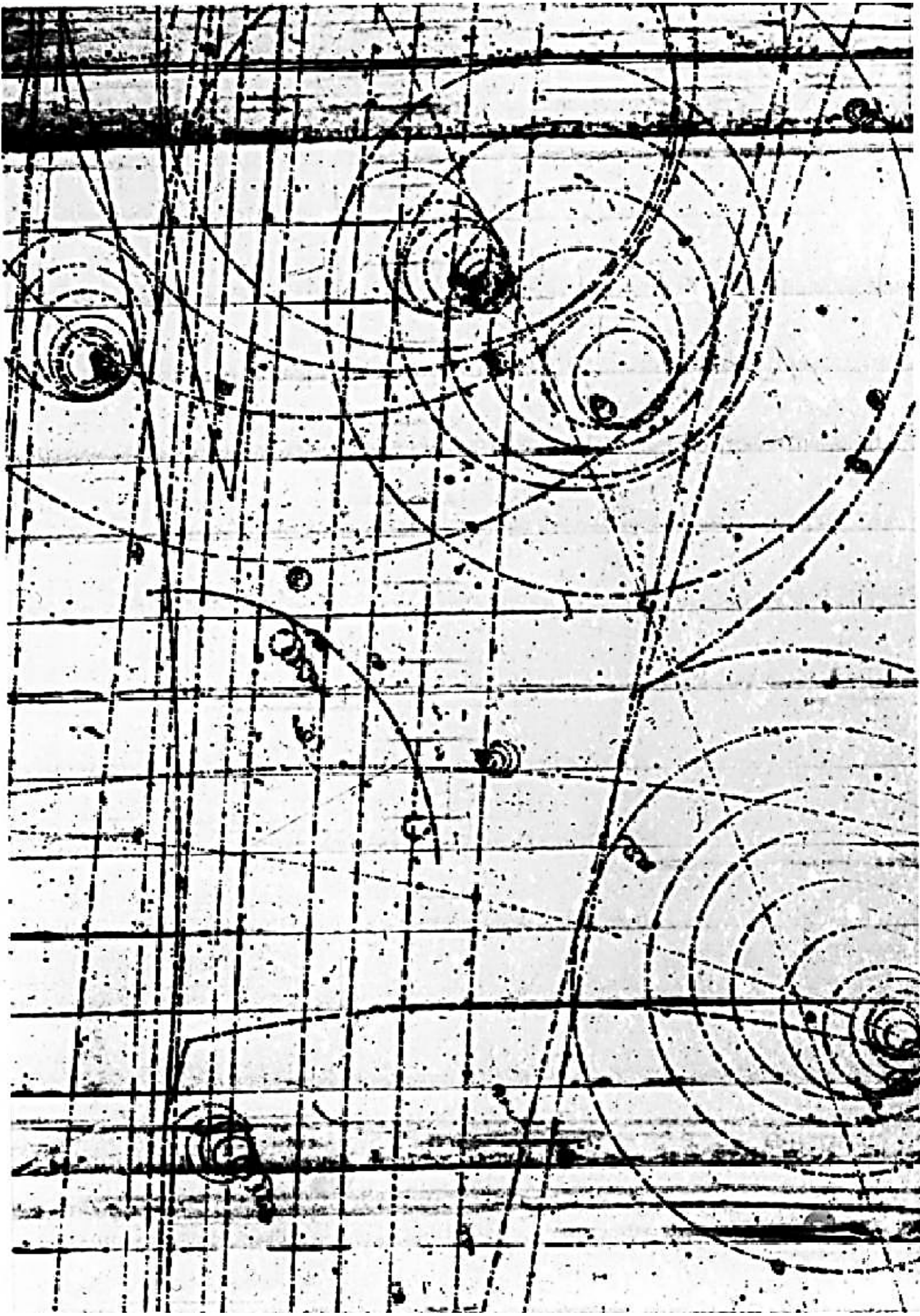
the missing link Ω^- discovered 1964

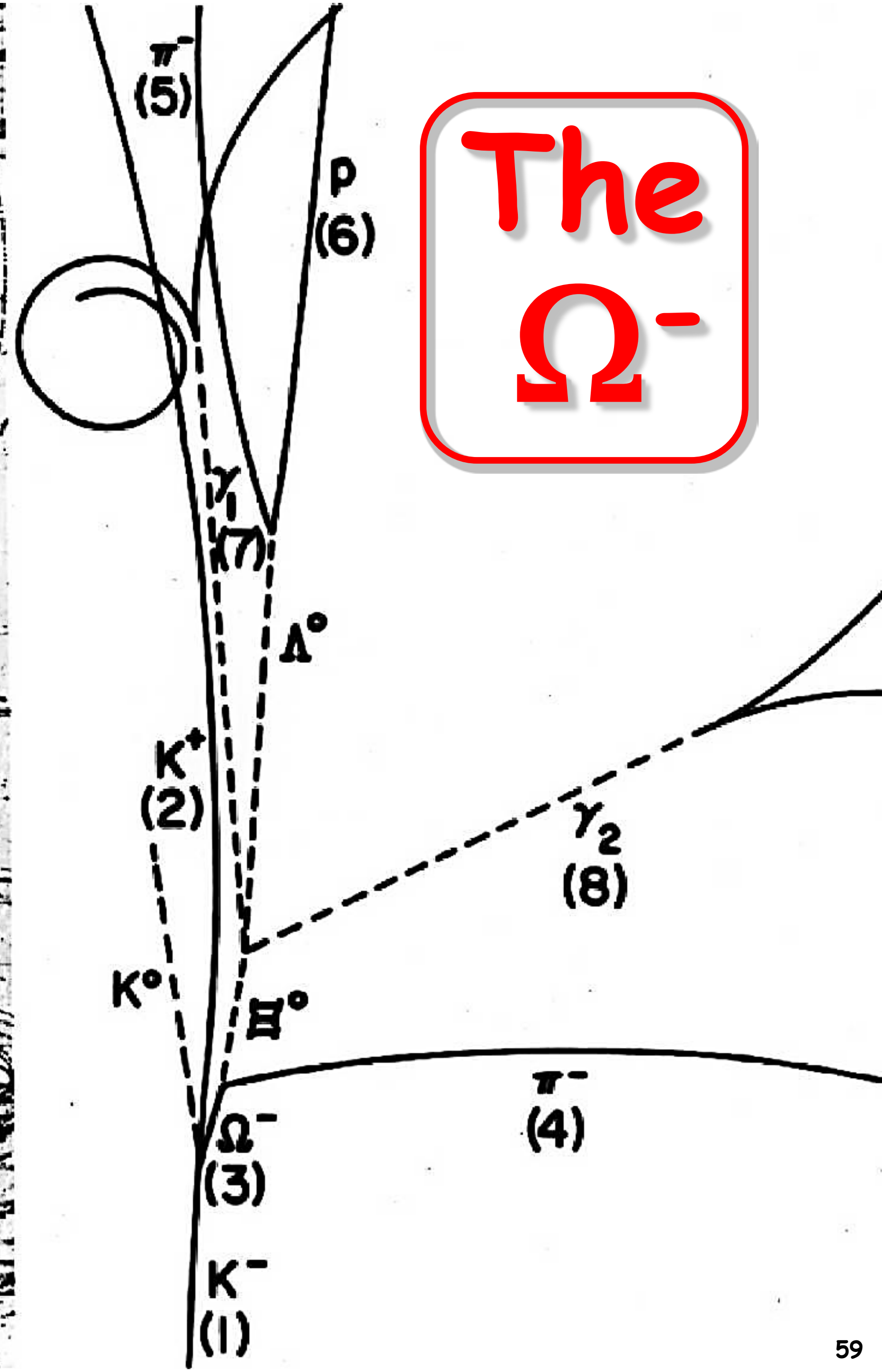
in Brookhaven 80 inch bubble chamber

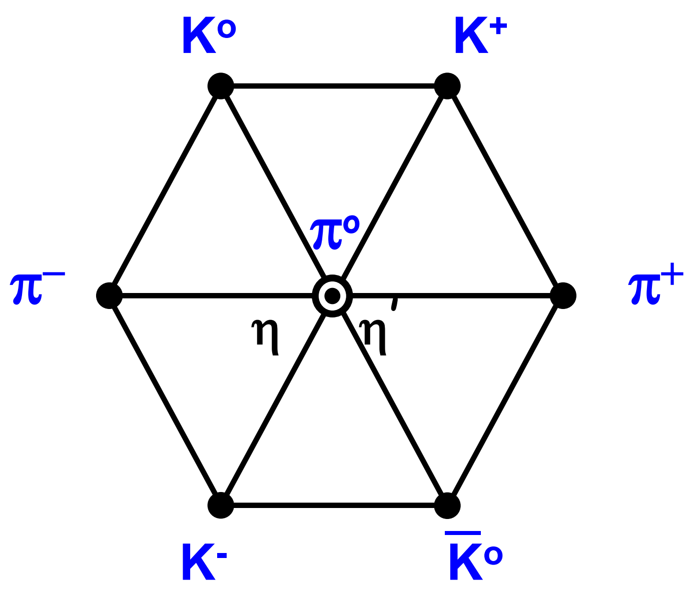
Nobel prizes:

1968 Alvarez for bubble chamber
1969 Gell-Mann for quark model

The Ω^-





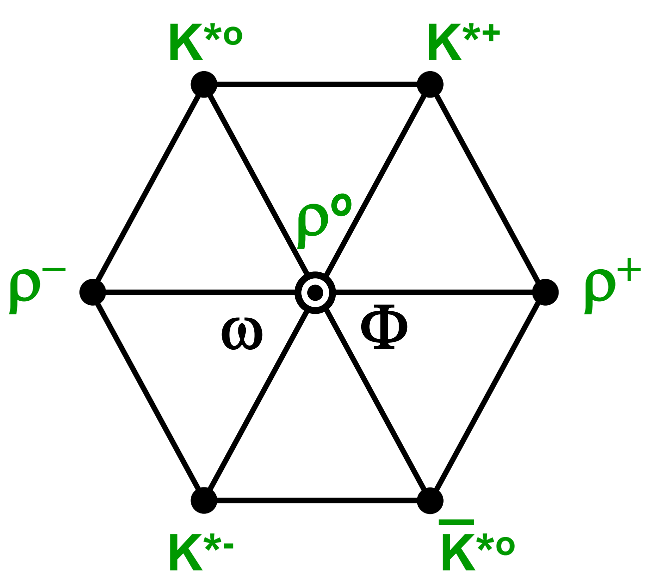


PSEUDO-SCALAR

↑↓ $J^P = 0^-$

weak / elm. decay

The Spin



VECTOR

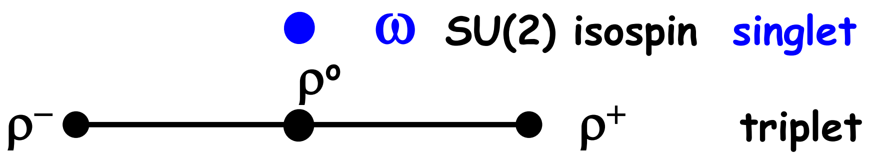
↑↑ $J^P = 1^-$

strong decay

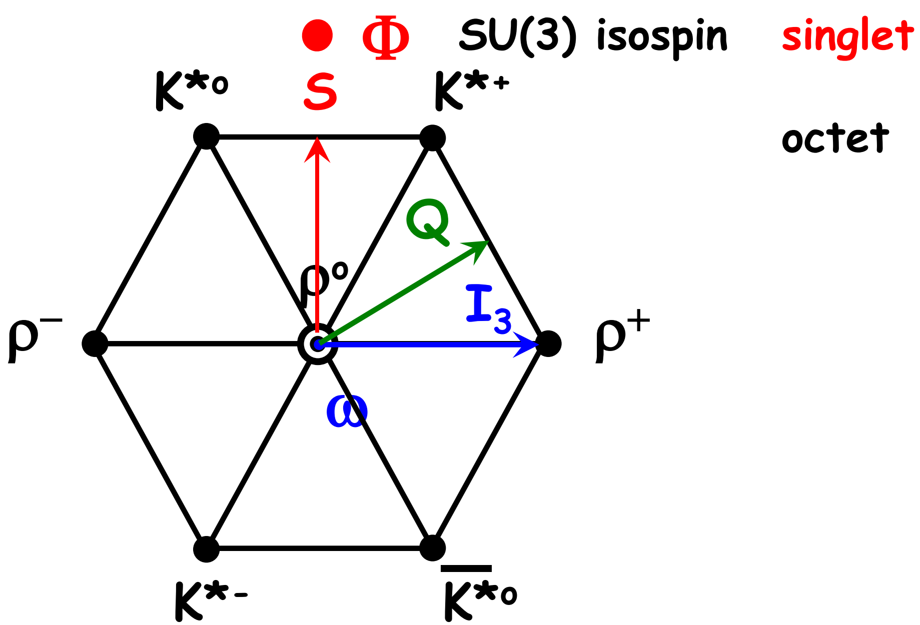
Meson Octet

$= \{q\} \otimes \{\bar{q}\} = \nabla \otimes \Delta$

$= \{2\} \otimes \{\bar{2}\} = \{1\} \oplus \{3\}$ in **SU(2)**



$= \{3\} \otimes \{\bar{3}\} = \{1\} \oplus \{8\}$ in **SU(3)**

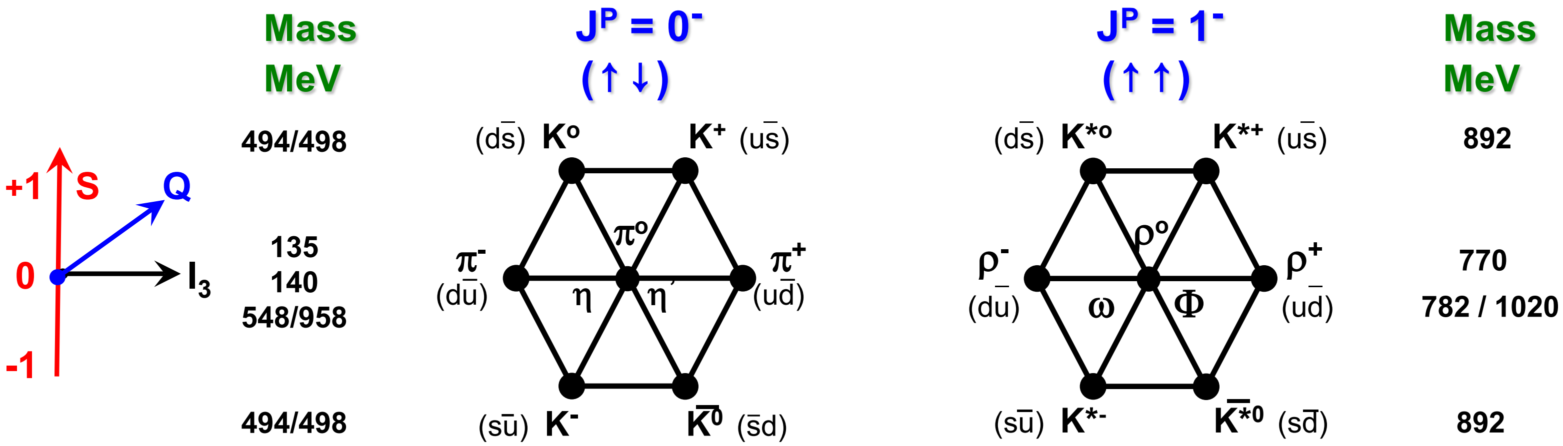


hyper-charge
$Y = B + S$

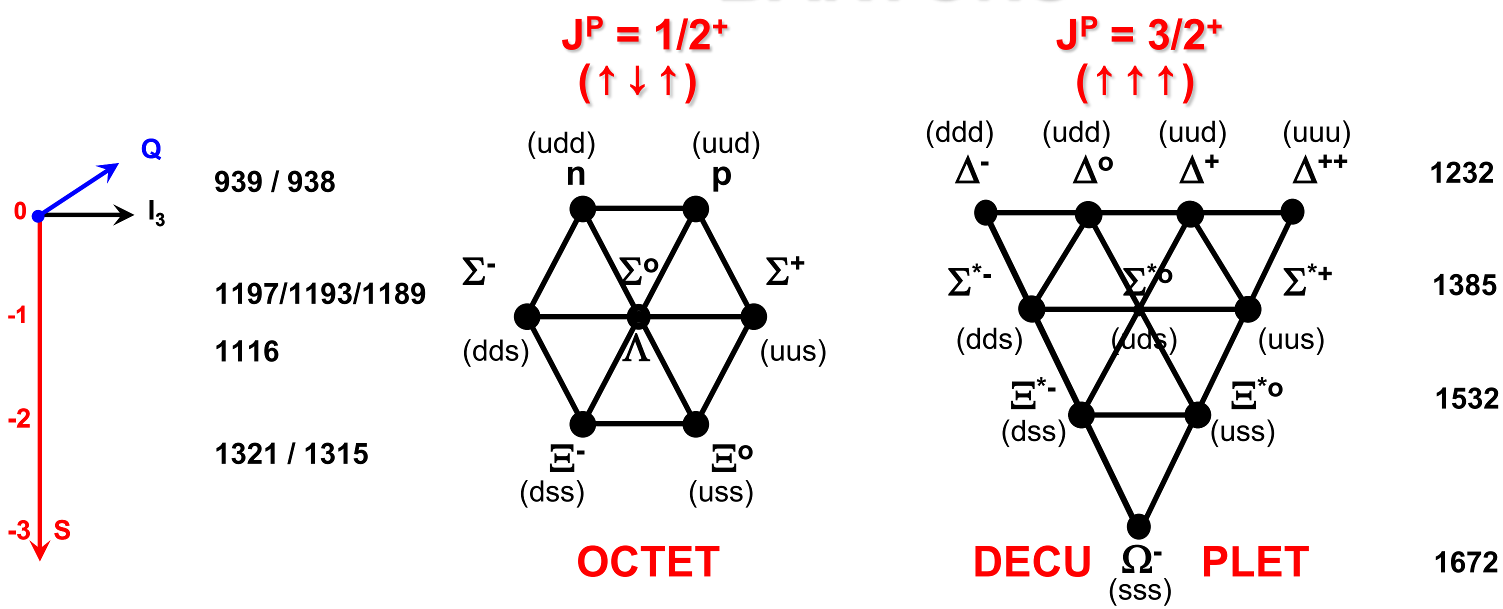
Gell-Mann - Nishijima:
$I_3 = Q - Y/2$

The Quark Model

MESONS



BARYONS

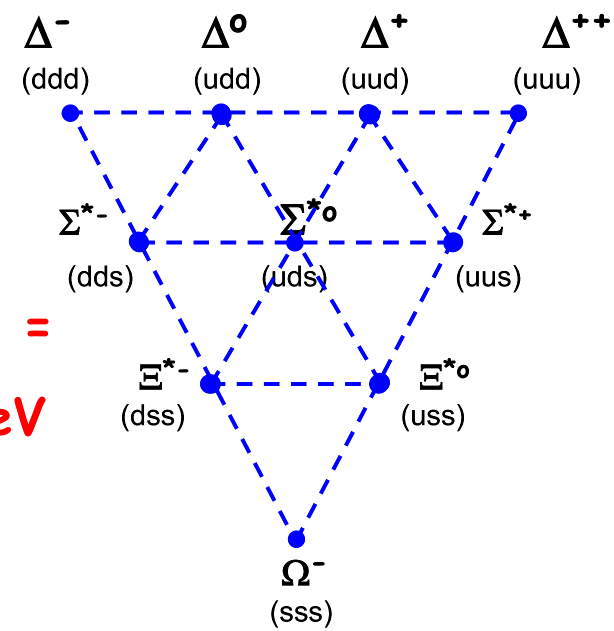


Quark Masses

1. $M_{\Sigma^-} + M_{\Sigma^o} - 2 M_{\Sigma^+} = 3m_d - 3m_u = 11$ MeV

   (dds) + (uds) - 2 (uus) = 3 d - 3 u

   $m_d - m_u \sim$ MeV



2. decuplet equal spacing rule:

   $M_{\Omega} - M_{\Xi^*} = M_{\Xi^*} - M_{\Sigma^*} = M_{\Sigma^*} - M_{\Delta} = m_s - m_d =$

   142 ~ 145 ~ 153 ~ 145 MeV

3. Gell-Mann–Okubo mass relation:

   $3 M_{\Lambda} + M_{\Sigma} = 2 (M_N + M_{\Xi})$

   3 (uds) + (uds) = 2 [(uud) + (dss)]

Hadron Spectroscopy

$SU(N)_F \otimes O(3)$: N flavors ⊗ spatial excitations

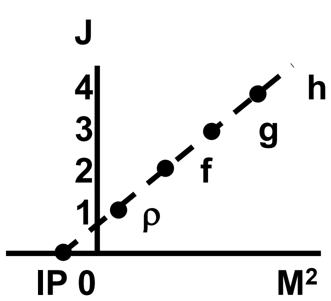
1. S hyperfine splitting: energy of spin flip:

↑↑		↑↓	mass split / GeV
K^*	-	K	~ 0.4
↑↑↑		↑↓↑	
Δ	-	N	~ 0.3
Σ^*	-	Σ	~ 0.2 ~ Ξ^* - Ξ

$M_{\Sigma*} - M_\Sigma = M_{\Xi*} - M_\Xi = M_\Delta - M_N$

196 ~ 214 ~ 294 MeV

2. L orbital momentum: $J = L + S \sim M^2$ Regge theory:



3. N radial excitations:

$\rho' - \rho$ dm ~ 0.5 GeV

heavy quarks: non-relativistic potential: harmonic oscillator:

$\Psi' - \Psi$ dm ~ 0.6 GeV

$\Psi'' - \Psi'$

$\Upsilon' - \Upsilon$

Light Meson Spectroscopy

$n\,^{2s+1}\ell_J$	J^{PC}	I = 1 $ud,\ \bar{u}d,\ \frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s},\ d\bar{s};\ \bar{d}s,\ -\bar{u}s$	I = 0 f'	I = 0 f	$\theta_{\rm quad}$ [°]	$\theta_{\rm lin}$ [°]
$1\,^1S_0$	0^{-+}	π	K	η	$\eta'(958)$	-11.5	-24.6
$1\,^3S_1$	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$	38.7	36.0
$1\,^1P_1$	1^{+-}	$b_1(1235)$	$K_{1B}{}^\dagger$	$h_1(1380)$	$h_1(1170)$		
$1\,^3P_0$	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$		
$1\,^3P_1$	1^{++}	$a_1(1260)$	$K_{1A}{}^\dagger$	$f_1(1420)$	$f_1(1285)$		
$1\,^3P_2$	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$	29.6	28.0
$1\,^1D_2$	2^{-+}	$\pi_2(1670)$	$K_2(1770)^\dagger$	$\eta_2(1870)$	$\eta_2(1645)$		
$1\,^3D_1$	1^{--}	$\rho(1700)$	$K^*(1680)^\ddagger$		$\omega(1650)$		
$1\,^3D_2$	2^{--}		$K_2(1820)^\ddagger$				
$1\,^3D_3$	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$	32.0	31.0
$1\,^3F_4$	4^{++}	$a_4(2040)$	$K_4^*(2045)$		$f_4(2050)$		
$1\,^3G_5$	5^{--}	$\rho_5(2350)$					
$1\,^3H_6$	6^{++}	$a_6(2450)$			$f_6(2510)$		
$2\,^1S_0$	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$	$\eta(1295)$	-22.4	-22.6
$2\,^3S_1$	1^{--}	$\rho(1450)$	$K^*(1410)^\ddagger$	$\phi(1680)$	$\omega(1420)$		

Heavy Quarks

flavor =

property to be up, down, strange, … quark

N flavors

SU(N) flavor space =

unitary N-dim. complex vector space

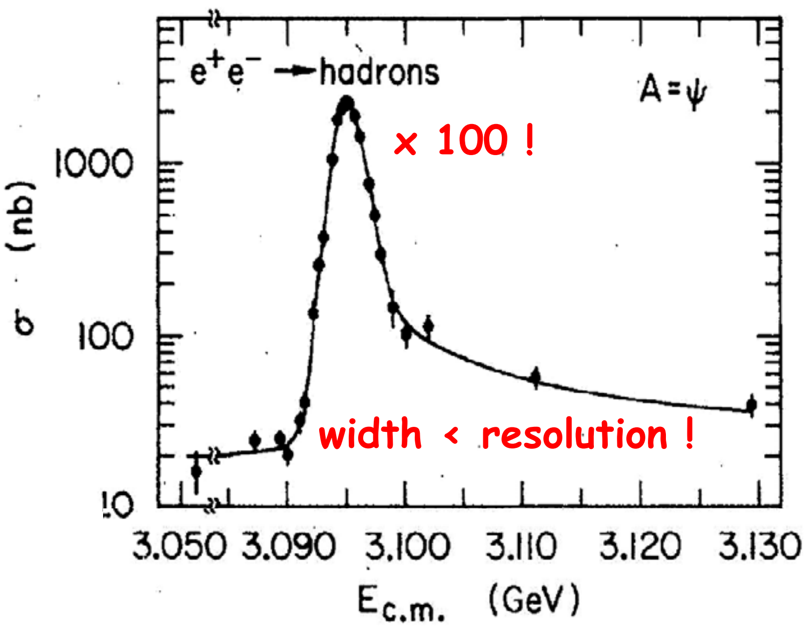
from SU(3) to SU(N):

Charm

B. Richter

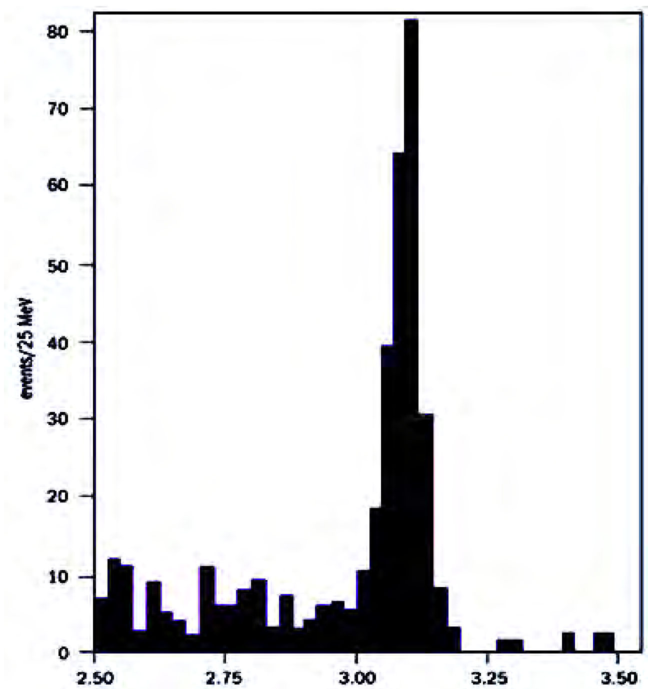
discovered
1974

S. Ting



Nobel

Prize
1976



$e^+ e^- \rightarrow \Psi \rightarrow$ hadrons

3.1 GeV

MARK-I @ SPEAR

Stanford Linear Accelerator Lab.

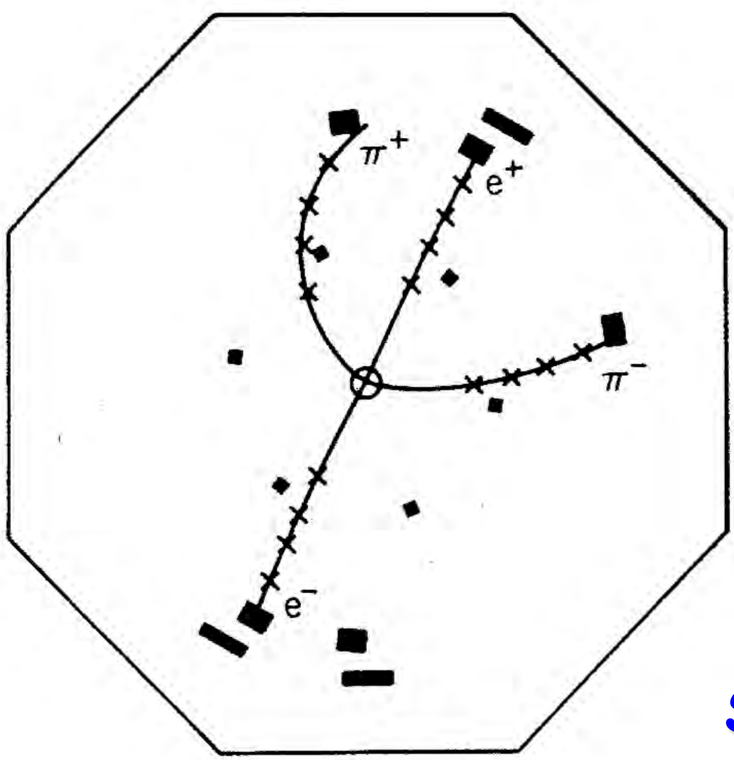
$p\,Be \rightarrow J\,X \rightarrow e^+ e^- X$

28.5 GeV

double arm spectrometer @ AGS

Brookhaven National Lab.

Charmonium



$e^+e^- \rightarrow \Psi' \rightarrow \Psi\, \pi^+\, \pi^-$

$\quad\quad\quad\quad\quad\quad\quad\quad\;\, \hookrightarrow e^+\, e^-$

m_Ψ = 3.1 GeV

m_c = 1.5 GeV

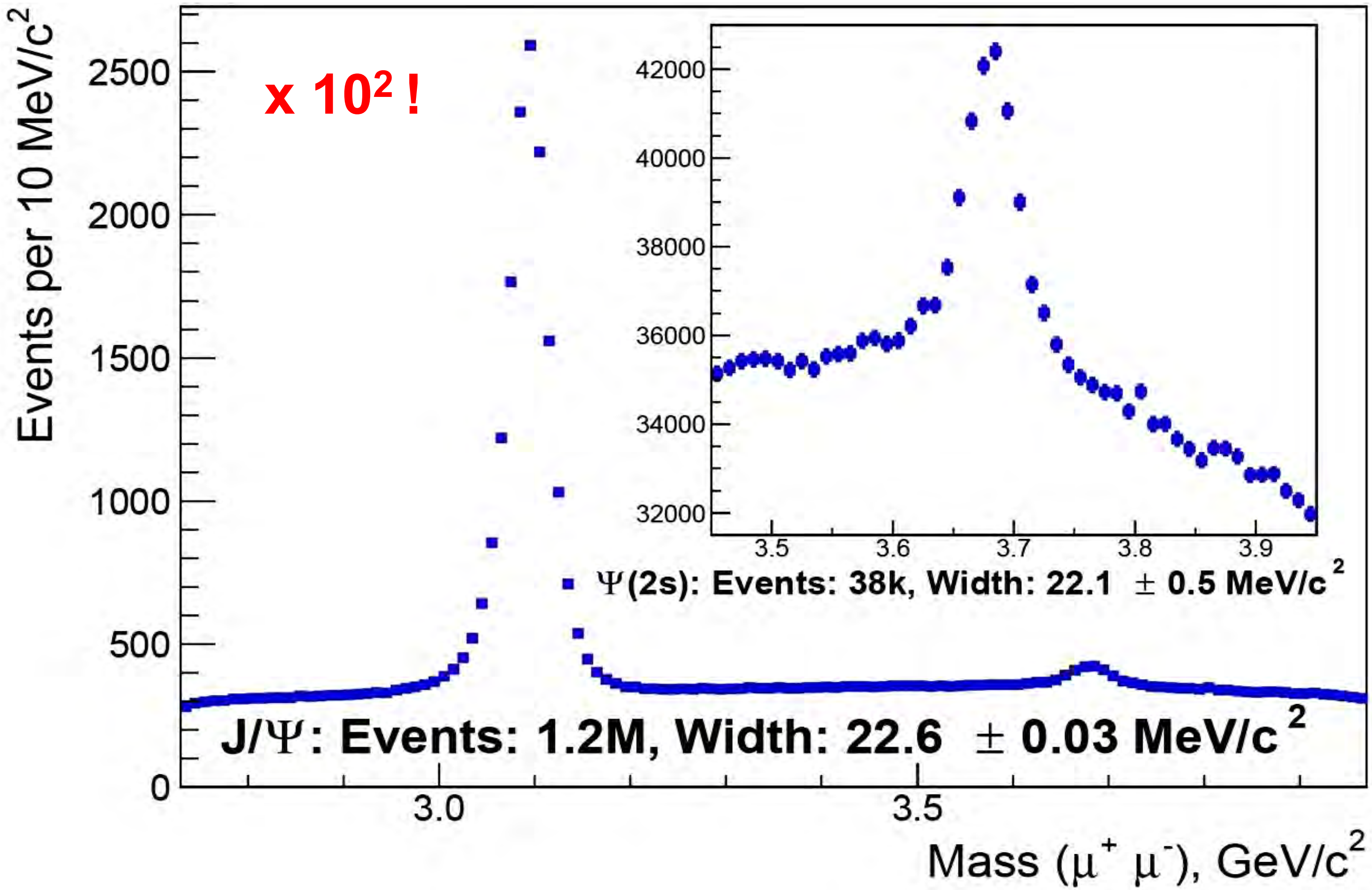
MARK-I @ SPEAR 1975

Stanford Linear Accelerator Lab.

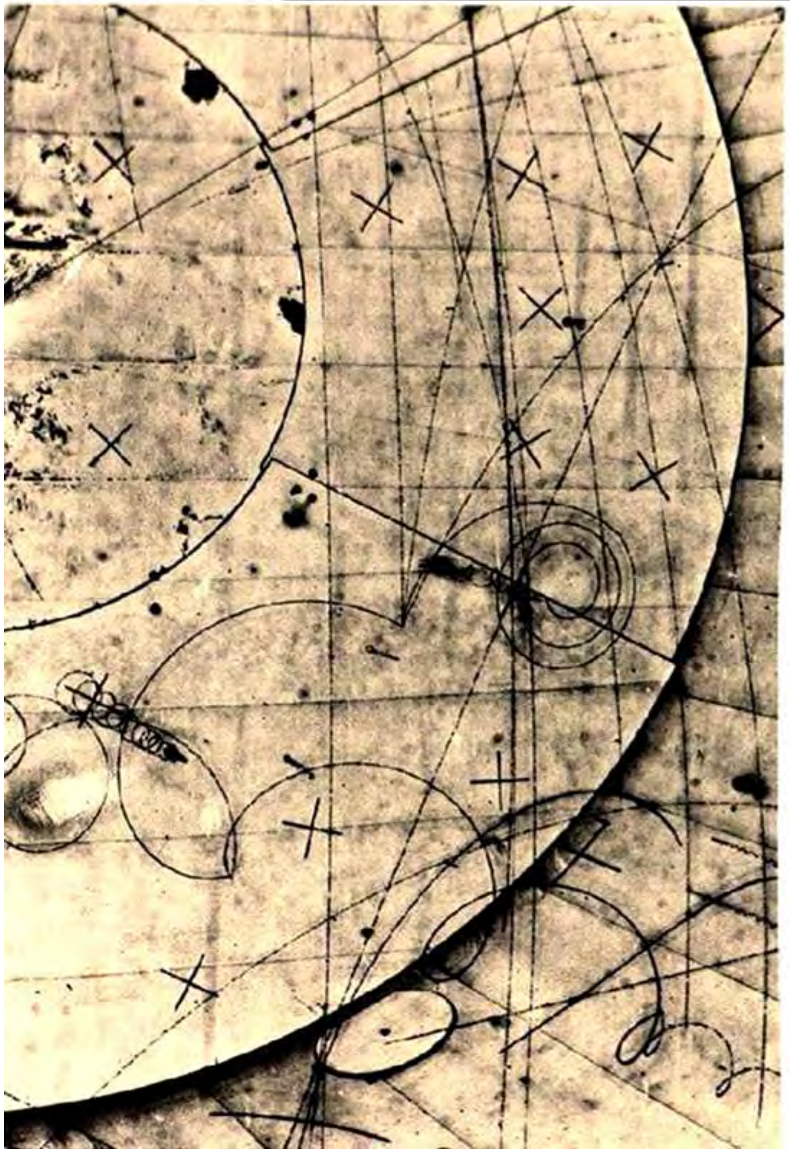
Charm

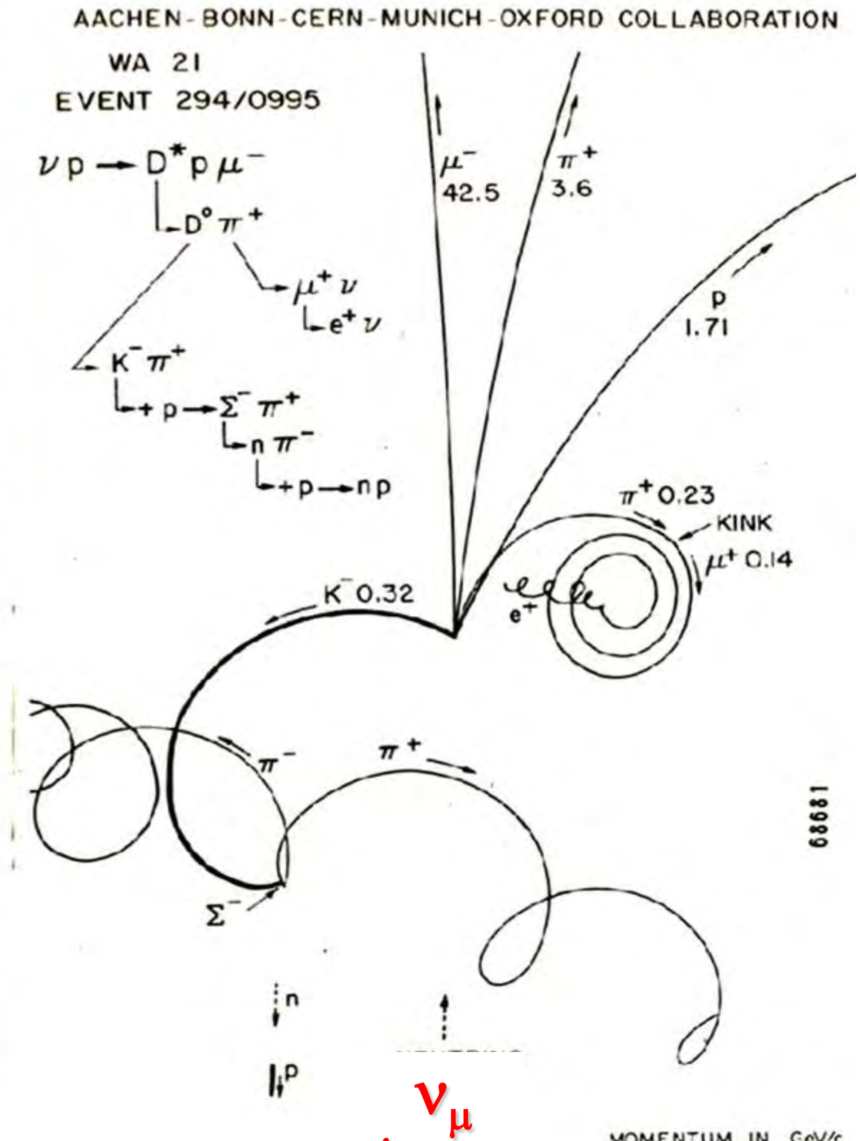
BESIII e^+e^-, Beijing 2012: 500 Ψ(1S) / s

1.3 billion Ψ(1S) + 0.6 billion Ψ(2S)



Open Charm Decay





ν_{μ} beam

production flavor violating = weak

Open Charm Decay

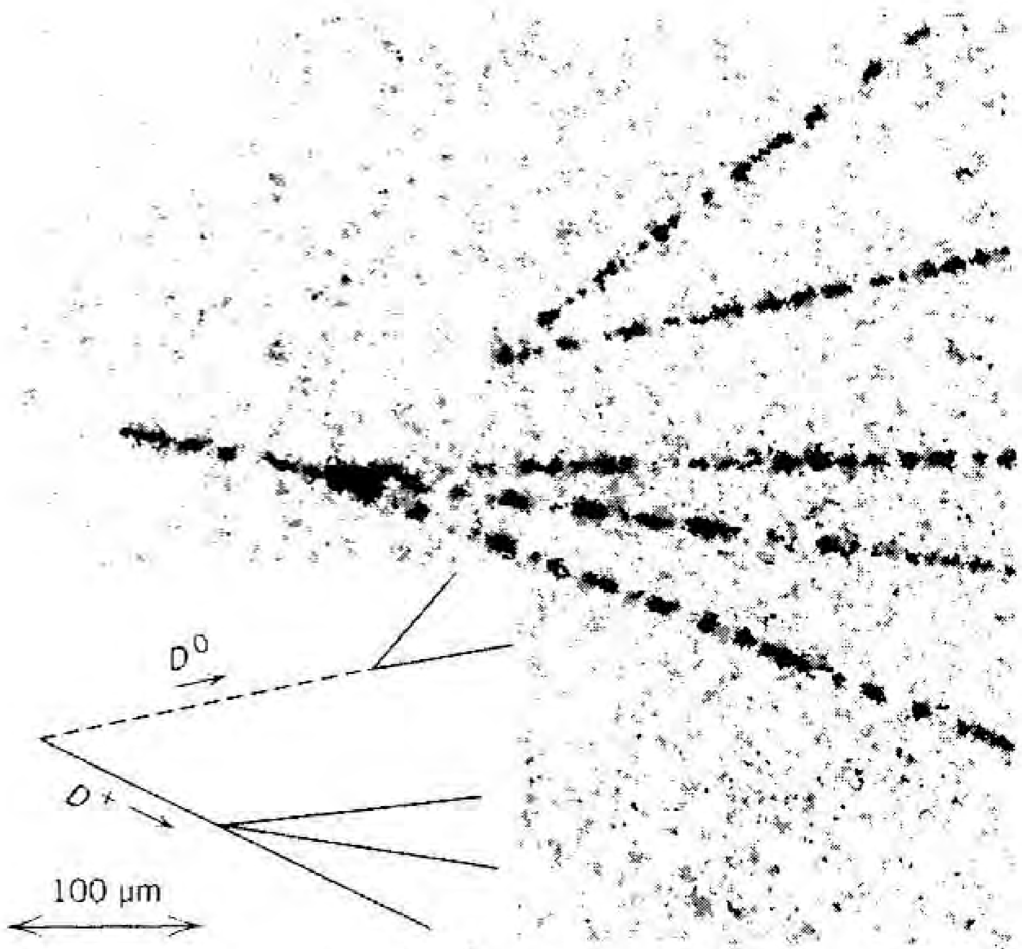


FIGURE 18-11 Example of *D* meson decays observed in a bubble chamber.
From K. Abe et al., *Phys. Rev. Lett.* **48**, 1526 (1982).

$D^0 \rightarrow K^- \; \pi^+$

τ = 0.4 ps

$c\tau$ = 123 μm

$D^+ \rightarrow K^+ \, \pi^+ \, \pi^-$

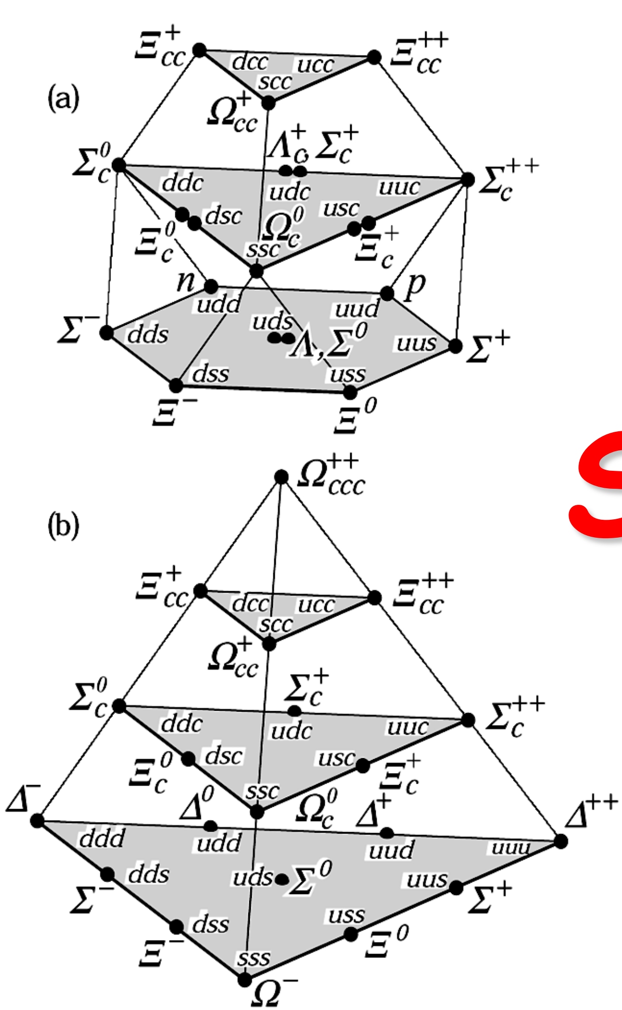
τ = 1.0 ps

$c\tau$ = 315 μm

Charm Spectroscopy

J=1/2
↑ ↓ ↑

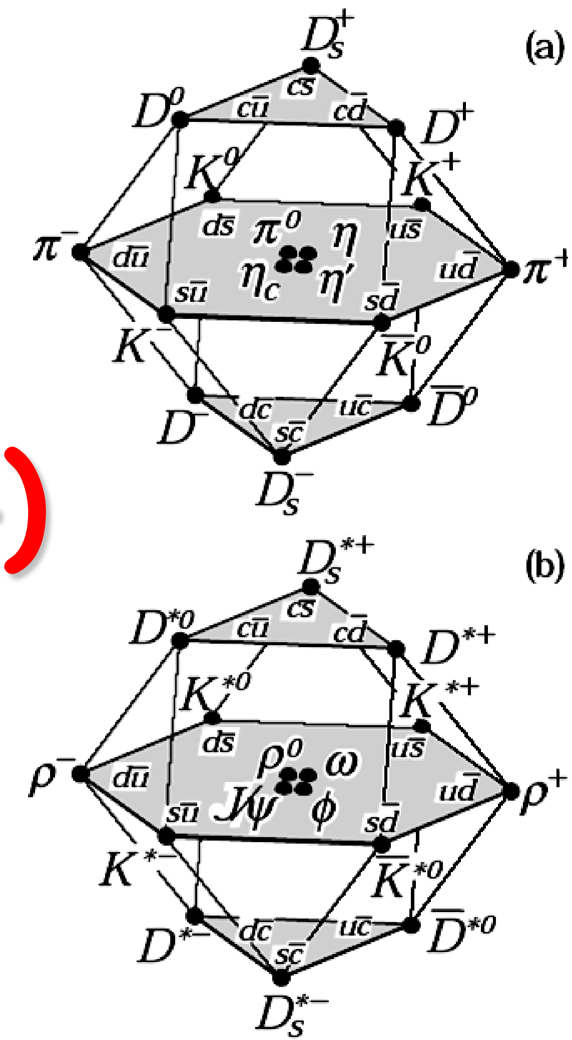
J=3/2
↑ ↑ ↑



SU(4)

J=0
↑ ↓

J=1
↑ ↑



Baryons

Mesons

Beauty

discovered 1977 by

L. Lederman et al.

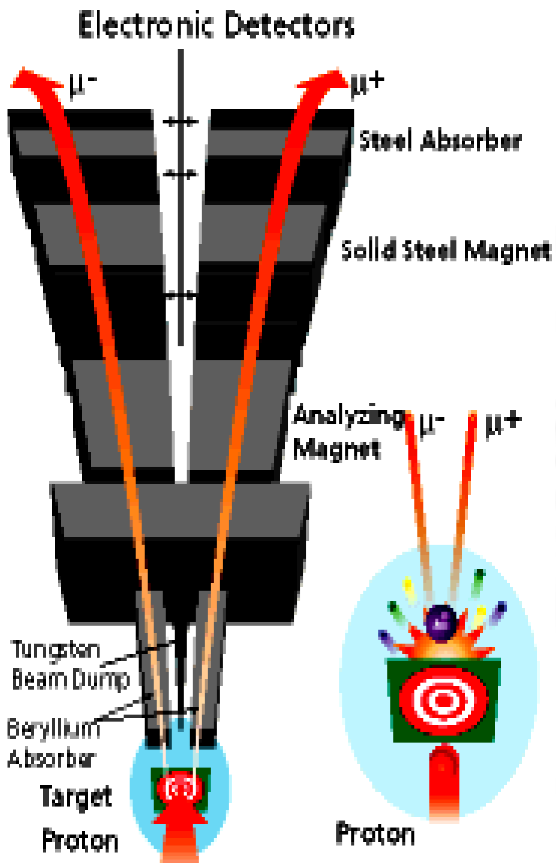
p N→ $\mu^+\mu^-$ X

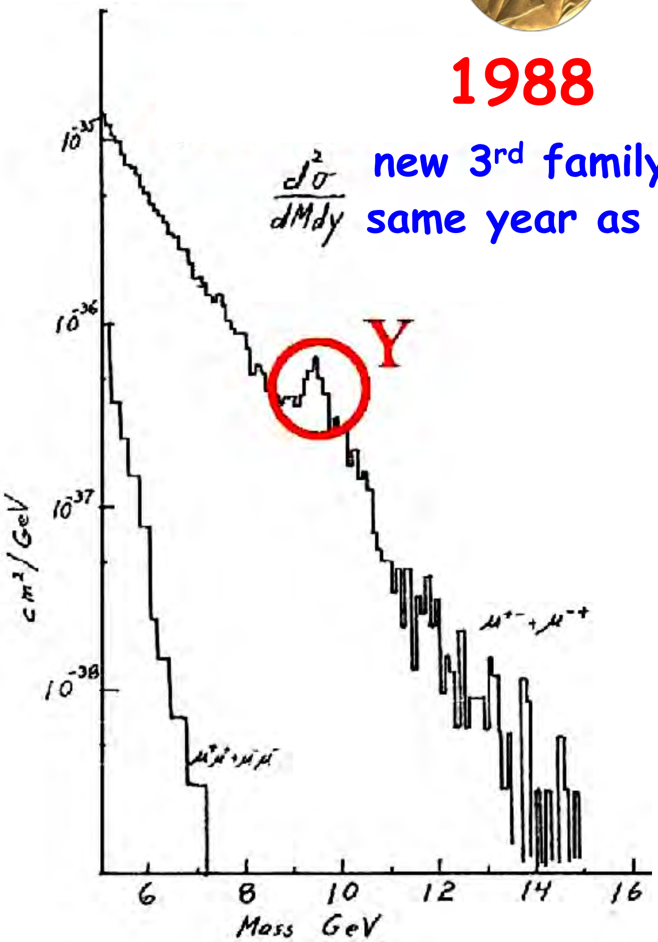
Fermilab, USA

Nobel  prize

1988

L.Lederman





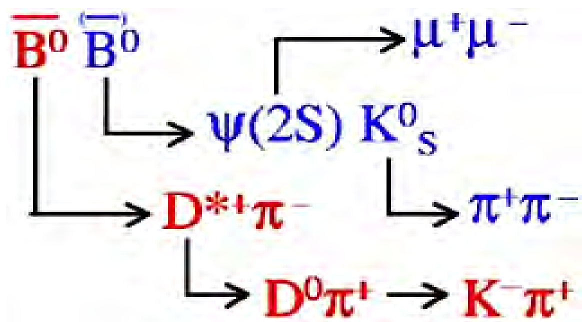
new 3rd family

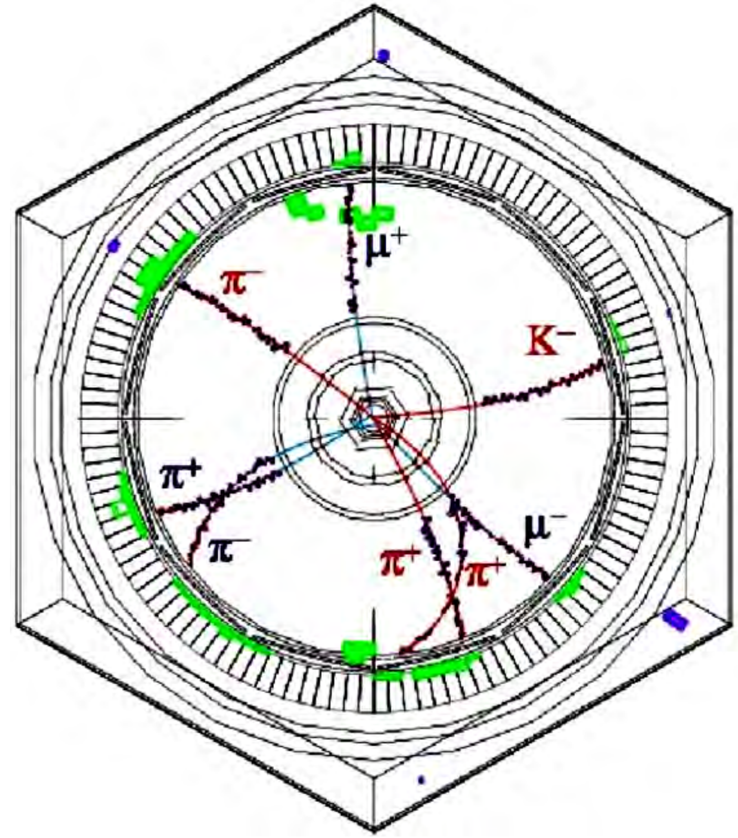
same year as τ !

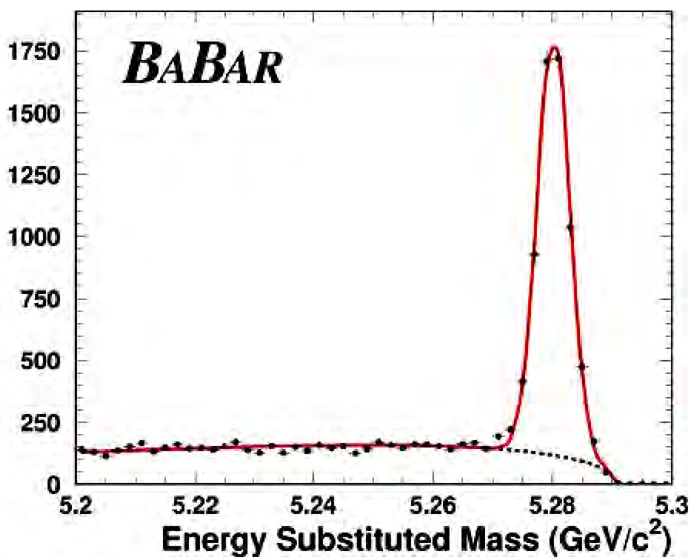
http://link.aps.org/doi/10.1103/PhysRevLett.39.252

Bottom quark

BaBar @ PEP, SLAC: $e^+e^- \to \Upsilon(4S, \bar{b}b) \to$





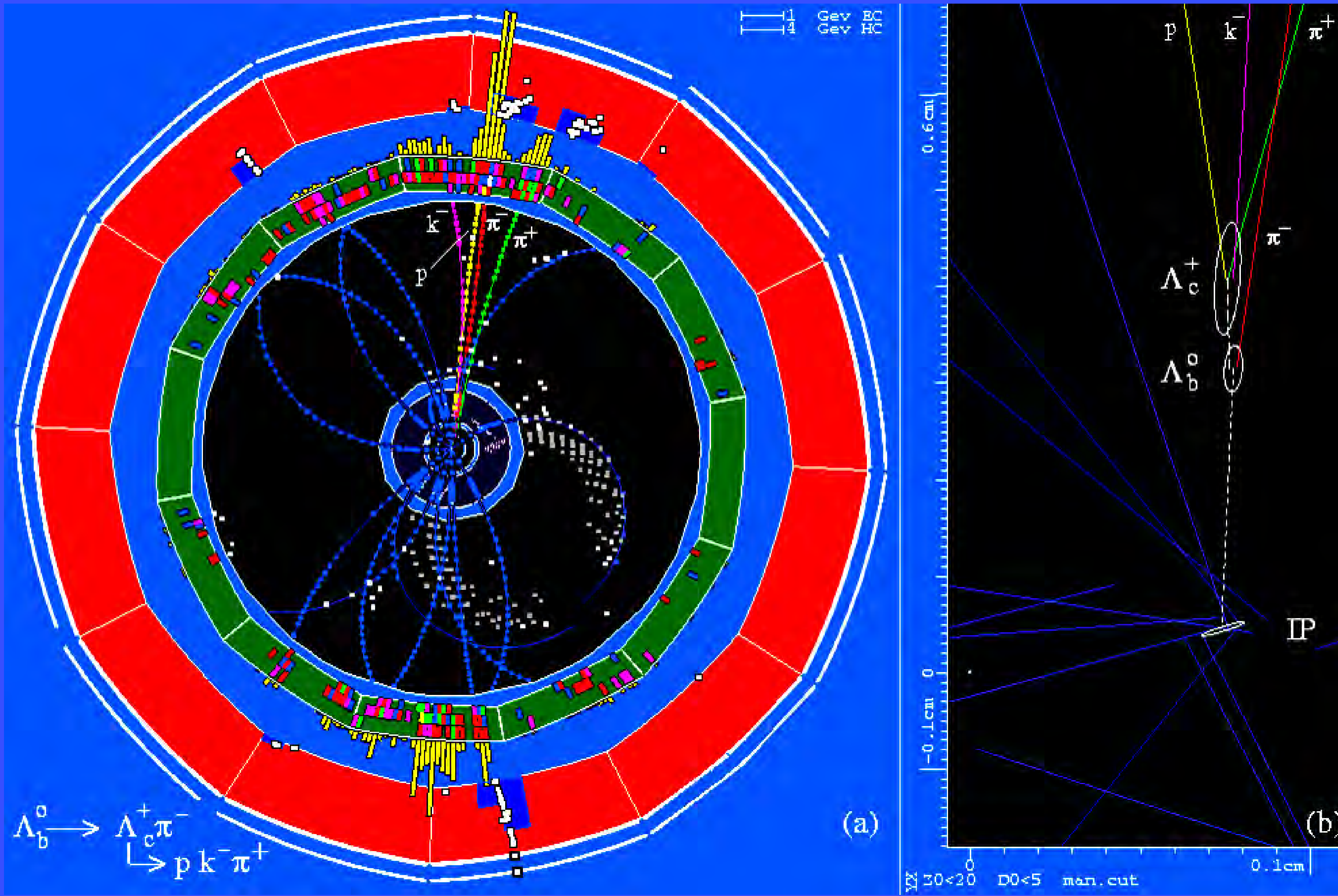


BaBar @ PEP, SLAC, Stanford, USA. 553 fb^{-1} 99-08
Belle @ KEK, Tsukuba, Japan: 1000 fb^{-1} 01-10
Belle2 @ KEK : 50 ab^{-1} 18-

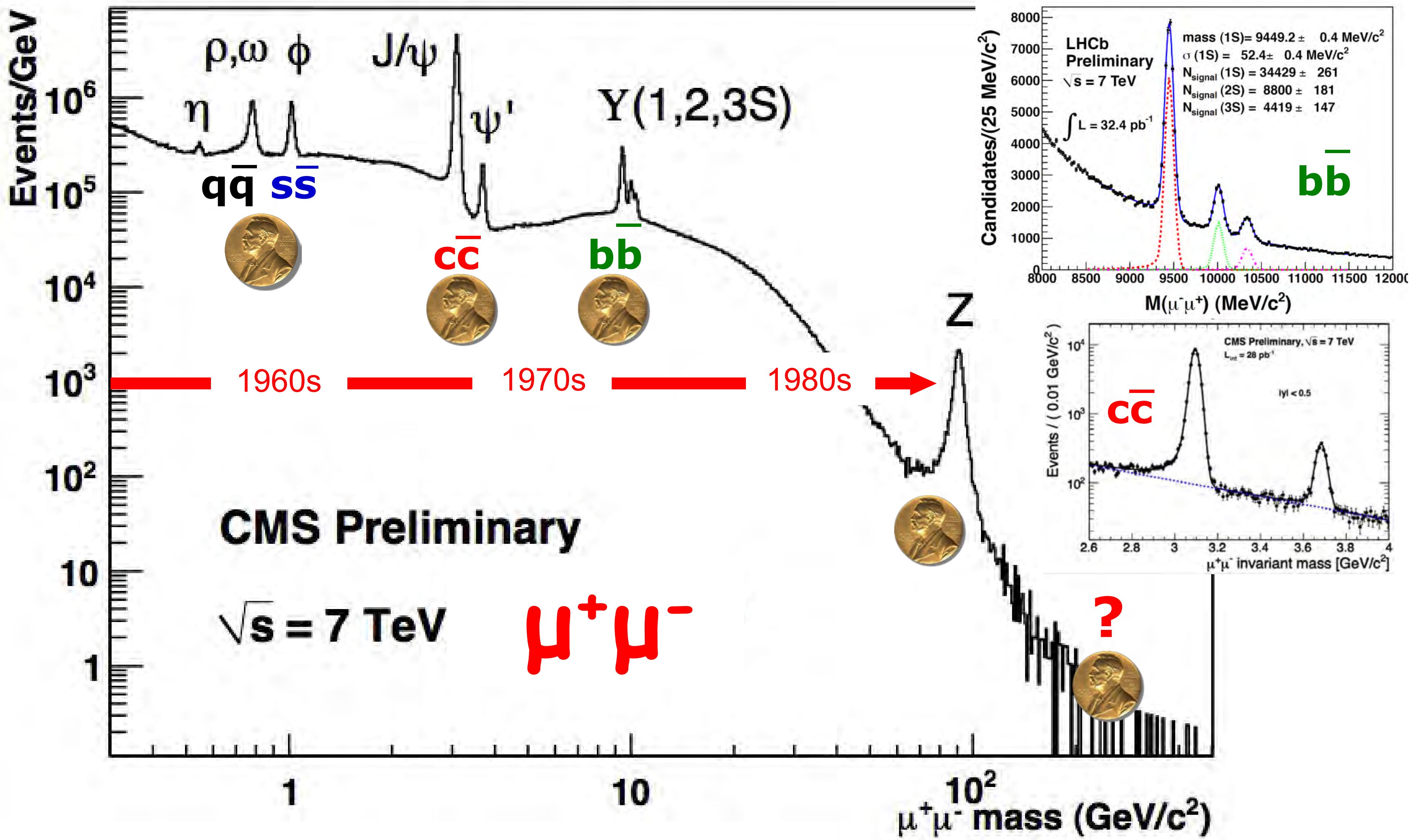
1.3 billion $\bar{B}B$ pairs in 1.5 ab^{-1} since 2001

LHCb@CERN: produce 10^6 $\bar{B}B$ pairs/s, $6 \cdot 10^{11}$ detected

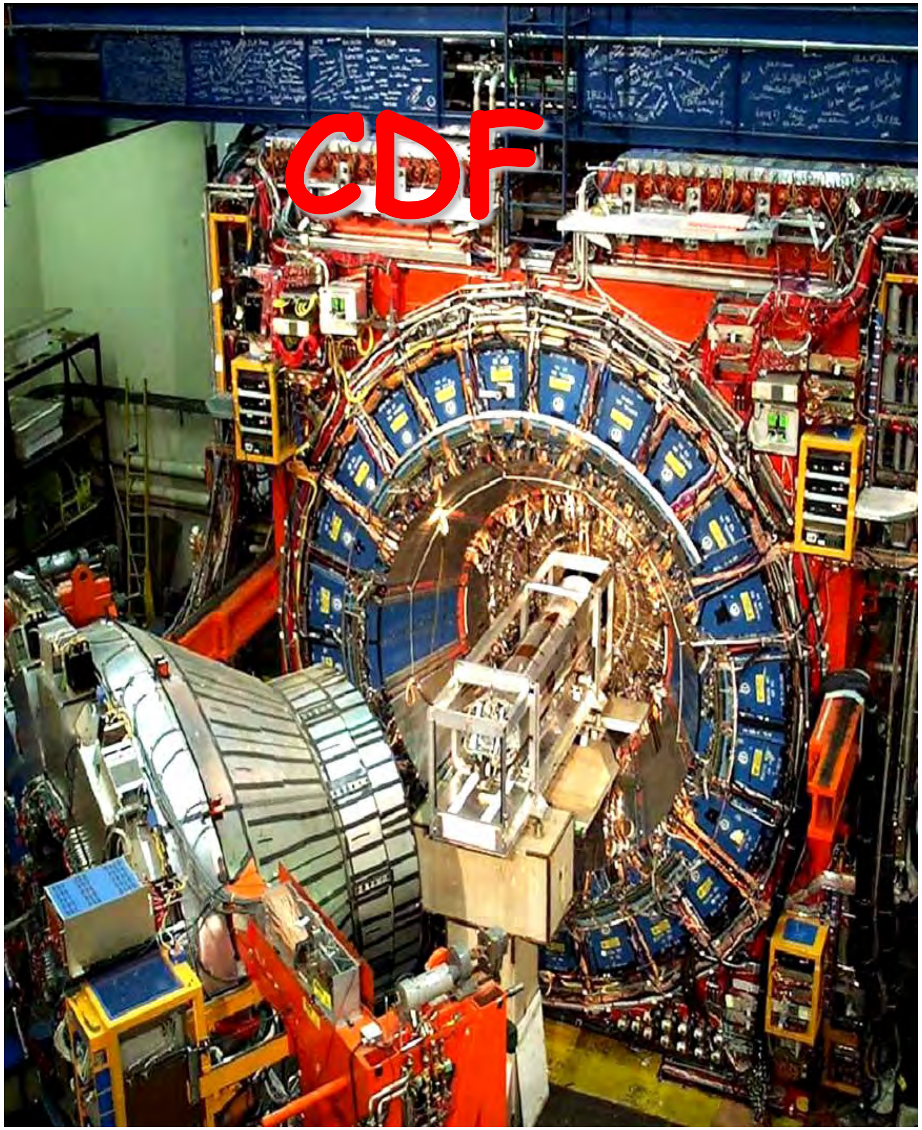
Heavy Quark Decays

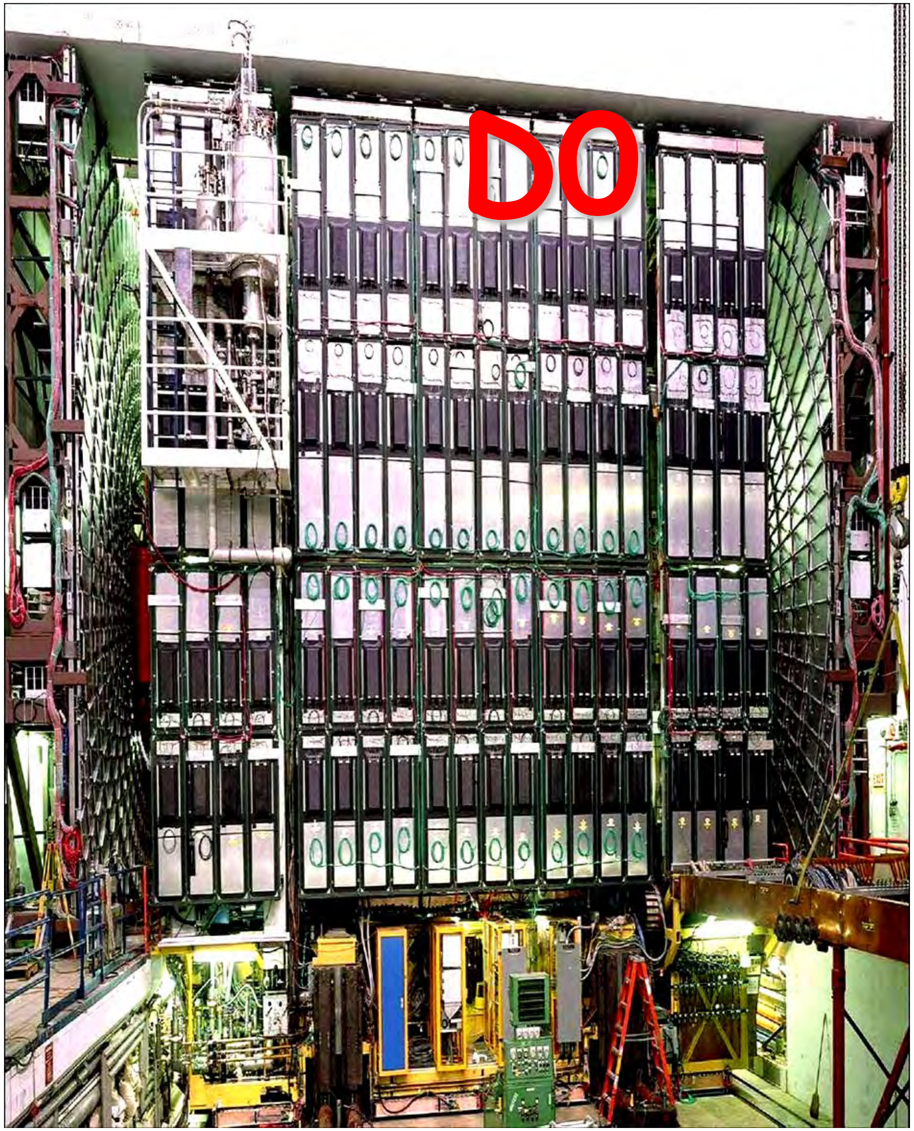


quarkonium & vector mesons



Tevatron



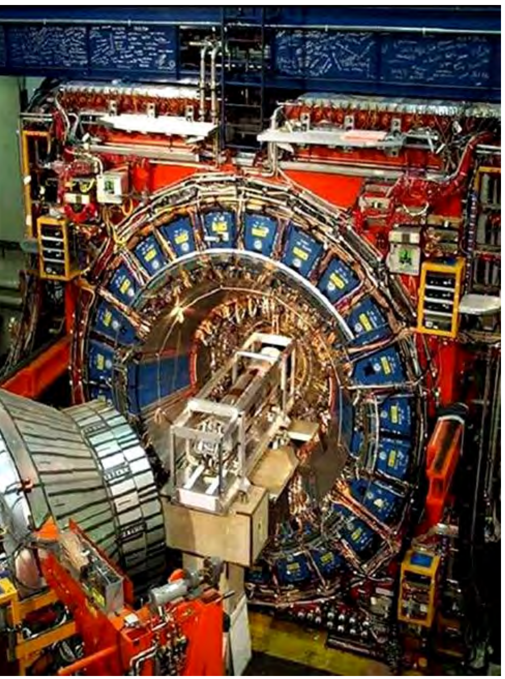


Fermi National Lab., Chicago: $\bar{p}\,p$ @ 1x1 TeV

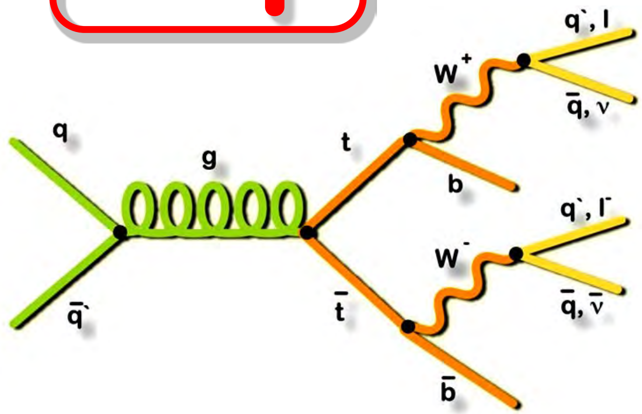
Top

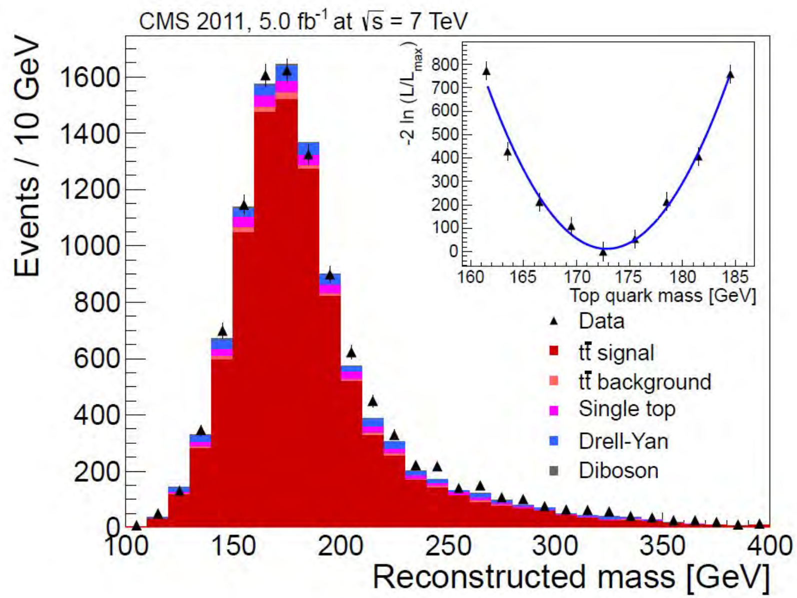
Phys.Rev.Lett. 74 (1995) 2626+2632

discovered
1995
CDF+D0



Tevatron
$\bar{p}$ p @ 1x1 TeV
Fermi Natl. Lab.
Chicago, USA





find 1 top in
10^{10} hadronic interactions !

as heavy as a
Gold atom:

m_t =
173.3 ± 0.8 GeV

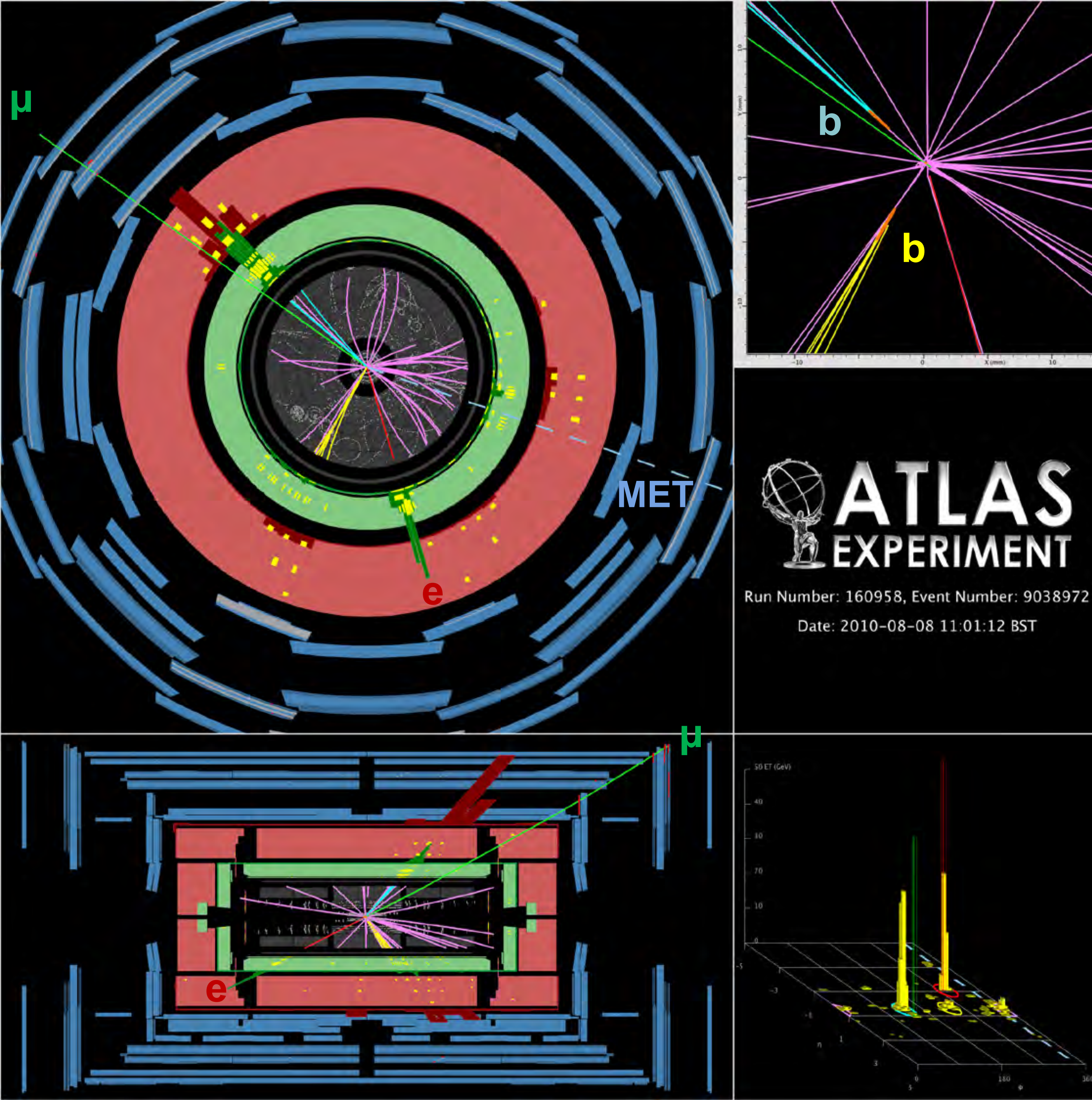
arXiv:1403.4427

width: 1.3 GeV
expt.: <2 GeV

weak decay
within $3 \cdot 10^{-25}$ s -

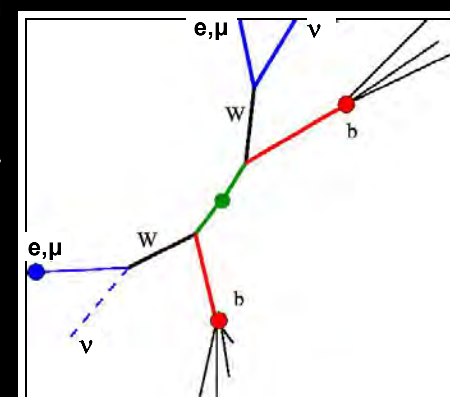
too fast
to find a partner -

top quark
never becomes
a dressed hadron !



LHC: top factory:

$\bar{t}\,t \rightarrow$ e + μ 2 b-jets



SUSY like

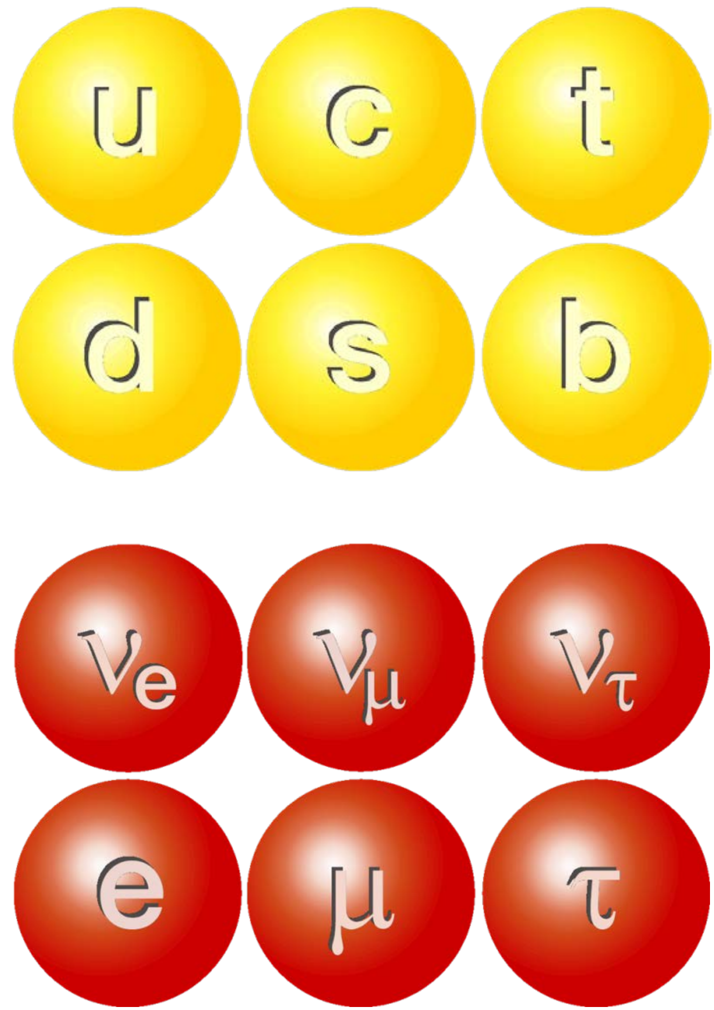
The Building Blocks

Great scheme,

BUT:

all 3 symmetries mysterious:

- up-down ((weak) isospin)
- lepton-quark
- 3 families



Fermions

QUARKS			Q	I_3	B	L
u d	c s	t b	+2/3 -1/3	+1/2 - 1/2	1/3	0
LEPTONS						
e^- ν_e	μ^- ν_μ	τ^- ν_τ	-1 0	- 1/2 +1/2	0	1

Spin
J=1/2

Questions:

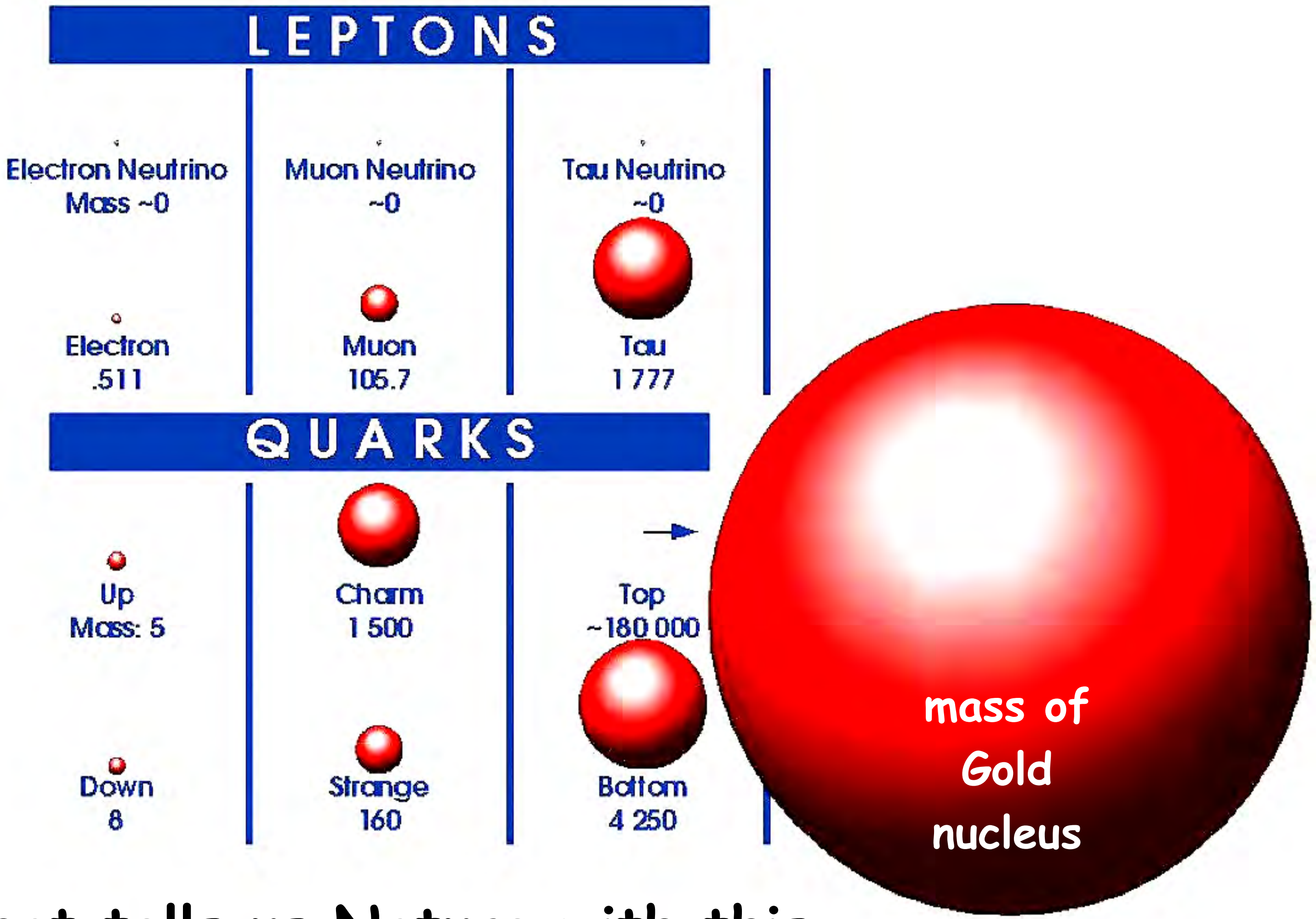
- why no free quarks ?
  => confinement, QCD
- fractional charge 1/3 ?

Masses

u	c	t	e^-	μ^-	τ^-
2-5 MeV	1.25 GeV	173 GeV	511 keV	106 MeV	1.78 GeV
d	s	b	ν_e	ν_μ	ν_τ
5-8 MeV	120 MeV	4.25 GeV	<2 eV	<190 keV	<18.2 MeV

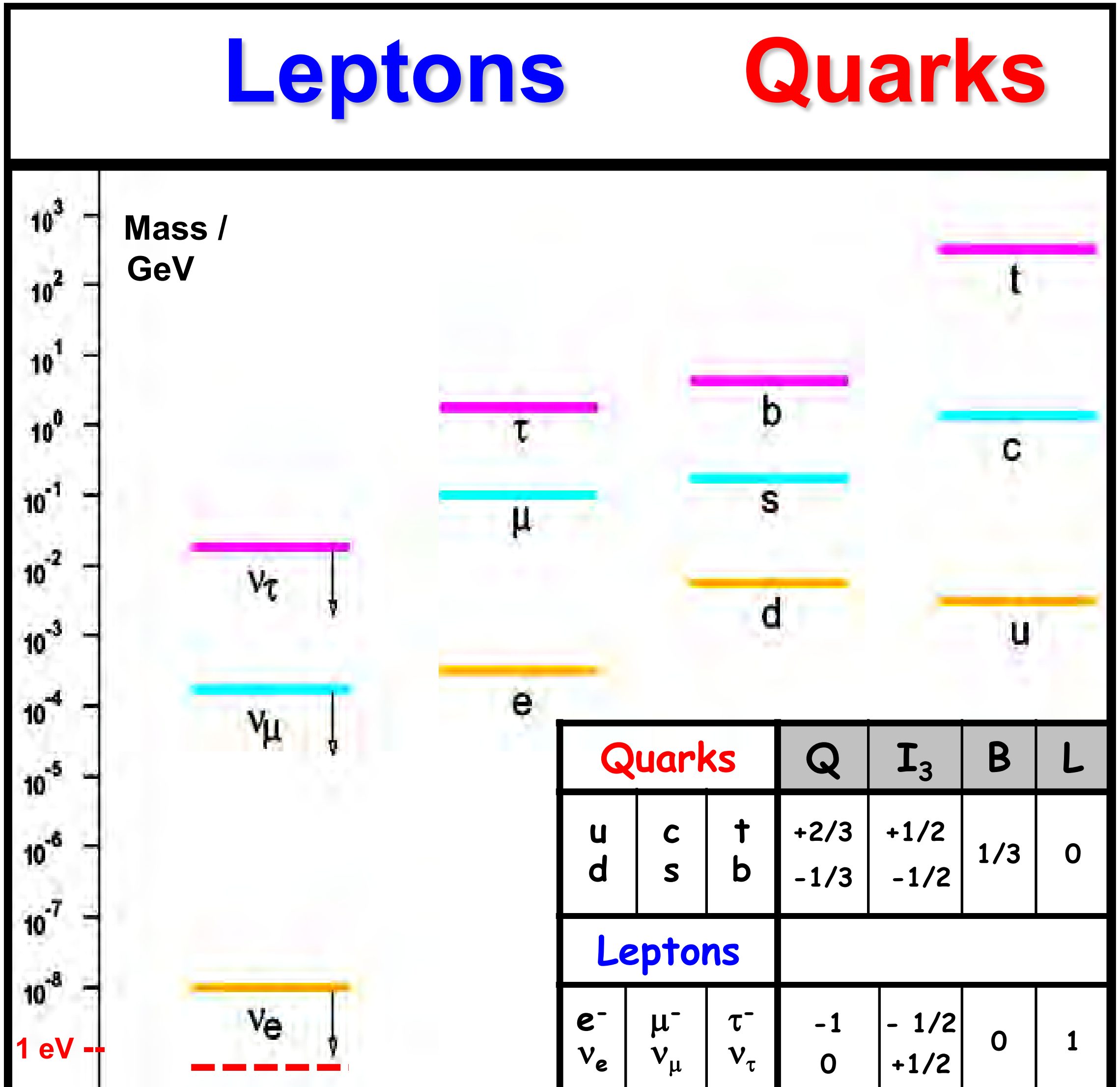
PARTICLES					FORCES			
					Electro-Magnet.	Weak	Nuclear	Gravi-tation
	Charge				Electric	Weak	Color	Mass
	Symmetry				U(1)	SU(2)	SU(3)	
Matter Particles	Fermions J=1/2							
Quarks	Up	u	c	t	+2/3	I_W, Y_W	r g b	
	Down	d	s	b	-1/3			
Leptons	Electrons	e	μ	τ	-1	I_W, Y_W		
	Neutrinos	ν_e	ν_μ	ν_τ	0			
Force Particles	Bosons J=1							
Photon		γ						
Weak Bosons		W^+, Z^0, W^-						
Gluons		$8\ g_{ij}$						
Graviton (J=2)		G						

Fermion Mass Spectrum



What tells us Nature with this
new spectroscopy ?

Fermion Mass Spectrum



Quarks			Q	I_3	B	L
u d	c s	t b	+2/3 -1/3	+1/2 -1/2	1/3	0
Leptons						
e^- ν_e	μ^- ν_μ	τ^- ν_τ	-1 0	- 1/2 +1/2	0	1

PLANCK, KATRIN, $0\nu2\beta$

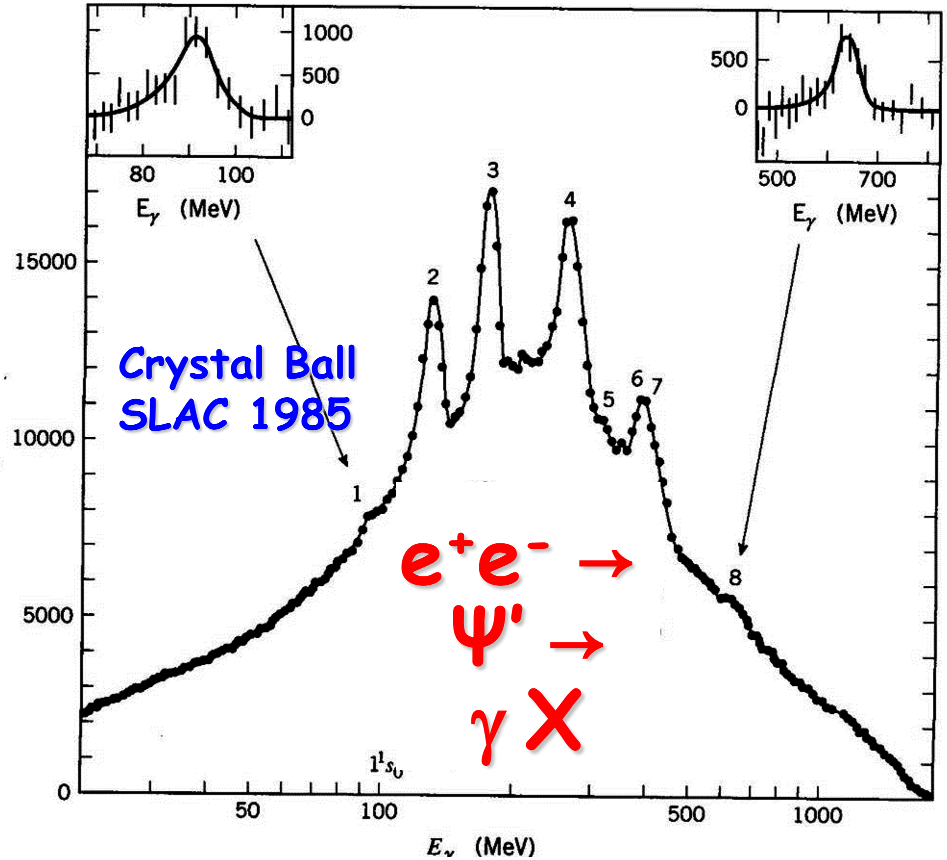
What tells us **Nature** with this **spectrum** ?

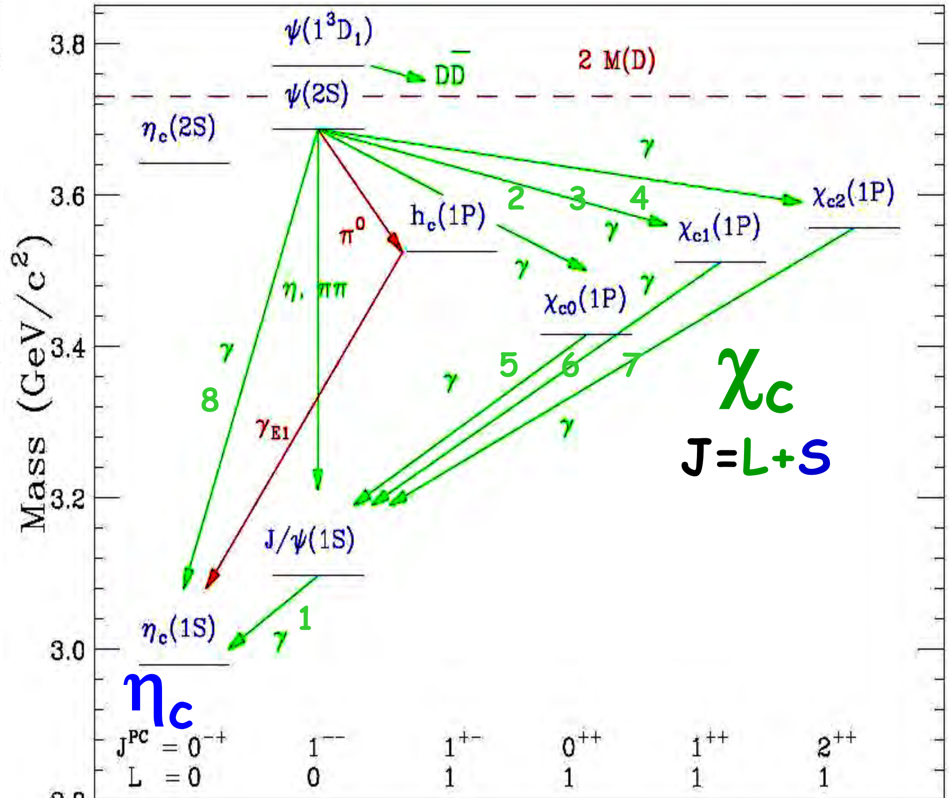
IF
$m_d < m_u$ =>
$m_n < m_p$ =>
p decay =>
neutral universe

Hadron

spectroscopy
+
mixing

Charmonium Spectrum





$N^{2S+1} L_{J=L+S}$

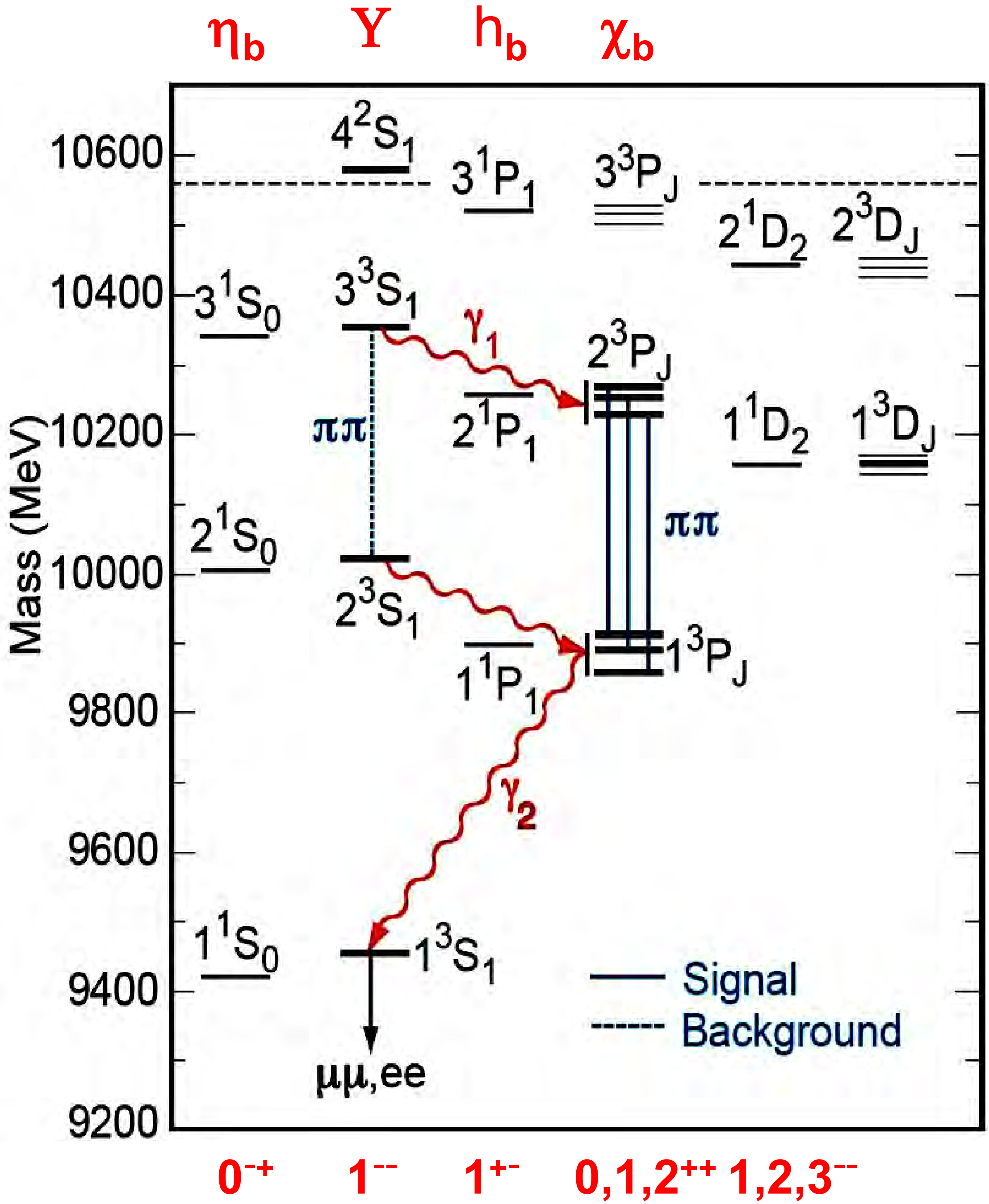
- input state fixed by Ψ', measure γ in NaI (Tl) crystals =>
- missing mass spectrum X
- $\Psi' \nrightarrow \Psi\ \gamma$: J^{PC} : $1^{--} \nrightarrow 1^{--}\ 1^{--}$ C violated
- $^1S_0$ and ^3P states η_c and χ_c only in decays, not in e^+e^-

Ψ-Ψ'	radial	N
Ψ-η_c	↑↑- ↑↓	S
Ψ-χ_c	orbital	L

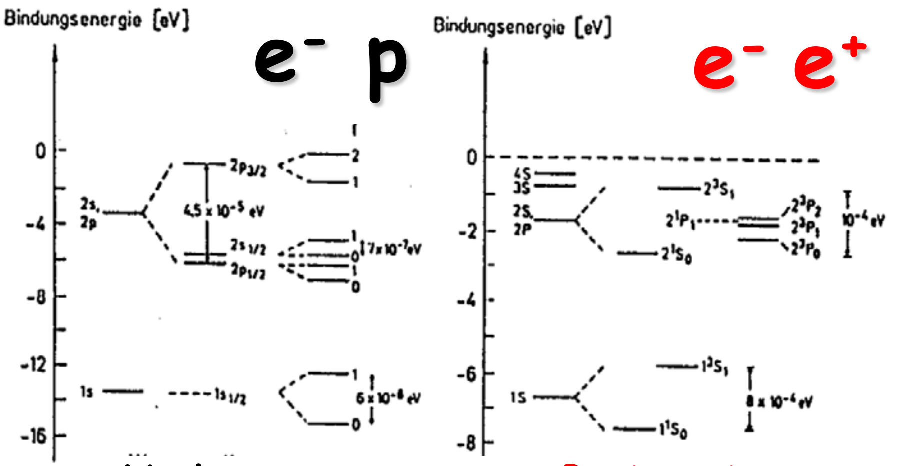
Bottonium

$N^{2S+1}L_{J=L+S}$

Υ'- Υ	radial	N
Υ - η	↑↑ - ↑↓	S
Υ - χ	orbital	L

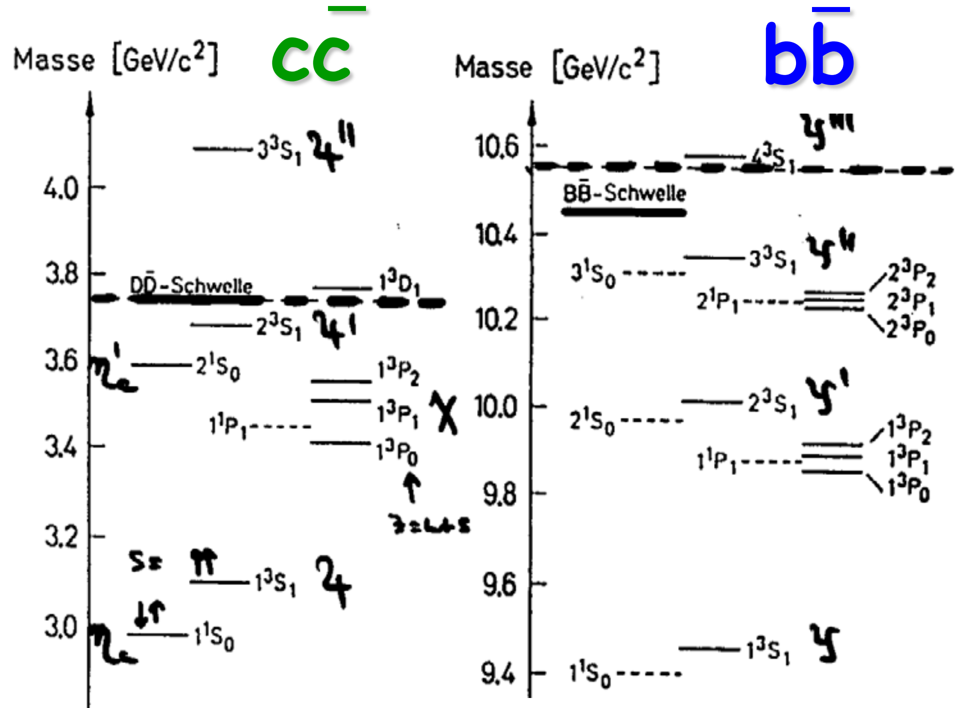


Quarkonium

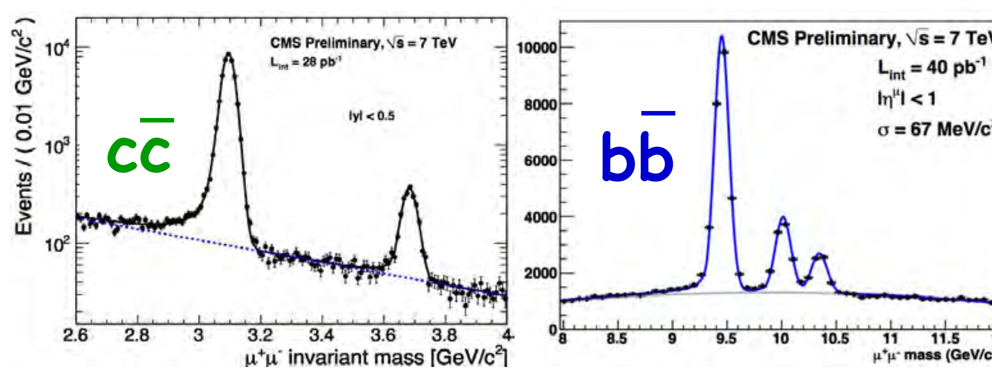


Hydrogen Positronium

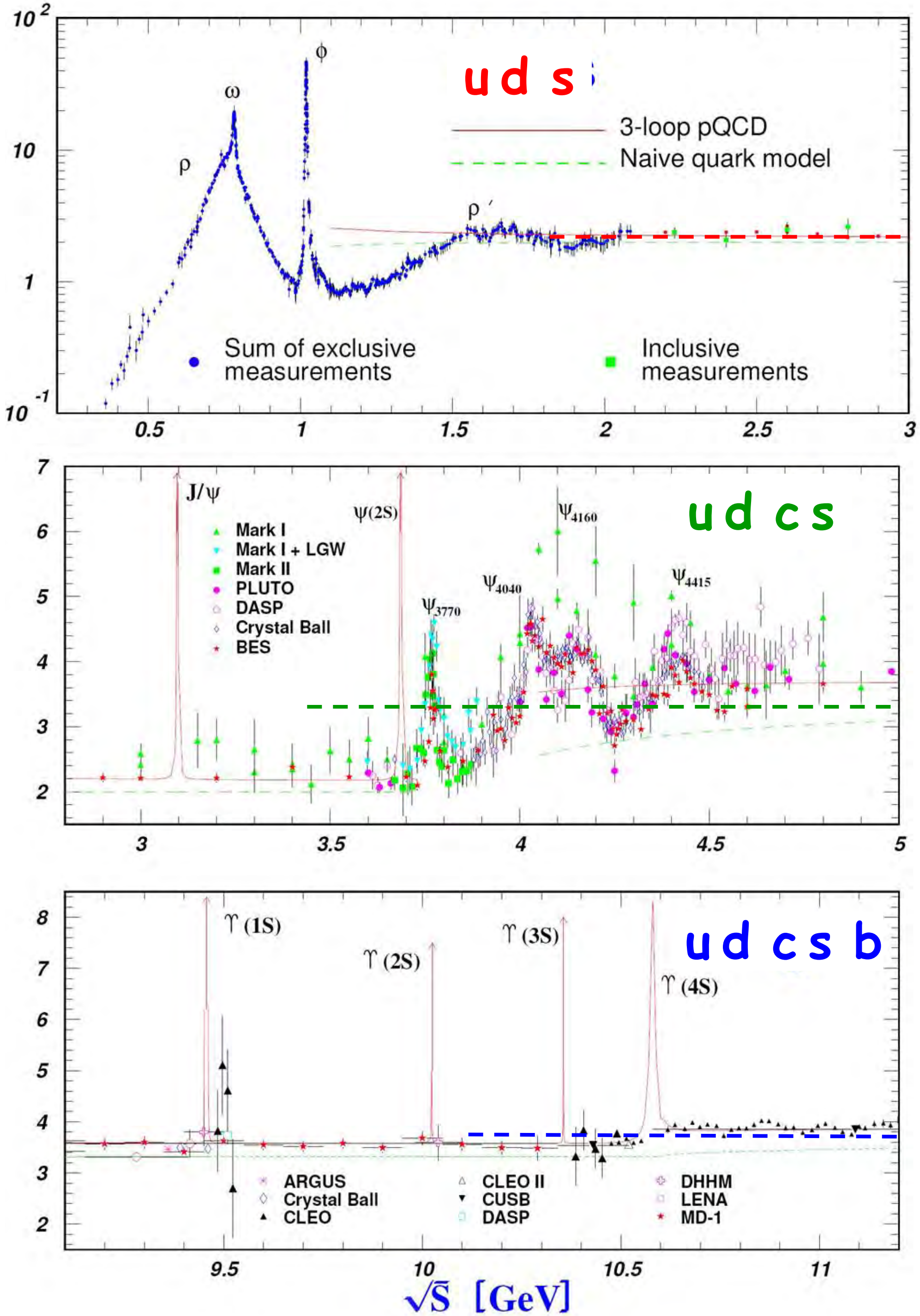
Abb. 13.1. Energieniveauschema von Wasserstoffatom und Positronium. Gezeigt sind der Grundzustand ($n = 1$) und der erste angeregte Zustand ($n = 2$), sowie ihre Feinstruktur- und Hyperfeinstrukturaufspaltungen. Die Aufspaltungen sind nicht maßstäblich eingezeichnet.

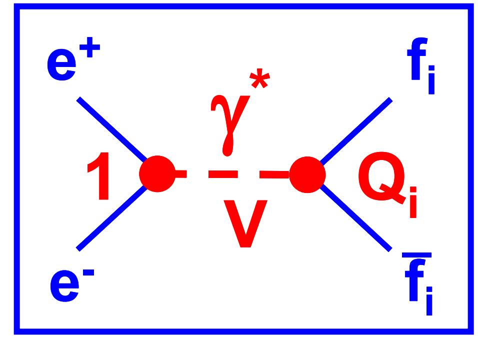


Charmonium Bottonium



quark charge



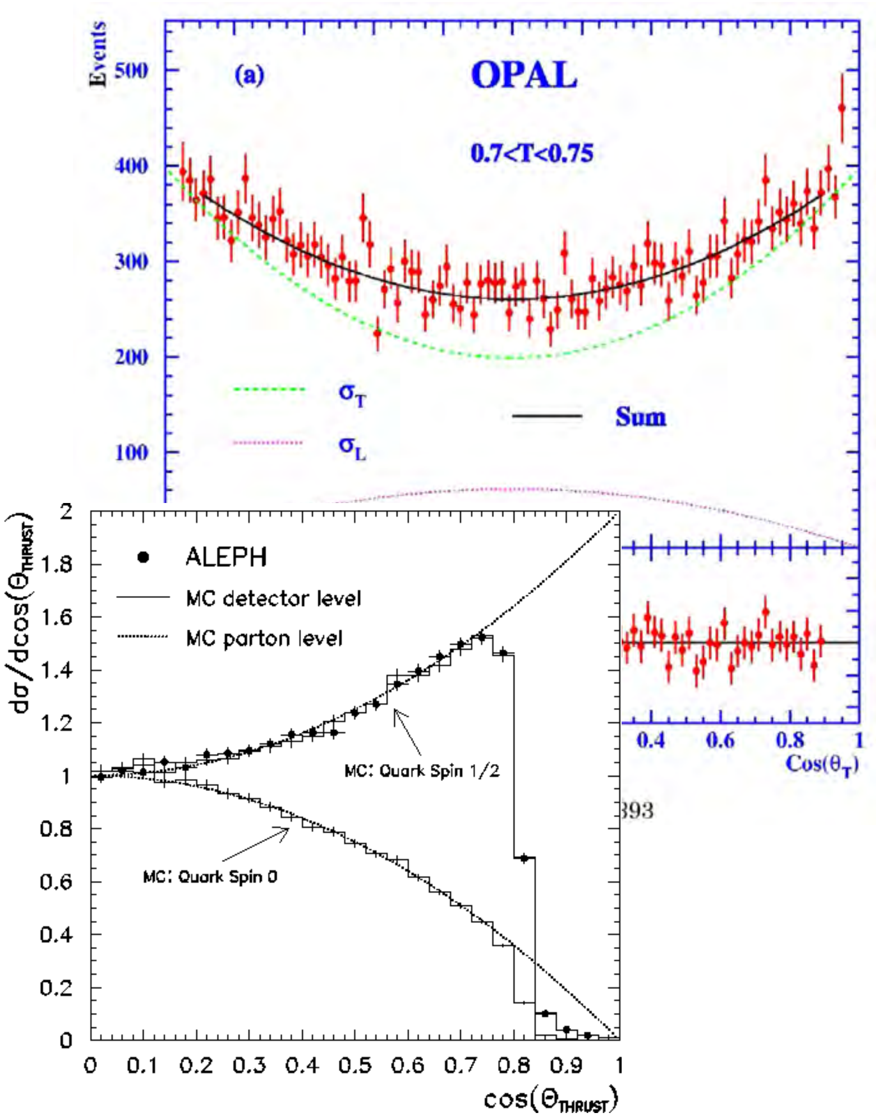


$$R = \frac{\sigma(e^+e^- \to q\,\bar{q} \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

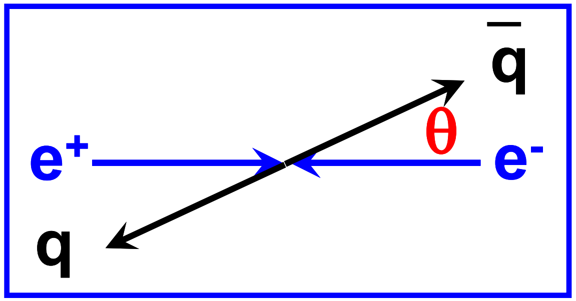
$$= \sum_{\substack{\text{flavors}\\ \text{colors}}} Q_i^2 \quad =$$

N_F	Q_i^2		N_C		
3	$\left(\frac{2}{3}\right)^2+\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^2$	$=\frac{6}{9}$	$\cdot 3$	$=\frac{6}{3}$	$\binom{u}{d}\binom{}{s}$
4	$\left(\frac{2}{3}\right)^2+\left(\frac{1}{3}\right)^2+\left(\frac{2}{3}\right)^2+\left(\frac{1}{3}\right)^2$	$=\frac{10}{9}$	$\cdot 3$	$=\frac{10}{3}$	$\binom{u}{d}\binom{c}{s}$
5	$\left(\frac{2}{3}\right)^2+\left(\frac{1}{3}\right)^2+\left(\frac{2}{3}\right)^2+\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^2$	$=\frac{11}{9}$	$\cdot 3$	$=\frac{11}{3}$	$\binom{u}{d}\binom{c}{s}\binom{}{b}$

Quark spin



PETRA @ DESY
LEP @ CERN



$e^+ e^- \rightarrow \bar{q}\, q \rightarrow$ 2 jets

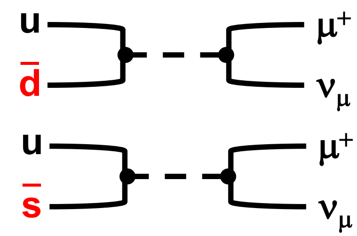
jet angular distribution

Spin 0:
$d\sigma / d \cos\theta \sim (1-\cos^2\theta)$

Spin 1/2:
$d\sigma / d \cos\theta \sim (1+\cos^2\theta)$

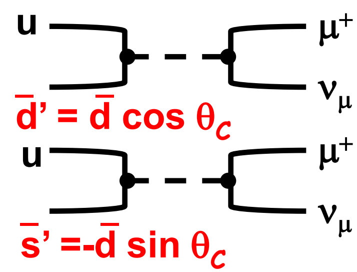
Quark Mixing

Decay	Δm/MeV	τ/ns
$\pi^+ \rightarrow \mu^+ \nu_\mu$	139-105 = 34	26
$K^+ \rightarrow \mu^+ \nu_\mu$	494-105 = 389	19



- kinematics: $\tau_\pi/\tau_K \sim 20$, observed: $\tau_\pi \sim \tau_K$
- d and s coupling different ?

- N.Cabbibo 1963:
  universal couplings, but weak interaction
  does not see eigenstates d and s of mass + strong interaction,
  but mixed states d', s' with mixing angle θ_C :



$$d' = d \cos \theta_C + s \sin \theta_C$$
$$s' = -d \sin \theta_C + s \cos \theta_C$$

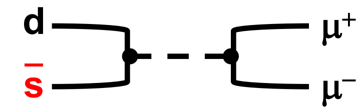
with **Cabbibo angle:** $\sin \theta_C = 0.220 \pm 0.002$

- $\sin \theta_C$: mixing strange - normal world or probability of family change

- SU(2) doublet: (u d') , $\Gamma(K^+ \rightarrow \mu^+ \nu_\mu)/\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu) \sim \sin^2\theta_C/\cos^2\theta_C \sim 0.05$

GIM Mechanism

Decay	Current	Γ_i / Γ
$K^+ \rightarrow \mu^+ \nu_\mu$	charged	64 %
$K^+ \rightarrow \pi^+ \nu \bar{\nu}$	neutral	$< 10^{-10}$ *
$K^0 \rightarrow l^+ l^-$	neutral	$< 10^{-8}$



- **Cabbibo theory:** weak neutral current

$$= (ud')(ud')^T = u\bar{u} + d'\bar{d'} =$$
$$= u\bar{u} + \cos^2\theta_c\, d\bar{d} + \sin^2\theta_c\, s\bar{s} + \ldots \qquad \Delta S = 0$$
$$\sin\theta_c \cos\theta_c\, (s\bar{d} + d\bar{s}) \qquad \Delta S = 1$$

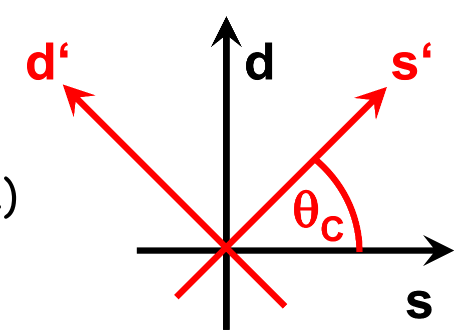
- $\Delta S = 1$ flavor changing neutral current not observed
- 1970: Glashow, Iliopoulos, Maiani: 2. quark doublet

$$d' = d\cos\theta_c + s\sin\theta_c$$
$$s' = -d\sin\theta_c + s\cos\theta_c$$

$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix}$$

neutral current $= u\bar{u} + d'\bar{d'} + c\bar{c} + s'\bar{s'}$

$$= u\bar{u} + d\bar{d} + c\bar{c} + s\bar{s} \qquad \Delta S = 0$$



- **1974: Charm discovered** by S.Ting (SLAC) and B.Richter (BNL)
- **1973:** Kobayashi + Maskawa postulate 3rd quark family mixing matrix with 1 complex phase -> CP violation

* LHCb: JHEP 01 (2013) 090.
NA 62, 2015-7: expect 10^2 in 10^{13} K^+ decays
expect $8\ 10^{-11}$, Buras et al, JHEP 1302 (2013) 116

Quark Mixing

1973: Cabbibo-Kobayashi-Maskawa postulate 3rd quark family

CKM matrix V_{CKM} transforms
mass eigen states (d s b) to
weak eigen states (d's'b')

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \equiv \hat{V}_{\mathrm{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Nobel prize

2008

represent in space of n=3 families by
n(n-1)/2 = 3 Euler angles $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, i=1,2,3:

$$\hat{V}_{CKM} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} . \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} . \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}$$

+ (n-1)(n-2)/2 = 1 complex phase δ (allows CP violation!)

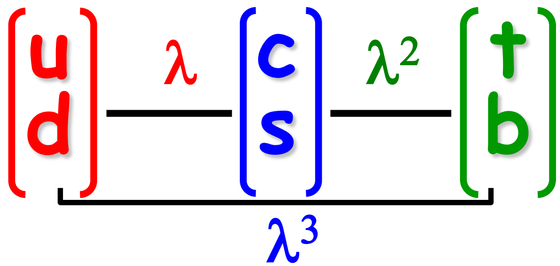
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Quark Mixing

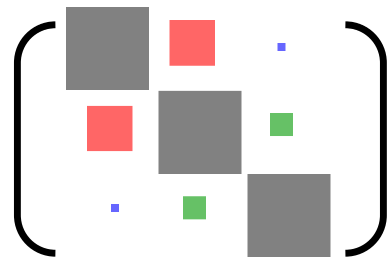
Experiment:

$\lvert V_{ud}\rvert =$	0.97424 ± 0.00022	$\lvert V_{us}\rvert =$	0.2252 ± 0.0009	$\lvert V_{ub}\rvert =$	$(4.07 \pm 0.38)\times 10^{-3}$
$\lvert V_{cd}\rvert =$	0.231 ± 0.010	$\lvert V_{cs}\rvert =$	1.03 ± 0.04	$\lvert V_{cb}\rvert =$	$(40.6 \pm 1.3)\times 10^{-3}$
$\lvert V_{td}\rvert =$	$(8.1 \pm 0.6)\times 10^{-3}$	$\lvert V_{ts}\rvert =$	$(38.7 \pm 2.3)\times 10^{-3}$	$\lvert V_{tb}\rvert =$	$(1.00 \pm 0.10)\times 10^{-3}$

hierarchic suppression of family change :



why ?



Wolfenstein parameterization :

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

mixing angles:

$s_{12} = V_{us} = \lambda = 0.2248 \pm 0.0004$

$s_{23} = V_{cb} = A\lambda^2 = 0.042 \pm 0.001$

$s_{32} = V_{ts} = A\lambda^2 = 0.039 \pm 0.002$

$s_{13} = V_{ub}\, e^{i\delta} = A\lambda^3\,(\ldots) = 0.0041 \pm 0.0004$

Cabbibo

$A = 0.82 \pm 0.01$

$\rho = 0.160 \pm 0.008$

$\eta = 0.350 \pm 0.006$

arXiv:1604.04940, 1702.08834

3. C+P Symmetries

C Parity

Dirac, QFT:

matter-antimatter: each particle has antiparticle !

C operator

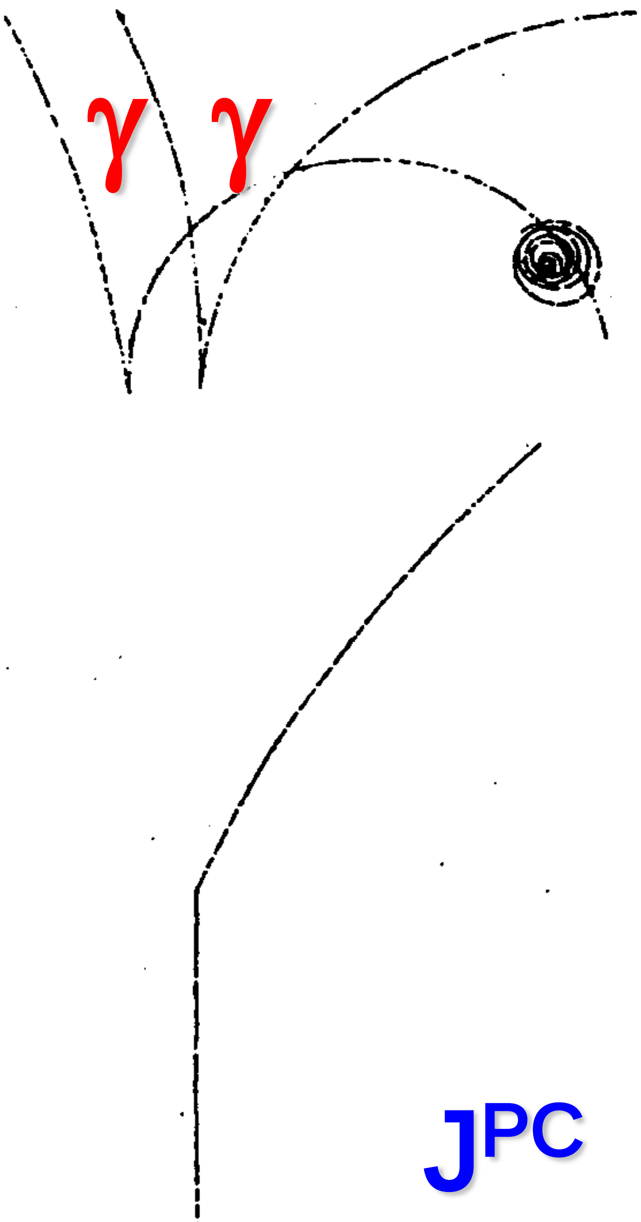
inverts all charge like additive quantum numbers:

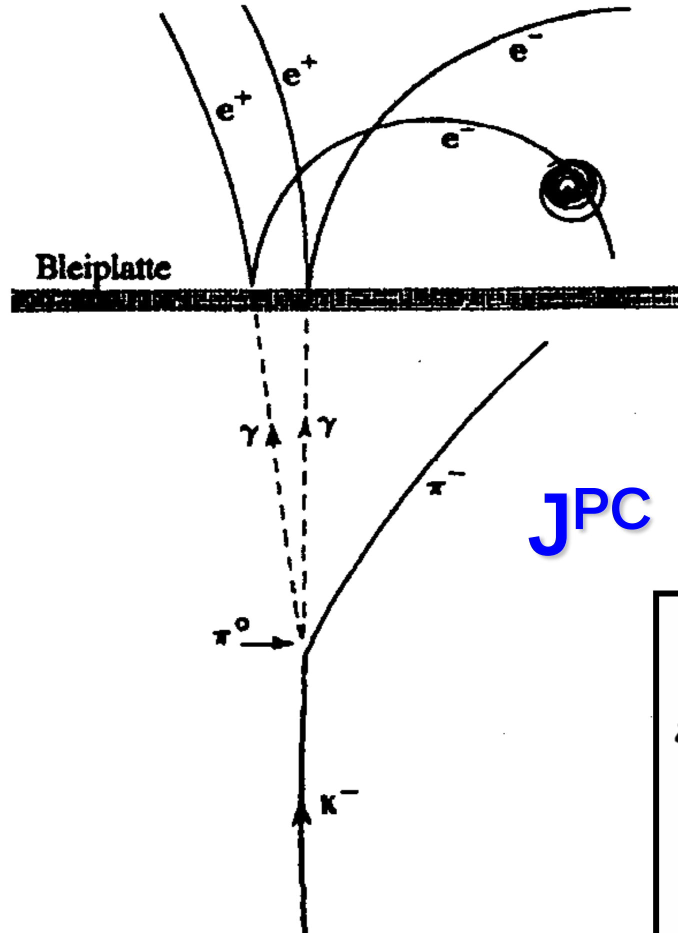
	C		
L,B	$\rightarrow$	-L, -B	lepton + baryon nr.
F	$\rightarrow$	-F	flavor: F = u, d, s, c, b, t
Y,Q	$\rightarrow$	-Y, -Q	((weak) hyper) charge
I_3	$\rightarrow$	$-I_3$	(weak) isospin
E,B	$\rightarrow$	-E, -B	electric + magnetic field

=> $C\,|\gamma\rangle = -|\gamma\rangle$

C: multiplicative, not charge like

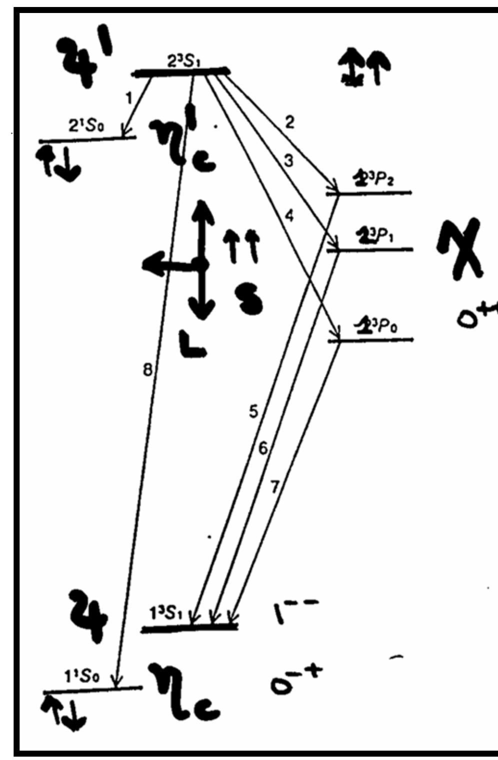
C Parity





$\Psi' \nrightarrow \Psi \gamma$

J^{PC} $1^{--} \nrightarrow 1^{--}\, 1^{--}$



$\pi^0 \rightarrow \gamma\ \ \gamma$

J^{PC} $0^{-+} \rightarrow 1^{--}\, 1^{--}$

C Parity

$\pi^0 \nrightarrow \gamma\gamma\gamma$ $\qquad$ $\Gamma(\pi^0 \to 3\gamma)/\Gamma(\pi^0 \to 2\gamma) < 3\ 10^{-8} \ll \alpha = 1/137$

$\pi^0 \to \gamma\gamma$

$C|\pi^0\rangle = C_\gamma^2 |\pi^0\rangle = +|\pi^0\rangle$ $\qquad$ $J^{PC} = 0^{-+}$ $\qquad$ pseudo-scalar mesons

$V \longrightarrow P\gamma$

$\rho^0 \to \pi^0\gamma$

$\rho^0 \nrightarrow$ 2 identical particles :

$\rho^0 \nrightarrow \pi^0\pi^0$

$\rho^0 \nrightarrow \gamma\gamma$

$C|\rho^0\rangle = +C|\gamma\rangle = -|\rho^0\rangle$ $\qquad$ $J^{PC} = 1^{--}$ $\qquad$ vector mesons

quarkonium spectroscopy:

$\Psi' \nrightarrow \gamma\Psi$	$1^{--} \nrightarrow 1^{--}1^{--}$
$\to \gamma\eta_C$	$1^{--} \to 1^{--}0^{-+}$
$\to \gamma\chi$	$1^{--} \to 1^{--}\ 0,1,2^{++}$

$\Phi, \Psi, \Upsilon \nrightarrow \gamma\gamma$, 2 gluons $\qquad$ OZI rule

$\to \gamma\gamma\gamma$, 3 gluons, photons

C Parity

- not eigenstates of C operator !
  only eigenstates have eigenvalue !

$C\ |\pi^+\rangle = |\pi^-\rangle$ (Q)
$C\ |n\rangle = |\bar{n}\rangle$ (B)
$C\ |K^0\rangle = |\bar{K}^0\rangle$ (S)
$C\ |\nu\rangle = |\bar{\nu}\rangle$ (L)

- only totally neutral states with
  all additive quantum numbers zero:
  Q, B, L, S, C = 0

$\gamma,\ \pi^0, \eta^0,\ \rho^0, \omega^0, \Phi^0,\ \Psi,\ \Upsilon, Z$

are eigenstates of C operator !

- spin like magnetic field = rotating charge:
  negative C parity

$$C\,|\,f\,\bar{f}\,\rangle = C\,|\,\text{Meson}\,\rangle = |\,f\,\bar{f}\,\rangle\ (-1)^{L+S}$$

L	S	J	C	Multiplet	Expl.
L=S			+		
0	0	0	+	Pseudo-Scalar	π^0
0	0	0	-	FORBIDDEN !	
0	1	1	-	Vector (like γ)	ρ^0
1	1	0	+	Scalar L=1 rare	A_0
1	1	1	+	Axial-Vector	A_1
1	1	2	+	Tensor	f^0

Pauli+Dirac Matrices

1927 Pauli: nonrelativistic spin 1/2 electron

SU(2) spin algebra in 2-D complex vector space isomorphic to
SO(3) group of 3D rotations

- H atom: spin $Y_l^m(\theta,\varphi)$ SO(3)
- strong iso-spin $Y_I^{I_3}$ $SU(2)_I$
- weak iso-spin $SU(2)_w$

2 representations: 3D rotation matrices OR 2D Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad [\sigma_i \sigma_j] = i\sigma_k \qquad J_i = \frac{1}{2}\sigma_i \qquad \sigma_i^2 = I$$

1928 Dirac: relativistic spin 1/2 electron (= fermion = matter part.)

$$(i\gamma^\mu \partial_\mu - m)\,\Psi = 0 \qquad (\mu = 0,\dots,3)$$

Ψ ... 4-component Dirac spinors of Lorentz group L(1,3) =
$2 \otimes 2$ = 2 spin orientations $\otimes$ particle-antiparticle

γ matrices describe space-time structure:

from Pauli to Dirac matrices:

$$\gamma_0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \qquad \gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ -\sigma_\mu & 0 \end{pmatrix} \qquad \gamma_5 = i\,\gamma_0\gamma_1\gamma_2\gamma_3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

Clifford algebra of γ *matrices* :

$$[\gamma_\nu \gamma_\mu]_+ = 2\delta_{\nu\mu} \qquad (\nu, \mu = 0,\ldots,3)$$

$$[\gamma_\mu \gamma_5]_+ = 0$$

or : $\gamma_1^2 = \gamma_2^2 = \gamma_3^2 = -I \qquad \gamma_o^2 = I = \gamma_5^2$

$$P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$$ ... right/left handed projection operators :

$$P_L + P_R = 1 \qquad P_L^2 = P_L \qquad P_R^2 = P_R \qquad P_L P_R = P_R P_L = 0$$

P Parity

P Operation	Type	Dirac	J^P	Example
$P\,\lvert\vec{r}\rangle = -\lvert\vec{r}\rangle$	vector	γ_μ	1^-	
$P\,\lvert\vec{p}\rangle = -\lvert\vec{p}\rangle$	vector	γ_μ	1^-	$\vec{p} = d\vec{r}/dt$
$P\,\lvert t\rangle = \lvert t\rangle$	scalar	1	0^+	
$P\,\lvert E\rangle = \lvert E\rangle$	scalar	1	0^+	
$P\,\lvert\vec{B}\rangle = \lvert\vec{B}\rangle$	axialvector	$\gamma_5\gamma_\mu$	1^+	$\vec{B} = \vec{v}\times\vec{E} = \vec{\nabla}\times\vec{A}$
$P\,\lvert\vec{\sigma}\rangle = \lvert\vec{\sigma}\rangle$	axialvector	$\gamma_5\gamma_\mu$	1^+	$\vec{L} = \vec{v}\times\vec{r}$
$P\,\lvert\vec{p}\,\vec{\sigma}\rangle = -\lvert\vec{p}\,\vec{\sigma}\rangle$	pseudoscalar	γ_5	0^-	$H = \vec{p}\vec{\sigma}/\lvert\vec{p}\rvert\lvert\vec{\sigma}\rvert$
$P\,\lvert F_{\mu\nu}\rangle = \lvert F_{\mu\nu}\rangle$	tensor	$\gamma_\mu\gamma_\nu$	2^+	$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$T\,\lvert t\rangle = -\lvert t\rangle$	time reversal			

H = helicity
helix = screw

$P^2 = 1$ P … unitary

combined CPT conserved in field theory

Parity

$P\, Y_L^M = (-1)^L\, Y_L^M$ parity H atom with L

atomic electric dipole transitions
flip atomic spin by L=1 :

$P\, |\gamma\rangle = P\, |\vec{E}\rangle = (-1)^{L=1}\, |\gamma\rangle = -|\vec{E}\rangle = -\, |\gamma\rangle$

spin S is axial-vector:
$P\, |\sigma\rangle = +|\sigma\rangle$

Dirac equation: $(i\gamma^\mu \partial_\mu - m)\, \Psi = 0$

$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$ ... parity operator =>

$P\, |\, f\, \rangle = +|\, f\, \rangle$
$P\, |\, \bar{f}\, \rangle = -|\, \bar{f}\, \rangle$

$P\, |\, f\, \bar{f}\, \rangle = P\, |\, \text{MESON}\, \rangle = -|\, f\, \bar{f}\, \rangle\ (-1)^L$
$P\, |\, f\, f\, f\, \rangle = P\, |\, \text{BARYON}\, \rangle = +|\, f\, f\, f\, \rangle\ (-1)^L$

L	S	J^P	multiplet
			MESONS
0	0	0^-	pseudoscalar
0	1	1^-	vector
1	1	0^+	scalar L=1 rare
1	1	1^+	axialvector
1	1	2^+	tensor
			BARYONS
0	1/2	$1/2^+$	nucleon octet
0	3/2	$3/2^+$	Δ decuplet
1	1/2	$1/2^-$	N^* octet

Parity Violation

Wu et al. (Co^{60}), Garwin + Lederman (polarized muons), Columbia Univ., subm. together 15.1.57, Telegdi + Friedman subm. 17.1.57: Phys. Rev. 105 (1957) 1413; 1415; 1681.
Nobel prize 1957 to Lee + Yang

nuclear β decay:

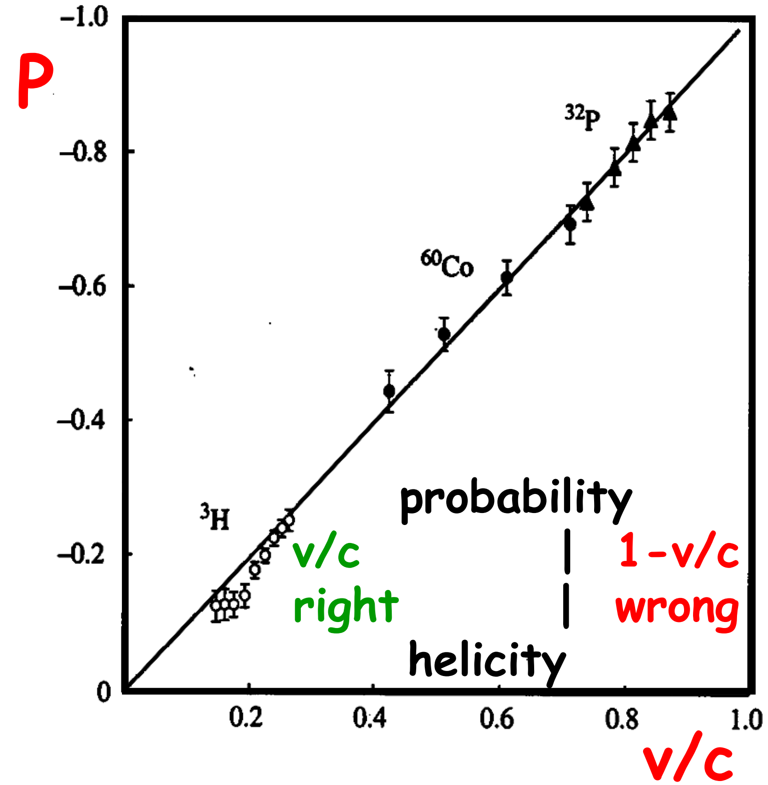
helicity $H = (\vec{\sigma}\cdot\vec{p})/|p|$

in a system with c>v'>v momentum p and helicity H are inverted:

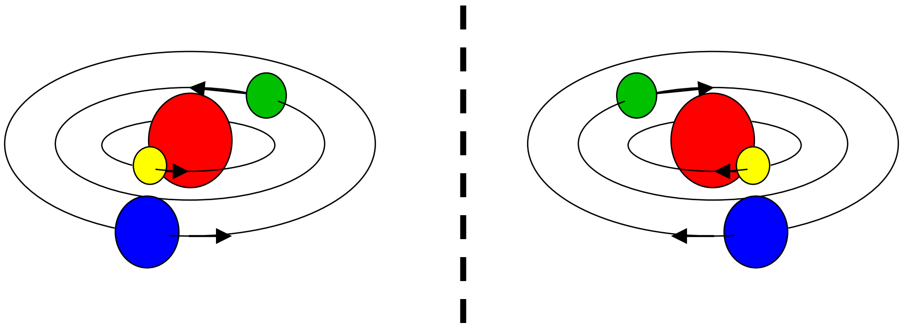
probability of 'wrong' helicity:
$1 - v/c$

polarization
$P = (N^+ - N^-)/(N^+ + N^-)$
$= \sigma \cdot pc / E = H\,v/c$

longitudinal polarization P vs. electron velocity v/c:



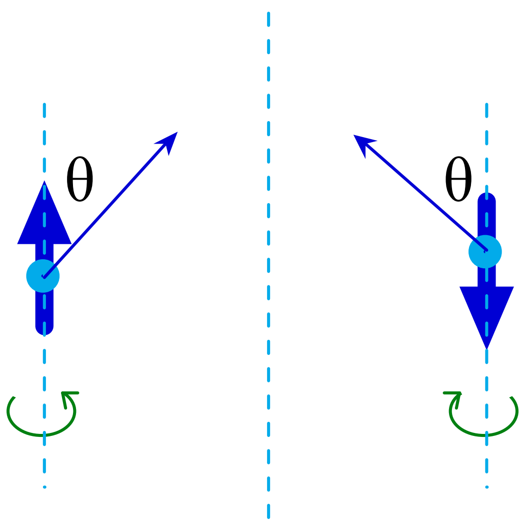
Parity violation



Gravitation + electromagnetism:

same physical laws
in mirror system,
e.g. planetary motion

parity conserved.



Weak interaction:

β-decay
of polarized ^{60}Co
violates parity !

discovered 1956

(V-A) Theory

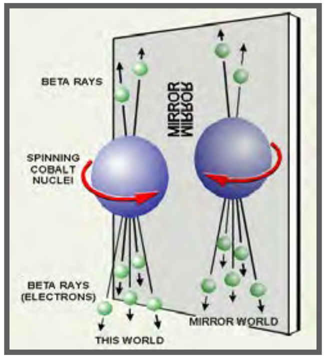
$\vec{\sigma}$ ... unit vector in spin direction
helicity operator H:

$$H = \frac{\vec{\sigma}\,\vec{p}}{|\vec{p}|} \qquad H|\psi\rangle = \pm|\psi\rangle$$

projection operator L / R left / right:

L = (1 − H) / 2 = 0,1 for H = ±1 or $(1\pm\gamma_5)/2$
R = (1+H) / 2 = 1,0

chirality = handedness



L = (1 − H) / 2 particle: left handed
L = (S − P) / 2 space structure: (Scalar-Pseudoscalar)

all interactions via vector bosons V :

L = V (S − P) / 2
L = (V − A) / 2
L = $\gamma^\mu(1-\gamma^5)$ / 2

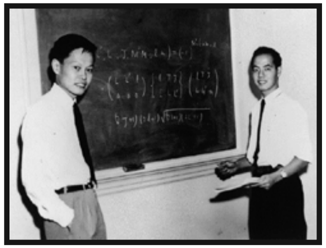
Nobel prize

1957

space structure of weak force :

(V-A) why ?

maximum parity violation !



T.D.Lee + C.N.Yang
Brookhaven, USA, 1956

Helicity + Chirality

$m \neq 0 \ , \ \beta = v/c < 1$ $\qquad$ $m = 0 \ , \ \beta = v/c = 1$

ν_L :	FERMIONS	are	LEFT	handed
$\bar{\nu}_R$:	ANTI-FERMIONS		RIGHT	handed

$$P \ | \nu_L \rangle = | \nu_R \rangle \qquad P \ \text{violated}$$

$$C \ | \nu_L \rangle = | \bar{\nu}_L \rangle \qquad C \ \text{violated}$$

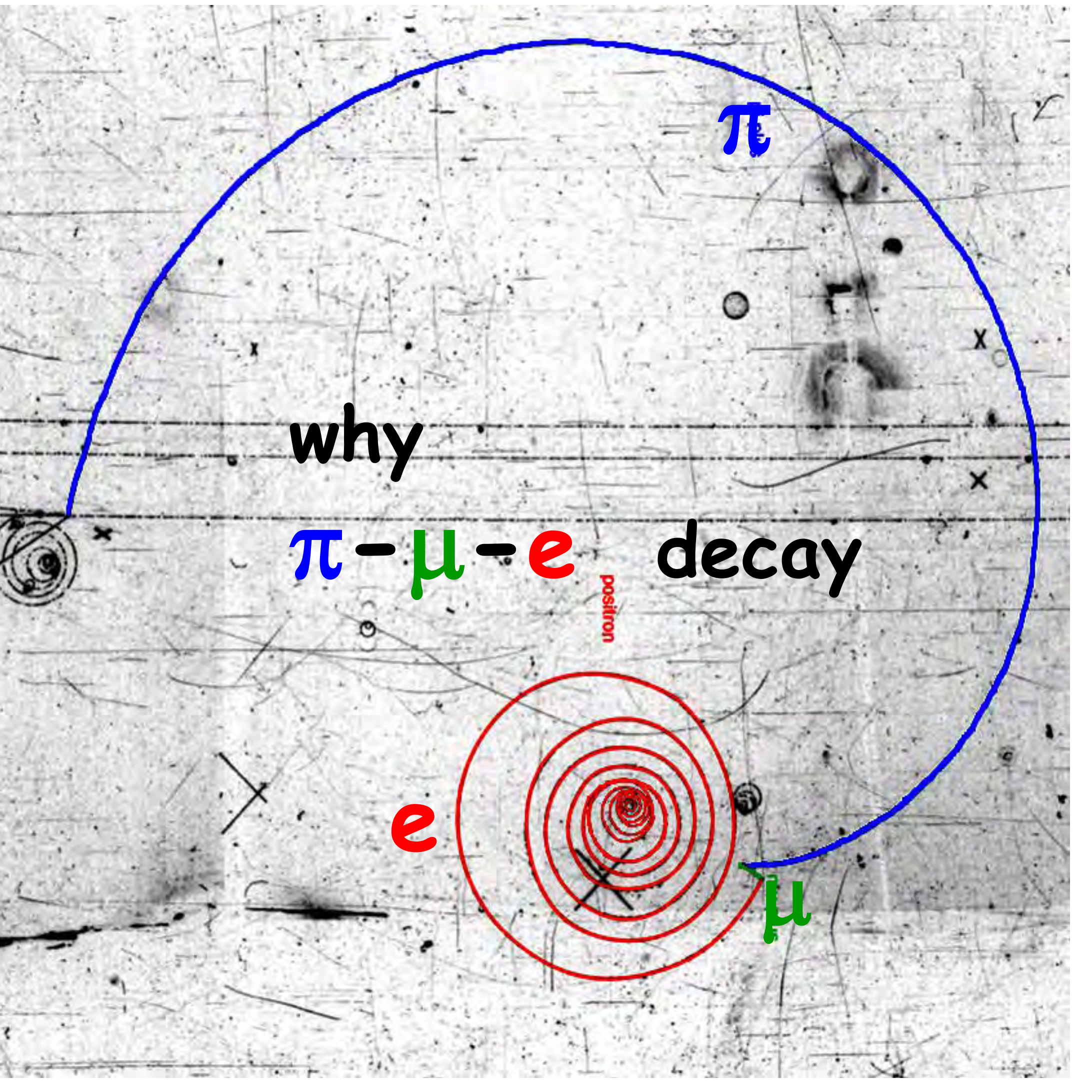
$$CP \ | \nu_L \rangle = | \bar{\nu}_R \rangle \qquad CP \ \text{conserved}$$

Manche meinen,
lechts und rinks
kann man nicht
velwechsern.

Werch ein Illtum !

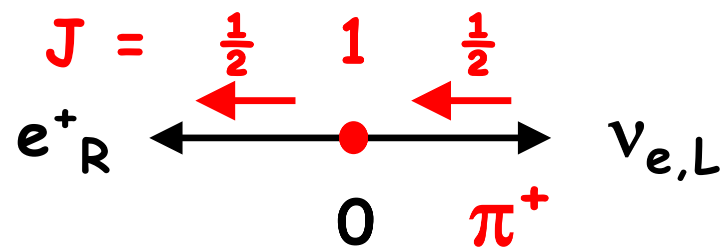
E.Jandl

Helicity Suppression



and not π-e directly?

π decay rest frame :



parity violation versus
angular momentum conservation

polarization P
of decay lepton vs
helicity H and velocity v/c

$P = H\, v/c$

$m_{e,\nu} \ll p_{e,\nu} \sim m_\pi/2 \Rightarrow$
$v \to c \Rightarrow P \to 1$

Helicity Suppression

$\pi \to \mu\, \nu_\mu$

$2\, E_\mu^{max} = m_\pi - m_\mu = 34$ MeV $< m_\mu$

$\mu \to e\, \nu_e\, \nu_\mu$

$2\, E_e^{max} = m_\mu - m_e = 105$ MeV

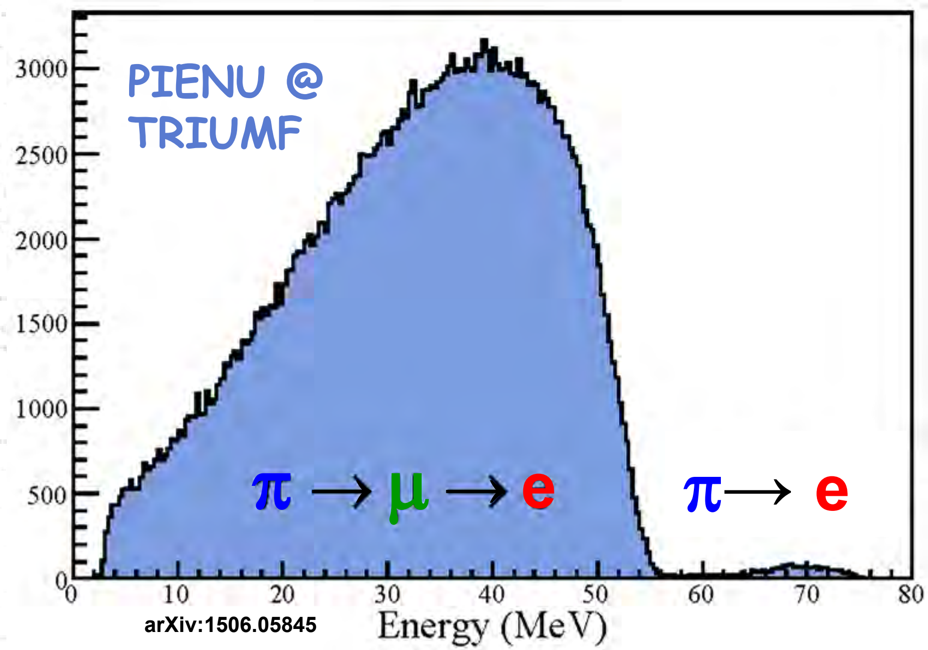
$\pi \to e\, \nu_e$

$2\, E_e^{max} = m_\pi - m_e = 139$ MeV $\gg m_e$

$$\frac{\Gamma(\pi \to e\, \nu_e)}{\Gamma(\pi \to \mu\, \nu_\mu)} = 1.234 \cdot 10^{-4}$$

$= (m_e^2/m_\mu^2)\,(m_\pi^2 - m_e^2)^2/(m_\pi^2 - m_\mu^2)^2(1+R_{QED})$

stopped pions: electron spectrum



parity violation !

also:

$\Gamma(K \to e\, \nu_e) \,/\, \Gamma(K \to \mu\, \nu_\mu) = 2.5\ 10^{-5}$

$\Gamma(B \to e\, \nu_e) \,/\, \Gamma(B \to \mu\, \nu_\mu) \,/\, \Gamma(B \to \tau\, \nu_\tau) =$

10^{-11} / 10^{-7} / 1

4. Weak Interaction

Neutrinos

most abundant fermion in Universe

pure sources and pure probes of weak interaction

Pauli's Neutrino hypothesis

$n \rightarrow p\, e^-$? continuous β spectrum

E conservation violated ?

Offener Brief an die Gruppe der Radioaktiven bei der Gauvereins-Tagung zu Tübingen.

Abschrift

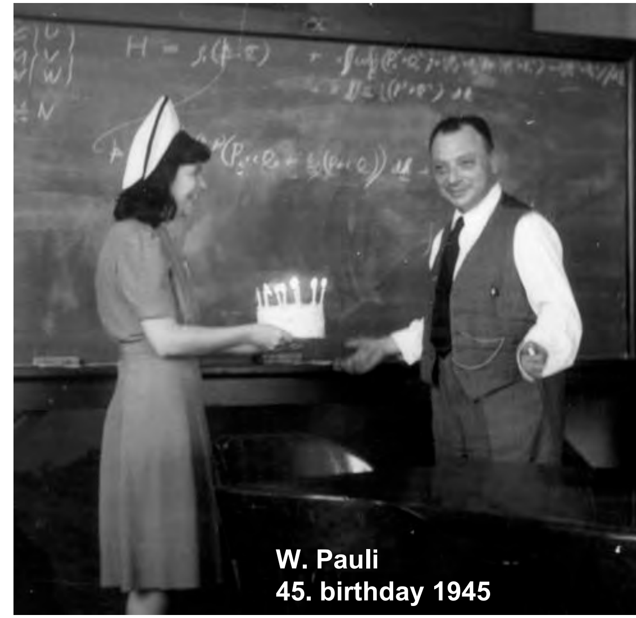
Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grossenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

Also, liebe Radioaktive, prüfet und richtet!
Leider kann ich nicht persönlich in Tübingen erscheinen, da ich infolge eines in der Nacht vom 6. zum 7. Dez. in Zürich stattfindenden Balles hier unabkömmlich bin.



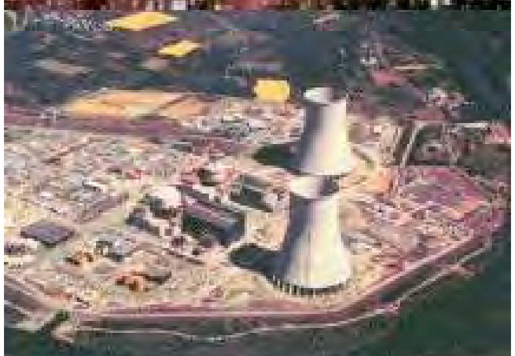
W. Pauli
45. birthday 1945

$n \rightarrow p\, e^-\, \bar{\nu}_e$

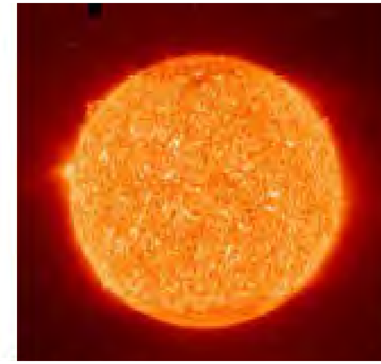
renamed
Neutrino
1933 by E. Fermi
after neutron
discovery

„Heute habe ich etwas Schreckliches getan, etwas, was kein theoretischer Physiker jemals tun sollte. Ich habe etwas vorgeschlagen, was nie experimentell verifiziert werden kann."

Pauli am selben Tag an den Astronomen W.Baade



Neutrinos



Supernovae

SN 1987A: 10^{38} s^{-1}

10^{58} total

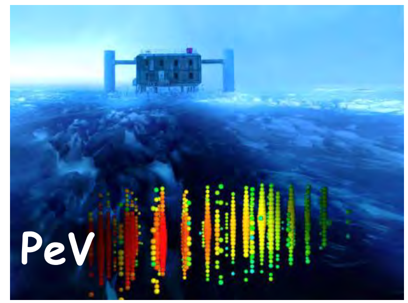
10^{14} m^{-2}

99% of E

~cosmic lumi

Sun

$6 \cdot 10^{14}$ m^{-2} s^{-1}



Reactor

$2 \cdot 10^{20}$ s^{-1} GW_{th}^{-1}

@ 10 m: 10^{17} m^{-2} s^{-1}

Atmospheric

10^{4} m^{-2} s^{-1}

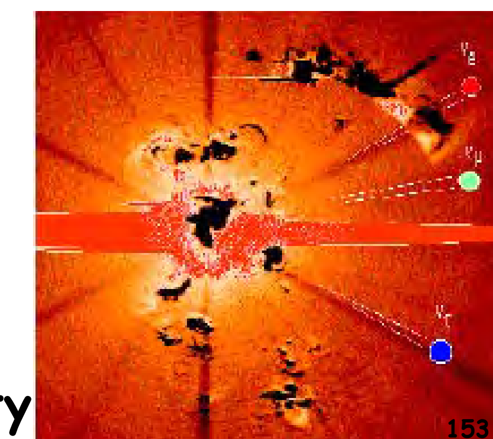
Accelerators

Big Bang

335 cm^{-3}

100 μs - 100 s

dominant cosmic energy



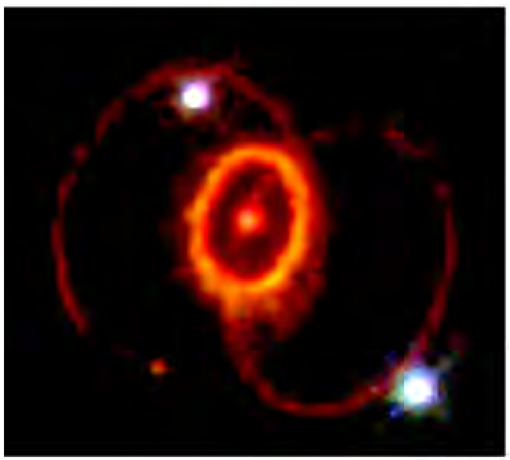
Earth

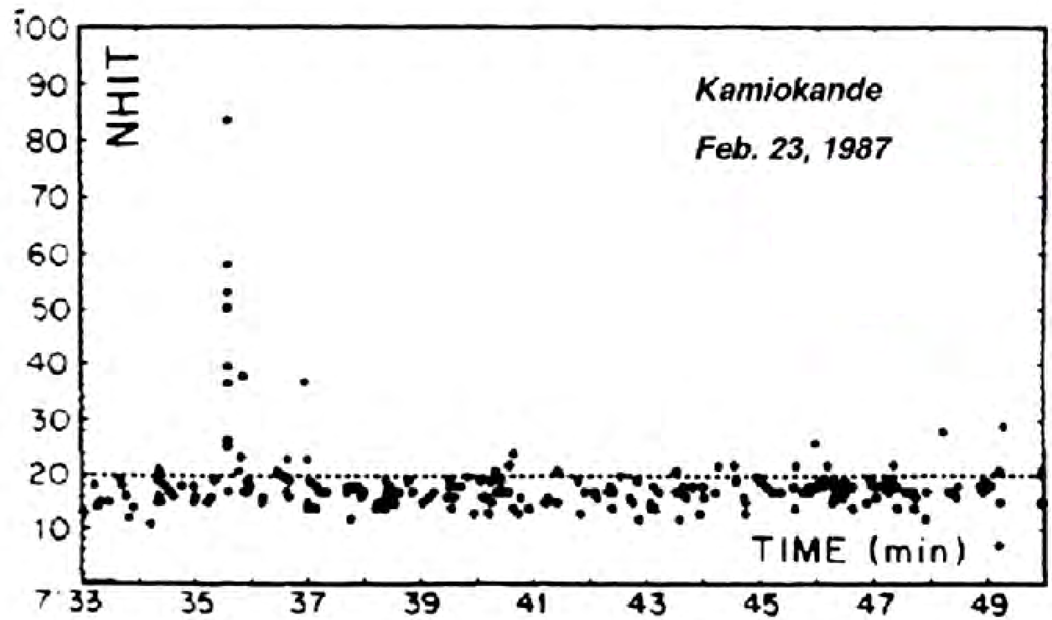
$2 \cdot 10^{11}$ m^{-2} s^{-1} radioactivity

Cosmic Neutrinos

Supernova SN 1987A: Magellan cloud, 168.000 ly

10^{58} ν total, 10^{15} ν/m^2

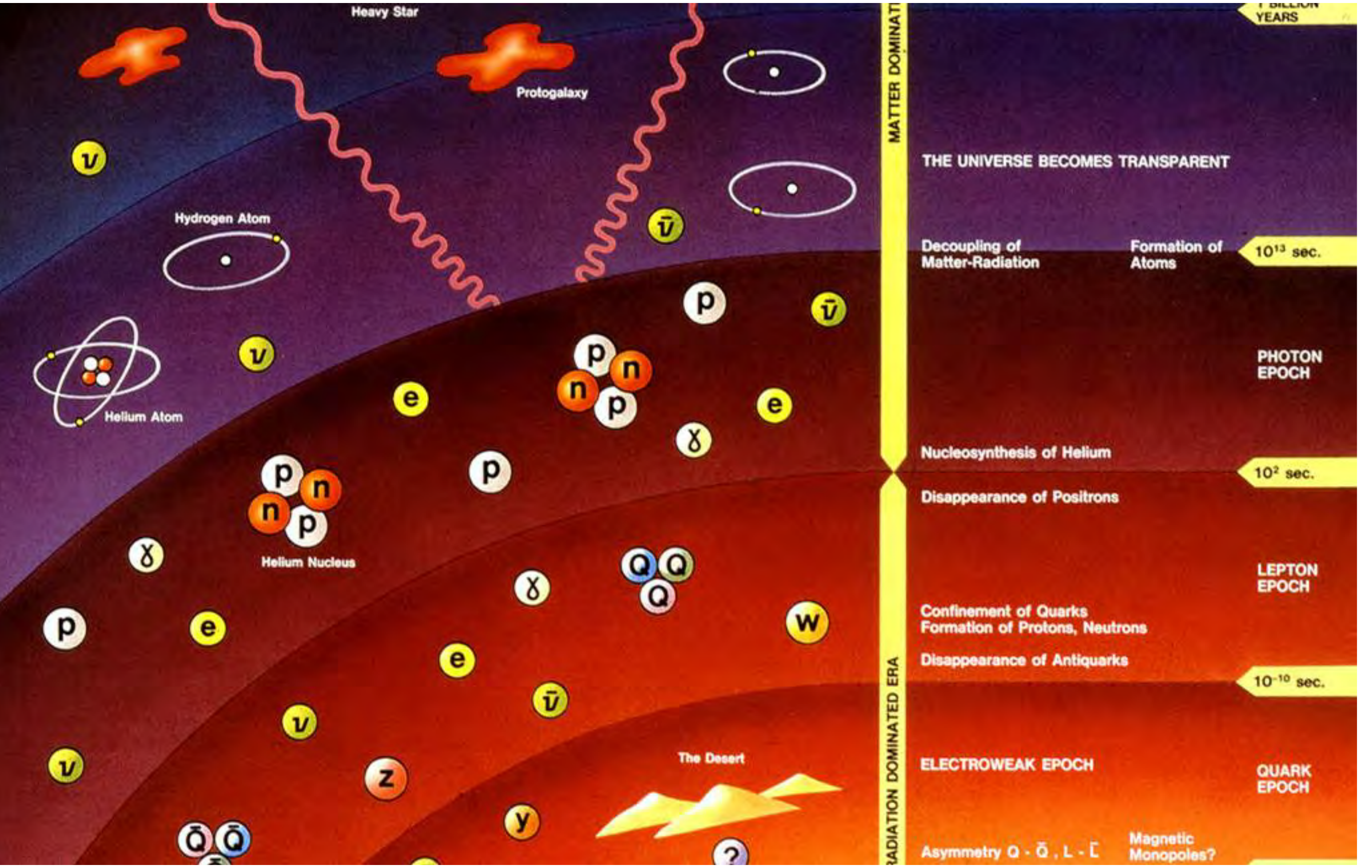

2002



first Supernova visible by naked eye
since Kepler 1604 at 20.000 ly in Milky Way!

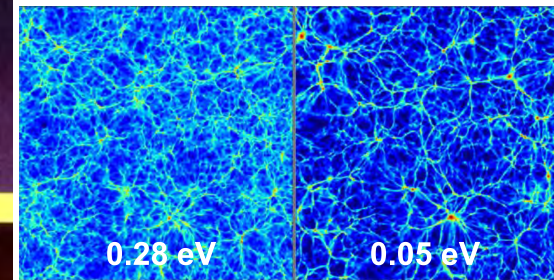
neutrinos seen ~2h before light => m_ν < 30 eV (Sun: light takes >10.000 yrs)

Big Bang Neutrinos



decoupling:

γ 380.000 a



hot ν's. $m_\nu > 1$ eV wipe out small-scale cosmic structures

CMBR (PLANCK) + BAO:

$\Sigma m(\nu_i) < 0.23$ eV

$N^{eff} = 3.2 \pm 0.2$

arXiv:1502.01582, 9.
Thomas, Abdalla, Lahav, PRL 2010
Putter et al, arXiv: 1201.1909. Phys.Rep. 517(2012)141

ν 1 s $\bar{\nu}\nu \nrightarrow e^+e^-$

equilibrium till 0.2 s / 2 MeV

$\bar{\nu}_e\, p \to n\, e^+$

$n \to p\, e^-\, \bar{\nu}_e$

now: γ: 2.7 K background radiation 410/cm³ — Penzias, Wilson 1964

ν: 1.9 K background radiation 336/cm³ — Nobel 1978

$T_\nu = (4/11)^{1/3}\, T_{CMB}$ $n_\nu = (9/11)^{1/3}\, n_{CMB}$

"dark photons": n_ν, m_ν

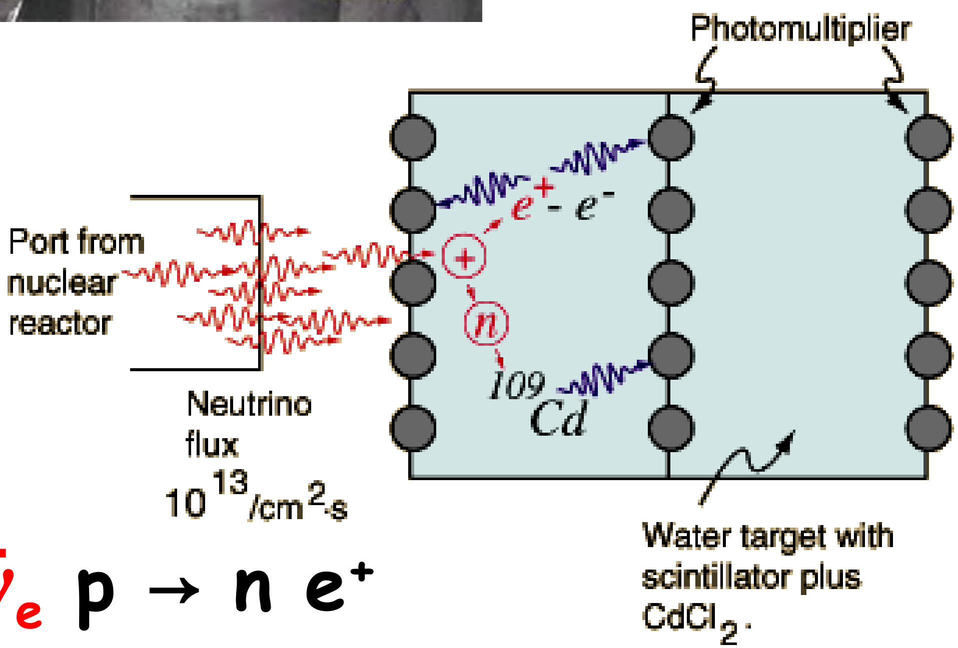
neutrino / baryon density: $\Omega_\nu / \Omega_b = 0.5\, \Sigma m_\nu /\text{eV}$

$\Omega_\nu h^2 = m_\nu / 94\ \text{eV} < 0.0025$

Neutrino Discovery

25 years after Pauli: Project Poltergeist:

C.Cowan and F.Reines, 1956 :
Savannah river reactor, USA



Reines:
Nobel prize

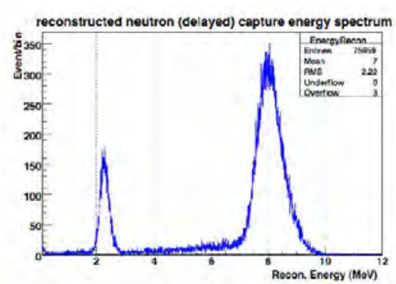
1995

$\bar{\nu}_e \; p \rightarrow n \; e^+$

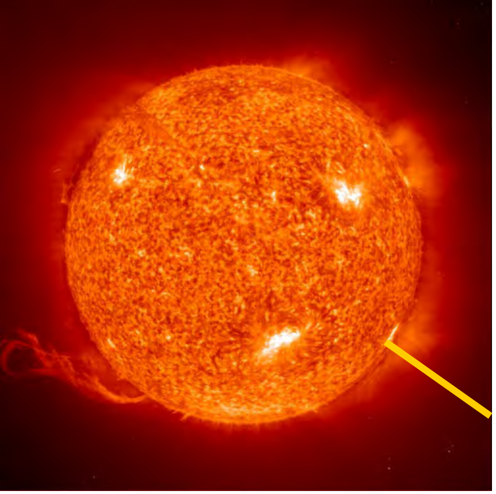
$e^+ e^- \rightarrow \gamma\gamma$ prompt, E_γ = some MeV

$n \; Cd \rightarrow Cd^* \gamma$

γ emission delay ~5 μs
look for $\gamma_n - \gamma_e$ coincidences



today Gd: 16t Daya Bay, 100t SuperK



Solar Neutrinos

Homestake Gold mine

ν_e

6×10^{14} $m^{-2}s^{-1}$

nuclear fusion in the Sun :

pppp → ppnn = He

$p \rightarrow n\ e^+\ \nu_e$

micro-radiochemistry: detect single decays

$\nu_e\ Cl^{37} \rightarrow Ar^{37}\ e^-$ τ = 35 d

$\nu_e\ n \rightarrow p\ e^-$

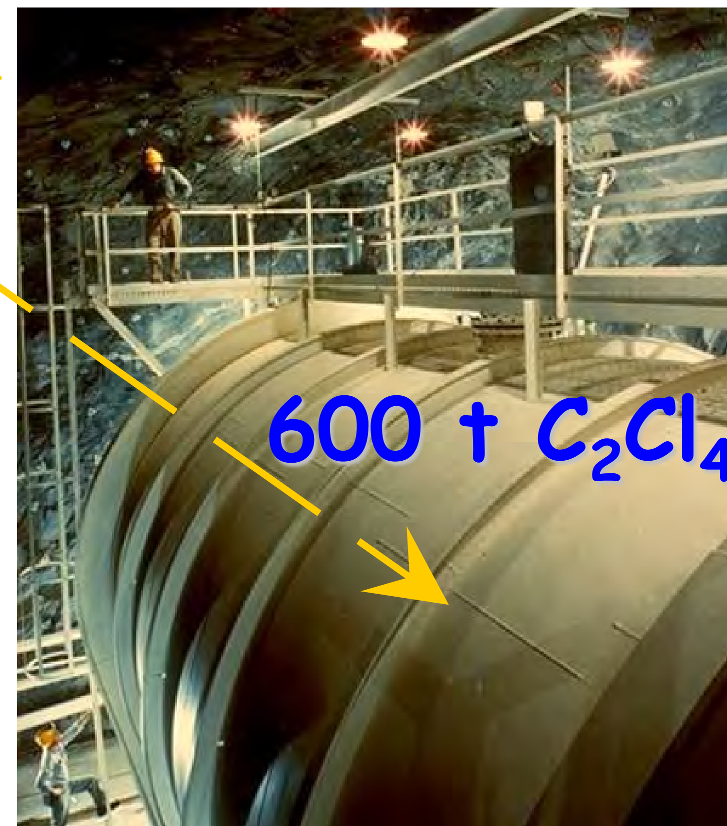
1967-94: ~ 2/3 neutrinos missing !

nuclear fission produces $\bar{\nu}_e$

1968 Savannah river reactor

$\bar{\nu}_e\ Cl^{37} \nrightarrow Ar^{37}\ e^-$ no signal =>

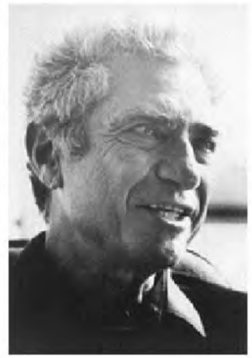
$\nu \neq \bar{\nu}$



Are neutrinos equal? The muon neutrino

L.Lederman

M.Schwartz

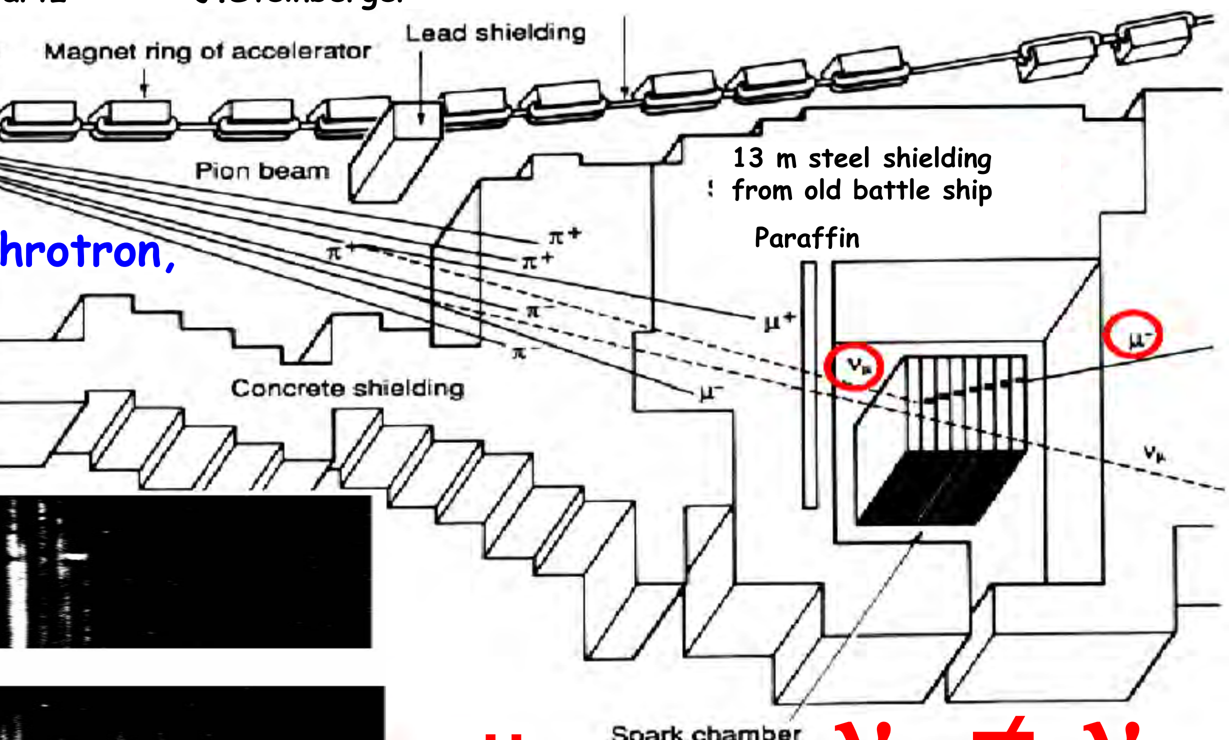


J.Steinberger

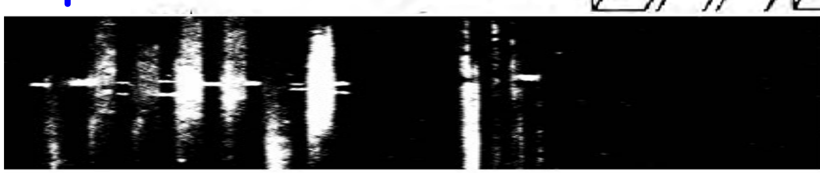
Nobel

prize 1988

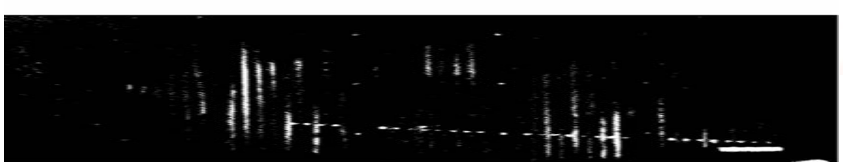
Brookhaven Proton Synchrotron, USA, 1961



10 t spark chamber



μ



ν_μ

$\nu_\mu \neq \nu_e$

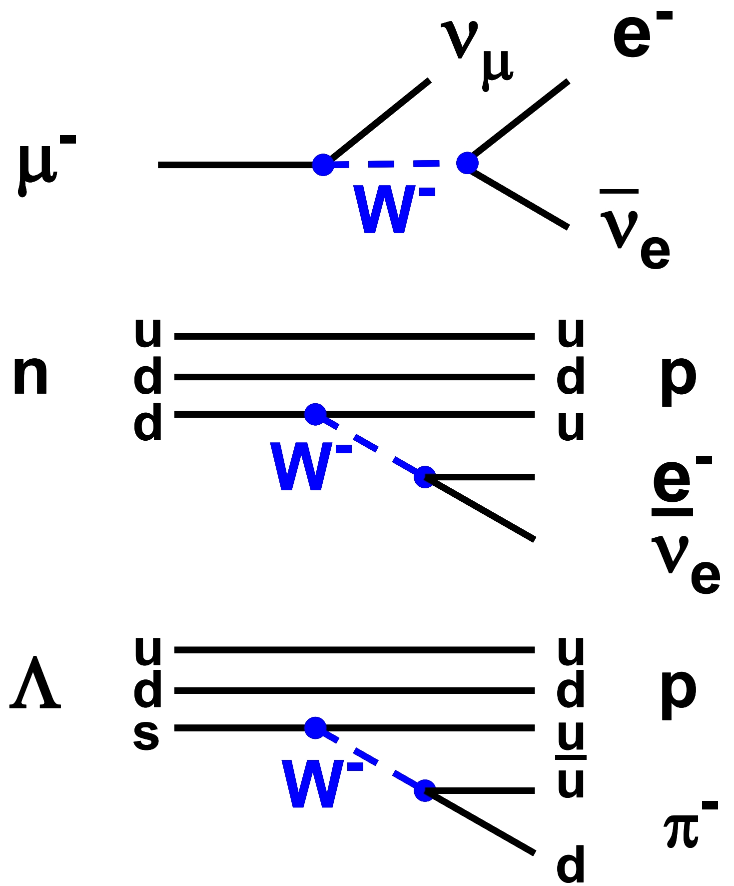
Weak Decays

DECAYS

leptonic

semi-leptonic

non-leptonic



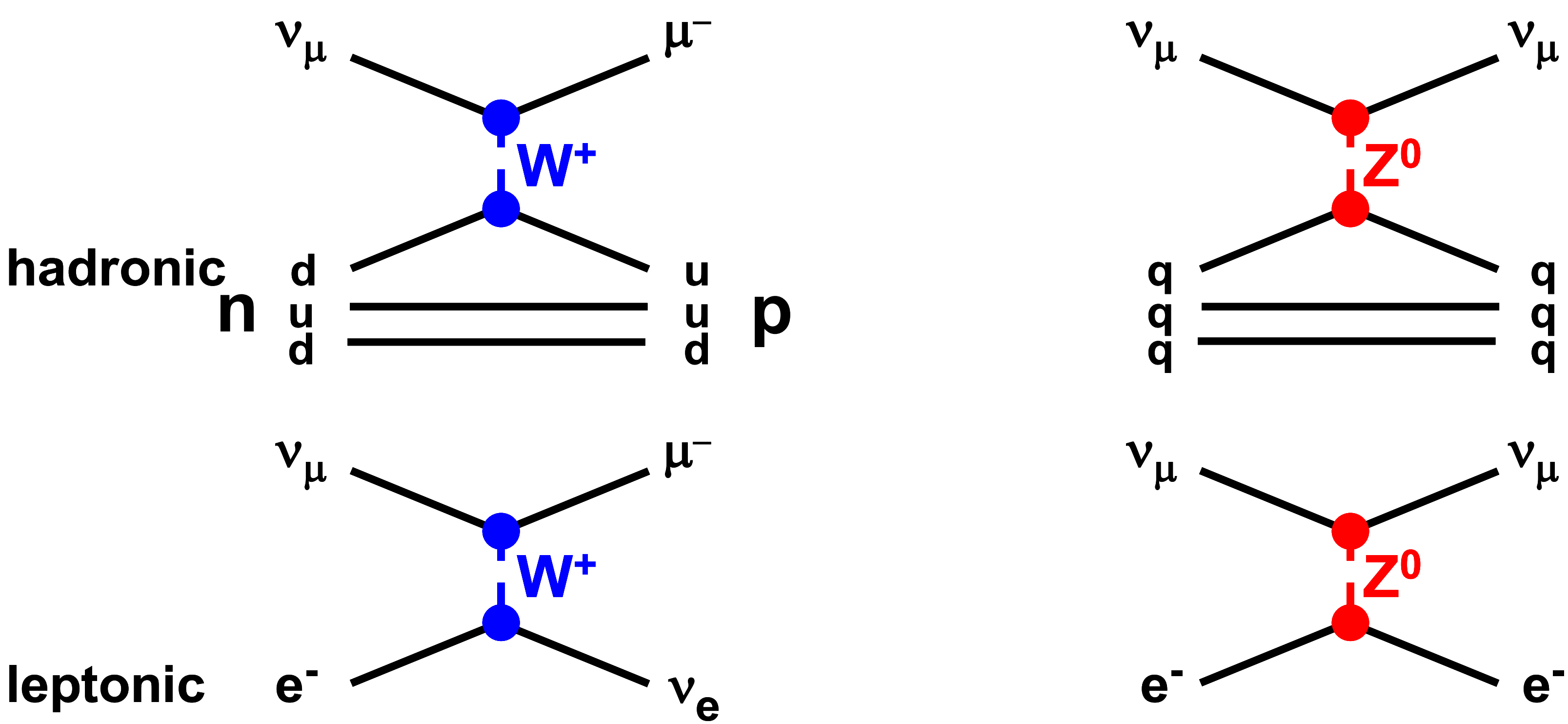
μ decay

β decay

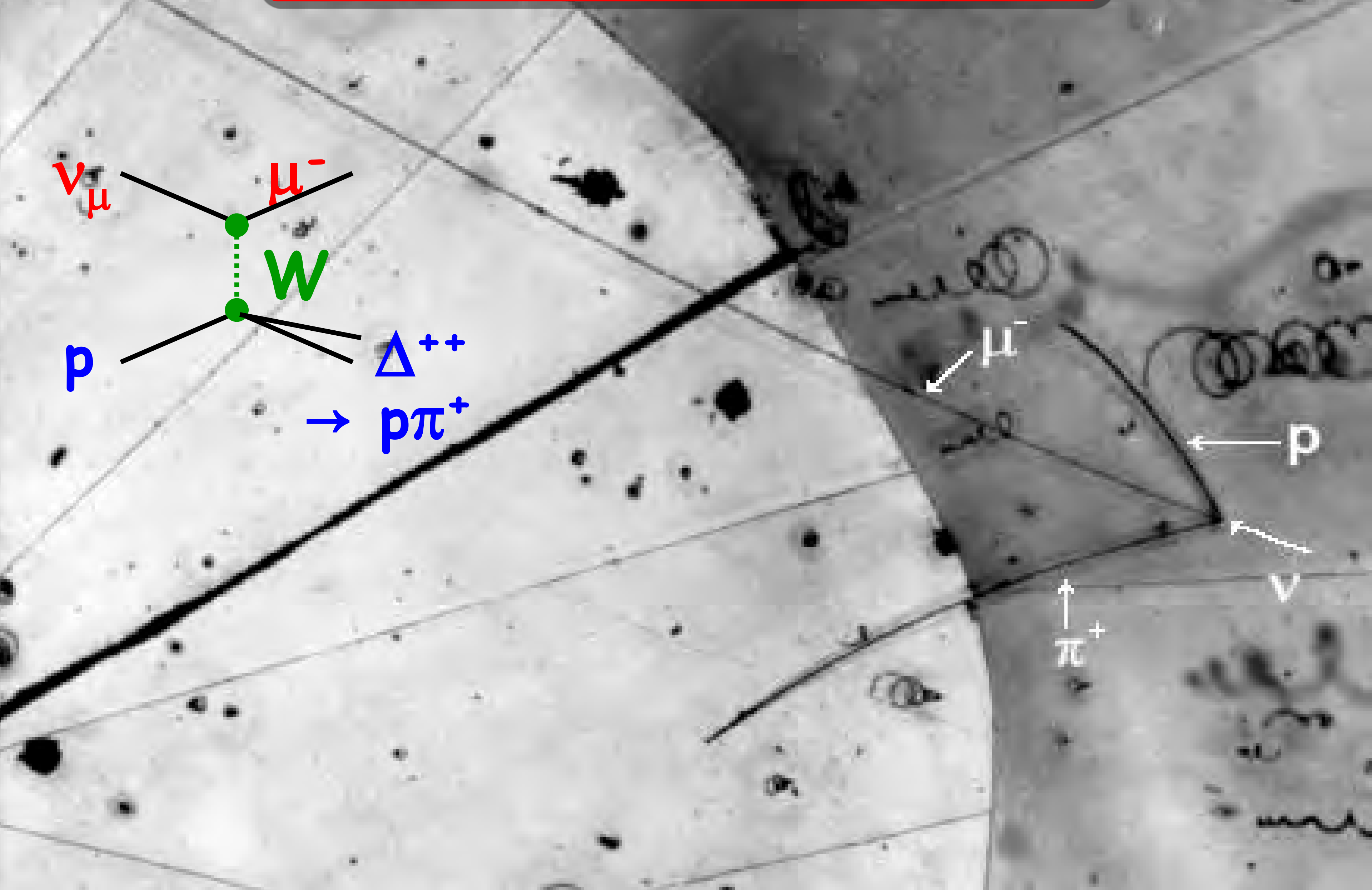
Weak Reactions

current

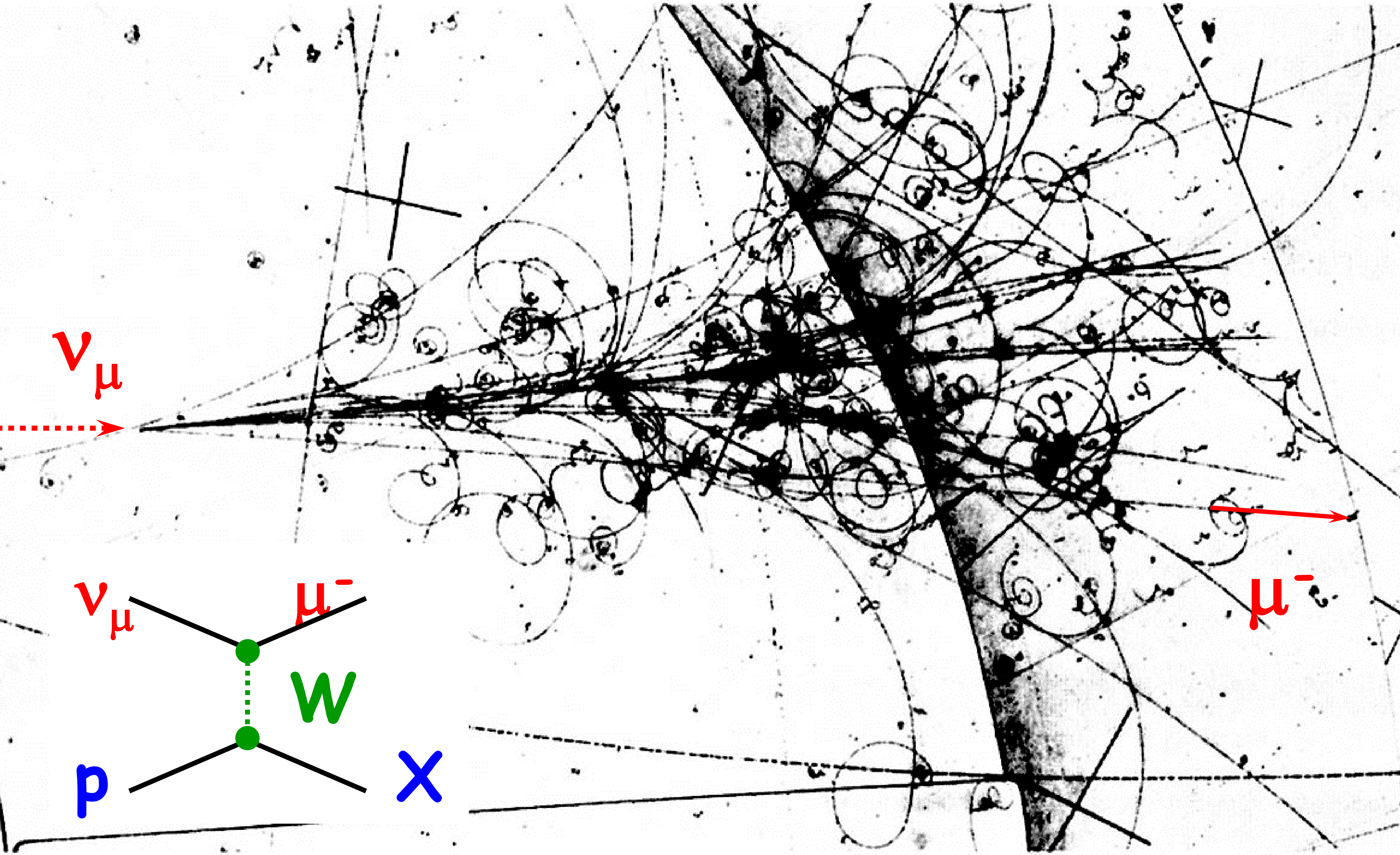
charged **neutral**



Charged Weak Current



Charged Weak Current

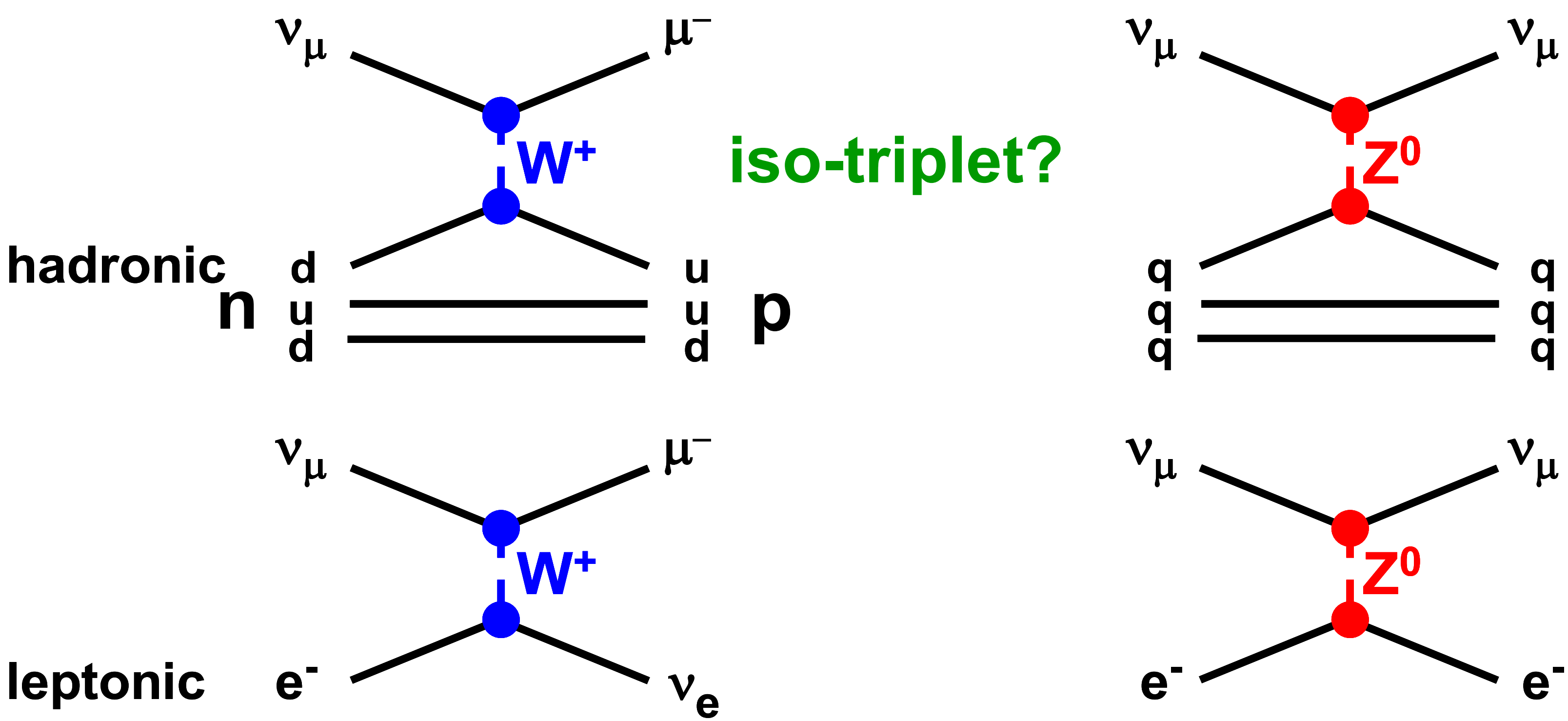


CERN **1984**: Big European Bubble Chamber BEBC

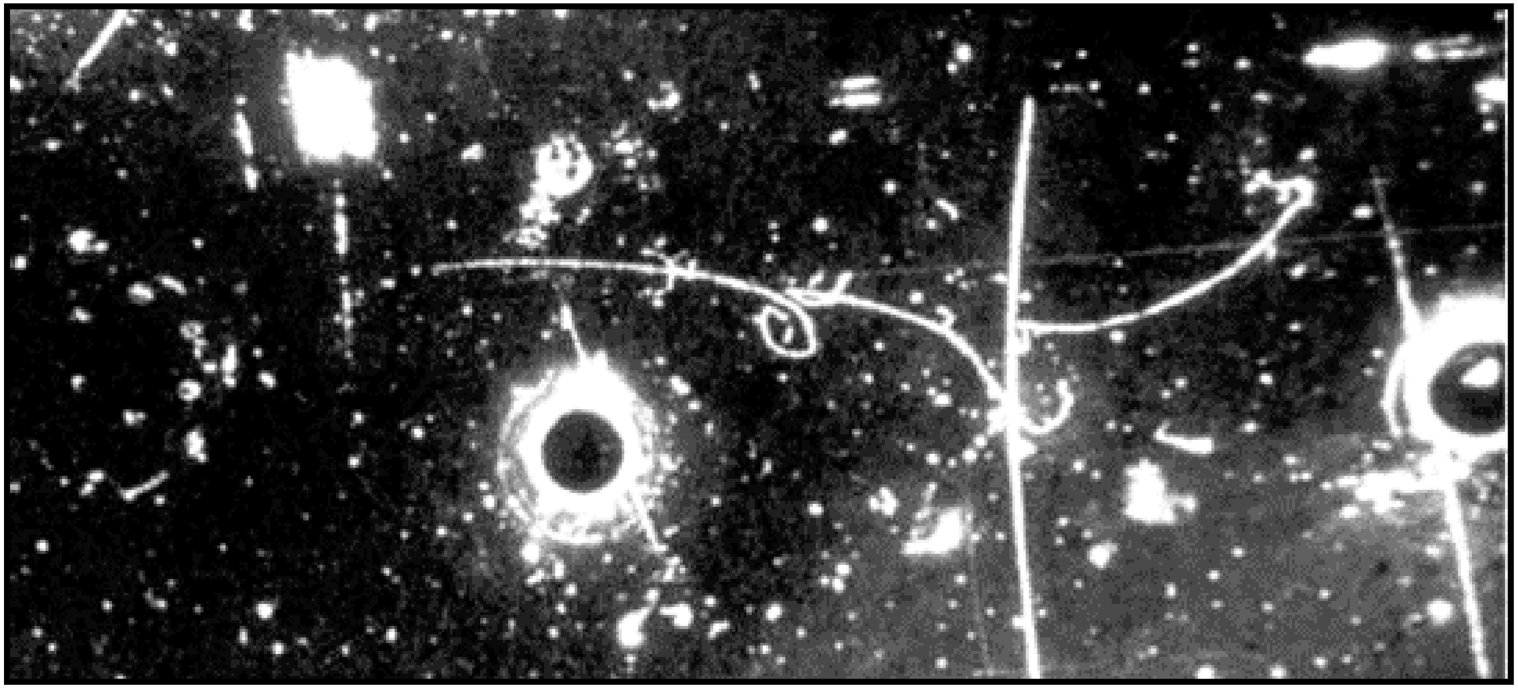
Weak Reactions

current

charged **neutral**



Neutral Weak Current

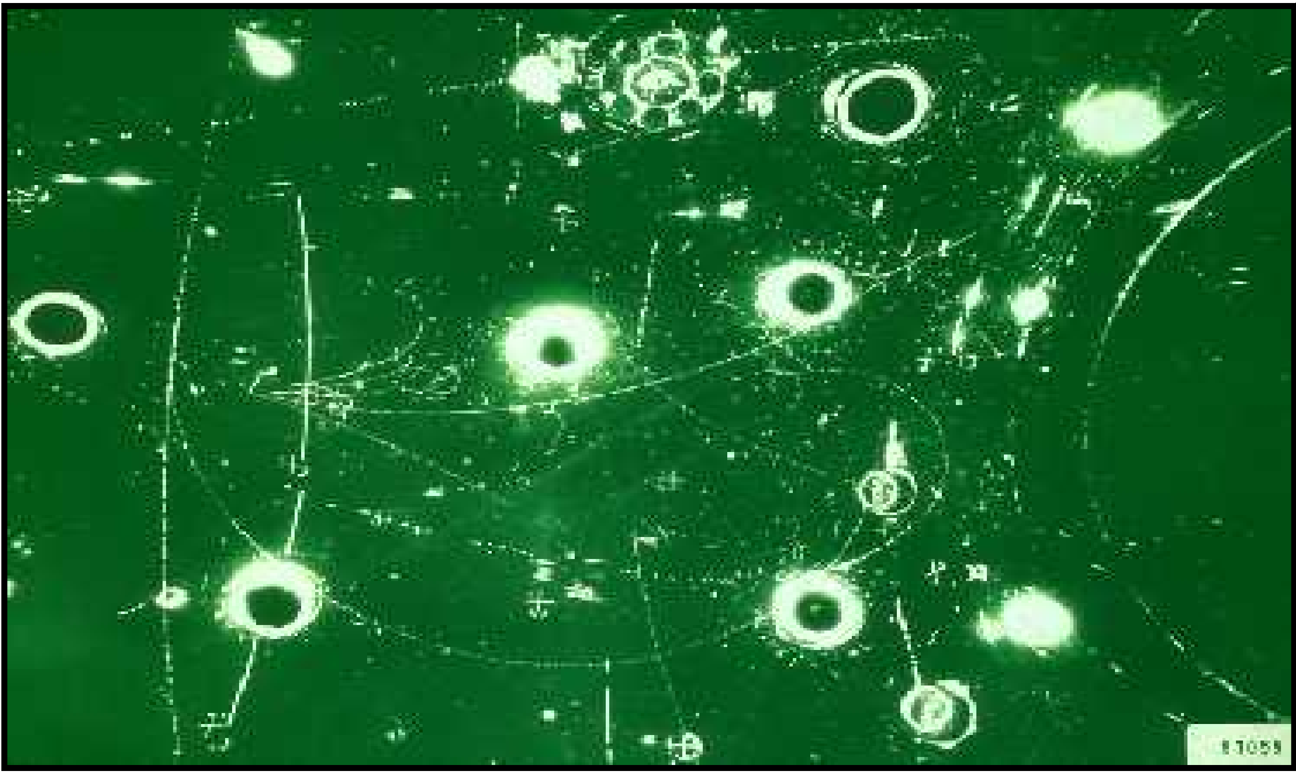


CERN 1973 : Gargamelle Heavy Liquid Bubble Chamber

Neutral Weak Current

$$\nu_\mu N \rightarrow \nu_\mu X$$

CERN 1973: Gargamelle Heavy Liquid Bubble Chamber

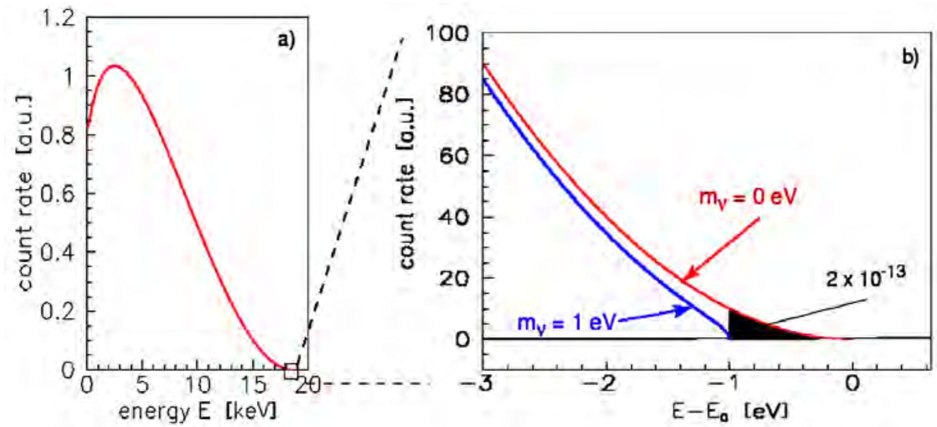


Neutrino Mass

Kurie plot: end point of Tritium β spectrum

$H^3 \rightarrow He^3\ e^-\ \bar{\nu}_e$

problem: nuclear models



MAINZ expt.: m (ν_e) < 2.2 eV

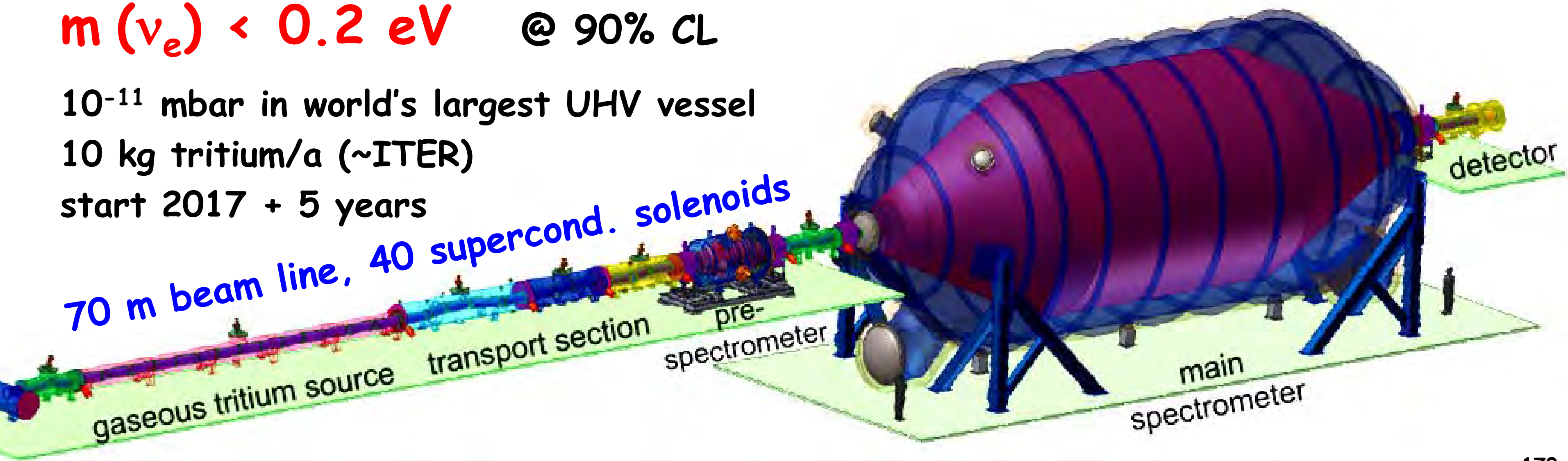
KATRIN: KArlsruhe TRItium Neutrino Experiment

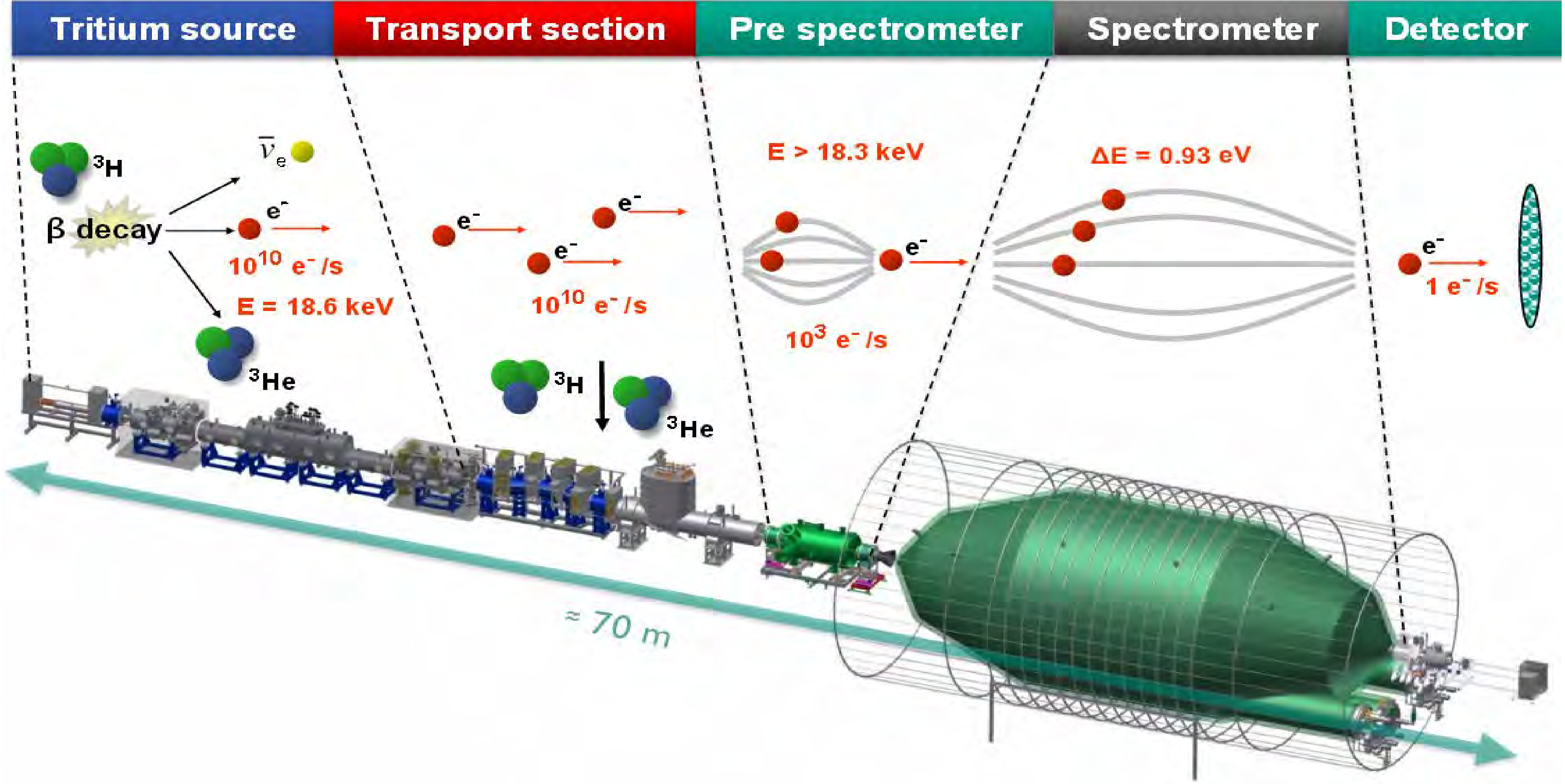
m (ν_e) < 0.2 eV @ 90% CL

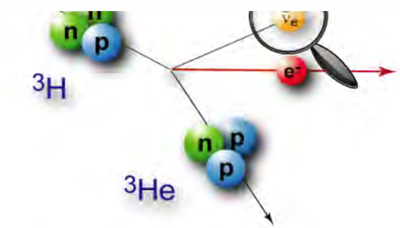
10^{-11} mbar in world's largest UHV vessel

10 kg tritium/a (~ITER)

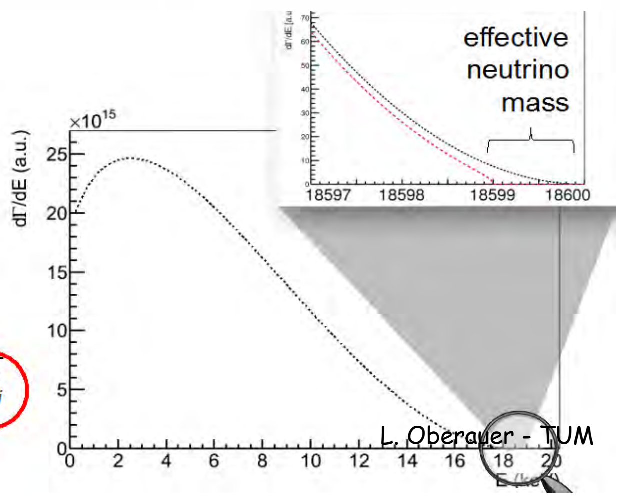
start 2017 + 5 years







$$\frac{d\Gamma}{dE} = C \cdot F(E,Z) \cdot p \cdot (E + m_e)$$
$$\cdot (E - E_0) \cdot \sum_i |U_{ei}|^2 \sqrt{(E - E_0)^2 - m_{\nu i}^2}$$



KATRIN at KIT

spectrometer: retarding MAC-E filter

transport of KATRIN's large spectrometer vessel from Leopoldshafen to FZ Karlsruhe

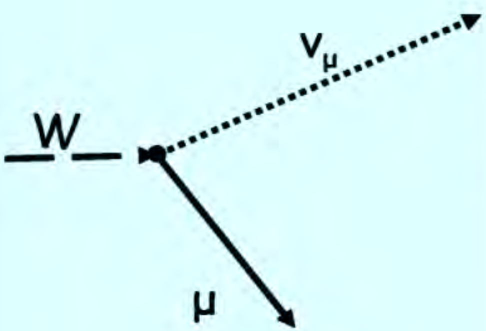
Neutrino oscillations

Flavor eigenstates
ν_μ, ν_τ

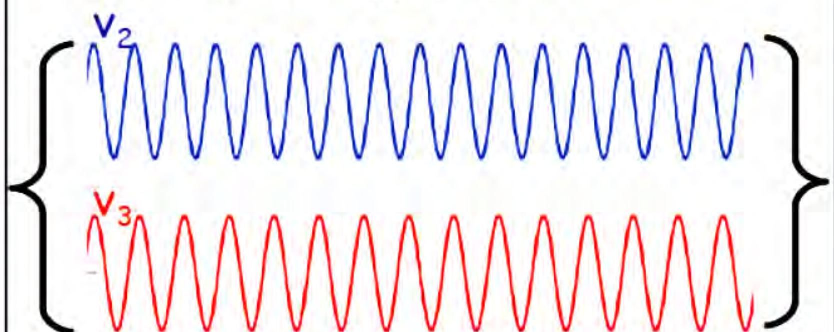
$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta_{23} & \sin\theta_{23} \\ -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}$$

Mass eigenstates
ν_2, ν_3 with m_2, m_3

source creates
flavor-eigenstates



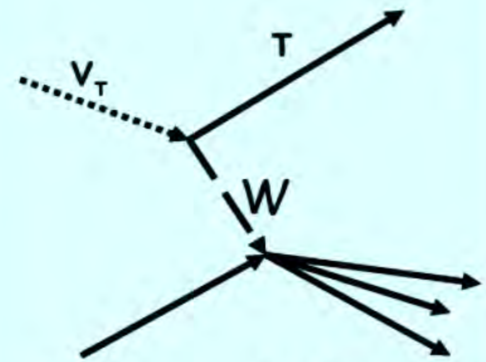
propagation determined by
mass-eigenstates



$$\omega_{2,3} = E_{2,3} = \sqrt{p^2 + m_{2,3}^2}$$

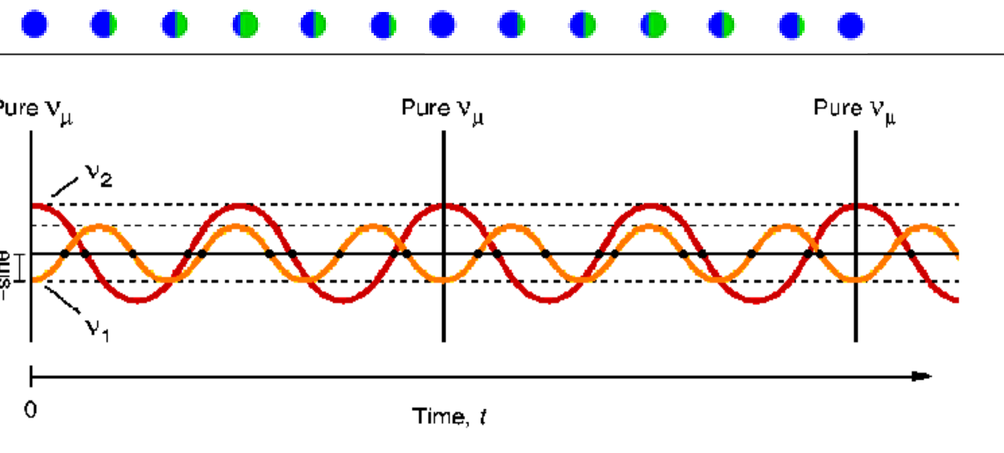
slightly different frequencies
→ phase difference changes

detector sees
flavor-eigenstates



θ=45° would be max. mixing: $\nu_e = \nu_1 - \nu_2$ $\nu_\mu = \nu_1 + \nu_2$

Neutrino oscillations



2 Neutrinos: ν_e,ν_μ

$$|\nu_e(0)\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$
$$|\nu_\mu(0)\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

$$|\nu_\mu(t)\rangle = -\sin\theta\;\exp[-\frac{iE_1t}{\hbar}]\;|\nu_1\rangle + \cos\theta\;\exp[-\frac{iE_2t}{\hbar}]\;|\nu_2\rangle$$

$$E_i = \sqrt{p_i^2 + m_i^2} \quad \xrightarrow{p_i=p\gg m_i} \quad \simeq p + \frac{m_i^2}{2p} \quad \simeq p + \frac{m_i^2}{2E}$$

$$L = c\cdot t \qquad \Delta m^2 = m_2^2 - m_1^2 \Rightarrow \quad E_2 - E_1 = \frac{\Delta m^2}{2E}$$

2ν-transition-probability:

$$P(\nu_\mu \to \nu_e) = |\langle\nu_\mu(t)|\nu_e(0)\rangle|^2 = \sin^2 2\theta\cdot\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$= \sin^2 2\theta\;\cdot\sin^2\,(1.27\;[\Delta m^2/\mathrm{eV}^2]\;[L/\mathrm{km}]\;/\;[E/\mathrm{GeV}])$$

Neutrino mixing matrix

quarks: Cabbibo-Kobayashi-Maskawa (CKM) matrix

neutrinos: Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\mathrm{U}_{\mathrm{MNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & +c_{23} & +s_{23} \\ 0 & -s_{23} & +c_{23} \end{pmatrix} \begin{pmatrix} +c_{13} & 0 & +s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & +c_{13} \end{pmatrix} \begin{pmatrix} +c_{12} & +s_{12} & 0 \\ -s_{12} & +c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Super-Kamiokande

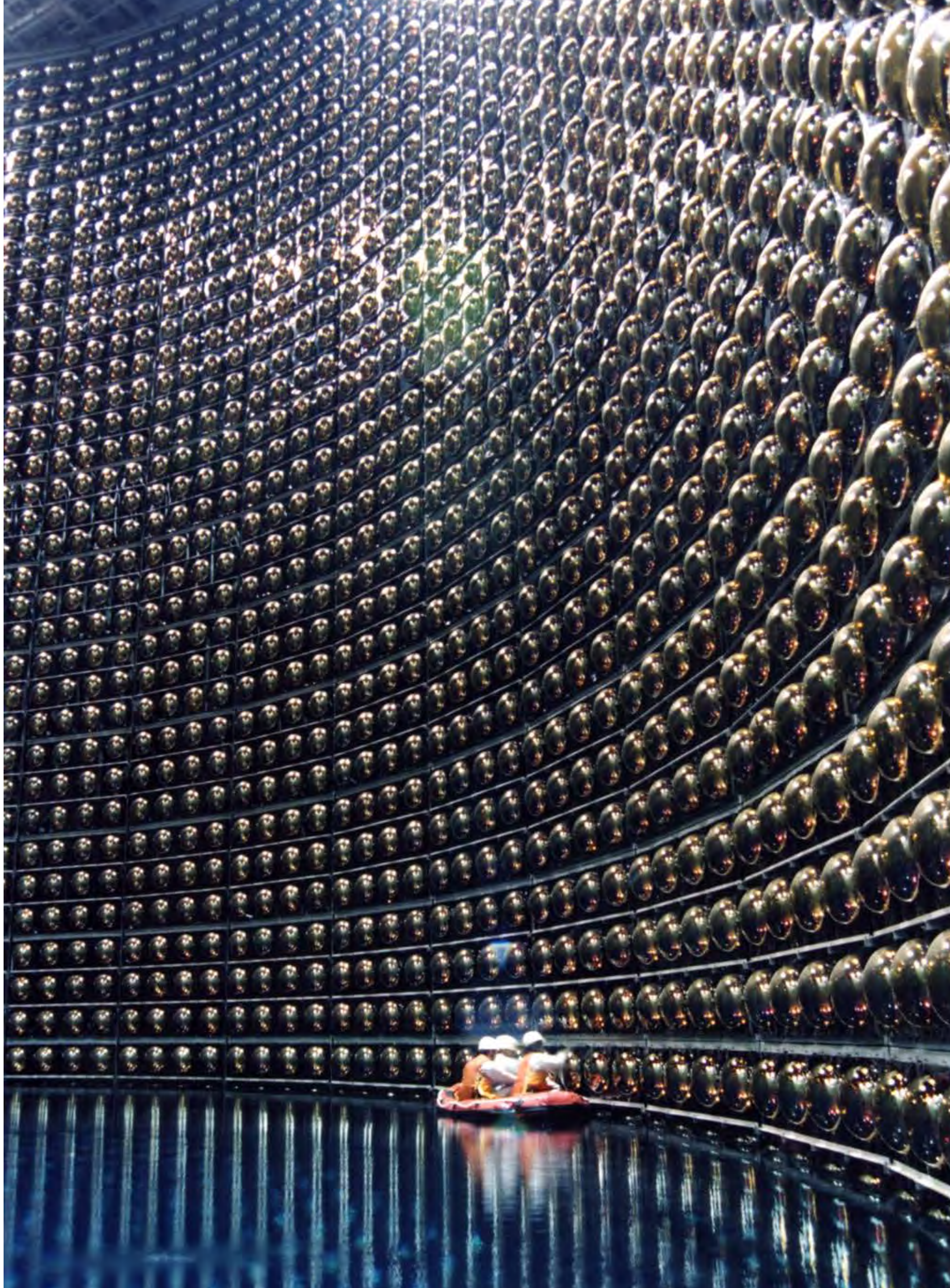
Kamioka mine, Japan:
1000 m underground

detect Cerenkov light
in 22.5 of 50 kt water
or $>10^{34}$ nucleons

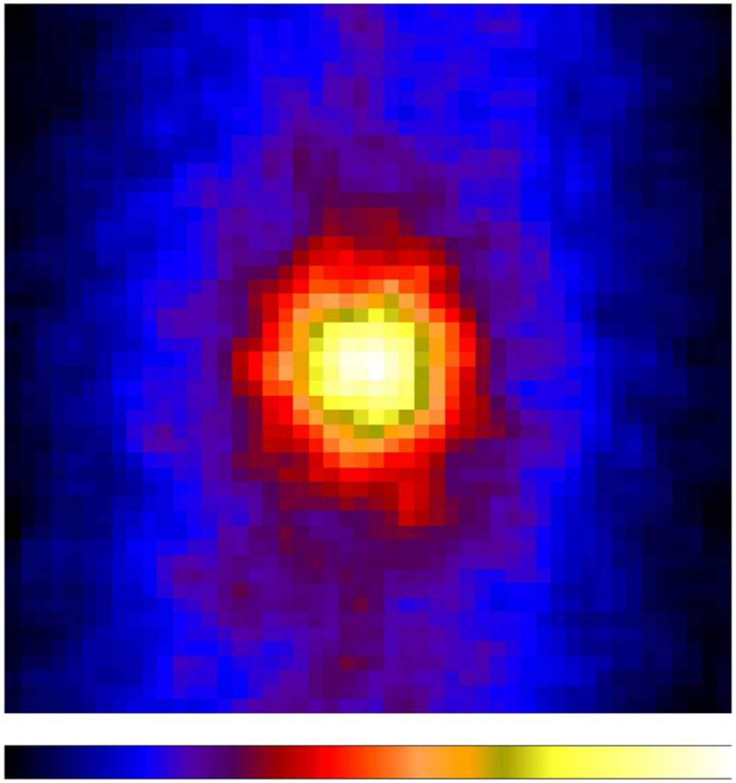
11.000 photomultipliers

built for p decay
1996-2015: 306 kt·y:
$\tau(p \to \pi^0 e^+) > 1.6\ 10^{34}$ a

arXiv:1610.03597



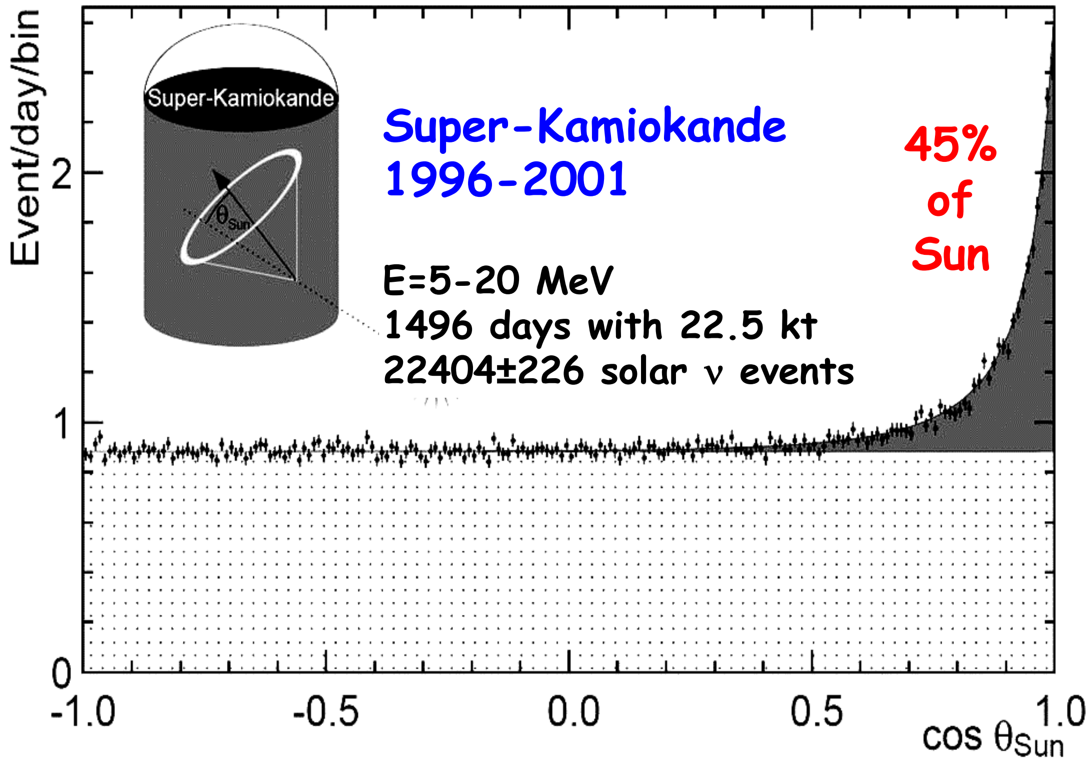
Solar Neutrinos



Sun light thermalized after >100.000 years

First Sun neutrino-graphy
$90^0 \times 90^0$
SuperKamiokande 1998

Borexino 2014: flux of 0.4 MeV ν's
from dominant pp fusion corresponds to photon flux



45±2% of Bahcall solar model

Where are the solar neutrinos?

Is the Sun ok ?

Atmospheric Neutrinos

Super-Kamiokande:

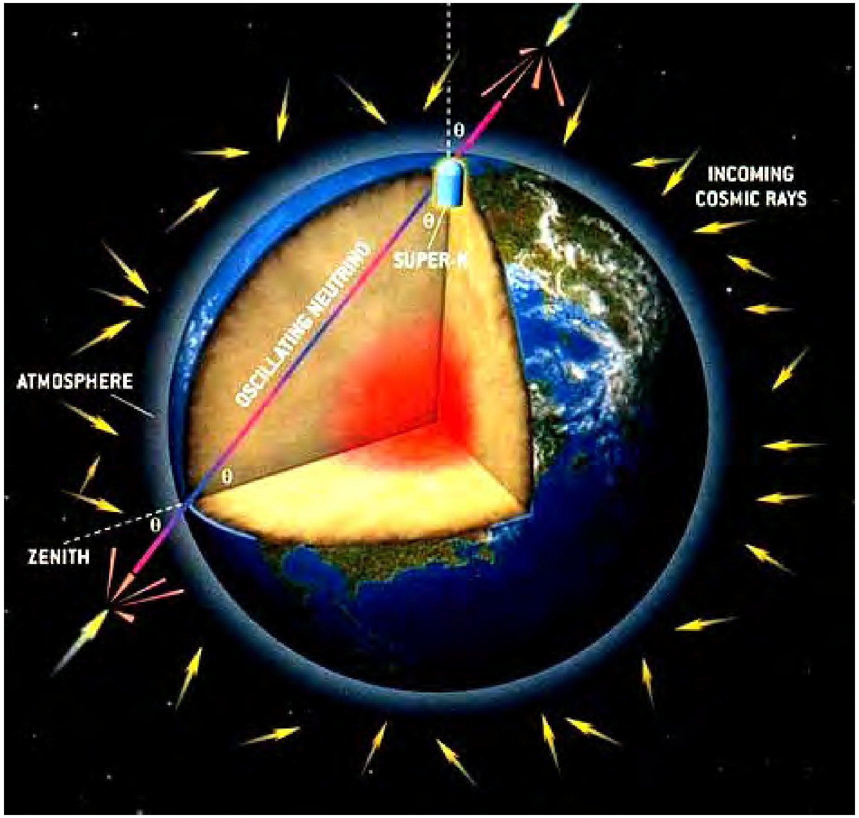
cosmic protons
react in atmosphere:

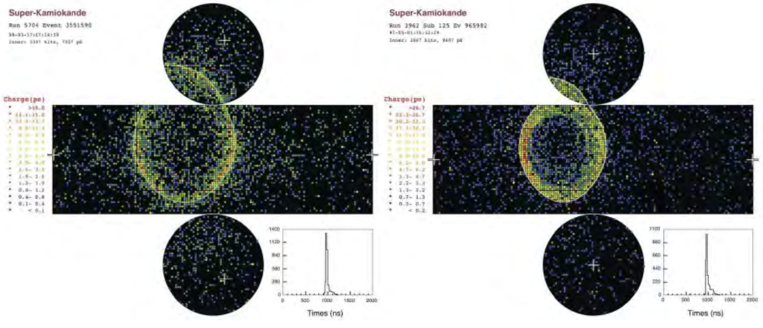
$p\,A \rightarrow \pi$'s + ...

$\pi \rightarrow \mu\, \nu_\mu$ $\qquad$ $\mu \rightarrow \nu_\mu\, e\, \nu_e$

expect $\nu_\mu/\nu_e \sim 2/1$
observe $\sim 1/1$ dep. on θ

$\nu_\mu \rightarrow \nu_e$
detect Cerenkov rings:
$\nu_\mu \rightarrow \mu$ disappear
$\nu_e \rightarrow e$ appear

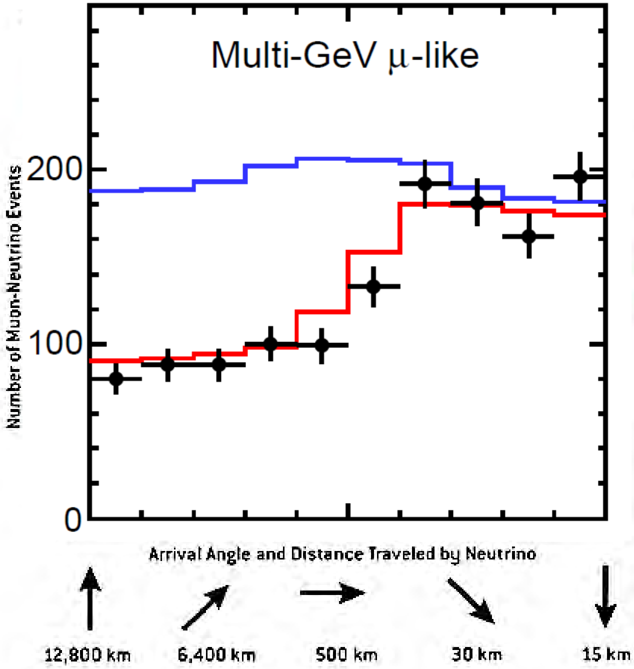


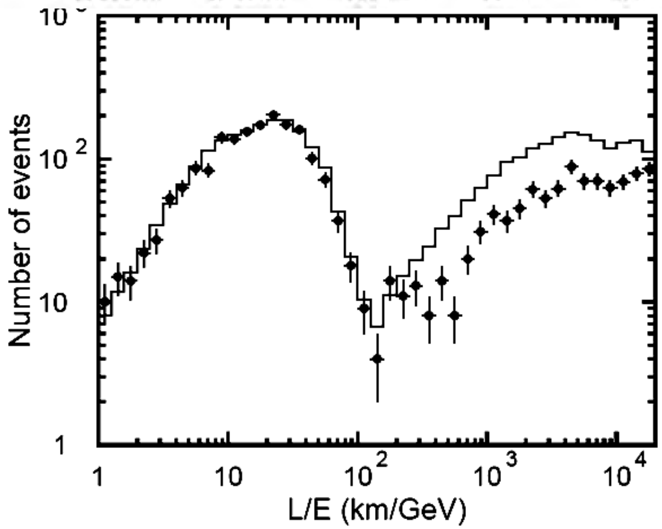


M.Koshiba

Nobel prize
2002

23-mix, GeV: Atmospheric Neutrinos





Super Kamiokande 2004:

Nobel 2015

Kajita McDonald (SNO)

rate too small as f (θ, E) !

ν_μ disappear !

ν oscillation with mass difference

$\Delta m_{23}^2 = 2.4 \pm 0.4 \cdot 10^{-3}$ eV^2

large mixing angle:

$\sin^2 2\theta_{23} > 0.92$ @ 90% CL

$\theta_{23} \sim 45°$ max. mixing ?

Solar neutrinos: SNO

Sudbury Neutrino Observatory, Canada

lent ~1 reactor load ~1000 t heavy water ~330 M$

CC: ν_e d -> p p e get ν_e flux: solar ν_e missing

NC: ν_x d -> p n ν_x get ν_{all} flux: solar models ok

CC: e in water Cerenkov, detect by photomultipliers

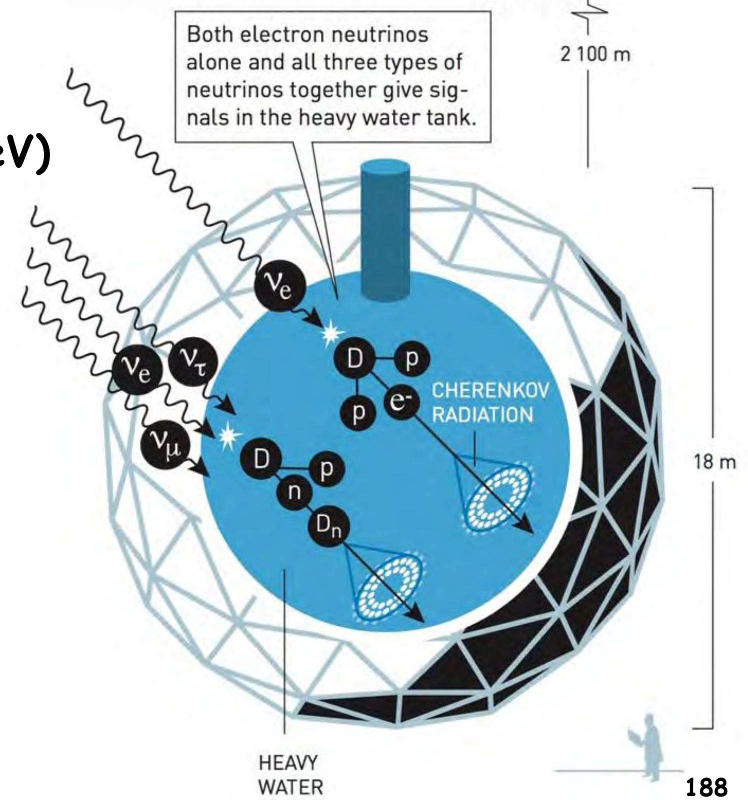
NC: n capture in salt: n ^{35}Cl -> ^{36}Cl -> Cl + γ's (8MeV)

~10 evts/s radioactive background

~30 evts/day solar neutrinos

2002-4: ν_{all} from Sun ok

but $\nu_e \sim \nu_{all}/3$



Nobel 2015

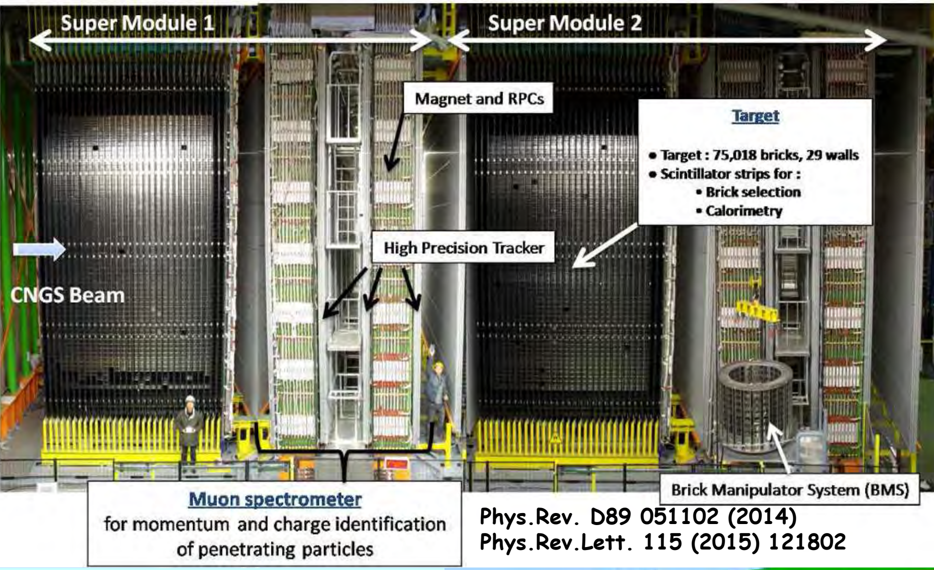
T.Kajita — Super-Kamiokande

A.McDonald — SNO

ν_τ appearance: CNGS+OPERA

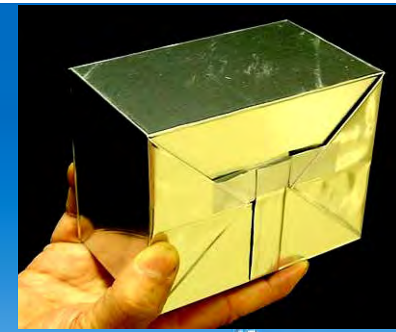
CERN **N**eutrino ν_μ beam to **G**ran **S**asso Underground Lab near Rome

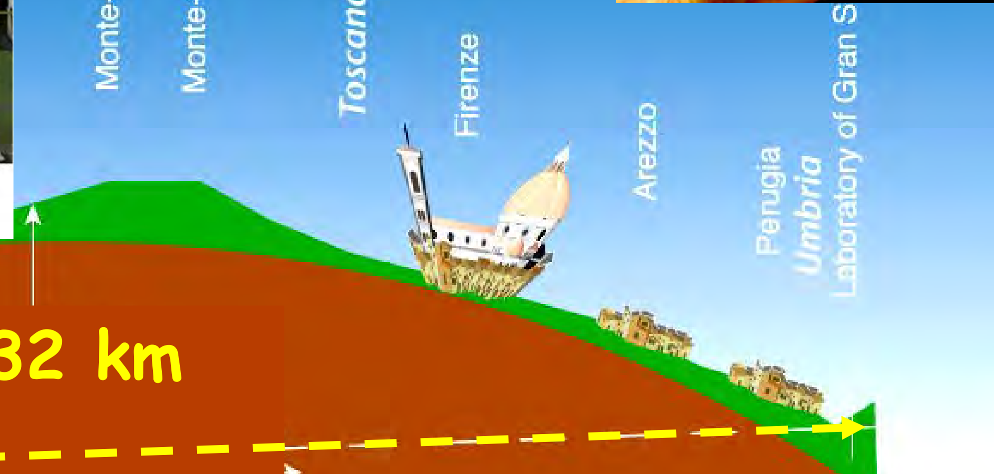
detect ν_μ-ν_τ osci's: τ leptons appear in active emulsion target



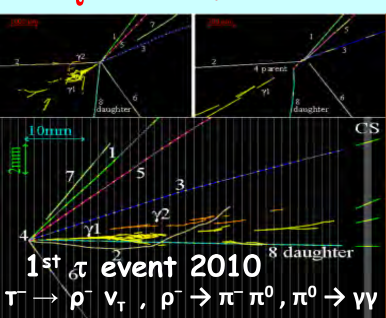
Phys.Rev. D89 051102 (2014)
Phys.Rev.Lett. 115 (2015) 121802

150.000 lead (56x1mm)-emulsion (57x44μm) tiles of 8 kg = 1.250 t





2008-12: 2 10^{20} p:
5 $\nu_\tau \rightarrow \tau$ seen
bkgr. 0.25
$c\tau_\tau$ = 87 μm



1st τ event 2010
$\tau^- \rightarrow \rho^- \nu_\tau$, $\rho^- \rightarrow \pi^- \pi^0$, $\pi^0 \rightarrow \gamma\gamma$

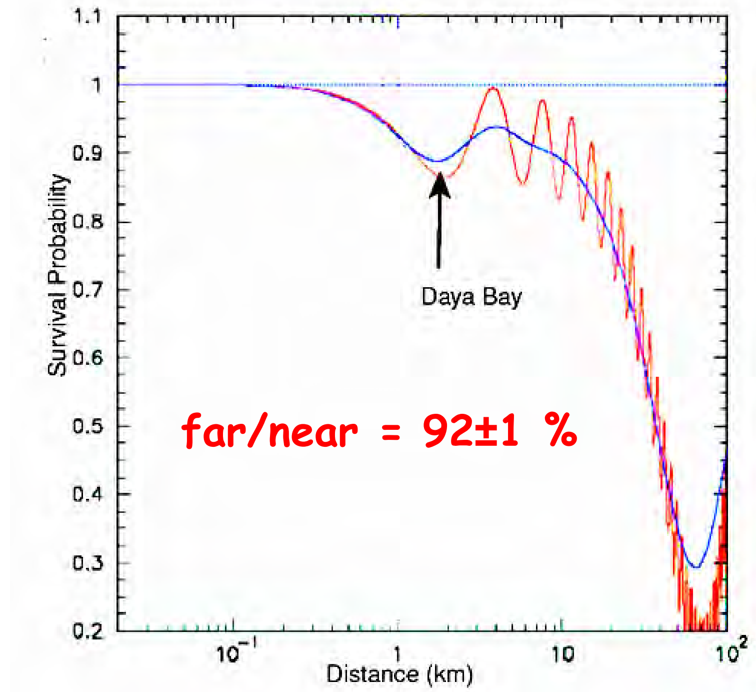
$\nu_\mu \rightarrow \nu_\tau$ $\nu_\tau N \rightarrow \tau X$

high E ! $\theta_{13} = 9°$

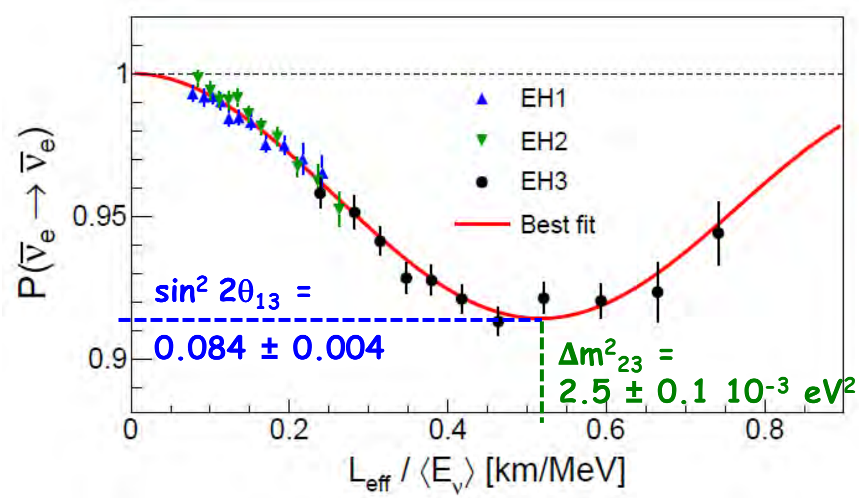
Daya Bay

$$P(\bar{\nu}_e \to \bar{\nu}_e) \approx 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{1.27\Delta m^2_{31} L}{E_\nu}\right)$$

55 km NE of Hongkong: 6 nuclear reactors
power 3x2x2.9 = 17.6 GW_{th} 3.6×10^{21} ν/s
2x2 near + 4 far detectors in 3 halls at 0.5, 2 km
total target mass 160 t
50 and 10 ν_e interactions/h, 2.5 million total

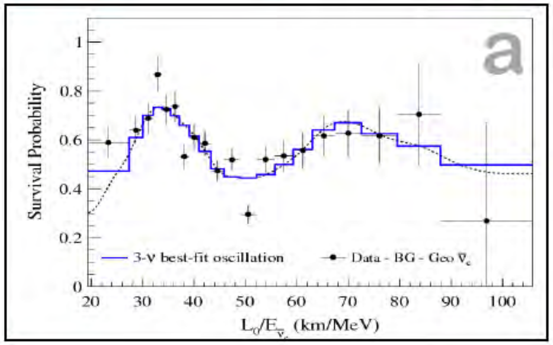


arXiv:1610.04802

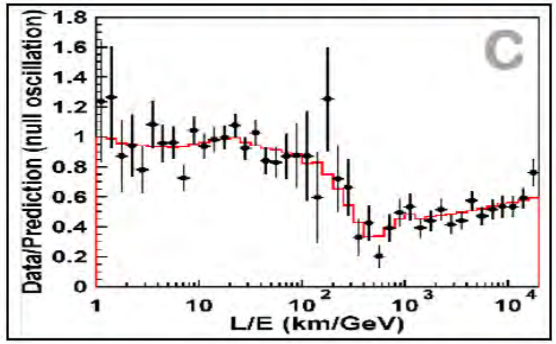


ν oscillation parameters

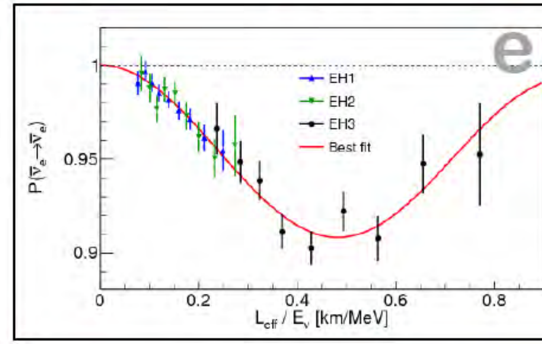
$e \to e$ (δm^2 , θ_{12})



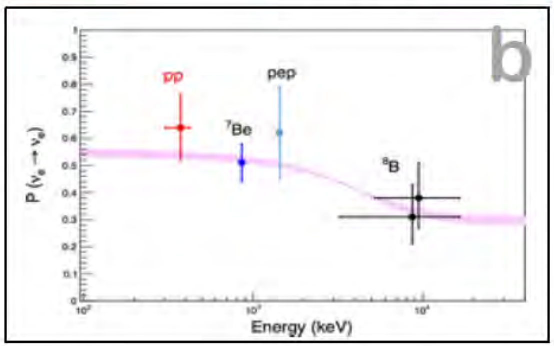
$\mu \to \mu$ (Δm^2 , θ_{23})



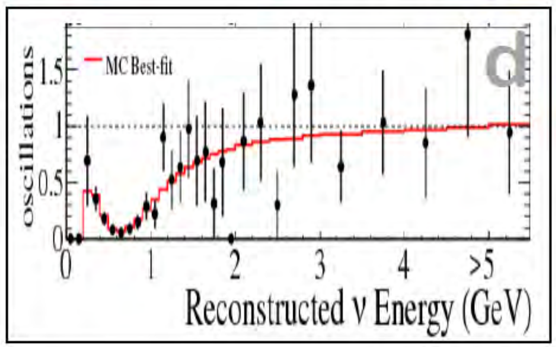
$e \to e$ (Δm^2 , θ_{13})



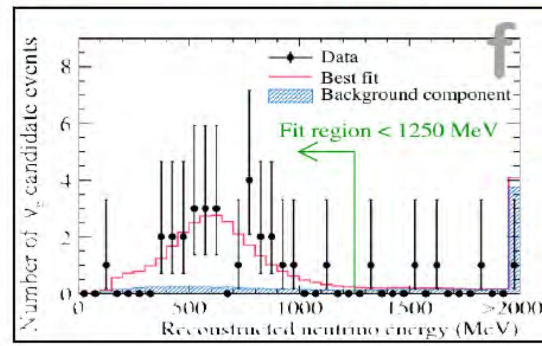
$e \to e$ (δm^2 , θ_{12})



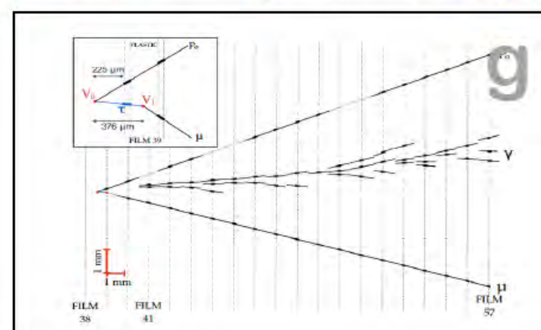
$\mu \to \mu$ (Δm^2 , θ_{23})



$\mu \to e$ (Δm^2 , θ_{13} , θ_{23})



$\mu \to \tau$ (Δm^2 , θ_{23})



Data from various types of neutrino experiments: (a) solar, (b) long-baseline reactor, (c) atmospheric, (d) long-baseline accelerator, (e) short-baseline reactor, (f,g) long baseline accelerator (and, in part, atmospheric).

(a) KamLAND [plot]; (b) Borexino [plot], Homestake, Super-K, SAGE, GALLEX/GNO, SNO; (c) Super-K atmosph. [plot], DeepCore, MACRO, MINOS etc.; (d) T2K (plot), MINOS, K2K; (e) Daya Bay [plot], RENO, Double Chooz; (f) T2K [plot], MINOS, NOvA; (g) OPERA [plot], Super-K atmospheric.

Neutrino oscillations: Results

$\Delta m^2_{13} = \Delta m^2_{12} + \Delta m^2_{32}$ $\Delta m^2_{12} \ll \Delta m^2_{32}$ => $\Delta m_{13}{}^2 \approx \Delta m_{23}{}^2$

12 mixing: MeV, solar + reactor neutrinos:

small: $\Delta m_{12}{}^2 = 7.6 \pm 0.2 \cdot 10^{-5}\ \mathrm{eV}^2$ (KamLAND)

large: $\sin^2\theta_{12} = 0.32 \pm 0.02$ $\theta_{12} = 34 \pm 1°$ (SNO, solar)

$m(\nu_\mu) < 9$ meV $< 10^{-7}\ m_\mu$ in simplest mass hierarchy - why ?

23 mixing: GeV, atmospheric + accelerator: ν_μ disappear

large: $\Delta m_{23}{}^2 = 2.55 \pm 0.04 \cdot 10^{-3}\ \mathrm{eV}^2 \approx \Delta m_{13}{}^2$ (MINOS, NOvA)

max: $\sin^2\theta_{23} = 0.43 \pm 0.02$ $\theta_{23} = 41 \pm 1°$ (SuperK, T2K)

13 mixing: reactors: ν_e disappear (Daya Bay, RENO, Chooz)

small: $\sin^2\theta_{13} = 0.022 \pm 0.001$ $\theta_{13} = 8.4 \pm 0.2°$

experiments: www.nu-fit.org, arXiv:1708.01186

FermiLab, USA	accelerator	MiniBoonE, MINOS, NOvA	ν_μ disappearance
KEK, Japan	accelerator	T2K, KamLand ν_μ-ν_e appearance	ν_μ disappearance
CERN, Geneva	accelerator	CERN-Gran Sasso OPERA	ν_μ-ν_τ appearance
France, China, Korea	reactor	Double-Chooz, Daya Bay, RENO	ν_e disappearance
South Pole, Mediterr.	atmospheric	IceCube, Antares	ν_μ disappearance

Neutrino Mixing

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$\theta_{23} = 43 \pm 2°$

$\theta_{13} = 9 \pm 1°$, δ?

$\theta_{12} = 34 \pm 1°$

Atmospheric, accelerator
- MINOS (precision)
- T2K
- OPERA (ν_τ appearance)

- Reactor:
  RENO, Daya Bay
  CHOOZ
- MINOS, T2K

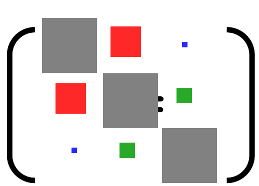
- Solar
- KAMLAND

sign of Δm^2_{23}?

CP-violation: Dirac CP-phase?

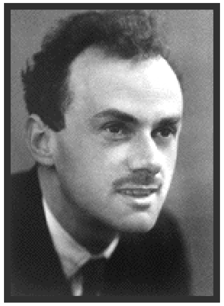
MSW matter effect

Quarks: $V_{CKM} = \begin{pmatrix} 1.0 & 0.2 & 0.001 \\ 0.2 & 1.0 & 0.01 \\ 0.001 & 0.01 & 1.0 \end{pmatrix}$



Neutrinos: $V_{PMNS} = \begin{pmatrix} 0.8 & 0.5 & 0.15 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix}$

Why so different ?



Majorana equation

E.Majorana
1933 Leipzig
1937 Neutrino

Dirac equation $(i\gamma^\mu \partial_\mu - m)\,\Psi = 0$
describes particles Ψ + antiparticles Ψ^*:
real γ matrices, complex spinor fields Ψ.

Bosons like photon etc. are their own antiparticles.

Majorana 1937:

Can also fermions be their own antiparticles:

$$\Psi = \Psi^* \quad ?$$

To get real Ψ define imaginary γ matrices:

$$\tilde{\gamma}^0 = \sigma_2 \otimes \sigma_1$$
$$\tilde{\gamma}^1 = i\sigma_1 \otimes 1$$
$$\tilde{\gamma}^2 = i\sigma_3 \otimes 1$$
$$\tilde{\gamma}^3 = i\sigma_2 \otimes \sigma_2$$

$$\tilde{\gamma}^0 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$$

$$\tilde{\gamma}^1 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}$$

$$\tilde{\gamma}^2 = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix}$$

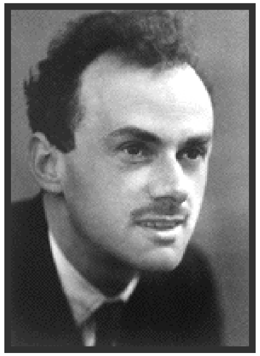
$$\tilde{\gamma}^3 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

satisfying Clifford algebra + Majorana equ.

$$(i\,\bar{\gamma}^\mu \partial_\mu - m)\,\bar{\Psi} = 0$$

with real fields $\Psi = \Psi^*$.

Supersymmetry: neutralinos (= photino x zino x higgsinos)

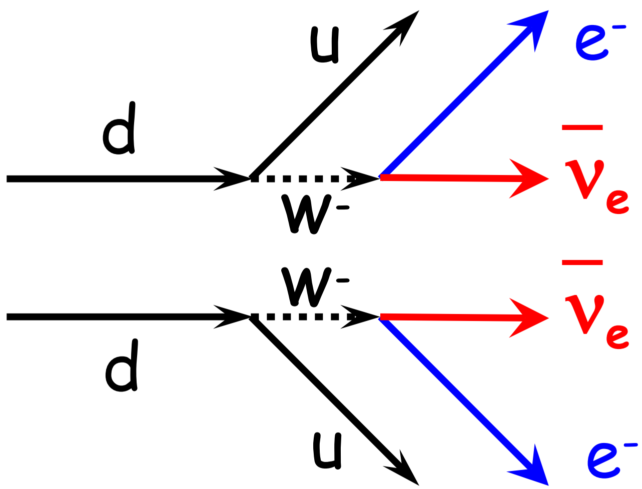


neutrino physics without neutrinos: Double Beta Decay

E.Majorana
1933 Leipzig
1937 Neutrino

$2\nu\ \beta\beta$:

$(A,Z) \rightarrow (A,Z+2) + 2e^- + 2\ \bar{\nu}_e$



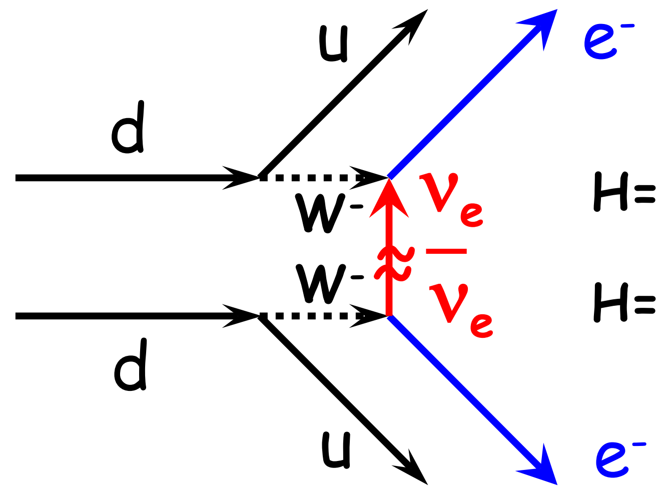
$\sim G_F^4$

ΔL=0 Dirac

Observed for several nuclei
Test of nuclear calculations
Dirac only ?

$0\nu\ \beta\beta$:

$(A,Z) \rightarrow (A,Z+2) + 2e^-$



H=L= -1

H=L= +1

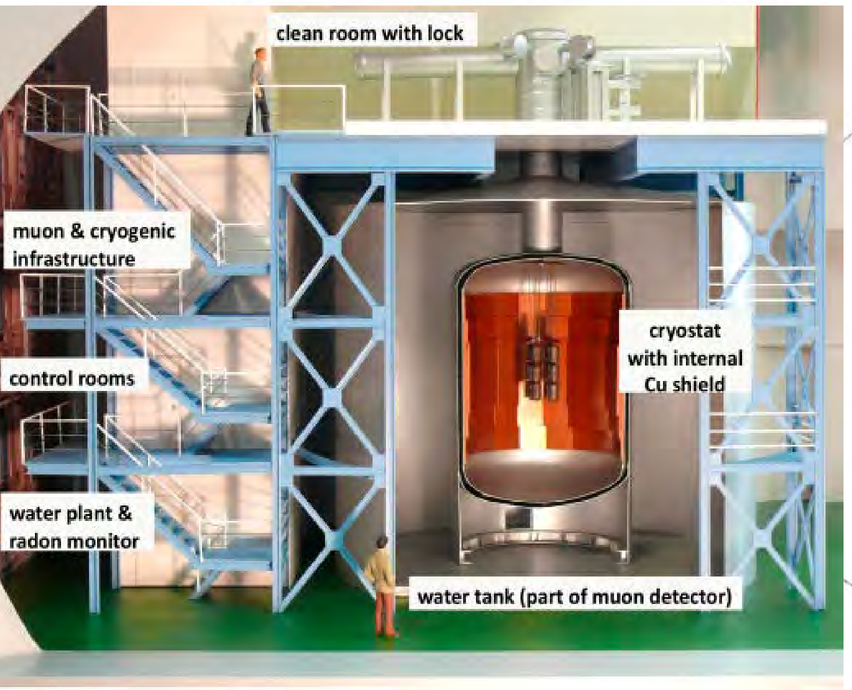
ΔL=2 Majorana

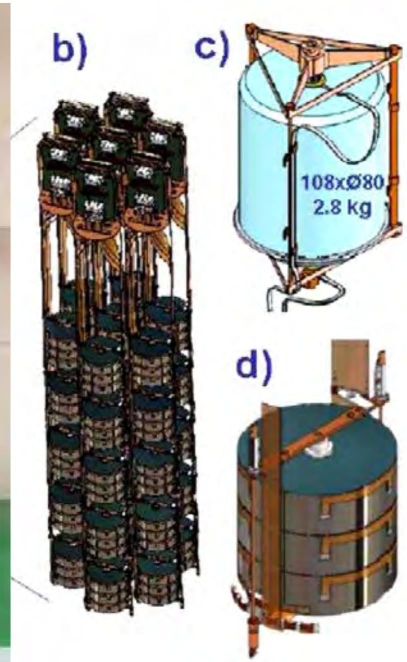
double-weak $\sim G_F^4$
helicity suppressed: $\sim m_\nu^2/E_\nu^2 < (eV/100MeV)^2$
leptogenesis: L(+B) asymmetry of Universe

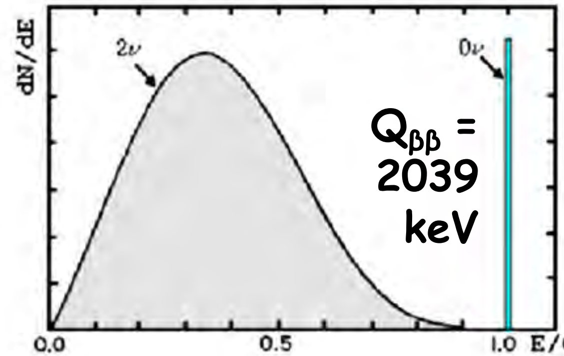
Double Beta Decay

Gran Sasso tunnel, Italy, 1400 m underground

GERDA: 36 kg high purity 86% enriched ^{76}Ge diodes in liquid Ar cryostat







<100 kg y ^{76}Ge

$2\nu\ \beta\beta$: $T_{1/2} = 2\ 10^{21}$ y

$0\nu\ \beta\beta$: $T_{1/2} > 5\ 10^{25}$ y

arXiv:1703.00570. Nature 554 (201) 47

$m_{\beta\beta} <$ 0.3-0.1 eV

2014: MAJORANA 40 kg Ge $m_{\beta\beta} <$ 0.1 eV

Double Beta Results

2ν ββ $m_{\beta\beta}$ < 0.3 eV (90% CL) 0ν ββ

Isotope	Technique	$\tau^{0\nu}_{1/2}$ (y)	$<m_{\beta\beta}>$ eV
^{48}Ca	CaF_2 scint	$>1.4\times10^{22}$	<7-45
^{76}Ge (HM)	Ge diode	$>1.9\times10^{25}$	<(0.3-1.27)
^{76}Ge (IGEX)	Ge diode	$>1.6\times10^{25}$	<(0.33-1.35)
^{76}Ge (Klapdor 2004)	Ge diode	1.2×10^{25}	.38
^{76}Ge (Klapdor 2006)	Ge diode	2.2×10^{25}	.28
^{76}Ge (GERDA I)	Ge diode	$>2.1\times10^{25}$	<(.29-1.1)
^{76}Ge (GERDA+HM+IGEX)	Ge diode	$>3\times10^{25}$	<(.25-.98)
^{82}Se	Foil&track	$>.6\times10^{23}$	<(0.89-2.)
^{96}Zr	Foil&track	$>9.2\times10^{21}$	<(7.2-19.5)
^{100}Mo	Foil&track	$>1.1\times10^{24}$	<(0.31-.79)
^{116}Cd	Scintillator	$>1.7\times10^{23}$	<1.7
^{128}Te	Geochem	$>7.7\times10^{24}$	<(1.1-1.35)
^{130}Te	Bolometer	$>2.8\times10^{24}$	<(0.3-.7)
^{136}Xe	EXO	$>1.6\times10^{25}$	<140-380
^{136}Xe	Kamland Zen	$>1.9\times10^{25}$	**<128-349**
^{136}Xe	EXO+Kamzen	$>3.4\ 10^{25}$ 0.	<120-250
^{150}Nd	Foil TPC	$>1.8\times10^{22}$	

E. Fiorini, Moriond Elw. 2014. EXO-200: arXiv:1402.6956
OM. Lindner, ICHEP 2014 Valencia. GERDA: arXiv:1307.4720
KamLAND Zen. 320 kg ^{136}Xe Phys.Rev.Lett. 110, 062502 (2013)

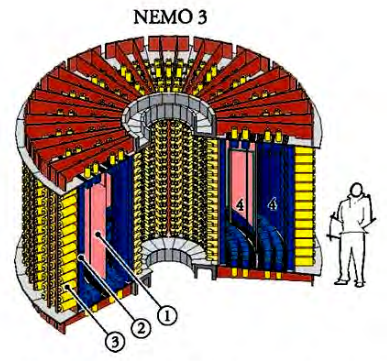
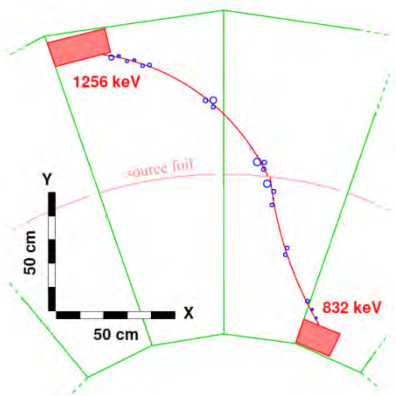


Figure 3. The NEMO-3 detector without shielding [120]. 1 – source foil; 2– plastic scintillator; 3 – low radioactivity PMT; 4 – tracking chamber.

NEMO-3:

7 kg ^{100}Mo foils 2003-10
34 kgy E(ee) = 3034 keV

0ν ββ candidate



EXO	200 kg liquid Xe^{136} 81% + TPC:	$T_{1/2} > 1.8\ 10^{25}$ y	$m_{\beta\beta}$ < 0.4-0.2 eV	arXiv:1707.08707
KamLand	320 kg liquid Xe^{136} 91% + scint:	$T_{1/2} > 1.1\ 10^{26}$ y	$m_{\beta\beta}$ < 0.5-0.2 eV	arXiv:1605.02889, PRL 117
GERDA	36 kg y Ge^{76} 86% :	$T_{1/2} > 5.3\ 10^{25}$ y	$m_{\beta\beta}$ < 0.6-0.2 eV	arXiv:1703.00570. Nature
combined:	Ge, Xe, Te, Mo		$m_{\beta\beta}$ < 0.31-0.13 eV	arXiv:1407.4357, 1504.036
CUORE	Te^{130}	$T_{1/2} > 1.5\ 10^{25}$ y	$m_{\beta\beta}$ < 0.4-0.14 eV	arXiv:1710.07988.

translation $T_{1/2} \rightarrow m_{\beta\beta}$ dominated by nuclear matrix element uncertainty

neutrinos - gate to New Physics

Neutrinos

- oscillate:
  - have mass
  - violate lepton nr - not locally,
    only due to mass diff at macroscopic oscillation lengths

Questions

- mass - not only difference ?
  Curie (tritium), cosmology, $0\nu\beta\beta$
- why $<10^{-6}$ of charged leptons?
  Higgs? see-saw?
- lepton nr violated: $0\nu\beta\beta$, $\mu \rightarrow e$ ... ?
- Dirac or Majorana - $0\nu\beta\beta$?

Weak Interactions

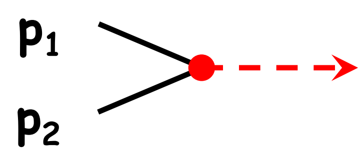
Kinematics

- four-momentum : $p = (E, \mathbf{p})$ $c=1$
- four-momentum^2 : relativistic invariant
- effective mass^2 : $m^2 := p^2 = E^2 - \mathbf{p}^2$

- ultra-relativist.: $m \ll E$ $E = p$
- classic: $p \ll m$ $E = m$

- **s = invariant reaction energy^2**

omit $\mathbf{p_x = p_y = 0}$



$\mathbf{s = (p_1+p_2)^2}$

- ***Center-of-Mass System :*** e^- -------><------- e^+

$(E, \mathbf{p})$ $(E, -\mathbf{p})$

$p_1+p_2 = (2E, 0)$

$s = 4E^2$

- **Lab system :** 1 --------> • 2

$(E_1, \mathbf{p_1})$ $(m_2, \mathbf{0})$

$m_{1,2} \ll E_1$

$p_1+p_2 = (E_1+m_2, E_1)$

$s = 2\, m_2 E_1$

target mass effect: $m_p = 2000\, m_e$

Cross Sections

- cross section: $\sigma = \pi R^2$

  $[\sigma]$ = barn 1 b = 10^{-24} cm^2

  1 mb = 10^{-27} cm^2

  1 fb = 10^{-39} cm^2

  strong interaction: $\sigma(\pi N) \sim 30$ mb

- interaction radius: $R = \sqrt{\sigma/\pi}$

  R (strong) $\sim \sqrt{10^{-30}\,\text{m}^2} \sim 10^{-15}$ m = 1 fm = 1 fermi

- lifetime: $\tau = R / c$ = reaction time

  τ (strong) = 1 fm / c = 3×10^{-24} s

- uncertainty relation: $\hbar = \Gamma\,\tau$

  $\hbar c$ = 200 MeV fm = $\Delta E\,\Delta R$

- decay width: $\Gamma = \hbar / \tau = \hbar c / R$

  energy scale $\Gamma = E$

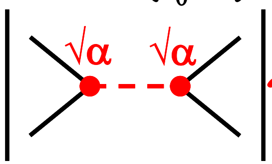
  Γ = 200 MeV fm / R

  E (strong) = 200 MeV Γ (strong: Δ, ρ) = 120-150 MeV

- coupling constant: $F = \alpha\,\hbar c / r^2$ electric force

  $\alpha = e^2 / (4\pi\,\hbar c)$ dimensionless (ε_0=1)

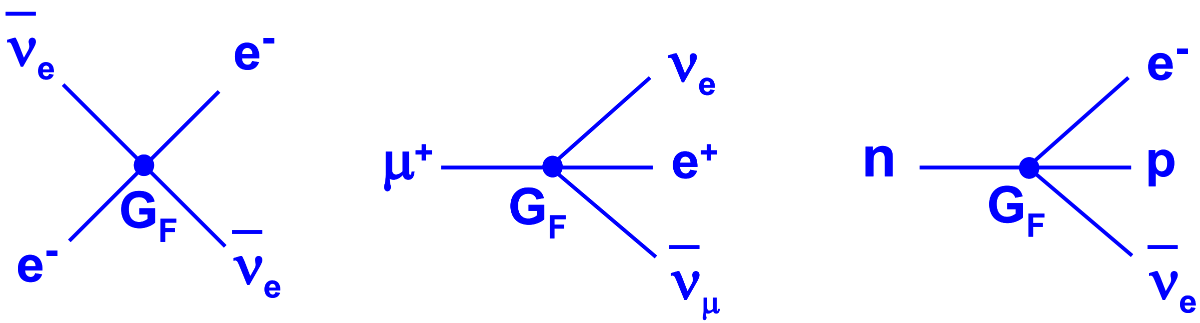
$E = \Gamma = \hbar/\tau = \hbar c / R = \hbar c$

$\sigma \sim$ 

$^2 \sim \alpha^2$

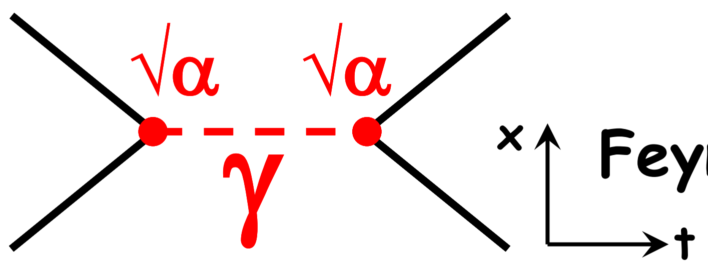
four-fermion theory

E.Fermi, Rome 1933:



four-fermion theory of weak interaction

R.Feynman,
USA 1948:

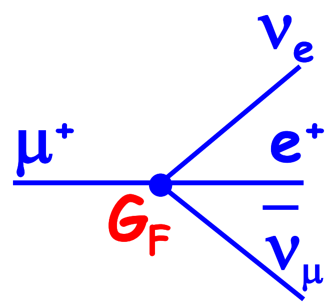


Feynman diagrams

Fermi constant

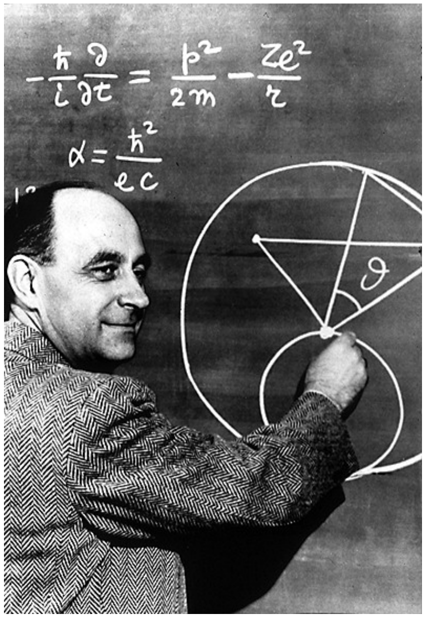
4 Fermi theory of weak interaction with coupling constant

$G_F = 1.1663788\ (7) \times 10^{-5}\ GeV^{-2}$



determined from muon lifetime

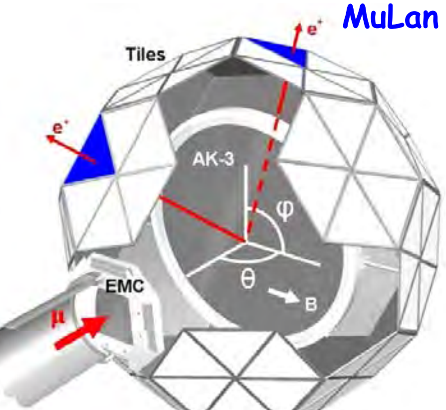
$\tau_\mu = 192\pi^3/(G_F^2 m_\mu^5)\ (1+\delta_{kin})\ (1+\delta_{weak})\ (1+\delta_{QED})$

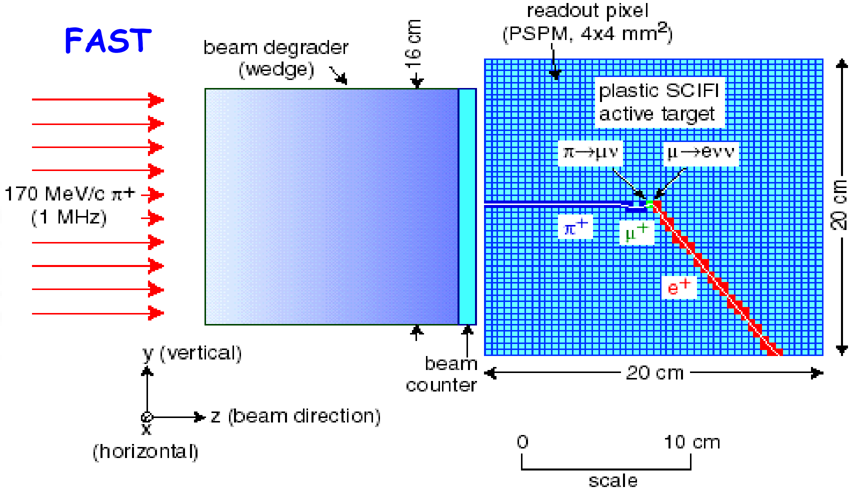


E. Fermi

Nobel  prize

1938





$>10^{12}$ μ decays

$\tau_\mu = 2\,196\,980\,(2)$ ps

get G_F to $<10^{-6}$

FAST, MuLan @ PSI

MuLan: arXiv:1211.0960
FAST: arXiv:0707.3904

Weak interaction

- **QED:** $F = \alpha \hbar c / r^2$ electric force ($\varepsilon_0 = 1$)

coupling constant $\alpha = e^2 / (4\pi \hbar c) = 1/137$ dimensionless !

$\sigma(e^+e^- \to \mu^+\mu^-) = 4\pi/3\ \alpha^2/s \approx 80\text{ nb} / (s/\text{GeV}^2)$

$s \to \infty \Rightarrow \sigma \to 0$ charge point like !

$[\alpha^2] = [\sigma\, s] = [l^2E^2] = [\hbar c]^2 = 1$ dimensionless ($\hbar c$ = 200 MeV fm)

- **weak force:**

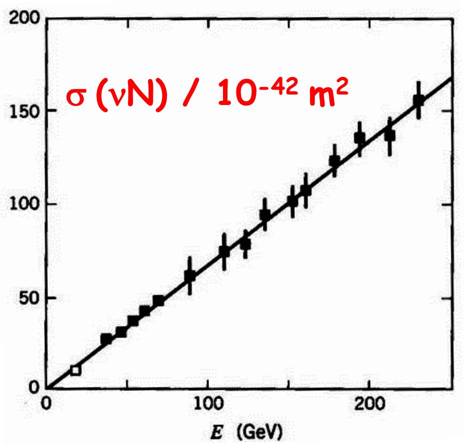
$\sigma(\nu_\mu e^- \to \nu_e \mu^-) = G_F^2\, s / \pi$

$s \to \infty \Rightarrow \sigma \to \infty$ Unitarity violated !

$\hbar^2c^2 = 0.4\text{ GeV}^2\text{ mb}$

$[G_F^2] = [\sigma/s] = [l^2/E^2] = [\hbar^2c^2/E^4] \Rightarrow$

$[G_F] = [E^{-2}] = \text{GeV}^{-2}$???



weak coupling constant G_F contains energy scale !

Weak Interaction

$$\sigma\,(\nu_\mu e^- \to \nu_e \mu^-) \;=\; G_F^2\, s\,/\,\pi$$

$$G_F = 1.2 \times 10^{-5}\ \mathrm{GeV}^{-2}$$

$s \approx 2\, m_e E_\nu$

$\sigma = 9.5 \times 10^{-11}\ \mathrm{GeV}^{-2}\ [m_e/\mathrm{GeV}]\ [E_\nu/\mathrm{GeV}]$

$\hbar c = 1 = 0.2\ \mathrm{GeV\ fm}$

$(\hbar c)^2 = 1 = 0.04\ \mathrm{GeV}^2\ \mathrm{fm}^2 = 0.4\ \mathrm{mb\ GeV}^2$

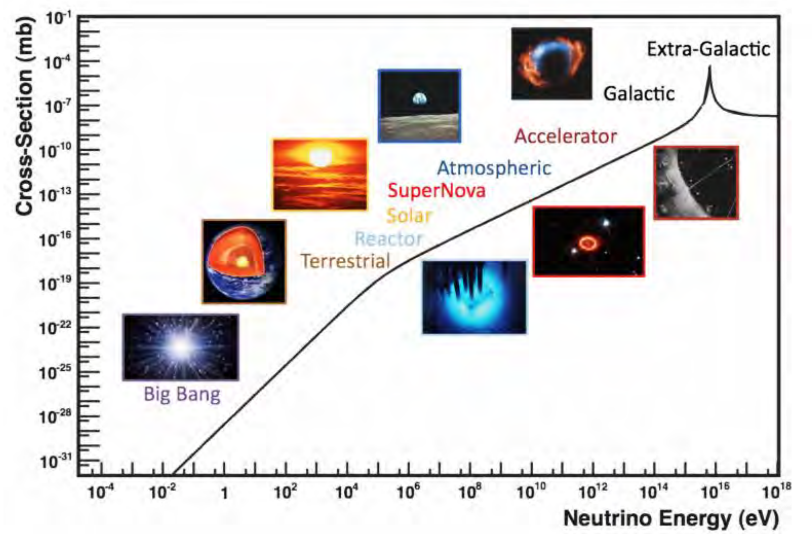
$\sigma = 3.8 \times 10^{-11}\ \mathrm{mb}\ [m_e/\mathrm{GeV}][E_\nu/\mathrm{GeV}] = 10^{-16}\ \sigma\,(\text{strong: } \pi p \to \Delta)$

$\sigma = 3.8 \times 10^{-12}\ \mathrm{fm}^2\ [m_e/\mathrm{GeV}][E_\nu/\mathrm{GeV}]$

$\sigma = 3.8 \times 10^{-42}\ \mathrm{m}^2\ [m_e/\mathrm{GeV}][E_\nu/\mathrm{GeV}]$

σ extremely weak !

$m_N/m_e = 2.000$, $m_A/m_N > 100$ =>
target mass effect !



neutrino interactions

Fermi constant very small:

mean free path of 1 MeV solar neutrino in Earth :

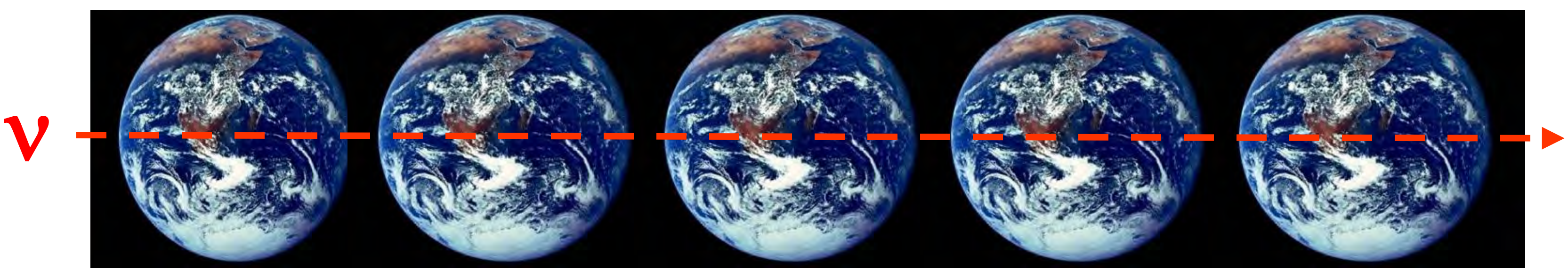
$\sigma = 3.8 \times 10^{-42}\ m^2\ [m_A/GeV][E_\nu/GeV]$

$1/L = \sigma\, \rho\ /\ m_N$

$\rho\ /\ m_N = 5.5\ g/cm^3\ /\ 1.67\ 10^{-24}\ g = 3.3\ 10^{24}\ /cm^3$

$1/L = 3.3\ 10^{24}/cm^3\ 2\ 10^{-44}\ cm^2 = 6.6\ 10^{-20}\ /cm$

$L = 1.5\ 10^{14}\ km = 2\ 10^{10}\ R_{Earth} \sim 10$ **light years:**



10 billion Earths

only 1 solar neutrino/person/human life reacts !

Weak Interaction

$[G_F] = GeV^{-2}$

Which energy scale hidden in G_F ?

assume

point like electro-weak coupling with α_W:

$$G_F^2\, s/\pi = \sigma = \alpha_W^2\, \pi/s$$

$$s^2 = \alpha_W^2\, \pi^2 / G_F^2$$

electro-weak energy scale:

$\sqrt{s} \sim \sqrt{G_F^{-1}} \sim \sqrt{10^5}$ GeV $\sim$ 300 GeV

collapse of Fermi theory

massive exchange boson W

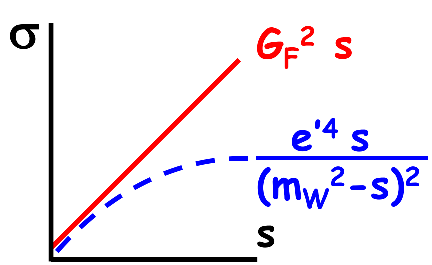
Weak Interaction

$G_F \quad \rightarrow \quad e'^2 / (m_W^2 - s)$

e' ... weak charge, dimensionless
massive W boson, correct s dependence.

- $s \rightarrow 0$: $\quad e'^2 = m_W^2\, G_F \;=\; e^2 = 4\pi\alpha \sim 0.1$
  weak ~ electric force !
  $m_W^2 = 4\pi\alpha / G_F \qquad m_W \sim 90$ GeV

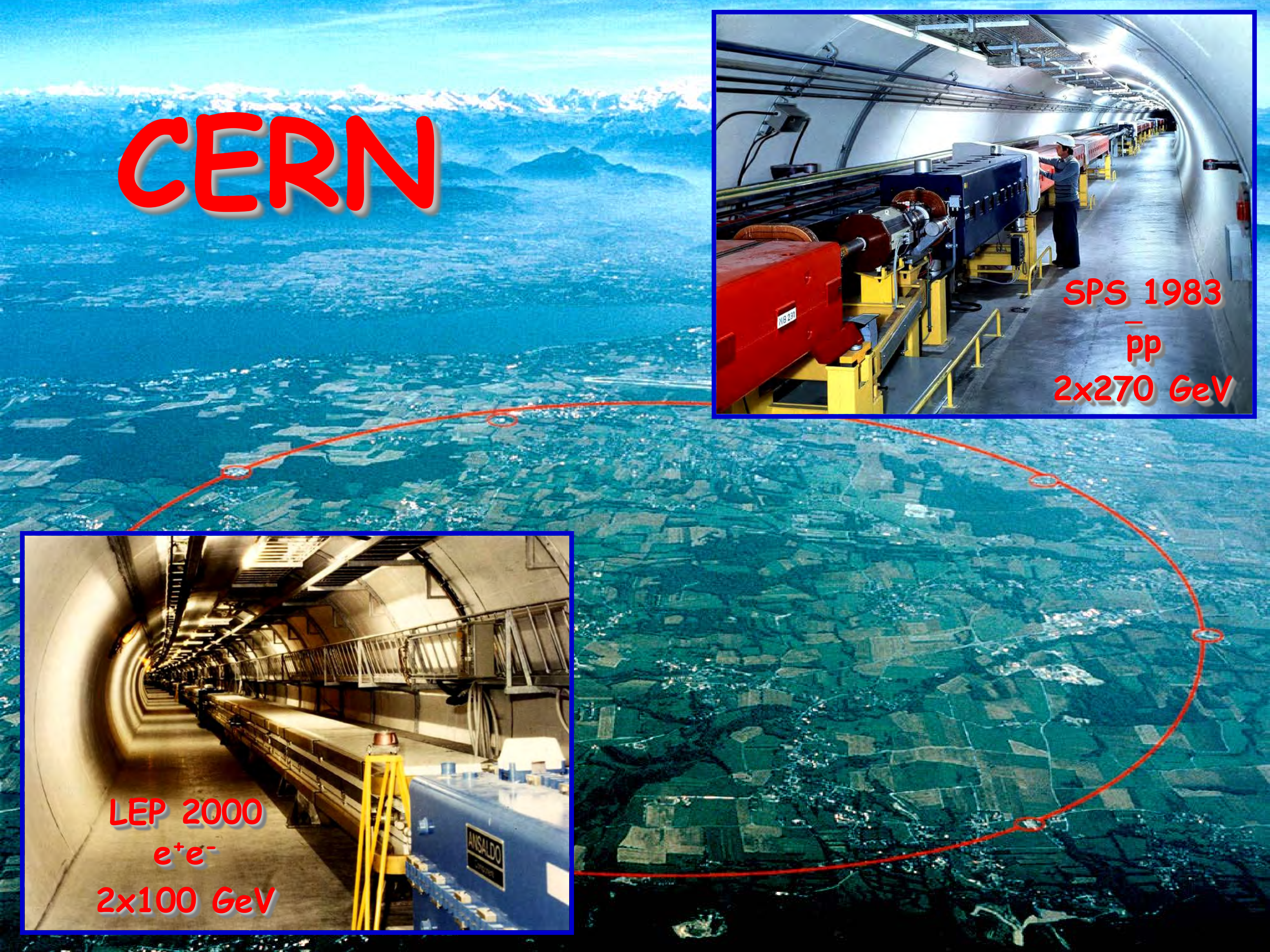
- $s \gg m_W^2$: $\quad \sigma \sim e'^4 / s \qquad$ scatter on pointlike charge



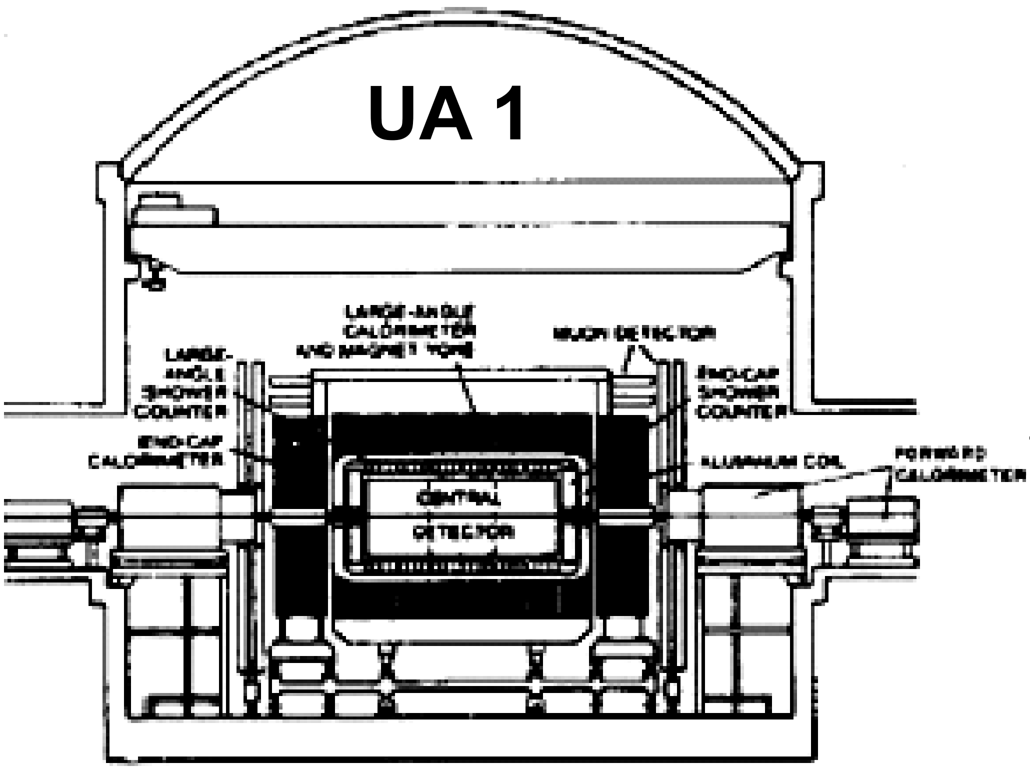
$s \rightarrow \infty \;\Rightarrow\; \sigma \rightarrow \infty \quad$ unitarity violated !

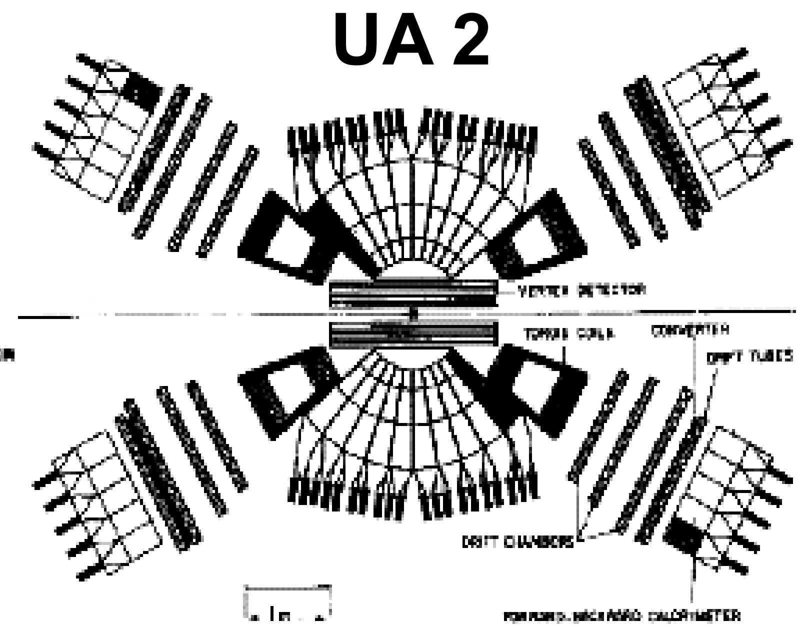
weak charge, dimensionless
massive W boson
correct s dependence

~1970: find $W^\pm + Z^0$ bosons !


CERN
SPS 1983
$\bar{p}p$
2x270 GeV
LEP 2000
e^+e^-
2x100 GeV
ANSALDO

W+Z Discovery



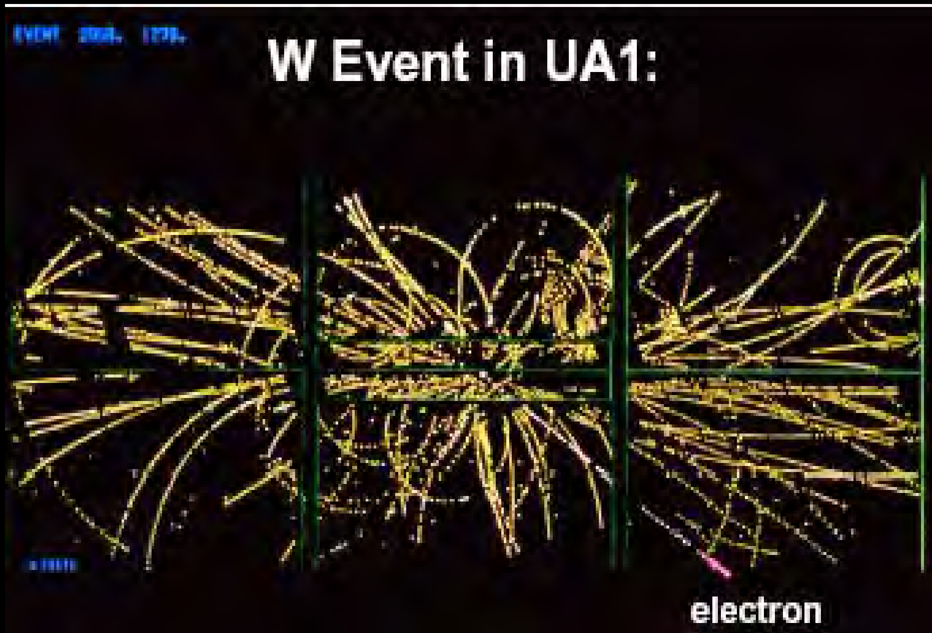


Expts. UA1+UA2 1983 — SPS $p\bar{p}$ collider at CERN

C. Rubbia — S. van der Meer

Nobel prize 1984

W+Z Discovery



$u\,\bar{d} \rightarrow W^+ \rightarrow e^+\,\nu_e$

$q\,\bar{q} \rightarrow Z^0 \rightarrow e^+e^-$

Expts. UA1+UA2 1983

C. Rubbia

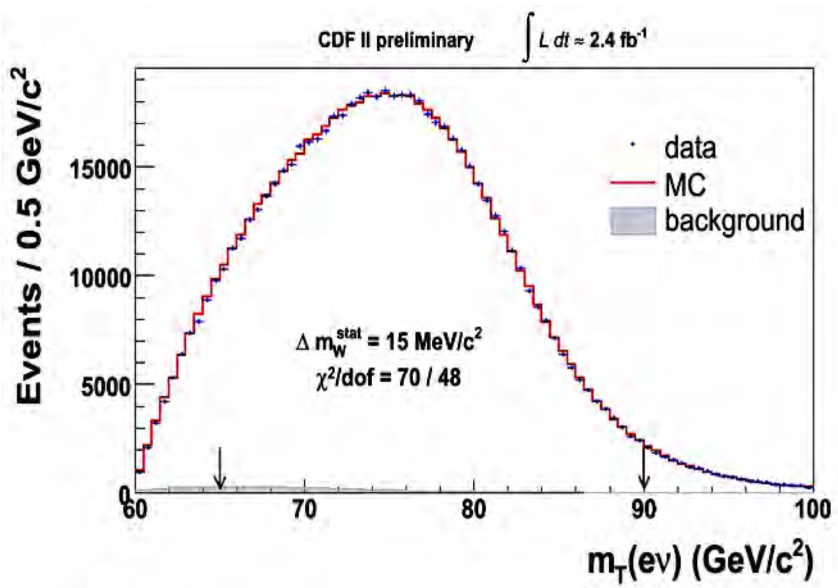
SPS $p\,\bar{p}$ collider at CERN

S. van der Meer

Nobel prize 1984

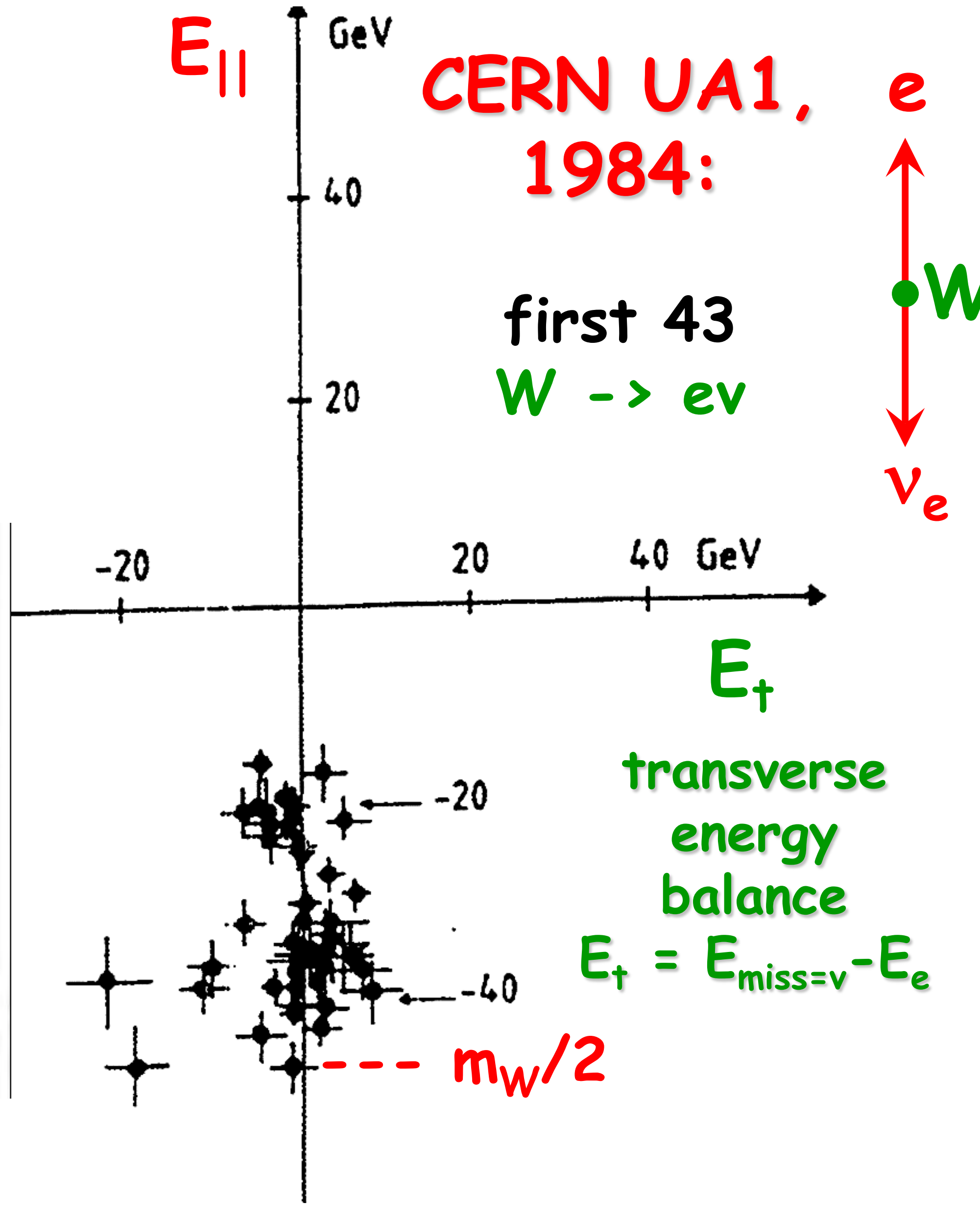
W discovery

CDF+D0 @ Fermilab
>1M $W \rightarrow l\nu$ each



m_W = 80.385 ± 0.015 GeV
Γ_W = 2.08 ± 0.04 GeV

http://tevewwg.fnal.gov/



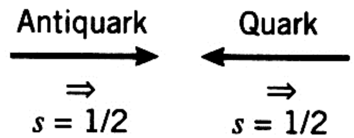
CERN UA1, 1984:

first 43
$W \rightarrow e\nu$

transverse energy balance
$E_t = E_{miss=\nu} - E_e$

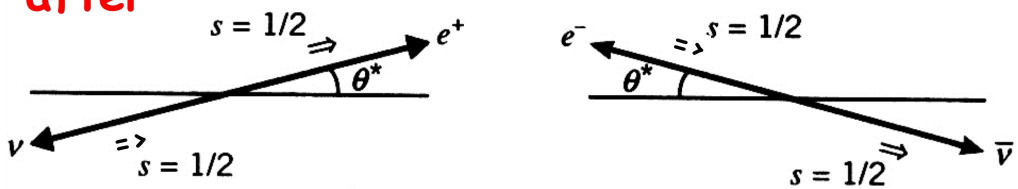
W spin

before



W => s = 1

after



$u\,\bar{d} \rightarrow W^{+} \rightarrow e^{+}\,\nu_e$ $\quad$ $d\,\bar{u} \rightarrow W^{-} \rightarrow e^{-}\,\bar{\nu}_e$

W=1

helicity conservation:

outgoing (anti)fermion keeps direction of incoming (anti)fermion

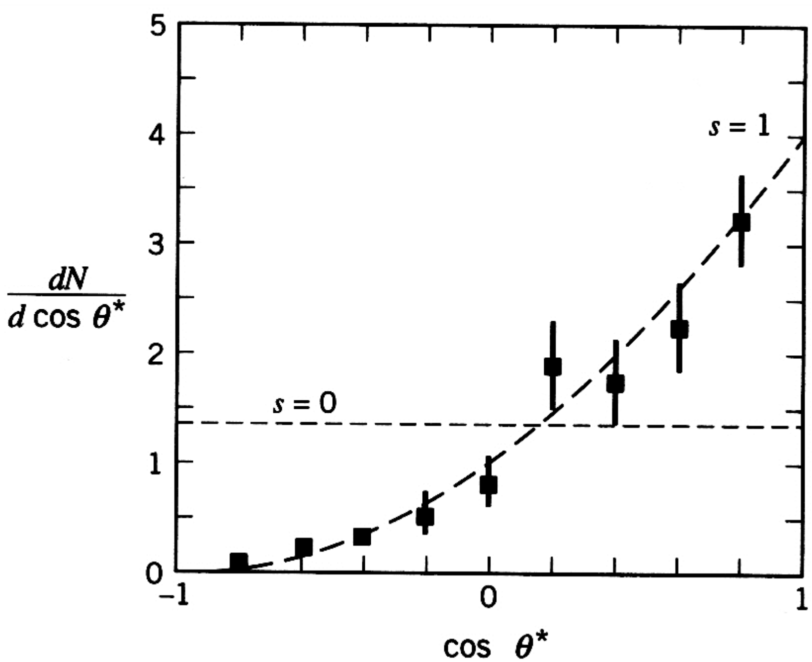
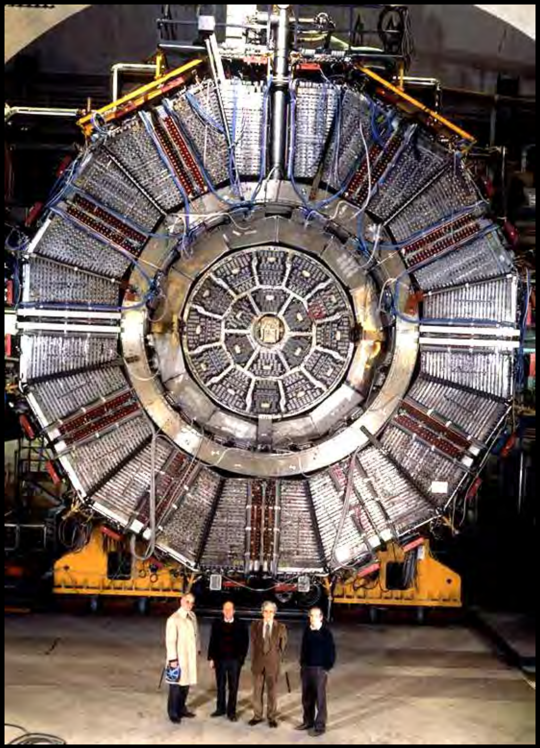


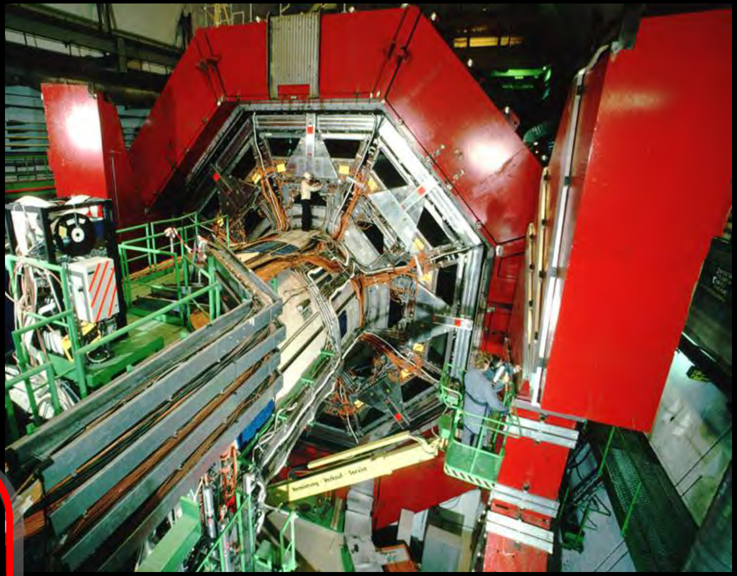
FIGURE 18-19 Measuring the intrinsic angular momentum of the *W* particle.
(*a*) Since the quarks and antiquarks are both polarized, the *W* spin tends to point along the direction of the antiquark. Therefore, the antiparticle from the *W* decay (positron or antineutrino) tends to be emitted in the direction of the antiquark. (*b*) Measurement of the angular distribution of electrons and positrons proves that the spin of the *W* particle is 1.

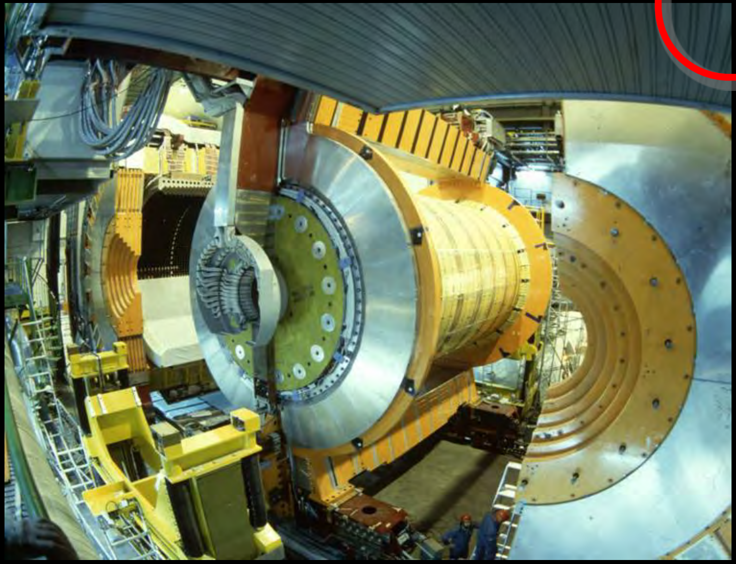
LEP detect ors



ALEPH

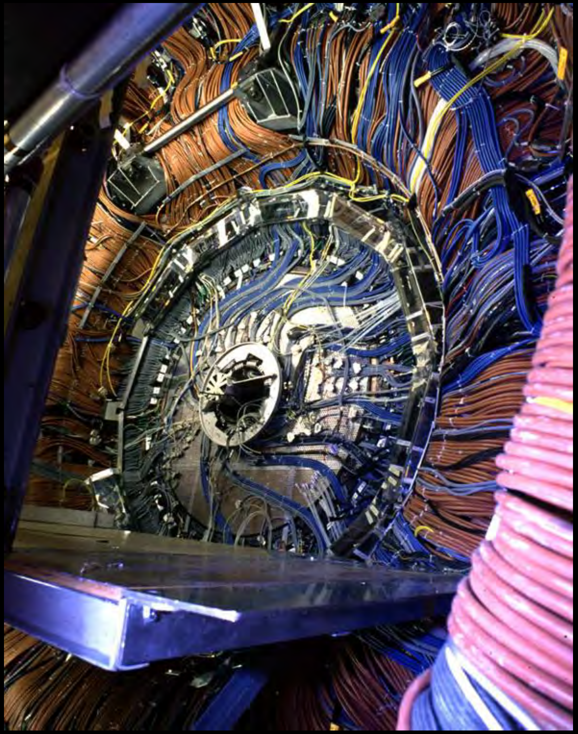
L3





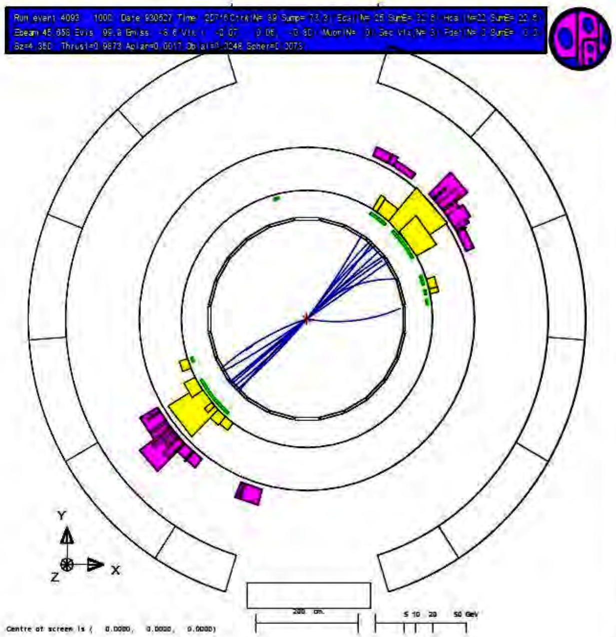
OPAL

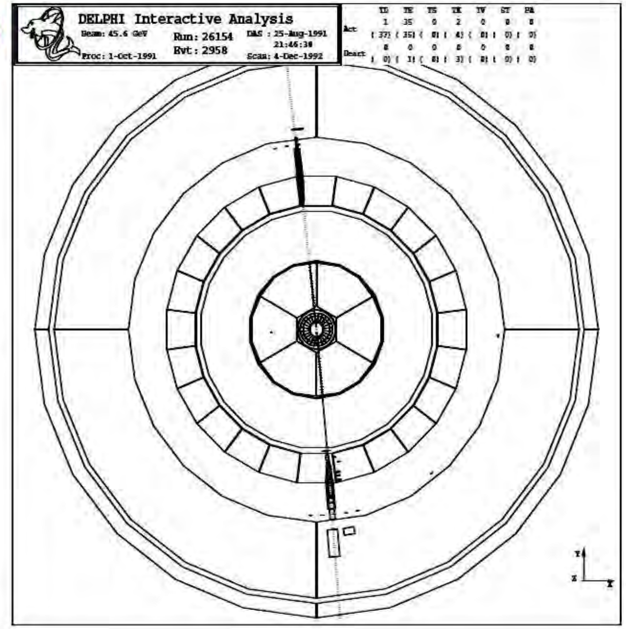
DELPHI



$q\bar{q}$

OPAL





e^+e^-

DELPHI

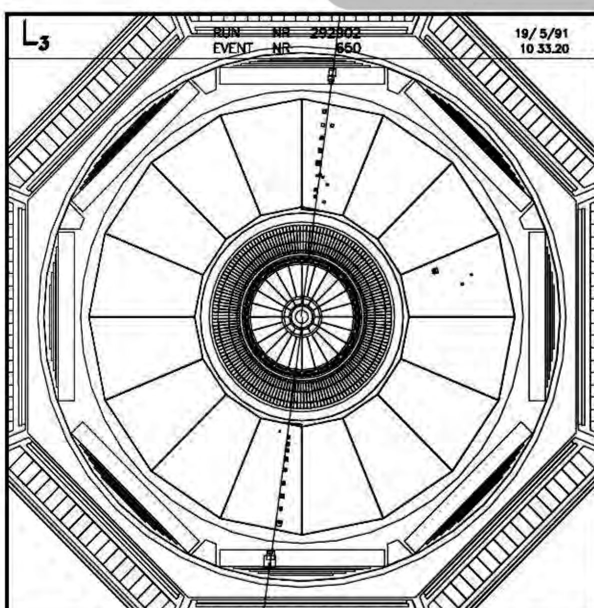
LEP

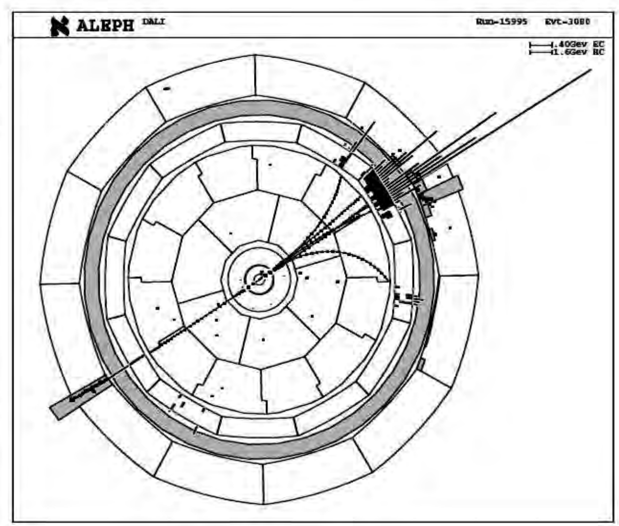
Z decays

CERN

L3

$\mu^+\mu^-$



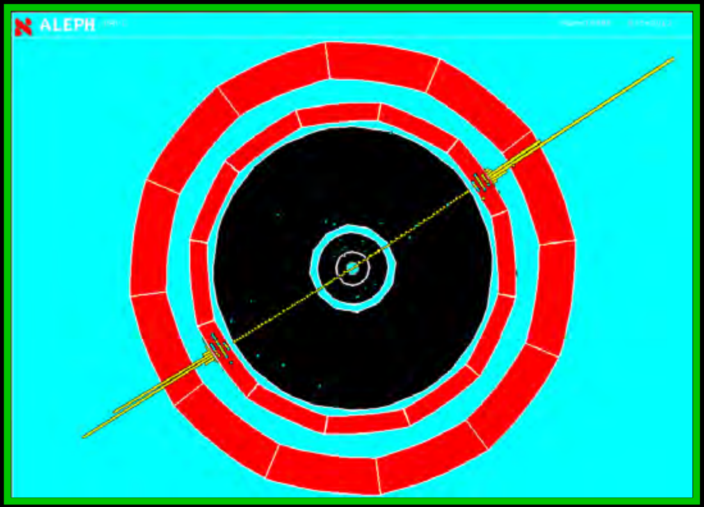


ALEPH

$\tau^+\tau^-$

Z decays

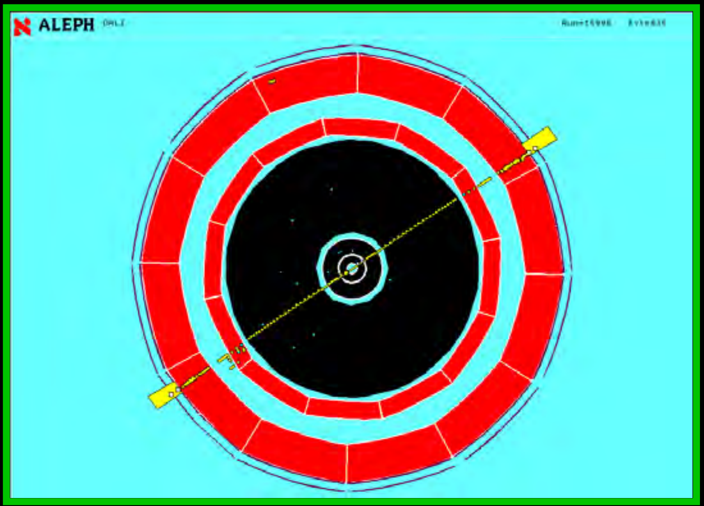
ALEPH @ LEP



$Z^0 \to e^+ e^-$

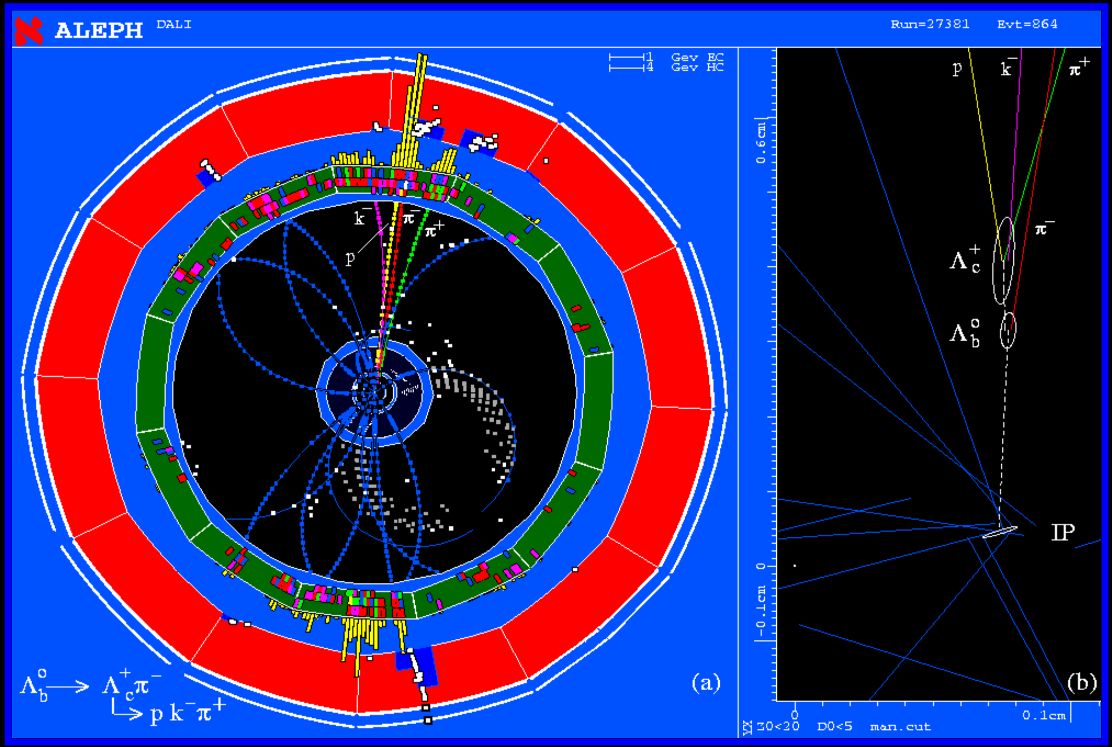
leptonic

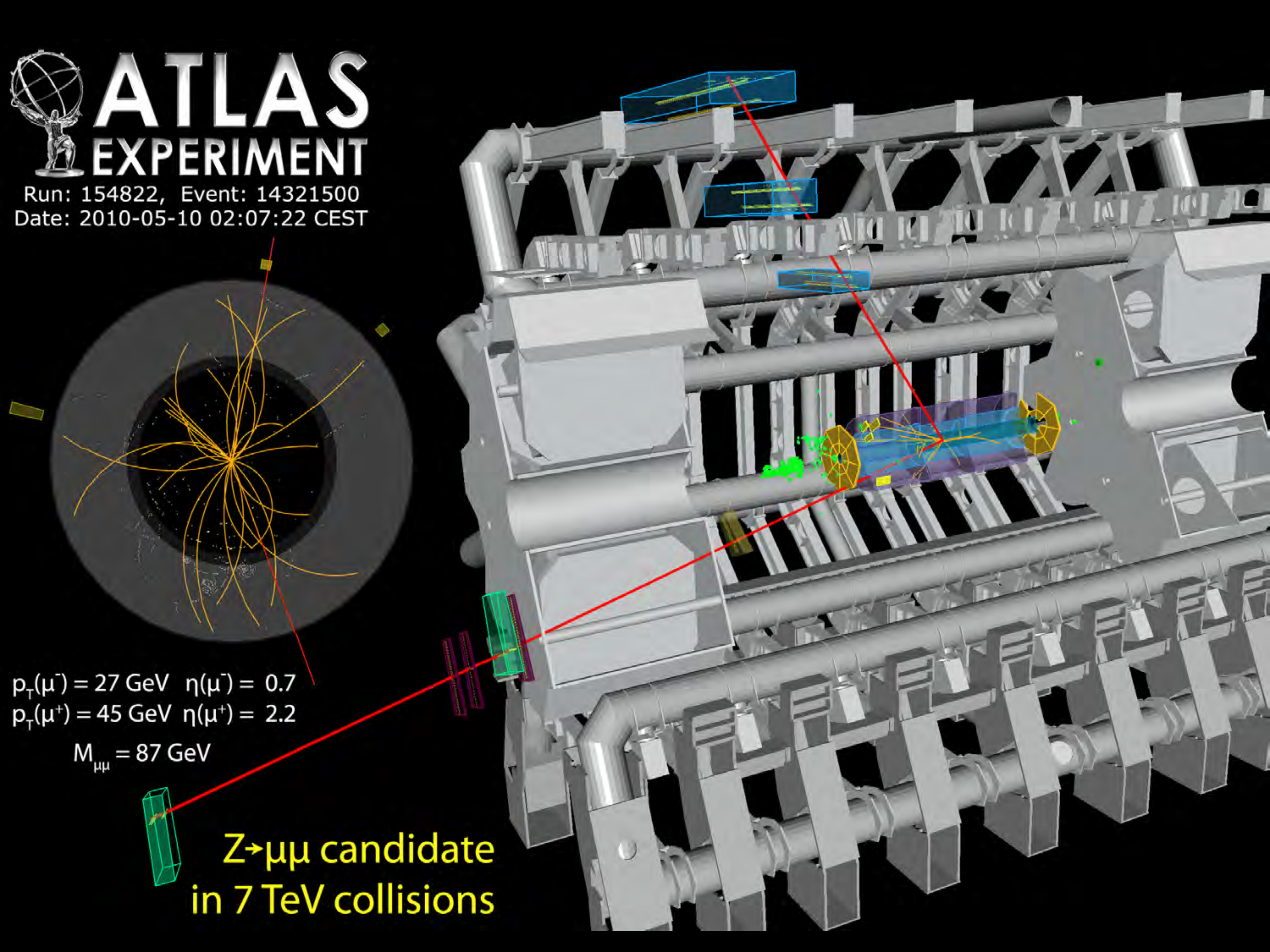
$Z^0 \to \mu^+ \mu^-$

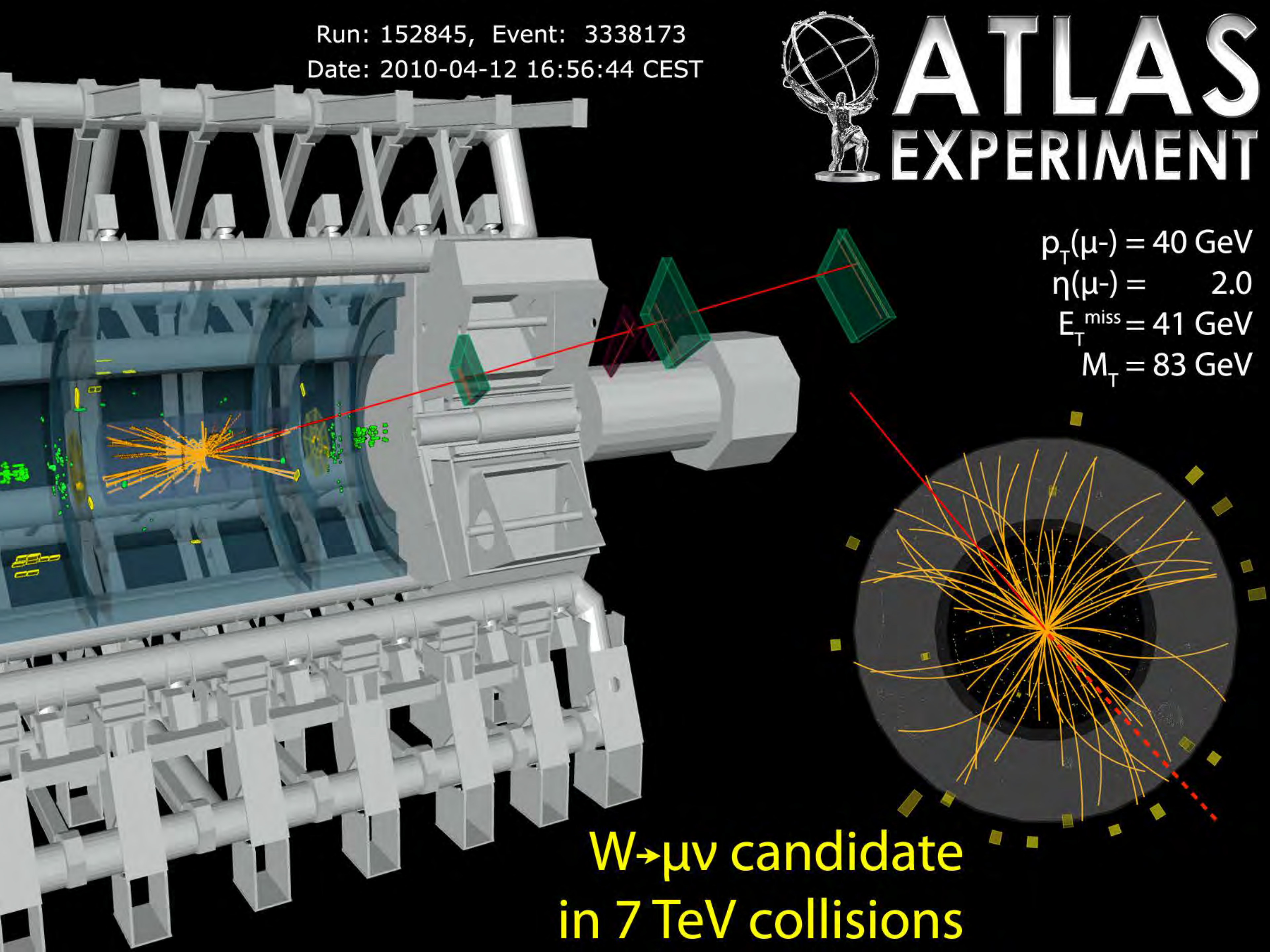


hadronic

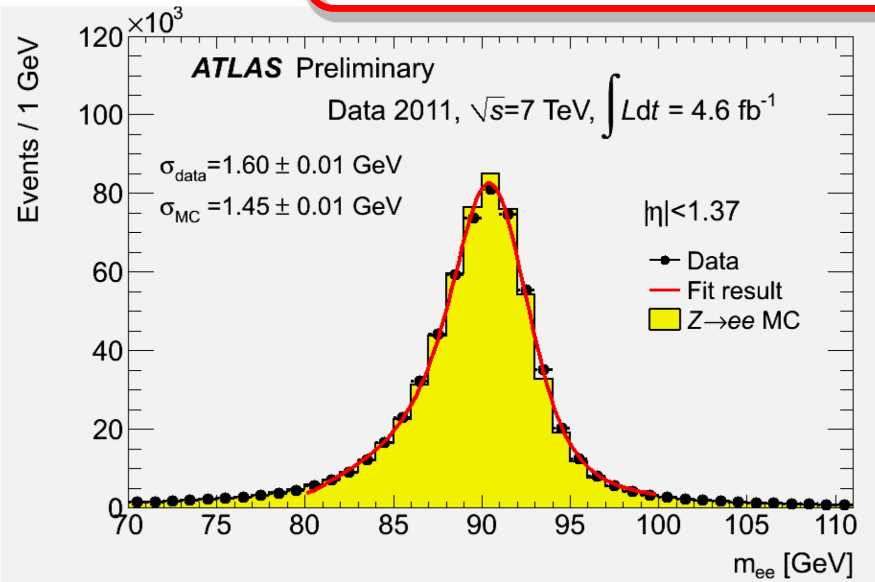
$Z^0 \to b\bar{b}$

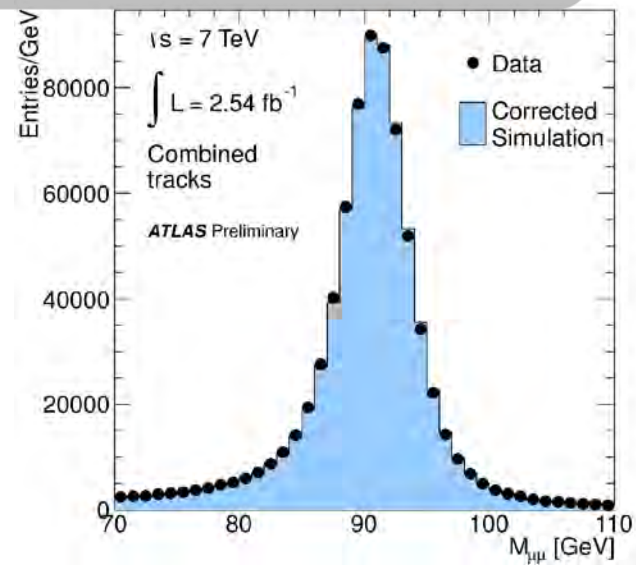



ATLAS EXPERIMENT
Run: 154822, Event: 14321500
Date: 2010-05-10 02:07:22 CEST
$p_T(\mu^-) = 27$ GeV $\eta(\mu^-) = 0.7$
$p_T(\mu^+) = 45$ GeV $\eta(\mu^+) = 2.2$
$M_{\mu\mu} = 87$ GeV
Z→μμ candidate
in 7 TeV collisions

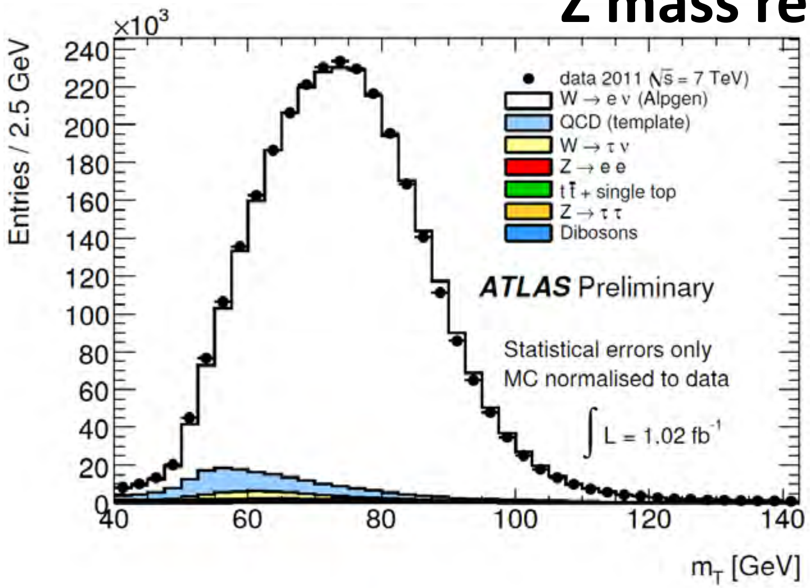

Run: 152845, Event: 3338173
Date: 2010-04-12 16:56:44 CEST
ATLAS
EXPERIMENT
$p_T(\mu-)$ = 40 GeV
$\eta(\mu-)$ = 2.0
E_T^{miss} = 41 GeV
M_T = 83 GeV
W→μν candidate
in 7 TeV collisions

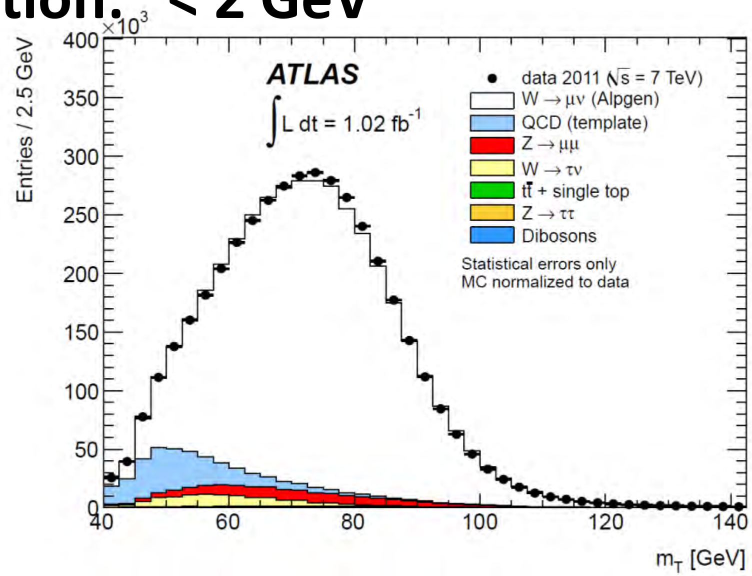
W,Z bosons @ LHC





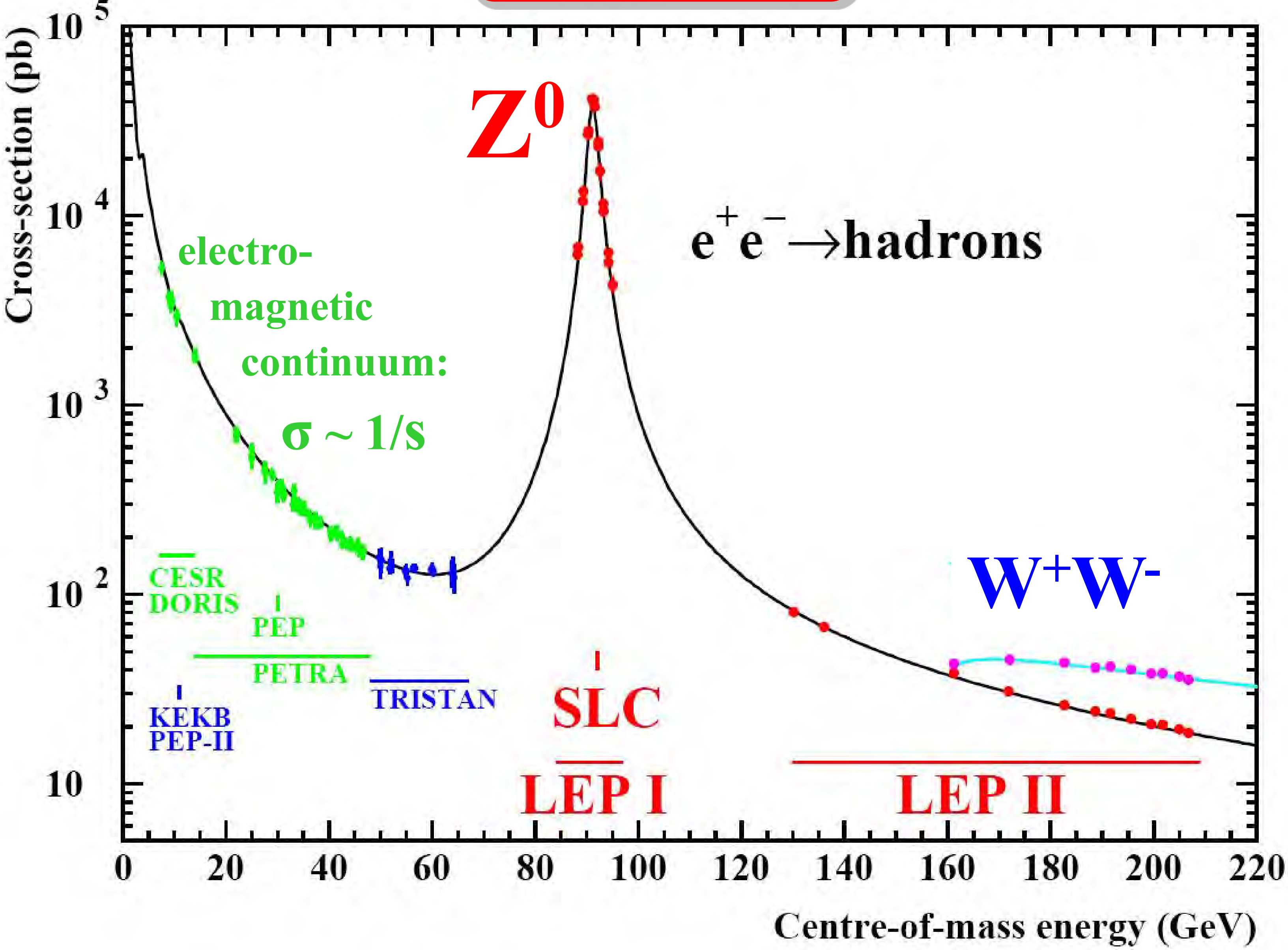
Z mass resolution: < 2 GeV





2010-2: 27 fb^{-1} 50 M $Z \rightarrow ll$, >100 M $W \rightarrow l\nu$

Z peak



Cross-section (pb)
10 5
10 4
10 3
10 2
10
Z0
e+e−→hadrons
electro-
magnetic
continuum:
σ ~ 1/s
CESR
DORIS
PEP
PETRA
KEKB
PEP-II
TRISTAN
SLC
LEP I
LEP II
W+W-
0
20
40
60
80
100
120
140
160
180
200
220
Centre-of-mass energy (GeV)

5. Gauge Theories

For an overview see e.g.:

A. Pich, The Standard Model of Electroweak Interactions, http://arxiv.org/pdf/hep-ph/0502010

The LEP Electroweak Working Group, http://arxiv.org/pdf/hep-ex/0612034 , http://arxiv.org/pdf/hep-ex/0509008

Gauge Invariance in Electro-Magnetism

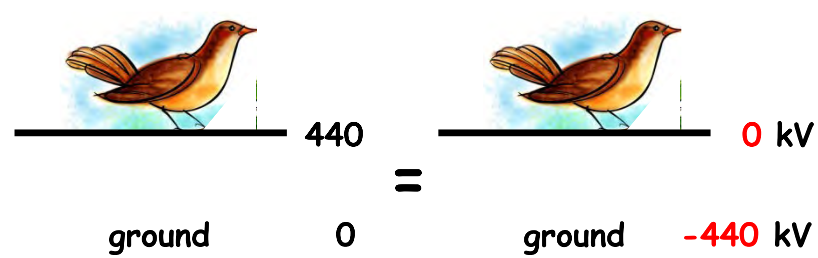
Electro-Static

rot $E = 0$

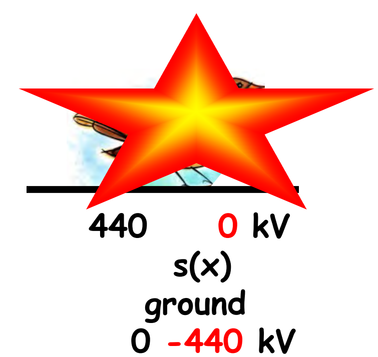
$E = -\,\text{grad}\ V$ scalar potential
$V' = V + s$ gauge freedom

global gauge trafo: $s = \text{const}$
$E' = E$

local gauge trafo: $s = s(x)$
$E' = E - \text{grad}\ s$ compensation ?



GLOBAL TRAFO.



LOCAL TRAFO.

Magneto-Static

div $B = 0$ find magnetic monopole !

$B = \text{rot}\ A$ vector potential

To get local gauge invariance
$B' = B$
and to compensate $-\text{grad}\ s$ choose :
$A' = A - \text{grad}\ s$ rot grad $s = 0$

Electro-Dynamics

Gauge Invariance in Electro-Magnetism

$\mathrm{rot}\; E + \dot{B} = 0$

$\mathrm{rot}\,(E + \dot{A}) = 0$

$E + \dot{A} = -\,\mathrm{grad}\, V$

$$E = -\,\mathrm{grad}\, V - \dot{A}$$
$$B = \mathrm{rot}\, A$$

(E, B) as a function of potentials (V, A)

Demand: Invariance of the theory under
local gauge transformations of the potentials !
Fulfilled by proper choice of gauge trafo.s:

$(V', A') \;\to\; (V, A)$

$V' = V + \dot{s}$

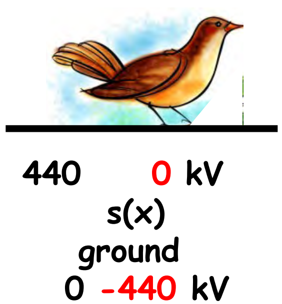
$A' = A - \mathrm{grad}\, s$

$E' = -\,\mathrm{grad}\, V - \mathrm{grad}\, \dot{s} \quad - \dot{A} + \mathrm{grad}\, \dot{s}$

$B' = \mathrm{rot}\,(A - \mathrm{grad}\, s)$

$$E' = E$$
$$B' = B$$

Invariance under gauge + Lorentz transformations
fully defines electrodynamics !



local gauge trafo. ok!

Relativistic Covariance

$x_\mu = (t, \vec{r})$ 4-dim. space-time

$\partial_\mu = (\partial/\partial t, \partial/\partial\vec{r})$ 4-derivative

$p_\mu = (E, \vec{p})$ 4-momentum

$A_\mu = (V, \vec{A})$ 4-potential

$j_\mu = (\rho, \vec{j})$ 4-current

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ elm. field tensor

$A_\mu' = A_\mu + \partial_\mu s(x_\mu)$ gauge transform.

$$F_{\mu\nu} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & B_3 & -B_2 \\ E_2 & -B_3 & 0 & B_1 \\ E_3 & B_2 & -B_1 & 0 \end{pmatrix}$$

Maxwell equations:

$$\partial_\mu F^{\mu\nu} = j^\nu \qquad \varepsilon^{\alpha\beta\mu\nu}\,\partial_\beta F_{\mu\nu} = 0$$

Gauge Invariance in Electro-Magnetism

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Maxwell equ.: $\partial^\mu F_{\mu\nu} = j_\nu$

$\nu=0$: div $E = \rho$
$\nu=1\text{-}3$: rot $B = j + \dot{E}$

$\partial^\nu j_\nu = \partial^\nu \partial^\mu F_{\mu\nu} = 0$ sym x asy
4-current conservation !

gauge fct. $s(x^\mu)$ free
Lorentz gauge condition (covariant!):

$\partial^\mu A_\mu = 0$ =>
$\partial^\mu F_{\mu\nu} = j_\nu = \partial^\mu \partial_\mu A_\nu - \partial^\mu \partial_\nu A_\mu$

$\partial^\mu \partial_\mu A_\nu = \Box A_\nu = j_\nu$ d'Alembert

free photon: $j_\mu = 0$

$\Box A_\nu = 0$ wave equ.

solution: $A_\nu = \varepsilon_\nu \exp(-iq_\mu x^\mu)$

$q_\mu q^\mu = 0 = q^2$
photon massless !

Coulomb gauge (not covariant!):

$\partial^i A_i = 0$ $(i,j = 1,3)$
$\partial^\mu A_\mu = 0$ Lorentz gauge =>

$\partial^0 A_0 = 0$ static scalar potential
$\nu=0$: $\Delta V = \rho$ Poisson equ.

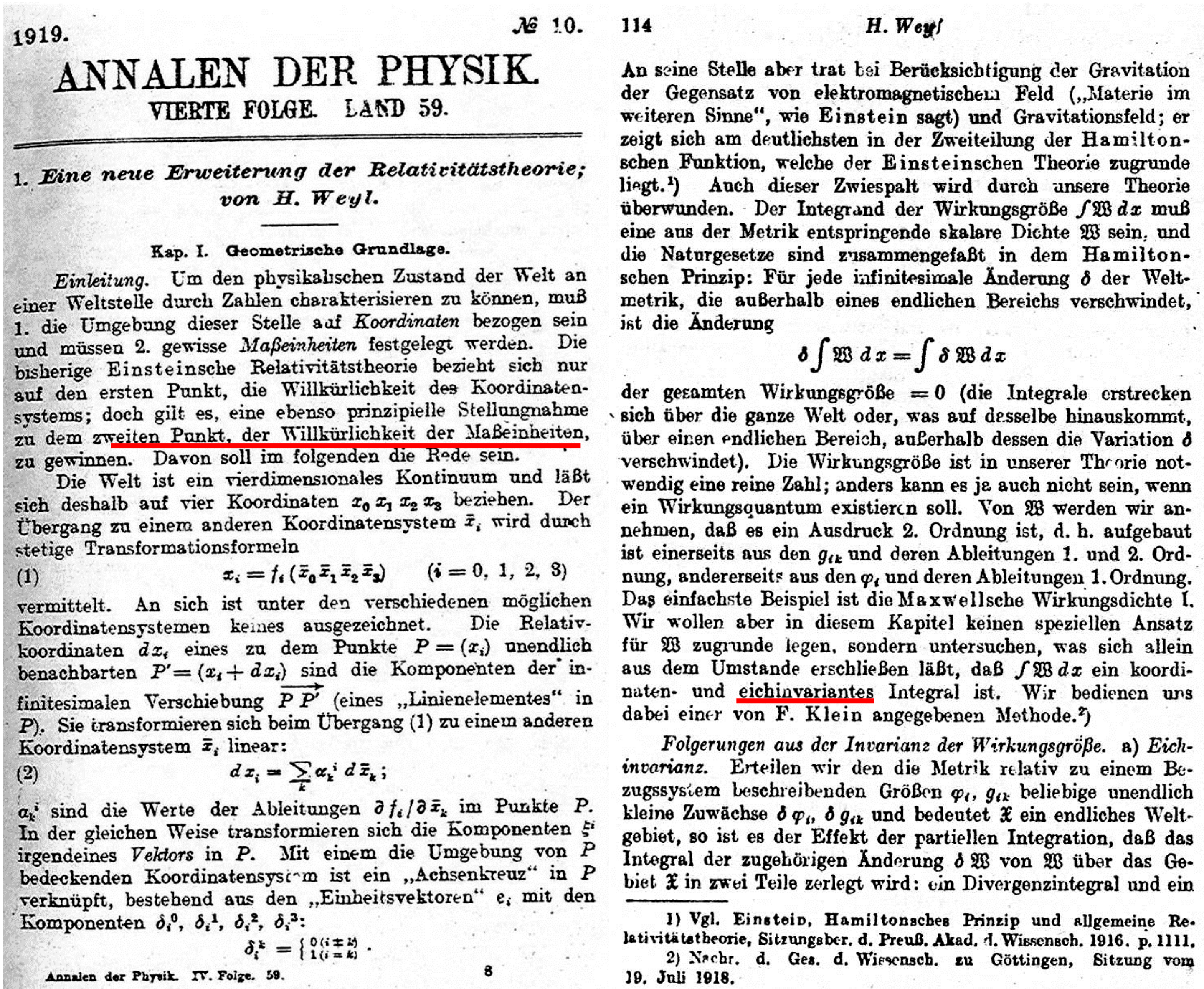
free photon: $\rho=0$
$\Box A_j = 0$ wave equ.

$A_i = \varepsilon_i \exp(-iq_\mu x^\mu) = \varepsilon_i \exp i(\omega t - k_j x^j)$
$\partial^i A_i = 0$ =>
$k^i \varepsilon_i = 0$ transversely polarized

$E = -\dot{A} = -i\omega A$ $\parallel \varepsilon$
$B = \text{rot } A = -ik \times A$

$E \perp B \perp k$

Gauge Invariance



1919. № 10.

ANNALEN DER PHYSIK.
VIERTE FOLGE. BAND 59.

1. *Eine neue Erweiterung der Relativitätstheorie;*
von H. Weyl.

Kap. I. Geometrische Grundlage.

Einleitung. Um den physikalischen Zustand der Welt an einer Weltstelle durch Zahlen charakterisieren zu können, muß 1. die Umgebung dieser Stelle auf *Koordinaten* bezogen sein und müssen 2. gewisse *Maßeinheiten* festgelegt werden. Die bisherige Einsteinsche Relativitätstheorie bezieht sich nur auf den ersten Punkt, die Willkürlichkeit des Koordinatensystems; doch gilt es, eine ebenso prinzipielle Stellungnahme zu dem zweiten Punkt, der Willkürlichkeit der Maßeinheiten, zu gewinnen. Davon soll im folgenden die Rede sein.

Die Welt ist ein vierdimensionales Kontinuum und läßt sich deshalb auf vier Koordinaten $x_0\, x_1\, x_2\, x_3$ beziehen. Der Übergang zu einem anderen Koordinatensystem $\bar{x}_i$ wird durch stetige Transformationsformeln

(1) $$x_i = f_i(\bar{x}_0\, \bar{x}_1\, \bar{x}_2\, \bar{x}_3) \qquad (i = 0,\ 1,\ 2,\ 3)$$

vermittelt. An sich ist unter den verschiedenen möglichen Koordinatensystemen keines ausgezeichnet. Die Relativkoordinaten $d x_i$ eines zu dem Punkte $P = (x_i)$ unendlich benachbarten $P' = (x_i + d x_i)$ sind die Komponenten der infinitesimalen Verschiebung $\overrightarrow{P P'}$ (eines „Linienelementes" in P). Sie transformieren sich beim Übergang (1) zu einem anderen Koordinatensystem $\bar{x}_i$ linear:

(2) $$d x_i = \sum_k \alpha_k^i\, d \bar{x}_k ;$$

α_k^i sind die Werte der Ableitungen $\partial f_i / \partial \bar{x}_k$ im Punkte P. In der gleichen Weise transformieren sich die Komponenten ξ^i irgendeines *Vektors* in P. Mit einem die Umgebung von P bedeckenden Koordinatensystem ist ein „Achsenkreuz" in P verknüpft, bestehend aus den „Einheitsvektoren" e_i mit den Komponenten $\delta_i^0,\ \delta_i^1,\ \delta_i^2,\ \delta_i^3$:

$$\delta_i^k = \begin{cases} 0\ (i \neq k) \\ 1\ (i = k) \end{cases}.$$

An seine Stelle aber trat bei Berücksichtigung der Gravitation der Gegensatz von elektromagnetischem Feld („Materie im weiteren Sinne", wie Einstein sagt) und Gravitationsfeld; er zeigt sich am deutlichsten in der Zweiteilung der Hamiltonschen Funktion, welche der Einsteinschen Theorie zugrunde liegt.[1] Auch dieser Zwiespalt wird durch unsere Theorie überwunden. Der Integrand der Wirkungsgröße $\int \mathfrak{W}\, dx$ muß eine aus der Metrik entspringende skalare Dichte $\mathfrak{W}$ sein, und die Naturgesetze sind zusammengefaßt in dem Hamiltonschen Prinzip: Für jede infinitesimale Änderung δ der Weltmetrik, die außerhalb eines endlichen Bereichs verschwindet, ist die Änderung

$$\delta \int \mathfrak{W}\, d x = \int \delta\, \mathfrak{W}\, d x$$

der gesamten Wirkungsgröße $= 0$ (die Integrale erstrecken sich über die ganze Welt oder, was auf dasselbe hinauskommt, über einen endlichen Bereich, außerhalb dessen die Variation δ verschwindet). Die Wirkungsgröße ist in unserer Theorie notwendig eine reine Zahl; anders kann es ja auch nicht sein, wenn ein Wirkungsquantum existieren soll. Von $\mathfrak{W}$ werden wir annehmen, daß es ein Ausdruck 2. Ordnung ist, d. h. aufgebaut ist einerseits aus den g_{ik} und deren Ableitungen 1. und 2. Ordnung, andererseits aus den φ_i und deren Ableitungen 1. Ordnung. Das einfachste Beispiel ist die Maxwellsche Wirkungsdichte $\mathfrak{l}$. Wir wollen aber in diesem Kapitel keinen speziellen Ansatz für $\mathfrak{W}$ zugrunde legen, sondern untersuchen, was sich allein aus dem Umstande erschließen läßt, daß $\int \mathfrak{W}\, d x$ ein koordinaten- und eichinvariantes Integral ist. Wir bedienen uns dabei einer von F. Klein angegebenen Methode.[2]

Folgerungen aus der Invarianz der Wirkungsgröße. a) *Eichinvarianz.* Erteilen wir den die Metrik relativ zu einem Bezugssystem beschreibenden Größen φ_i, g_{ik} beliebige unendlich kleine Zuwächse $\delta \varphi_i$, δg_{ik} und bedeutet $\mathfrak{X}$ ein endliches Weltgebiet, so ist es der Effekt der partiellen Integration, daß das Integral der zugehörigen Änderung $\delta\, \mathfrak{W}$ von $\mathfrak{W}$ über das Gebiet $\mathfrak{X}$ in zwei Teile zerlegt wird: ein Divergenzintegral und ein

1) Vgl. Einstein, Hamiltonsches Prinzip und allgemeine Relativitätstheorie, Sitzungsber. d. Preuß. Akad. d. Wissensch. 1916. p. 1111.

2) Nachr. d. Ges. d. Wissensch. zu Göttingen, Sitzung vom 19. Juli 1918.

H. Weyl. Eine neue Erweiterung der Relativitätstheorie. Ann. d. Phys., 59:101, 1919.

H. Weyl
Berlin
1919

Gauge invariance classic

Einstein:
„grandiose achievement of the mind"

H.Weyl, Gauge Invariance Princeton 1929:

in Quantum Mechanics: covariant derivative

Elektron und Gravitation. I.

Von Hermann Weyl in Princeton, N. J.

(Eingegangen am 8. Mai 1929).

Einleitung. Verhältnis der allgemeinen Relativitätstheorie zu den quantentheoretischen Feldgleichungen des spinnenden Elektrons: Masse, Eichinvarianz, Fernparallelismus. Zu erwartende Modifikationen der Diracschen Theorie. — I. Zweikomponententheorie: Die Wellenfunktion ψ hat nur zwei Komponenten. — § 1. Bindung der Transformation der ψ an die Lorentztransformation des normalen Achsenkreuzes in der vierdimensionalen Welt. Asymmetrie von Zukunft und Vergangenheit, von rechts und links. — § 2. In der allgemeinen Relativitätstheorie wird die Metrik in einem Weltpunkt festgelegt durch ein normales Achsenkreuz. Komponenten von Vektoren relativ zu den Achsen und den Koordinaten. Kovariante Differentiation von ψ. — § 3. Allgemein invariante Fassung der Diracschen Wirkungsgröße, welche für das Wellenfeld der Materie charakteristisch ist. — § 4. Die differentiellen Erhaltungssätze von Energie und Impuls und die Symmetrie des Impulstensors folgen aus der doppelten Invarianz: 1. gegenüber Koordinatentransformation, 2. gegenüber Drehungen des Achsenkreuzes. Impuls und Impulsmoment der Materie. — § 5. Einsteins klassische Gravitationstheorie in der neuen analytischen Formulierung. Gravitationsenergie. — § 6. Das elektromagnetische Feld. Aus der Unbestimmtheit des Eichfaktors in ψ ergibt sich die Notwendigkeit der Einführung der elektromagnetischen Potentiale. Eichinvarianz und Erhaltung der Elektrizität. Das Raumintegral der Ladung. Einführung der Mass . Diskussion und Zurückweisung einer anderen Möglichkeit, in welcher die Elektrizität nicht als Begleitphänomen der Materie, sondern der Gravitation erscheint.

Einleitung.

In dieser Arbeit entwickle ich in ausgeführter Form eine Gravitation, Elektrizität und Materie umfassende Theorie, von der eine kurze Skizze in den Proc. Nat. Acad., April 1929, erschienen ist. Es ist von verschiedenen Autoren der Zusammenhang der Einsteinschen Theorie des Fernparallelismus mit der Spintheorie des Elektrons bemerkt worden *. Trotz gewisser formaler Übereinstimmungen unterscheidet sich mein Ansatz in radikaler Weise dadurch, daß ich den Fernparallelismus ablehne und an Einsteins klassischer Relativitätstheorie der Gravitation festhalte.

Um zweier Gründe willen verspricht die Adaption der Pauli-Diracschen Theorie des spinnenden Elektrons an die allgemeine Relativität zu physikalisch fruchtbaren Ergebnissen zu führen. 1. Die Diracsche Theorie, in welcher das Wellenfeld des Elektrons durch ein Potential ψ mit vier Komponenten beschrieben wird, gibt doppelt zu viel Energieniveaus: man sollte darum, ohne die relativistische Invarianz preiszugeben, zu den zwei Komponenten der Paulischen Theorie zurück-

* E. Wigner, ZS. f. Phys. 53, 592, 1929; u. a.

kehren können. Daran hindert das die Masse m des Elektrons als Faktor enthaltende Glied der Diracschen Wirkungsgröße. Masse ist aber ein Gravitationseffekt; es besteht so die Hoffnung, für dieses Glied in der Gravitationstheorie einen Ersatz zu finden, der die gewünschte Korrektur herbeiführt. 2. Die Diracschen Feldgleichungen für ψ zusammen mit den Maxwellschen Gleichungen für die vier Potentiale f_p des elektromagnetischen Feldes haben eine Invarianzeigenschaft, die in formaler Hinsicht derjenigen gleicht, die ich in meiner Theorie von Gravitation und Elektrizität vom Jahre 1918 als Eichinvarianz bezeichnet hatte; die Gleichungen bleiben ungeändert, wenn man gleichzeitig

$$\psi \text{ durch } e^{i\lambda}\cdot\psi \quad\text{und}\quad f_p \text{ durch } f_p - \frac{\partial\lambda}{\partial x_p}$$

ersetzt, unter λ eine willkürliche Ortsfunktion in der vierdimensionalen Welt verstanden. Dabei ist in f_p der Faktor $\frac{e}{ch}$ aufgenommen ($-e$ Ladung des Elektrons, c Lichtgeschwindigkeit, $\frac{h}{2\pi}$ Wirkungsquantum). Auch die Beziehung dieser „Eichinvarianz“ zum Erhaltungssatz der Elektrizität bleibt unangetastet. Es ist aber ein wesentlicher und für den Anschluß an die Erfahrung bedeutungsvoller Unterschied, daß der Exponent des Faktors, den ψ annimmt, nicht reell, sondern rein imaginär ist. ψ übernimmt jetzt die Rolle, welche in jener alten Theorie das Einsteinsche ds spielte. Es scheint mir darum dieses nicht aus der Spekulation, sondern aus der Erfahrung stammende neue Prinzip der Eichinvarianz zwingend darauf hinzuweisen, daß das elektrische Feld ein notwendiges Begleitphänomen nicht des Gravitationsfeldes, sondern des materiellen, durch ψ dargestellten Wellenfeldes ist. Da die Eichinvarianz eine willkürliche Funktion λ einschließt, hat sie den Charakter „allgemeiner“ Relativität und kann natürlich nur in ihrem Rahmen verstanden werden.

An den Fernparallelismus vermag ich aus mehreren Gründen nicht zu glauben. Erstens sträubt sich mein mathematisches Gefühl a priori dagegen, eine so künstliche Geometrie zu akzeptieren; es fällt mir schwer, die Macht zu begreifen, welche die lokalen Achsenkreuze in den verschiedenen Weltpunkten in ihrer verdrehten Lage zu starrer Gebundenheit aneinander hat einfrieren lassen. Es kommen, wie ich glaube, zwei gewichtige physikalische Gründe hinzu. Gerade dadurch, daß man den Zusammenhang zwischen den lokalen Achsenkreuzen löst, verwandelt sich der Eichfaktor $e^{i\lambda}$, der in der Größe ψ willkürlich bleibt, notwendig

Gauge invariance in SU(N)

C.N. Yang + R.L. Mills, 1954:

The difference between a neutron and a proton is then a purely arbitrary process. As usually conceived, however, this arbitrariness is subject to the following limitations:

once one chooses what to call a proton, what a neutron, at one space-time point, one is then not free to make any choices at other space-time points.

It seems that this is not consistent with the localized field concept that underlies the usual physical theories. In the present paper we wish to explore the possibility of requiring all interactions to be invariant under independent rotations of the isotopic spin at all space-time points…

C.N. Yang and R.L. Mills. Conservation of Isotopic Spin and Isotopic Gauge Invariance. Phys. Rev. 96:191, 1954.

C.N. Yang

Nobel prize

1957

Gauge Theory

U(1): H.Weyl, 1918, 1929.

- covariant derivative :

$$D_\mu = \partial_\mu - ieA_\mu$$

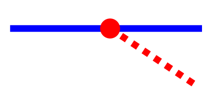
- gauge transformation :

$$A'_\mu = A_\mu + 1/e\, \partial_\mu \alpha(x_\mu) \qquad \text{exchange boson}$$
$$\psi' = e^{\,i\alpha(x_\mu)}\, \psi \qquad \text{matter fermion}$$

- Dirac fermion $\mathcal{L}$ gauge invariant :

$$\begin{aligned}
\mathcal{L} &= \psi'^* (D'_\mu - m)\, \psi' \\
&= \psi'^* \quad (\partial_\mu - ie\, A_\mu - i\, \partial_\mu \alpha(x_\mu)) \quad \psi\, e^{i\alpha(x_\mu)} \quad - \; m\, \psi^*\psi \\
&= \psi'^*\, e^{i\alpha}\, (\partial_\mu - ie\, A_\mu - i\, \partial_\mu\alpha \; + i\, \partial_\mu\alpha)\, \psi \quad - \; m\, \psi^*\psi \\
&= \psi^* \; (D_\mu - m)\; \psi \\
&= \psi^* \,(\partial_\mu - m)\, \psi \;\; - \;\; ie\, \psi^* A_\mu\, \psi
\end{aligned}$$

fermion propagation + fermion-boson-interaction



- fermion mass term $m\;\psi^*\psi$ gauge invariant !
- boson mass term $m^2\, A_\mu A^\mu$ gauge violating ! $m_\gamma < 10^{-16}$ eV

$SU(2)_W$: weak isospin

QUARKS: doublets of strong isospin I
FERMIONS: doublets of weak isospin I_W

Y_w ... weak hypercharge
I_w ... weak isospin

weak Gell-Mann-Nishijima:
$Q = I_{3,W} + Y_w / 2$

	Q	I_{3W}	$Y_w/2$	I_w
ν_L	0	+1/2	-1/2	1/2
e_L	-1	-1/2	-1/2	1/2
$W^{\pm}$	±1	±1	0	1
Z^0	0	0	0	1
γ	0	0	0	0

(W^+, Z^0, W^-) : iso-triplet of weak $SU(2)_W$!
γ : singlet of elm. $U(1)$!

$U(1) \rightarrow SU(N)$

U(1): Weyl, 1918,1929
SU(2): Yang, Mills, 1954

- **covariant derivative :**

$$D_\mu = \partial_\mu + ig\, T_i\, A_\mu^i$$

- **gauge transformations:**

$$\psi' = e^{iT_i \alpha_i}\, \psi$$
$$A'^i_\mu = A^i_\mu - 1/g\, \partial_\mu \alpha_i$$

T_i … generators of SU(N) with $i = 1 \ldots N^2-1$
f_{ijk} … structure constants of SU(N)

$$[T_i, T_j] = i\, f_{ijk}\, T_k$$

non-commutative, SU(N) non-abelian

SU(2): $2^2-1=3$ Pauli matrices:

$$[\sigma_i, \sigma_j] = 2i\, \varepsilon_{ijk}\, \sigma_k$$

Self-Interaction of Gauge Bosons

- **U(1):**
$$D_\mu = \partial_\mu - ie\, A_\mu$$
$$F_{\mu\nu} = -i/e\, [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- **SU(N):**
$$D_\mu = \partial_\mu + ig\, T_i\, A_\mu^i$$
$$G_{\mu\nu} = i/g\, [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\, [A_\mu, A_\nu] \equiv T_i\, G_{\mu\nu}{}^i$$
$$G_{\mu\nu}{}^i = \partial_\mu A_\nu{}^i - \partial_\nu A_\mu{}^i + g\, f_{ijk}\, A_\mu{}^j\, A_\nu{}^k = F_{\mu\nu}{}^i + g\, f_{ijk}\, A_\mu{}^j\, A_\nu{}^k$$

- boson $\mathcal{L}$ gauge invariant:

$$\mathcal{L} = \tfrac{1}{4}\, G'_{\mu\nu}{}^i\, G'^{\mu\nu}{}_i = \tfrac{1}{4}\, G_{\mu\nu}{}^i\, G^{\mu\nu}{}_i$$

- self-interaction :

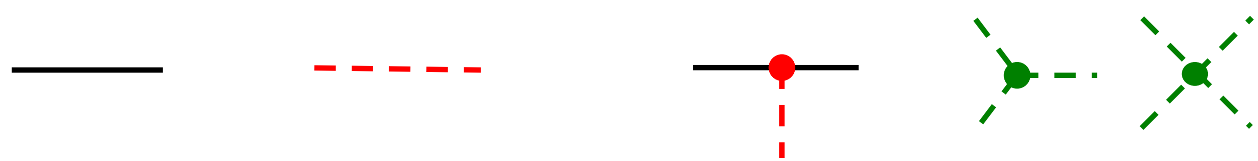
$$G_{\mu\nu}{}^i\, G^{\mu\nu}{}_i = F_{\mu\nu}{}^i\, F^{\mu\nu}{}_i + 2g\, f_{ijk}\, F_{\mu\nu}{}^i\, A^\mu{}_j\, A^\nu{}_k + g^2\, f_{ijk} f^{ilm}\, A_\mu{}^j A_\nu{}^k A^\mu{}_l A^\nu{}_m$$

free boson propagation — vector boson self-interaction

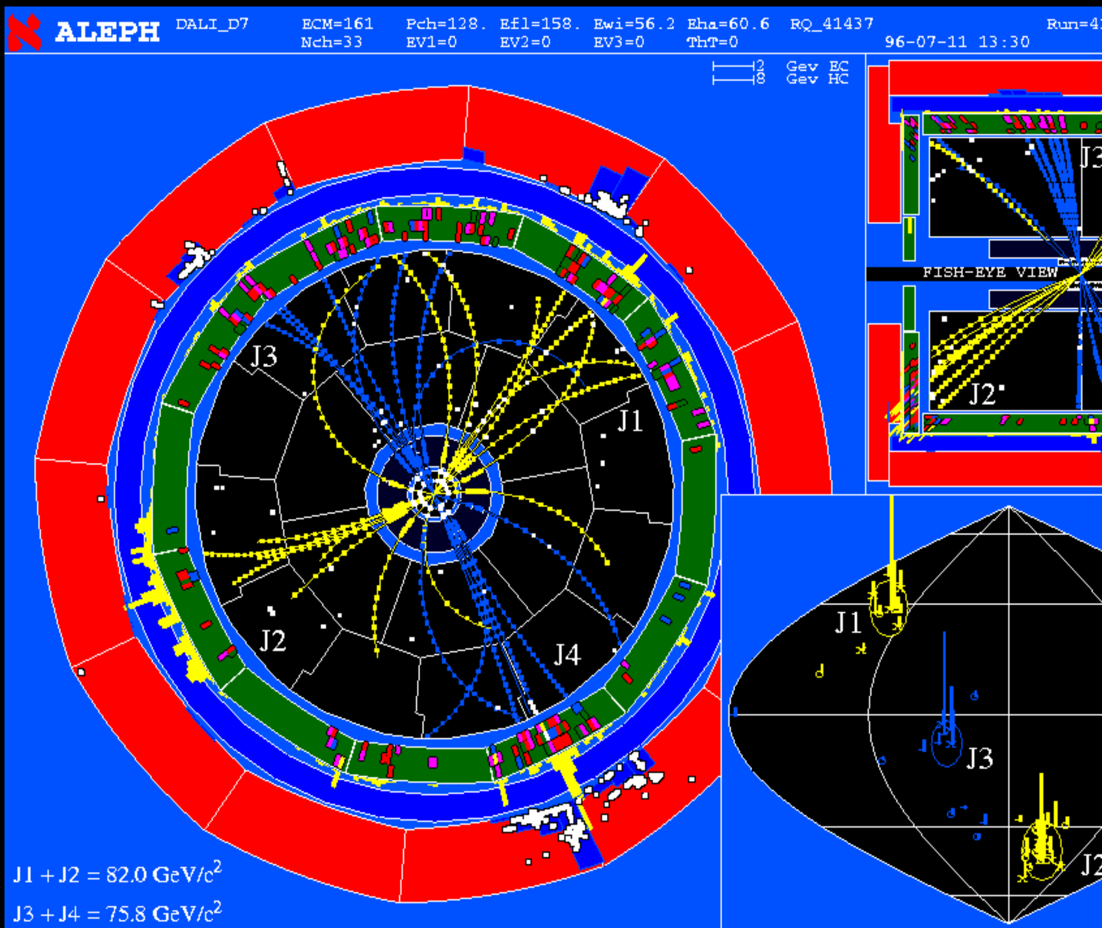
- Lagrangian $\mathcal{L}$:

$$\mathcal{L} \sim \bar{f}(\partial - m)f + (\partial A)^2 - m^2 A^2 + g\, \bar{f} A f + g\, A^3 + g^2\, A^4$$

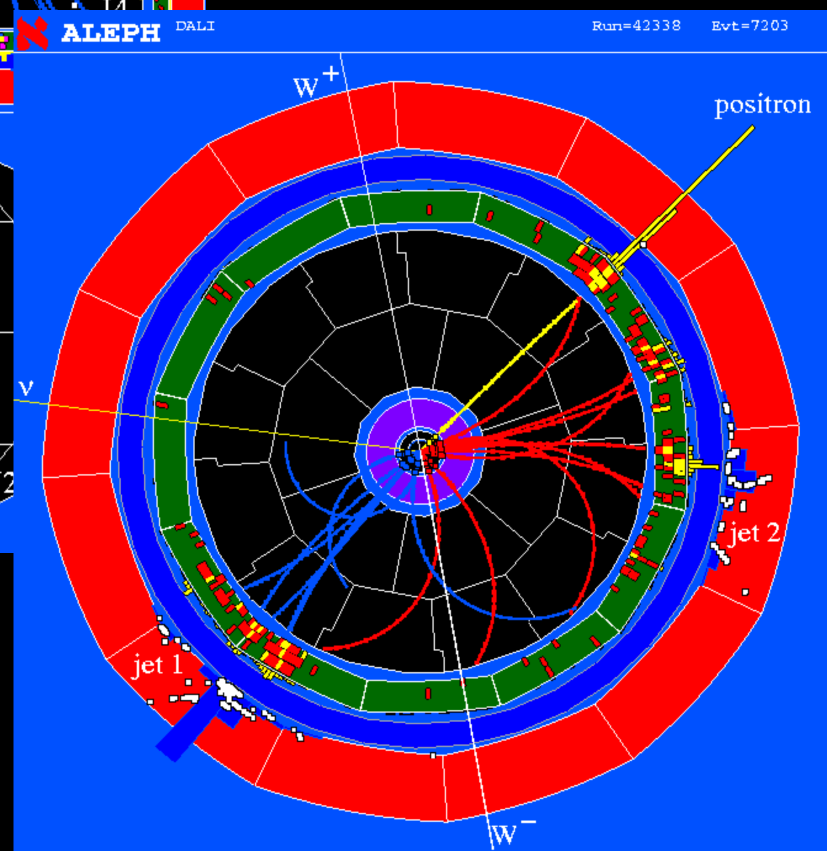
free fermion + propagation; boson gauge violating!; fermion-boson interaction; vector boson self-interaction



$e^+e^- \rightarrow W^+W^-$

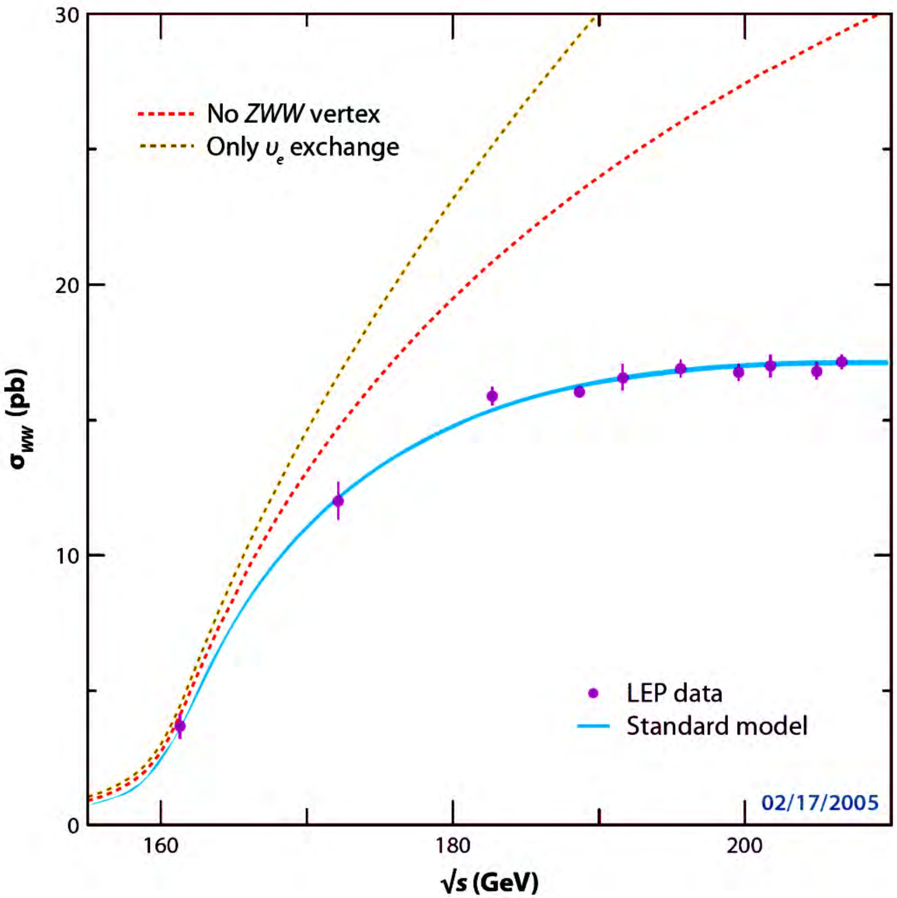


$e^+e^- \rightarrow$
$W^+W^- \rightarrow$
$j_1 j_2\ e^+\nu$



$e^+e^- \rightarrow W^+W^-$
$\rightarrow \bar{q}q\ \bar{q}q \rightarrow$ 4 jets

Gauge Boson Self-Interaction



$e^+e^- \rightarrow W^+W^-$

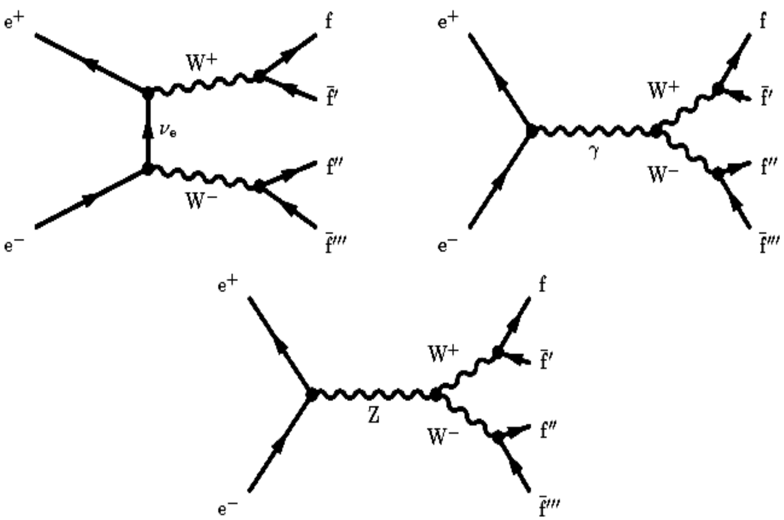


Figure 1: Feynman diagrams describing W-boson pair-production at LEP.

- each diagram separately violates unitarity
- cross section kept finite only through
  - GAUGE CANCELLATIONS
  - HWW vertex
- not present in elm. U(1): photons do not self-interact ! no $\gamma\gamma\gamma$ vertex !

$e^+e^- \rightarrow Z^0 Z^0$

$\varepsilon_{ijk} W_i W_j W_k = 0$: no ZZZ + γZZ vertex !

Electroweak Unification

QUARKS: doublets of strong isospin I
FERMIONS: doublets of weak isospin I_W

Y_w ... weak hypercharge
I_w ... weak isospin

weak Gell-Mann-Nishijima:
$Q = I_{3,W} + Y_w / 2$

	Q	I_{3W}	$Y_w/2$	I_w
ν_L	0	+1/2	-1/2	1/2
e_L	-1	-1/2	-1/2	1/2
e_R	-1	0	-1	0

$W^{\pm}$ conserve I_W, Y_W - γ+Z violate I_W, Y_W ?!

Electroweak Unification

$W^{\pm}$ conserve I_W, Y_W, γ+Z violate I_W, Y_W

mix neutral sector:

W^0, V^0 conserve I_W, Y_W, γ+Z conserve Q

	operator	group	boson
weak	Y_w I_w	U(1) SU(2)	V^0 W^+ W^0 W^-
real	Q $a\, I_{3,W} + b\, Y_w$	U(1) SU(2)	γ W^+ Z^0 W^-

ELECTROMAGNETIC + WEAK INTERACTIONS

MIX :

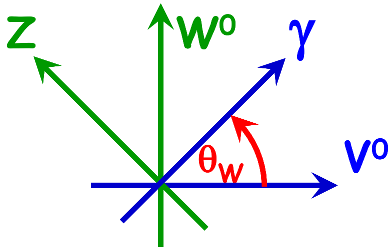
ELECTRO-WEAK INTERACTION

Electroweak Mixing

observed fields (γ , Z) =
orthogonal transformations
of (V^0, W^0)

$$\begin{pmatrix} \gamma \\ Z \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} V^0 \\ W^0 \end{pmatrix}$$

with $s_w = \sin\theta_w$
$c_w = \cos\theta_w$



θ_w ... **Weinberg angle**

=

electro-weak mixing angle


S.Glashow


A.Salam


S.Weinberg

develop 1961-73

Standard Model
of particle physics

Nobel Prize 1979

Weak Neutral Current

$NC = g I_3 \; W^0 \qquad + g' \, Y/2 \; V^0$

$= (g I_3 \, s_W + g' \, Y/2 \, c_W) \, \gamma + (g I_3 \, c_W - g' \, Y/2 \, s_W) \, Z$

fix g,g' by 2 demands:

elm. current: $eQ = g I_3 \, s_W + g' \, Y/2 \, c_W$

Gell-Mann - Nishijima: $Q = I_3 + Y/2$

compare coefficients:

$$e = g \, s_W = g' \, c_W$$

for Z couplings choose (g,g') or (e,s_W):

$NC = eQ \, \gamma + (g I_3 \, c_W - g' \, (Q - I_3) \, s_W) \, Z$

$= eQ \, \gamma + e/(s_W c_W)(I_3 - Q \, s^2_W) \, Z$

$CC = \quad g \quad I_3 \quad W$

$\qquad e/s_W \quad I_3 \quad W$

	Coupling	Space Structure		
γ	e	V	γ_μ	P conserved
$W^\pm$	$g = e/s_w$	V-A	$\gamma_\mu (1 - \gamma_5)$	P max. violated
Z	g/c_w	mixed	$\gamma_\mu (g_V - g_A \gamma_5)$	P mixed

conserve charge in Z mass term: Q=0

$M_W^2/M_Z^2 = g^2/(g^2 + g'^2)$ or $M_W/M_Z = \cos \theta_W$

Z^0 electroweak couplings

left and right handed, vector and axial couplings :

$P\,|H_L\rangle = -\,|H_R\rangle \qquad P\,|L+R\rangle = -\,|L+R\rangle = P\,|V\rangle$

$P\,|H_R\rangle = -\,|H_L\rangle \qquad P\,|L-R\rangle = +\,|L-R\rangle = P\,|A\rangle$

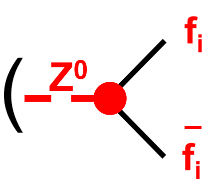
$g_L = I_3 - Q\,s_w^2 \qquad g_V = g_L + g_R = I_3 - 2Q\,s_w^2$

$g_R = \quad - Q\,s_w^2 \qquad g_A = g_L - g_R = I_3$ no mix with V elm.

$s_w^2 = \sin^2\theta_w \sim 0.23$ ... Weinberg angle = electro-weak mixing angle

	Q	I_3	g_A	g_V	g_V
ν	0	+1/2	+1/2	+1/2	0.50
e	-1	-1/2	-1/2	$-1/2 + 2\ s_W^2$	-0.04
u	+2/3	+1/2	+1/2	$+1/2 - 4/3 s_W^2$	0.20
d	-1/3	-1/2	-1/2	$-1/2 + 2/3 s_W^2$	-0.35

	$g_V^2+g_A^2$	N_{col}	Γ_i / Γ
ν	1/2	1	20 %
e	1/4+ε		10 %
u	0.29	3	5 q's 70 %
d	0.35		

Γ_i (

) $\sim g_L^{i2} + g_R^{i2} \sim g_V^{i2} + g_A^{i2}$

Try: $\sin^2\theta_W = 0, ¼$

$\sin^2\theta_W = 0$: no γ-Z mixing

$Z^0 = W^0$, $\gamma = V^0$

- Z^0 feels only weak charge
- γ feels only electric charge
- $\Gamma_\nu = \Gamma_l$ Z^0 blind to electric charge

$g_R = 0$

- no Z^0 coupling to e_R
- purely left-handed
- max. parity violation
- Z coupling pure (V-A) as for W

$\sin^2\theta_W = 1/4$:

- γ charged lepton coupling purely vector
- Z charged lepton vector coupling = 0
- Z charged lepton coupling purely axial

- P = +1 =>
- no P violation
- no asymmetries

W purely weak, V-A, P viol. max.

γ purely electric, V, P cons.

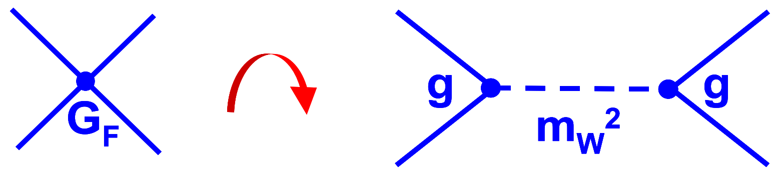
Z mixed, hybrid

Electro-Weak Parameters

couplings:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{e^2}{8\sin^2\theta_W M_W^2}$$

4-Fermion | W-propagator | (g, g') $(e, \sin^2\theta_W)$



$e^2 = 4\pi\alpha$

$\sin\theta_W^2 M_W^2 = (\pi\alpha)/(\sqrt{2}\,G_F) = (\pi\alpha)(246\text{ GeV})^2 = (37.2805\text{ GeV})^2$ very precise !

electroweak Standard Model defined by 2 couplings + 1 mass scale =

3 out of α , G_F , M_Z ; M_W , $\sin^2\theta_W$, ...

value $1/137$, 10^{-5} , 91 GeV ; 81 GeV , 0.231

relative precision $3\cdot10^{-10}$, $5\cdot10^{-7}$, $2\cdot10^{-5}$; $2\cdot10^{-4}$, $6\cdot10^{-4}$

over-constrained: test consistency of Standard Model !

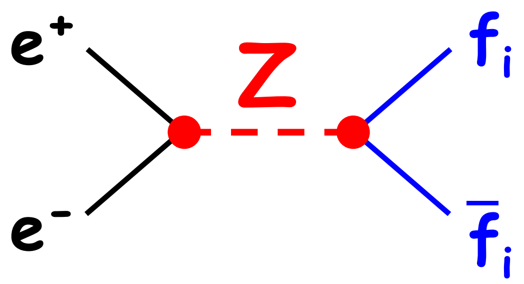
Nr of families

$\begin{pmatrix} l \\ \nu \end{pmatrix}_i \begin{pmatrix} U \\ D \end{pmatrix}_i \quad i=1,\ldots,N$ nr of fermion families

neutrinos much lighter than quarks+leptons
if there are >3 fermion families
4^{th} ν type might show up in Z decays if $m(\nu_4) < M_Z/2$

$\Gamma_Z = N_\nu \Gamma_\nu + 3\,\Gamma_L + \Gamma_h$ total Z width

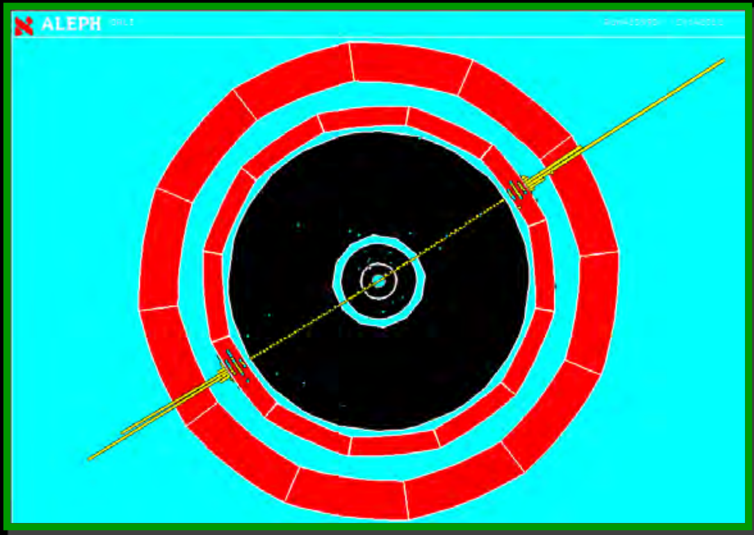
$N_\nu \Gamma_\nu/\Gamma_L = \Gamma_Z/\Gamma_L - 3 - \Gamma_h/\Gamma_L$



- $\Gamma_h/\Gamma_L = R = 20.76 \pm 0.02$
- $\Gamma_\nu/\Gamma_L = 2/[1+(1-4s_W^2)^2] = 1.991 \pm 0.001$
- Γ_Z/Γ_L from R + Z line shape:

Z decays

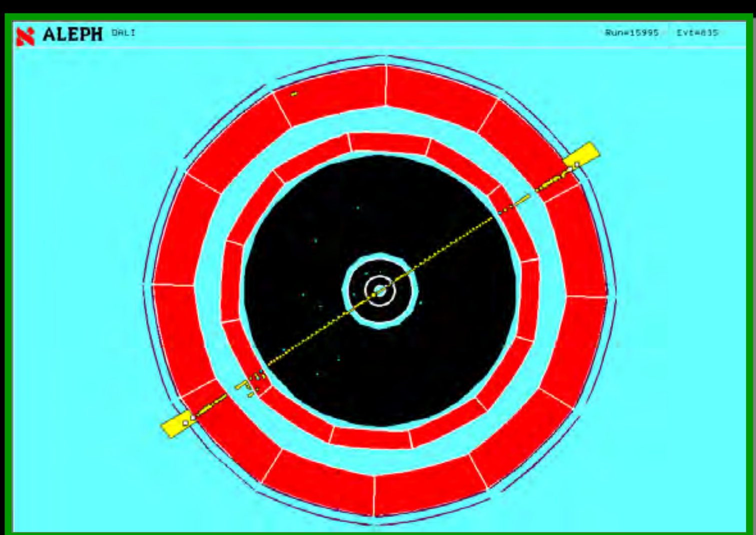
ALEPH @ LEP



$Z^0 \to e^+ e^-$

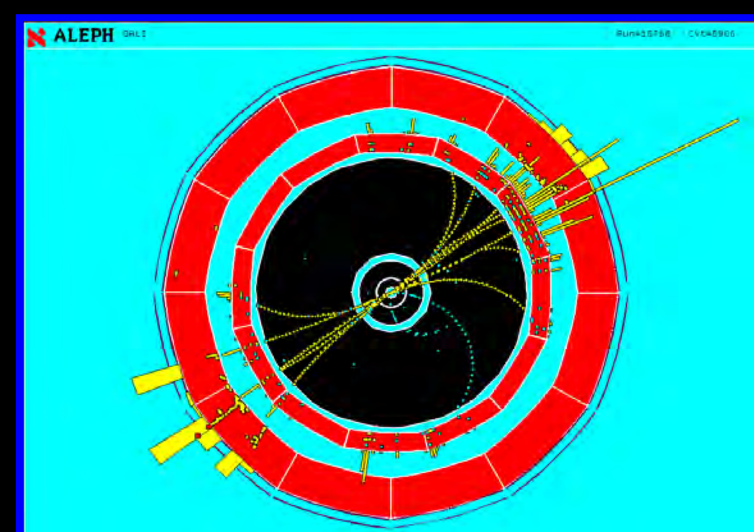
leptonic

$Z^0 \to \mu^+ \mu^-$



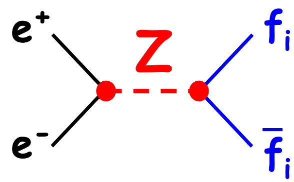
hadronic

$Z^0 \to q\ \bar{q}$



Nr of families

$$\sigma_{elm} = \frac{4\pi}{3}\frac{\alpha^2}{s} \qquad s = m_Z^2$$



analog:

$$\sigma_h = \frac{12\pi}{m_Z^2}\frac{\Gamma_L\ \Gamma_h}{\Gamma_Z^2} = \frac{12\pi}{m_Z^2} R \frac{\Gamma_L^2}{\Gamma_Z^2}$$

$$\Gamma_L/\Gamma_Z = \sqrt{\frac{m_Z^2\sigma_h}{12\pi R}} = (3.37\pm0.07)\ \%$$

$$N_\nu\ \Gamma_\nu/\Gamma_L = \Gamma_Z/\Gamma_L - 3 - \Gamma_h/\Gamma_L$$

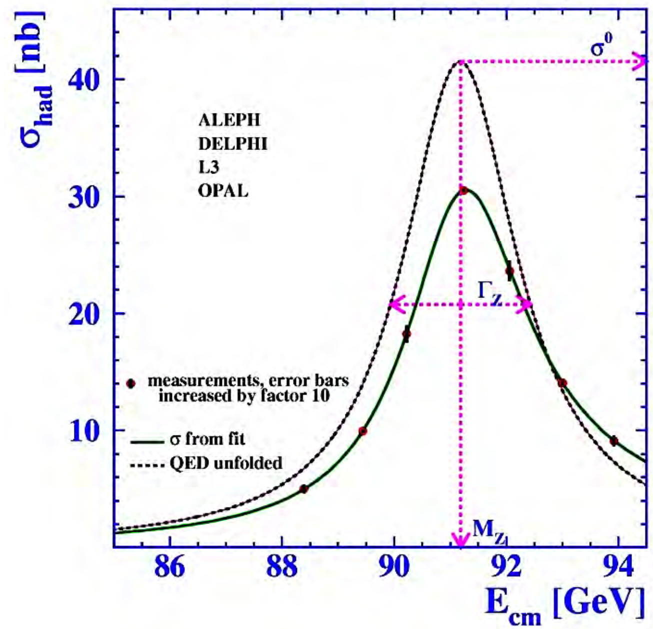
$$1.99\ N_\nu = 29.7 - 3 - 20.8$$

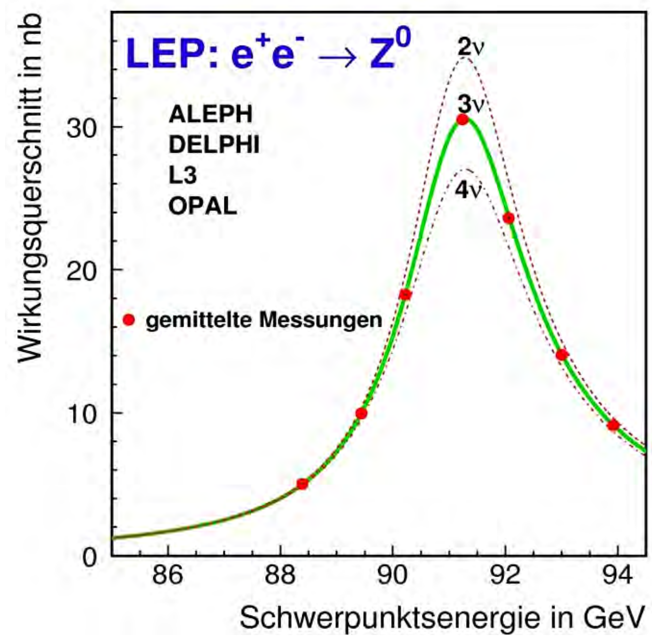
$$N_\nu = 2.992 \pm 0.007$$

J.Erler, A.Freitas, PDG: Chin. Phys. C, 40, 100001 (2016).

Nature has only
3 fermion generations !

PLANCK satellite CMBR: $N_\nu = 3.0 \pm 0.3$





Electroweak coupling:

$\sin^2\theta_W$ -

the Weinberg angle

$\sin^2\theta_W$

Get $\sin^2\theta_W$ from parity violation of Z coupling to leptons+hadrons

Standard Model test:

- lepton universality: $e=\mu=\tau$?
- lepton-hadron universality: $(ud) = (e\nu)$?

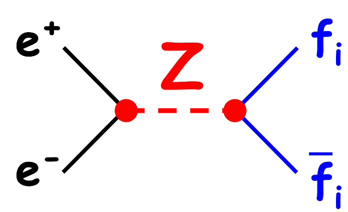
CERN LEP, 1990-96: 17 million Z decays

- 16 million hadronic
- 1 million leptonic

1. Forward-Backward Asymmetry

asymmetries: systematic errors cancel

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$



fermion:

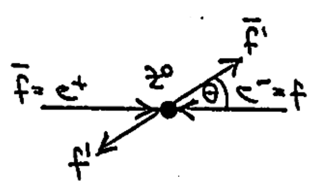
- ν invisible
- q->jets: identify
- leptons ok !

$A^{ff'}_{FB} = 3 \; \tfrac{1}{2}A^f \; \tfrac{1}{2}A^{f'}$ where

$$A_f = 2\,\frac{g_V/g_A}{1+(g_V/g_A)^2}$$ with

$$g_V/g_A = 1-4\sin^2\theta_W$$

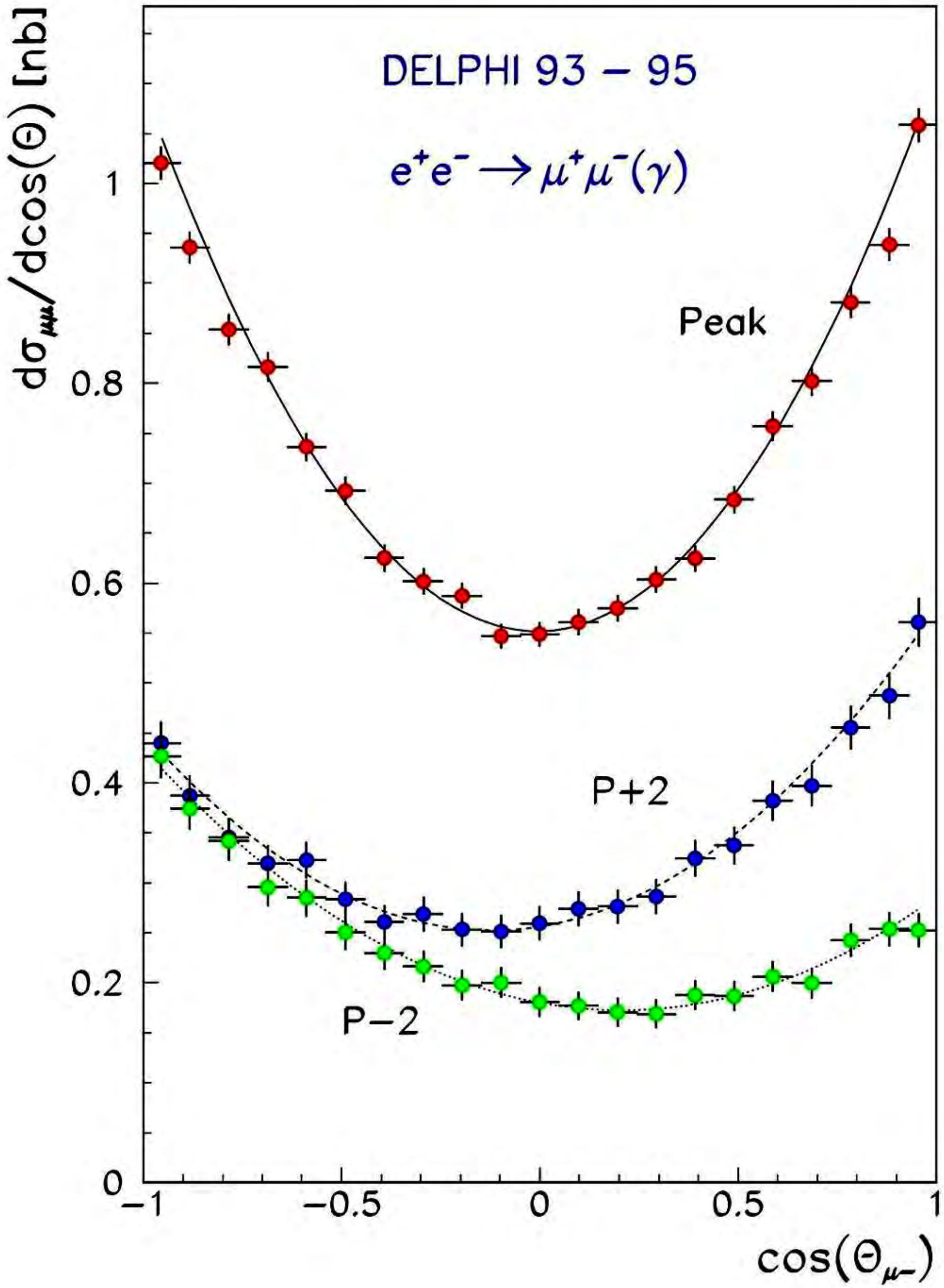
Z rest system, lepton scattering angle θ dependence:



$\sigma_F = \int_0^1 d\cos\Theta \; d\sigma/d\cos\Theta$

$\sigma_B = \int_{-1}^0 d\cos\Theta \; d\sigma/d\cos\Theta$

$d\sigma/\sigma \; d\cos\Theta = 3/8\,(1+\cos^2\Theta) + A_{FB}\cos\Theta$



Forward-Backward Asymmetry

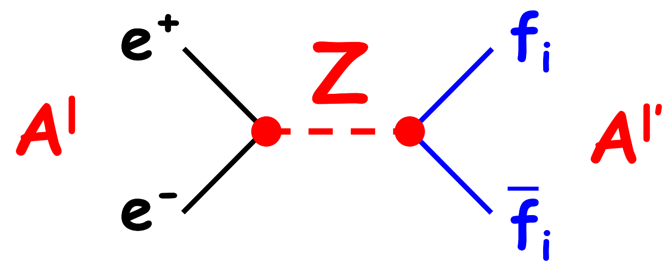
$A^{ll'}_{FB}$ = (1.7±0.1) %

small, but contains electroweak coupling s_W^2:

$g_V/g_A = 1\text{-}4\sin^2\Theta_W = 0.07$

$\sin^2\theta_W$ from leptons

- $A^{ff'}_{FB} = 3\ \tfrac{1}{2}A^f\ \tfrac{1}{2}A^{f'}$
- $A^f\ = 2\,(g_V/g_A)\ /\ (1+(g_V/g_A)^2)$
- $g_V/g_A = 1-4s_W^2$



$s_W^2 \sim 1/4 \quad \Rightarrow \quad g_V/g_A \ll 1 \quad \Rightarrow$
$A^{ll'}_{FB} \sim 3\ g_V^2/g_A^2 = 3\ (1-4s_W^2)^2$

CERN, LEP, 4 expts. 1990-96: >1 million leptonic Z decays

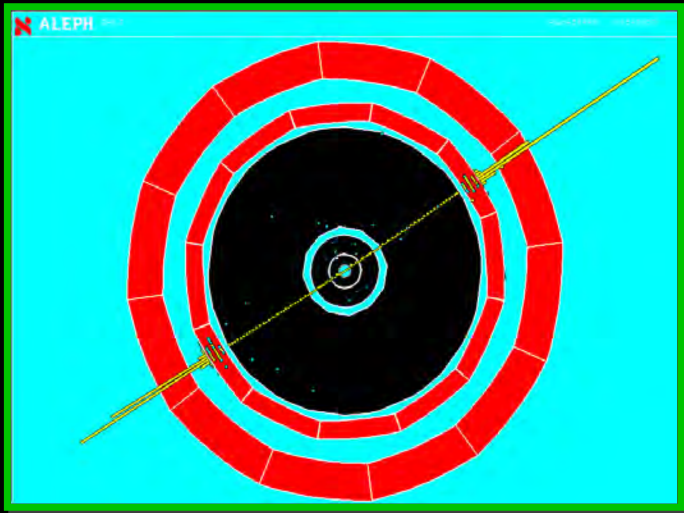
$A^{ll'}_{FB} = (1.7\pm0.1)\ \%$ => $\sin^2\theta_W = 0.2310 \pm 0.0003$

- all charged leptons couple universally: $A_f = A_l = A_e = A_\mu = A_\tau$ at ‰ level!

- $s_W^2 \sim 1/4$ => asymmetry + P violation small
  $s_W^2 = 1/4$ => $g_V = 0$ => $A^{ll'}_{FB} = 0$
  - no asymmetry
  - no P violation (in Z coupling)
  - purely axial coupling: P=+1

Z decays

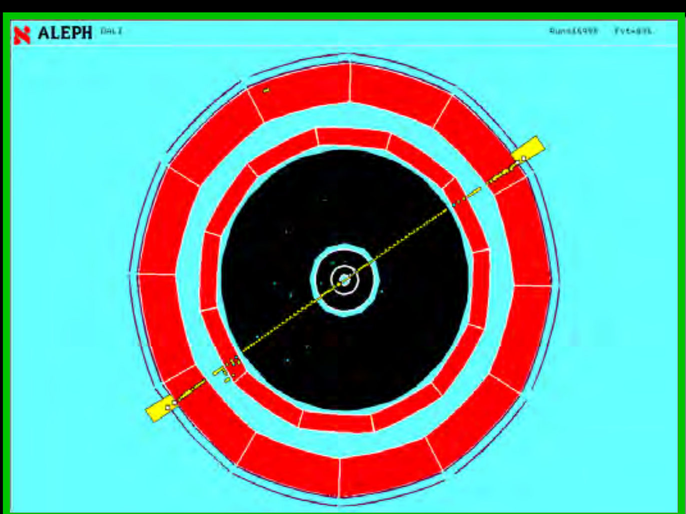
ALEPH @ LEP



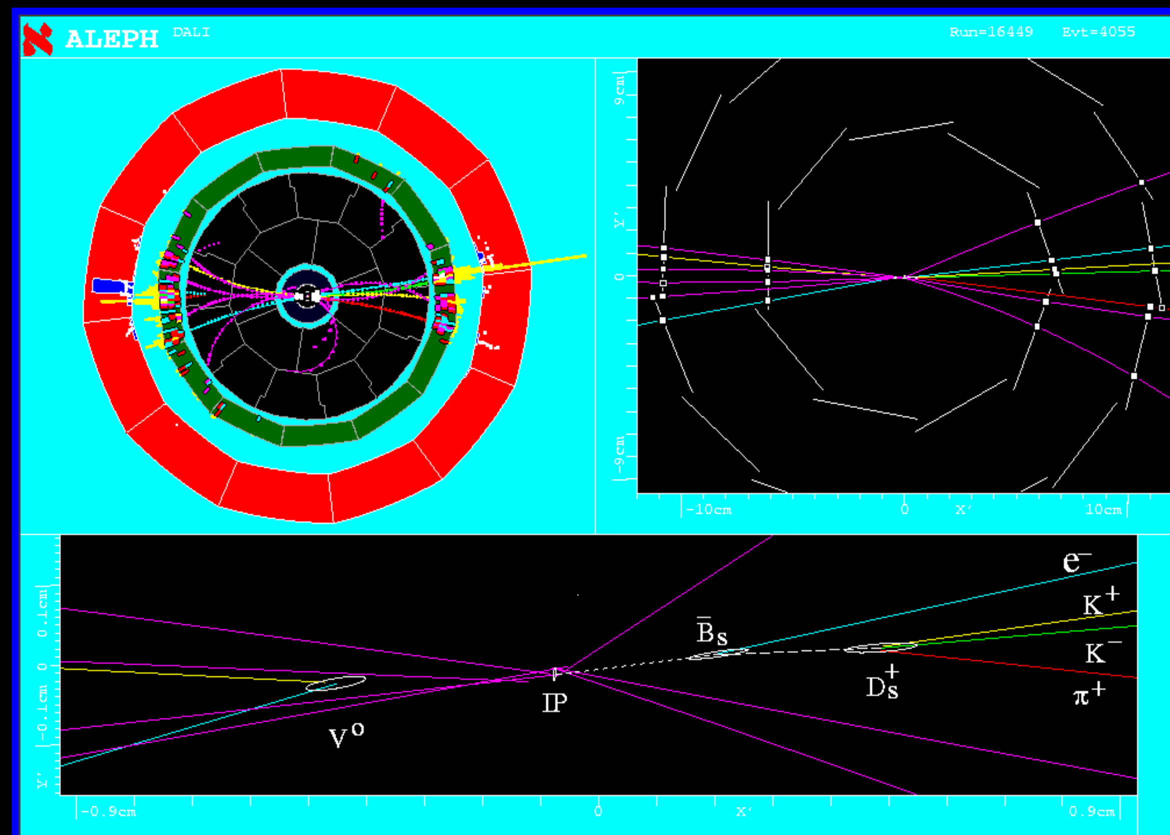
$Z^0 \rightarrow e^+ e^-$

leptonic

$Z^0 \rightarrow \mu^+ \mu^-$



hadronic $Z^0 \rightarrow b\,\bar{b}$



Z -> bb

DELPHI @ LEP

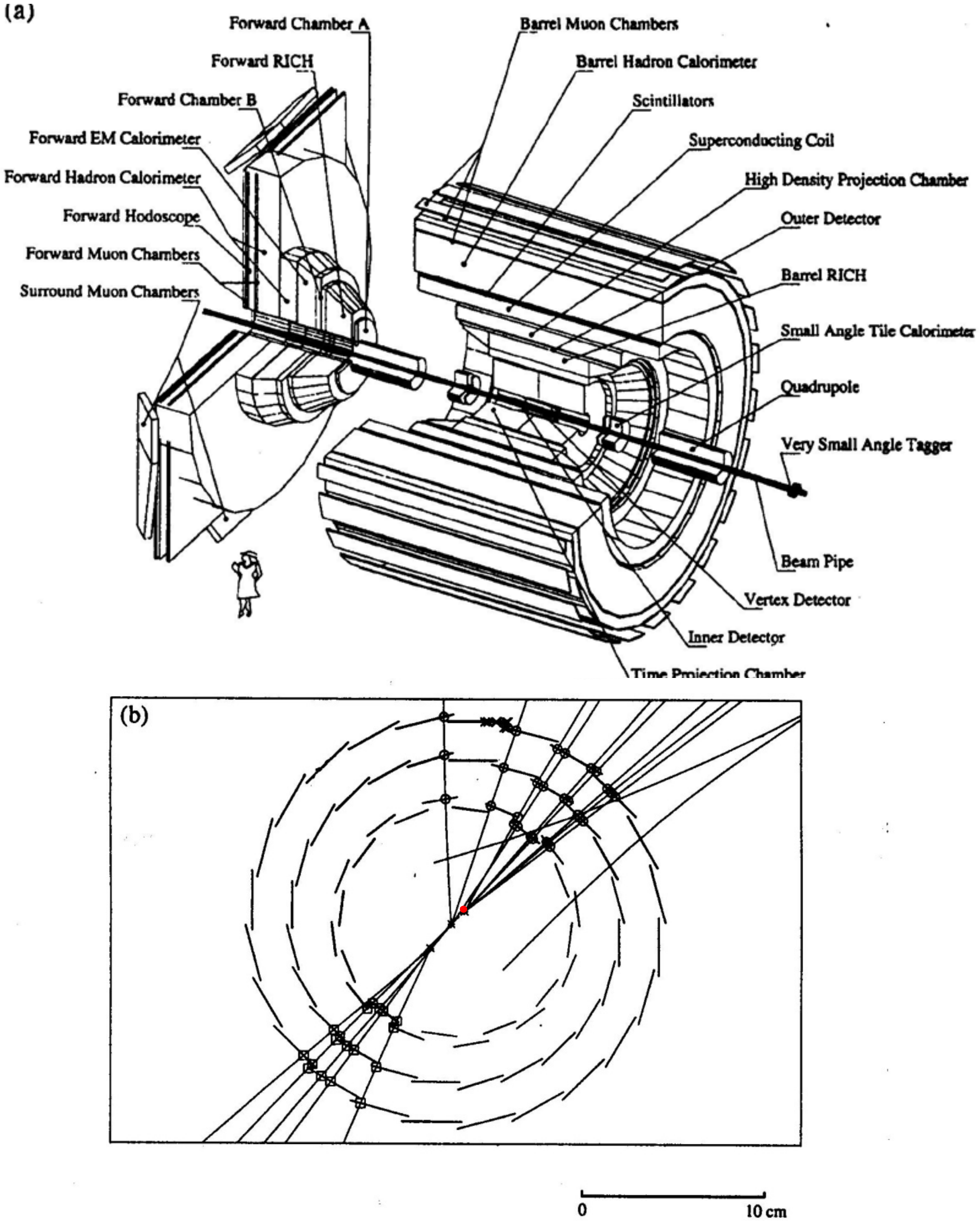
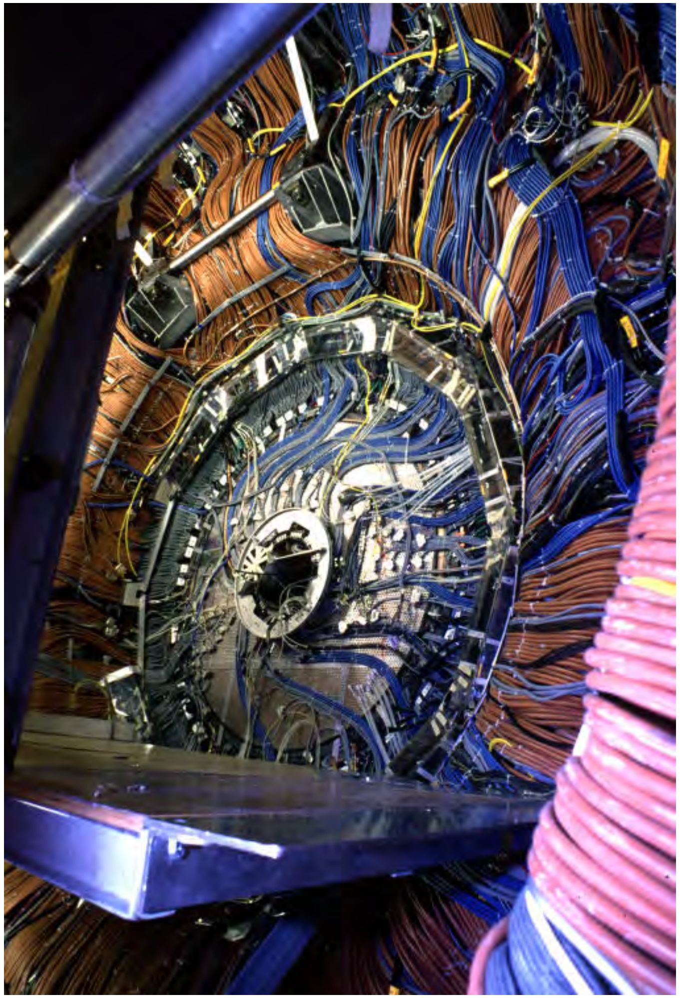
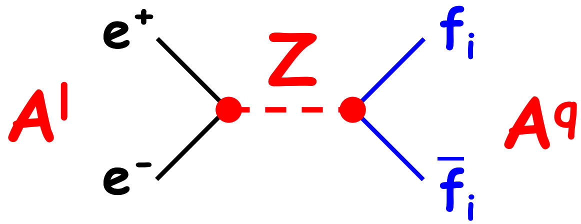


Fig. 8.5. (a) Layout of the detector DELPHI employed at the LEP collider; (b) example of $Z^0 \to b\bar{b} \to$ two jets event, showing the displaced vertices corresponding to the B meson decays.



$\sin^2\theta_W$ from quarks

- $A^{ff'} = 3\ \tfrac{1}{2}A^{f}\ \tfrac{1}{2}A^{f'}$
- $A^{f} = 2\,(g_V/g_A)\,/\,(1+g_V^2/g_A^2)$
- $g_V/g_A = 1 - 4\,q\,s^2_W$



- **Leptons:** q=1 => small effect

$g^l_V/g^l_A = 1 - 4\ s^2_W = 0.075$ => $A^l = 15\%$ => $A^{ll'} = 1.7\%$

☺ good lepton identification
☹ small P violation + asymmetries

- **Hadrons:** q<1 => large effect

$g^b_V/g^b_A = 1 - 4/3\ s^2_W = 0.69$ => $A^b = 92\%$ => $A^{lb} = 10\%$

☺ large P violation + asymmetries, esp. for b
☹ flavor lifetime tag: charm+bottom jets difficult to identify

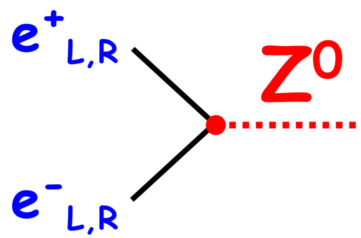
- CERN, LEP, 1990-96: 16 million hadronic Z decays

$A^{lb} = (9.9 \pm 0.2)\ \%$ => $\sin^2\theta_W = 0.2322 \pm 0.0003$

- lepton-hadron universality of couplings !

$\sin^2\theta_W$

2. Polarized e^- scattering



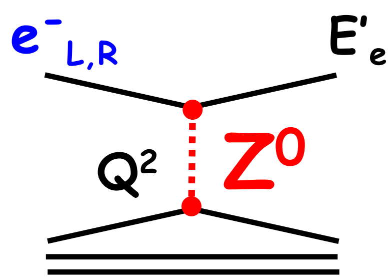
SLAC, Stanford, CA, USA:

- longitudinally polarized $e^+ e^-$ beams

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = A_e = 2\,\frac{g_V/g_A}{1+(g_V/g_A)^2} = 2\,\frac{1-4\sin^2\Theta_W}{1+(1-4\sin^2\Theta_W)^2} \qquad (P_e = 1)$$

$\sin^2\theta_W = 0.2310 \pm 0.0003$

- polarized electron-deuteron scattering:



$e^-_{L,R}\,d$: virtual Z, isoscalar target

$\sigma\,(Q^2, E'_e) = f(1-4\sin^2\Theta_W)$

$\sin^2\theta_W = 0.222 \pm 0.018$

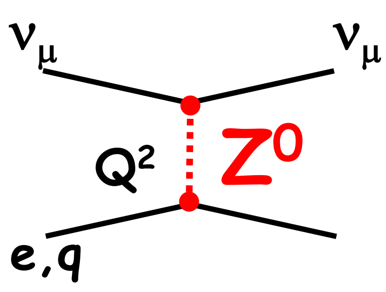
3. νe scattering

$\sin^2\theta_W$

Z spacelike, ν energy spectrum

$\sigma^{\bar{\nu},\nu} \sim (g_V \pm g_A)^2 + (g_V \mp g_A)^2/3$

$$R = \frac{\sigma^{\nu e}}{\sigma^{\bar{\nu} e}} = \frac{(2-4\sin^2\Theta_W)^2 + (4\sin^2\Theta_W)^2/3}{(4\sin^2\Theta_W)^2 + (2-4\sin^2\Theta_W)^2/3}$$



- $\sin^2\theta_W = 0$ =>
  $g_V = g_A$ 100% P violation $\sigma^\nu = 3\,\sigma^{\bar{\nu}}$ R=3
  pure (V-A) like weak charged current W exchange
  $\bar{\nu}$: helicity suppression of backscattering
  ν: J=0 , flat scattering angle distribution

- $\sin^2\theta_W = 1/4$ =>
  $g_V = 0$ no P violation $\sigma^\nu = \sigma^{\bar{\nu}}$ R=1

Paschos-Wolfenstein relation:

$$R^- = \frac{\sigma^\nu_{NC} - \sigma^{\bar{\nu}}_{NC}}{\sigma^\nu_{CC} - \sigma^{\bar{\nu}}_{CC}} = \rho^2\left(\tfrac{1}{2} - \sin^2\theta_W\right) = g_L^2 - g_R^2 \qquad g^2_{L,R} = u^2_{L,R} + d^2_{L,R}$$

Fermilab, Chicago, NuTeV expt, CC+NC $\bar{\nu}/\nu$N
$\sin^2\theta_W = 0.228 \pm 0.002$

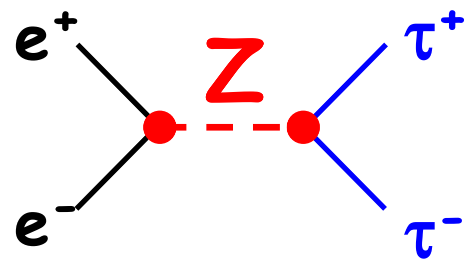
CDF+D0 m_W:
$\sin^2\theta_W = 0.23179 \pm 0.00035$

CERN CHARMII νe: $\sin^2\theta_W = 0.231 \pm 0.008$

4. τ polarization

$\sin^2 \theta_W$

$\tau \rightarrow \pi \, \nu_\tau$ get $\theta = \angle(e,\pi)$



measure τ polarization from
τ decay angular distribution

$$P_\tau = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = \frac{A_\tau (1+\cos^2\Theta) + 2 \; A_e \cos\Theta}{(1+\cos^2\Theta) + 2A_\tau A_e \cos\Theta} \quad => \quad A_\tau, \; A_e \quad =>$$

$\sin^2 \theta_W = 0.2316 \pm 0.0004$

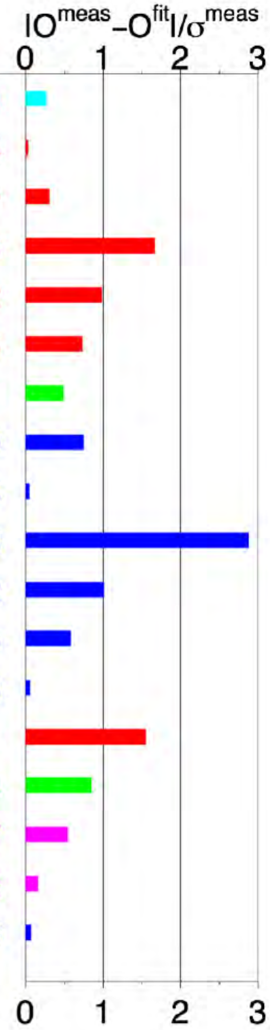
World average of electro-weak Weinberg mixing angle:

$\sin^2 \theta_w (M_Z) = 0.23149 \pm 0.00016$

Gfitter, MS scheme, rel. precision $< 10^{-3}$!

Standard Model Parameters

	Parameter	Measurement	Fit	relative precision
	$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02750 ± 0.00033	0.02759	
L	m_Z [GeV]	91.1875 ± 0.0021	91.1874	10^{-5}
	Γ_Z [GeV]	2.4952 ± 0.0023	2.4959	10^{-3}
	σ_{had}^0 [nb]	41.540 ± 0.037	41.478	10^{-3}
	R_l	20.767 ± 0.025	20.742	10^{-3}
E	$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01645	
	$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481	
	R_b	0.21629 ± 0.00066	0.21579	10^{-3}
	R_c	0.1721 ± 0.0030	0.1723	
P	$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038	
	$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742	
	A_b	0.923 ± 0.020	0.935	
	A_c	0.670 ± 0.027	0.668	
SL	A_l(SLD)	0.1513 ± 0.0021	0.1481	
AC	$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314	10^{-3}
FN	m_W [GeV]	80.385 ± 0.015	80.377	10^{-4}
AL	Γ_W [GeV]	2.085 ± 0.042	2.092	
	m_t [GeV]	173.20 ± 0.90	173.26	10^{-2}



March 2012

α, G_F
or
$\sin^2\theta_W$, M_Z:

system
over-constrained

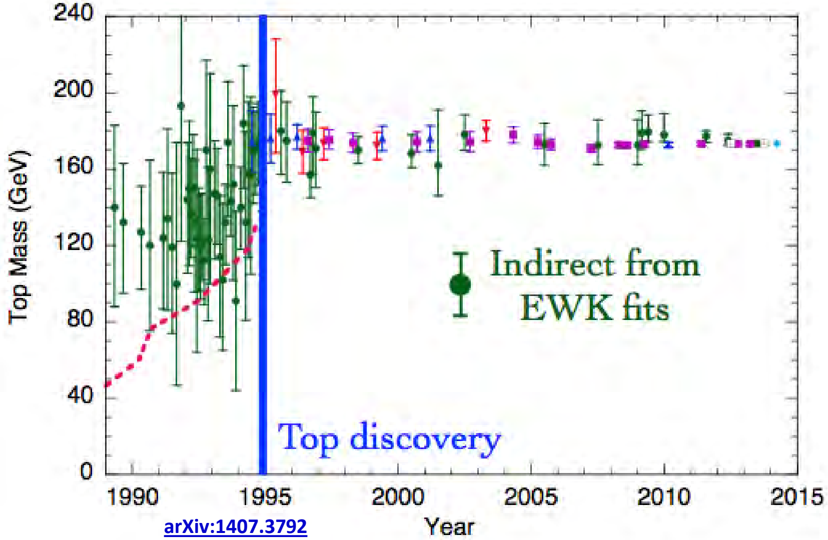
Electro-
Weak
Theory

consistent

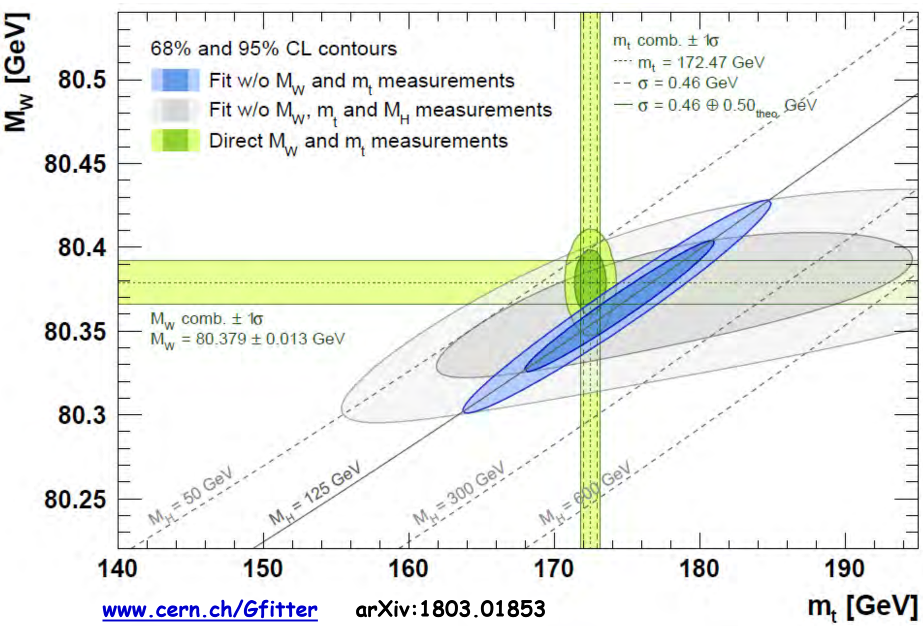
at the per mille level

incl. radiative corrections
incl. top + Higgs mass !

predictions: from top to Higgs



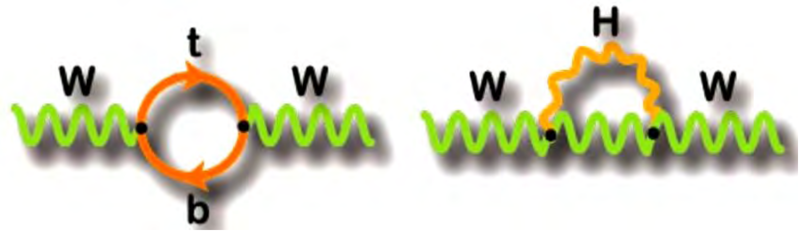
arXiv:1407.3792



www.cern.ch/Gfitter arXiv:1803.01853

predict top mass from

- per mille exptl. precision
- higher order electroweak radiative corrections incl. Higgs



$\sim (m_t/m_W)^2$, $\ln (m_H/m_W)$

	experiment	electroweak fit
1994	176 ± 13 GeV	169 ± 25 GeV
2014	172.4 ± 0.7 GeV	177 ± 2 GeV

predict W mass

2013 80.385 ± 0.015 80.362 ± 0.008 GeV

arXiv:1608.01509

The Higgs Mechanism

Higgs Mechanism

- **Fermion mass term gauge invariant :**

  $m\,\psi'^{*}\,\psi' = m\,\psi^{*}\,\psi$

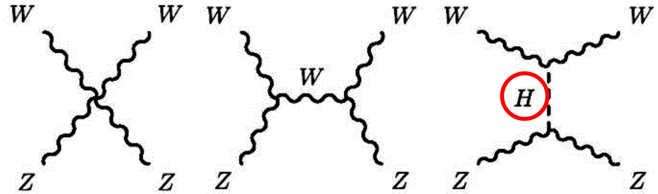
- **Boson mass term gauge violating :**

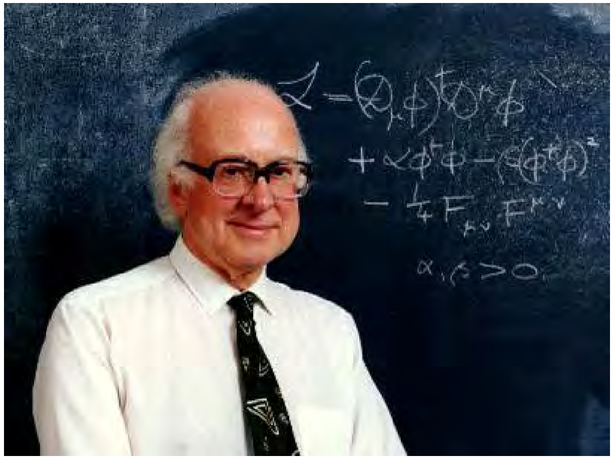
  $m^2 A'_\mu A'^\mu = m^2 (A_\mu + \partial_\mu\alpha)(A^\mu + \partial^\mu\alpha) =$
  $m^2(A_\mu A^\mu + \ldots)$

- **$W_L^+W_L^-$ scattering violates unitarity :**

  $\mathrm{Re}\, A_J \leq 1/2$ for each partial wave J
  $A_0\,(W_L^+W_L^- \to Z_L\,Z_L) = s\,G_F\,/\,(8\pi\sqrt{2})$
  at $s > 4\pi\sqrt{2}\,/\,G_F = (1.2\ \mathrm{TeV})^2$

- **Solution:**
  new scalar (Higgs) boson field restores gauge invariance and unitarity





Peter Higgs 1964

Nobel prize 2013
with J. Englert

Broken Symmetries,
Massless Particles
and Gauge Fields
Physics Letters 12 132

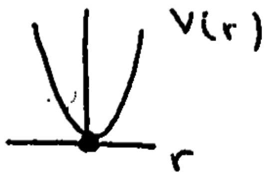
but also:
Brout, Kibble, Hagen, Guralnik
Anderson, Nambu, Goldstone
t'Hooft, Veltman, Weinberg

Higgs Mechanism

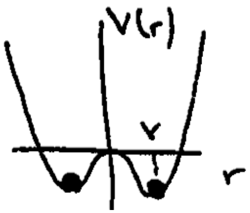
1. Toy model:

scalar potential $V(r) = \mu^2 r^2/2 + \lambda r^4/4$

$\lambda>0$: stable for large r



- $\mu^2>0$:
  trivial minimum at $r=0$



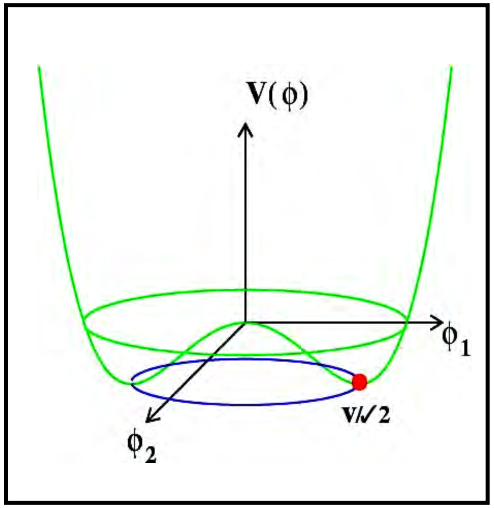
- $\mu^2<0$:
  minima at $r = \pm\sqrt{-\mu^2/\lambda}$

break P symmetry $\pm r$
select vacuum state $r = +\sqrt{-\mu^2/\lambda} = v$

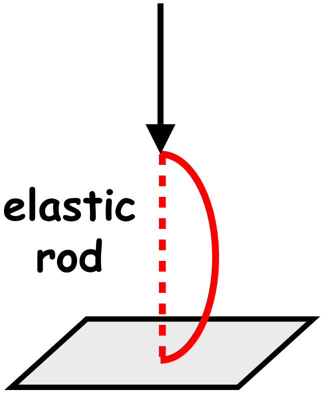
$V(r) = V(-r)$ Lagrangian conserves P
$P|0\rangle \neq |0\rangle$ vacuum states violate P

Spontaneous Symmetry Breaking !

mechanics: breaking of elastic rod:
- problem: symmetric
- solution: symmetry breaking



Spontaneous Symmetry Breaking



Higgs Mechanism

2. Toy model:

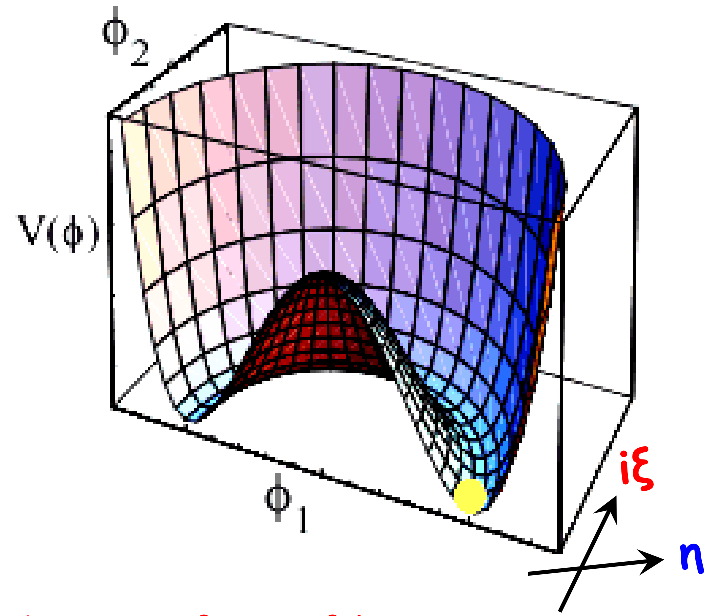
replace scalar real potential V(r) ->
scalar complex Higgs field in abelian U(1):

$D_\mu = \partial_\mu - iq A^\mu$ covariant derivative

$\Phi(x_\mu) = [v+\eta(x_\mu)] \exp(i\, \xi(x_\mu)/v)$

small excitations η, $i\xi$ around v:

$\Phi(x_\mu) = v + \eta(x_\mu) + i\xi(x_\mu)$



$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{1}{2} (D_\mu \Phi)^*(D^\mu \Phi)$
$- \frac{1}{2}\mu^2\, \Phi^*\Phi - \frac{1}{4}\lambda\, (\Phi^*\Phi)^2$ Higgs potential, vev. $v^2 = -\mu^2/\lambda$

$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu}$ 0 free propagation of vector bosons
$+ \frac{1}{2} q^2v^2 A_\mu A^\mu$ + mass term for vector bosons A_μ
$+ \frac{1}{2} (\partial^\mu\eta)(\partial_\mu\eta) + \mu^2 \eta^2$ 0 kinetic + mass term for new scalar boson η
$+ \frac{1}{2} (\partial^\mu\xi)(\partial_\mu\xi)$ - massless Goldstone field ξ (symmetry breaking)
$- qv A_\mu (\partial_\mu\xi)$ - mixed term: $A_\mu p^\mu$ … longit. gauge boson polar.
$+ O(\eta^3, \xi^3, A\eta^2, \ldots) + \text{const.}$ omit

Higgs mechanism

Higgs trick: specific local gauge transformation of massive fields

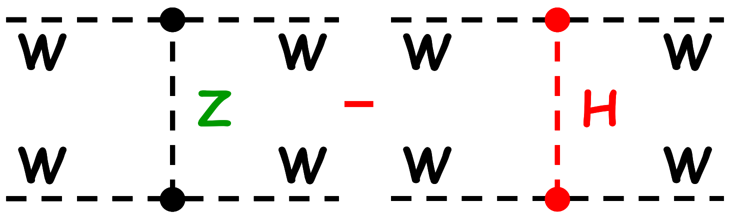
with unphysical complex rotational degree of freedom ξ of the Higgs field:

$A'_\mu = A_\mu + 1/qv\ \partial_\mu \xi(x_\mu)$

trick: gauge violating terms in mass term $A_\mu A^\mu$

- $A_\mu \partial^\mu \xi = A_\mu p^\mu \xi$ — longitudinal polarization
- $(\partial^\mu \xi)^2$ — kinetic ξ term

cancel with ξ terms of Higgs potential !



theory gauge invariant:

$$\mathcal{L} = -F_{\mu\nu}F^{\mu\nu}/4 + q^2v^2/2\, A_\mu{}' A^{\mu\prime} + (\partial^\nu \eta)(\partial_\nu \eta)/2 + \mu^2 \eta^2 + O(\eta^3, A\eta^2, \ldots)$$

particle spectrum:

- $q^2v^2 \sim m_A{}^2 > 0$: massive vector bosons A_μ
- $\mu^2 \sim m_H{}^2 > 0$: massive Higgs field η
- Goldstone boson ξ gauged away against long. polarization of vector bosons

Higgs in SU(2)

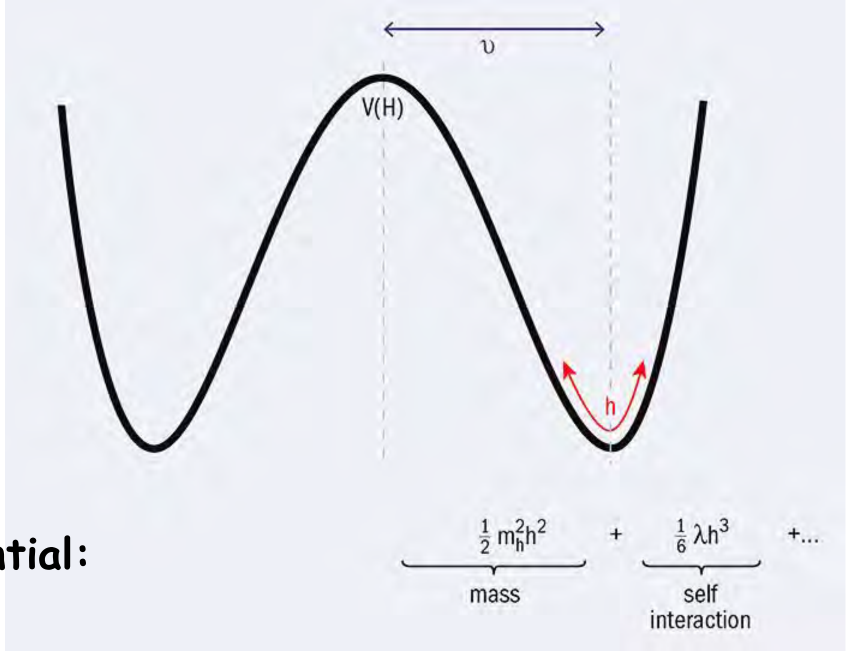
Higgs produces BOSON masses:

- mass term for 2 W bosons in Lagrangian:

  $g^2v^2/2\ A_\mu A^\mu = (2m_W)^2/2\ A_\mu A^\mu$

  scale of electroweak unification:

  $v = 2m_W/g = (\sqrt{2}\ G_F)^{-1/2} = 246\ \text{GeV} = \mu / \sqrt{\lambda}$



- m_H connected to quadratic term in Higgs potential:

  $m_H = 125\ \text{GeV} = \sqrt{2}\mu = \sqrt{2}\sqrt{\lambda}\ v$

  parameter λ determined by Higgs self-interactions or μ and v:

  $\lambda = \mu^2/v^2 = m_H^2/2v^2 = G_F\ m_H^2/\sqrt{2} = 0.13$

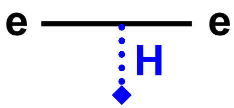
Higgs generates fermion masses

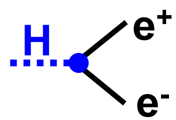
Higgs-lepton coupling: $\Phi = (v+\eta)\, e^{i\xi}$

S.Weinberg, *A Theory of Leptons*, 1967.

$\mathcal{L}_{eHe} = G_e\ e^+ \Phi\ e^-$

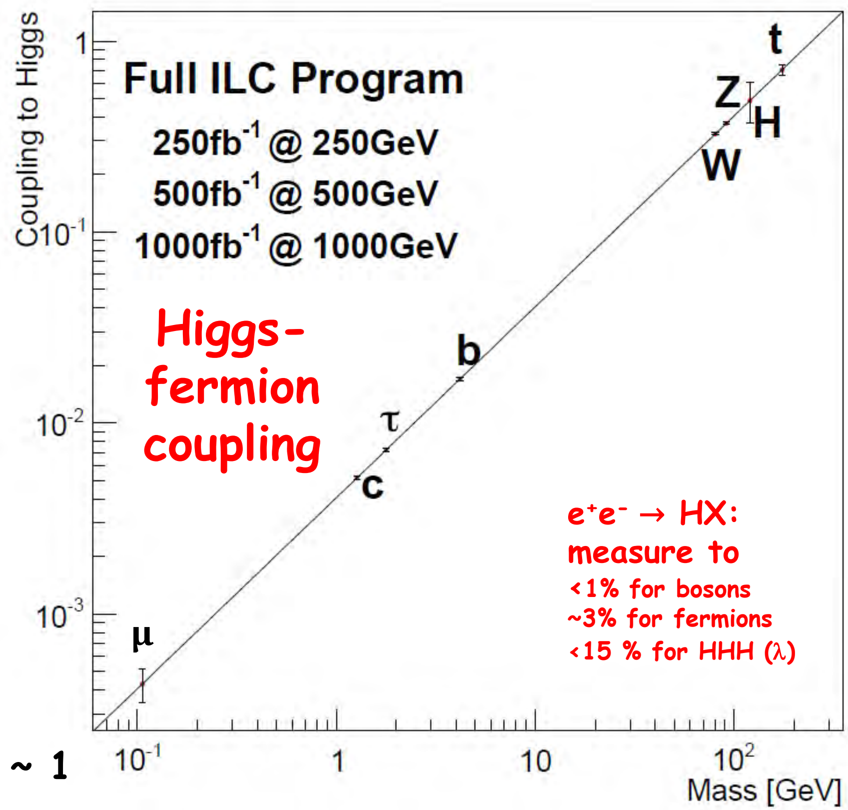
$= G_e v\ e^- e^+ + G_e\ e^- \eta\ e^+$





$G_e v := m_e$

$= m_e\ e^- e^+ + m_e/v\ e^- \eta\ e^+$



- Higgs produces **FERMION** masses !
- Higgs-fermion coupling ~ fermion masses:

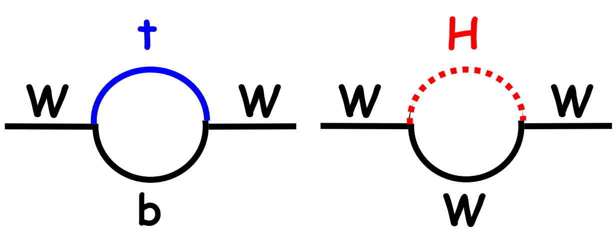
$\sqrt{2}\, m_e/v = 511\ \text{keV}\ \sqrt{2}/246\ \text{GeV} \sim 10^{-6}$

$\sqrt{2}\, m_t/v = 173\ \text{GeV}\ \sqrt{2}/246\ \text{GeV} = 0.996 \pm 0.004 \sim 1$

$\sqrt{2}\, m_\nu/v < 500\ \text{meV}\ \sqrt{2}/246\ \text{GeV} \sim 10^{-12}$

Higgs prediction

m_W radiative corrections $\Delta r \sim 3\%$

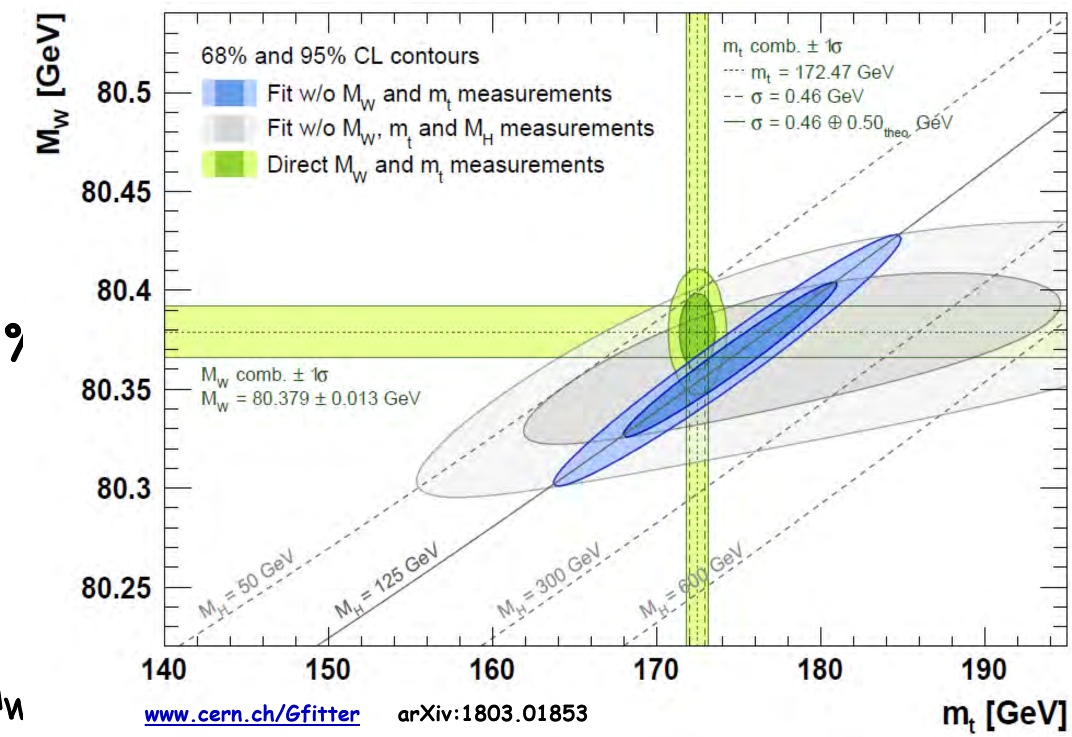


$\Delta m_W \sim (m_t/m_W)^2$ $\Delta m_W \sim \ln (m_H/m_W$

mostly t-b + Higgs

$$\Delta\rho = 0.0026 \frac{M_t^2}{M_Z^2} - 0.0015 \ln\left(\frac{M_H}{M_W}\right)$$

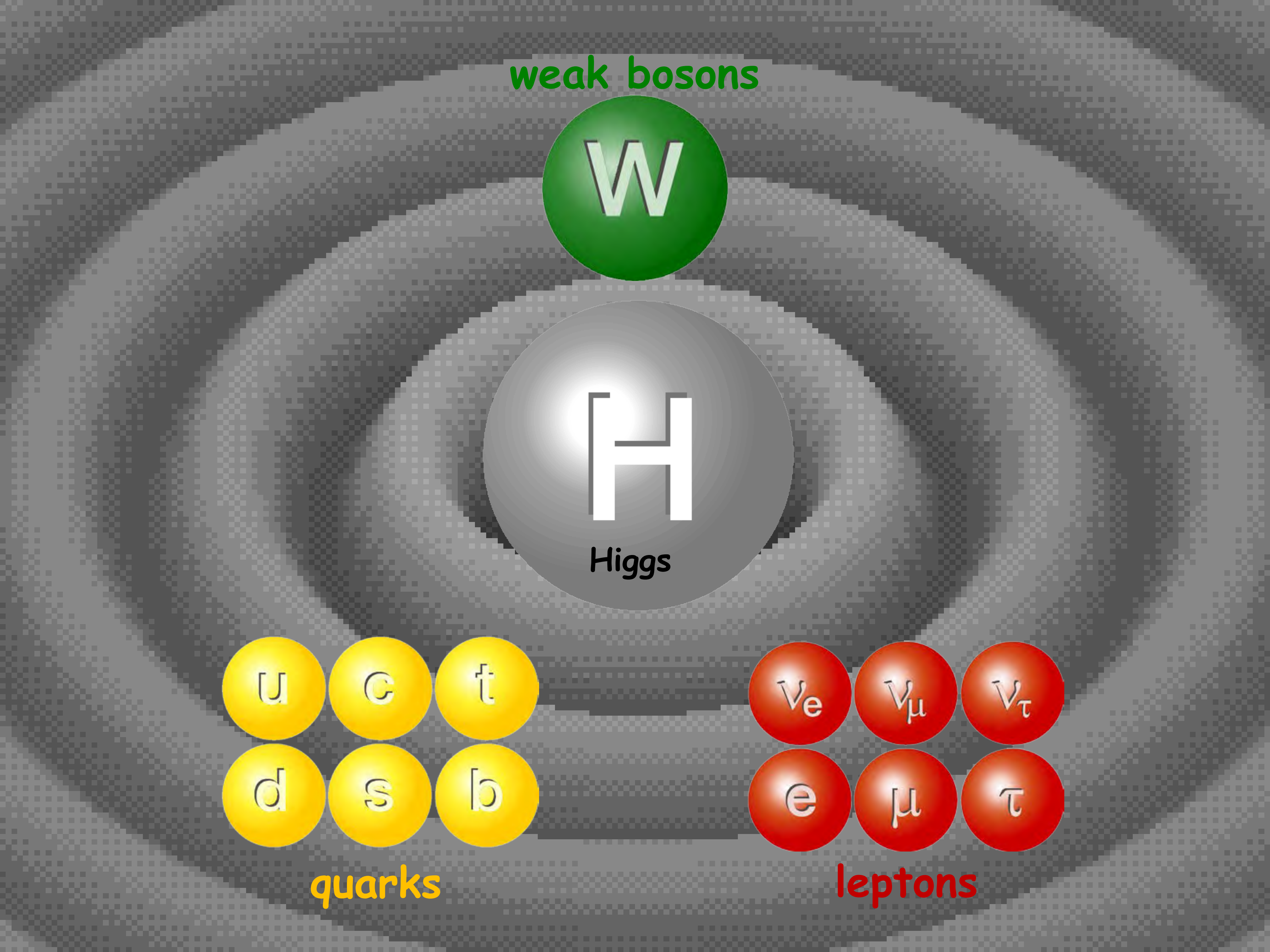
vice versa: SM m_W prediction 2x better than direct measurement: 80358 ± 8 MeV



Standard Model fits

work excellently

for top + Higgs !


weak bosons
W
H
Higgs
u
c
t
d
s
b
quarks
ν_e
ν_μ
ν_τ
e
μ
τ
leptons

Higgs Detection

https://twiki.cern.ch/twiki/bin/view/AtlasPublic/HiggsPublicResults
https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsHIG

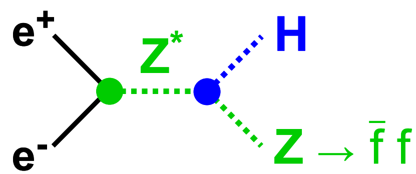
Higgs Production

Higgs couples to heaviest possible particle !

Higgs coupling $y_f = \sqrt{2}\, m_f / v$ $h_Z = \sqrt{2}\; 91/246 \sim 0.5$ $y_t = \sqrt{2}\; 173/246 \sim 1$

1. e^+e^-:

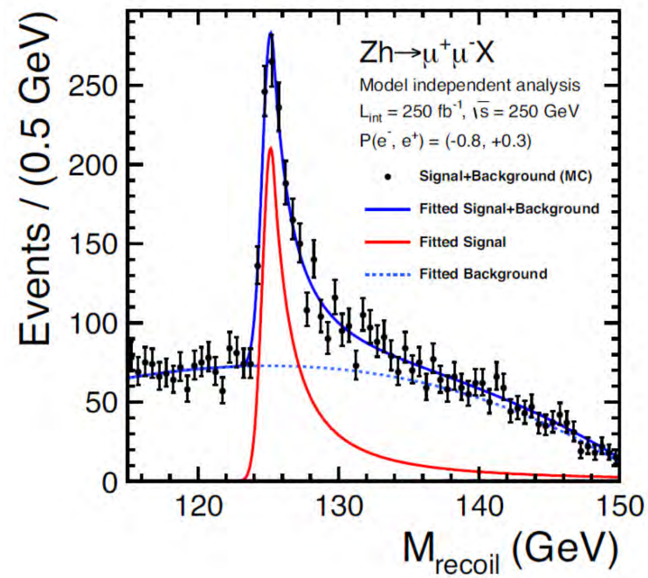
Higgs-strahlung



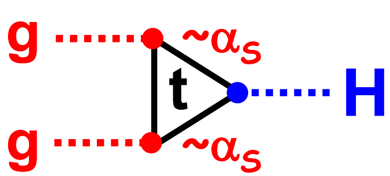
detect Higgs in missing mass: $m_H < \sqrt{s} - m_Z$

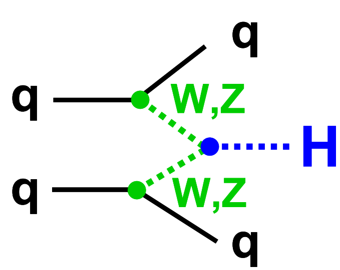
Linear Collider: $\sqrt{s} = 250$ GeV $> m_H + m_Z$

>2025: 30 k Higgs/ y



2. pp: CERN LHC 2x7 TeV





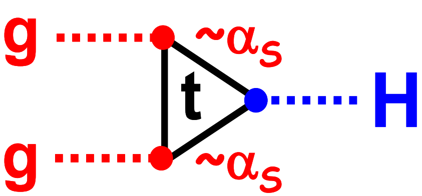
- Higgs coupling
- strong coupling
- weak coupling

gluons carry only tiny fraction of p momentum

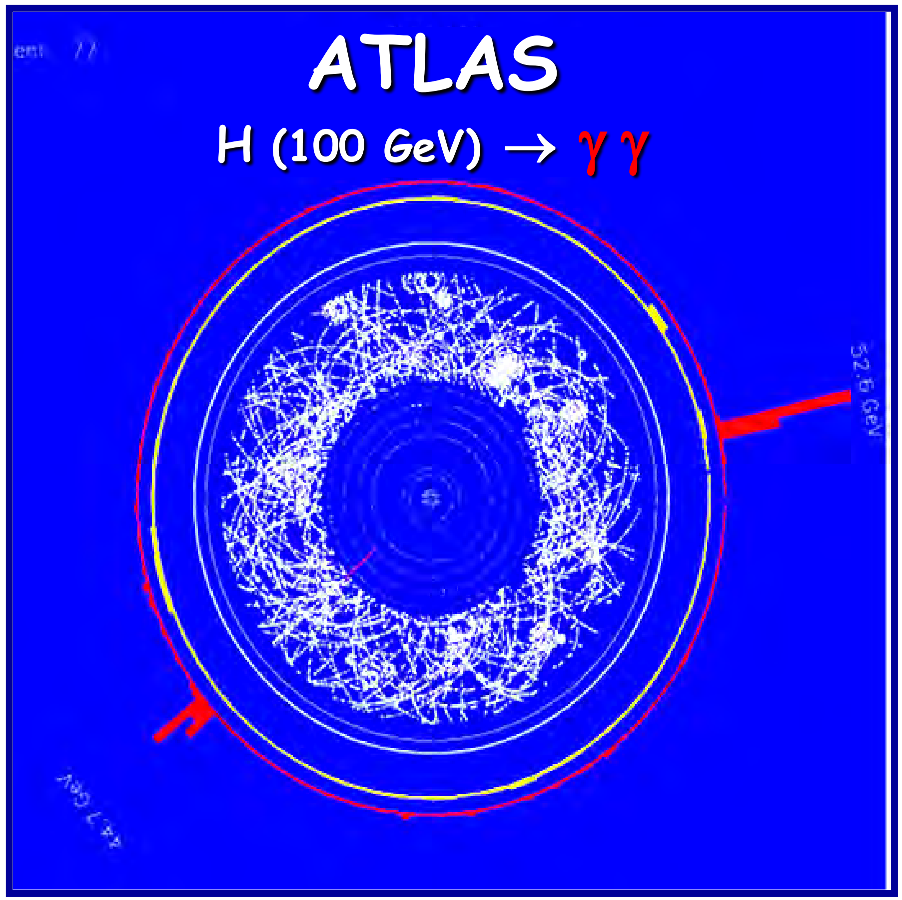
quarks carry ~1/6 of p momentum: ~1 TeV @ LHC

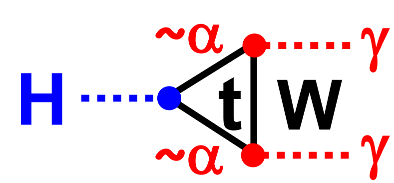
Higgs production + decay

$$pp \rightarrow H\,X \rightarrow \gamma\,\gamma\,X$$



production

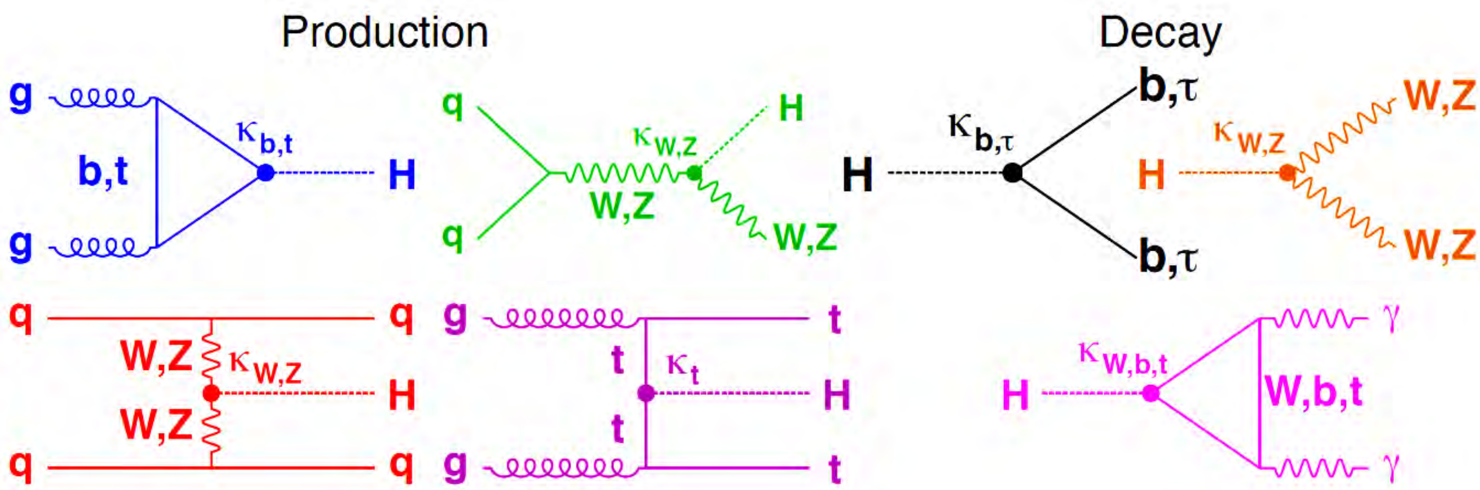


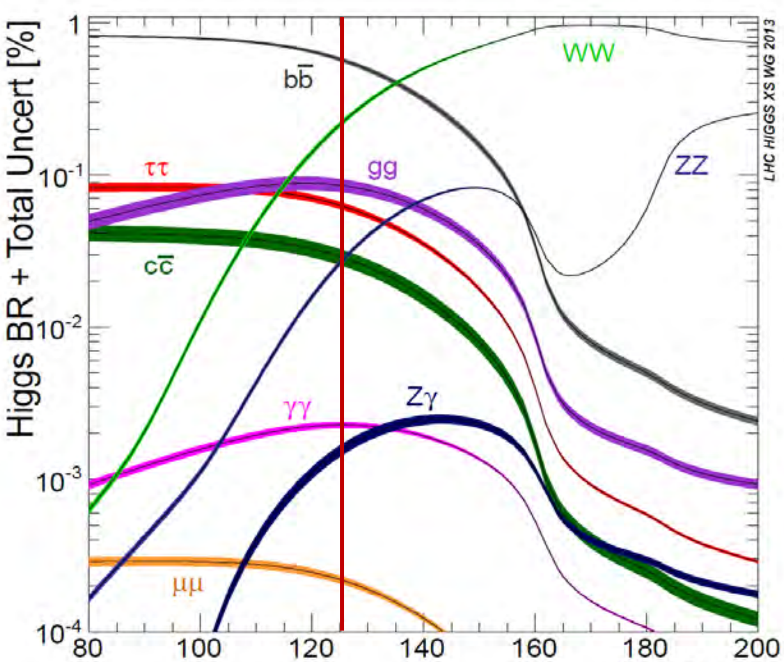


decay

simulation in the ATLAS detector at the LHC at CERN

Higgs production + decay





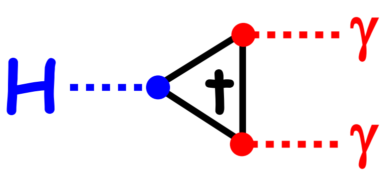
arXiv:1307.1347

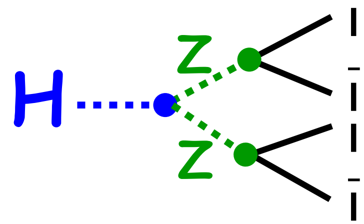
$B_b \,/\, B_\tau = N_C\, m_b/m_\tau$

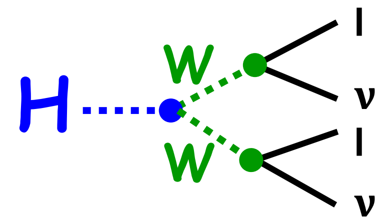
$B_{WW} \,/\, B_{ZZ} = 2$

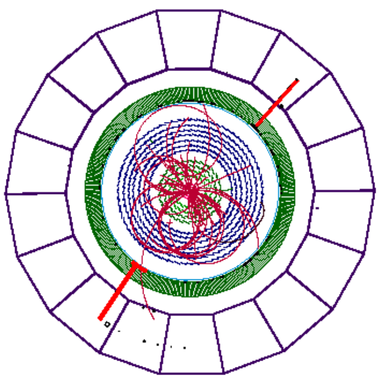
Higgs-fermion coupling =
Yukawa coupling ~
fermion mass

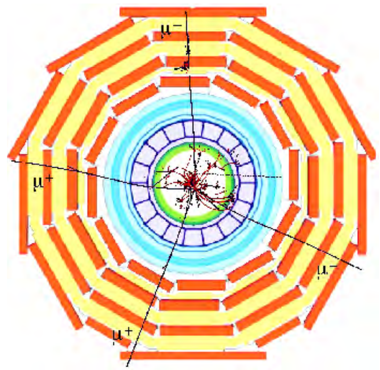
Higgs decay channels

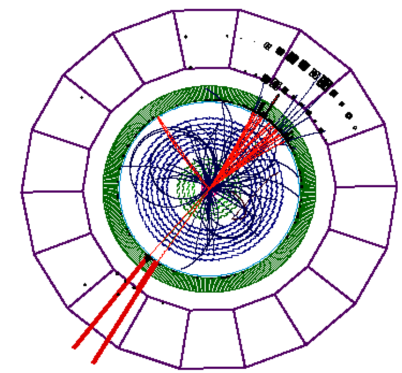








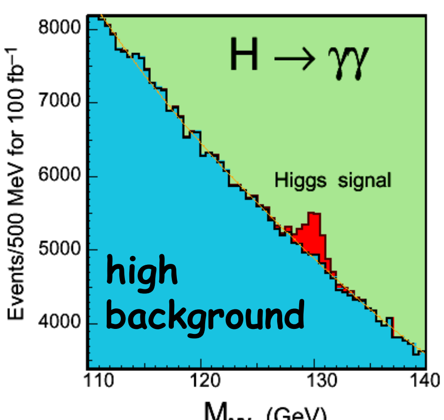


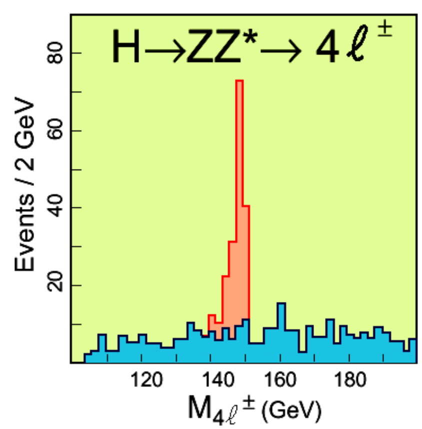


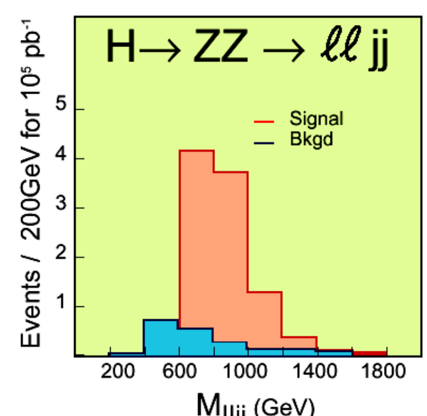
easy to detect but rare:
$\sim \alpha^2 < 10^{-4}$
Br (H→γγ) = 0.2 %

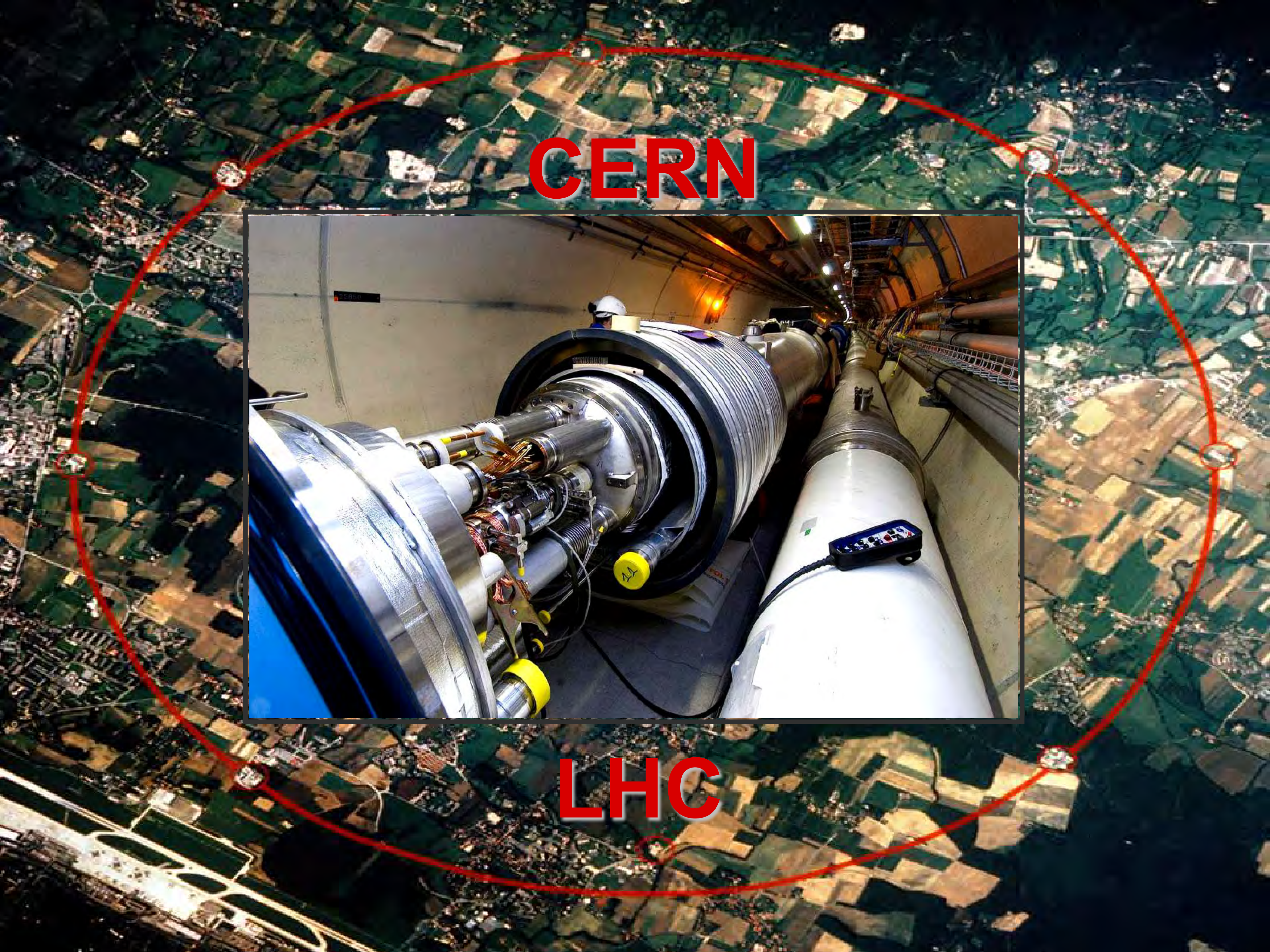
clean, but rare
$(\Gamma_l/\Gamma_Z)^2 \sim (3\%)^2 \sim 10^{-3}$
Br (H→ZZ→4l) ~ 3 10^{-5}

leptons, jets + neutrinos: broad
Br (H→WW) = 22 %







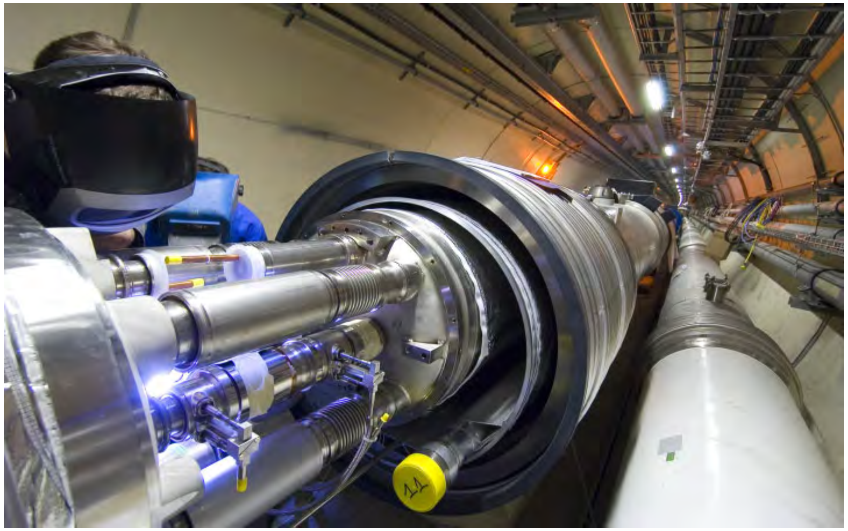

CERN
LHC

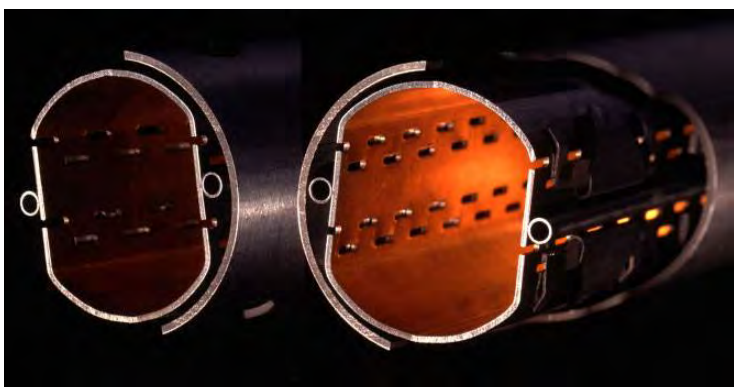
LHC

10^{14} **protons** in **2808 bunches** circle the 27 km ring **11.245** times a second.
The bunches cross 40 million times/s. Up to **a billion protons/s** interact.

LHC

emptier and colder than outer space





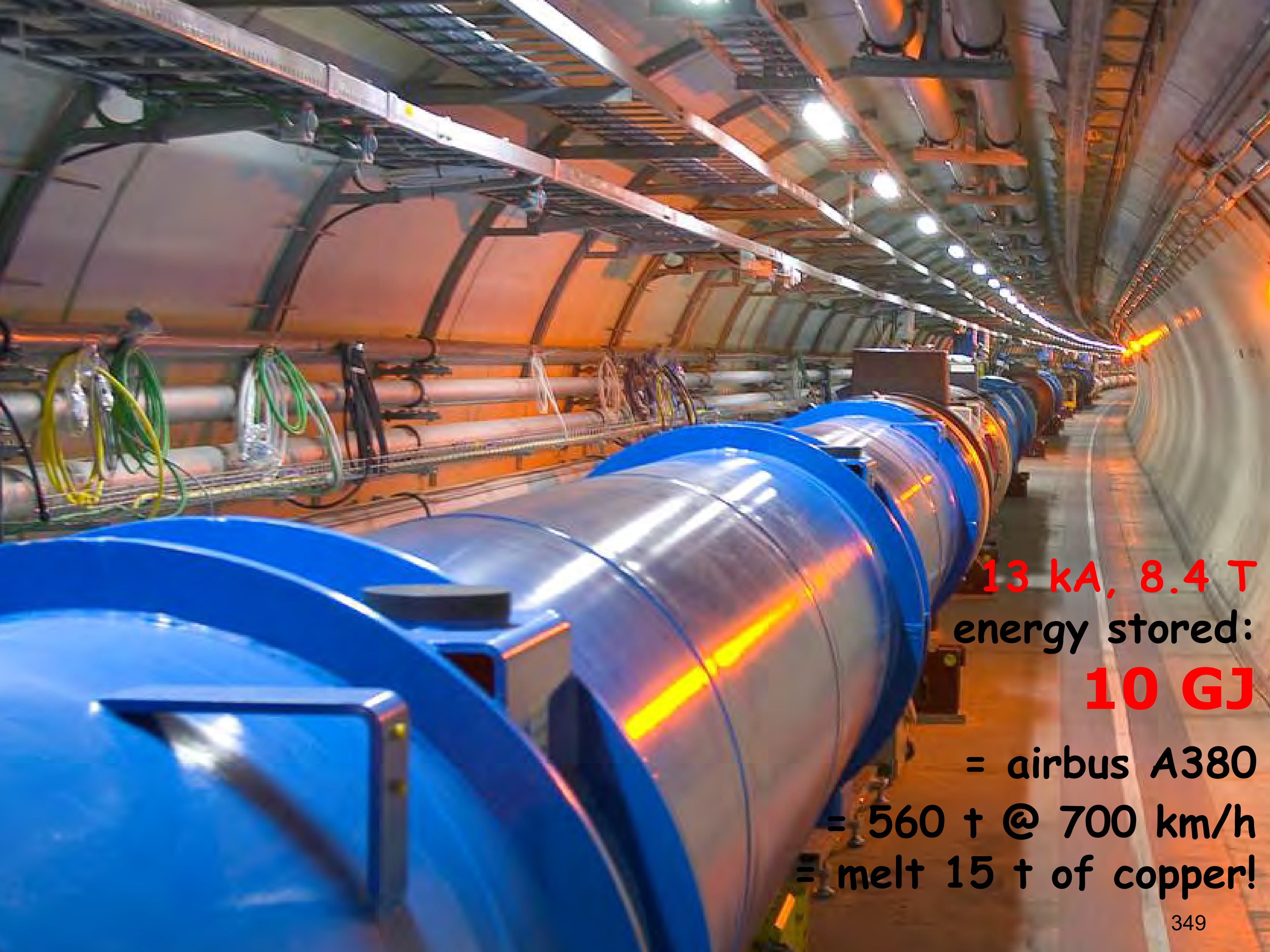
The pressure in the beam pipes of 10^{-10} Torr is about 10 times lower than on the moon.

The vacuum volume of 9.000 m³ is as large as a cathedral.

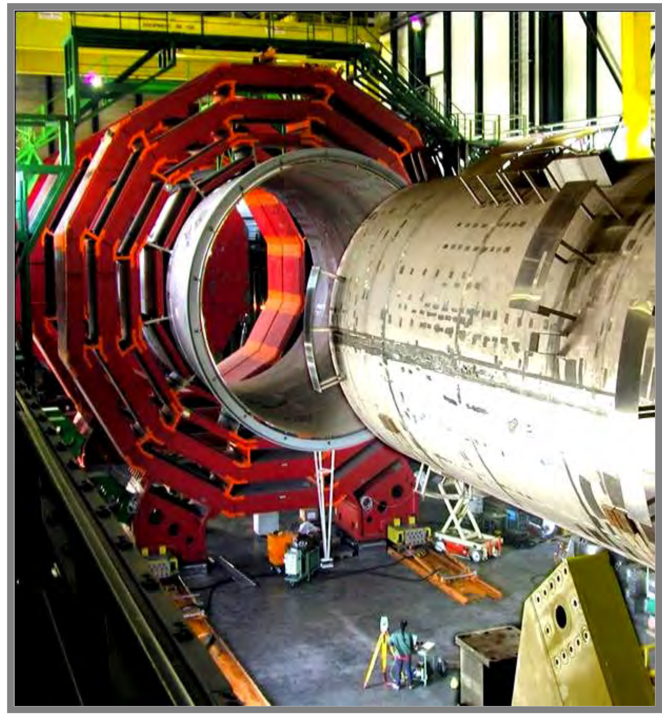
470 trucks, one/hour

With a temperature of 1.9 K the LHC is colder than outer space with 2.7 K.

10.000 t nitrogen and 120 t helium cool down 37.000 t of material.

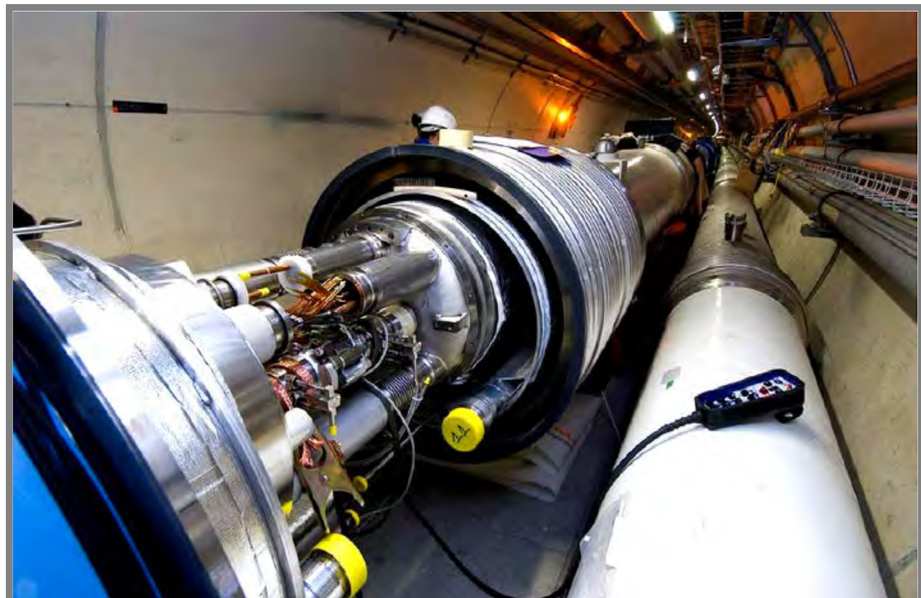

13 kA, 8.4 T
energy stored:
10 GJ
= airbus A380
= 560 t @ 700 km/h
= melt 15 t of copper!

LHC beam + magnets



CMS magnet: world's largest SC magnet!
- 6x12.5 m @ 19 kA, **4T = 2.6 GJ**
  = melt 4 t of Cu !

LHC magnets: world's largest cryosystem !
- 1232 dipoles: 15 m long, 30 t
- 37 kt cold mass: 130 t He @ 1.8 K (superfluid)
- 13 kA @ 1 ppm precision, B = **8.4 T**
- energy stored **10 GJ**
  = airbus A380 = 560 t @ 700 km/h
  = melt 15 t of Cu !



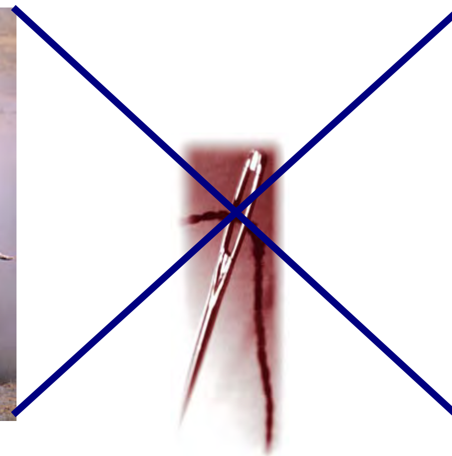
LHC beam: macroscopic energy:
- $3 \cdot 10^{14}$ p = **0.5 A @ 7 TeV**:
- E = **362 MJ** (HERA, Tevatron: 2 MJ)
  = 80 kg TNT
  = 400 t ICE @ 160 km/h
  = melt 0.5 t Cu /beam
- size <20 μm
- <0.1 J/cm^3 or 10^{-10} beam loss quench

LHC Beam

total energy of proton beams: 2x360 MJ

240 colliding elephants

120 elephants with 40 km/h

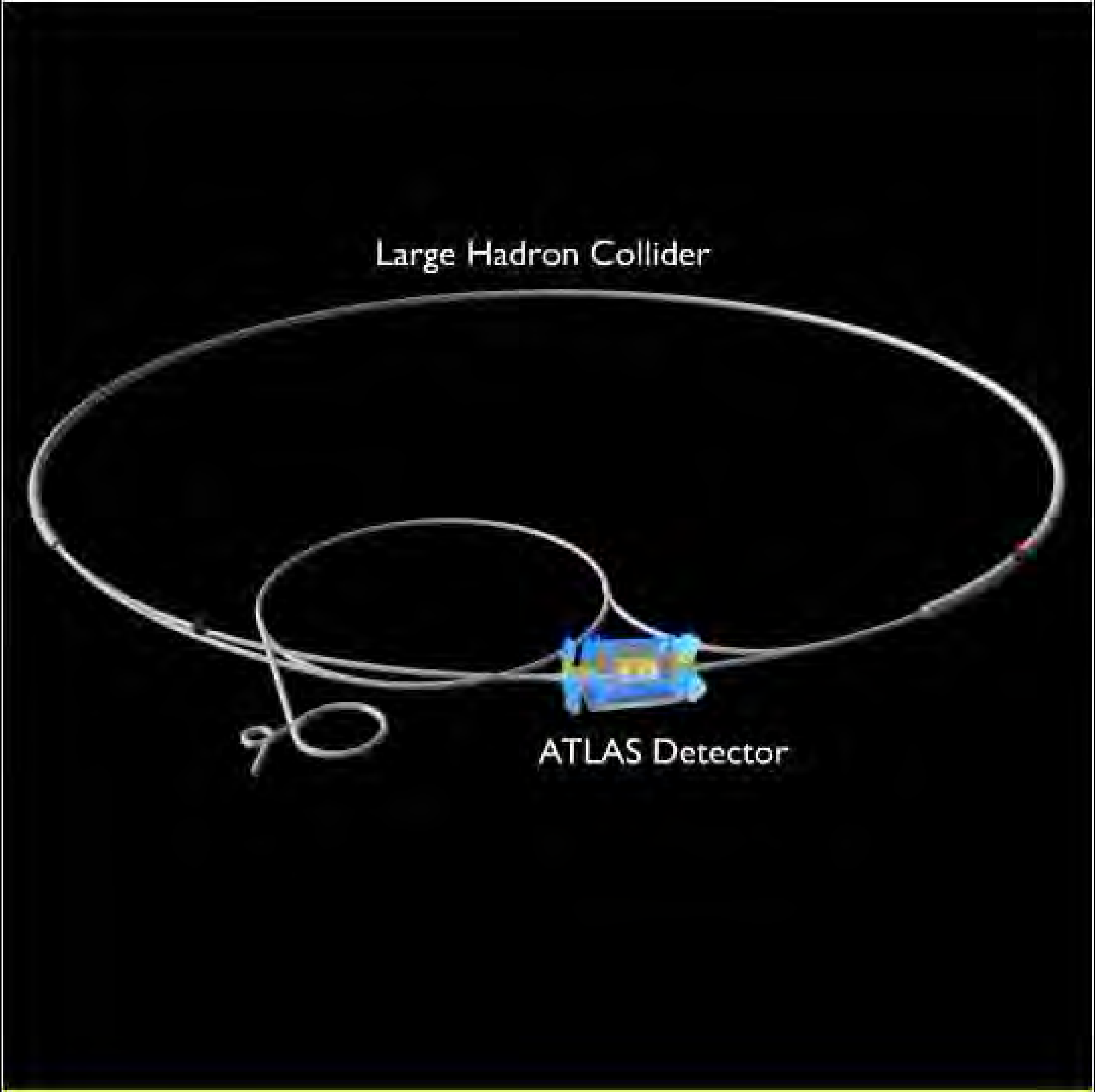


120 elephants with 40 km/h

needle eye:
0.3 mm diameter

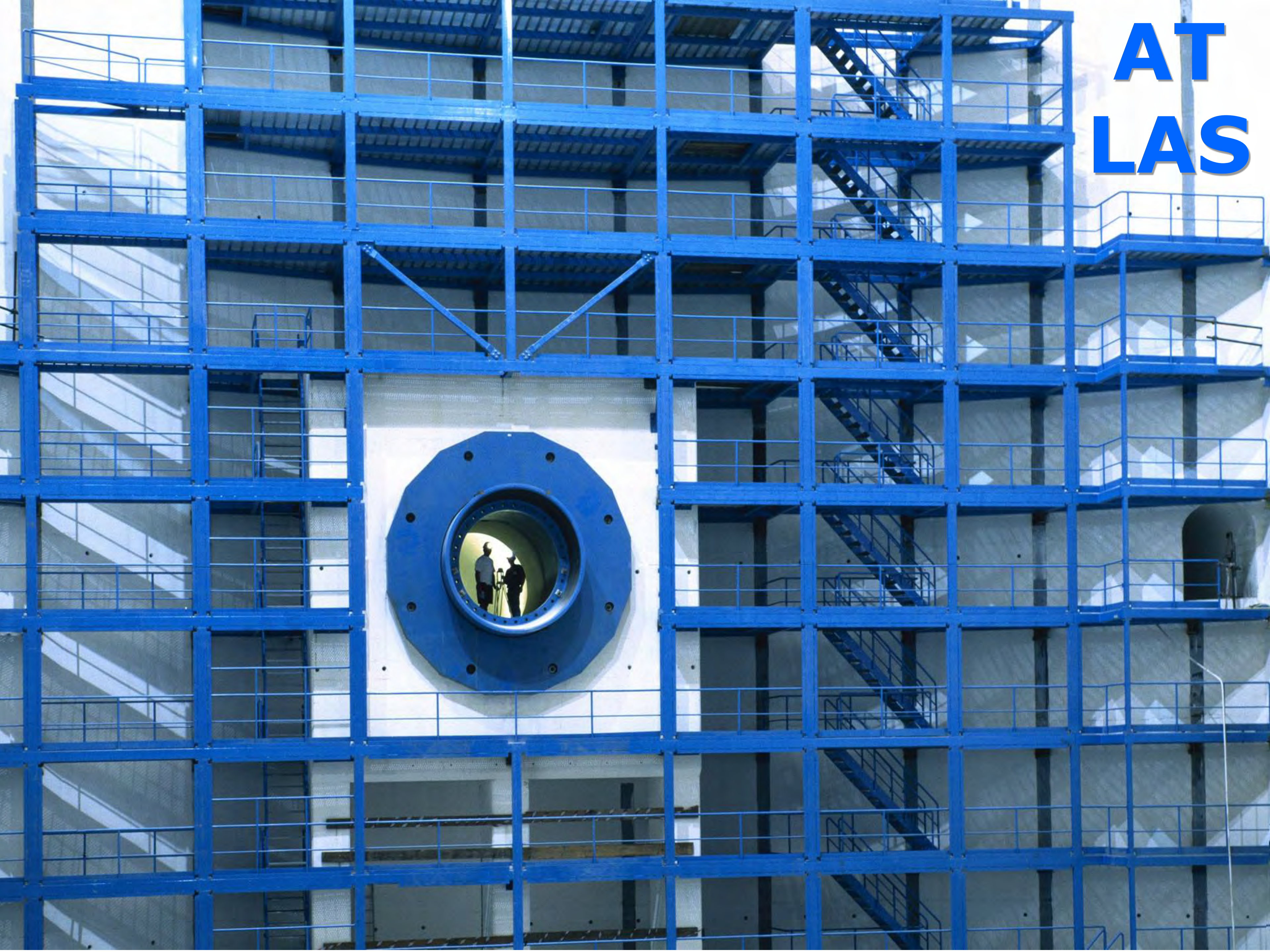
proton beam at collision point:
0.02 mm diameter

proton energy:
flying mosquito
(μJ)

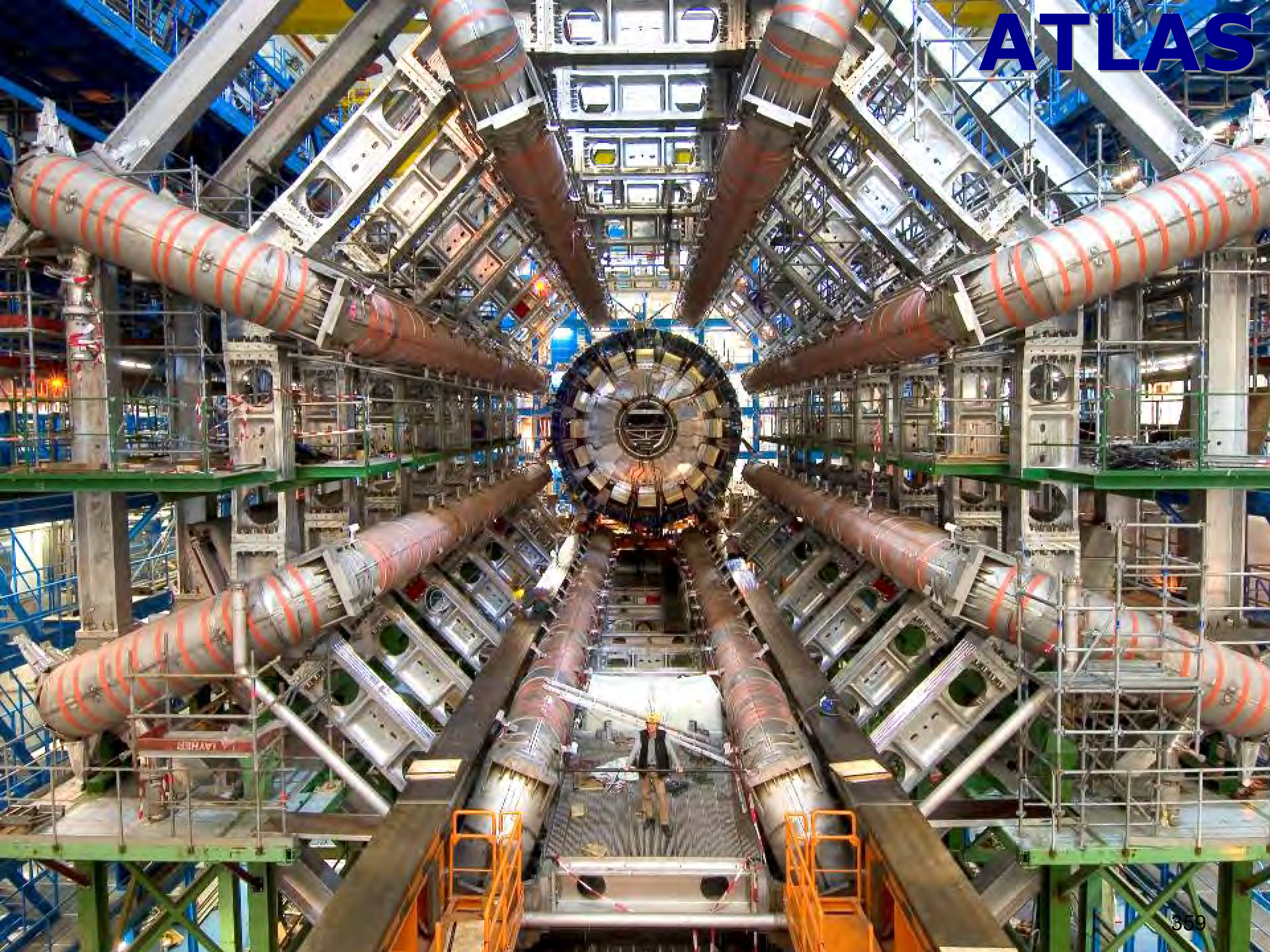

Large Hadron Collider
ATLAS Detector


ATLAS
>3000 physicists
177 institutions
38 states
C
D

Cathedrals of Science

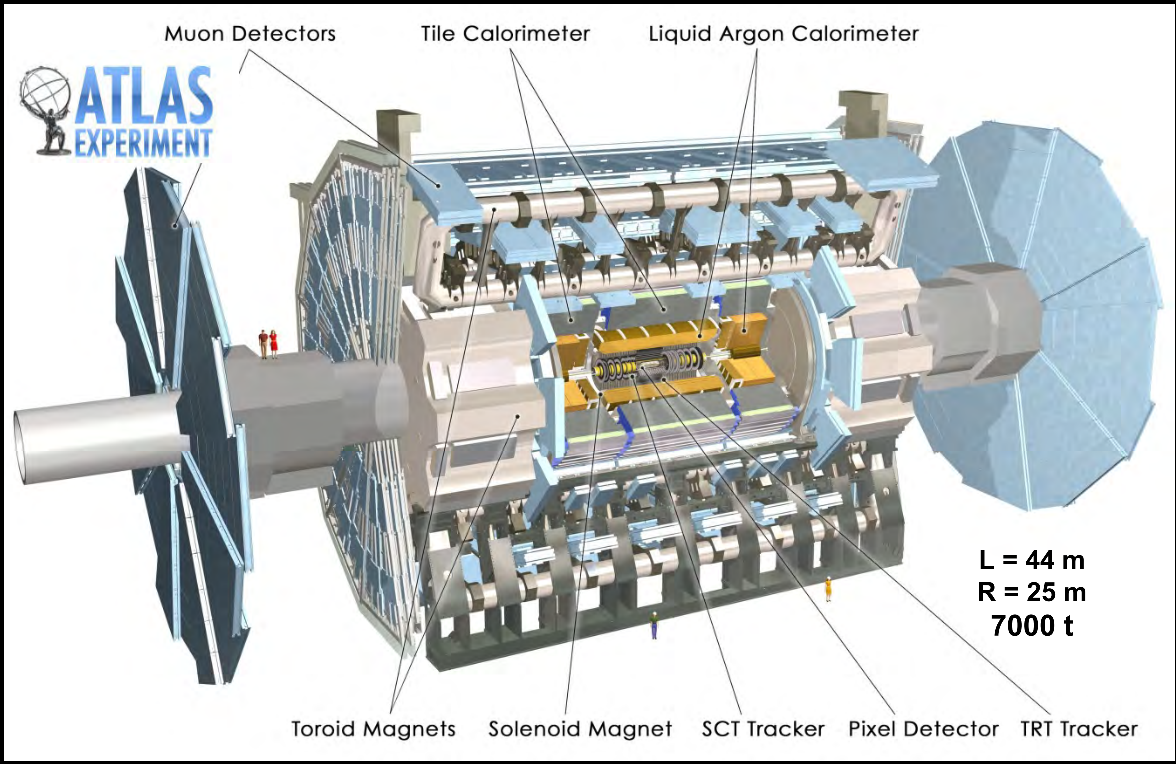

AT
LAS

ATLAS



ATLAS

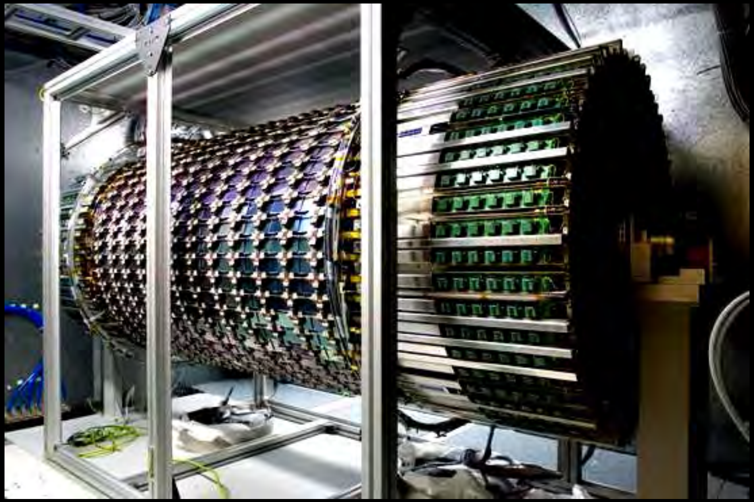
Forward Muon Chambers



Liquid Argon Calorimeter

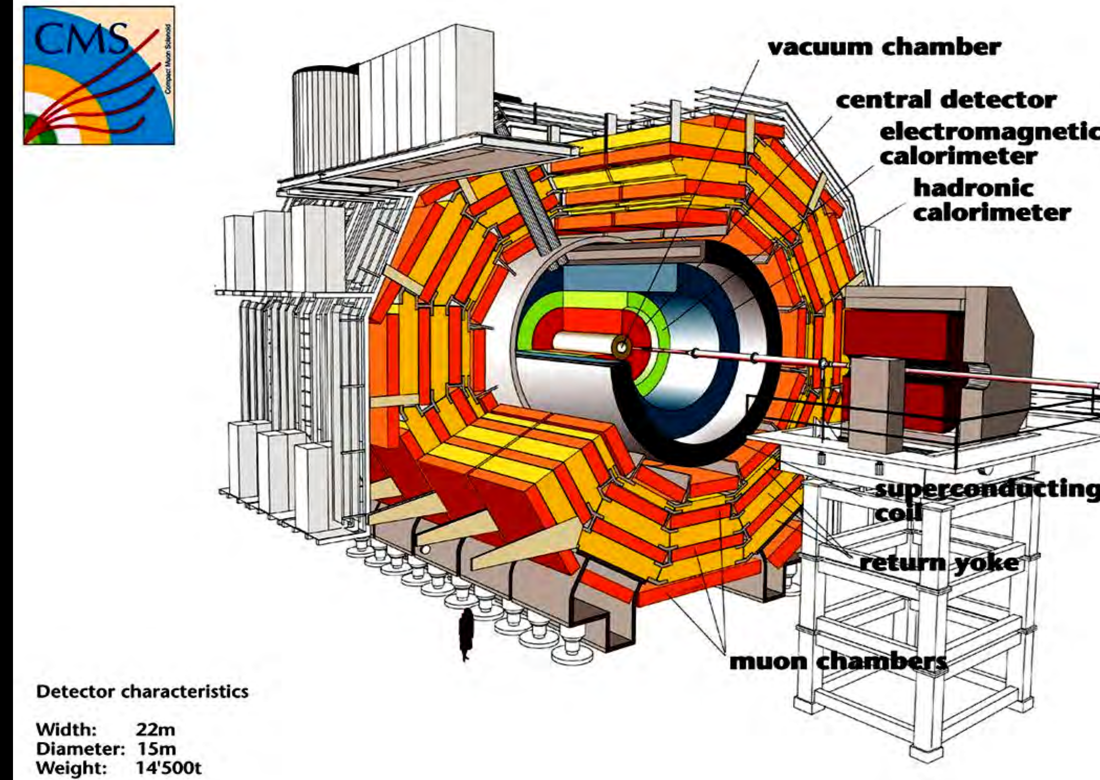
83, 15, 11 m^3 liquid Ar, N, He

Silicon Pixel Tracker



~100 million channels >100 m^2 Si

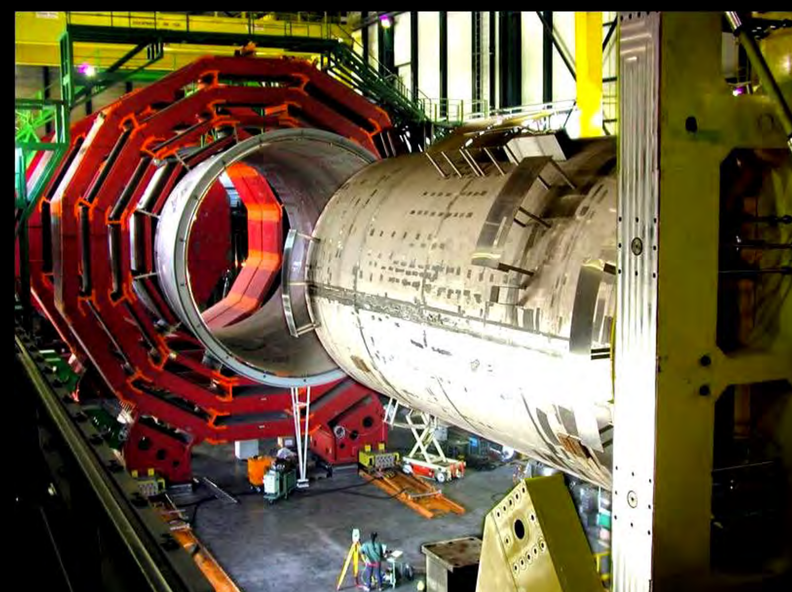
CMS



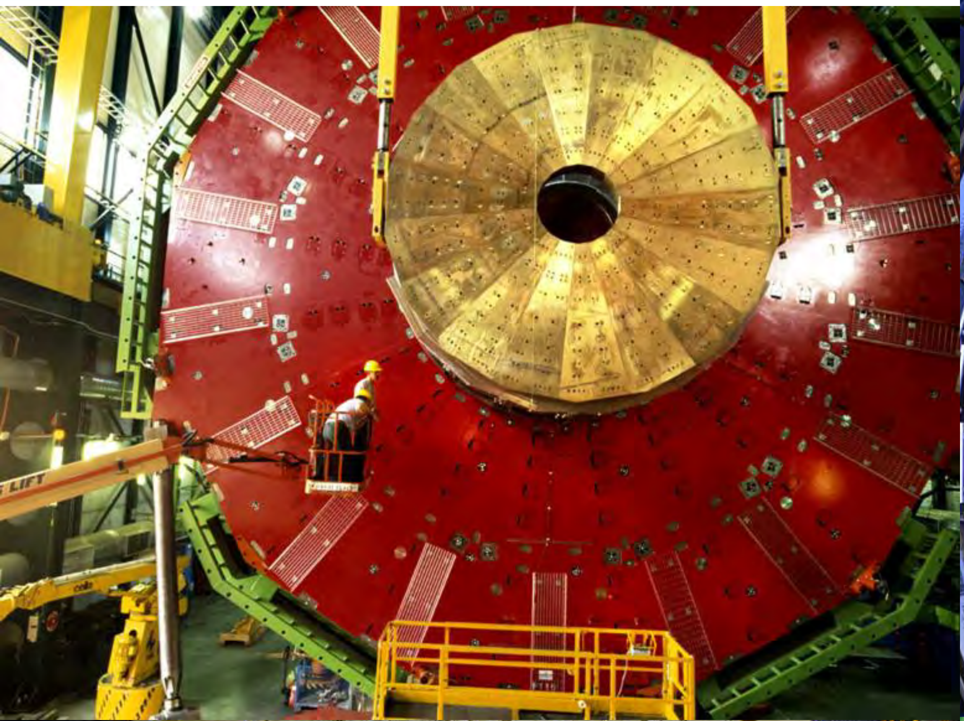
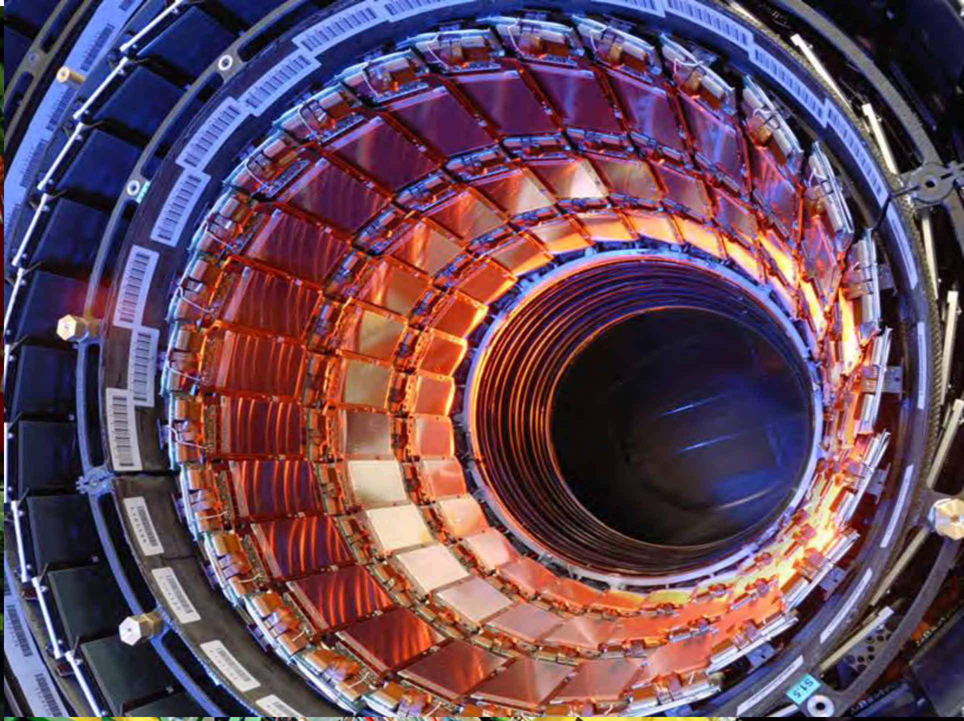
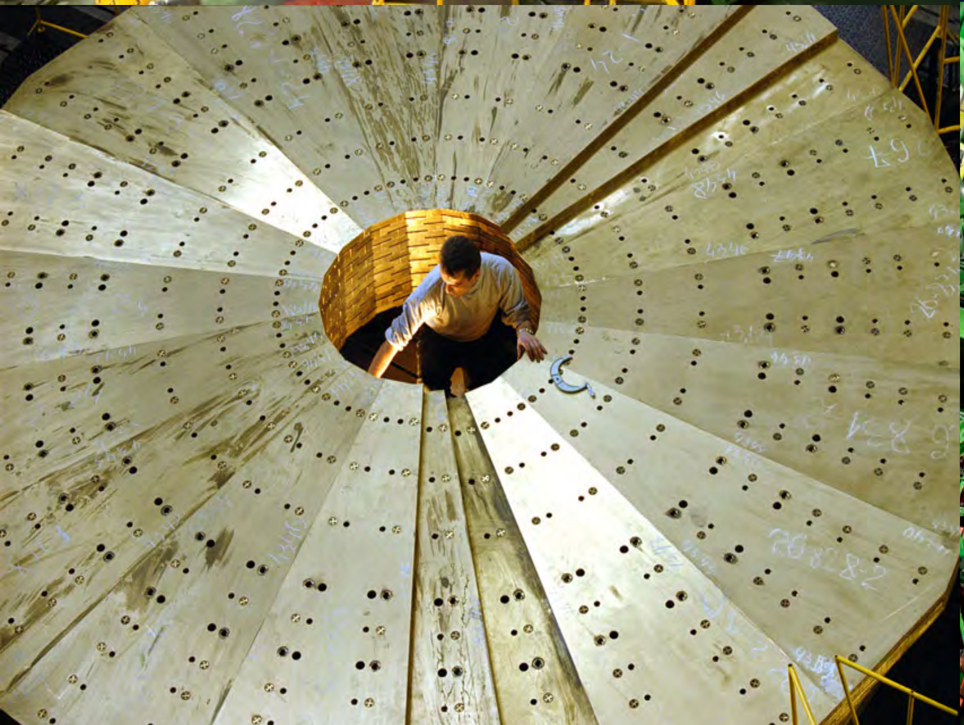
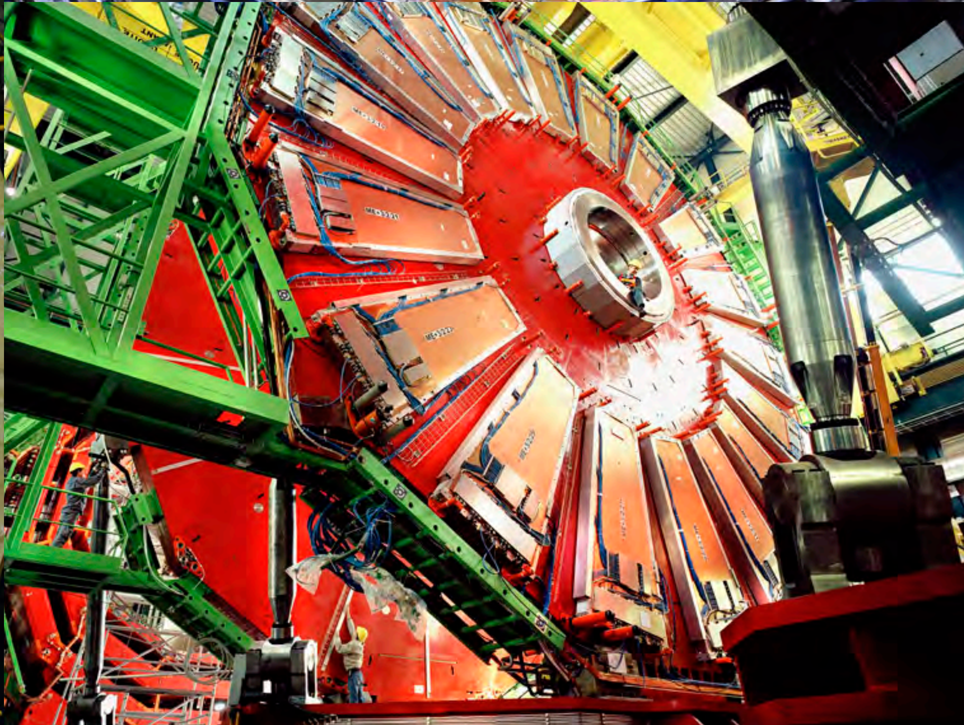
SC magnet

6x12 m
20 kA
4 T
2.6 GJ

world record



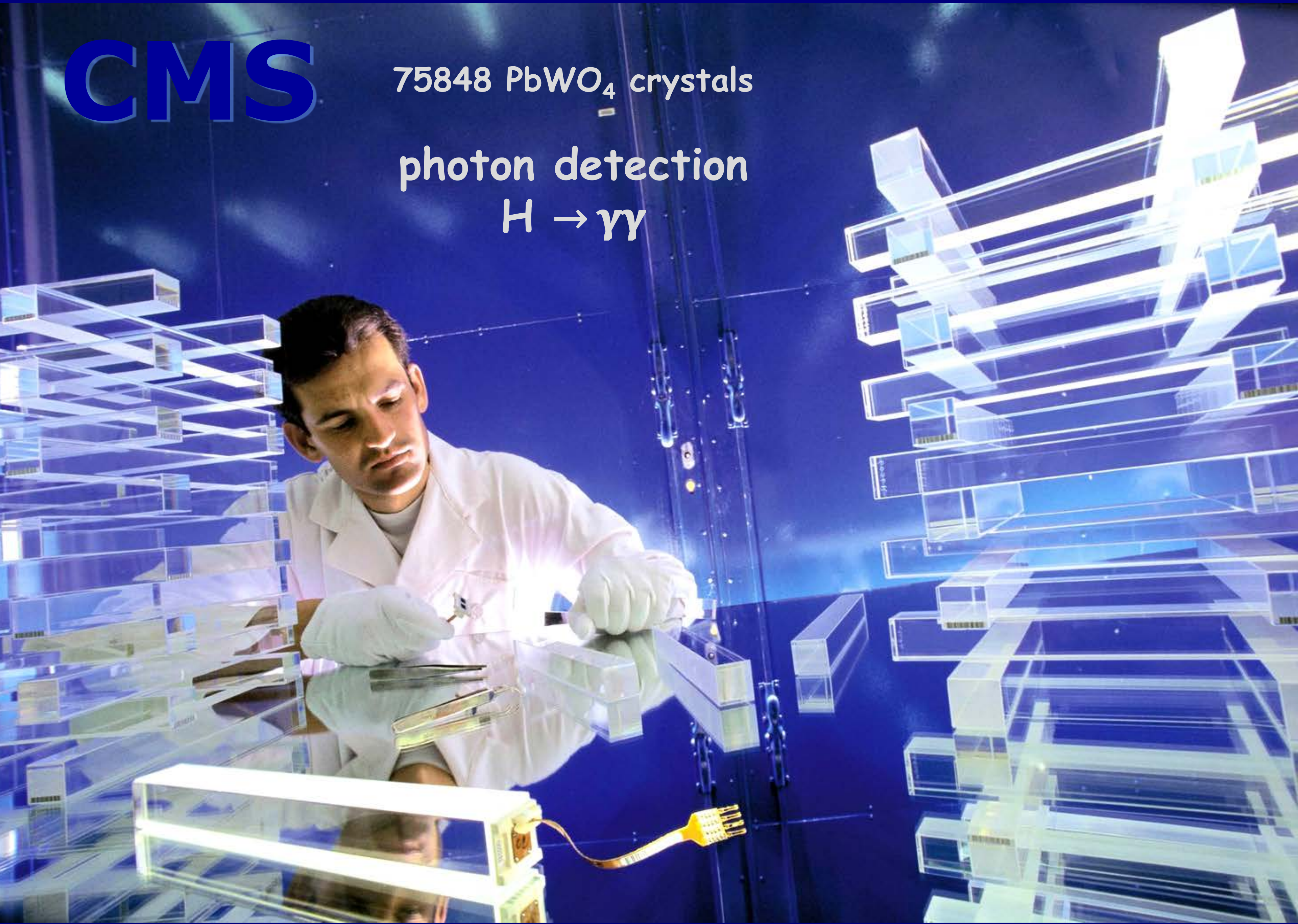
CMS

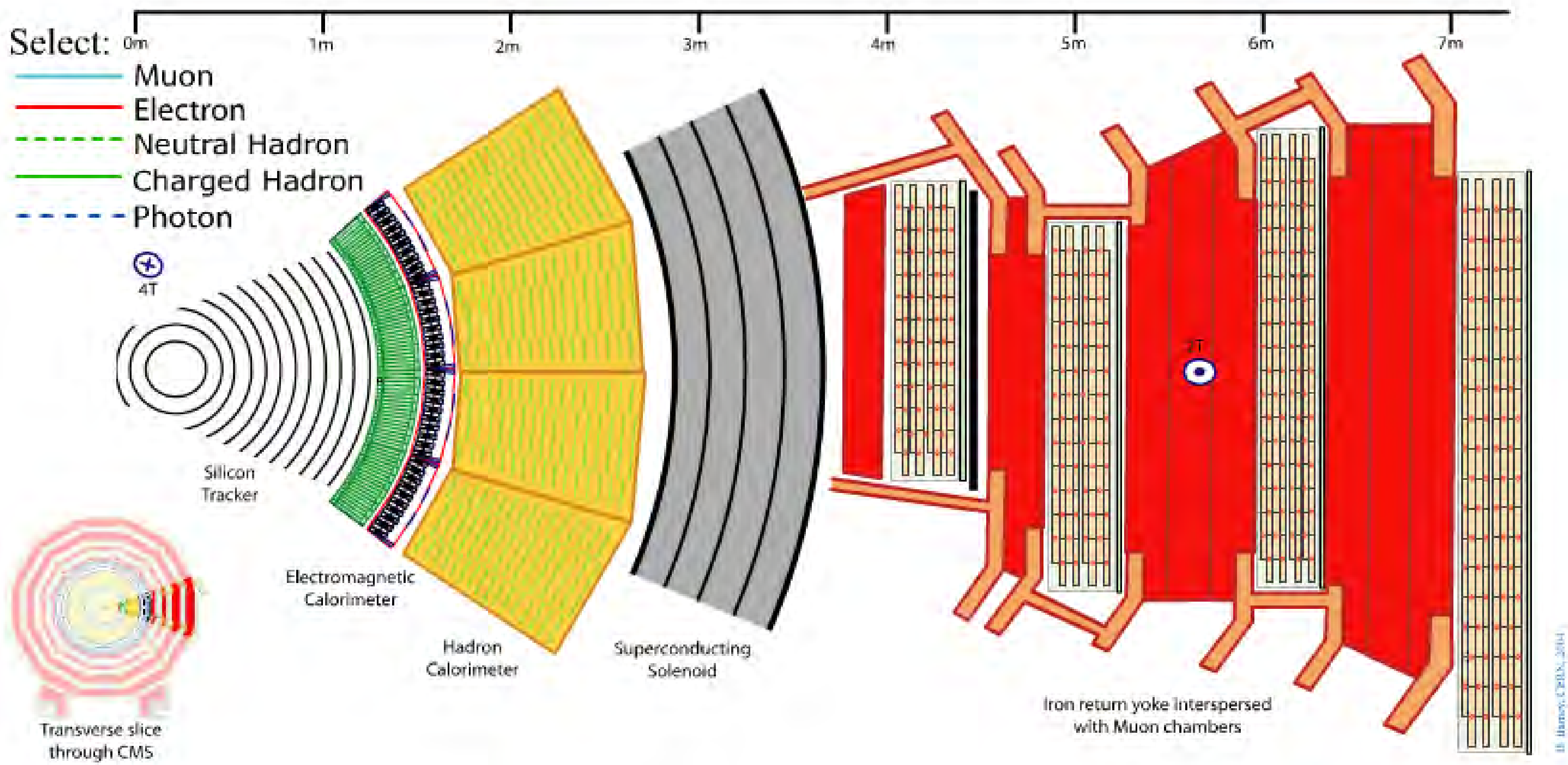
CMS

75848 $PbWO_4$ crystals

photon detection
$H \rightarrow \gamma\gamma$



Particle detection



Click on a particle type to visualise that particle
Press "escape" to exit

[CERN Videos\ATLAS Episode2 800x600 14.10.mov]
[CERN Videos\ATLAS Episode2 800x600 14.10 Ger.mp4]

from Web to Grid

raw data:

100 million channels
10^9 events/s * 1 MB =

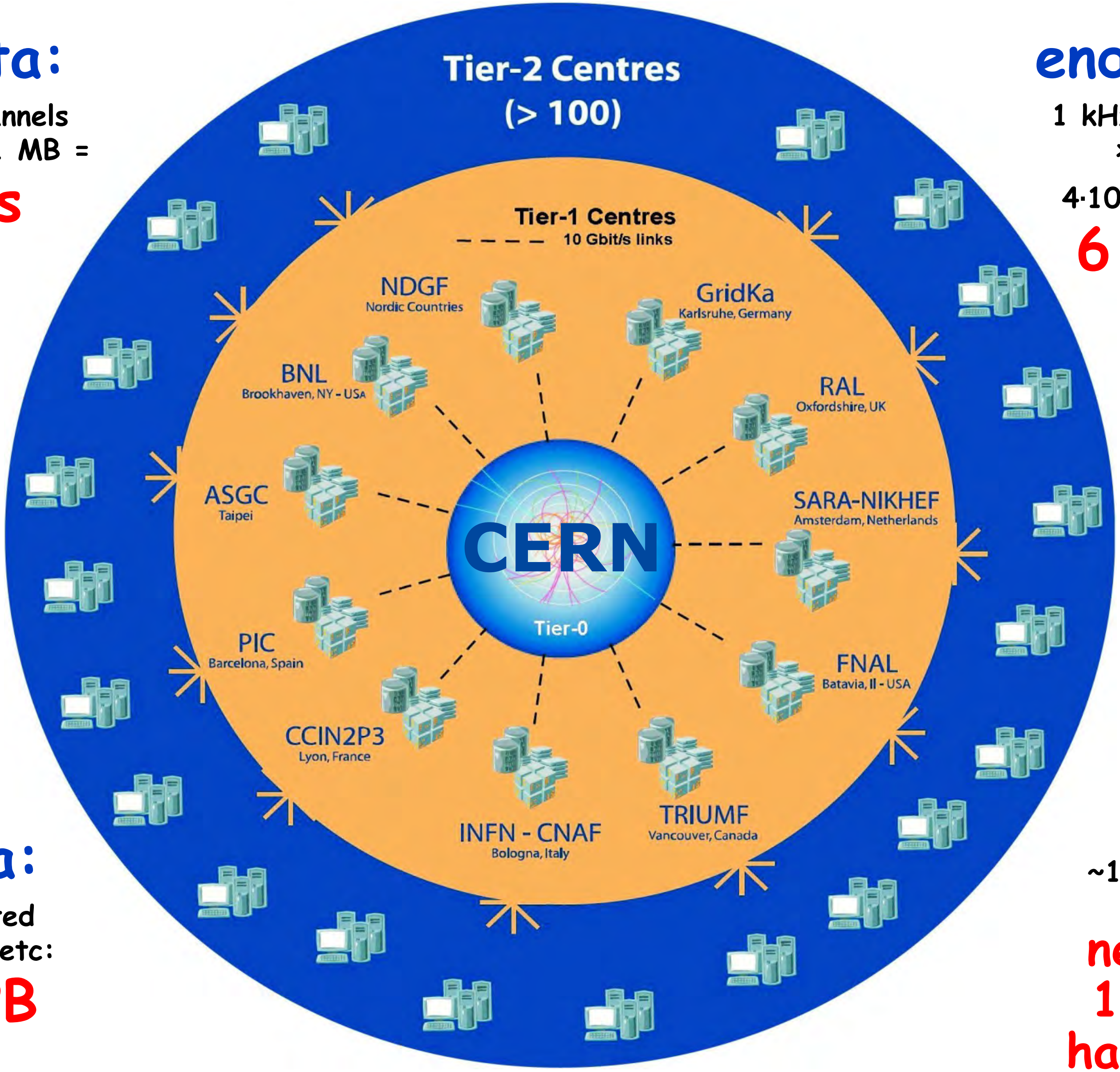
1 PB/s

end data:

1 kHz · >1 MB =
>1 GB/s

$4 \cdot 10^9$ events /a

6 PB/a



all data:

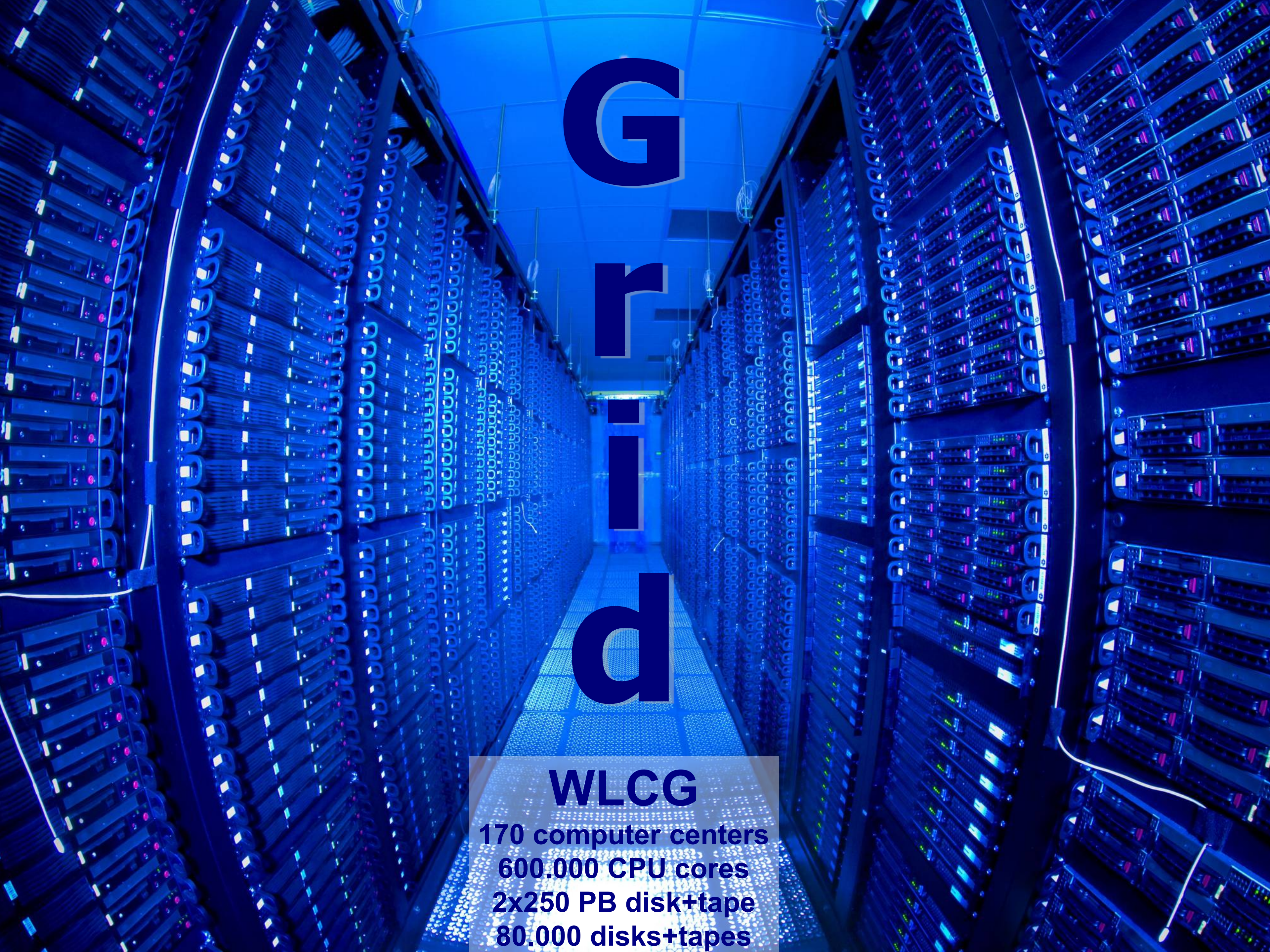
real + simulated
4 experiments etc:

~200 PB

find

~100 signals/a

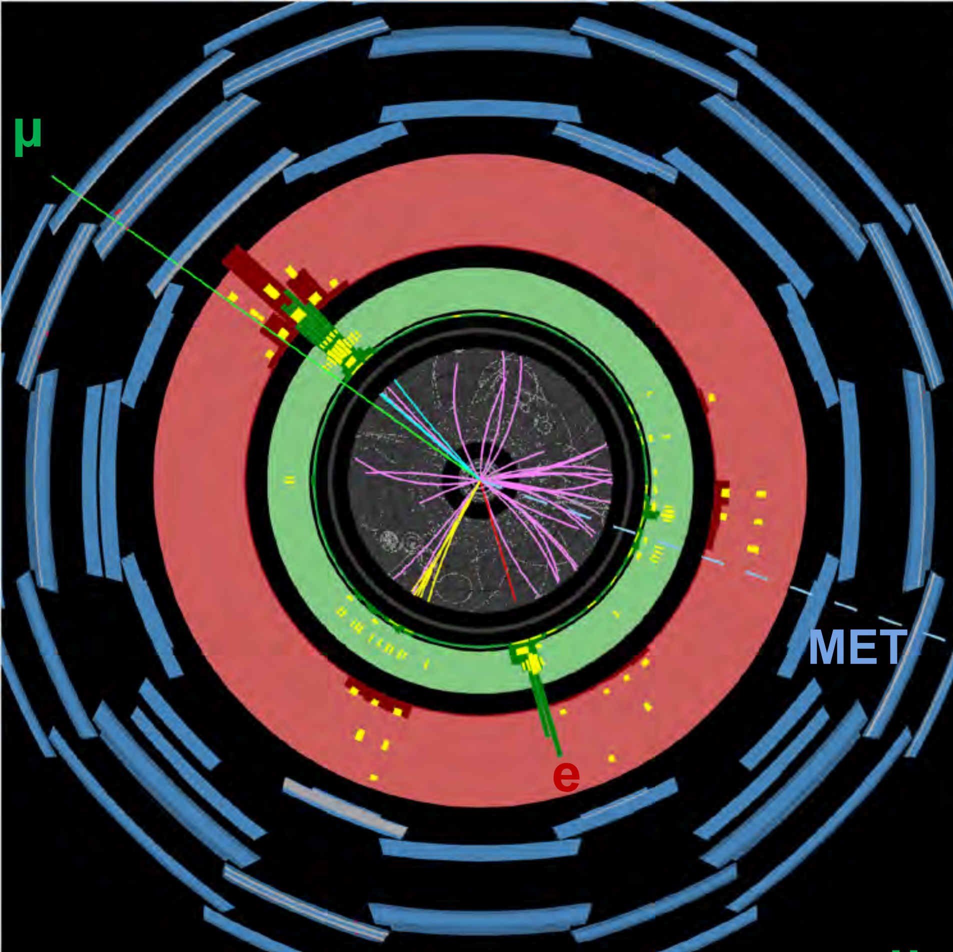
needle in 100.000 hay stacks

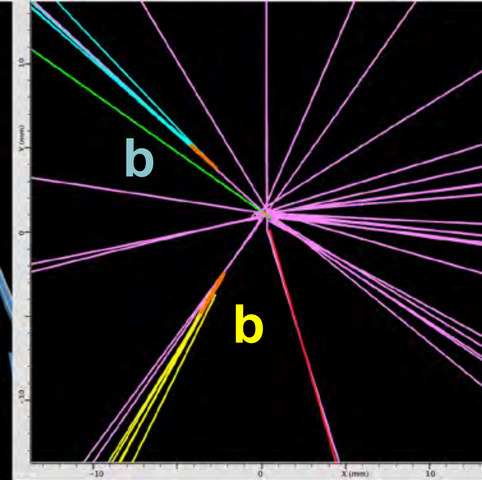

G
r
i
d
WLCG
170 computer centers
600.000 CPU cores
2x250 PB disk+tape
80.000 disks+tapes

LHC top factory 2012

4 million top pairs

$\bar{t}t \rightarrow$ e + μ 2 b-jets

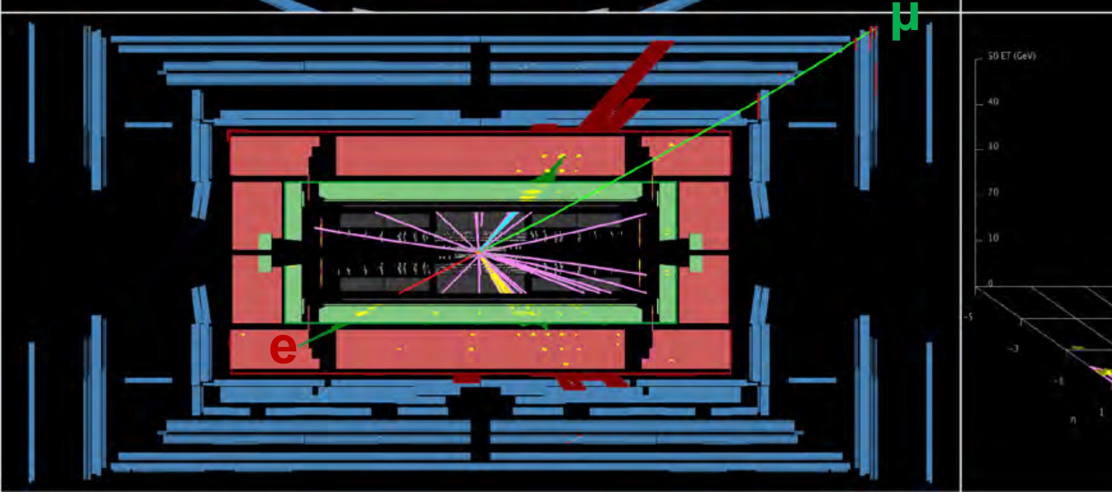




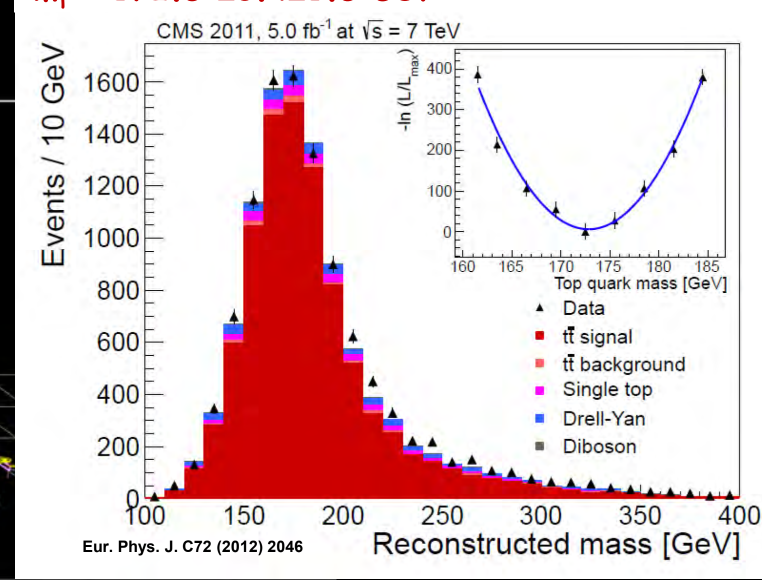
ATLAS EXPERIMENT

Run Number: 160958, Event Number: 9038972

Date: 2010-08-08 11:01:12 BST



m_t = 172.5 ±0.4±1.5 GeV



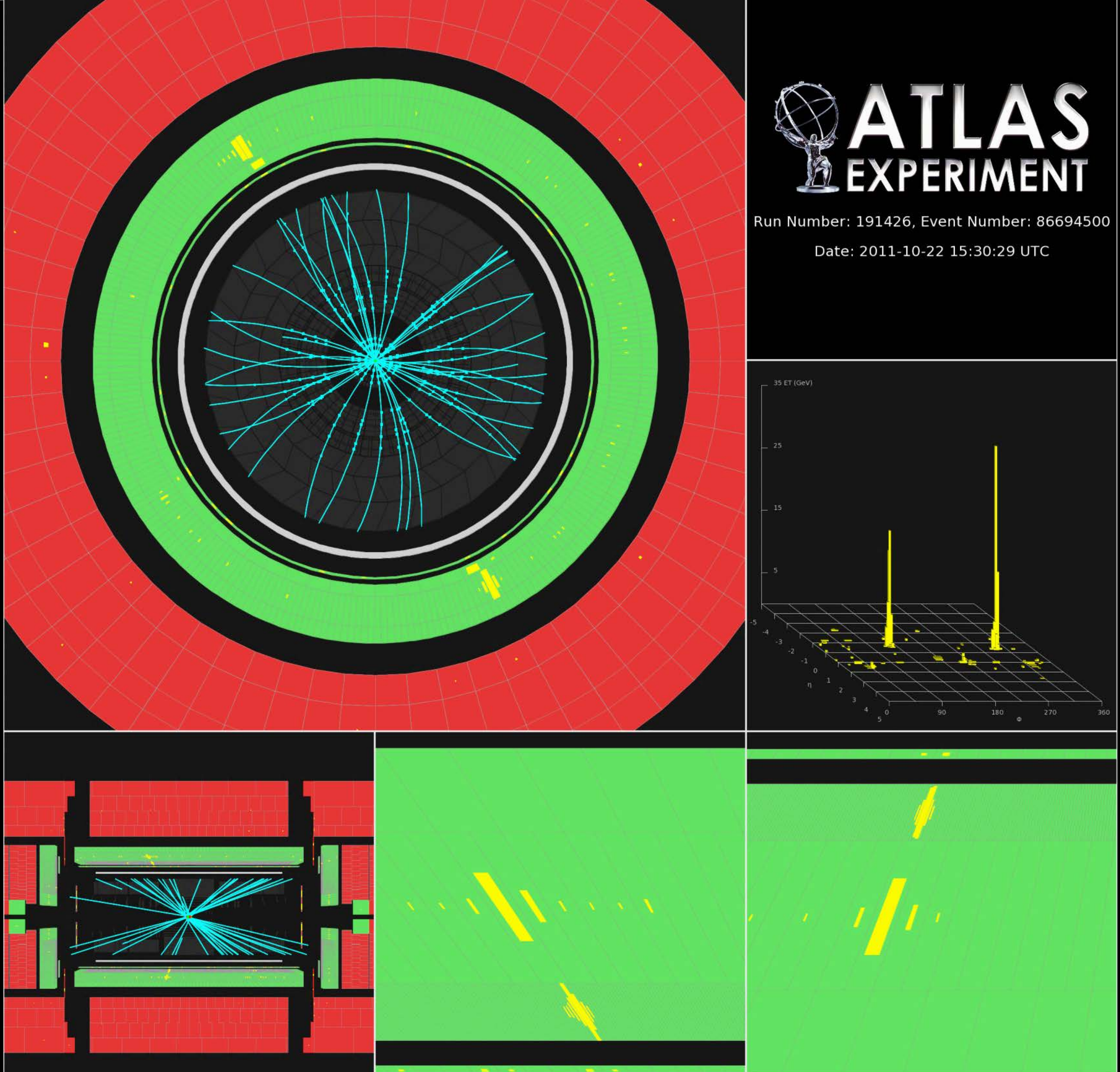
Nobel laureates Peter Higgs (right) and François Englert at CERN in July 2012.

NOBEL PRIZE

Higgs theorists amass physics prize

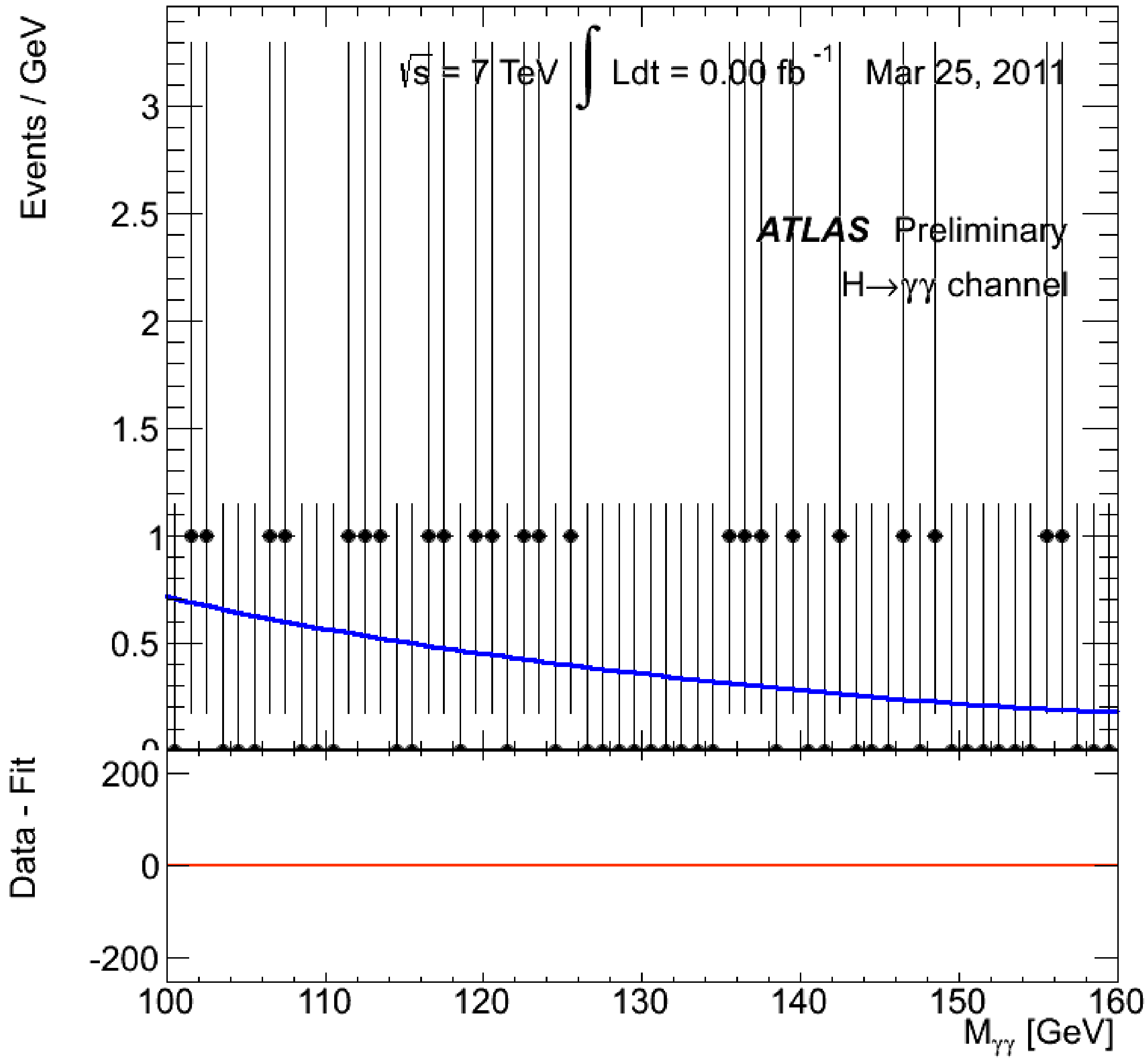
François Englert and Peter Higgs rewarded with Nobel 50 years after hunt for boson began.


Science
21 December 2012 | $10
BREAKTHROUGH
of the YEAR
The HIGGS
BOSON


ATLAS
EXPERIMENT
Run Number: 191426, Event Number: 86694500
Date: 2011-10-22 15:30:29 UTC
35 ET (GeV)
25
15
5
η
φ
0
90
180
270
360

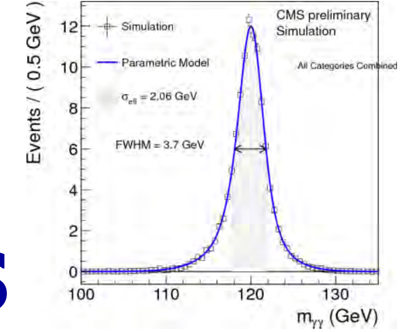
H → γ γ

Higgs → γγ

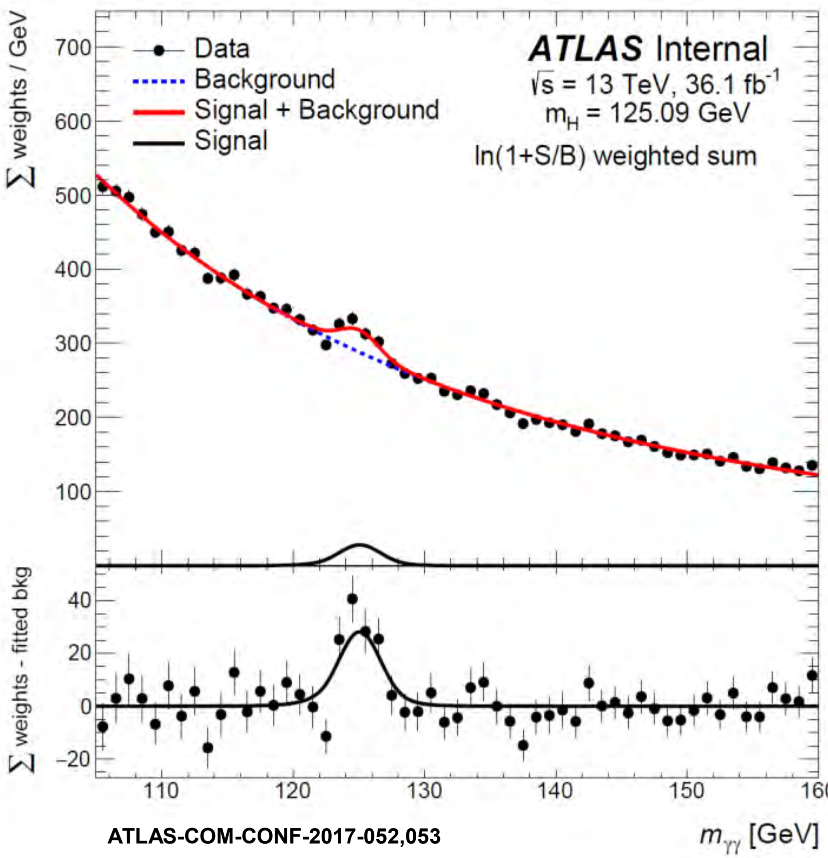


√s = 7 TeV ∫ Ldt = 0.00 fb⁻¹ Mar 25, 2011
ATLAS Preliminary
H→γγ channel
Events / GeV
3
2.5
2
1.5
1
0.5
0
Data - Fit
200
0
-200
100
110
120
130
140
150
160
Mγγ [GeV]

Higgs → γγ



ATLAS

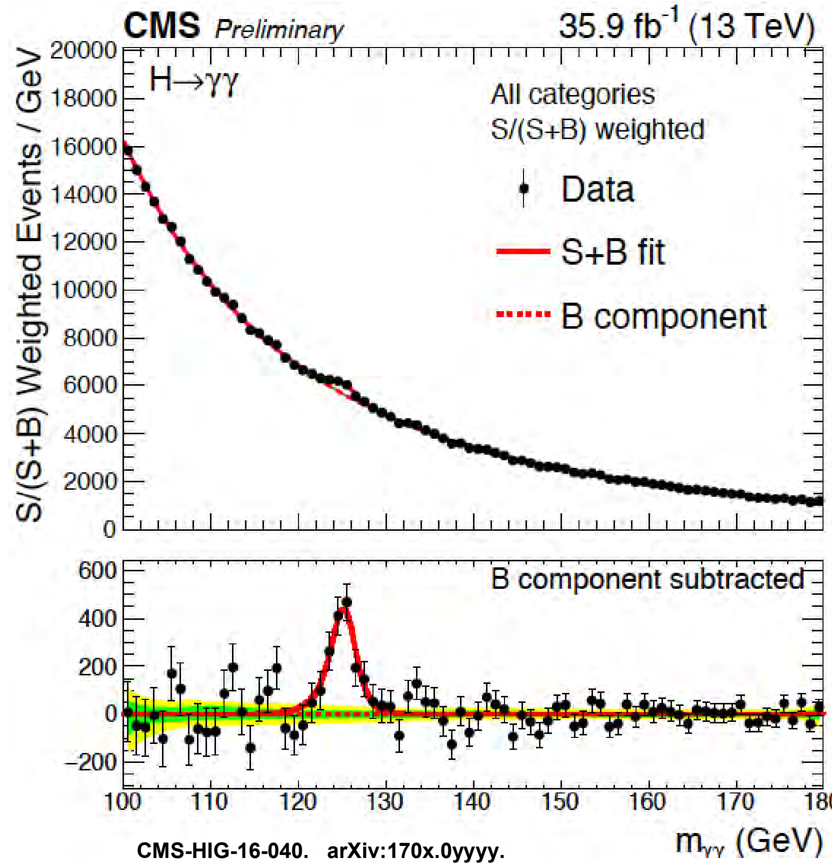


1.0±0.1 Standard Model expectation

mass = 125.1 ± 0.2_{stat} ± 0.4_{syst} GeV

https://twiki.cern.ch/twiki/bin/view/AtlasPublic/HiggsPublicResults

CMS

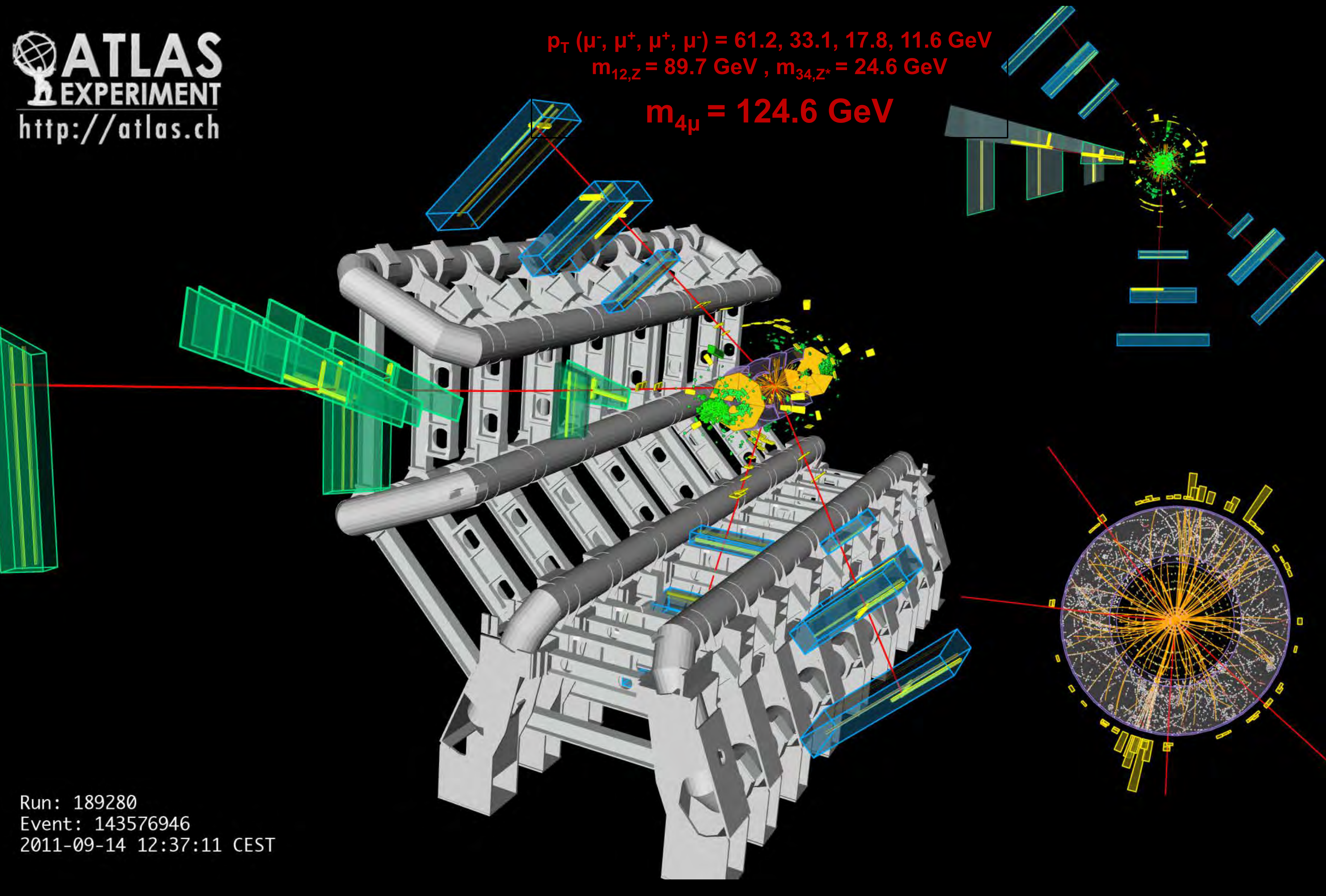


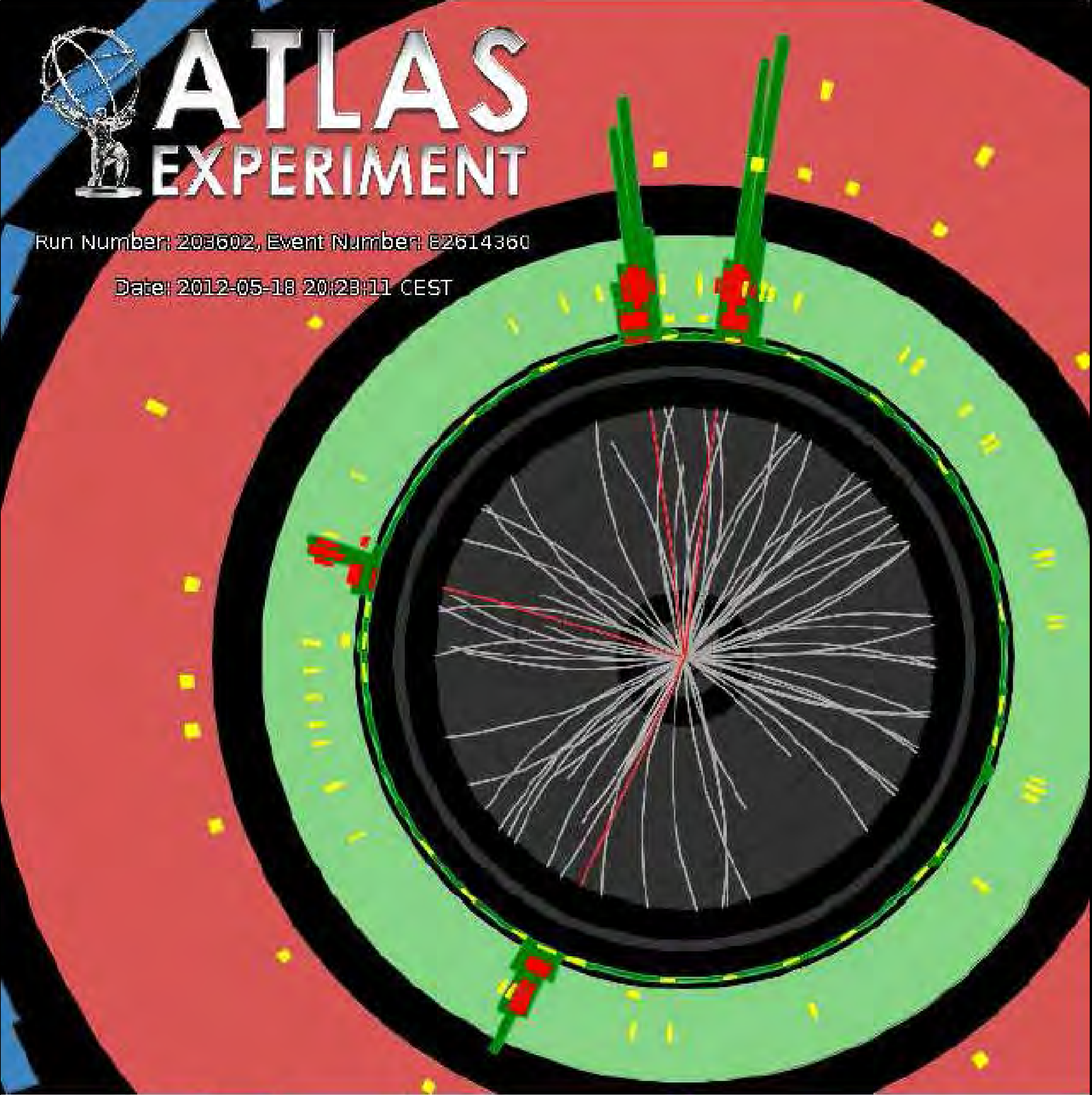
1.2±0.2 Standard Model expectation

mass = 124.7 ± 0.3_{stat} ± 0.2_{syst} GeV

https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsHIG

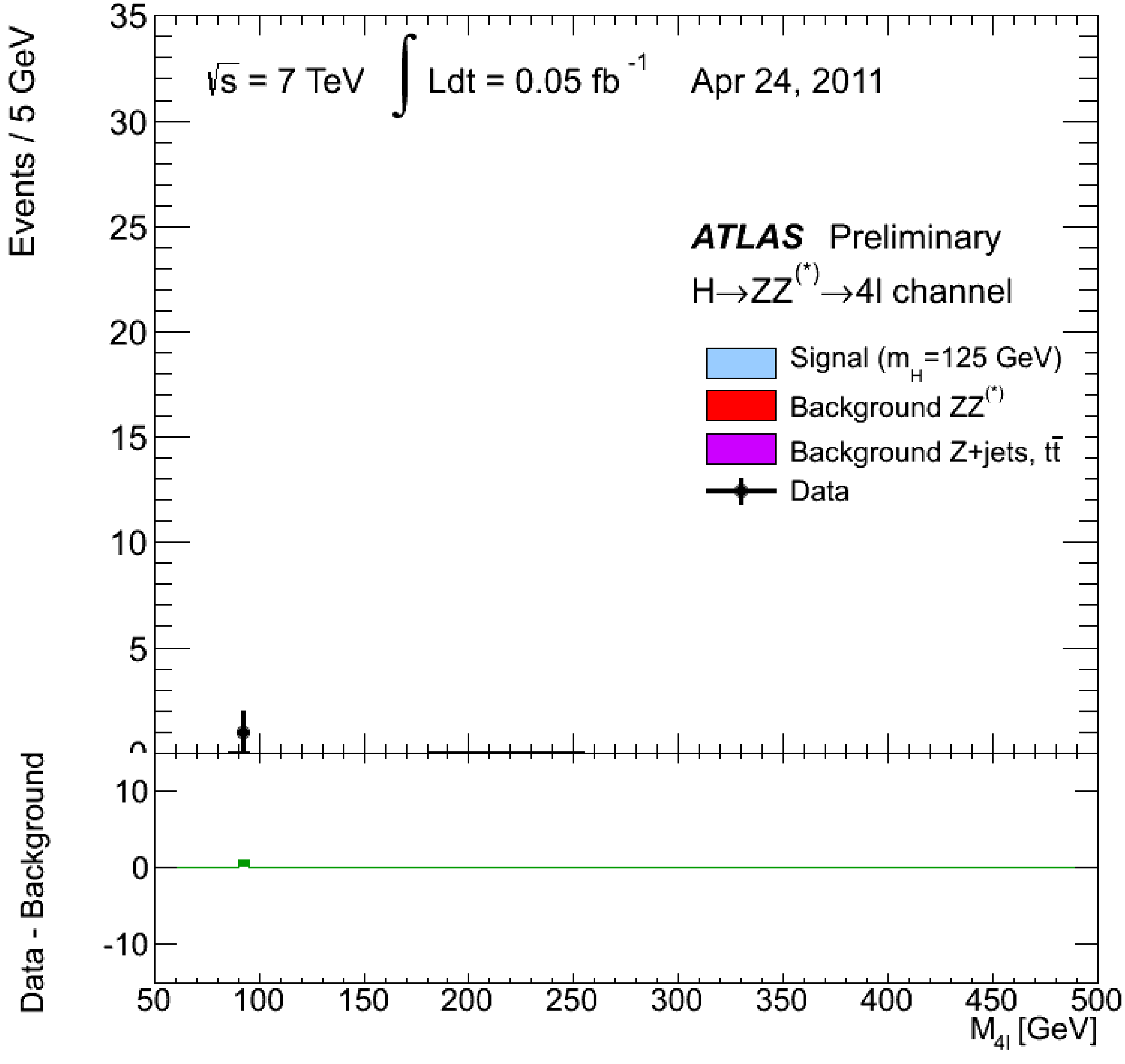
H → ZZ*→ 4μ candidate




ATLAS
EXPERIMENT
Run Number: 203602, Event Number: 82614360
Date: 2012-05-18 20:28:11 CEST

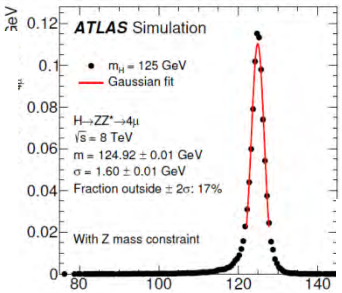
Higgs → ZZ → 4e

Higgs → ZZ → 4ℓ



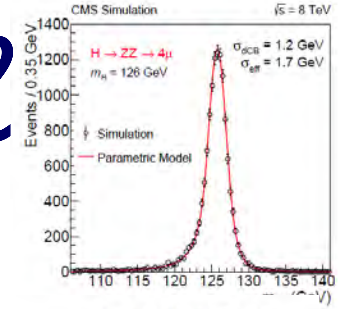
Events / 5 GeV
$\sqrt{s}$ = 7 TeV
$\int$ Ldt = 0.05 fb^{-1}
Apr 24, 2011
ATLAS Preliminary
H→ZZ$^{(*)}$→4l channel
Signal (m_H=125 GeV)
Background ZZ$^{(*)}$
Background Z+jets, $t\bar{t}$
Data
Data - Background
M_{4l} [GeV]

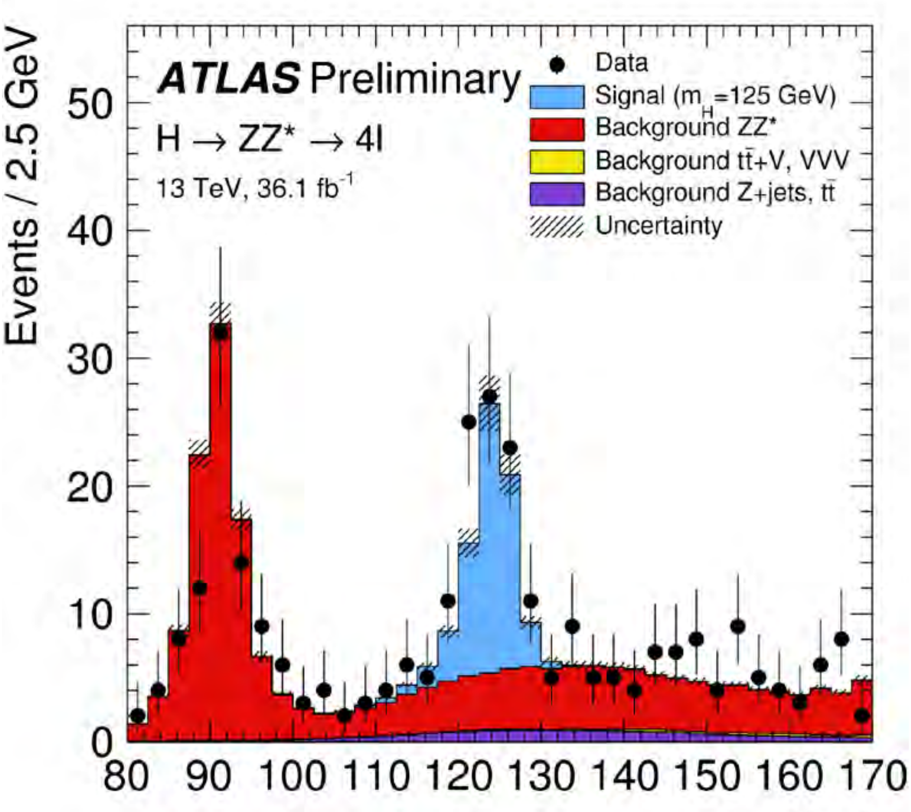
Higgs → ZZ → 4ℓ



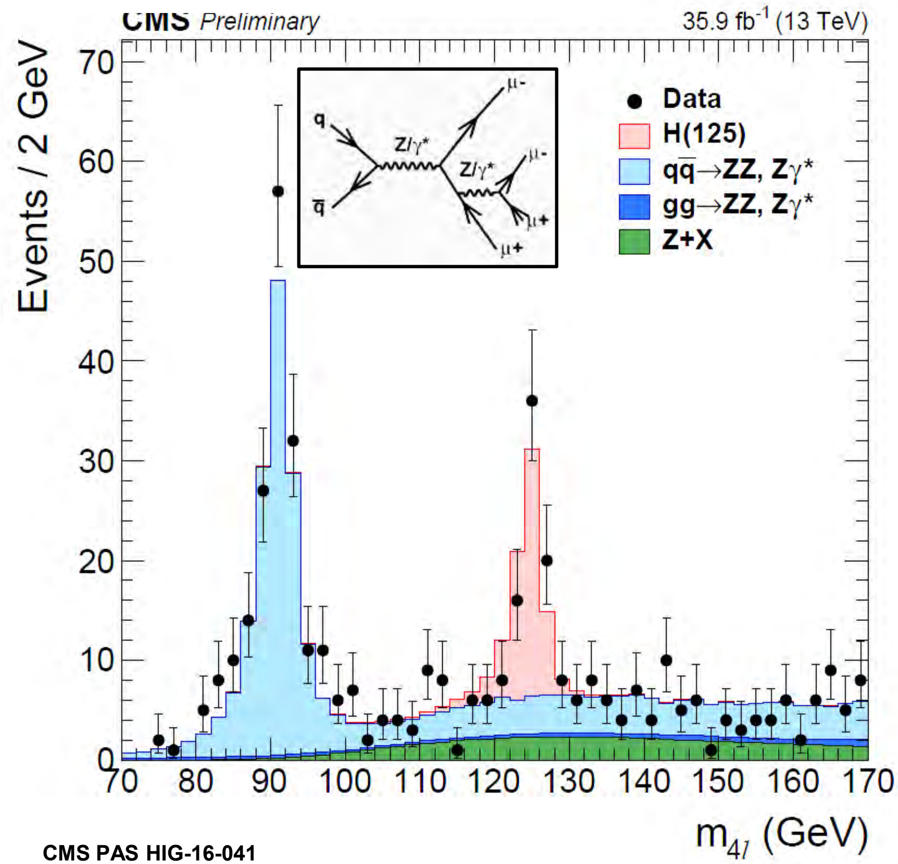
ATLAS

CMS





ATLAS-HIGG-2016-22



CMS PAS HIG-16-041

1.3±0.2 Standard Model expectation

mass = 124.9 ± 0.4_{stat} ± 0.1_{syst} GeV

1.1±0.2 Standard Model expectation

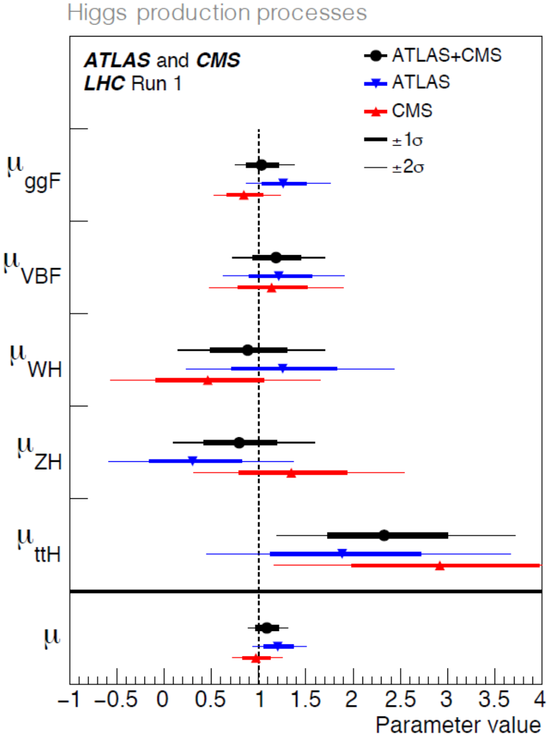
mass = 125.3 ± 0.2_{stat} ± 0.1_{syst} GeV

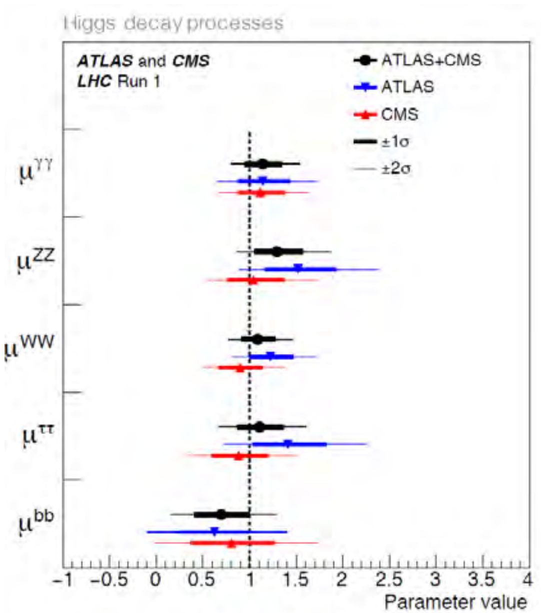
both: spin 0^+ favored over 0^-, 1,2 by 2-4 σ

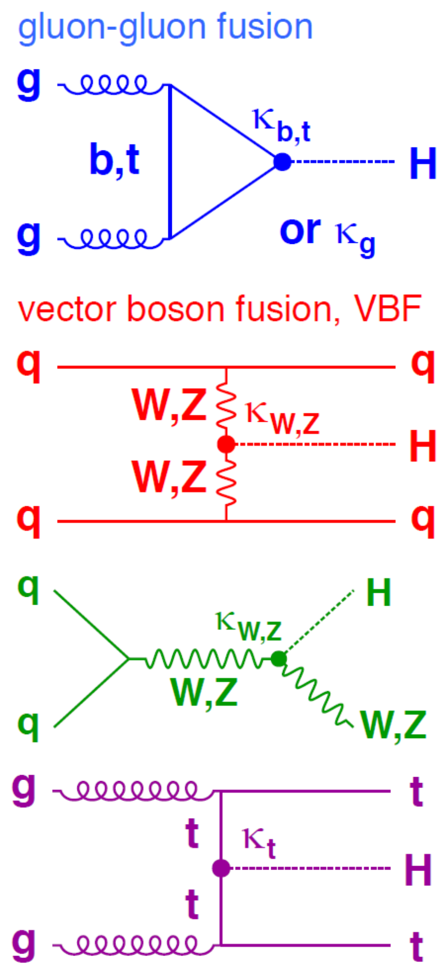
https://twiki.cern.ch/twiki/bin/view/AtlasPublic/HiggsPublicResults

https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsHIG

Is it the Higgs ?







coupling ~ mass

μ = seen / SM

= 1.1 $\pm$ 0.1

Run1 ATLAS+CMS comb JHEP 1608 (2016) 045

= 1.0 $\pm$ 0.1

Run2 ATLAS arXiv:1802.04146

Fermion coupling:

to 3rd generation

b+τ: 1.1 $\pm$ 0.3

to 1st + 2nd generation

small coupling

no H → ee, μμ :

H coupling

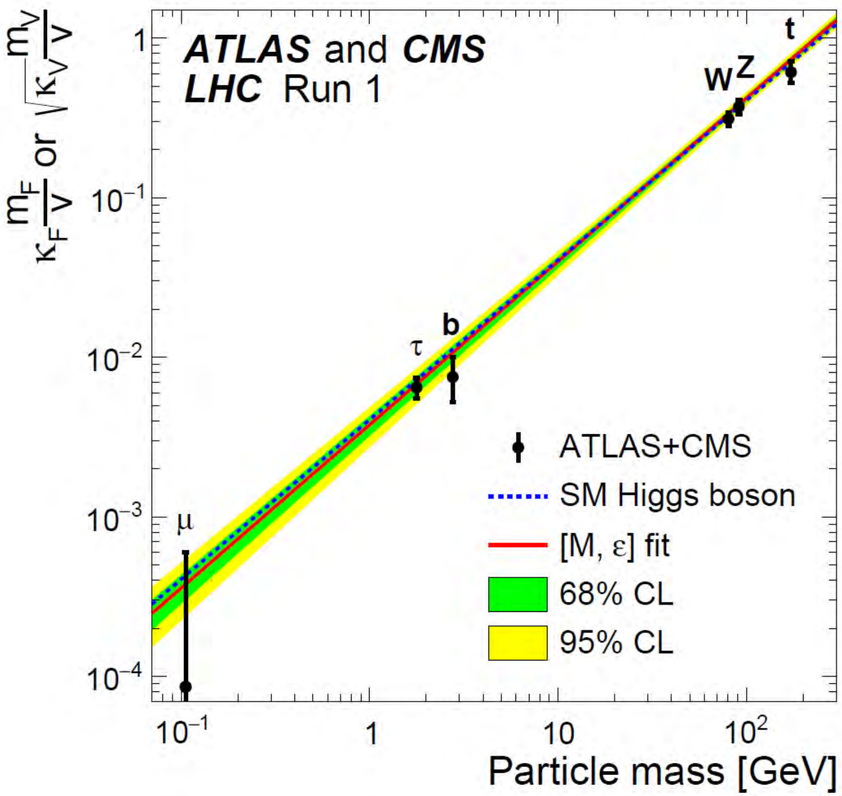
NON-UNIVERSAL !

JHEP 1608 (2016) 045. arXiv:1606.02266.

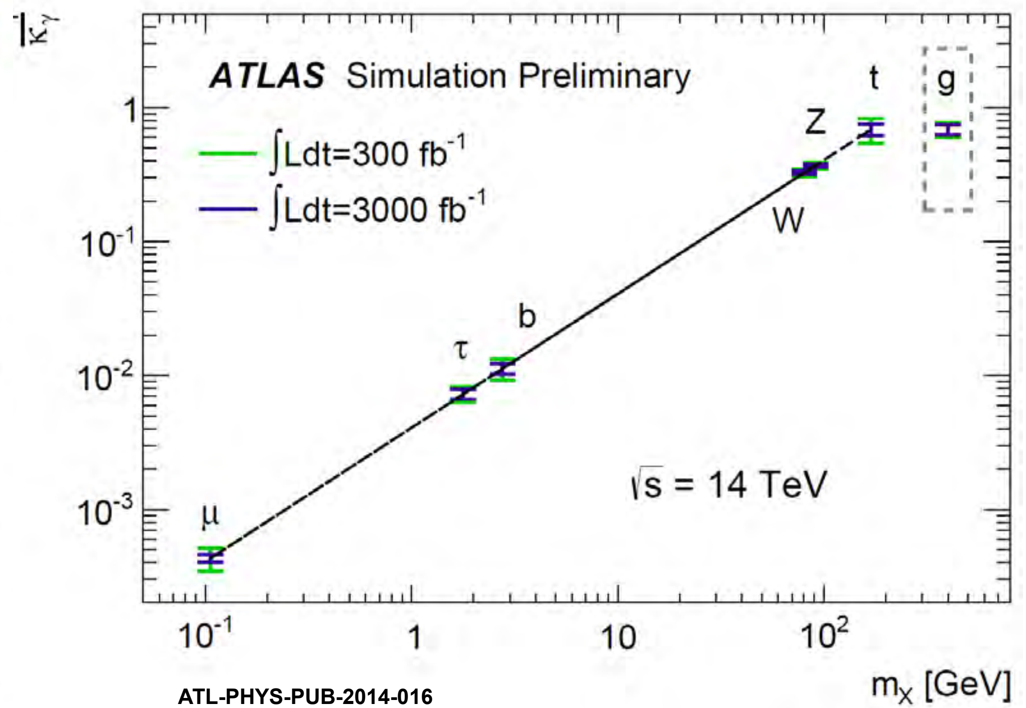
Is it the Higgs?

check the couplings !

2017 LHC >2022



arXiv:1606.02266 Journal High Energy Phys. 08 (2016) 045



ATL-PHYS-PUB-2014-016

Higgs J^P

- electroweak symmetry breaking **global** in space-time
- **no** preferred direction in the vacuum
- **no spin** of vacuum ground state !
- **Higgs =** first **fundamental scalar !**
- **Landau-Yang*** theorem: **J=1 forbidden**

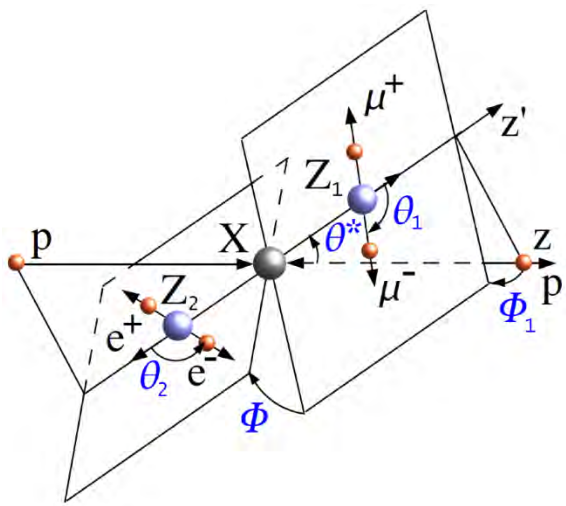
J^P	CL reject	channels
0^-	97.8 %	ZZ
1^+	99.97 %	ZZ, WW
1^-	99.7 %	ZZ, WW
2^+	>99.9 %	ZZ, WW, γγ

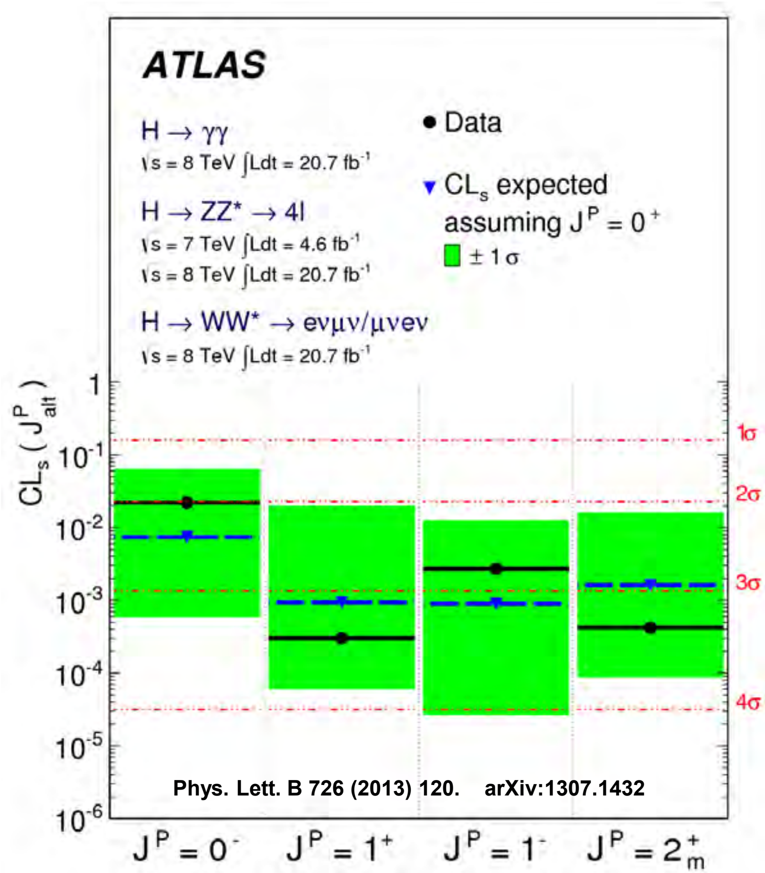
$H \to \gamma\gamma$, $ZZ^* \to 4l$, $WW^* \to ll\nu\nu$

spin other than scalar

excluded at 2-4 σ

$J^P = 0^+$ favored





* L.D. Landau, Dokl. Akad. Nauk, USSR 60, 207-209 (1948); C.-N. Yang, Phys. Rev. 77, 242 (1950).

What is Mass?

Higgs – a new field

spin	field	emergent	fundamental
0	scalar	temperature, pressure, ...	Higgs. inflaton, cosmolog. const. / Dark Energy
1	vector	flow: wind, water	forces: electromagnetism
2	tensor	elasticity	gravity
½	spinor		building blocks = fermions: electron, quarks

LHC Rap

6. QCD

G. Sterman et al., Handbook of Perturbative QCD, www.phys.psu.edu/~cteq/#Handbook
W.Tung, Perturbative QCD, www.physics.smu.edu/~olness/cteqpp/tung2003/IntroPqcd.pdf

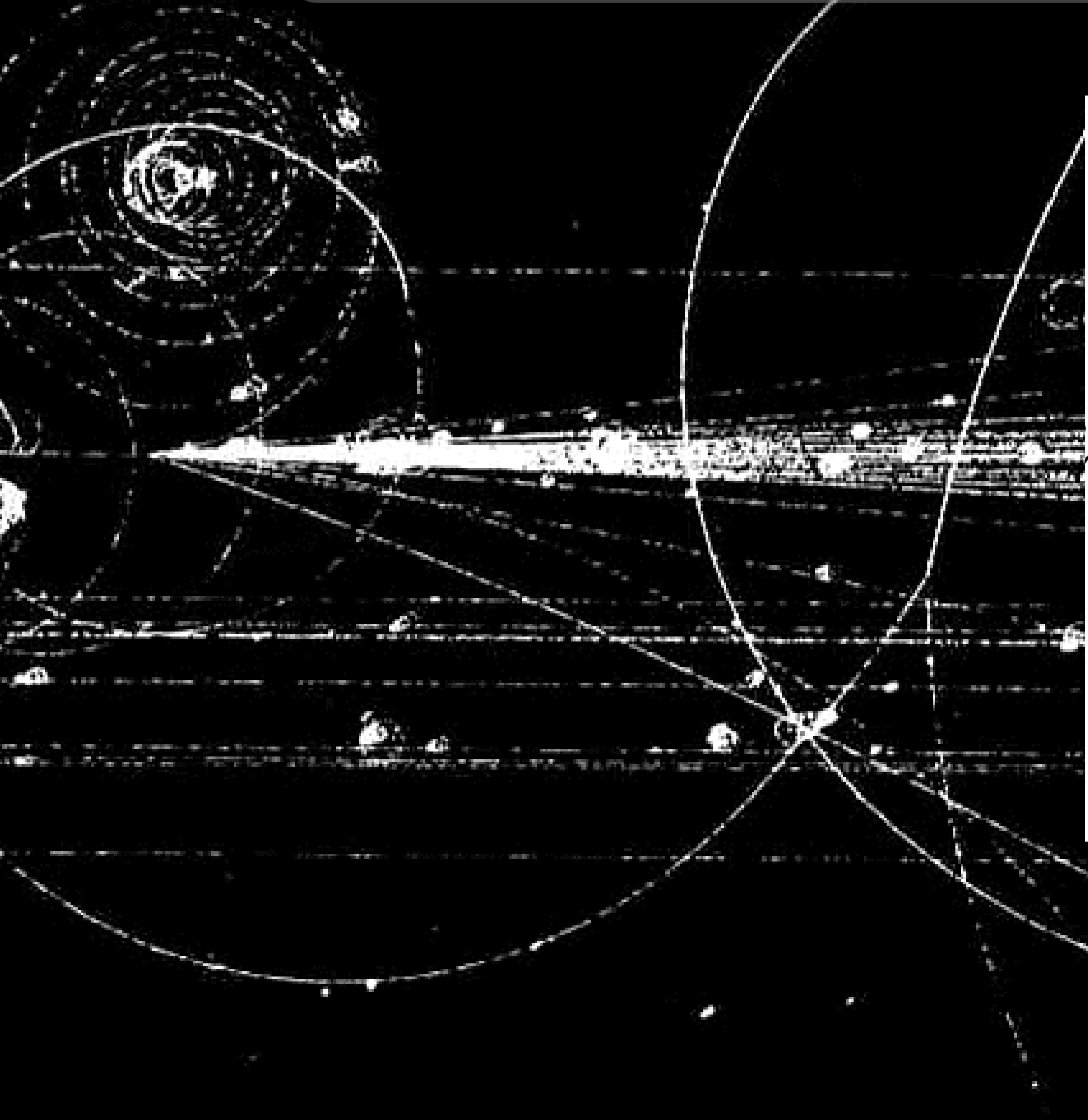
Quantum Chromo-Dynamics

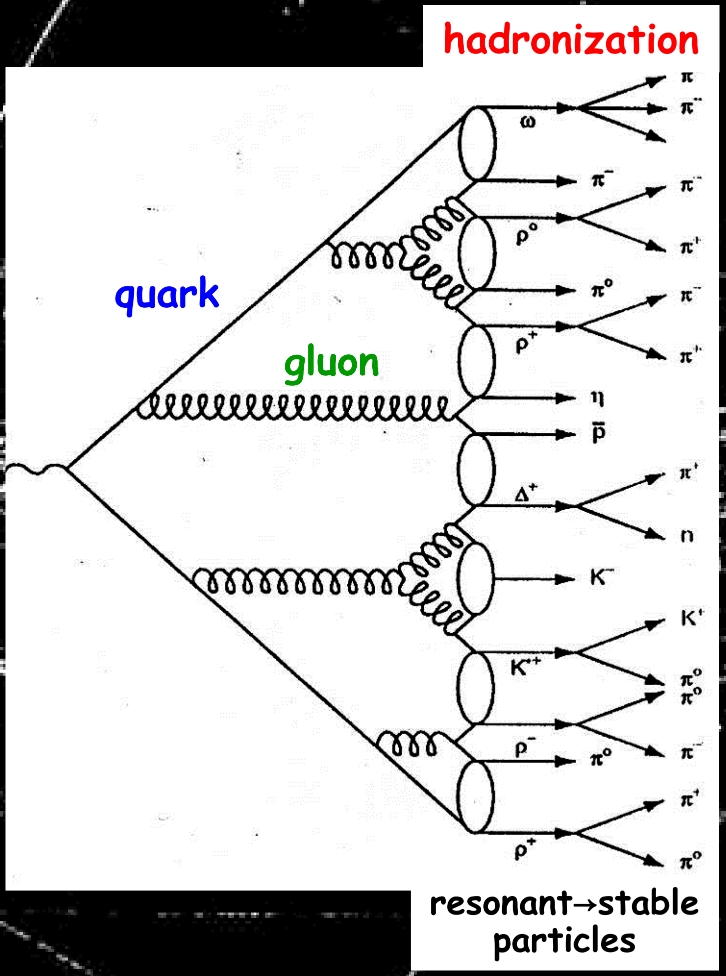
Gauge field theory of strong interactions

in $SU(3)_{COLOR}$

1. Color
2. Gluons
3. Quarks
4. Confinement + asymptotic freedom
5. Running coupling constant

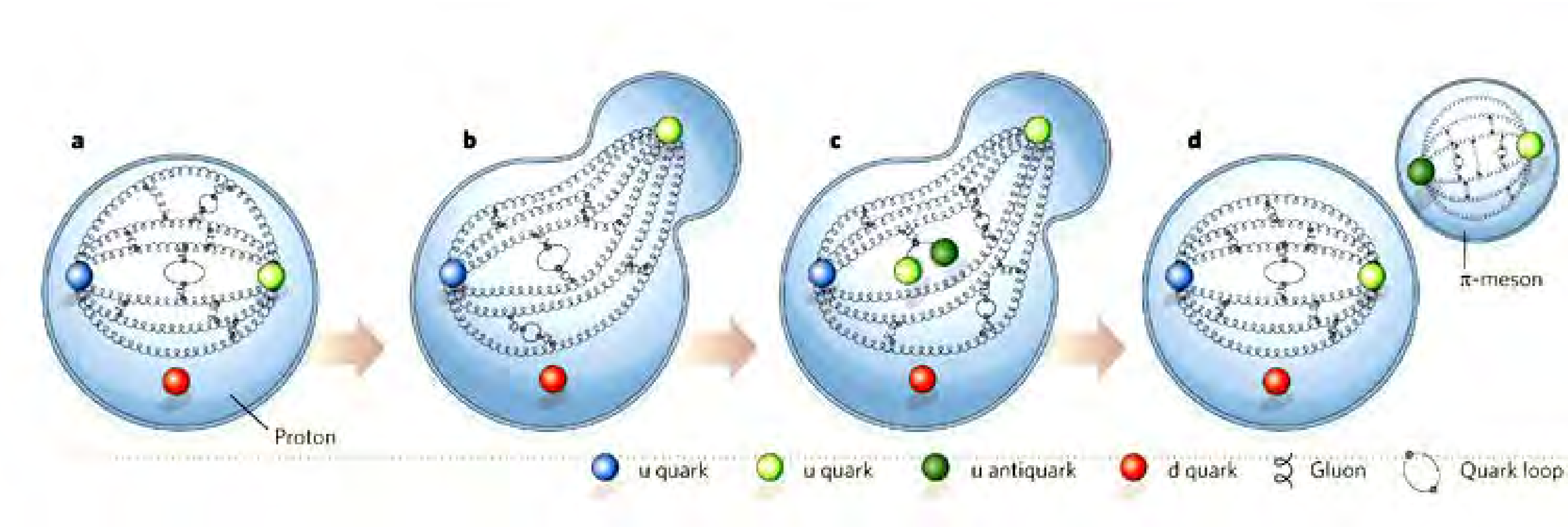
Hadronic interactions



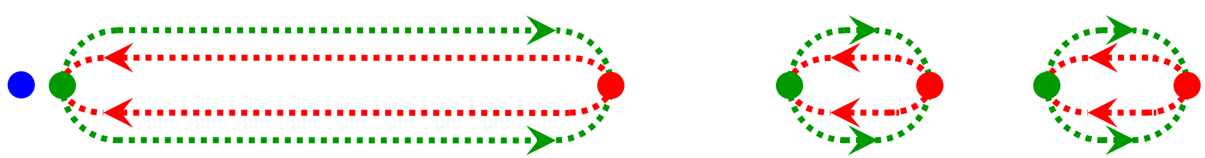


Confinement

Why no free quarks, no fractional charge observed ?!



color string: tension 200 MeV/fm ~ m_π



p π π ...

1. Confinement:

Color

no free quarks, no 1/3 charge observed !

which law forbids that + enforces trinity (qqq) ?

3 colors = (r,g,b)

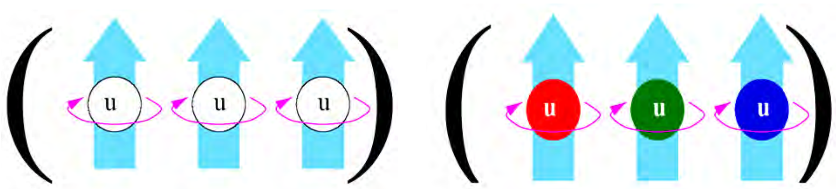
hadrons have to be color singlets

Mesons: $\{N\} \otimes \{\bar{N}\} = \{1\} \oplus \{N^2-1\}$

Baryons: $\{3\} \otimes \{3\} \otimes \{3\} = \{1\} \oplus \ldots$

2. Spin-Statistics problem: Nambu 1964, Nobel prize  2008

baryon decuplet $J^P = 3/2^+$

Δ^{++} : 

also $\Omega^- =$ (sss)

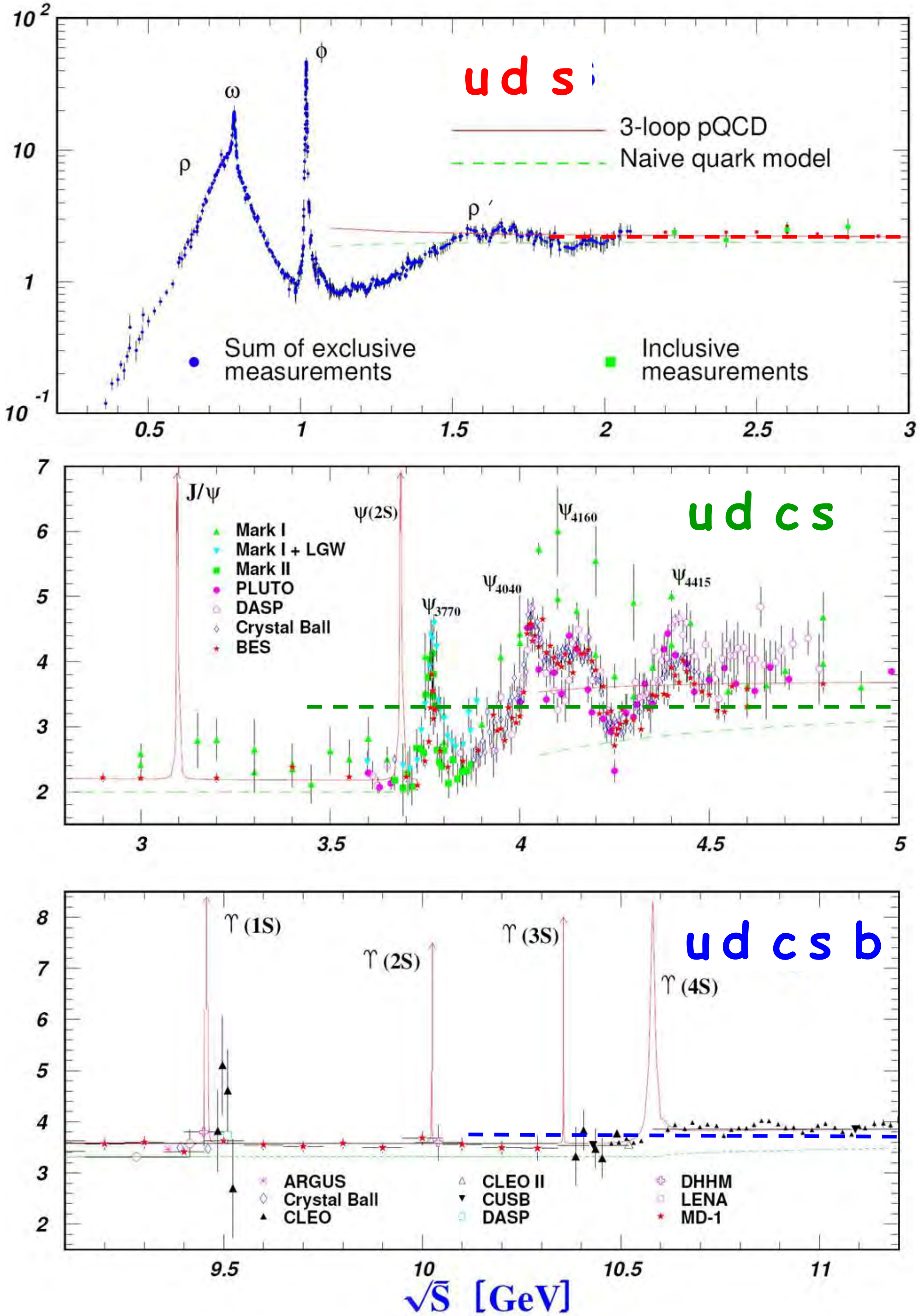
$\Psi(qqq) = \Psi(\text{space})\ \Psi(\text{spin})\ \Psi(\text{flavor})\ \ \Psi(\text{color})$

asy = sym × sym × sym × asy

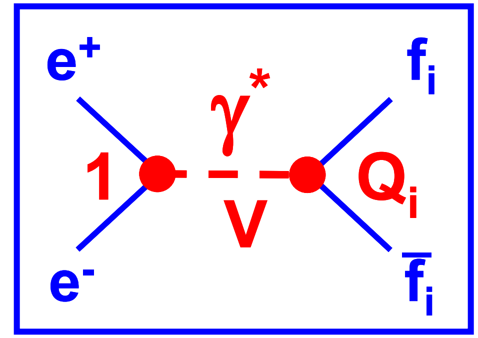
$\Psi_c\,(q\bar{q}) = (r\bar{r}+g\bar{g}+b\bar{b})\ /\sqrt{3}$

$\Psi_c\,(qqq) = (rgb-grb+gbr-rbg+brg-bgr)\ /\sqrt{6}$ asym. color wave fct.!

3. $R_{QCD} = \sigma\,(e^+e^- \to q\,\bar{q})\ /\ \sigma\,(e^+e^- \to \mu^+\mu^-) \sim N_C$:



color counting



$$R = \frac{\sigma(e^+e^- \to q\ \bar{q} \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

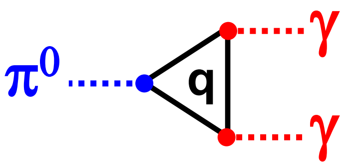
$$= \sum_{\substack{\text{flavors}\\ \text{colors}}} Q_i^2 \quad =$$

N_F	Q_i^2	N_C	
3	$\left(\frac{2}{3}\right)^2+\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^2=\frac{6}{9}$	$\cdot 3 = \frac{6}{3}$	$\binom{u}{d}\binom{}{s}$
4	$\left(\frac{2}{3}\right)^2+\left(\frac{1}{3}\right)^2+\left(\frac{2}{3}\right)^2+\left(\frac{1}{3}\right)^2=\frac{10}{9}$	$\cdot 3 = \frac{10}{3}$	$\binom{u}{d}\binom{c}{s}$
5	$\left(\frac{2}{3}\right)^2+\left(\frac{1}{3}\right)^2+\left(\frac{2}{3}\right)^2+\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^2=\frac{11}{9}$	$\cdot 3 = \frac{11}{3}$	$\binom{u}{d}\binom{c}{s}\binom{}{b}$

4. Triangle anomaly (Adler,Bell,Jackiw 1969)

$$\Gamma\ (\pi^0 \to \gamma\gamma) = (\alpha/2\pi)^2\ (Q_u^{\,2}-Q_d^{\,2})\ N_C^{\,2}\ \ m_\pi^{\,2}/(8\pi\ f_\pi)$$

Γ = 0.86	eV	THEORY	$N_C=1$
Γ = 7.8±0.5	eV	EXPT.	
Γ = 7.75	eV	THEORY	$N_C=3$



5. τ branching ratios

$$\frac{\Gamma\ (\tau^- \to e^-\ \nu_e \nu_\tau)}{\Gamma_{tot}} = \frac{\Gamma_e}{\Gamma_e + \Gamma_\mu + \Gamma_{had} \cdot N_C} = \frac{1}{1+1+1\times 3} = \frac{1}{5} = (17.8 \pm 0.1)\ \%\ \ (+\ \alpha_S/\pi)$$

6. Okubo-Zweig-Iizuka rule:

Why are $\Phi(s\bar{s})$, $\Psi(c\bar{c})$, … so narrow? Why is

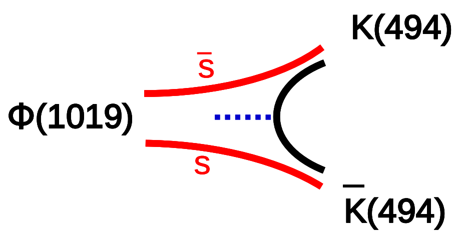
suppressed ?

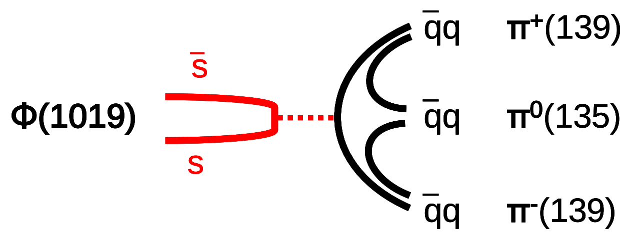
OZI rule

Quarkonium decay: Heavy vector mesons

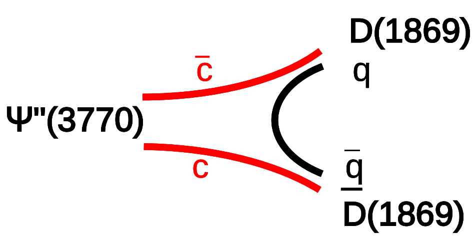
$M \to m_1 \ldots m_i$ $Q = M - \sum m_i$ free energy

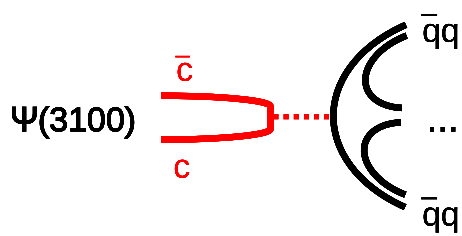
...... $E \sim \Lambda_{QCD} \sim \hbar c/\text{fm} \sim 200$ MeV ...... $E = m_{\bar{Q}Q}$





Q = 32 MeV Γ = 3.7 MeV Q = 606 MeV Γ = 0.1 MeV





Q = 39 MeV Γ = 24 MeV Q = 2684 MeV Γ_{tot} = 0.1 MeV

Γ > MeV Γ « MeV **WHY ???**

empirical rule: no hair pin diagrams

1964: Okubo-Zweig-Iizuka (OZI) rule

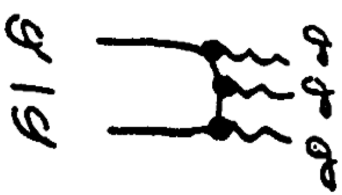
Vector Mesons

$J^{PC}=1^{--}$

OZI rule

α_s^1 forbidden: free color !

α_s^2 forbidden: $J^{PC} = 1^{--} \neq (\ldots)^2$ C parity



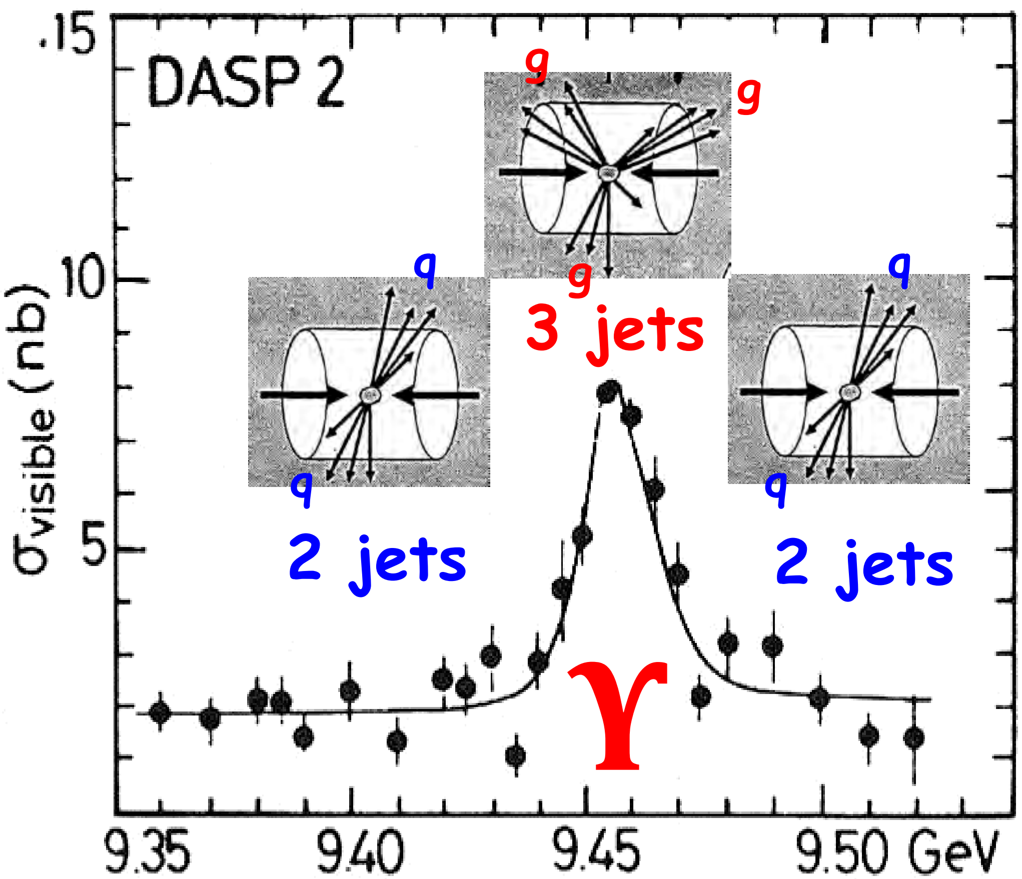
α_s^3 ok: measure α_s

$\Upsilon(b\bar{b}) \to ggg \to 3$ gluon jets

$$\frac{\Gamma(V \to 3g \to \text{hadrons})}{\Gamma(V \to \gamma \to l^+l^-)} = \frac{\alpha_s^3(m_V)}{\alpha^2\, q_i^2}\ \frac{10(\pi^2-9)}{81\pi}\ (1+\ldots)$$

$\alpha^2\, q_i^2$ measure quark charges q_i

OZI: $\Upsilon(b\bar{b}) \rightarrow$ 3 gluons



event sphericity on/off Υ

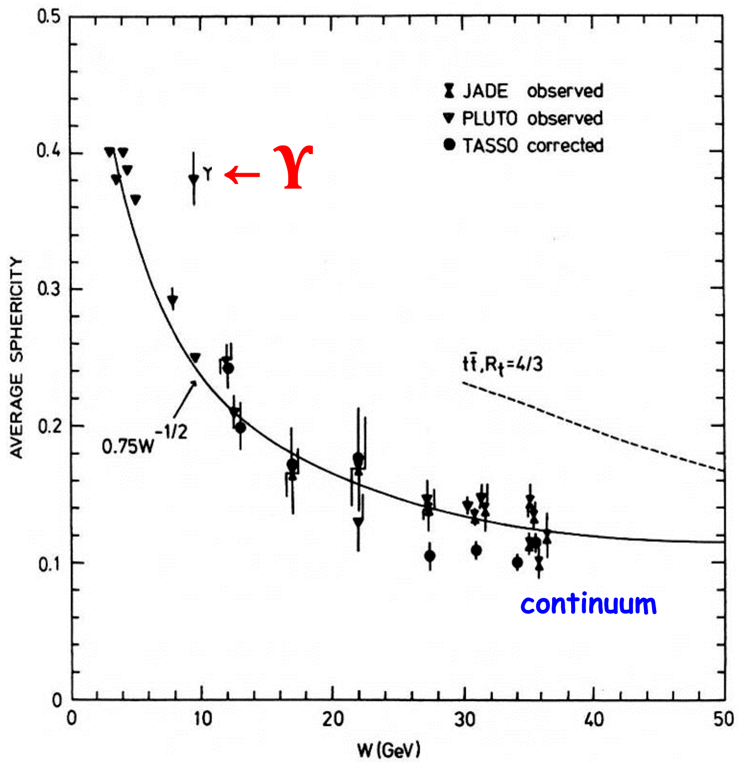


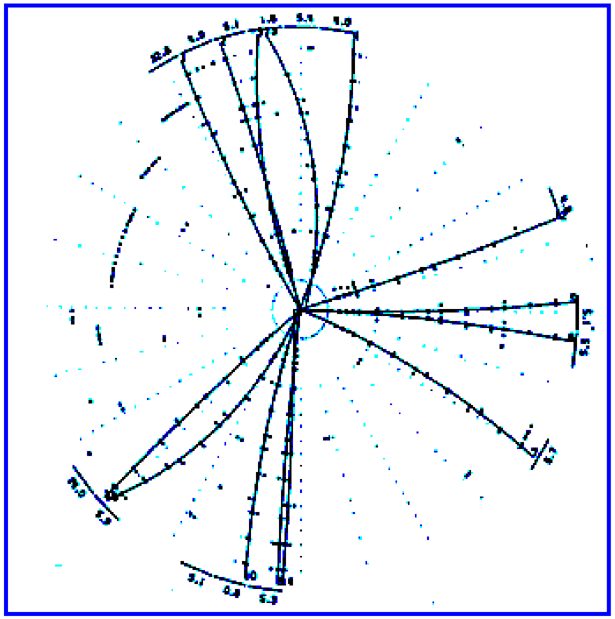
Figure 14 The average sphericity as measured by the JADE, PLUTO, and TASSO groups

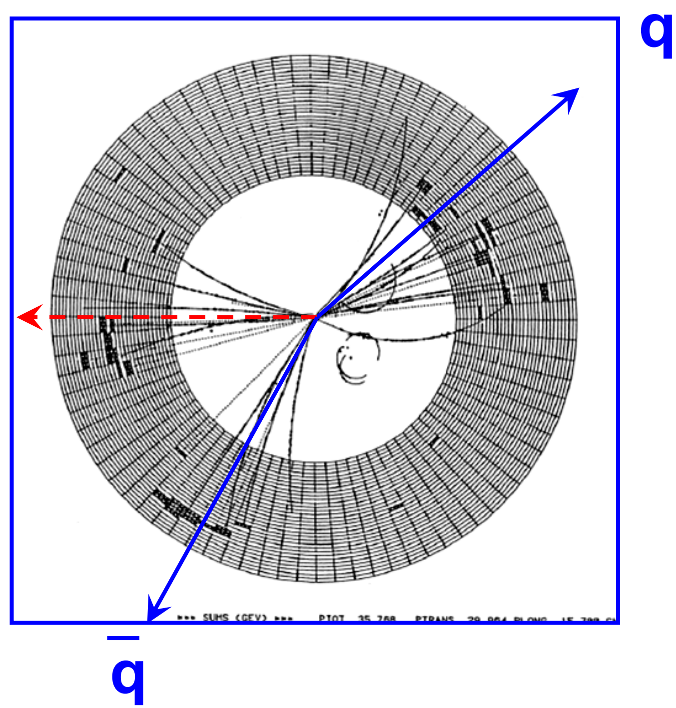
C. Berger et al. [PLUTO Collaboration]
Jet Analysis of the Υ(9.46) Decay Into Charged Hadrons
Phys. Lett. B 82, 449 (1979).

Gluon

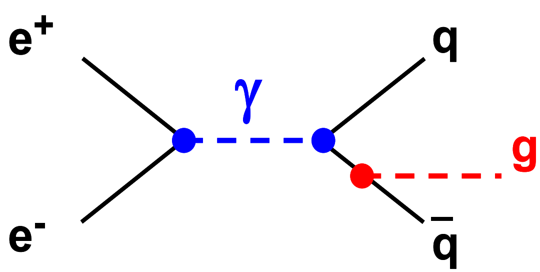
discovered 1979 at PETRA at DESY in

TASSO





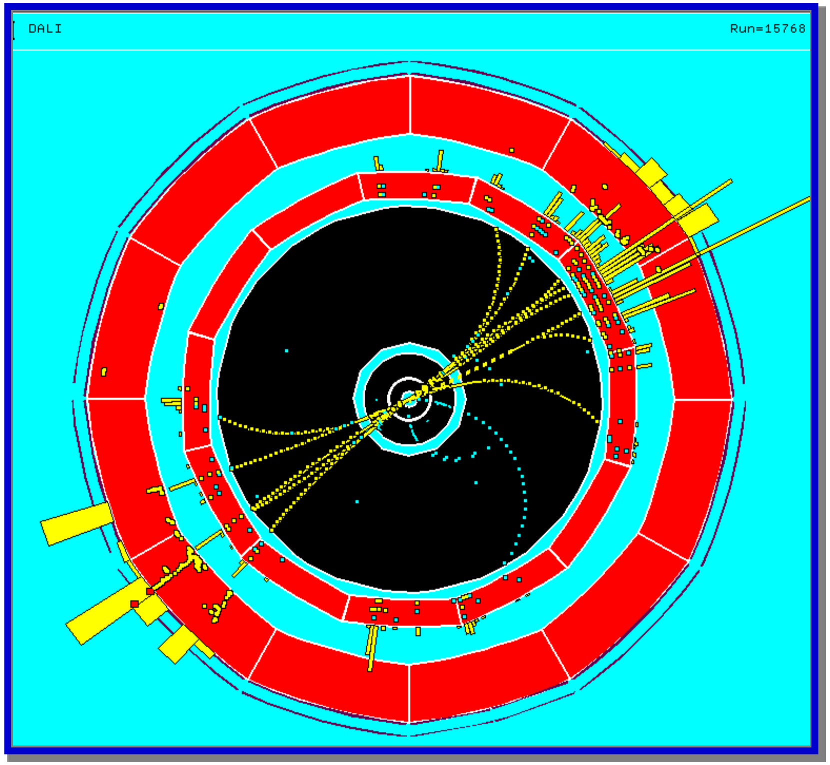
JADE



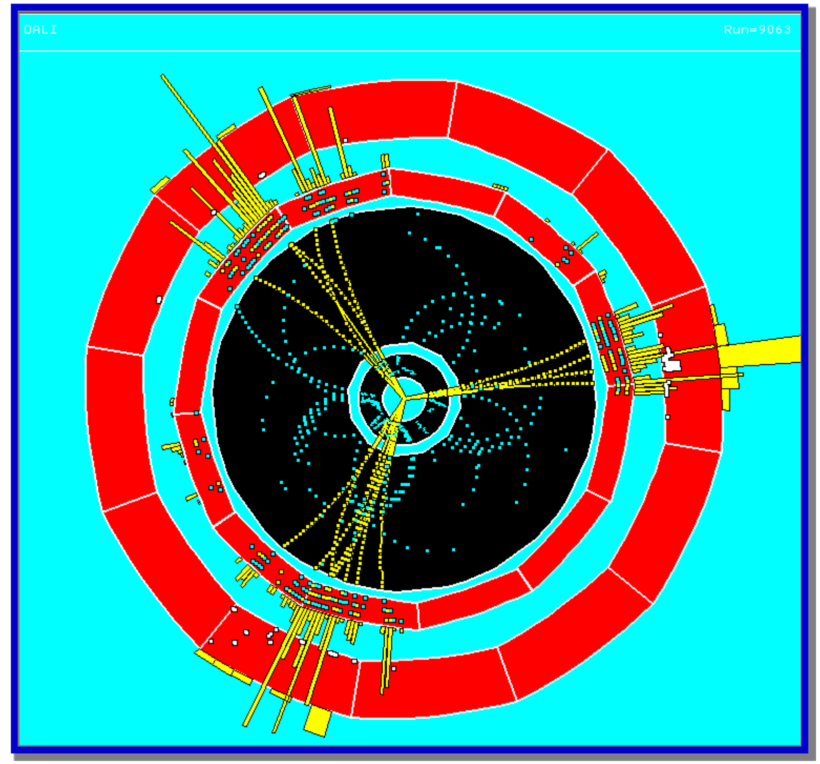
$$e^+ e^- \rightarrow q\ \bar{q}\ \ g$$

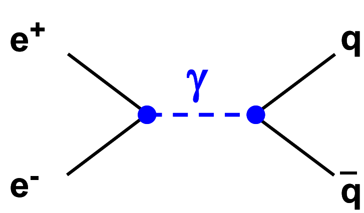
second gauge boson after the photon
Europhysics Prize 1995

Gluon

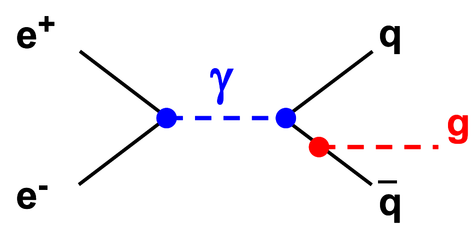


A L E P H





LEP

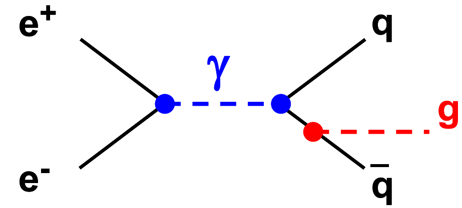


$e^+ e^- \to q\,\bar{q}$

$e^+ e^- \to q\,\bar{q}\,g$

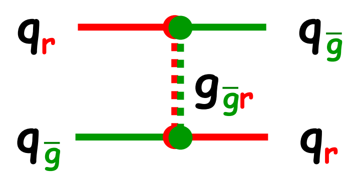
Gluons

1. Existence: DESY PETRA 1978:
3 jet events



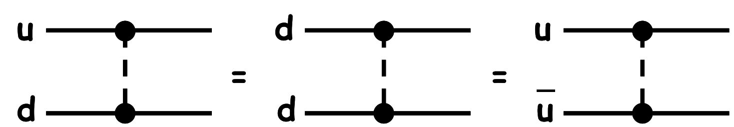
2. Couplings

SU(3): N=3 color charges
group has N^2-1 = 8 generators



$$\{3\} \otimes \{\bar{3}\} = \{1\} \oplus \{8\}$$

strong int. conserves isospin,
not flavor dependent



$\{1\} = (r\bar{r}+g\bar{g}+b\bar{b})\,/\sqrt{3}$ color singlet, color blind

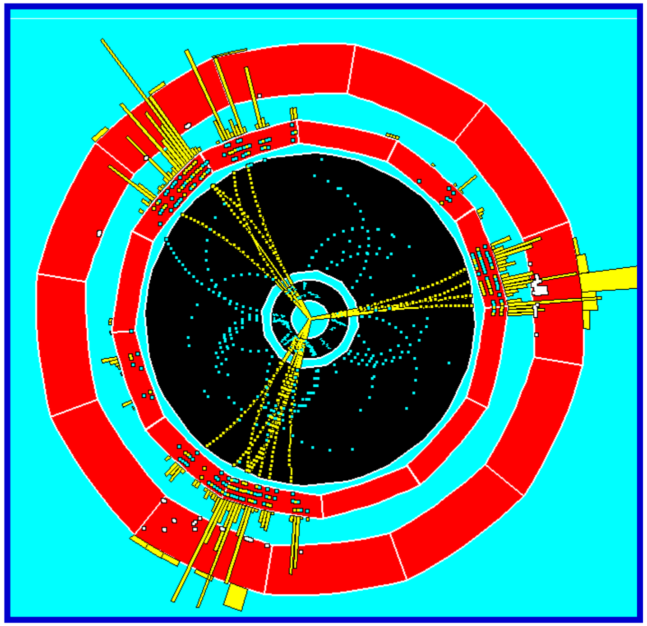
$\{8\} = [(r\bar{g}+g\bar{r})-i(r\bar{g}-g\bar{r}) + (r\bar{b}+b\bar{r})-i(r\bar{b}-b\bar{r}) + (g\bar{b}+b\bar{g})-i(g\bar{b}-b\bar{g})$
$+(r\bar{r}-g\bar{g}) + (r\bar{r}+g\bar{g}-2b\bar{b})/\sqrt{3}]\ /\sqrt{2}$

8 colored gluons represented by Gell-Mann matrices

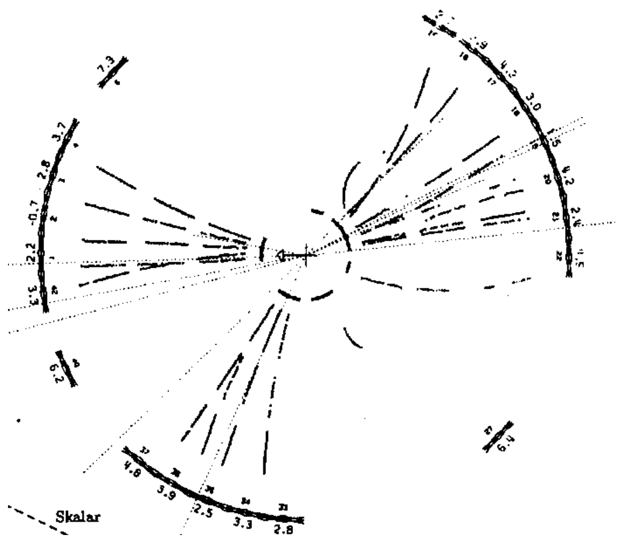
3. Spin: jet-jet angle => $J^P=1^-$

Gluon spin

CERN LEP ALEPH 200 GeV



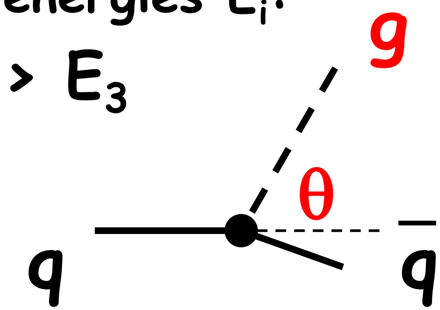
DESY PETRA TASSO 32 GeV

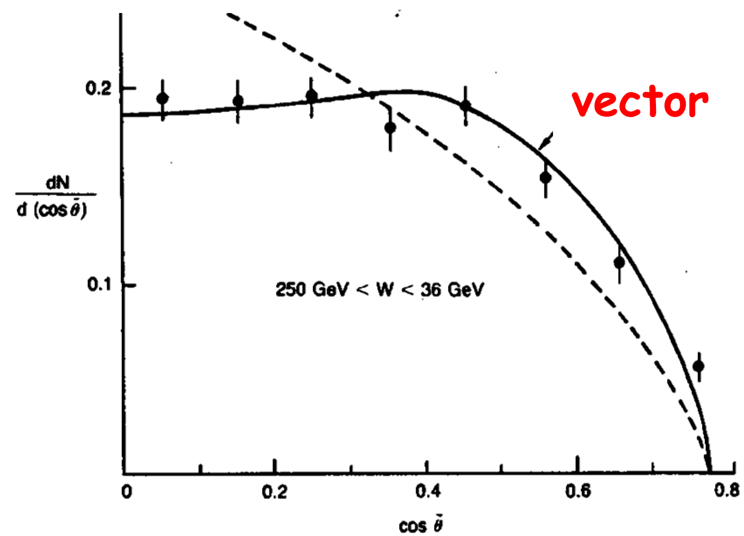


$e^+ e^- \rightarrow q \, \bar{q} \, g$

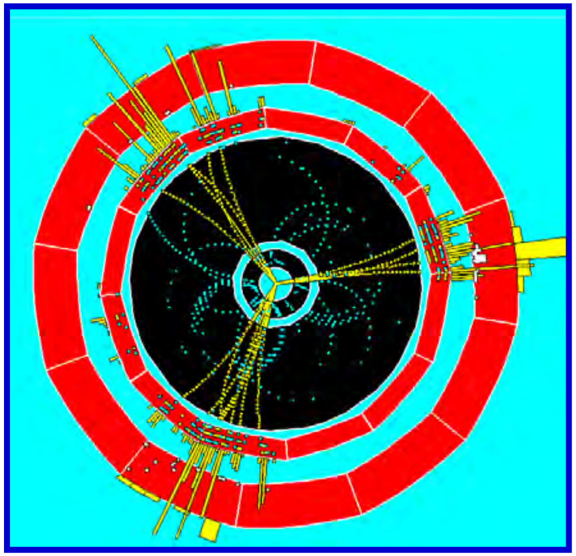
order the 3 jet energies E_i:

$E_1 > E_2 > E_3$





CERN LEP ALEPH 200 GeV



Gluon spin

LEP

$e^+ e^- \to$ 3 jets:

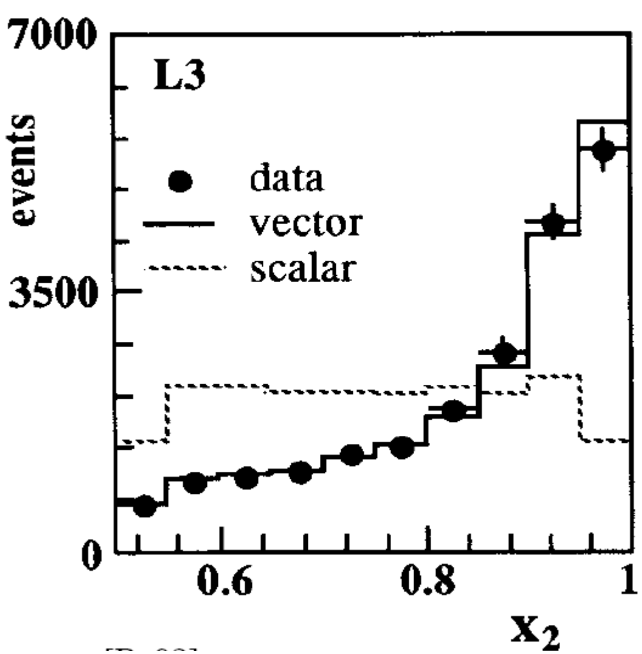
order normalized

jet energies x_i:

$$x_i = 2E_i / \sqrt{s}$$

$$x_1 > x_2 > x_3$$

collinearity of 2nd energetic jet:



infrared collinear singularity
of gluon bremsstrahlung

S.Bethke, J.Pilcher, Ann.Rev.Nucl.Part.Sci. 42 (1992) 251.

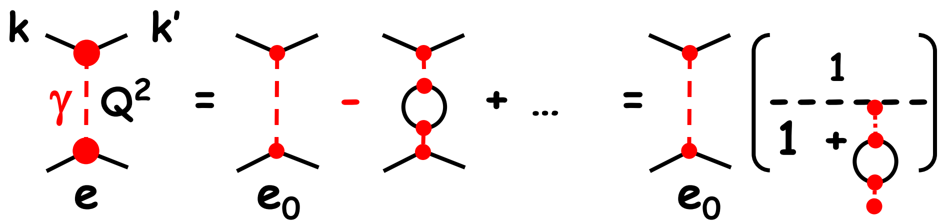
Running coupling

A constant is not constant:

- running coupling constant
- asymptotic freedom
- confinement

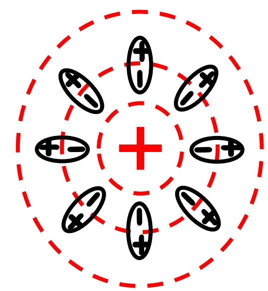
QED

Vacuum polarization in e-e scattering:



physical charge | naked charge | vacuum polarization | $Q^2 = -(k-k')^2$

dielectric screening:



- coupling ~ charge²: $F = \alpha/r^2$ $\alpha = e^2/4\pi$
- infrared stable: $\alpha = 1/137$
  ultraviolet divergent - naked charge infinite !?
- cutoff at arbitrary scale: renormalization !
  energy scale: $Q^2 = -(k-k')^2$
- consider only evolution from energy scale Q to scale μ
  UV divergences cancel

classical electron radius
$r = \alpha/m_e \sim 3$ fm
$(m_e = \alpha/r)$

Compton wavelength
$\lambda_C = 1/m_e$

ALRIGHT RUTH, I ABOUT GOT THIS ONE RENORMALIZED

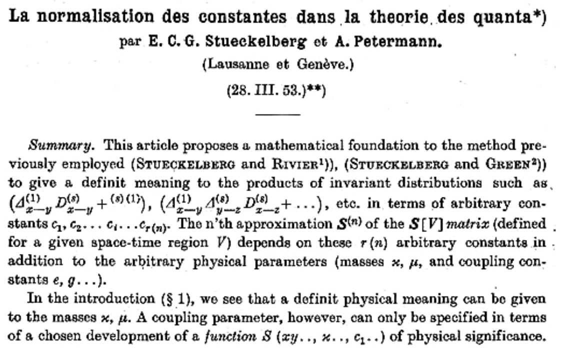
Nobel Prize 1990
for first
Quark Evidence.

Physics Today,
Jan. 1991.

R.Feynman, Nobel Lecture, 1965:

I think that the renormalization theory is simply a way to sweep the difficulties of the divergences of electrodynamics under the rug.

Renormalization


La normalisation des constantes dans la theorie des quanta*)

par E. C. G. Stueckelberg et A. Petermann.

(Lausanne et Genève.)

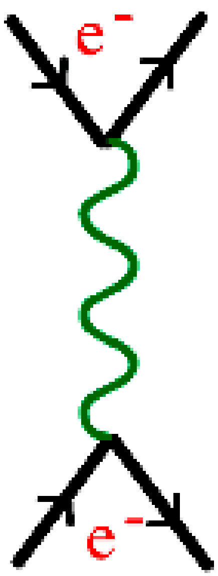
(28. III. 53.)**)

Summary. This article proposes a mathematical foundation to the method previously employed (STUECKELBERG and RIVIER[1]), (STUECKELBERG and GREEN[2])) to give a definit meaning to the products of invariant distributions such as $(\Delta^{(1)}_{x-y} D^{(s)}_{x-y} + {}^{(s)(1)})$, $(\Delta^{(1)}_{x-y} \Delta^{(s)}_{y-z} D^{(s)}_{x-z} + \ldots)$, etc. in terms of arbitrary constants $c_1, c_2 \ldots c_i \ldots c_{r(n)}$. The n'th approximation $S^{(n)}$ of the $S[V]$ *matrix* (defined for a given space-time region V) depends on these $r(n)$ arbitrary constants in addition to the arbitrary physical parameters (masses $\varkappa$, μ, and coupling constants $e, g \ldots$).

In the introduction (§ 1), we see that a definit physical meaning can be given to the masses $\varkappa$, μ. A coupling parameter, however, can only be specified in terms of a chosen development of a *function* $S(xy.., \varkappa.., c_1..)$ of physical significance.

Petermann, Stückelberg, 1943, 1951. Gell-Mann, Low 1954.
Bogoljubov, Shirkov 1956. Callan, Symanzik 1970.
Wilson 1971. Particle + solid state physics. Lattice. Nobel prize 1982.

renormalize charge + cutoff such that physics
does not depend on arbitrary energy scale μ:



β function: $2\beta = \partial\, \alpha(\mu) / \partial \ln(\mu)$

renormalization group equ.:

$\beta=0$ for $\mu \to 0$ IR stable: $\alpha(0) = 1/137$

problem:

point interaction means UV stable ! $\alpha(\infty) = ?$

$\beta=0$ for $\mu \to \infty$

Are there point interactions (on elementary particles) ?

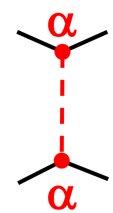
Are there asymptotically free gauge theories ?

Running coupling

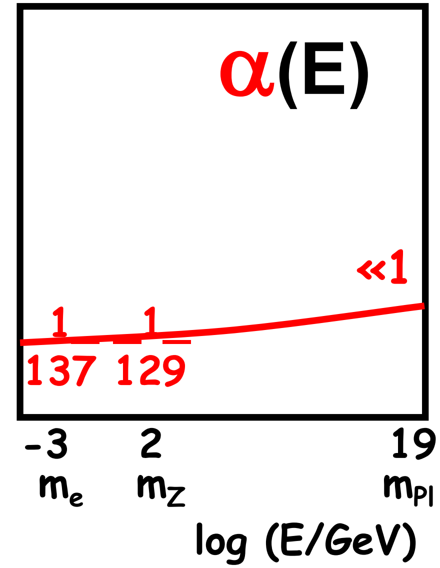
$\alpha = e^2/4\pi$ charge ~ coupling = fine structure constant

constant not constant:

$$2\beta = \partial\alpha(\mu)/\partial \ln\mu = \frac{2}{3\pi}\alpha^2 + \ldots$$



$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log\frac{Q^2}{\mu^2}}$$



α (E) running (or crawling):

$$\partial\, \alpha(Q^2) / \partial Q^2 > 0$$

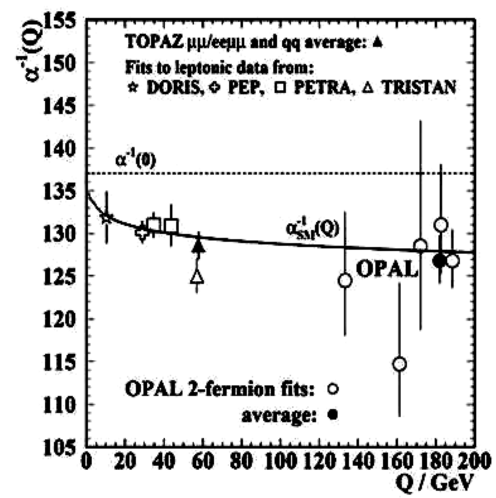
α (m_e = 0.5 MeV) = 1/137

α (m_Z = 91 GeV) = 1/128.9 CERN LEP

α (m_{Pl} = 10^{19} GeV) « 1 more Fermion-Loops

α ($e^{3\pi/2\alpha} m_0 \sim e^{646} m_0 \sim 10^{280} m_0 \gg m_{all}$) -> 1 (more loops)

α (∞) undefined no electric point interaction ?!

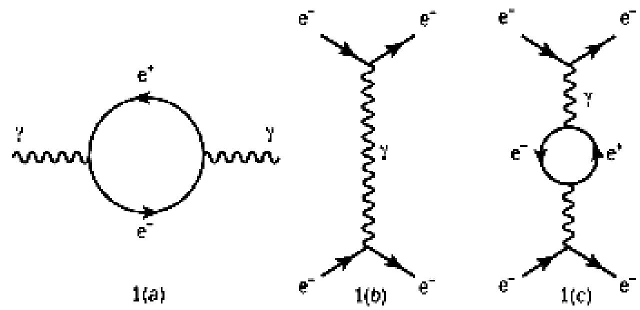


Renormalization

Landau 1955:

„weak coupling electrodynamics is … fundamentally logically incomplete.”

„within the limits of formal electrodynamics a point interaction is equivalent … to no interaction at all.”



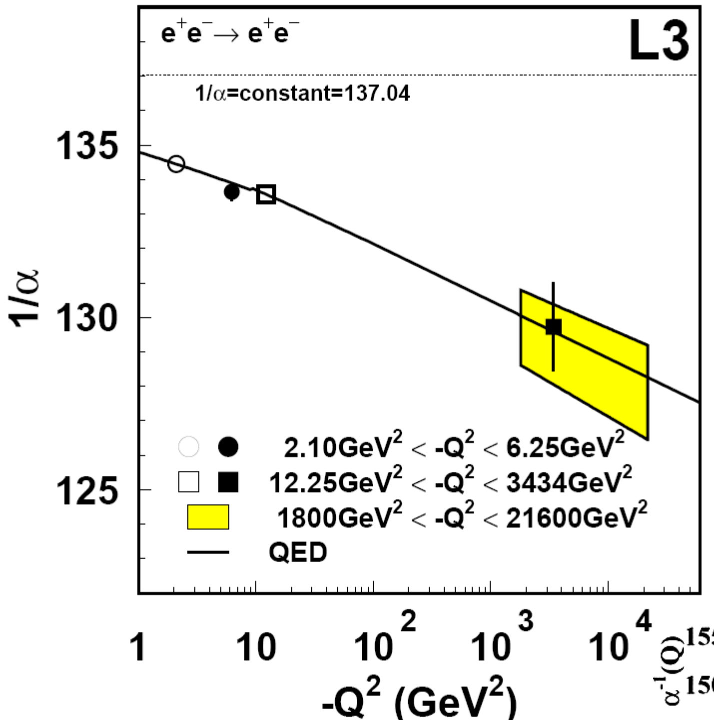
Dyson 1960:

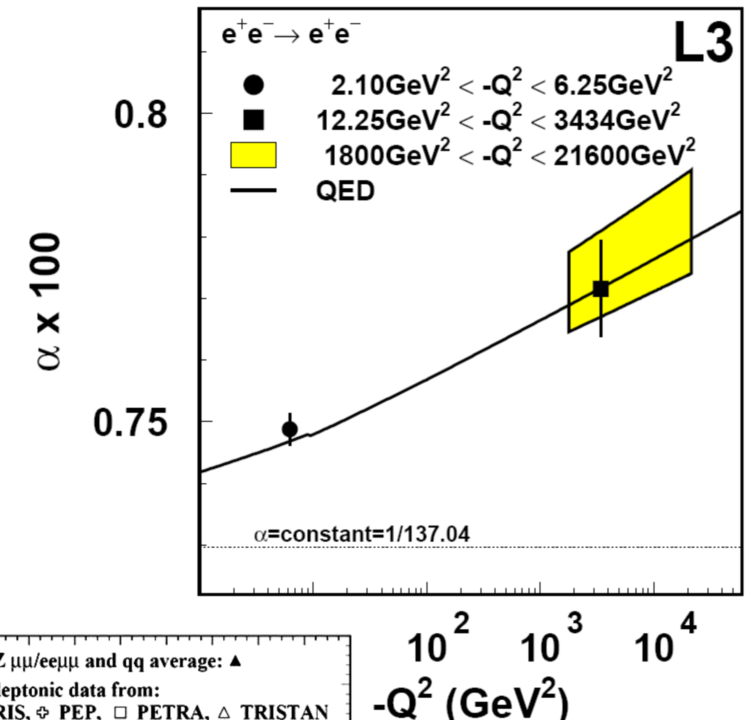
“The correct theory will not be found within the next 100 years.”

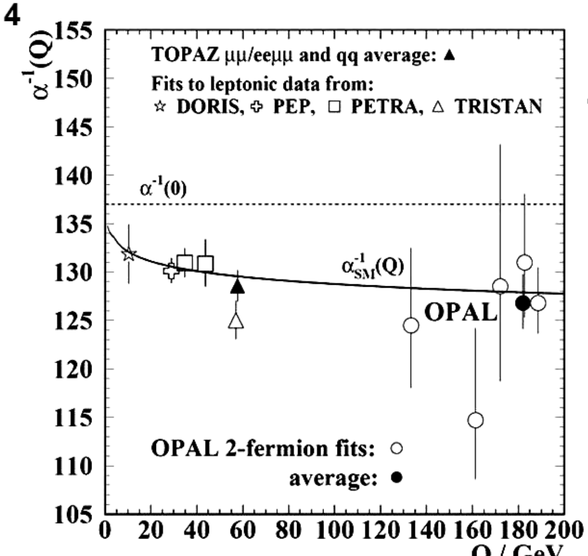
Feynman 1961:

“I still … do not subscribe to the philosophy of renormalization.”

QED: running coupling







QCD: the Lagrangian

$$\mathcal{L} = \frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_j \bar{q}_j \left(i\gamma^\mu D_\mu + m_j \right) q_j$$

where $$G^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + i f^a_{bc} A^b_\mu A^c_\nu$$

and $$D_\mu \equiv \partial_\mu + i t^a A^a_\mu$$

<u>That's it!</u>

j ... quark flavors

a,b,c ... colored gluons

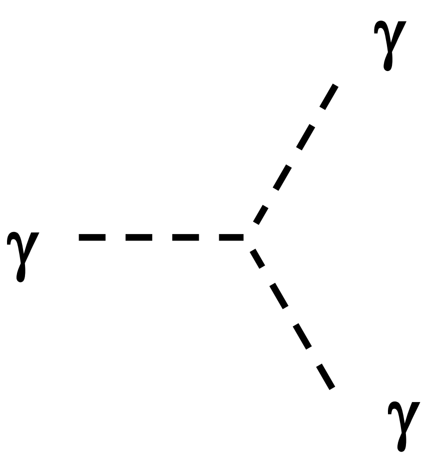
μ,ν ... space-time

F.Wilczek, Physics Today, August 2000.

Quantum Chromo-Dynamics

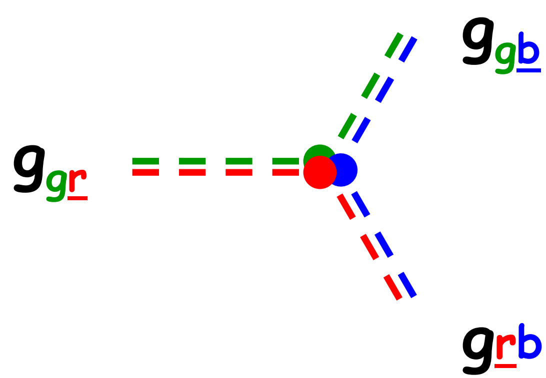
QED

- U(1), abelian
- 1 charge type
- 1 photon:
- electric neutral
- no photon-photon coupling:
  - light does not clump !



QCD

- $SU(3)_{COLOR}$, non-abelian
- 3 charge types: r,g,b
- $\{3\}\otimes\{\bar{3}\} = \{1\}\oplus\{8\}$: 8 gluons:
- carry color charges
- gluon-gluon self-coupling
  - gluonium, glue balls



β function

expand β in powers of coupling + find zeros:

$$\mu \frac{\partial \alpha_s}{\partial \mu} = 2\beta(\alpha_s) = -\frac{\beta_0}{2\pi}\alpha_s^2 - \frac{\beta_1}{4\pi^2}\alpha_s^3 - \frac{\beta_2}{64\pi^3}\alpha_s^4 - \cdots$$

$\beta_0 = (11\, N_C - 2\, N_F) / 3$

N_C … nr of colors N_F … nr of flavors

Casimir operators of gauge group SU(N)

for $N_F \leq 16$ fermion flavors $N_C = 3$ boson colors win:

$\partial\, \alpha(\mu) / \partial \ln \mu < 0$

non-abelian gauge theories asymptotically free !

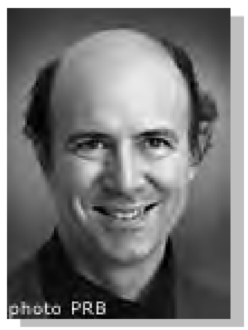
D.Gross on occasion of Nobel Prize 2004:

The discovery of asymptotic freedom was totally unexpected …

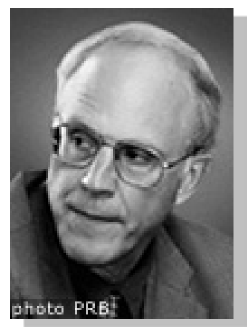
Field theory was not wrong.

Nobel prize 2004

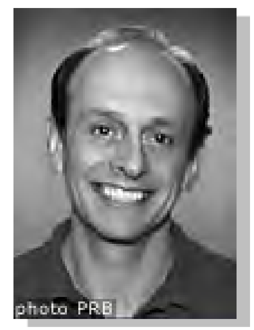
for the discovery of asymptotic freedom
in non-abelian gauge theories,
in particular in Quantum Chromo-Dynamics



F.Wilczek



D.Gross

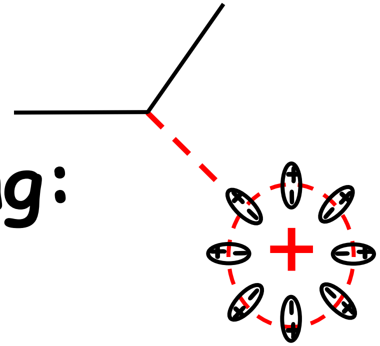


D.Politzer

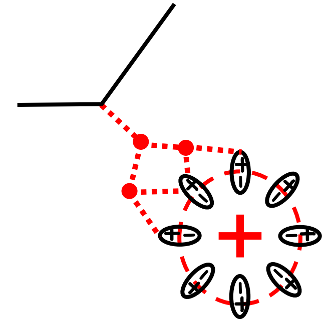
H. D. Politzer, Reliable Perturbative Results for Strong Interactions?, Phys. Rev. Lett. 30, 1346 (1973).
D. J. Gross and F. Wilczek, Ultraviolet Behavior of Nonabelian Gauge Theories, Phys. Rev. Lett. 30, 1343 (1973).

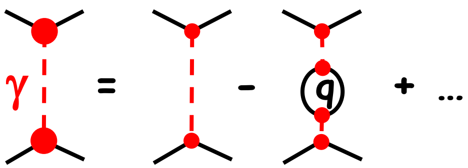
QED and QCD

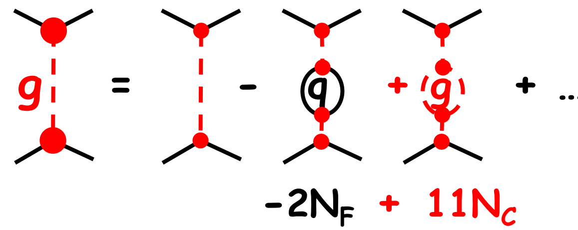
Screening:



Anti-Screening:







$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 + b_0 / 4\pi \; \alpha(\mu^2) \log(Q^2/\mu^2)}$$

$b_0 = -4/3$

$\partial\, \alpha(Q^2) / \partial Q^2 > 0$

$b_0 = (-2N_F + 11N_C)/3$

$\partial\, \alpha(Q^2) / \partial Q^2 < 0$

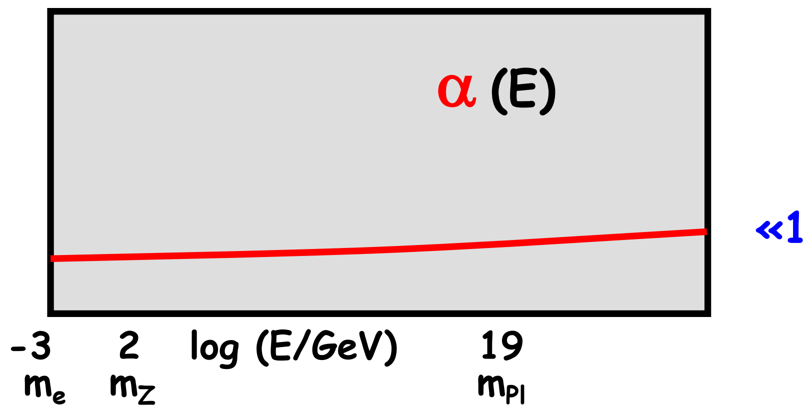
gluon massless ! $SU(2)_W$: $m_W > 10^5\, m_e$

QED and QCD

QED: $\partial\, \alpha(\mu^2) / \partial \mu^2 > 0$ **screening**

IR:

$\alpha = 1/137$



UV:

Landau Singularity

QCD: $\partial\, \alpha(\mu^2) / \partial \mu^2 < 0$ **anti-screening**

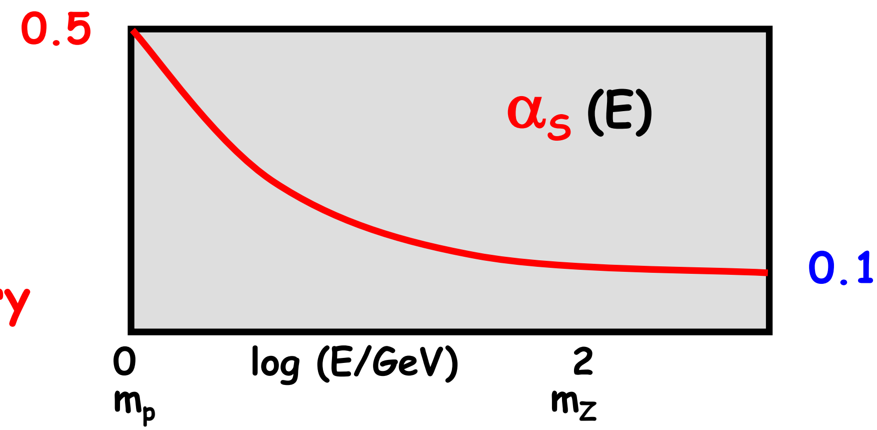
IR:

$\alpha_S \to \infty$

collapse of perturbation theory

infrared slavery

confinement



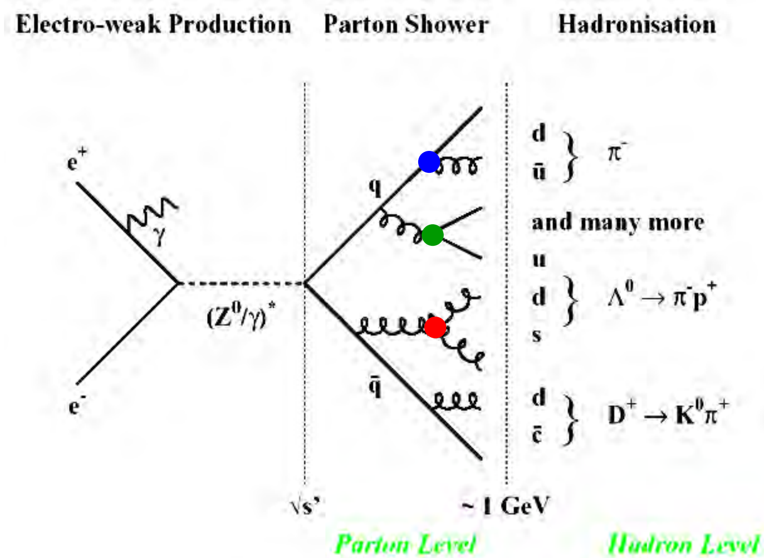
UV:

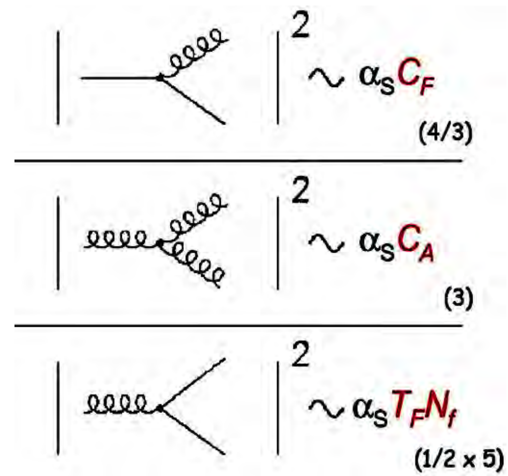
$\alpha_S \to 0$

asymptotic freedom

QCD = SU(3) ?

CERN LEP: $e^+e^- \to$ 4 jets event shape NLO QCD

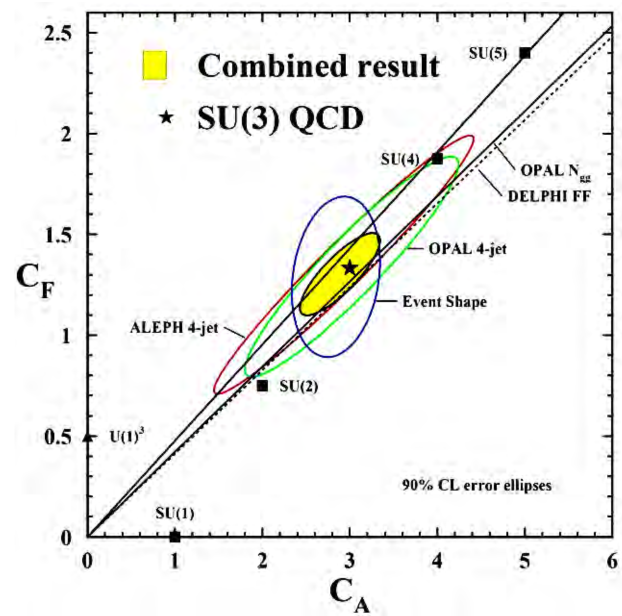




QCD color factors C_A, C_F

for SU(N). In SU(3):

C_A= N_C	C_F= $(N_C^2-1)/2N_C$	C_A/C_F
3	4/3	9/4
2.9±0.2	1.3±0.1	2.26±0.08



Confinement

instead of $\alpha_s(\mu^2)$ define

$$\Lambda = \mu \exp\left[-2\pi/(b_0\alpha_s(\mu^2))\right]$$

$$\alpha_s(Q^2) = \frac{4\pi}{9 \ln (Q^2/\Lambda^2)} + \ldots \quad (N_F=3)$$

$$\alpha_s(Q^2 \rightarrow \Lambda^2) \rightarrow \infty$$

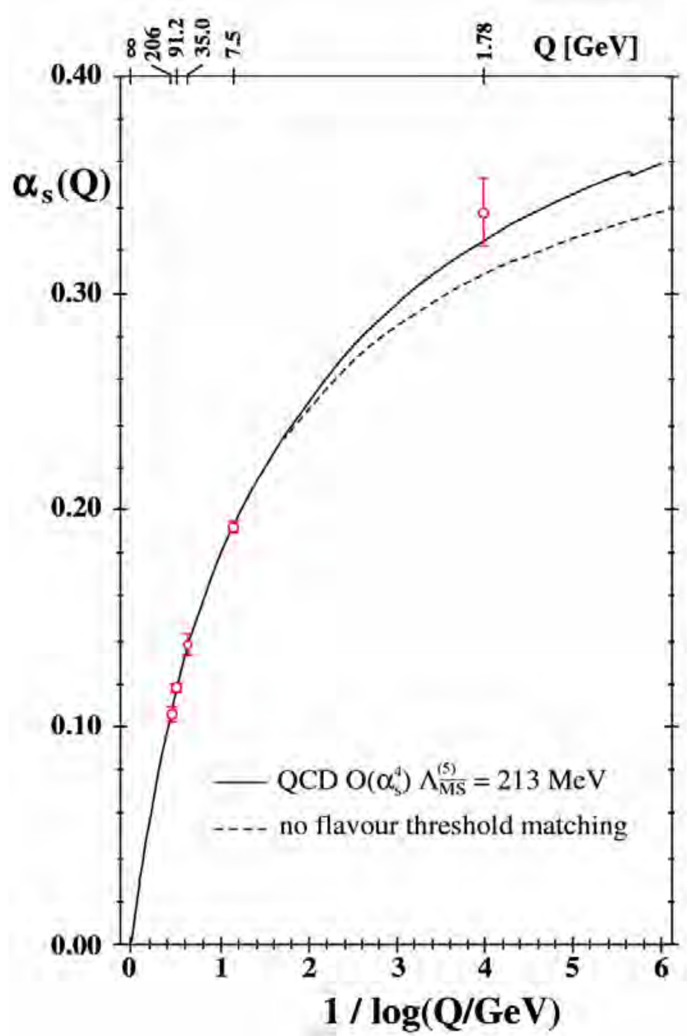
collapse of perturbation theory
nuclear force confines - infrared slavery:
no free quarks !

$\hbar c \approx$ 200 MeV • fm
QCD scale Λ • proton radius

proton = 'QCD black hole'

From asymptotic freedom to infrared slavery

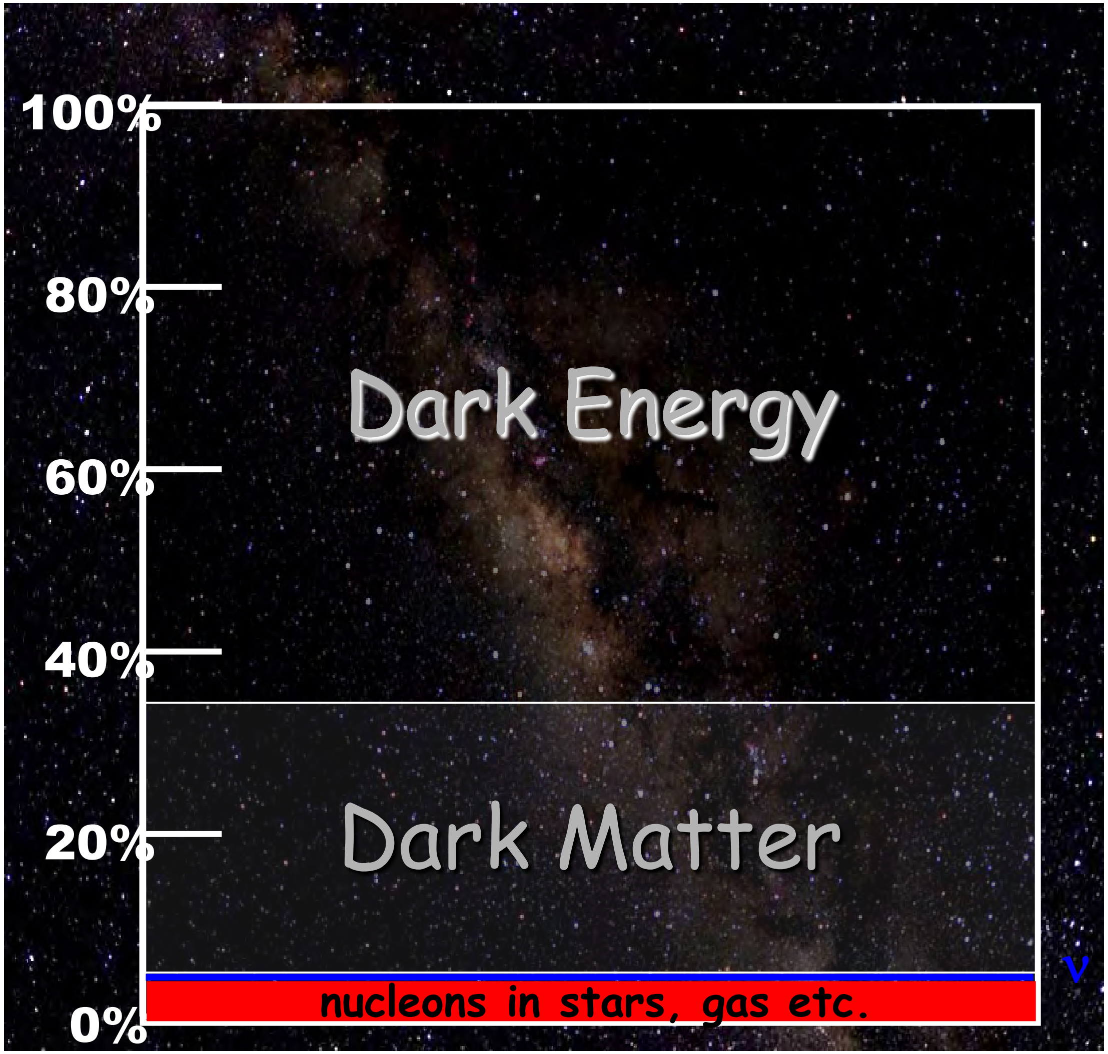
Quarks are
born free,
but everywhere
they are
in chains.



F.Wilczek, Nobel talk, 2004.

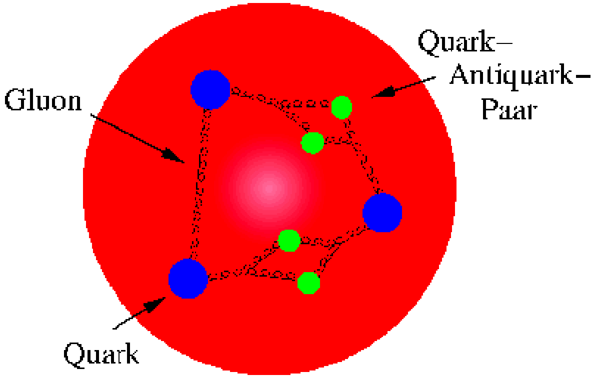
J.J. Rousseau, Du Contrat Social, 1762:
«L'homme est né libre et partout il est dans les fers.»

Mass vs Energy



5%

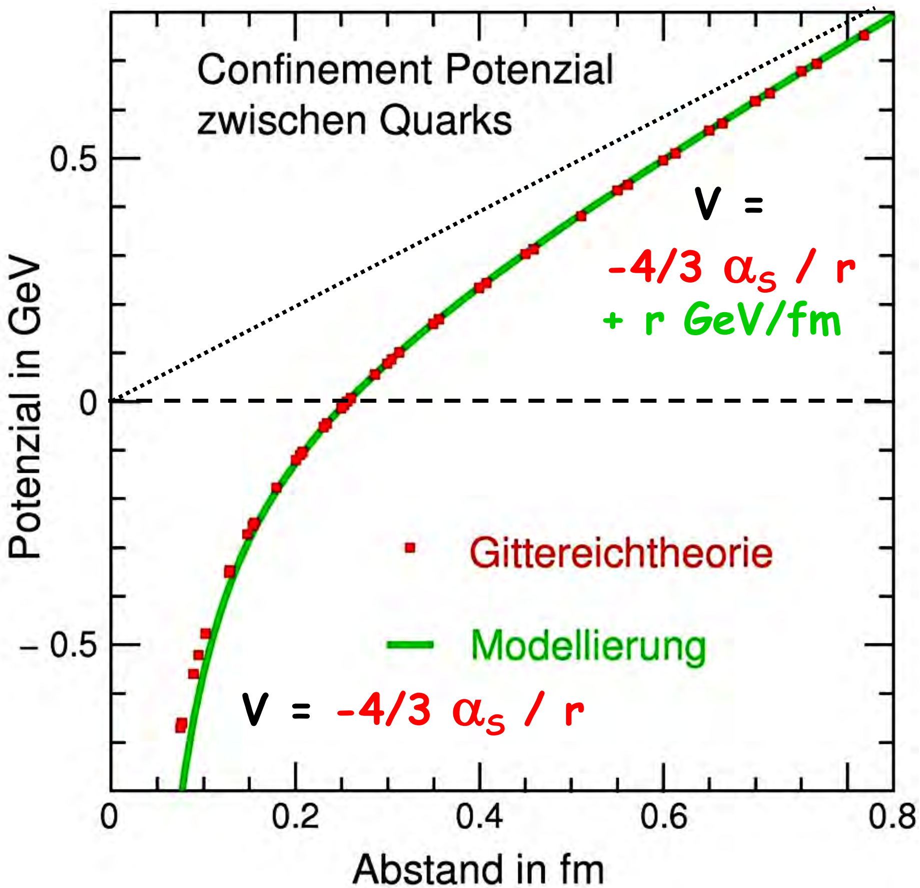
Nucleon mass



~ % due to
quark masses
electrons negligible

dominant:
binding energy
of partons

Confinement



hadron radius:

confinement

color string: constant force = energy/length: $k = 1$ GeV / fm

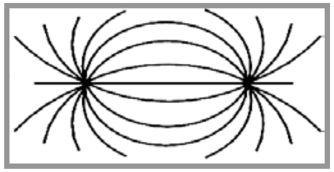
describes spectroscopy of heavy quark bound states:

$\Psi, \Psi', \Psi'', \ldots = (c\ \bar{c})$
$\Upsilon, \Upsilon', \Upsilon'', \ldots = (b\ \bar{b})$

like positronium $= (e\ \bar{e})$

asymptotic freedom

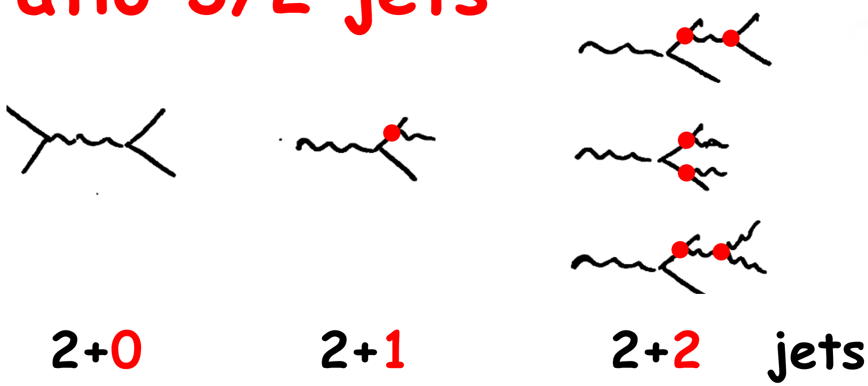
at short distances = high energies: Coulomb law

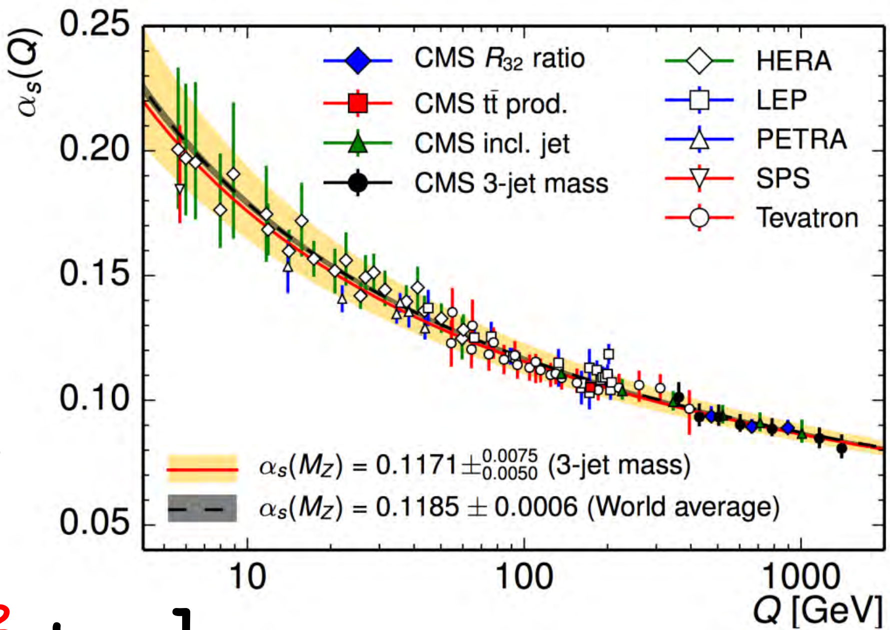


$\alpha_S(Q^2)$

α_s measurement

ratio 3/2 jets





$$R_Z = \Gamma_h/\Gamma_l = R_0 \left[1^0 + (\alpha_s/\pi)^1 + 0.76(\alpha_s/\pi)^2 + \ldots\right]$$

$R_0 = 19.943$, $R_Z = 20.768 \pm 0.0024$

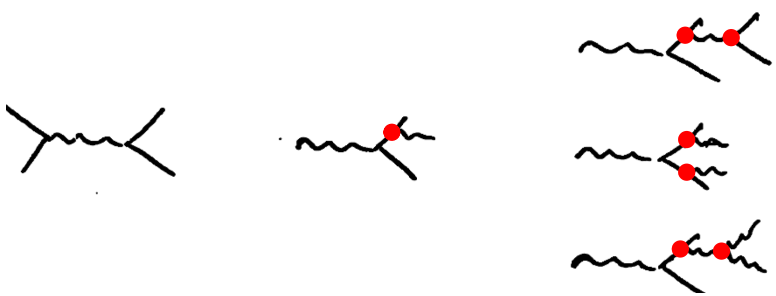
$\delta R/R \sim 10^{-3}$ syst. errors cancel: luminosity, ...

$\delta\alpha_s/\alpha_s = \pi/\alpha_s \, \delta R/R \sim 25 \, \delta R/R$

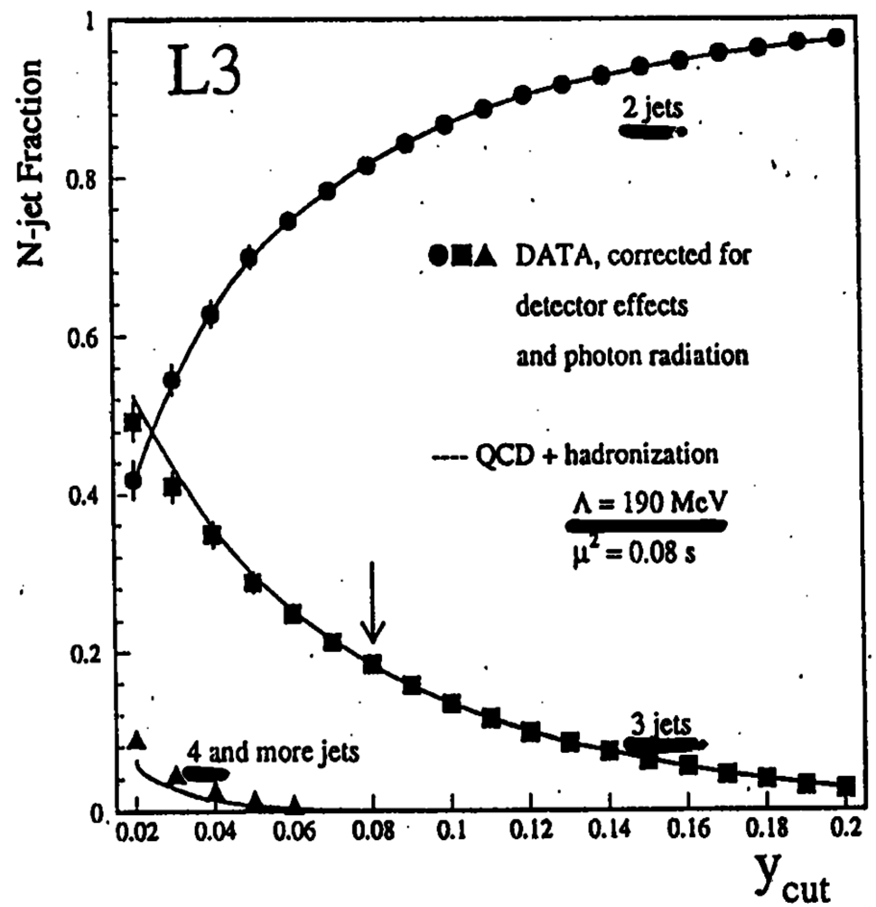
- CLEO,PEP,PETRA,TRISTAN: α_s (34 GeV) = 0.15 ± 0.03
- jets world average: α_s (91 GeV) = 0.120 ± 0.003
- LEP2: α_s (172 GeV) = 0.102 ± 0.006
- LHC CMS: α_s (800 GeV) = 0.090 ± 0.006

α_s from jets

$e^+e^- \rightarrow$ 2+n jets



2+0 2+1 2+2 jets

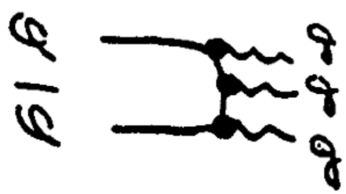


Heavy Vector Mesons

α_s in Quarkonium

α_s^1 forbidden: free color !

α_s^2 forbidden: $1^{--} \neq (\ldots)^2$



α_s^3 ok: measure α_s

$\Upsilon \rightarrow ggg \rightarrow$ 3 gluon jets: $R = \dfrac{\Gamma(V \rightarrow 3g \rightarrow \text{hadrons})}{\Gamma(V \rightarrow \gamma \rightarrow l^+ l^-)} = \dfrac{\alpha_s^3(m_V)}{\alpha^2\, q_i^2} \; \dfrac{10(\pi^2-9)}{81\pi} \; (1+\alpha_s/\pi\,[\ldots])$

$\alpha^2 q_i^2$ measure quark charges q_i

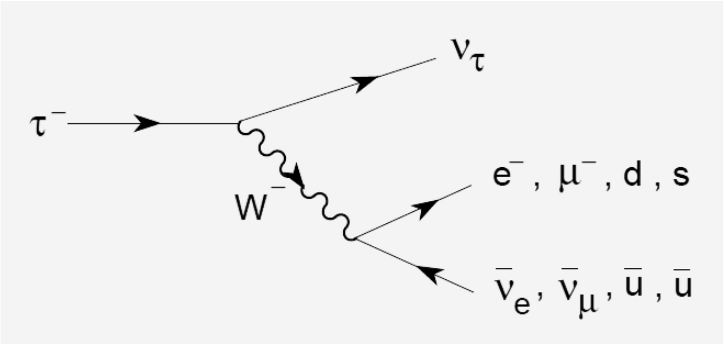
$\alpha_s (m_\psi = 3.1 \text{ GeV}) = 0.256$

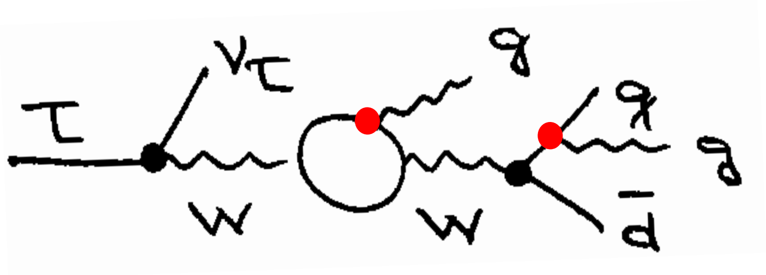
$\alpha_s (m_\Upsilon = 9.5 \text{ GeV}) = 0.184 \pm 0.015$

$\alpha_s (m_Z = 91 \text{ GeV}) = 0.118 \pm 0.001$

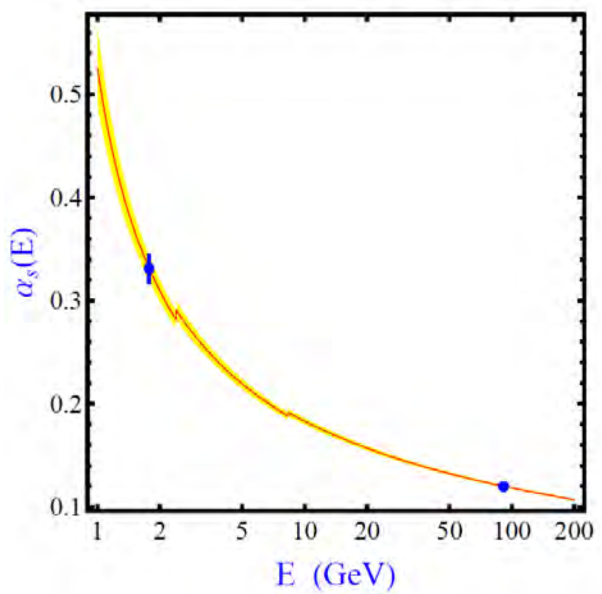
τ decays

$$R_\tau = \frac{\Gamma(\tau^- \to \nu_\tau \ \text{hadrons}_{ud})}{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} =$$





$$= 3 \ V_{ud}^2 \ (1 + \alpha_S/\pi + \ldots)$$



$\alpha_S \ (m_\tau = 1.78 \text{ GeV}) = 0.32 \ \pm 0.02$

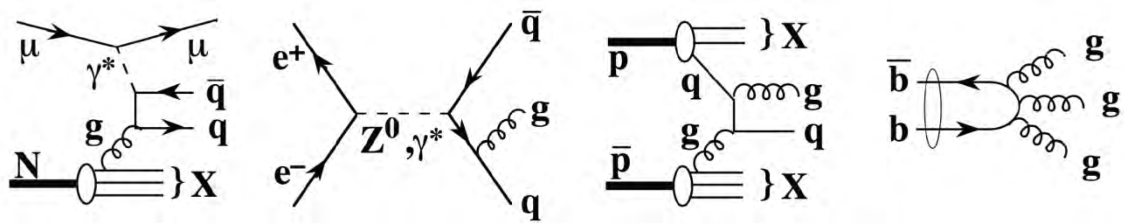
$\alpha_S \ (m_Z = 91 \text{ GeV}) = 0.118 \pm 0.002$

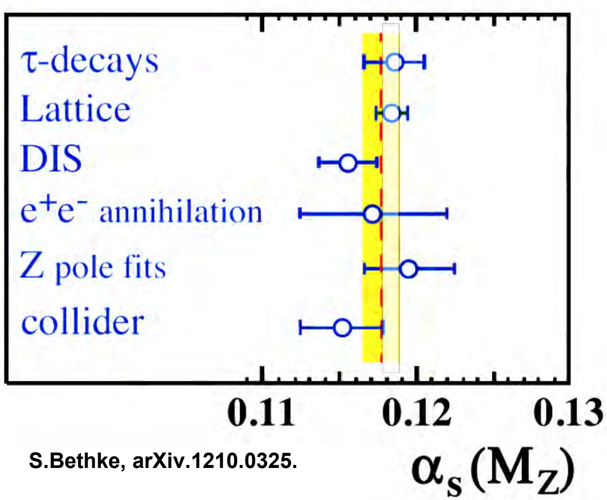
PDG, arXiv:1606.07764,1611.03457

QCD evolution, arXiv:1612.05010

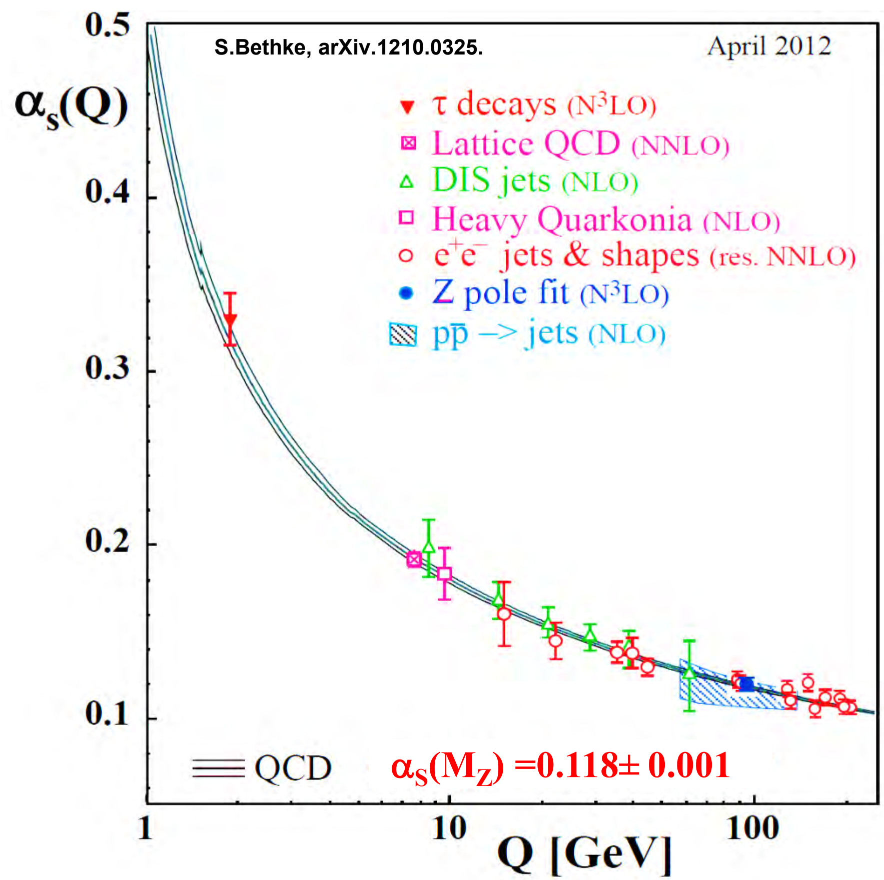
α_s measurement

- e^+e^- collisions
  - R_{QCD}
  - jet multiplicity
  - jet shapes
  - Z pole fits
- heavy quarkonia
- τ decay
- deep inelastic scattering
- lattice calculations





S.Bethke, arXiv.1210.0325.

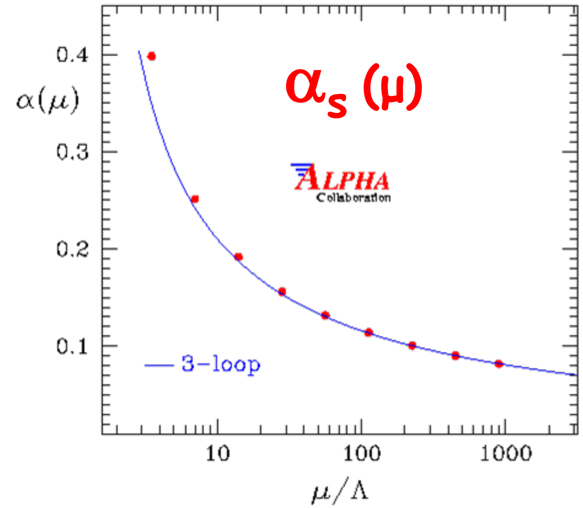


Running coupling

$$\alpha_s(Q^2) = \frac{4\pi}{9\ln(Q^2/\Lambda^2)} + \ldots$$

$$\alpha_s(M_Z) = 0.118 \pm 0.001$$

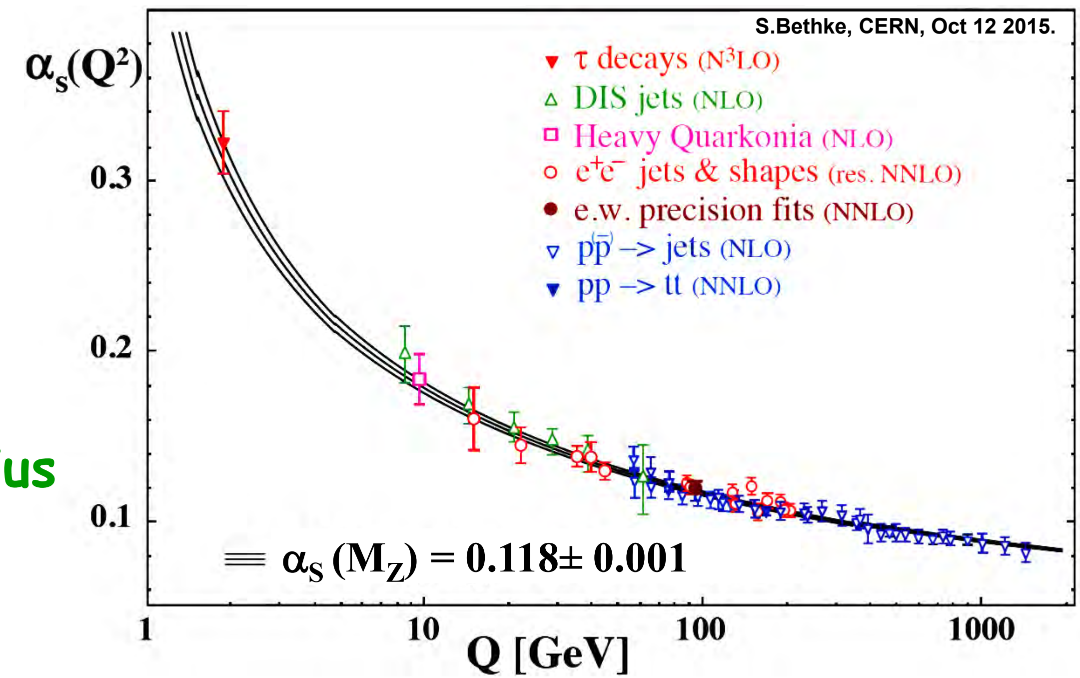
QCD on a 3D discrete lattice:



QCD scale:

Λ = 206±14 MeV

(MS, N_F=5)



$\hbar c \approx$ 200 MeV • fm

QCD scale • proton radius

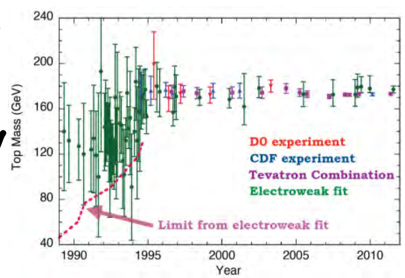
7. Unification

PARTICLES					FORCES			
					Electro-Magnet.	Weak	Nuclear	Gravi-tation
	Charge				Electric	Weak	Color	Mass
	Symmetry				U(1)	SU(2)	SU(3)	
Matter Particles	Fermions J=1/2							
Quarks	Up	u	c	t	+2/3	I_W, Y_W	r g b	
	Down	d	s	b	-1/3			
Leptons	Electrons	e	μ	τ	-1	I_W, Y_W		
	Neutrinos	ν_e	ν_μ	ν_τ	0			
Force Particles	Bosons J=1							
Photon		γ						
Weak Bosons		W^+, Z^0, W^-						
Gluons		8 g_{ij}						
Graviton (J=2)		G						

The Standard Model

PREDICTION		DISCOVERY	
1963	Quarks: Gell-Mann	1968	SLAC
1970	Charm: GIM model	1974	SLAC, BNL
1973	Beauty: CKM matrix	1977	FNAL
1994	Top: radiative corrections	1994	FNAL
1994:	m_t = 169 ± 25 GeV		m_t = 176 ± 13 GeV
2017:	m_t = 177 ± 2 GeV		m_t = 173.3 ± 0.8 GeV
1958	weak neutral current	1977	CERN
1958-71	W,Z bosons: GSW model	1983	CERN
1964	Gluons, Color, QCD	1978	DESY
1964	Self coupling of gauge bosons	1996	CERN
1964	Higgs	2012	CERN LHC
1971	Supersymmetry ?	201X	CERN LHC
1974	Grand Unification	202X	HyperK

Top quark mass



prediction + meas.t vs time

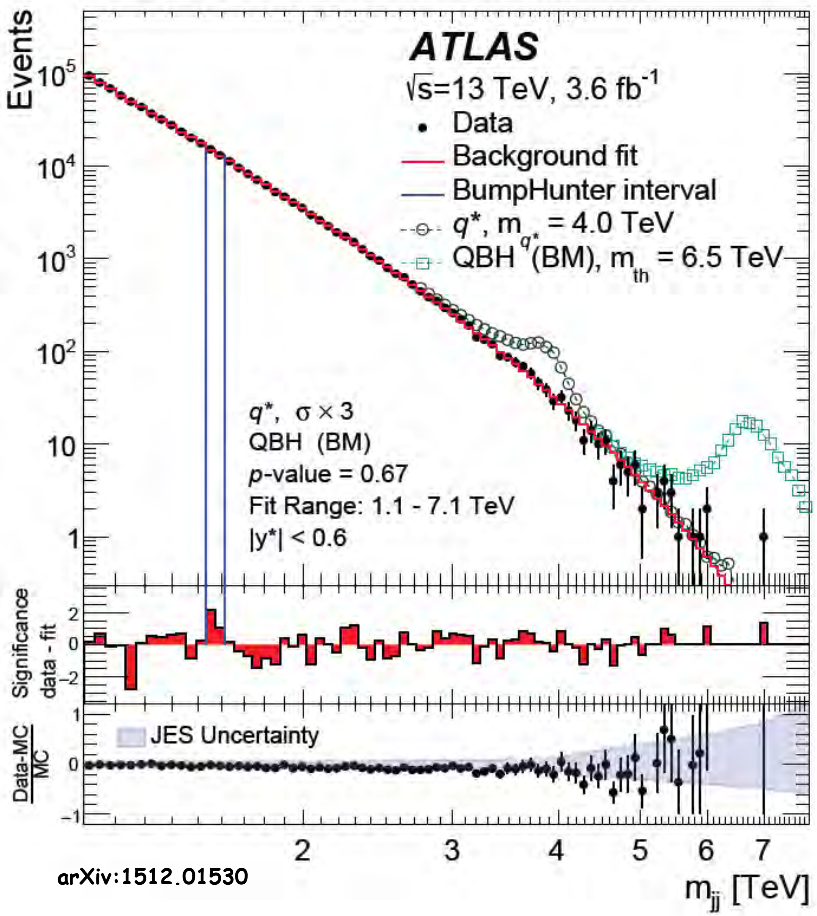
The Standard Model

- predictive power: c, b, t, ... ok
- theory + experiment agree to 10^{-3}
  with 3rd order radiative corr.: test of theory
- consistency of all parameters
- Higgs discovered !
- no new building blocks :
  - quarks (>3 families)
  - leptons "
  - bosons: W´ , Z´
- no new structure level :
  - composite leptons: e*, ...
  - quarks: q*
  - bosons: W*, Z*
  - lepto-quarks
- no new couplings:

lepton-quark universality

- no proton decay: baryon nr ok
- neutrino oscillations: lepton nr violated NEW PHYSICS !!!
- no magnetic monopole

excited quarks



arXiv:1512.01530

$g\ q \to q^* \to$

$g\ q \to$ jet jet

or $g\ q \to g$ jet

cannot excite pointlike object -

new substructure of matter ?!

excited composite quark q*: m > 5.2 TeV

quantum black holes: m > 7.8 TeV

contact interactions: Λ > 11-15 TeV

ATLAS: arXiv:1512.01530. Phys. Rev. D 91, 052007 (20
CMS: arXiv:1501.0419. Phys. Rev. D 91, 052009 (20

ATLAS 13 TeV:

CMS: arXiv:1406.5171
ATLAS: arXiv:1407.2410. Eur. Phys. J. C (2014) 74:3

radii quark / proton < proton / atom: $r_q < 10^{-19}$ m ~ $10^{-4}\ r_p$

The Standard Model

DESCRIBES THE PROPERTIES OF

ELEMENTARY PARTICLES

AND THEIR

- WEAK
- ELEKTRO-MAGNETIC
- STRONG

INTERACTIONS

PRECISELY + COMPLETELY.

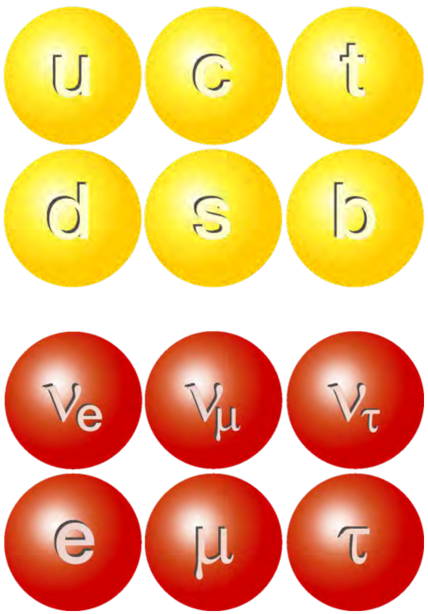
HOWEVER ,

MANY QUESTIONS REMAIN OPEN ...

Questions

PARTICLES:

- NR OF FAMILIES = 3 ?   WHY ?
- LEPTON-QUARK SYMMETRY ?
- SUBSTRUCTURE of Quarks + Leptons ?
- MASS spectrum: Higgs for Quarks + Leptons (+ Neutrinos) ?
- NEUTRINO: Dirac or Majorana ?
- MIXING ANGLES of Quarks + Neutrinos ?
- Dark Matter = SUSY ?



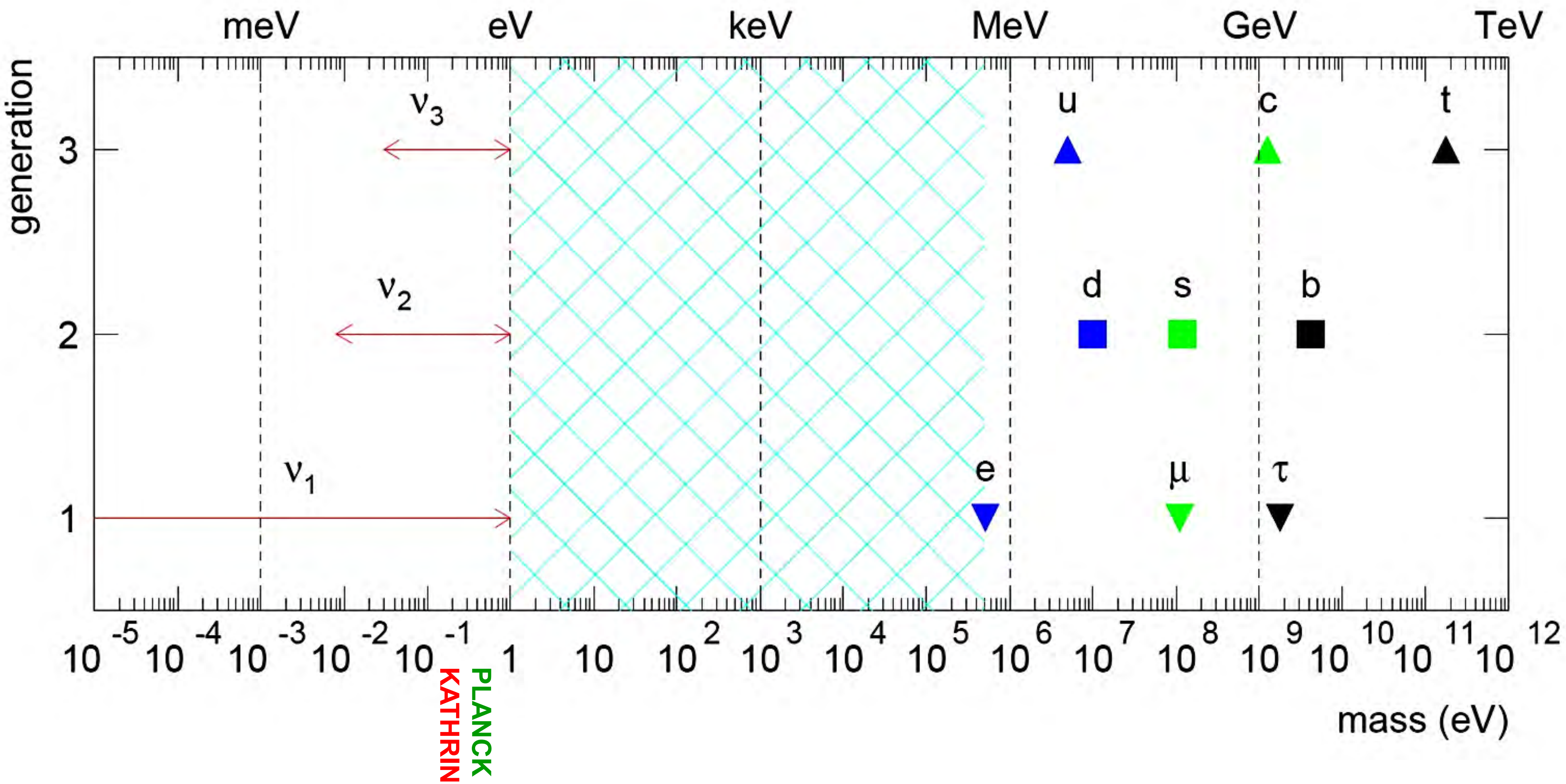
FORCES:

- STRUCTURE: $U(1)_{elm} \otimes SU(2)_{weak} \otimes SU(3)_{strong}$?
- COUPLINGS: Values ?
- GRAND UNIFICATION: Scale + Scheme ?
- GRAVITATION AND SUPER-STRINGS ?
- EXTRA DIMENSIONS ?

SYMMETRIES:

- P-VIOLATION ?
- CP-VIOLATION ?
- BARYON-NR ?   Baryon Asymmetry of Universe ?
- LEPTON-NR ?   Neutrino Oscillations !
- MAGNETIC MONOPOLES ?
- SYMMETRY BREAKING: HOW ?
- SUPER-SYMMETRY ?

Mass spectrum



$m_\nu / m_e \sim m_e / m_t \sim 10^{-6}$

What tells us Nature with this **mass spectrum ?**

Natural Units

M.Planck, 1899:

besides speed of light c and
Newton's constant G
find a third quantity b=h that allows

„Einheiten für Länge, Masse, Zeit und Temperatur aufzustellen, welche ... ihre Bedeutung für alle Zeiten und für alle, auch außerirdischen und außermenschlichen Culturen nothwendig behalten.“

one year before Planck's law !

Planck

mass = $(\hbar c/G_N)^{1/2}$ = $1.2 \cdot 10^{19}$ GeV/c^2
time = $(\hbar G_N/c^5)^{1/2}$ = $5.4 \cdot 10^{-44}$ s
length = $(\hbar G_N/c^3)^{1/2}$ = $1.6 \cdot 10^{-35}$ m

M. Planck, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin 1899 - Erster Halbband, S. 479 f.

Die Mittel zur Festsetzung der vier Einheiten für Länge, Masse, Zeit und Temperatur werden gegeben durch die beiden erwähnten Constanten a und b, ferner durch die Grösse der Lichtfortpflanzungsgeschwindigkeit c im Vacuum und durch die der Gravitationsconstante f. Bezogen auf Centimeter, Gramm, Secunde und Celsiusgrad sind die Zahlenwerthe dieser vier Constanten die folgenden:

$$a = 0.4818 \cdot 10^{-10} [\text{sec} \times \text{Celsiusgrad}]$$

$$b = 6.885 \cdot 10^{-27} \left[\frac{\text{cm}^2 \text{gr}}{\text{sec}}\right] \quad = \hbar$$

$$c = 3.00 \cdot 10^{10} \left[\frac{\text{cm}}{\text{sec}}\right]$$

$$f = 6.685 \cdot 10^{-8} \left[\frac{\text{cm}^3}{\text{gr.}\,\text{sec}^2}\right]^1.$$

Wählt man nun die »natürlichen Einheiten« so, dass in dem neuen Maasssystem jede der vorstehenden vier Constanten den Werth 1 annimmt, so erhält man als Einheit der Länge die Grösse:

$$\sqrt{\frac{bf}{c^3}} = 4.13 \cdot 10^{-33}\,\text{cm},$$

als Einheit der Masse:

$$\sqrt{\frac{bc}{f}} = 5.56 \cdot 10^{-5}\,\text{gr},$$

als Einheit der Zeit:

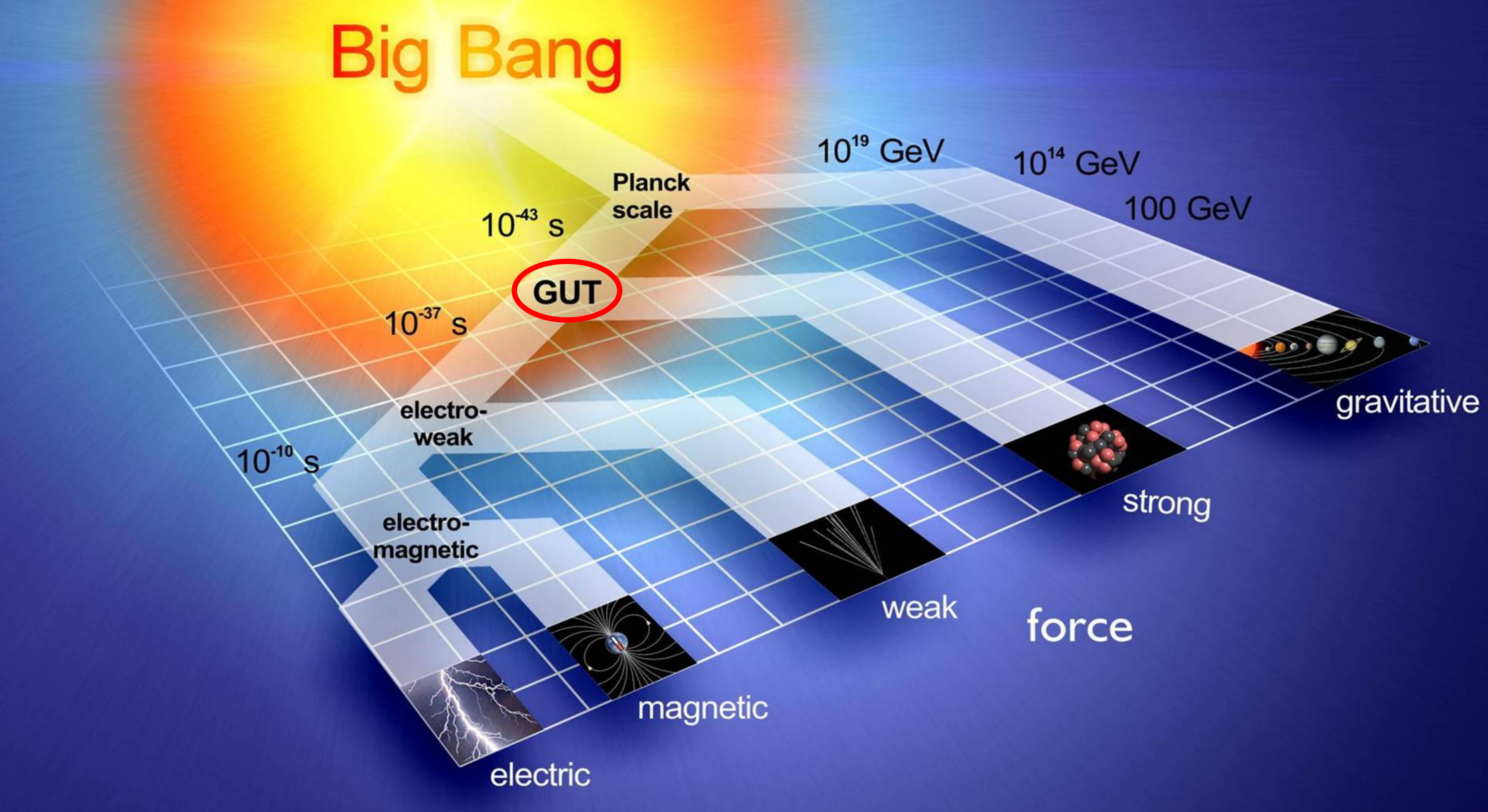
$$\sqrt{\frac{bf}{c^5}} = 1.38 \cdot 10^{-43}\,\text{sec},$$

als Einheit der Temperatur:

$$a\sqrt{\frac{c^5}{bf}} = 3.50 \cdot 10^{32}\,{}^\circ\,\text{Cels.}$$

Diese Grössen behalten ihre natürliche Bedeutung so lange bei, als die Gesetze der Gravitation, der Lichtfortpflanzung im Vacuum und die beiden Hauptsätze der Wärmetheorie in Gültigkeit bleiben, sie müssen also, von den verschiedensten Intelligenzen nach den verschiedensten Methoden gemessen, sich immer wieder als die nämlichen ergeben.

Unification of Forces



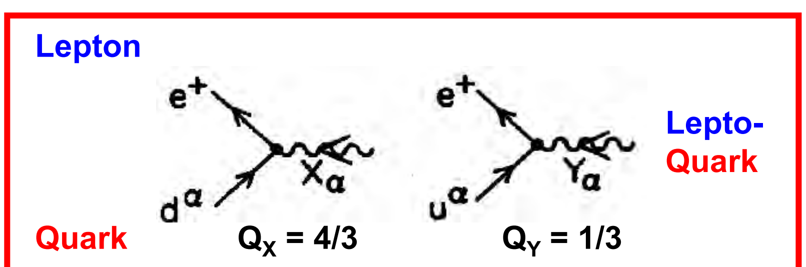
Grand Unification

elm.	weak	strong			
$U(1) \otimes$	$SU(2) \otimes$	$SU(3)$	$\subset$	$SU(5)$	SYMMETRIES

$$\{ e^- \ \nu_e \ , \ \bar{d}_r \ \bar{d}_g \ \bar{d}_b \}_L \qquad \{\bar{5}\} \qquad \text{MULTIPLETS}$$

$$\gamma \qquad W^{\pm 0} \qquad g_{ij} \qquad \text{BOSONS}$$

$$X, Y$$



$$SU(2) \updownarrow \begin{pmatrix} \nu_L^0 & \overset{X,Y}{\longleftrightarrow} & d_L^{0c} \\ e_L^{-0} & & \end{pmatrix} \qquad \begin{pmatrix} e_L^{+0} & \overset{X,Y}{\longleftrightarrow} & u_L^0 \ d_L^0 & \longleftrightarrow & u_L^{0c} \end{pmatrix} \updownarrow SU(2)$$

$$5^* \qquad\qquad 10$$

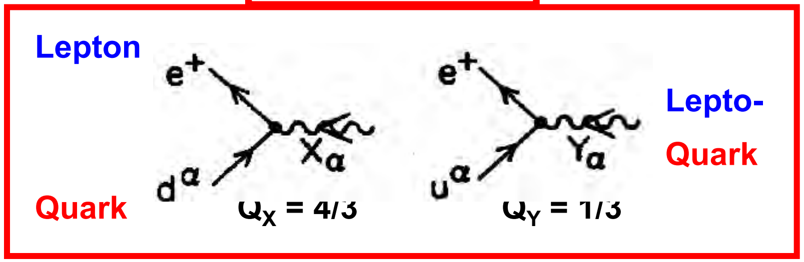
Grand Unification

SU(N)	N^2-1	gauge bosons
SU(5)	25-1 =	24
SU(3)	9-1 =	-8 gluons
SU(2)	4-1 =	-3 Z,W
U(1)	1 =	-1 photon
	6 (X+Y) =	12 X,Y bosons

Grand Unification

HERA

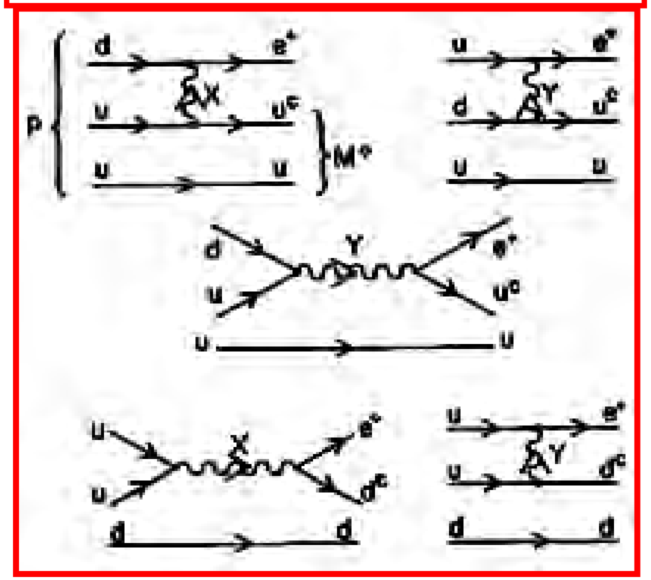
$e\ p \to LQ$



$m_{LQ} > 300$ GeV

Super-Kamiokande

proton decay: $p \to e^+ \pi^0$



$\tau_p > 10^{34}$ a

Charge quantization + $\sin^2\theta_W$

- SU(N): *generators traceless!*
- U(1)xSU(2):

weak isospin conserved: $Tr_2\,(I_3^w) = 0$

$Q = I_3^w + Y^w/2$ not conserved (only mixing, no unification)

- SU(5):

$Tr_5\,(Q) = N_C\,Q_d - Q_e = 0$ charge quantization =>

$Q_d = e/N_C$ or $Q_p = -Q_e$

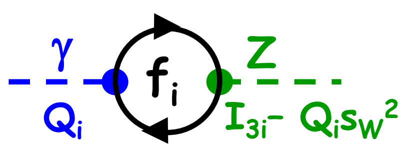
relate fractional quark charges and nr. of colors !

	I_{3W}	Q
ν_e	+1/2	0
e^-	-1/2	-1
$\bar{d}_r$	0	1/3
$\bar{d}_g$	0	1/3
$\bar{d}_b$	0	1/3

$\bar{d}_L$ ... $\{1\}_{SU(2)}$

- prediction of electroweak mixing angle:

γ,Z orthogonal => no coupling, but:



$\Sigma_i\, Q_i\,(I_{3i} - Q_i s_W^2) = 0$ true unification

$s_W^2 = \Sigma_i\, Q_i\, I_{3i} \,/\, \Sigma_i\, Q_i^2$ = "SU(2)/U(1)" = 3/8 @ m_{GUT}

$= g'^2 / (g^2 + g'^2) = 3/5 \,/\, (1+3/5) = 3/8$

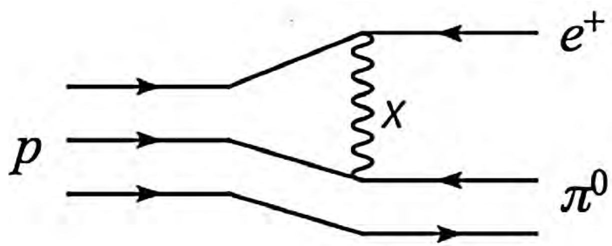
Grand Unification

- quark-lepton symmetry:
  quarks + leptons in one multiplet
- quantization of electric charge:

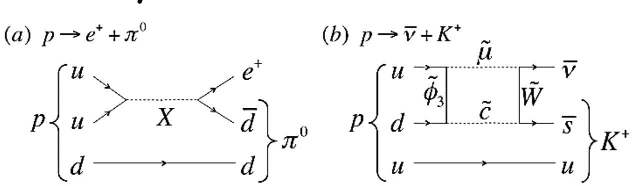
  $N_c\, Q_q - Q_e = 0 \;=\; 3 \times 1/3 - 1$
  or $\quad Q_p = Q_e$

$$\{e^-\ \nu_e\ ,\ \bar{d}_r\ \ \bar{d}_g\ \ \bar{d}_b\ \}_L$$

- prediction for electro-weak mixing angle:
  - $\sin^2\theta_W\,(M_X) \;=\; g'^2 / (g^2 + g'^2) \;=\; 3/5\ /\ (1+3/5) \;=\; 3/8$
  - $\sin^2\theta_W\,(M_Z) \;=\; 0.20$ GUT $\quad (M_X \to M_Z)$
  - $\sin^2\theta_W\,(M_Z) \;=\; 0.22$ expt.
- lepton number violation:
  neutrino masses + oscillations
- baryon number violation: birth + death of Universe:
  baryon asymmetry: $N_p / N_\gamma = 6 \cdot 10^{-10}$
  proton decay: $\tau_p \sim M_X^4 / g^4\, m_p^5$



- SU(5) GUT: $M_X \sim 10^{15}$ GeV $\quad \tau_p \sim 10^{29\pm2}$ a
- SUSY GUT: $M_X \sim 10^{16}$ GeV
  $\tau\,(p \to e^+\pi^0) \sim n \cdot 10^{35}$ a $\quad \tau\,(p \to K^+\nu) \sim n \cdot 10^{34}$ a
- SuperK 1996-2015: $\tau\,(p \to e^+\pi^0) > 1.6\ 10^{34}$ a $\quad \tau\,(p \to K^+\nu) > 6 \cdot 10^{33}$ a $\quad$ 306 kt yrs $\quad$ arXiv:1610.03597. PRD 95
- HyperK >2026: $\tau\,(p \to e^+\pi^0) > 2\ 10^{35}$ a $\quad \tau\,(p \to K^+\nu) > 3 \cdot 10^{34}$ a
- JUNO, DUNE >2026: $\tau\,(p \to K^+\nu) > 3 \cdot 10^{34}$ a



- GUT magnetic monopoles