RELATIVISTIC KINEMATICS

- four-vector: $r_i = (t\ ,\ \vec{r})$ **c = 1**

  Minkowski metrics: $r_i^2 = t^2 - \vec{r}^{\,2}$

- Lorentz transformation: *x'* moves with velocity *v* w.r.t. *x*:

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \gamma \begin{pmatrix} 1 & -v \\ -v & 1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \qquad \begin{pmatrix} x \\ t \end{pmatrix} = \gamma \begin{pmatrix} 1 & v \\ v & 1 \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix} \qquad \gamma = 1/\sqrt{1-v^2} > 1$$

- transformation $t\,(x'=0)$, $x'(t=0)$:

$t = \gamma t'$	time dilatation:	t' = time in moving system
$x' = \gamma x$	length contraction:	x = length in rest system

- $r_i'\, r_i' = r_i\, r_i$ square of 4-vectors Lorentz invariant

  $r_i'\, s_i' = r_i\, s_i$ scalar product of " "

- line element

$$ds = \sqrt{dt^2 - d\vec{r}^{\,2}} = dt\,\sqrt{1-\vec{v}^{\,2}} = dt/\gamma$$

- four-velocity:

$$v_i = dr_i/ds = \gamma\, dr_i/dt = \gamma(1,\vec{v}\,) \qquad |\cdot m \text{ ... invariant mass}$$

- four-momentum:

$$p_i = mv_i = (\gamma m, \gamma m \vec{v}\,) = (E, \vec{p}\,)$$

$$\gamma = E/m$$
$$\vec{v} = \vec{p}/E$$
$$p_i\, p^i = E^2 - \vec{p}^{\,2}$$

- **rest system:** $\vec{p} = 0 \;\Rightarrow\; \vec{v} = 0$, $\gamma = 1$

  $p_i^2 = E^2 = m^2$ four-momentum^2 = invariant mass^2

- **moving system**: $E^2 = m^2 + \vec{p}^{\,2} > 0$

  - non-relativistic: $\vec{p}^{\,2} \ll m^2$:

    $E = \sqrt{m^2 + \vec{p}^{\,2}} \approx m + \vec{p}^{\,2}/2m = m + E_{kin}$

  - ultra-relativistic: $\vec{p}^{\,2} \gg m^2$:

    $E = p$