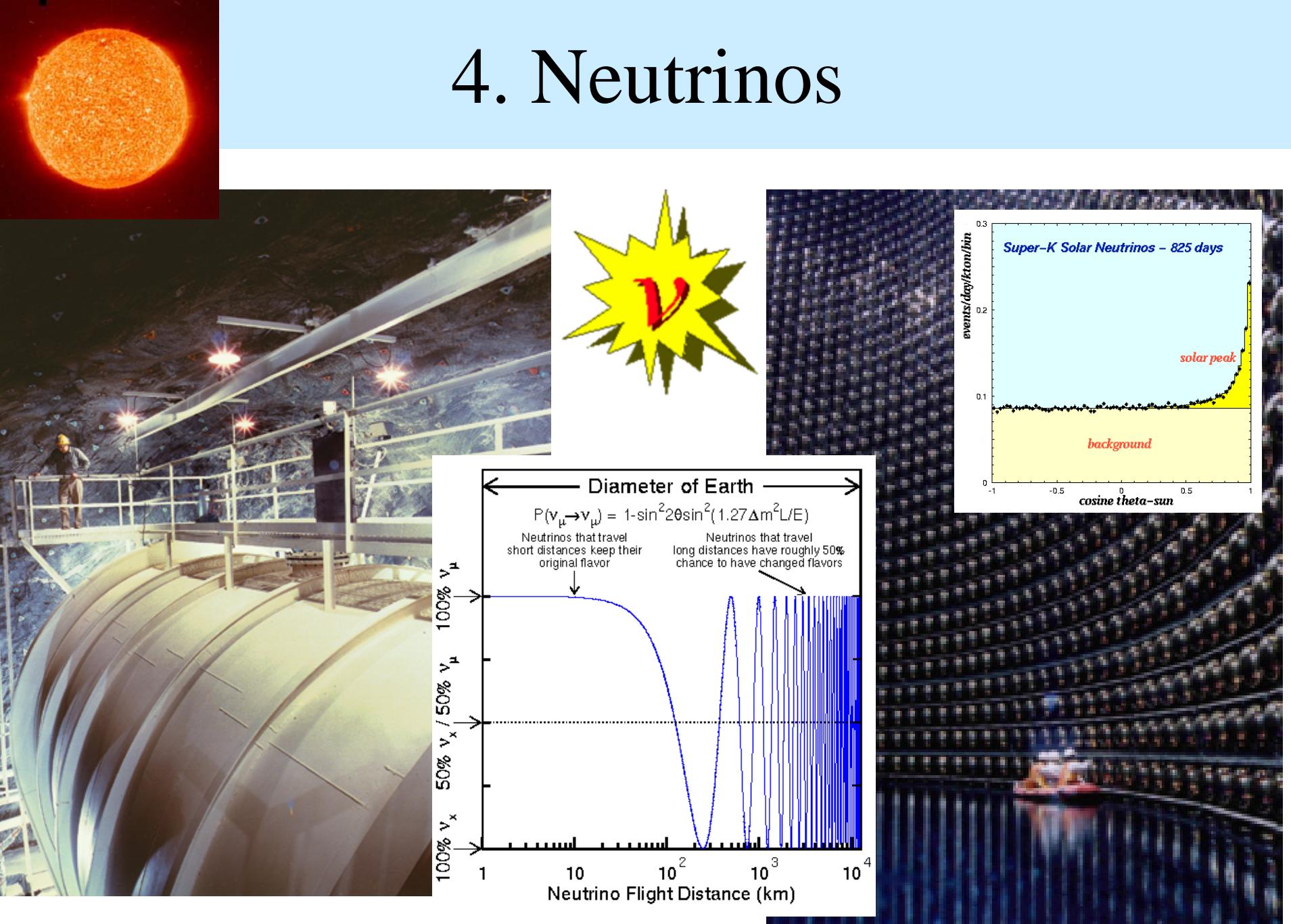
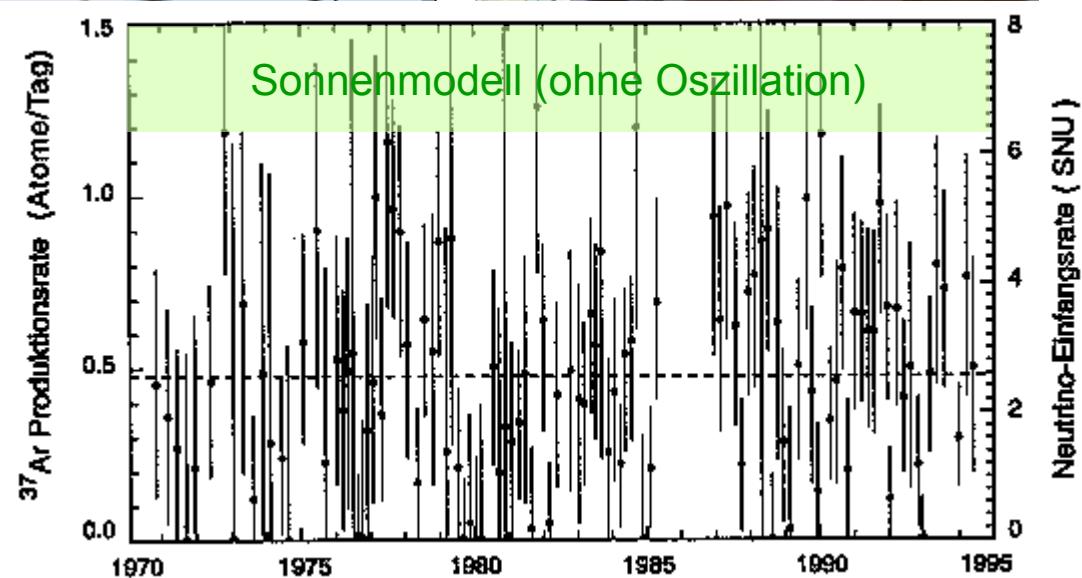
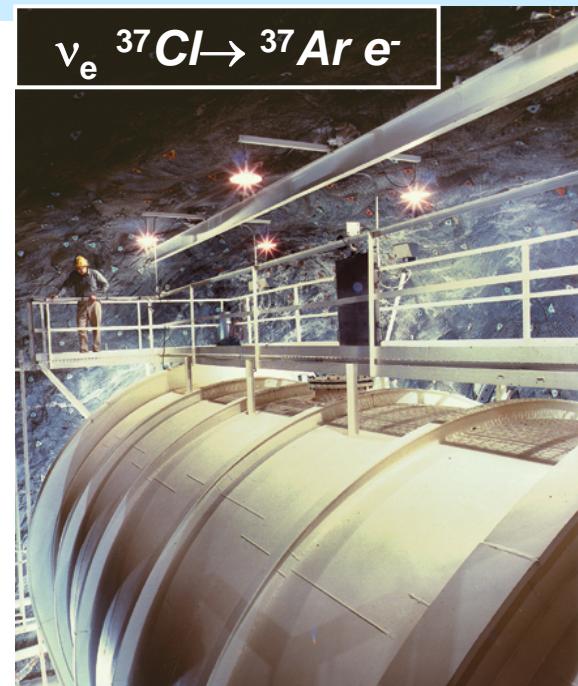
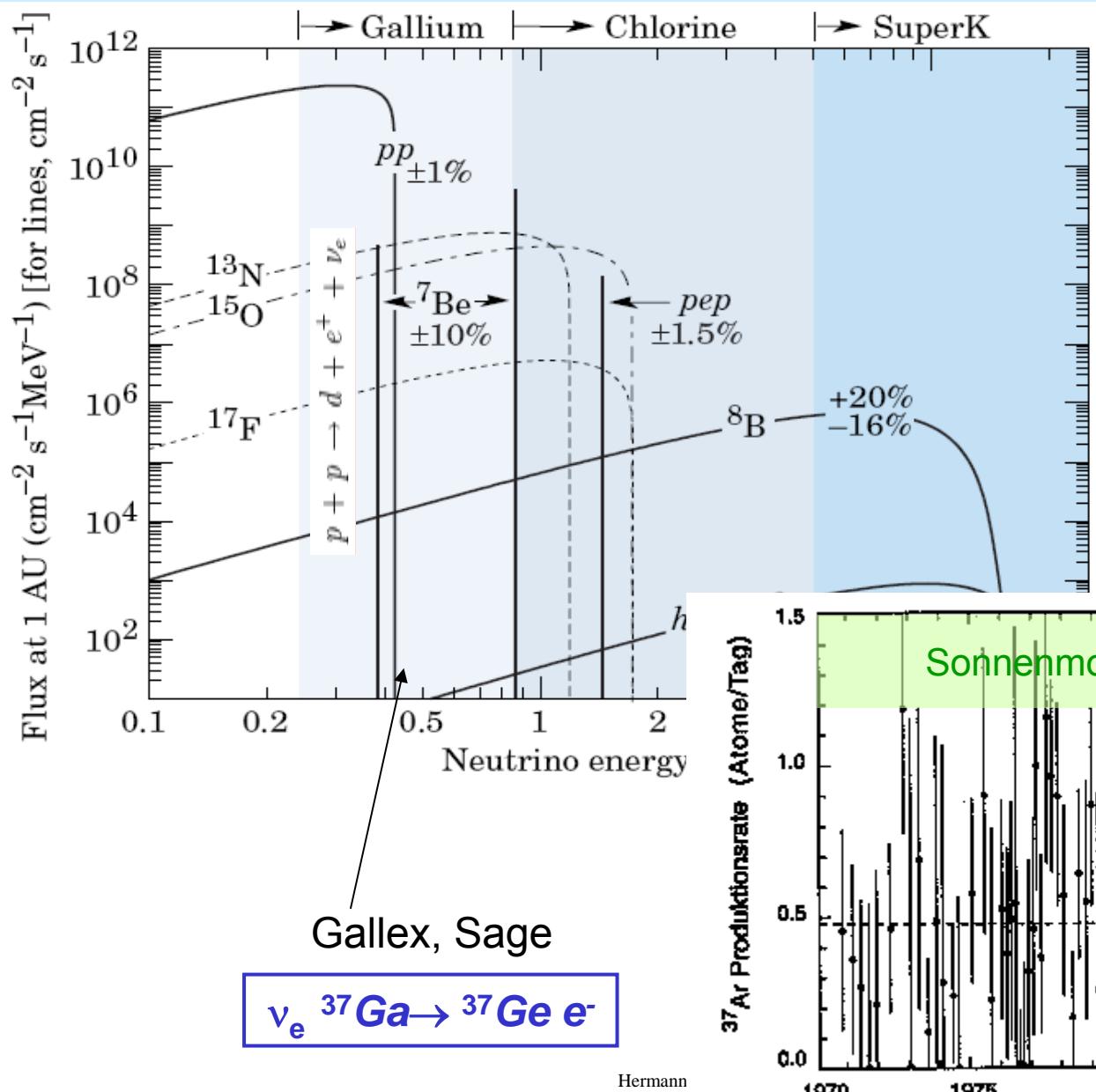


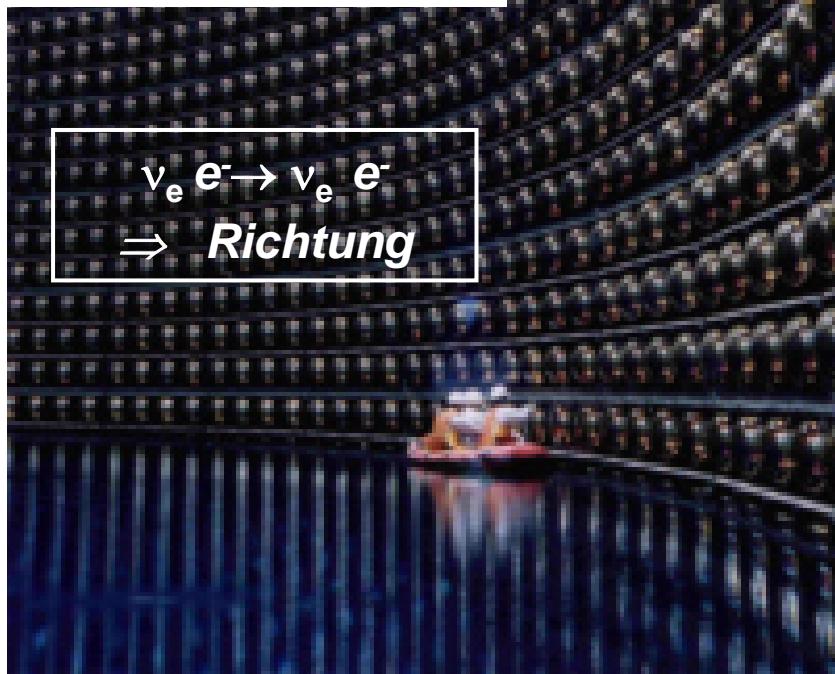
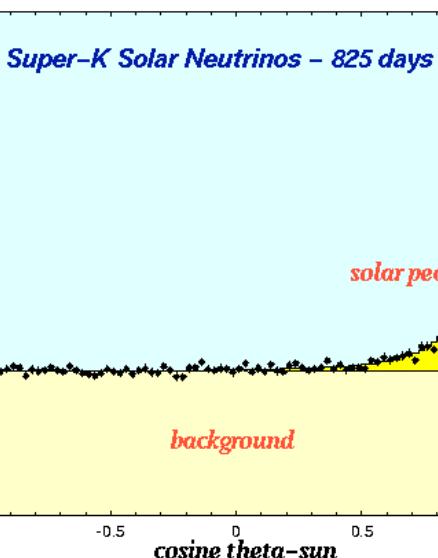
4. Neutrinos



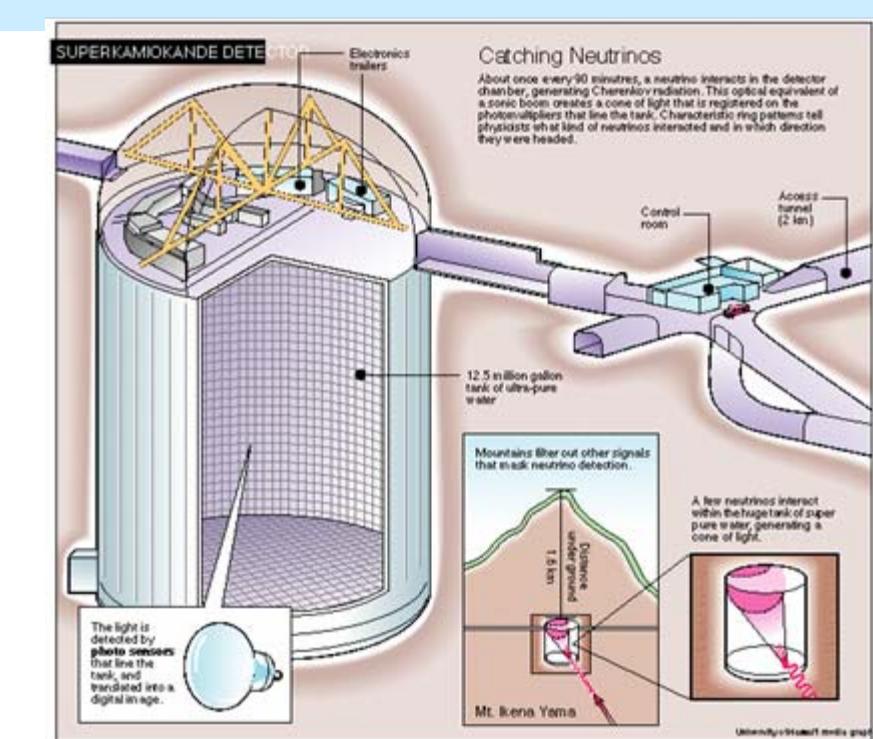
Sonnenneutrino-Defizit



Super-Kamiokande



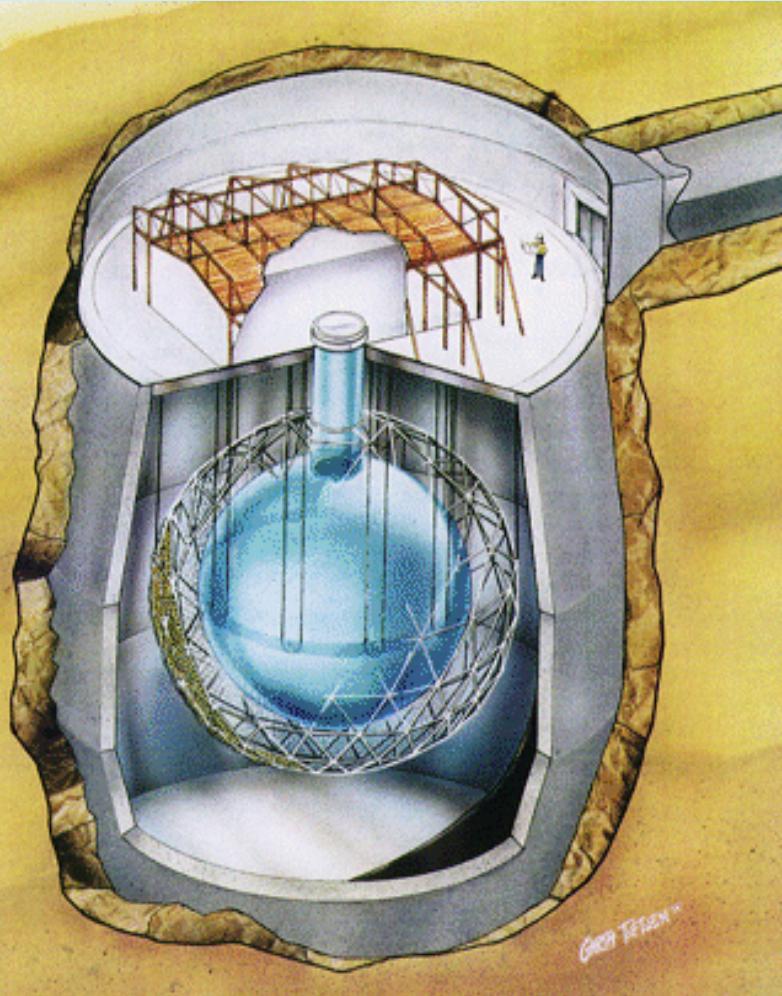
Hermann Kolanoski, EEP SS06 - 4.Neutrino-Oszillationen



ν_e -Defizit von der Sonne

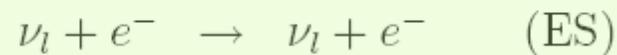
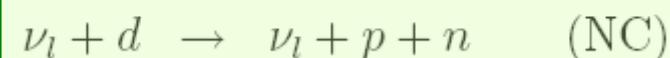
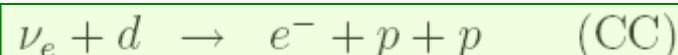
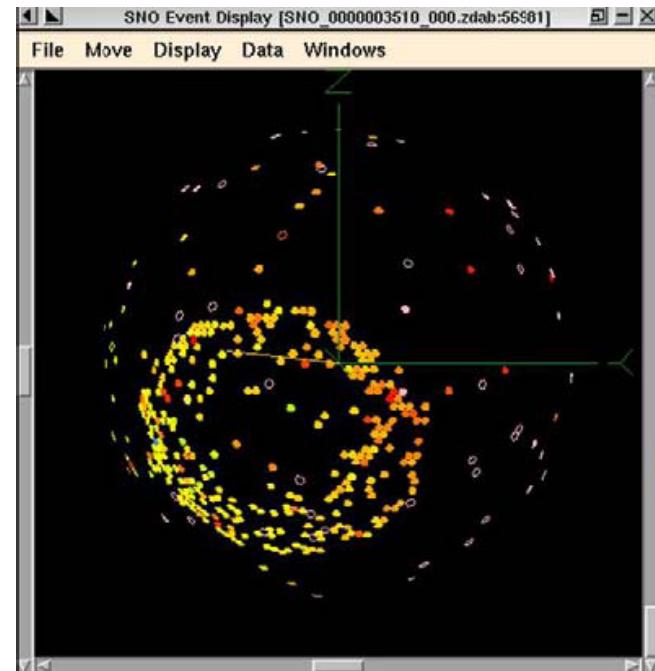
	Homestake	Kamiokande	Super-Kamiokande	Gallex	Sage
Schwelle [MeV]	0.814	7.5	7.0	0.233	0.233
Laufzeit	1970 - 1994	1987 - 1995	1996 - 1998	1991 - 1997	1990 - 1997
Vorhersage	$7.7^{+1.2}_{-1.0}$ U	$5.15^{+0.98}_{-0.72}$	$5.15^{+0.98}_{-0.72}$	129^{+8}_{-6}	129^{+8}_{-6}
Experiment	2.56 ± 0.22	2.82 ± 0.38	2.42 ± 0.08	77.5 ± 8	66.6 ± 8
S_{th}/S_{exp}	3.0	1.8	2.1	1.7	1.9

SNO: totaler ν -Fluss von der Sonne



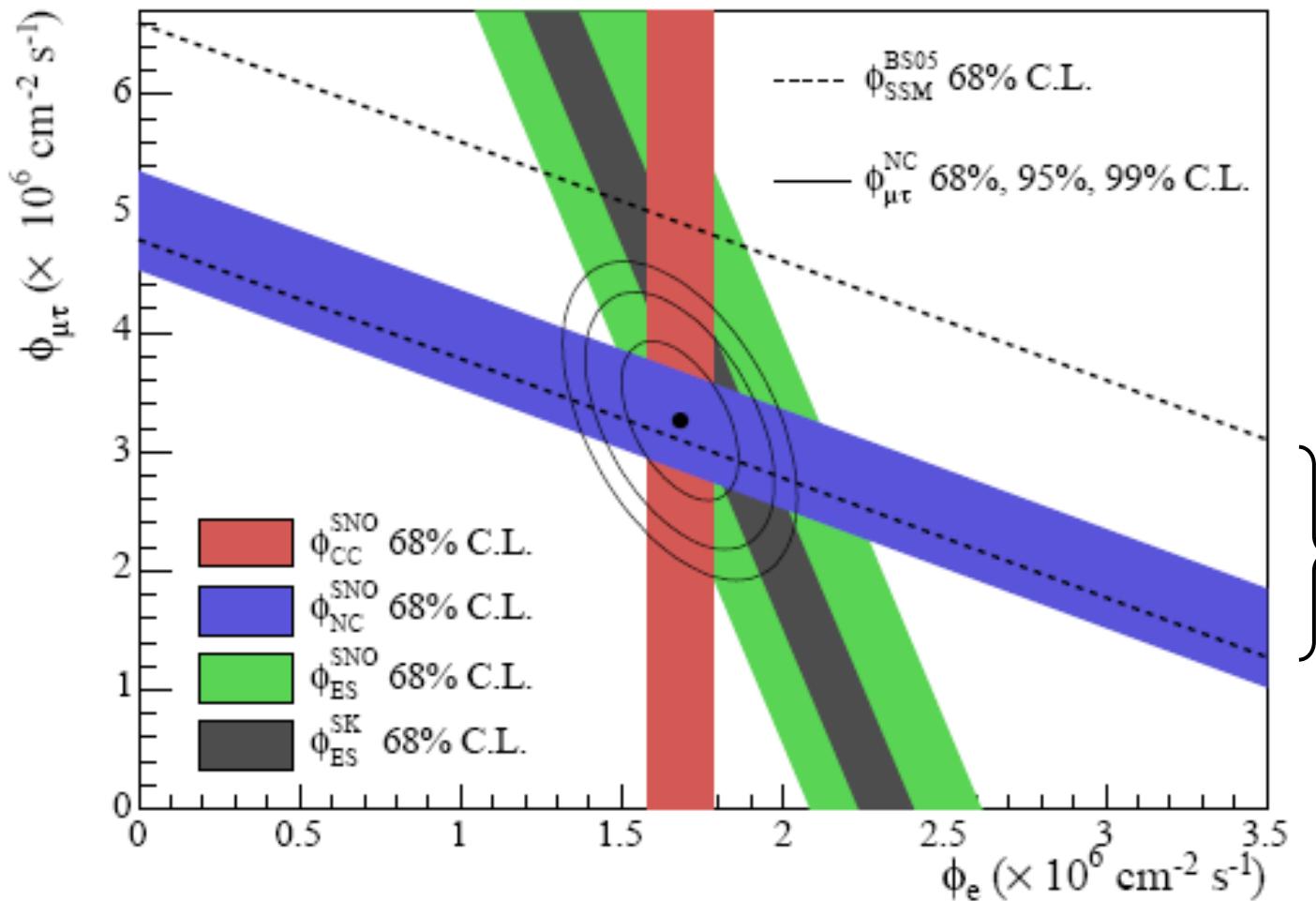
Sudbury Neutrino Observatory

Messe ${}^8\text{B}$ - ν 's in
 D_2O
CC und NC
(had.& lept.)



$$\frac{\phi(\nu_e)}{\phi(\nu_e) + \phi(\nu_{\mu,\tau})} = 0.306 \pm 0.026 \pm 0.024$$

SSM verglichen mit $\Phi_{\mu\tau} - \Phi_e$



SSM
= Standard-
sonnen-
modell

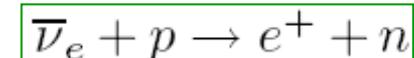
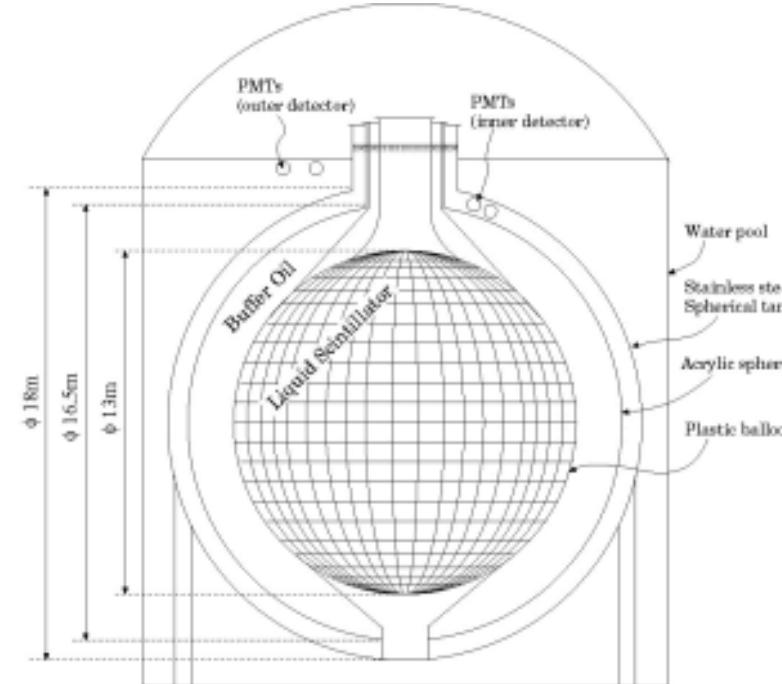
Reaktor-Antineutrinos: KamLand

Notwendige Information zur Interpretation der Sonnenneutrinos



KamLAND is surrounded by 53 Japanese power reactor units

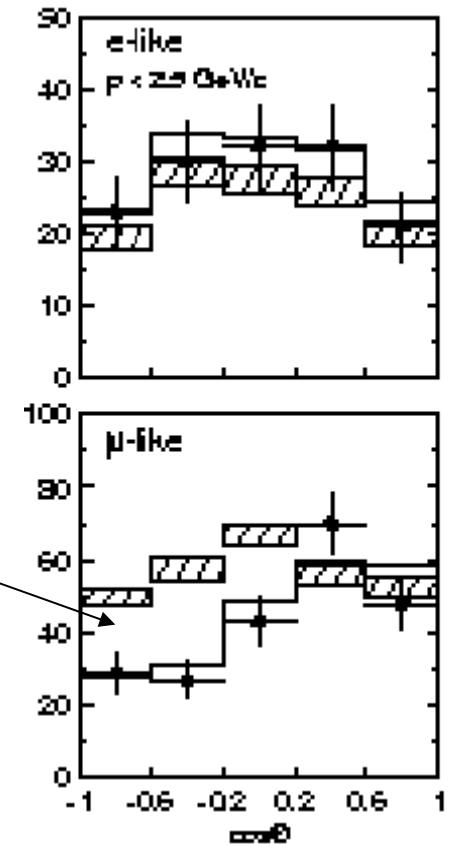
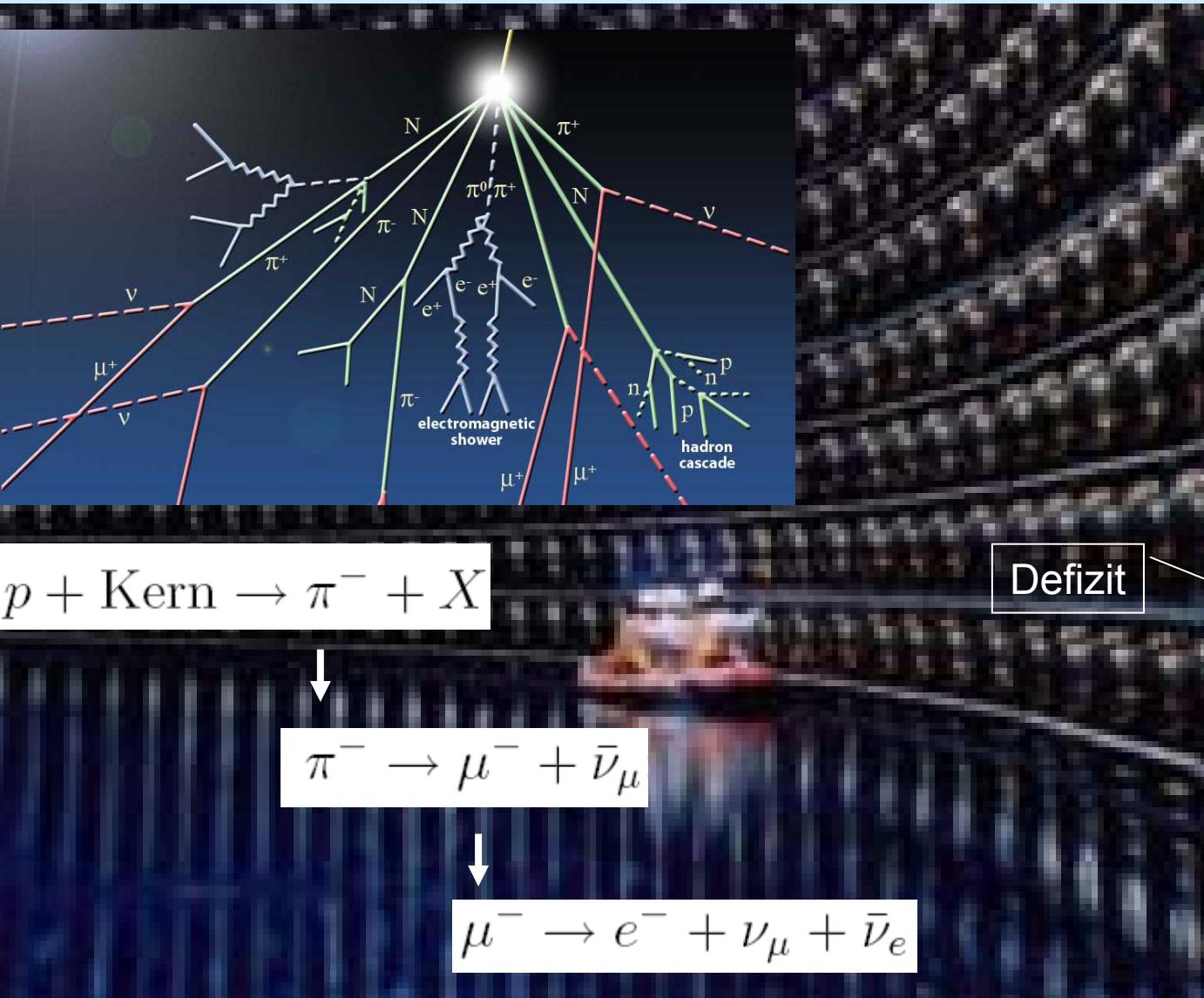
Erwartet:	365.2 ± 23.7 Ereignisse
Beobachtet:	258 Ereignisse



when *CPT* holds

$$P(\overline{\nu}_\alpha \rightarrow \overline{\nu}_\beta; U) = P(\nu_\alpha \rightarrow \nu_\beta; U^*)$$

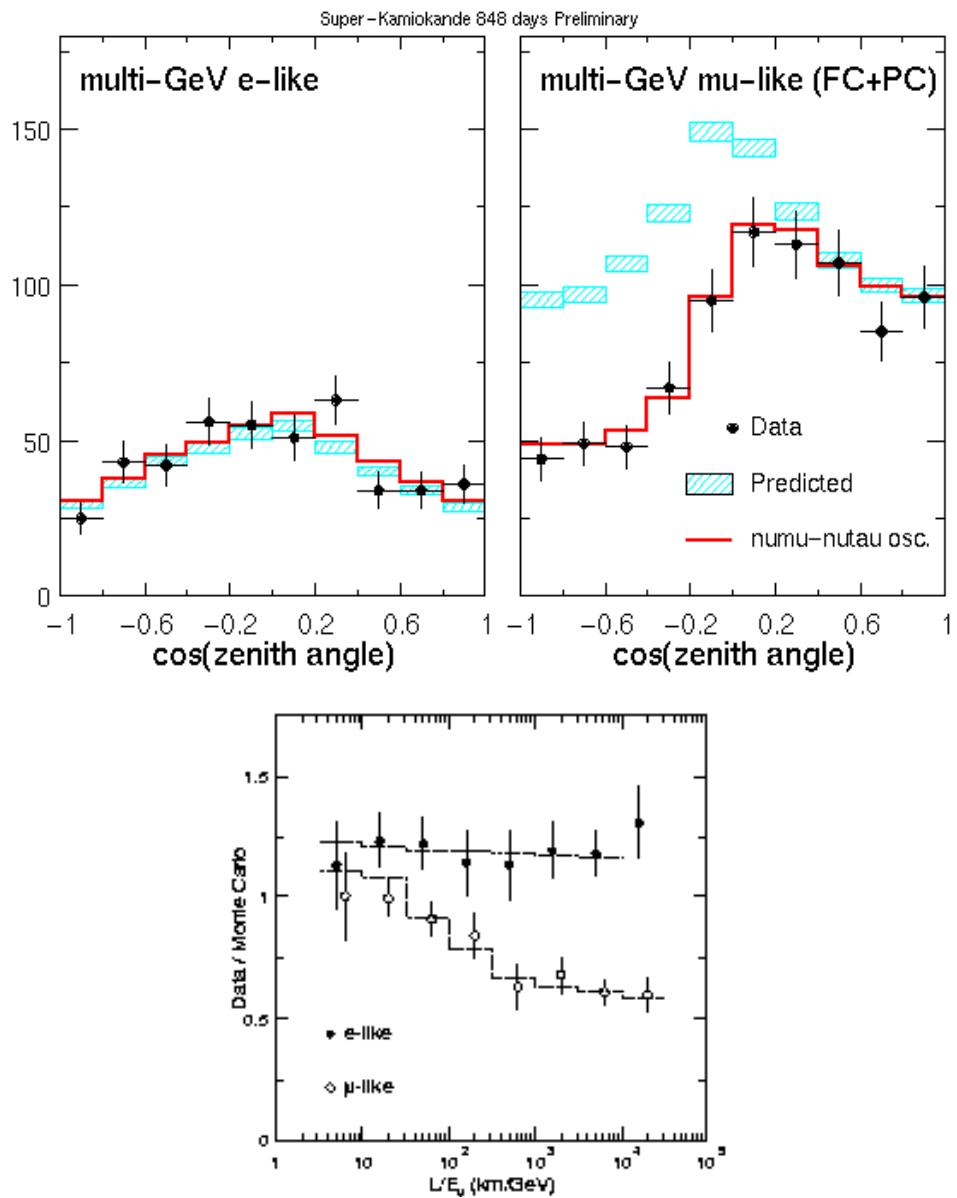
Atmosphärische Neutrinos



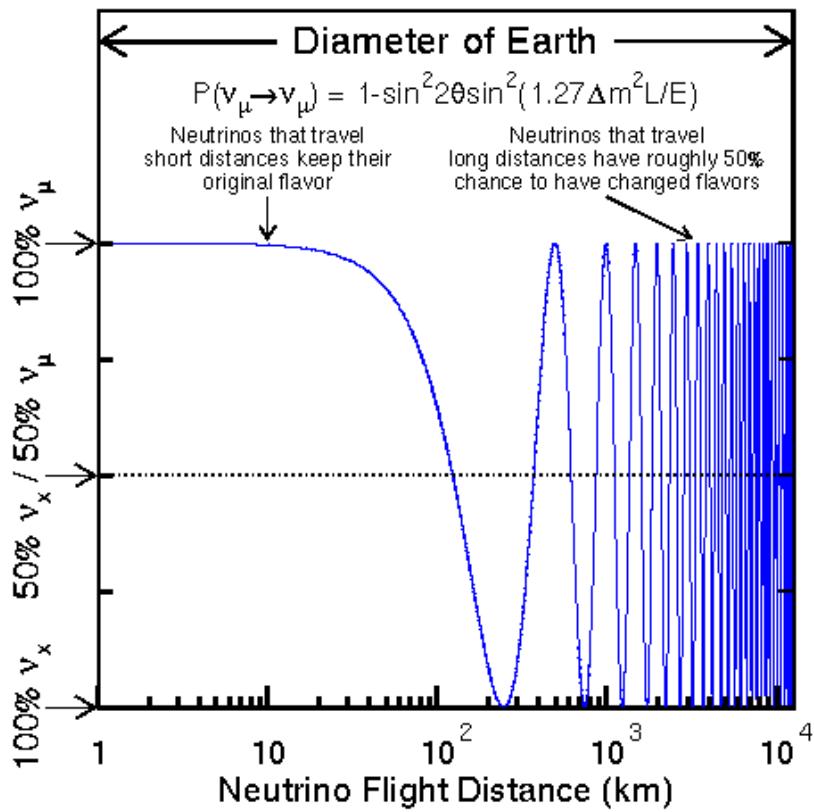
Defizit

aufwärts abwärts

Atmosphärische Neutrinos



$\nu_\mu \rightarrow \nu_x, \quad \nu_x \neq \nu_e !!$



Neutrino-Oszillationen (Formalismus)

Favour-EZ
(def. durch CC-Kopplung an I_α^\pm)

Massen-EZ

$$\nu_\alpha = \sum_{i=1}^3 U_{\alpha i}^* \nu_i$$

zeitl. Entwicklung der Massen-EZ:

$$\nu_i(x) = e^{-i(\frac{m_i^2}{2p_i})x} \nu_i(0)$$

zeitl. Entwicklung der Flavour-EZ:

$$\nu_\alpha(x), = \sum_{i=1}^3 U_{\alpha i}^* e^{-i(\frac{m_i^2}{2E})x} \nu_i(0)$$

Flavour-Oszillationen:

$$\nu_\alpha(x), = \sum_{\beta=1}^3 \sum_{i=1}^3 U_{\alpha i}^* e^{-i(\frac{m_i^2}{2E})x} U_{\beta i} \nu_\beta$$

$$P(\nu_\alpha \rightarrow \nu_\beta; x) = |\langle \nu_\beta(0) | \nu_\alpha(x) \rangle|^2$$

$$P(\nu_\alpha \rightarrow \nu_\beta; x) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \sin^2 \frac{\Delta m_{ij}^2}{4E} x) \\ + 2 \sum_{i>j} \text{Im}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \sin \frac{\Delta m_{ij}^2}{2E} x)$$

3-Generation-Neutrinomischung

PMNS Mischungsmatrix (ohne Majorana Phasen)

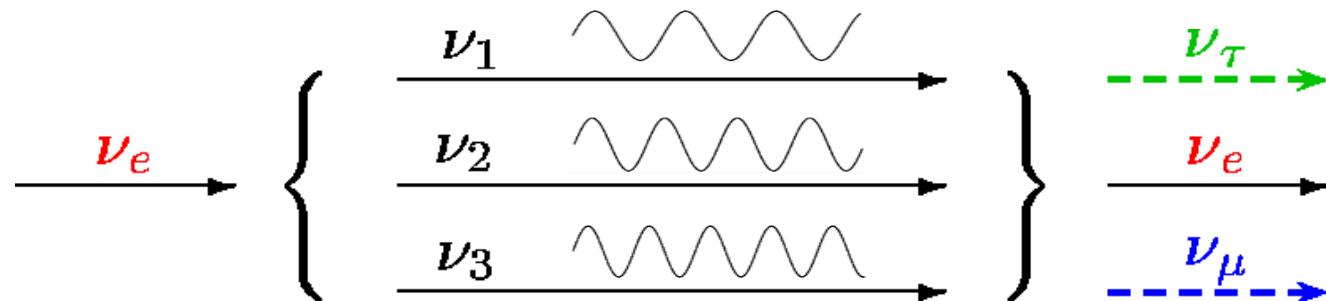
- 3 Mischungswinkel: $\theta_{12}, \theta_{23}, \theta_{13}$
- 1 CP-verletzende Dirac-Phase: δ

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

θ_{atm}

θ_{13}, δ

θ_{sol}



3-Flavour-Mischungsmatrix

wie bei CKM-Matrix: 3 Winkel + 1 Phase

$$U^* = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 1 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

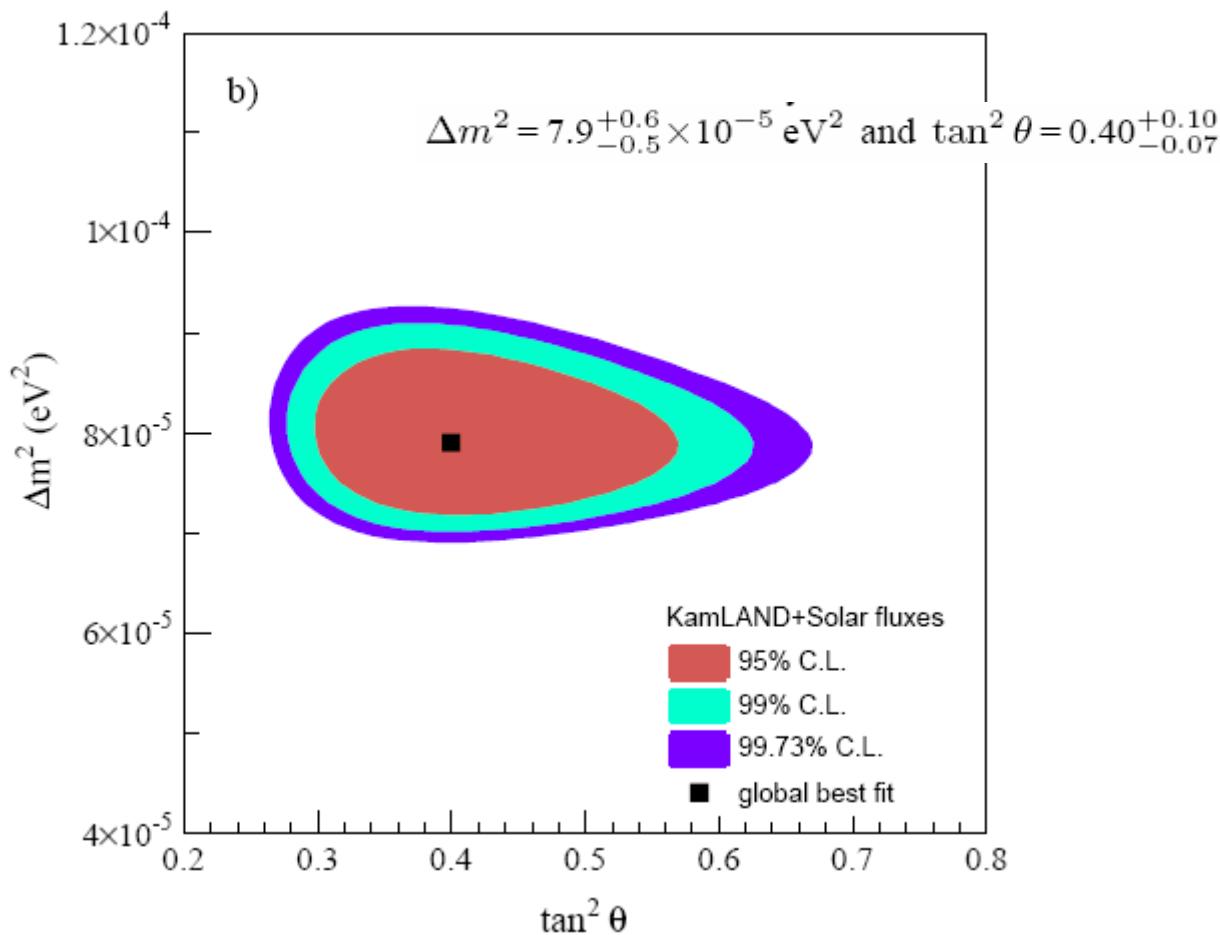
Ergebnisse $\sin^2 \theta_{12} \approx 0.31$, $\sin^2 \theta_{23} \approx 0.50$, $\sin^2 \theta_{13} \approx 0$

$$\theta_{12} \approx 33.8^\circ, \quad \theta_{23} \approx 45.0^\circ, \quad \theta_{13} \approx 0^\circ$$

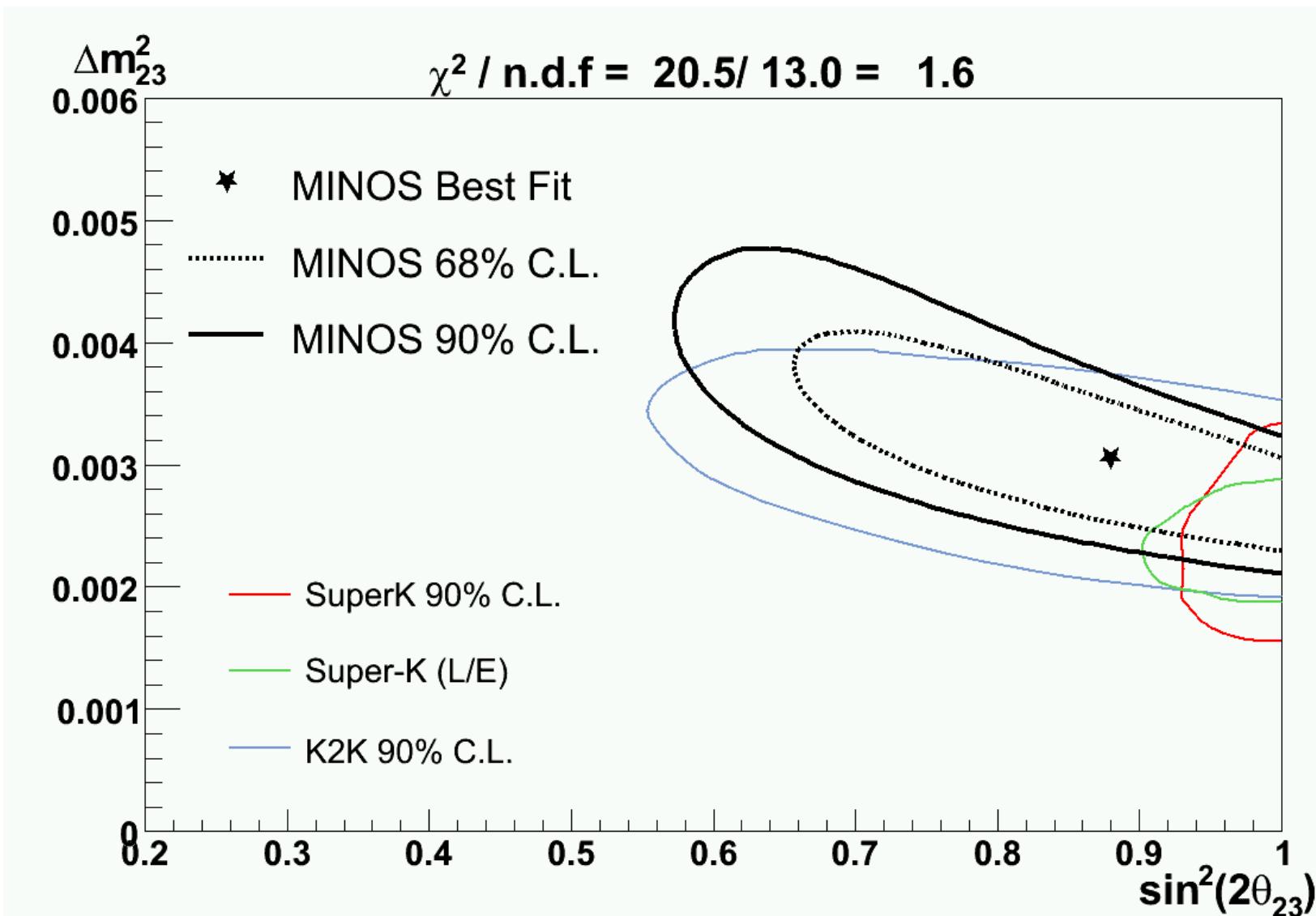
⇒ ganz andere Situation als bei Quarkmischung:
2 große, 1 kleiner Winkel (oder =0)

⇒ Wolfenstein-Parametrisierung nicht möglich

Reaktor- und Sonnenneutrinos



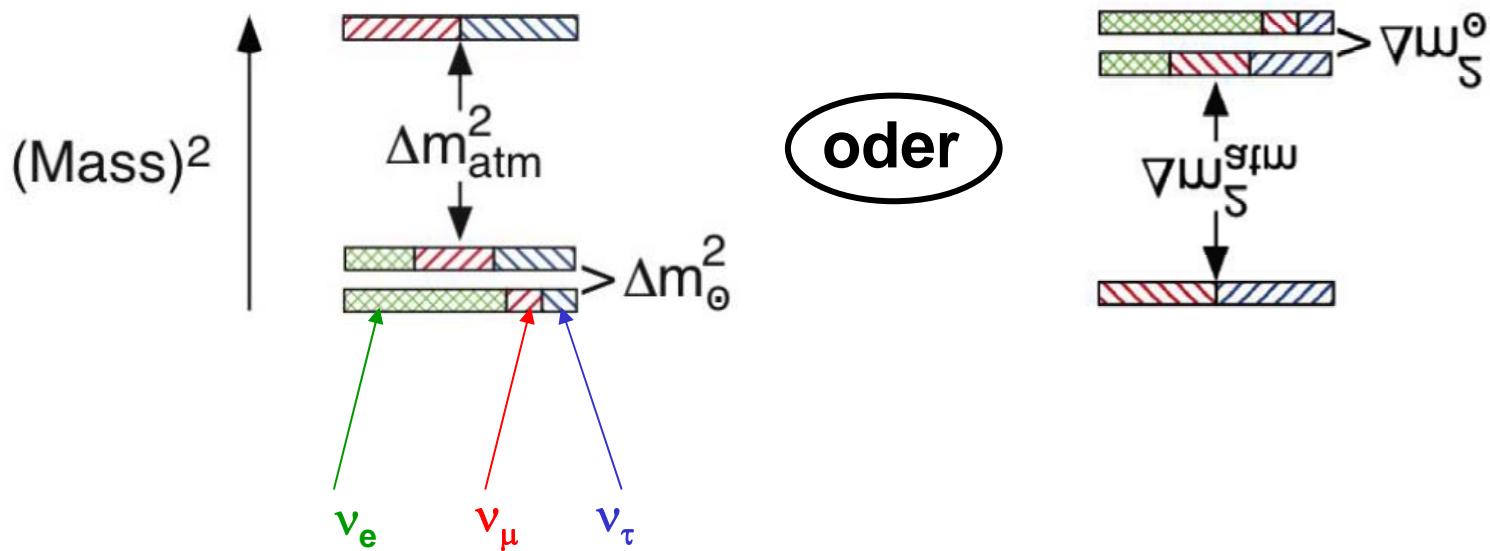
Atmosphärische und Beschleuniger- Neutrinos



Massenhierarchie

$$\Delta m_{12}^2 = m_2^2 - m_1^2 \approx 7.9 \cdot 10^{-5} \text{ eV}^2 \quad (> 0, \text{ festgelegt})$$

$$\Delta m_{(12)3}^2 = \left| m_3^2 - \frac{m_2^2 - m_1^2}{2} \right| \approx 2.4 \cdot 10^{-3} \text{ eV}^2$$



$\Delta m_{\text{atm}}^2 \gg \Delta m_\odot^2$ & $\theta_{13} \approx 0$ \Rightarrow Aufspaltung in effektiv 2-kompon. Mischung für jeweils solare und atm. Neutrinos

2-Komponenten-Mischung

Vakuum-Lösung:

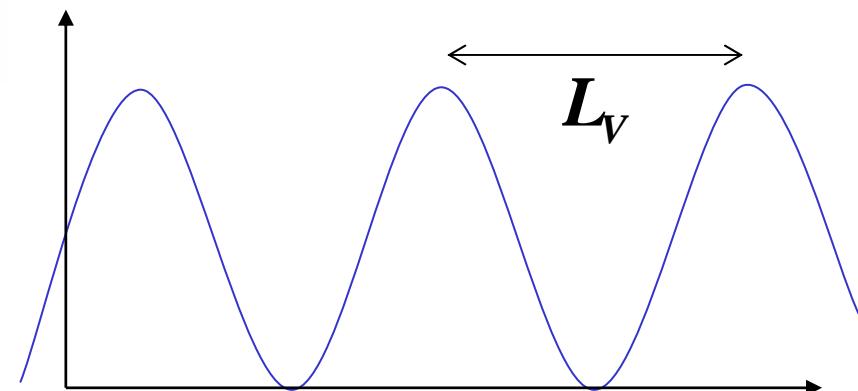
$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_V & \sin \theta_V \\ -\sin \theta_V & \cos \theta_V \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$P(\nu_e \rightarrow \nu_e; x) = |<\nu_e(x)|\nu_e(0)>|^2 = 1 - \sin^2 2\theta_V \sin^2 \frac{\Delta m^2}{4E} x$$

$\underbrace{\sin^2 \frac{\pi}{L_V} x}_{\text{sin}^2 \frac{\pi}{L_V} x}$

$$L_V = \frac{4\pi E \hbar}{\Delta m^2 c^3} = 2.48 \text{ m} \left(\frac{E}{\text{MeV}} \right) \left(\frac{\text{eV}^2}{\Delta m^2} \right)$$

$$\frac{\pi}{L_V} = 1.27 \frac{1}{\text{m}} \cdot \left(\frac{\Delta m^2}{\text{eV}^2} \right) \cdot \left(\frac{\text{MeV}}{E} \right)$$

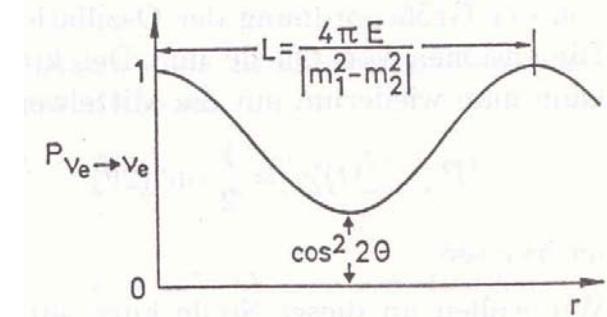


$$\Delta m_{ij}^2(L/4E) \simeq 1.27 \Delta m_{ij}^2(\text{eV}^2) \frac{L(\text{km})}{E(\text{GeV})}$$

Pendel-Neutrino-Korrespondenzen

Pendel	Neutrinos
Lineare Schwingung	Kreisbewegung des „Phasenzeigers“
Feste Moden	Massezustände
Mischung wegen Kopplung => $\Delta\omega^2 \sim k/M$	Flavor Mischungsmatrix => Δm^2
Oszillationsfrequenz der Pendel $\sim \Delta\omega^2$ der festen Moden	Oszillationsfrequenz der Neutrinos $\sim \Delta m^2$ der Massezustände
Amplitude ² jedes Pendels	Wahrscheinlichkeit, Neutrino zu finden

Anmerkung:
 nicht-maximale Neutrino Mischung
 entspräche Pendel mit $\ell_I \neq \ell_{II}$



$$L_{12} = 30 \text{ km} \times E(\text{MeV}) \quad \text{Derzeitige Werte} \quad L_{23} = 1 \text{ km} \times E(\text{MeV})$$

$\Delta m^2_{23} = 2.4 \times 10^{-3} \text{ eV}^2$	$\Delta m^2_{13} = 2.5 \times 10^{-3} \text{ eV}^2$	$\Delta m^2_{12} = 0.08 \times 10^{-3} \text{ eV}^2$
„schnelle“ Oszillation		„langsame“ Oszillation
$L_{23} = 1 \text{ km} \times E/\text{MeV}$		$L_{12} = 30 \text{ km} \times E/\text{MeV}$
$\theta_{23} = 45^\circ$	$\theta_{13} < 10^\circ$	$\theta_{12} = 32^{+3}_{-2}^\circ$

- konsistent mit tri/bi-maximaler Mischung**

$$U_{\text{PMNS}} \approx \begin{pmatrix} \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

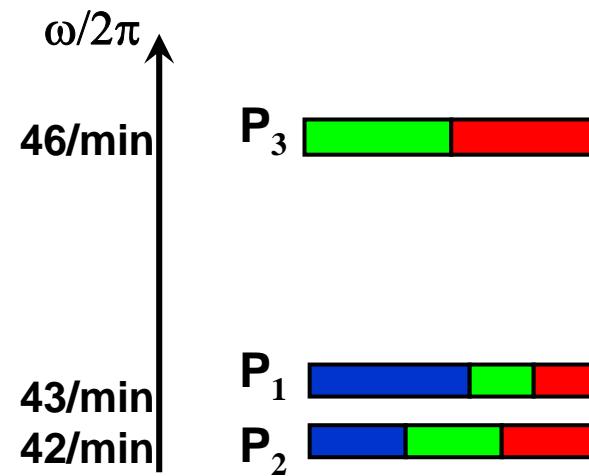
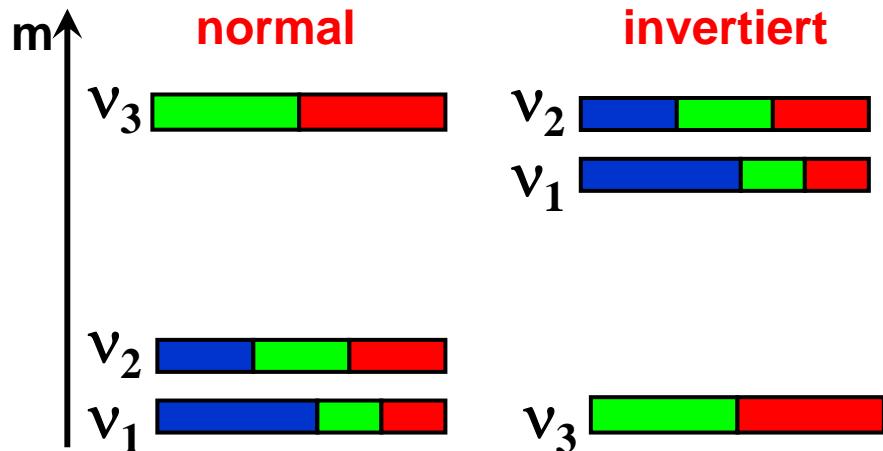
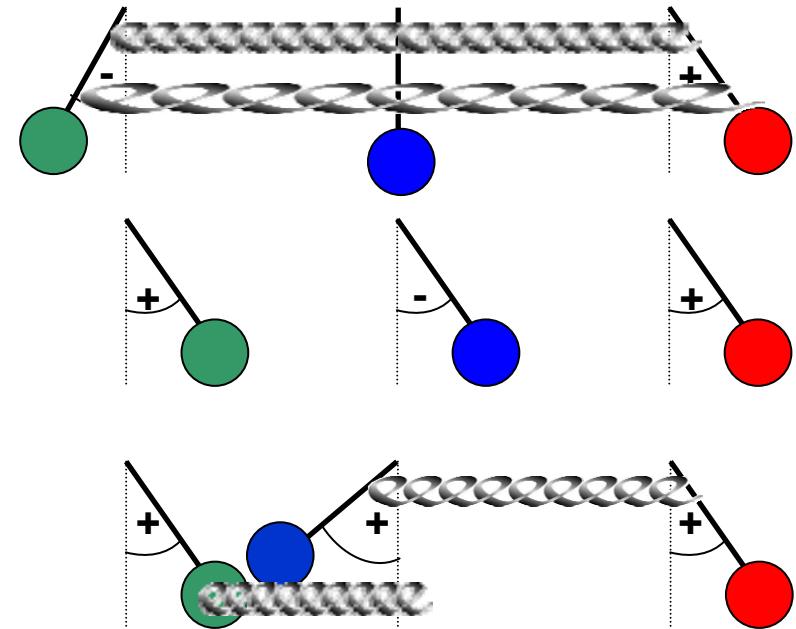
Harrison, Perkins, Scott '99,'02
 Z.Xing,'02, He, Zee, '03, Koide '03
 Chang, Kang, Kim '04, Kang '04

Realisation als gekoppelte Pendel

$$v_3 = (-v_e - v_\mu + v_\tau)/\sqrt{2}$$

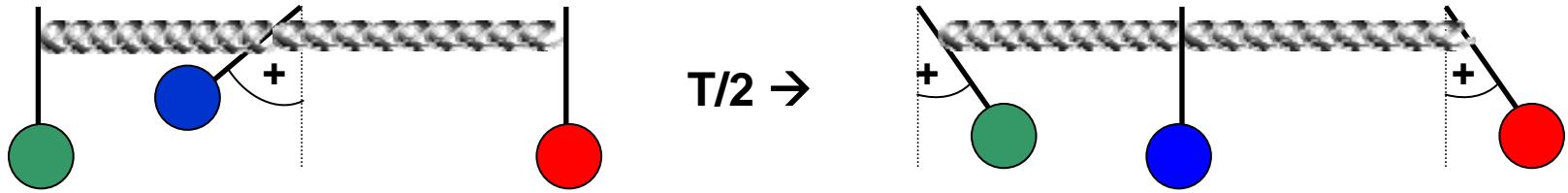
$$v_2 = (-v_e + v_\mu + v_\tau)/\sqrt{3}$$

$$v_1 = (2v_e + v_\mu + v_\tau)/\sqrt{6}$$



Solare Neutrinos

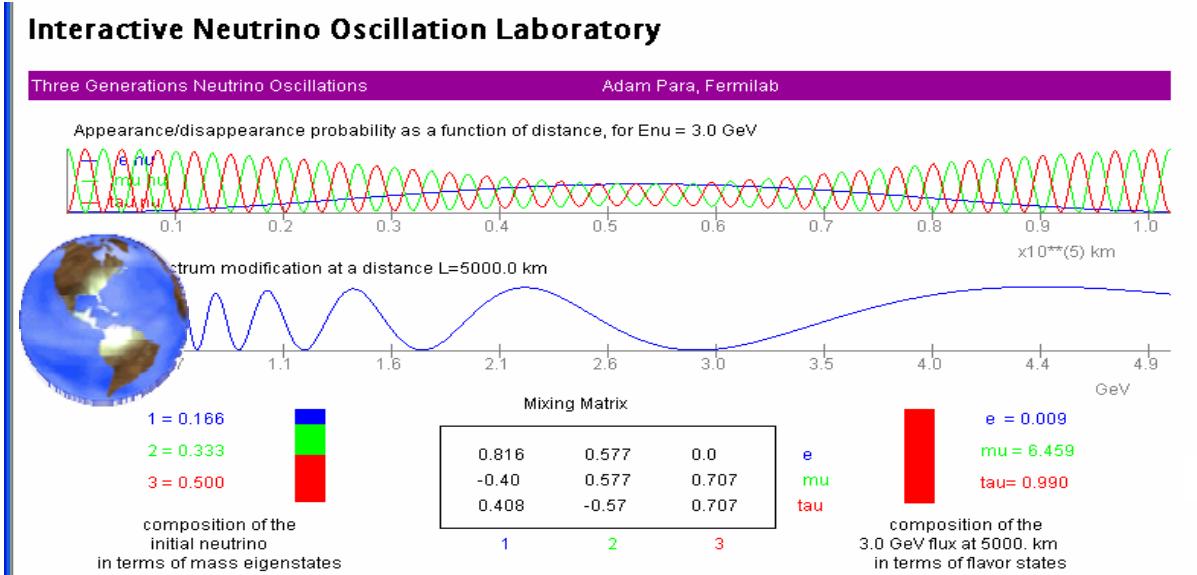
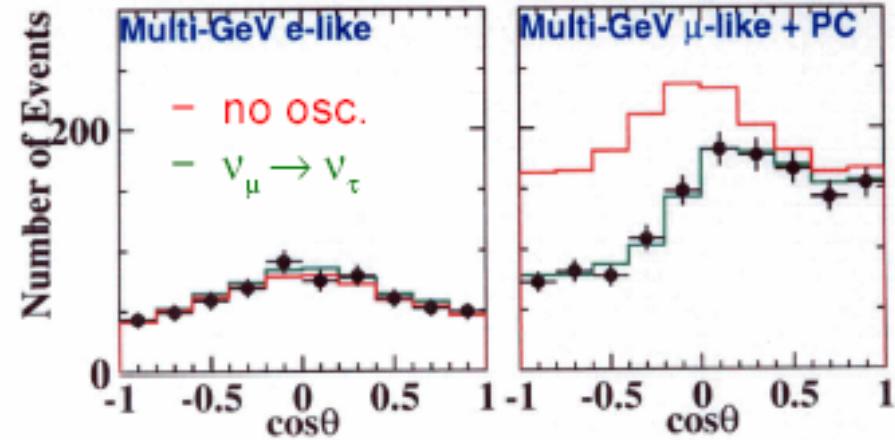
- nukleare Fusion: (ohne MSW Effekt: Produziere 100% ν_e)
 $4p \rightarrow {}^4He + 2e^+ + 2\nu_e + 27\text{ MeV}$
- Vakuum: langsame Oszillation mit θ_{12} , Pendel: schwache Federn



- Anregung von $(\nu_\tau - \nu_\mu)/\sqrt{2}$ nicht möglich, weil ν_e nicht in ν_3
- Oszillation nur zu $(\nu_\tau + \nu_\mu)/\sqrt{2}$
- $P(\nu_e \rightarrow \nu_e) > 50\%$ da nur ν_1 und ν_2 beteiligt

Atmosphärische Neutrinos

- Kamiokande 2000:
beschrieben als $\nu_\mu \rightarrow \nu_\tau$
- Pendel:
 ν_e : schwache Feder zu ν_μ, ν_τ
 ν_μ : schwache Feder zu ν_e
starke Feder zu ν_τ



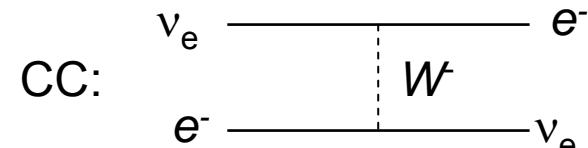
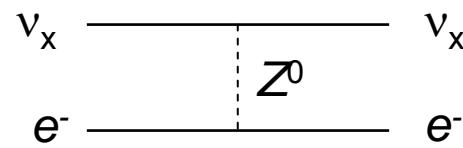
MSW-Effekt

$$i \frac{\partial}{\partial x} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \frac{1}{2p} \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = H_0 \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \quad \text{Massen-EZ}$$

Vakuum:

$$i \frac{\partial}{\partial x} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{\Delta m_{12}^2}{4p} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = H_V \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \quad \text{Flavour-EZ}$$

NC ($x = e, \mu, \tau$):



CC-Wechselwirkung der ν_e in der Sonne $\sim N_e$

$$V(x) = \sqrt{2} G_F N_e(x)$$

$$N_e \approx 6 \cdot 10^{25} \text{ cm}^{-3} \quad \text{bei } r = 0$$

$$i \frac{\partial}{\partial x} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \left[\frac{\Delta m_{12}^2}{4p} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \frac{V(x)}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

MSW-Effekt: adiabatische Lösung

$$i \frac{\partial}{\partial x} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \left[\frac{\Delta m_{12}^2}{4p} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \frac{V(x)}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$$V(x) = \sqrt{2}G_F N_e(x) \quad N_e \approx 6 \cdot 10^{25} \text{ cm}^{-3}$$

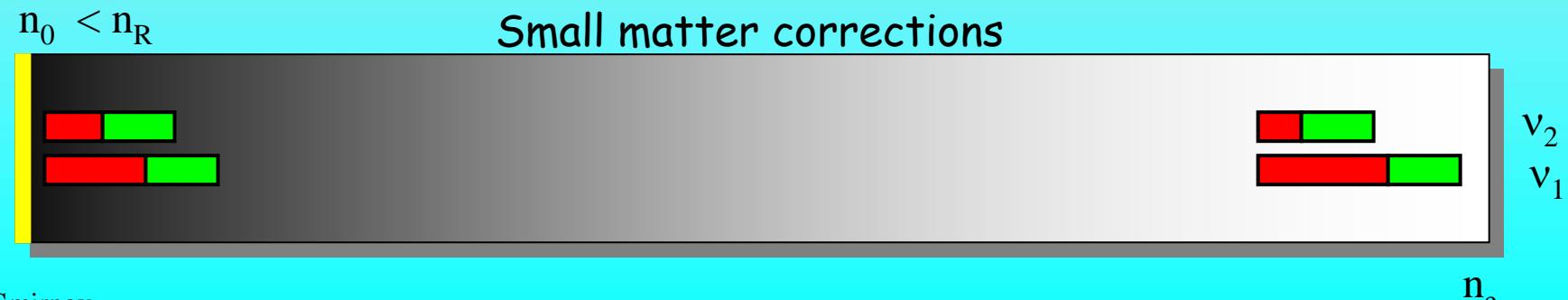
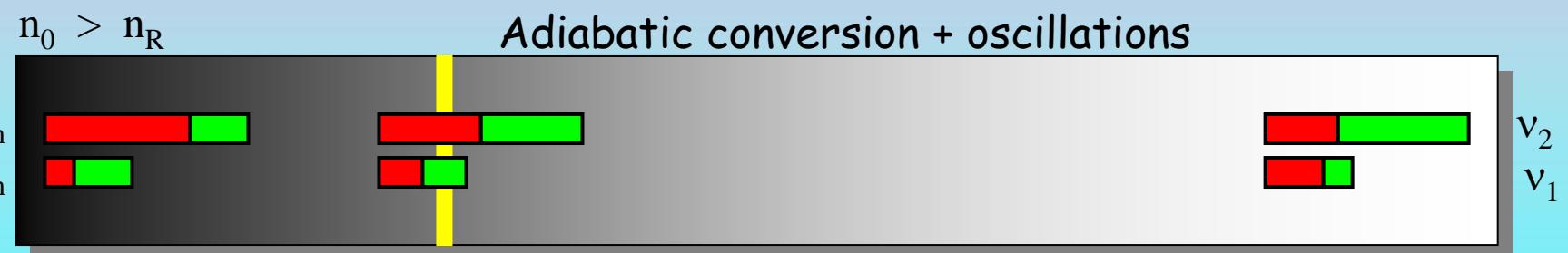
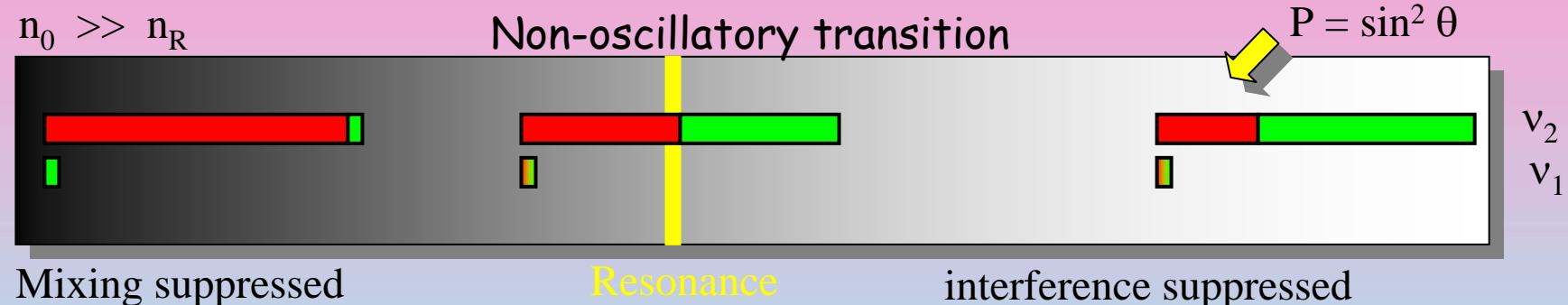
Falls H im Sonneninneren quasi-diagonal ($V(x)$ gross) $\Rightarrow \approx$ Massen-EZ = ν_{2m}
bei **adiabatischer Entwicklung** $V(x) \rightarrow 0$ am Sonnenrand

$$\Rightarrow \quad \nu_2 = \sin \theta \nu_e + \cos \theta \nu_\mu$$

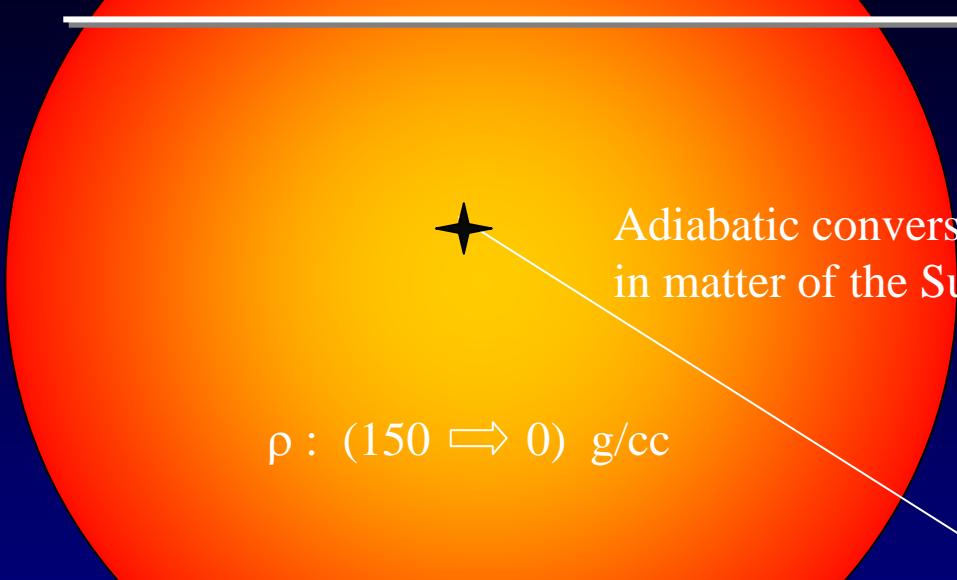
$$\Rightarrow \quad P(\nu_e) = \sin^2 \theta, \quad P(\nu_\mu) = \cos^2 \theta.$$

Adiabatic conversion

$$v_{1m} < \cancel{>} v_{2m}$$



Solar Neutrinos



Adiabatic conversion
in matter of the Sun

$$\rho : (150 \Rightarrow 0) \text{ g/cc}$$



electron neutrinos are produced

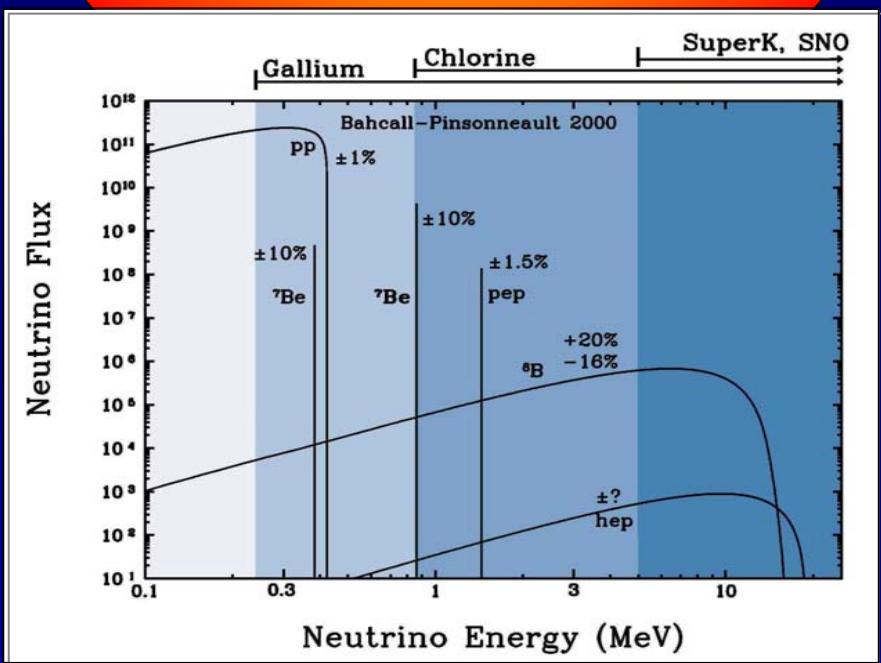
$$F = 6 \cdot 10^{10} \text{ cm}^{-2} \text{ s}^{-1}$$

total flux at the Earth

Oscillations
in vacuum

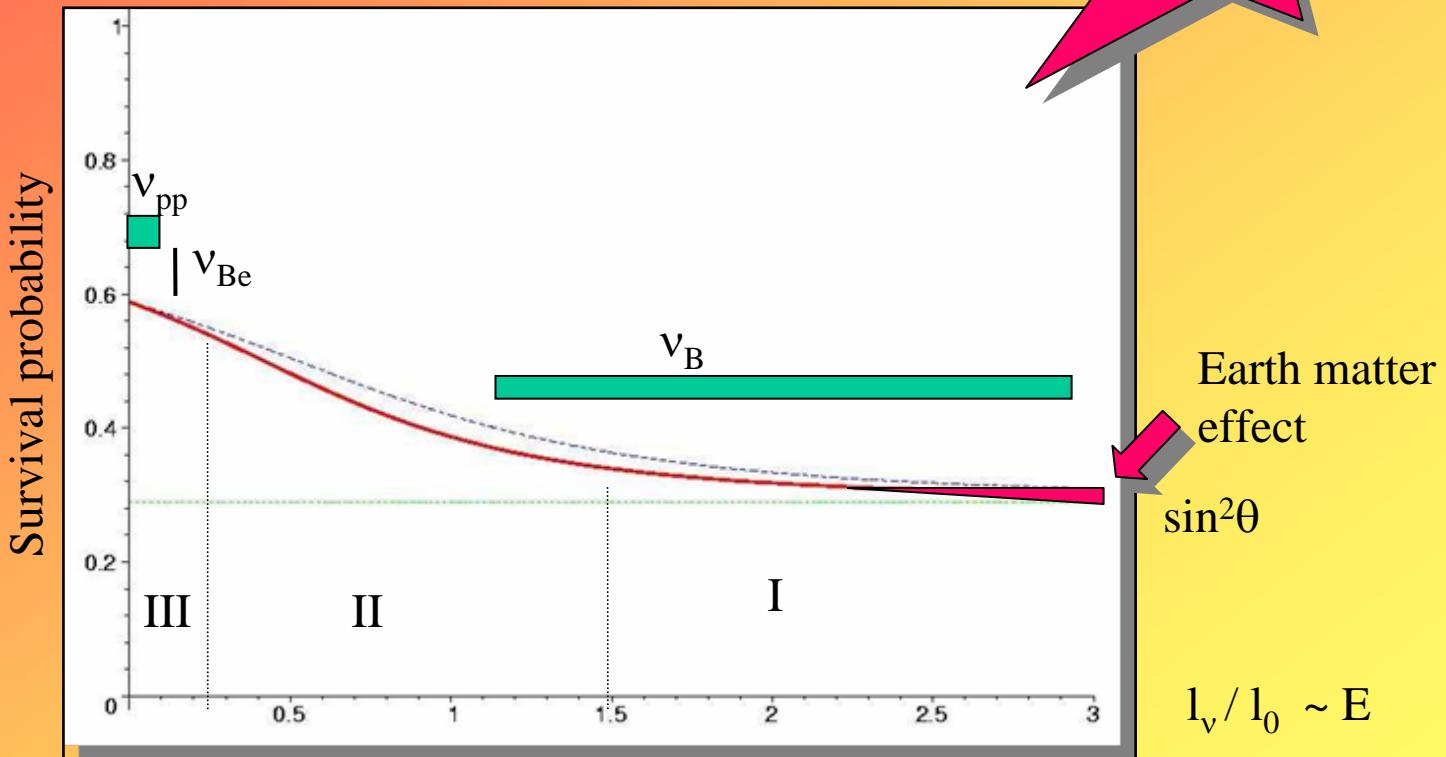
ν

Oscillations
in matter
of the Earth



J.N. Bahcall

Profile of the effect



Oscillations with
small matter effect

Conversion +
oscillations

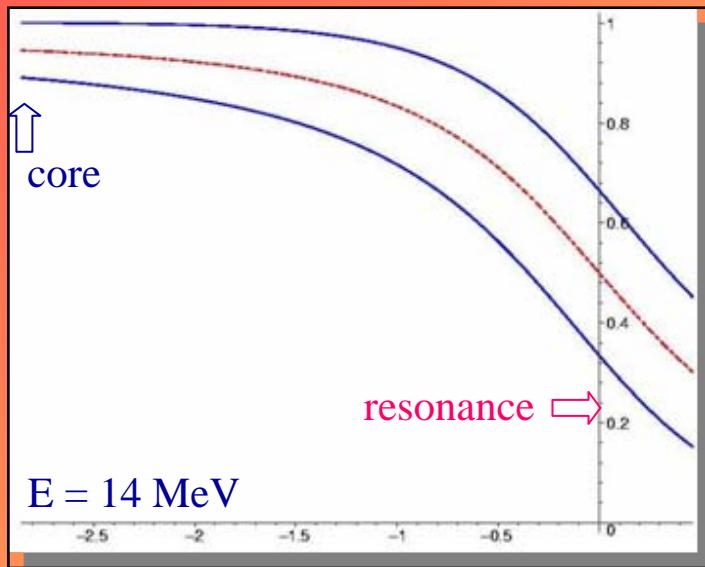
Conversion with
small oscillation
effect

Non-oscillatory
transition

Conversion inside the Sun

$$\tan^2\theta = 0.41, \Delta m^2 = 7.3 \cdot 10^{-5} \text{ eV}^2$$

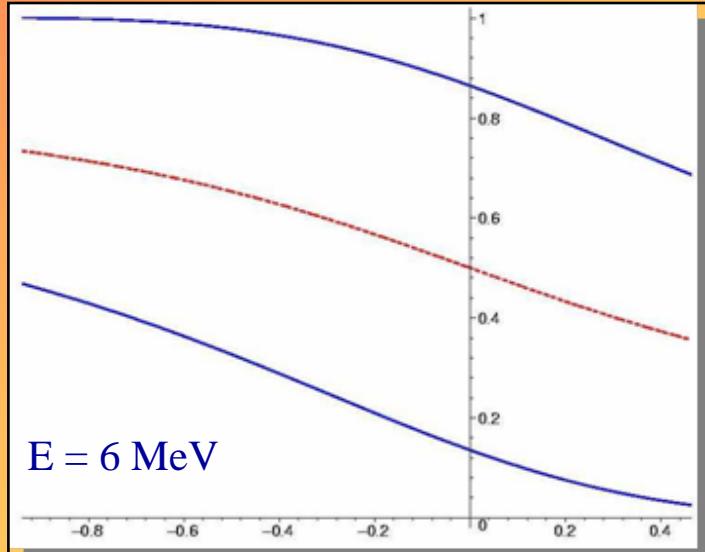
survival probability



y

distance

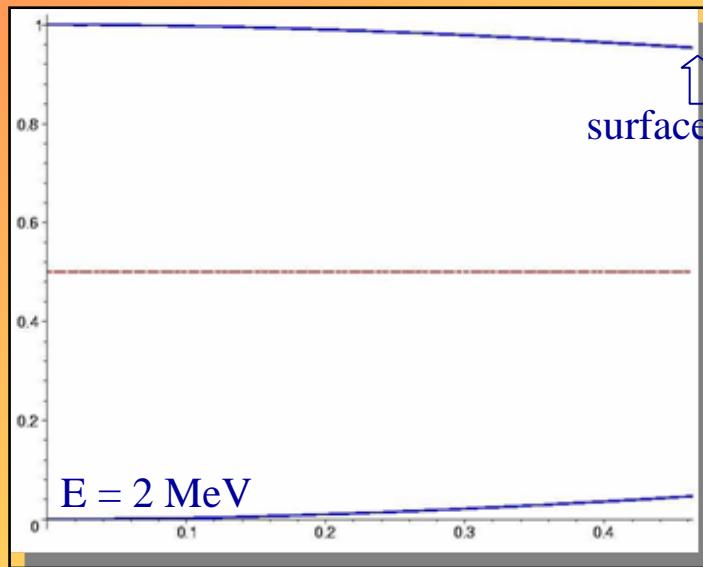
survival probability



y

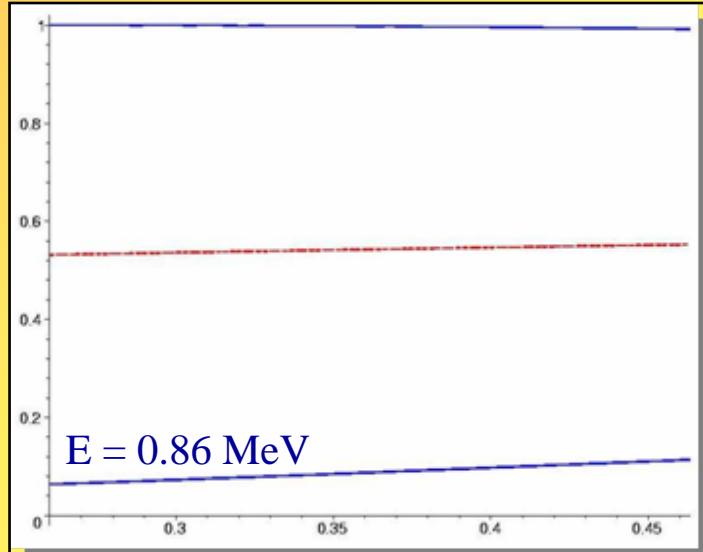
$E = 6 \text{ MeV}$

survival probability



y

distance



y

$E = 0.86 \text{ MeV}$

surface