THE TRIVIALITY BOUND OF THE FOUR-COMPONENT Φ^4 MODEL ☆

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Received 21 September 1987

The four-dimensional O(4) Φ^4 scalar theory is investigated in the broken phase at different values of the quartic coupling λ. The scalar mass, the field expectation value and the wave function renormalization constant are calculated. We demonstrate the λ dependence of the ratio $R_s = m_s/\langle \varphi_s^{ren} \rangle$ and estimate its upper bound to be around 2.7(1).

1. Introduction. It is almost rigorously proven that the renormalized coupling of the scalar Φ^4 theory in four dimensions vanishes in the infinite cut-off limit and the theory is trivial [1]. However, the presence of the marginally irrelevant quartic coupling can result in a non-trivial, interacting effective theory at energies below the cut-off. In such an effective theory the ratio of the scalar mass m_s and the field expectation value $\langle \varphi_s \rangle$ is basically a free parameter. It is expected, though, that $R_s \equiv m_s/\langle \varphi_s \rangle$ is bounded from above as one varies the bare quartic coupling at any fixed value of the cut-off and that the bound is increasing as the cut-off decreases. When $\Lambda_{cut} \sim m_s$, the effective theory loses its meaning. As a consequence the physically sensible value of R_s is bounded from above by $R_{s,max}$ [2–4].

The four-component scalar model O(4) is the heart of the spontaneous symmetry breaking and mass generation mechanism in the standard SU(2) Higgs model. The triviality of the scalar model may have important consequences for the GWS model. There are indications that the presence of the SU(2) gauge interaction does not change the basic properties of the scalar sector and the standard Higgs model is still a free field theory [5]. However, with a large but finite cut-off at lower energy it describes an effectively interacting model, where the ratio $R \equiv m_H/m_W$ is arbitrary but bounded from above as R_s is bounded in the scalar model.

There is an increasing interest towards the Φ^4 scalar and the SU(2) Higgs model. Recent Monte Carlo (MC) calculations of the standard Higgs model obtained an upper bound $R \sim 9$ [6]; in a situation where two typical masses differ by almost an order of magnitude finite-size effects make it rather difficult to study the system in detail. It is a feasible alternative to first study only the scalar sector non-perturbatively and then adding the gauge interaction accord-

☆ Supported by the Florida State University Supercomputer Computations Research Institute which is partially funded by the US Department of Energy through Contract No. DE-FC05-85ER250000; supported by Deutsche Forschungsgemeinschaft, and by Fonds zur Föderung der Wissenschaftliche Forschung in Österreich, Projekt P5965.

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ing to perturbative formulas. For example, R and R_s are related according to $R=R_s\times 2/g_{ren}$ corresponding to the formula $m_W^2=\frac{1}{4}g_{ren}^2\langle\varphi_s^{ren}\rangle^2$ from tree level renormalized perturbation theory. As was demonstrated in MC studies of gauge–Higgs systems [7] this relation holds surprisingly well even for the bare quantities at sufficiently small gauge coupling.

In ref. [3] an approximate renormalization group transformation was used to study the O(4) model. The calculation reproduced all the qualitatively expected features of the model giving $R_{s,max}\sim 3.2$. The one-component model has been studied recently by several groups. In ref. [4] the symmetric and the broken phase of the model are investigated by analytical methods focusing on the scaling behaviour and the consequences of triviality. In ref. [8] the authors' conclusions concerning the field renormalization constant Z and the irrelevance of the bare self-coupling disagree with the mentioned results [1,3,4] especially for small masses. A recent precise MC calculation [9] of the Ising limit ($\lambda\to\infty$) studies the finite-volume dependence and find good agreement with the behaviour expected from the results of ref. [4].

Here we report on a study of the lattice regularized four-component scalar model in the broken phase focusing on the question of the upper bound of R_s. We show that the field renormalization constant Z is close to 1 in the continuum limit. Work on the scaling behaviour of different observables and a more detailed finite-size study is in progress and will be presented in a forthcoming publication [10].

2. Definitions and notations. The lattice action we have studied by MC calculation on a lattice Λ of size L^4 ($L=12$ and 14) is

$$S=-\kappa\sum_{x\in\Lambda}\sum_{\mu=1}^{4}(\Phi_x^\alpha\Phi_{x+\mu}^\alpha+\Phi_x^\alpha\Phi_{x-\mu}^\alpha)$$
$$+\lambda\sum_{x\in\Lambda}(\Phi_x^\alpha\Phi_x^\alpha-1)^2+\sum_{x\in\Lambda}\Phi_x^\alpha\Phi_x^\alpha\,, \qquad (1)$$

where κ, $\lambda\geqslant 0$ are the bare coupling parameters; Φ_x^α, $\alpha=1, ..., 4$ is a real field and we use the convention that summation over the index α is implied whenever it occurs pairwise. For the usual continuum normalization, where the coefficient of the kinetic term is 1, the Φ fields should be rescaled to $\varphi=\sqrt{2\kappa}\Phi$.

For given λ (and, strictly speaking, for infinite lattice size) there is spontaneous symmetry breaking above a critical value of κ. In the broken phase the spectrum contains one massive scalar particle and three massless Goldstone particles with an unbroken SO(3) symmetry. Our goal was to determine the scalar mass m_s and the renormalized field expectation value $\langle\varphi_s^{ren}\rangle$. For the latter one has to calculate the bare field expectation value, $\langle\varphi_s\rangle=\sqrt{2\kappa}\langle\Phi\rangle$ and the wave function renormalization constant Z. A possible approach is to determine Z from the residue of the propagator of the massive scalar,

$$2\kappa G_2(p)=\sum_x \exp(ipx)\langle\varphi_s(0)\varphi_s(x)\rangle$$
$$=Z(p)/[p^2+m^2(p)]\,. \qquad (2)$$

We determine Z from non-zero values of the momentum. (In the infinite-volume limit this propagator diverges at $p=0$ due to coupling Goldstone modes, cf. e.g. the discussion of the longitudinal propagator in ref. [11].) In analogy with the one-component model [4] one expects a weak momentum dependence of $Z(p)$ and $m(p)$ up to $p^2\simeq m^2$. Furthermore, analytic calculations for the one-component model suggest that Z itself is close to 1 [4,9].

The presence of the massless modes makes the calculation of O(4) more problematic than for the one-component model. On a finite lattice the direction of symmetry breaking erratically moves around in group space related to the fact that strictly speaking there is no symmetry breaking. This makes it difficult to disentangle the massive scalar and the light Goldstone modes. Introducing an external field helps to stabilize the direction and also gives additional mass to the Goldstone particles. However, we found [10] that close to the phase transition on a 12^4 or similar size lattice one needs a sizeable external current to achieve this effect which makes the necessary extrapolation back to vanishing external field problematic.

For this reason we tried another approach. On a sufficiently large lattice for a given configuration the the sum over fields $\Phi^\alpha\equiv(1/L^4)\sum_x\Phi_x^\alpha$ is an estimator for the direction of the spontaneously chosen vacuum on an infinite lattice. Thus we introduce a field operator $\Phi_{s,x}^\alpha$ by performing a global rotation such that the direction of Φ^α is rotated to the direction of the one-axis in group space separately for each configuration. In the infinite-volume limit $\Phi_{s,x}^{\alpha=1}$ (in short, $\Phi_{s,x}$) corresponds to the scalar operator, its correlation function and expectation value gives the scalar mass and field expectation value, respectively.

We determined the scalar mass from the exponential decay of two different operators. The first one is the field $\Phi_{s,x}$ as defined above while the second one is a composite field operator.

$$O_1(\tau,\boldsymbol{p}) = \sum_{x\in\Lambda_\tau} \exp(\mathrm{i}\boldsymbol{x}\cdot\boldsymbol{p})\Phi_{s,x} \quad (3)$$

$$O_2(\tau,\boldsymbol{p}) = \sum_{x\in\Lambda_\tau} \sum_{\mu=1}^{3} \exp(\mathrm{i}\boldsymbol{x}\cdot\boldsymbol{p})\Phi^\alpha(x)\Phi^\alpha(x+\mu)\,, \quad (4)$$

where Λ_τ denotes the three-dimensional sublattice ("timeslice") at euclidean time τ [#1]. The mass is obtained from a fit of the connected correlation function $\langle O(0,\boldsymbol{p})O(\tau,\boldsymbol{p})\rangle^c$ to the form

$$a+b\{\exp(-m\tau)+\exp[-m(L-\tau)]\}\,. \quad (5)$$

The constant a is proportional to $\exp(-mL)$ and accounts for the finite extension in time when $\langle O\rangle\neq 0$ [9].

3. MC calculation and results. We have performed calculations at quartic coupling value $\lambda=\infty$, 1.0 and 0.05 varying the hopping parameter κ in the broken phase in the region where the scalar mass is between

[#1] The operator $\Phi^\alpha\Phi^\alpha$ couples to the scalar massive states and, for finite λ, could be used. It turned out, however, that its correlation function gives a poor signal and thus we did not include it in the analysis.

Table 1
Summary of our results for $\lambda=\infty$, 1.0 and 0.05 for lattice size 12^4 and 16^4

λ	Lattice	κ	$\frac{\text{Statistics}}{1000}$	$\langle\Phi_s\rangle$	$G_2(k_\tau=1)$	m_s	Z	$R=\frac{m_s}{\langle\varphi_s^{ren}\rangle}$
∞	12^4	0.3050	470	0.1330 (2)	4.46 (9)	0.32 (1)	1.00 (2)	3.03 (6)
		0.3075	410	0.1780 (1)	3.76 (6)	0.38 (1)	0.95 (2)	2.63 (6)
		0.3100	540	0.2140 (1)	3.16 (3)	0.47 (1)	0.96 (2)	2.74 (7)
		0.3175	500	0.2920 (1)	1.98 (1)	0.67 (1)	0.92 (2)	2.77 (5)
		0.3250	740	0.3468 (4)	1.44 (1)	0.80 (1)	0.88 (2)	2.69 (4)
		0.3330	500	0.3920 (4)	1.08 (1)	0.92 (2)	0.85 (3)	2.65 (7)
		0.3550	660	0.480 (3)	0.61 (1)	1.22 (4)	0.85 (5)	2.78 (12)
∞	14^4	0.3075	400	0.169 (2)	4.79 (11)	0.35 (1)	0.95 (3)	2.57 (8)
		0.3100	400	0.209 (1)	3.81 (4)	0.43 (1)	0.91 (2)	2.50 (7)
		0.3175	400	0.289 (1)	2.27 (1)	0.64 (1)	0.90 (2)	2.63 (5)
		0.3200	295	0.310 (1)	1.99 (1)	0.71 (1)	0.92 (2)	2.75 (5)
		0.3250	400	0.345 (1)	1.56 (1)	0.81 (2)	0.90 (3)	2.77 (9)
		0.3300	240	0.374 (1)	1.32 (1)	0.87 (2)	0.87 (5)	2.67 (8)
		0.3350	150	0.400 (1)	1.11 (1)	0.92 (2)	0.82 (3)	2.54 (7)
1.0	12^4	0.2520	350	0.307 (2)	3.53 (2)	0.50 (1)	0.93 (2)	2.21 (5)
		0.2530	270	0.330 (3)	3.22 (2)	0.54 (2)	0.92 (3)	2.21 (5)
		0.2540	330	0.351 (2)	2.98 (2)	0.58 (2)	0.93 (3)	2.24 (5)
		0.2570	240	0.406 (6)	2.36 (2)	0.70 (3)	0.94 (4)	2.33 (6)
		0.2600	150	0.452 (2)	1.96 (1)	0.78 (3)	0.89 (5)	2.26 (7)
0.05	12^4	0.1495	270	0.367 (2)	9.05 (27)	0.32 (1)	1.00 (3)	1.60 (6)
		0.1500	350	0.442 (2)	8.00 (15)	0.37 (1)	0.98 (3)	1.51 (5)
		0.1505	350	0.507 (2)	7.27 (10)	0.42 (1)	0.98 (2)	1.49 (4)
		0.1510	230	0.564 (1)	6.53 (8)	0.47 (1)	0.97 (2)	1.49 (4)
		0.1530	390	0.746 (1)	4.62 (3)	0.63 (1)	0.96 (2)	1.49 (3)
		0.1540	1110	0.820 (5)	4.02 (2)	0.70 (1)	0.96 (2)	1.51 (3)

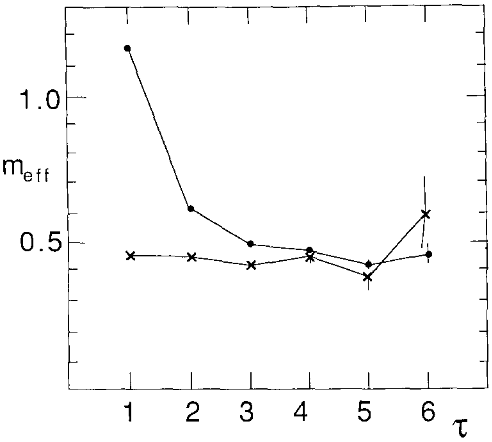

Fig. 1. m_{eff} versus τ as discussed in the text is given for the two operators O_1 (full circles) and O_2 (crosses), for $\kappa=0.31$, $\lambda=\infty$ for lattice size 14^4.

0.3 and 1.0. Most of the calculations were done on a 12^4 lattice, and only a few points for lattice size 14^4. The κ values, the statistics and our results are summarized in table 1 for the three different λ values.

We determined the energy of the operators projected to states with lattice momentum $k_x=0$ and $k_x=1$ ($p_\mu=2\pi k_\mu/L$) and found very good agreement with the expected spectral behaviour. Both operators, O_1 and O_2, couple to Goldstone intermediate states but the data generally was consistent with pure one-particle decay. O_1 gave very stable results for the mass depending only very little on the interval in τ chosen for the fit; the propagator of O_2, on the other hand, deviates from the exponential decay at small τ and was noisier. To give an impression on the amount of systematic error we show the variation of an effective mass $m_{\text{eff}}(\tau)$ which is obtained with the fit (5) using only data points at $\tau-1$, τ and $\tau+1$. Fig. 1 gives an example for the typical behaviour of this effective mass for both operators on a 14^4 lattice, for $\lambda=\infty$ and a value of κ close to the phase transition. For these reasons we decided to take the scalar mass from the correlation function of O_1 for $k_x=0$ (table 1) and to use O_2 only as a consistency check in our analysis.

We have accounted for the trivial lattice grid effects by substituting the lattice equivalents in (2), i.e. $p^2\to 2\sum_{\mu=1}^{4}(1-\cos p_\mu)$ and $m\to 2\sinh(m_s/2)$. We determined Z from our data according to (2) at lattice momentum $k_\tau=1$. Using (2) even at zero momentum leads to values consistent with those obtained at non-zero momentum. On a lattice of finite size the singular contributions are logarithmically divergent with $\log L$ and the observed behaviour indicates the coefficient of this infrared singularity may be very small. The value of Z is consistent with 1.0 at the phase transition and decreases slowly when moving away from the phase transition. The last entry in the table gives the ratio $R_s=m_s/\langle\varphi_s^{\text{ren}}\rangle=Z^{1/2}m_s/\langle\varphi_s\rangle$.

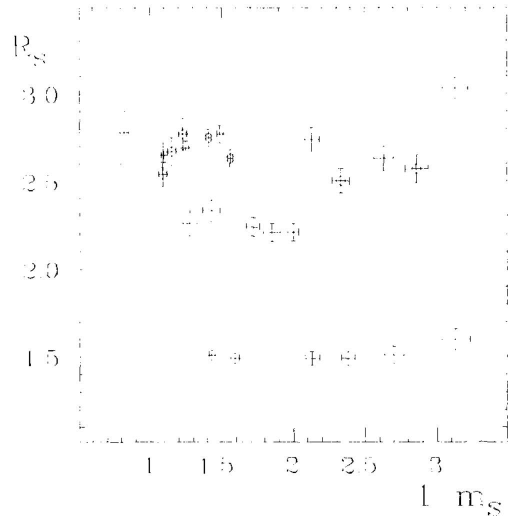

Fig. 2. Summary of our results for $R_s\equiv m_s/\langle\varphi_s^{\text{ren}}\rangle$ versus the inverse dimensionless scalar mass m_s as obtained for lattice size 12^4 (vertical crosses) and 14^4 (diamonds); the three vertically separated clusters correspond to the different values of $\lambda=0.05$, 1.0, and ∞, moving upwards.

As concerns the results for $\lambda=1.0$ and 0.05, it is interesting to note that the correlation functions of Φ_s at finite λ were more stable than at $\lambda=\infty$ giving a more reliable mass value. This indicates that Φ_s gives a better signal when the length of the field is allowed to fluctuate.

4. Conclusion. Our calculation demonstrates that Φ_s can be used to determine the field expectation values and the scalar mass at least in the region $0.3<m_s<1.0$ on 12^4 or larger lattices. The wave function renormalization constant Z may be determined from the propagator of Φ_s at non-zero momentum. Z is close to unity in the investigated region.

We have also studied the κ-dependence of the scalar mass and other quantities and see clear signals of the expected leading scaling behaviour and a somewhat weaker signal of the leading logarithmic corrections. We observed a systematic finite-size dependence; this part of the analysis together with further results will be presented elsewhere [10].

In fig. 2 we plot R_s versus $1/m_s$. The points corresponding to $\lambda=\infty$, 1.0 and 0.05 form three distinct clusters. One clearly sees the approach to an upper bound giving $R_{s,max}\approx 2.7$ (1). Assuming the validity of the perturbative relation to the W-mass and using $g^2_{ren}=0.4$ this corresponds to an upper bound for the ratio between the Higgs mass and the massive vector-boson mass of $R\approx 8.5$ (3). This is consistent with the result of ref. [3]. R_s is also close to the upper bound of the one-component model [4] indicating a weak dependence on the number of components of the Φ field.

During the completion of this manuscript we learned of an effective potential calculation for the one-component model [12] which is in agreement with the results of ref. [4] too.

We are much indebted to P. Hasenfratz for discussions and helpful criticism. Discussions with J. Jersák, H. Kastrup and P. Weisz and gratefully appreciated. We acknowledge the support of the Computer Centers of Florida State University, Universität Bochum, and KFA Jülich where the necessary computations were performed. C.B.L. wants to thank SCRI for the kind hospitality granted during the completion of this work.

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