3pts correlators:

$$C_\mu^{K\pi}(t_x,t_y)=\sum_{\vec{x},\vec{y}}\left\langle O_\pi(t_y,\vec{y})\,\hat{V}_\mu(t_x,\vec{x})\,O_K^\dagger(0)\right\rangle e^{-i\vec{p}_K\cdot\vec{x}+i\vec{p}_\pi\cdot(\vec{x}-\vec{y})}$$

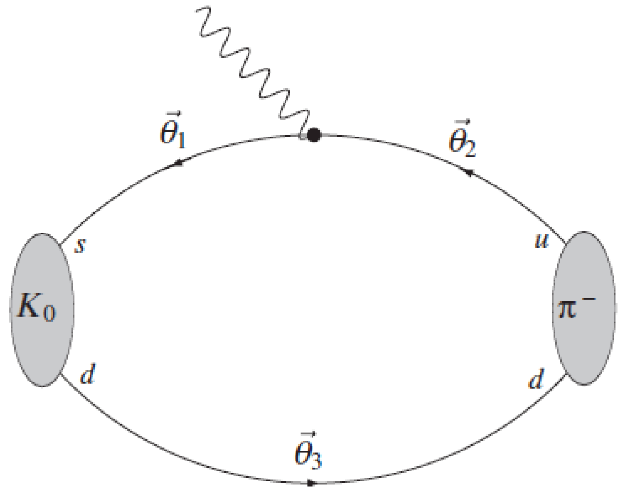
$$\xrightarrow[\substack{t_x\to\infty\\ t_y-t_x\to\infty}]{}\frac{\sqrt{Z_K Z_\pi}}{4E_K E_\pi}\langle\pi|\hat{V}_\mu|K\rangle e^{-E_K t_x-E_\pi(t_y-t_x)}$$

2pts correlators:

$$C^{K(\pi)}(t_x)=\sum_{\vec{x}}\left\langle O_{K(\pi)}(t_x,\vec{x})\,O_{K(\pi)}^\dagger(0)\right\rangle e^{-i\vec{p}_{K(\pi)}\cdot\vec{x}}$$

$$\xrightarrow[t_x\to\infty]{}\frac{Z_{K(\pi)}}{2E_{K(\pi)}}e^{-E_{K(\pi)}t_x}$$

$$O_K=\bar{d}\gamma_5 s,\; O_\pi=\bar{d}\gamma_5 u,\; \hat{V}_\mu=Z_V\;\bar{s}\gamma_\mu u$$



$$\vec{p}_K=\vec{\theta}_3-\vec{\theta}_1,\qquad \vec{p}_\pi=\vec{\theta}_3-\vec{\theta}_2,\qquad \vec{q}=\vec{p}_K-\vec{p}_\pi=\vec{\theta}_2-\vec{\theta}_1$$

* **head-on kinematics:** $\vec{p}_K=-\vec{p}_\pi$ ⇒ $\vec{\theta}_2=-\vec{\theta}_1\equiv\vec{\theta}$, $\vec{\theta}_3=0$ $\qquad \vec{q}=2\vec{\theta}$

* **twisted boundary conditions:** quark with momentum $2\pi\vec{\theta}/L$

$$\sum_z D^{\vec{\theta}}(x,z)\,S^{\vec{\theta}}(z,y)=\delta_{x,y}\;e^{2\pi i\vec{\theta}\cdot\vec{x}/L}\qquad \text{rephasing: } U_\mu^{\vec{\theta}}(x)=e^{2\pi i a\theta_\mu/L}\,U_\mu(x)$$

* **stochastic method:**

$$S^{\vec{\theta}}(x,y) \to \frac{1}{N_s}\sum_{r=1}^{N_s} \Phi_r^{\vec{\theta}}(x)\left[\eta_r(y)\right]^{\dagger} e^{2\pi i\vec{\theta}\cdot\vec{y}/L}$$

$$\sum_z D^{\vec{\theta}}(x,z)\,\Phi_r^{\vec{\theta}}(z) = \eta_r(x)$$

(drop color, spin and flavor labels)

$$\begin{cases} \left[\eta^r(x)\right]^{\dagger}\eta^r(x) = 1 \\ \lim\limits_{N_s\to\infty}\frac{1}{N_s}\sum_{r=1}^{N_s}\left[\eta^r(x)\right]^{\dagger}\eta^r(y) = \delta_{x,y} \end{cases}$$

* 2pts correlators: $C^{K(\pi)}(t_x) \to \frac{1}{N_s}\sum_{r=1}^{N_s}\sum_{\vec{x}}\left\langle Tr\left\{\gamma_5\left[\Phi_r^{\mp\vec{\theta}}(t_x,\vec{x})\right]^{\dagger}\gamma_5\Phi_r^{0}(t_x,\vec{x})\right\}\right\rangle$

* 3pts correlators: $C_\mu^{K\pi}(t_x,t_y) \to \frac{1}{N_s}\sum_{r=1}^{N_s}\sum_{\vec{x}}\left\langle Tr\left\{\gamma_5\left[\Phi_r^{-\vec{\theta}}(t_x,\vec{x})\right]^{\dagger}\gamma_\mu\Sigma_r^{\vec{\theta}}(t_x,\vec{x};t_y)\right\}\right\rangle$

$$\sum_z D^{\vec{\theta}}(x,z)\,\Sigma_r^{\vec{\theta}}(z;t_y) = \gamma_5\Phi_r^{0}(x)\,\delta_{t_x,t_y}$$

* O(a)-improvement: see Roberto's contribution (also for disconnected diagrams)

checkable by changing sign to $\vec{\theta}$

* ETMC gauge confs at β = 3.9, V T = 24^3 48 and a μ_{sea} = 0.0040

five different values of $\vec{\theta} = (\theta_x,\theta_y,\theta_z) \to$ 10 fermion inversions

$\theta_x = \theta_y = \theta_z = 0.0, 0.227, 0.289, 0.330, 0.361$ ⟹

$$q^2 = \left(\sqrt{M_K^2 + \left(\frac{2\pi}{L}\theta\right)^2} - \sqrt{M_\pi^2 + \left(\frac{2\pi}{L}\theta\right)^2}\right)^2 - \left(\frac{2\pi}{L}2\theta\right)^2$$

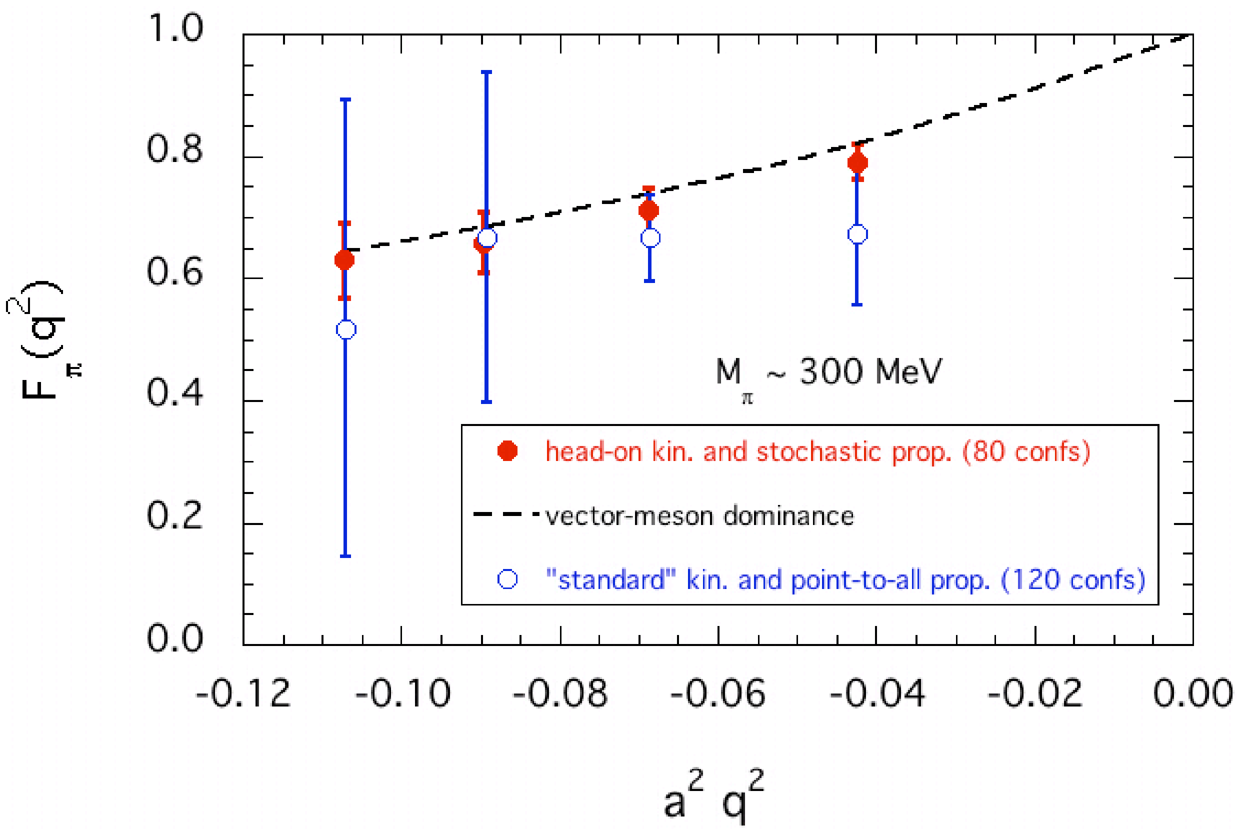
for $M_K = M_\pi$: $q^2 = -\left(\frac{2\pi}{L}2\theta\right)^2$

* **"standard" kinematics:** $\vec{p}_\pi = 0, \quad \vec{q} = \vec{p}_K$

* periodic boundary conditions in space and point-to-all quark propagator: $\sum_z D(x,z)\, S(z,0) = \delta_{x,0}$

five values of $\vec{q} = \frac{2\pi}{L}\{(0,0,0),(1,0,0),(1,1,0),(1,1,1),(2,0,0)\} \to$ 2 fermion inversions (but the cost is 3 times more)

$$q^2 = \left(\sqrt{M_K^2 + \left(\frac{2\pi}{L}\right)^2 n} - M_\pi\right)^2 - \left(\frac{2\pi}{L}\right)^2 n, \qquad n = 0, 1, 2, 3, 4$$



vector-meson dominance:

$$F_\pi(q^2) = 1/(1 - q^2/M_V^2)$$

$$a\, M_V = 0.440$$

The Double Ratio: (initial and final mesons at rest)

scalar form factor $f_0[q^2_{max}]$

$$R_0 \equiv \frac{C^0_{K\pi}\left(t_x,t_y\right)\cdot C^0_{\pi K}\left(t_x,t_y\right)}{C^0_{KK}\left(t_x,t_y\right)\cdot C^0_{\pi\pi}\left(t_x,t_y\right)} \xrightarrow[t_x-t_y\to\infty]{t_y\to\infty,} \frac{\langle\pi|V^0|K\rangle\cdot\langle K|V^0|\pi\rangle}{\langle K|V^0|K\rangle\cdot\langle\pi|V^0|\pi\rangle} \propto \left[f_0\left(q^2_{max}\right)\right]^2$$

essential features of the double ratio

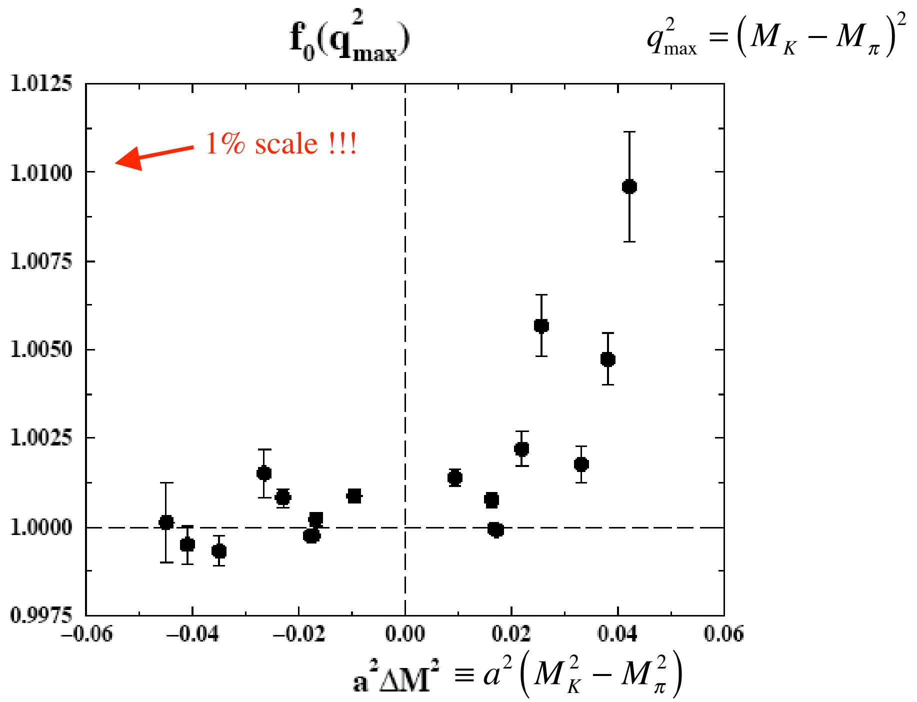
1. independent of renormalization constants;
2. exactly normalized at 1 in the SU(3) limit;
3. statistical fluctuations strongly cancel out;
4. improved at $O[a^2(m_s - m_\ell)^2]$ thanks to the $K \Leftrightarrow \pi$ symmetry.

$\beta = 6.2\ (a^{-1} \simeq 2.6\ GeV)$

$230\ (quenched)\ gauge\ confs.$

Clover fermions

$0.5 < M_{PS}(GeV) < 1$



*** preliminary ETMC results (80 confs) for the scalar form factor $f_0(q^2_{max})$**