RI-MOM DETERMINATION OF RENORMALIZATION CONSTANTS

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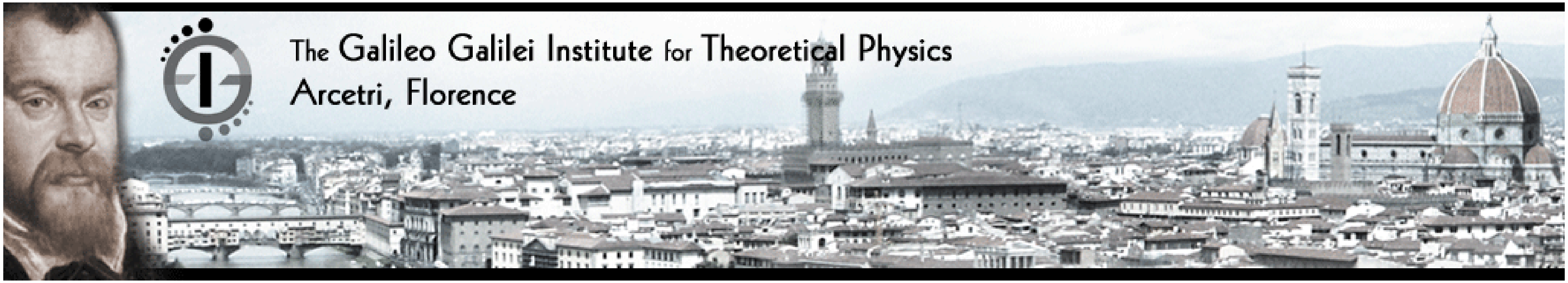
European Twisted Mass Collaboration

ETMC meeting

Arcetri, February 6-7 2007

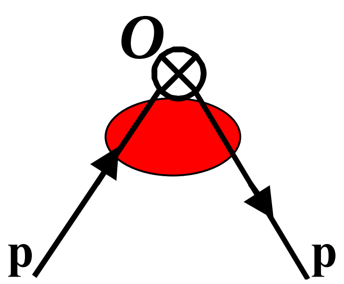


THE RI-MOM METHOD

$$G_\Gamma^{ud}(p,p') = \sum_{x,y} \left\langle u(x) (\bar{u}\Gamma d)_0 \bar{d}(y) \right\rangle e^{-ip\cdot x + ip'\cdot y}$$

$$S_u(p) = \sum_x \left\langle u(x)\bar{u}(0) \right\rangle e^{-ip\cdot x}$$

$$\Gamma_\Gamma^{ud}(p,p') = Tr\left[S_u(p)^{-1} G_\Gamma^{ud}(p,p') S_d(p')^{-1} P_\Gamma \right]$$



$$Z_\Gamma Z_q^{-1} \Gamma_\Gamma^{ud}(p,p)\big|_{p^2=\mu^2} = 1$$

$$Z_q \frac{i}{12} Tr\left[\frac{\not{p} S(p)^{-1}}{p^2} \right]_{p^2=\mu^2} = 1$$

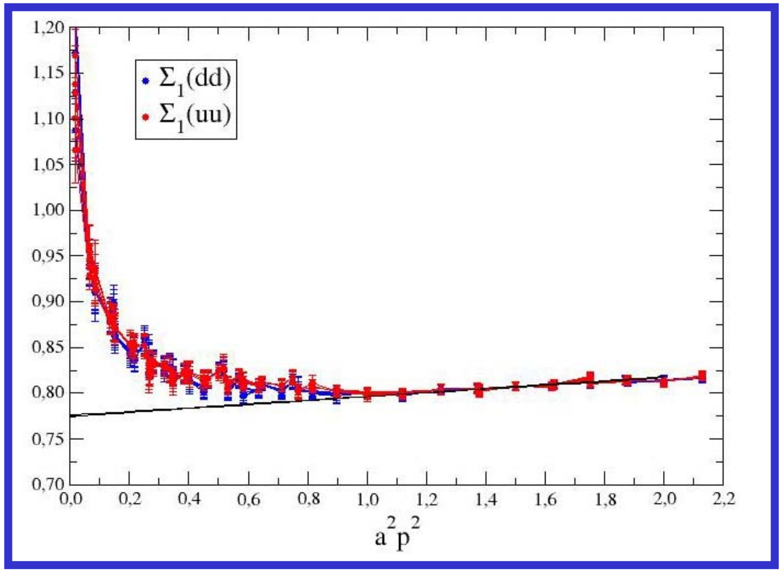
SIMULATION DETAILS:

- $\beta = 3.9$, $V = 24^3 \times 48$
- 240 gauge configurations
- 4 masses :$(a\,\mu) = 0.0040, 0.0064, 0.0100, 0.0150$
- periodic boundary conditions

THE QUARK PROPAGATOR

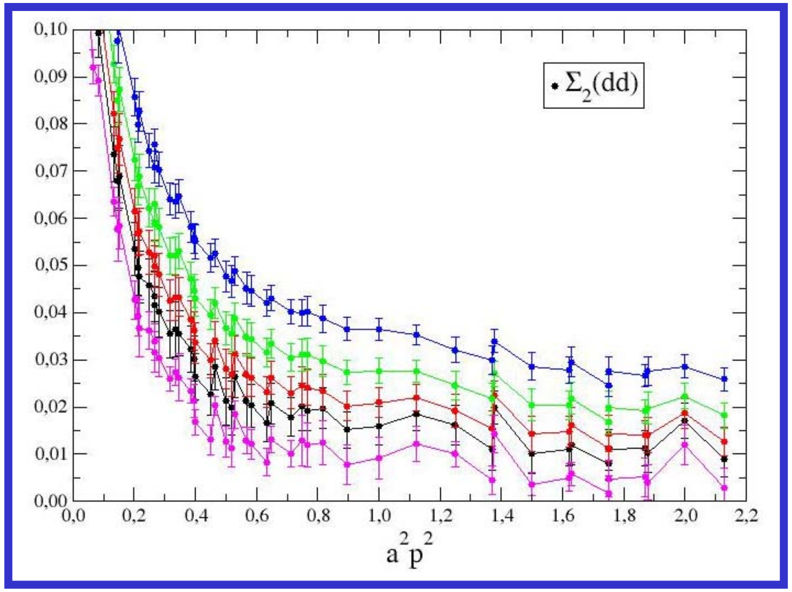
$$S(p) = \frac{i\not p}{p^2}\sigma_1(p^2) + \frac{1}{p^2}\sigma_2(p^2) + \frac{i\gamma_5}{p^2}\sigma_3(p^2)$$

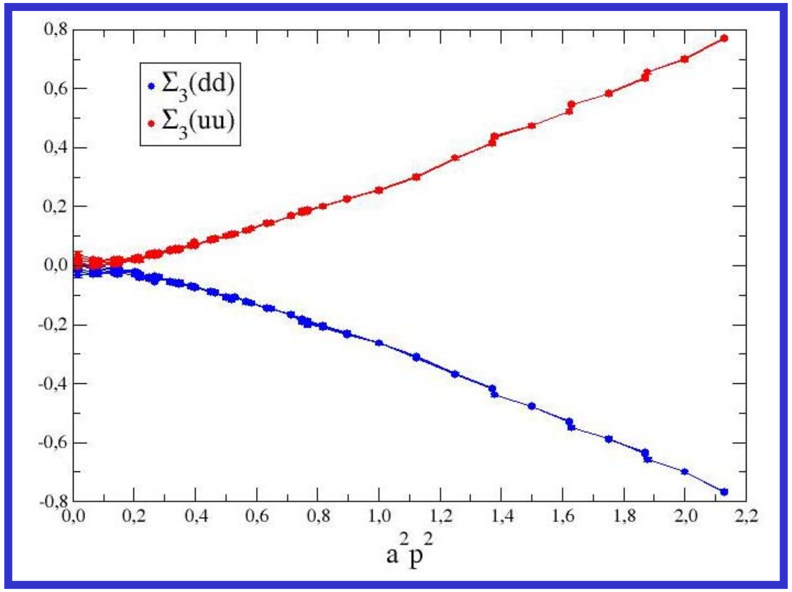
$$S(p)^{-1} = -i\not p\Sigma_1(p^2) + \Sigma_2(p^2) - i\gamma_5\Sigma_3(p^2)$$



At tree-level:

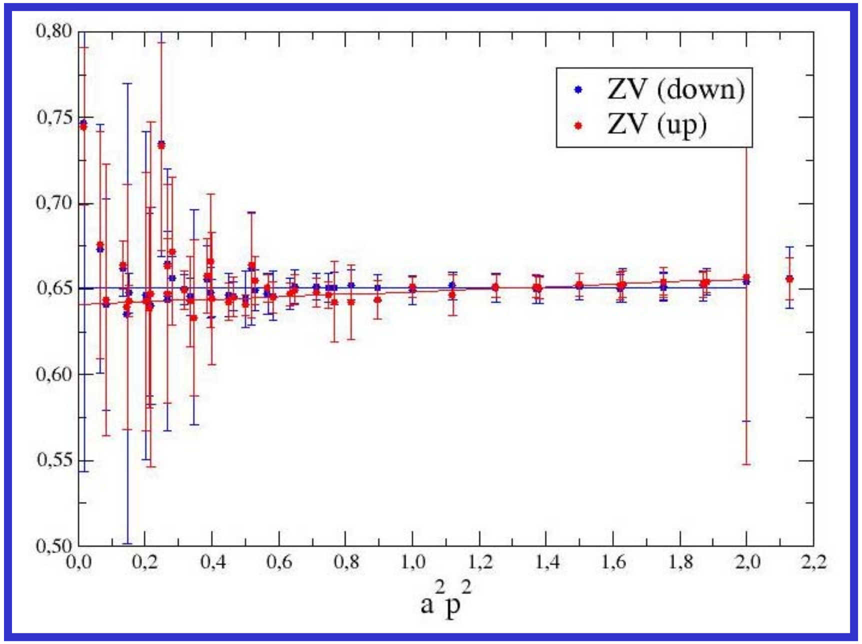
$$\Sigma_1(p^2) = 1\,,\ \Sigma_2(p^2) = m\,,\ \Sigma_3(p^2) = \pm\frac{ar}{2}p^2$$

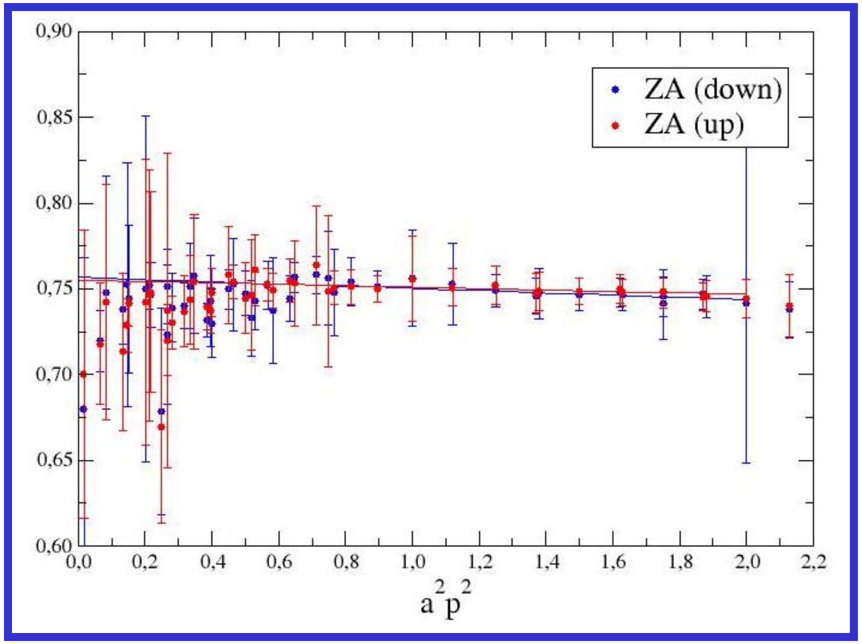




RENORMALIZATION CONSTANTS

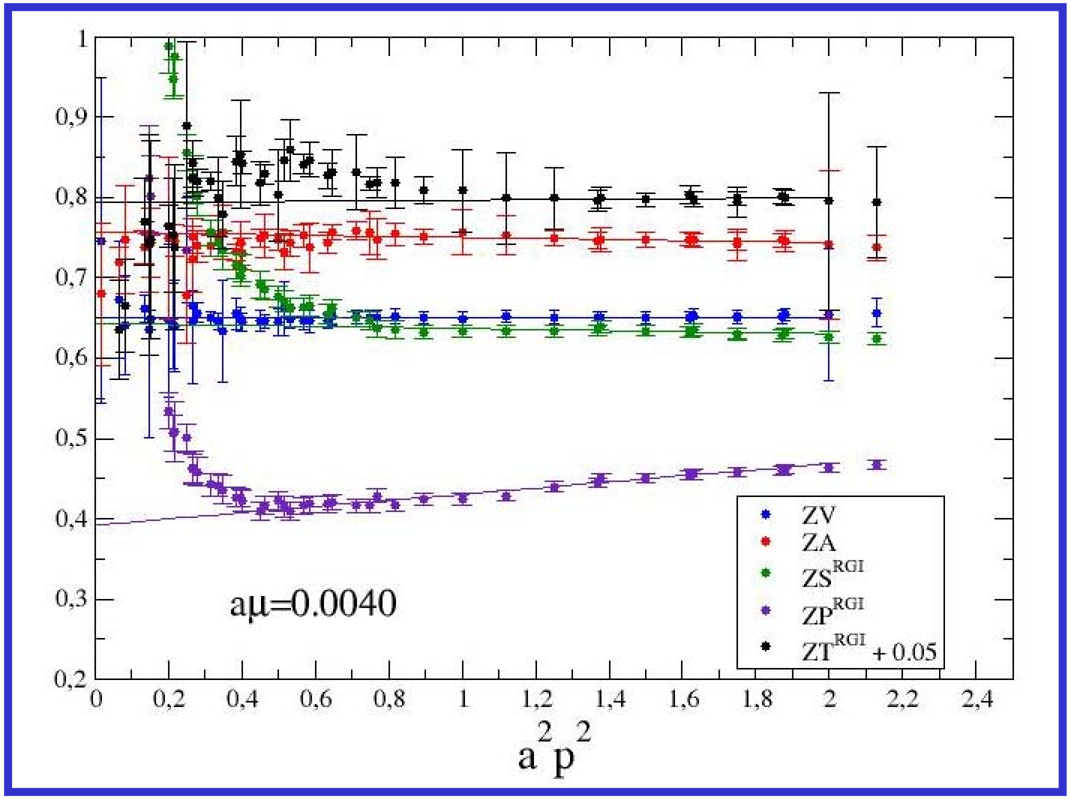
Comparison between "u-d" and "d-u" results





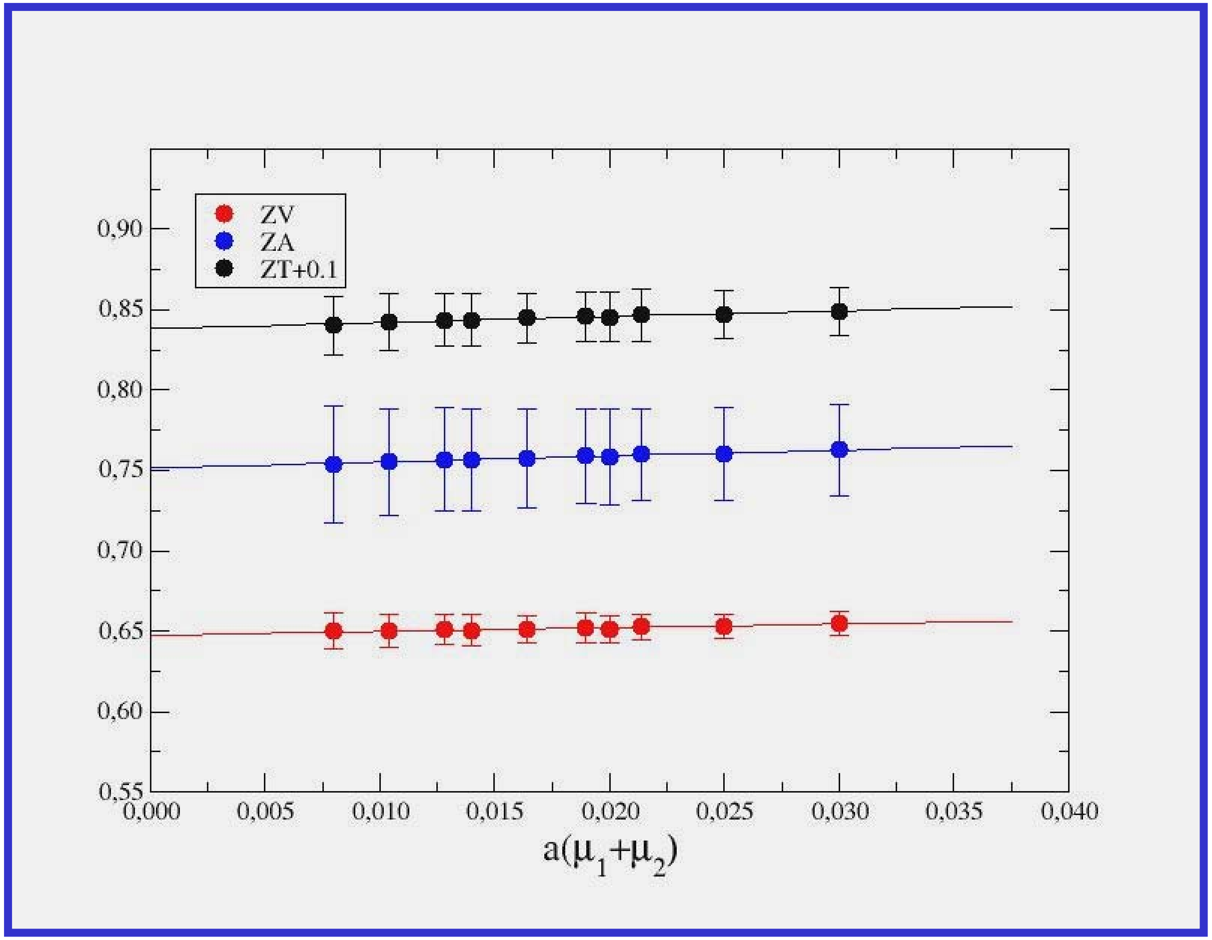
Similar results are obtained for the other RCs

RENORMALIZATION SCALE DEPENDENCE



For the scale dependent RCs: $Z(\mu) = C(\mu)\, Z^{RGI}$

CHIRAL EXTRAPOLATIONS

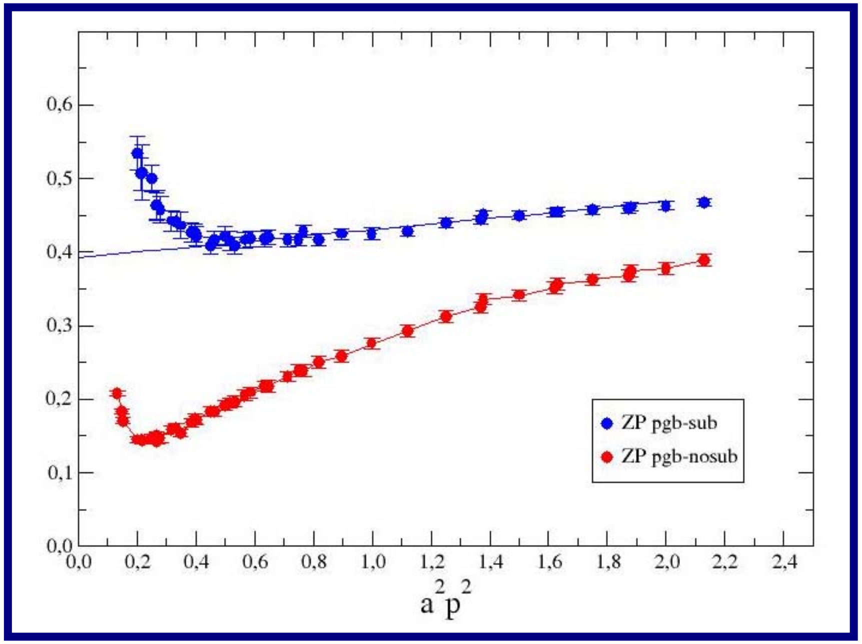


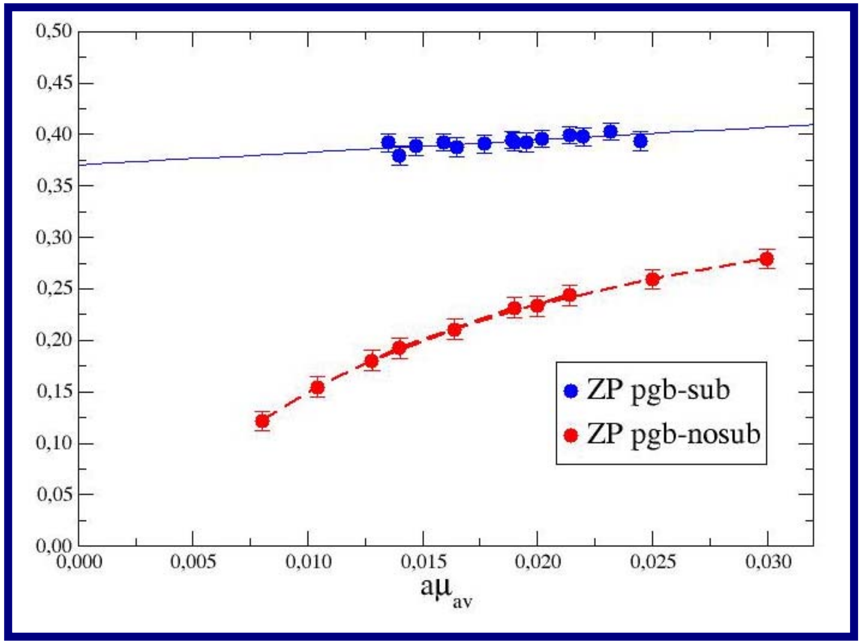
THE GOLDSTONE POLE

The pseudoscalar vertex couples to the PGB

$$\Gamma_P(p^2) \approx A\,(p^2) + B\,(p^2)\frac{<\bar{\psi}\psi>}{m_q p^2} + \ldots$$

J.Cudell *et al.*, '98



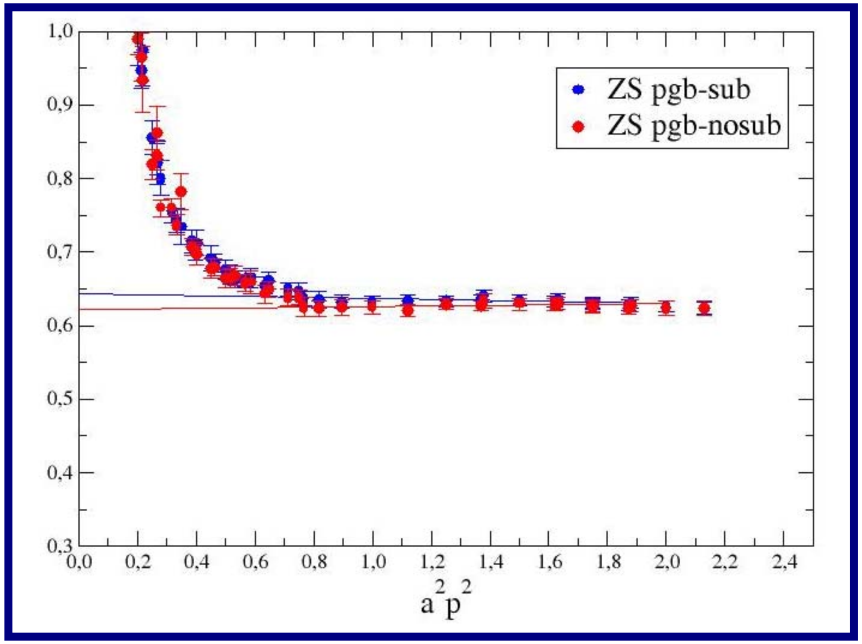


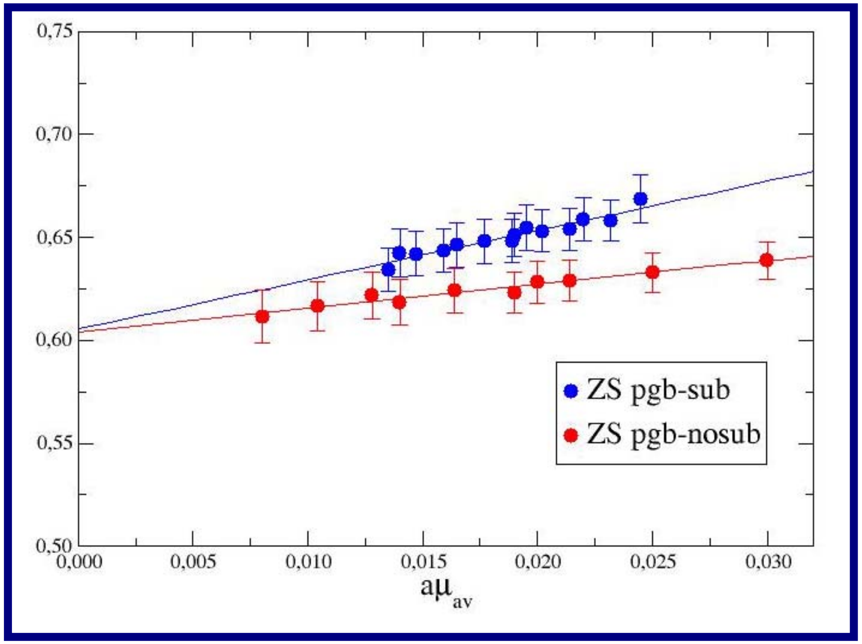
$$\Gamma_P^{SUB} = \frac{m_1\Gamma_P(m_1) - m_2\Gamma_P(m_2)}{m_1 - m_2} \approx \Gamma_P(m_1) + m_1\frac{\partial\,\Gamma_P}{\partial\, m_1} = A + \ldots$$

L.Giusti, A.Vladikas, '00

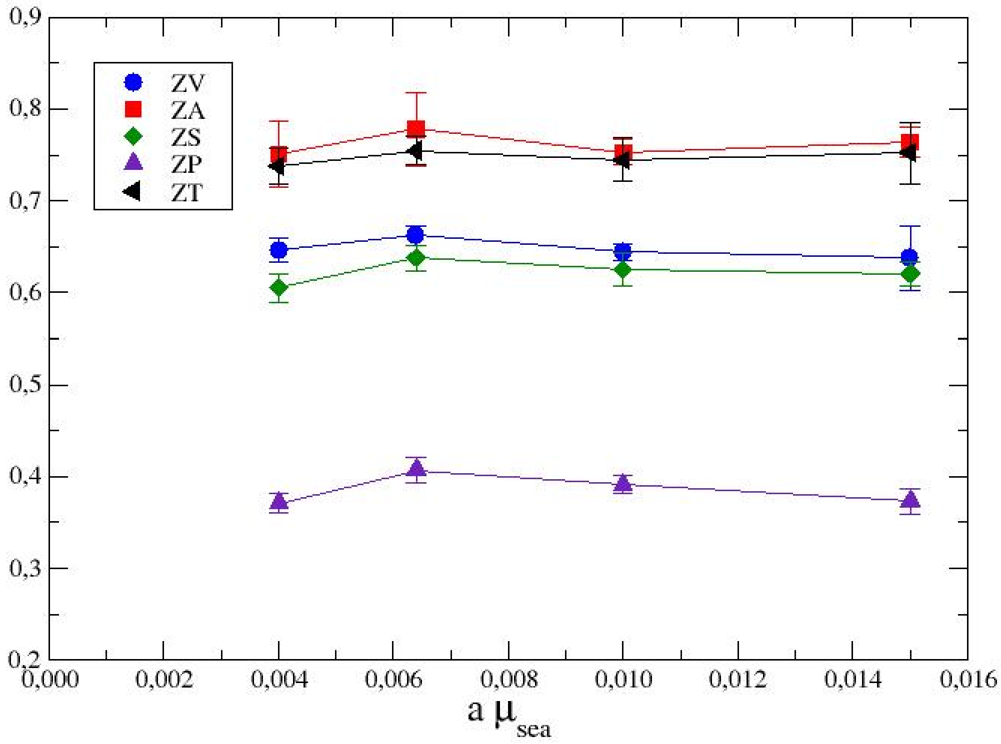
THE GOLDSTONE POLE (cont.)

With twisted mass at maximal twist the coupling of the scalar density to the PGB is suppressed by a factor of a^2





SEA QUARK MASS DEPENDENCE



O(a)-IMPROVEMENT

(see talk by Roberto)

- "Asymptotic" O(a)-improvement holds at large p^2 (and $m \to 0$)

- "Automatic" O(a)-improvement holds for parity-even correlators (even when gauge fixing is implemented)

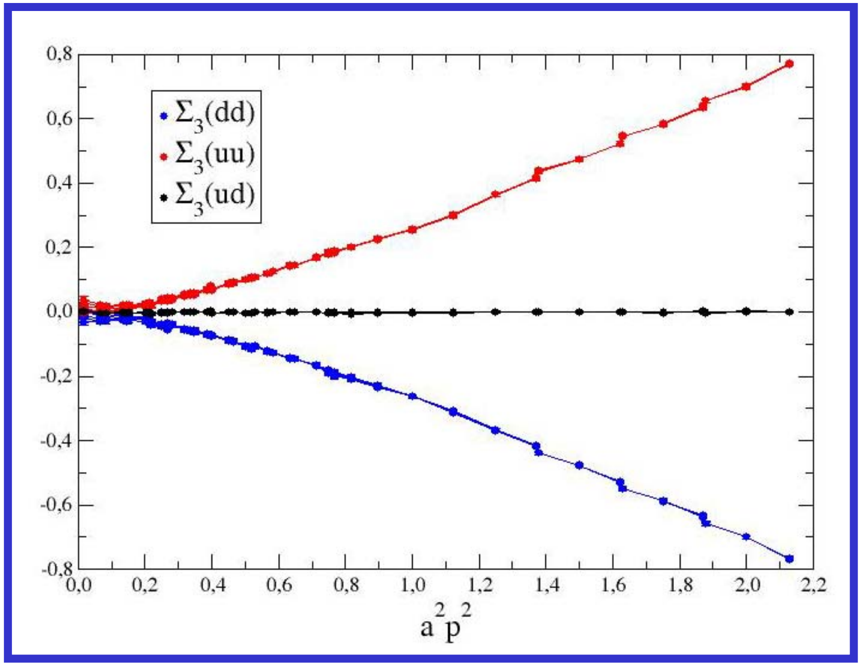
- Because the action is invariant under $P \times (u \leftrightarrow d)$, O(a)-improved results can be obtained by averaging over $(u \leftrightarrow d)$

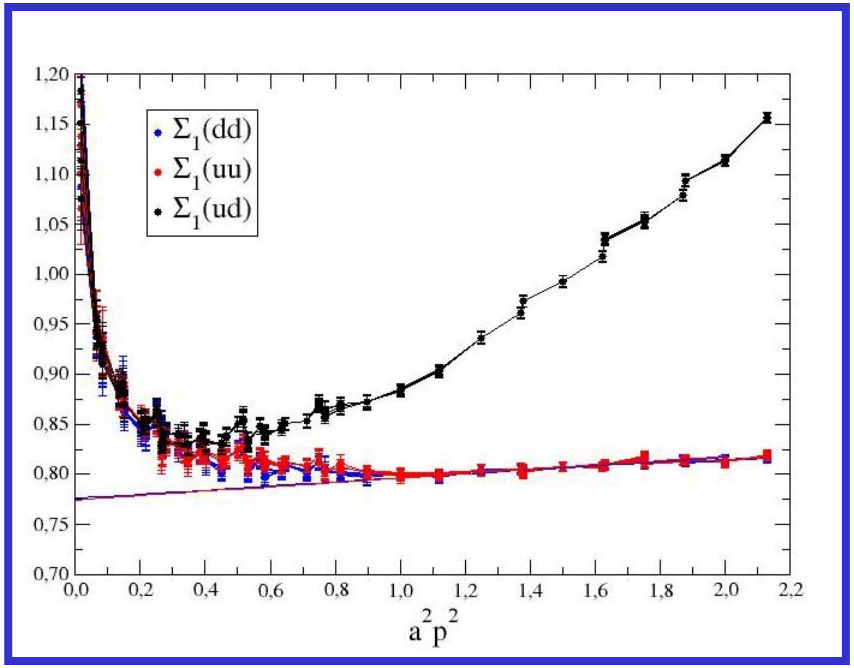
TWO STRATEGIES:

1) $\overline{G}(p)=[G^{ud}(p)+ G^{du}(p)]/2$, $\overline{S}(p)=[S_u(p)+ S_d(p)]/2$

2) $Z=[Z^{ud}+ Z^{du}]/2$

STRATEGY 1): $\bar{S}(p)=[S_u(p)+ S_d(p)]/2$





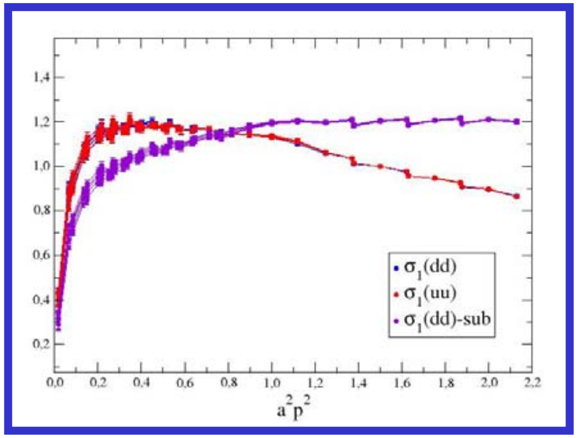
$$\bar{\Sigma}_1 \underset{p^2\to\infty}{\simeq} \Sigma_1^{u(d)}\left[1+\frac{1}{p^2}\left(\frac{\sigma_3}{\sigma_1}\right)^2\right] \simeq \Sigma_1^{u(d)}\left[1+O(a^2p^2)\right]$$

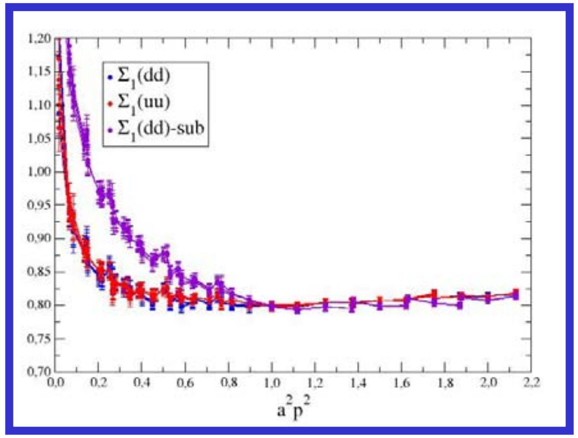
Large $O(a^2p^2)$ corrections.

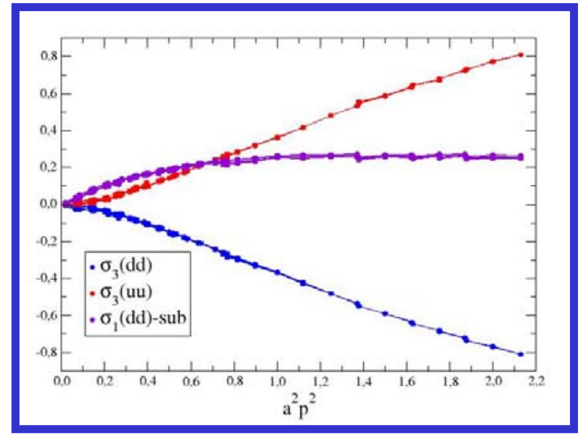
Use strategy 2) : $Z=[Z^{ud}+ Z^{du}]/2$

O(a²)-CORRECTIONS

[see D.Becirevic et al, hep-lat/9909082]



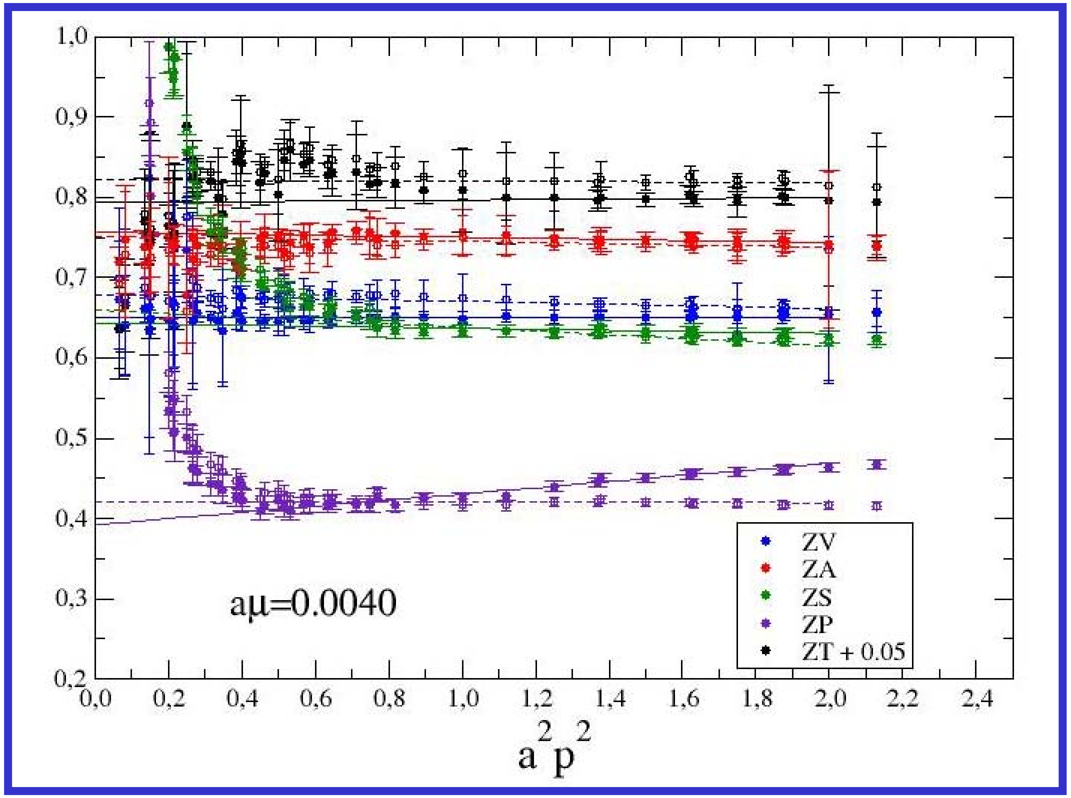




$$\sigma_1 = \frac{1}{\Sigma_1}\left[1+\frac{1}{p^2}\left(\frac{\Sigma_2}{\Sigma_1}\right)^2+\frac{1}{p^2}\left(\frac{\Sigma_3}{\Sigma_1}\right)^2\right]^{-1} \simeq \frac{1}{\Sigma_1}\left[1+z_3a^2p^2\right]^{-1}$$

$$\tilde{S}(p) = S(p)\left[1+z_3a^2p^2\right] - iCa\gamma_5$$

O(a²)-CORRECTIONS (cont.)



The largest effect is observed in ZP. But the final estimates of ZP are very close.

RESULTS

Z_V	0.65 (1)(2)
Z_A	0.75 (2)(2)
Z_S	0.61 (2)(2)
Z_P	0.39 (1)(2)
Z_T	0.74 (1)(2)
Z_q	0.78 (1)(1)

Note: $Z_V \simeq 0.61$ with other methods

	RI-MOM				BPT
	$a\mu$=0.0150	$a\mu$=0.0100	$a\mu$=0.0064	$a\mu$=0.0040	1-loop
ZV	0.638(35)	0.644(9)	0.662(10)	0.647(13) 0.635(23) 0.670(11)	0.63-0.71
ZA	0.764(17)	0.753(14)	0.778(40)	0.751(36) 0.750(19) 0.749(40)	0.72-0.78
ZS	0.621(13)	0.625(18)	0.638(14)	0.605(15) 0.600(15) 0.622(20)	0.70-0.76
ZP	0.373(14)	0.391(10)	0.407(14)	0.371(11) 0.377(14) 0.399(10)	0.52-0.62
ZT	0.752(34)	0.745(24)	0.755(16)	0.738(19) 0.743(18) 0.766(25)	0.70-0.76
ZP/ZS	0.581(24)	0.633(23)	0.632(25)	0.613(21) 0.617(26) 0.642(20)	0.74-0.82
Zq	0.783(13)	0.775(9)	0.789(10)	0.776(10) 0.774(10) 0.771(10)	0.71-0.77

TWISTED vs UNTWISTED

Wilson Plaquette + Clover, Nf=0, β=6.2

	Twisted	Untwisted
Z_V	0.788(42)	0.783(3)
Z_A	0.809(7)	0.819(3)
Z_S	0.546(8)	0.564(4)
Z_P	0.540(9)	0.642(3)
Z_T	0.875(4)	0.876(2)
Z_q	0.853(2)	0.850(2)

D.Becirevic et al, hep-lat/0401033

Large $O(a^2)$-corrections

CONCLUSIONS

- **Non-perturbative renormalization** is a crucial ingredient to achieve a percent precision on quark masses and hadronic matrix elements

- **O(a)-improvement of RCs is (almost) "automatic"** and can be achieved by following several strategies

- Determinations which differ by **$O(a^2)$ terms** agree better than 5%. But the issue of **ZV** should be clarified. Further studies are possible (e.g. $Z(a\mu)/Z(a'\mu)$ independent of μ up to discretization effects).

- **Future improvements**: implement i) the Liverpool stochastic method and ii) antiperidodic boundary conditions.