Matrix element of the electromagnetic operator between Kaon and pion states

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Outline

Kaon rare semileptonic decays as new physics probes

Rare Kaon decays have not been detected yet:

$$BR(K_L \to \pi^0 \ell^\pm \ell^\mp)_{exp} < 6.6 \cdot 10^{-10}$$

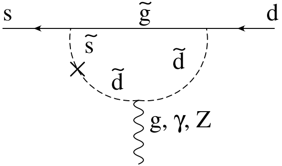
In the SM they are estimated to be

$$BR(K \to \pi ee)_{SM} \sim 1.5 \cdot 10^{-12}$$

$$BR(K \to \pi \mu\mu)_{SM} \sim 3 \cdot 10^{-10}$$

New physics can be the leading contribution, mediated through the Electro-magnetic and Chromo-magnetic operators:

$$\mathcal{Q}^+_{EM} = \overline{s} F_{\mu\nu} \sigma^{\mu\nu} d$$
$$\mathcal{Q}^+_{CM} = \overline{s} G_{\mu\nu} \sigma^{\mu\nu} d$$



- Sensitive to hadronic matrix elements.

Previous work

First lattice calculation [Becirevic et al. 2001] of the EM form factor

$$f_T(q^2 = 0) = 0.77 \pm 0.06 \pm 0.03$$

With the slope in q^2

$$\lambda = 1.21 \pm 0.05 \text{ GeV}^{-2}$$

- Quenched $(n_f = 0)$
- High pion masses $(530 < m_\pi < 800 MeV)$
- One lattice size $(a^{-1} = 2.7(1)\ GeV)$

Lattice details

- ETMC lattice QCD simulations [ETMC 0701012, 0911.5061]
- Dynamical flavors: $n_f = 2$
- Pion mass range: $270 < m_\pi < 600$ MeV
- Lattice sizes: $24^3 \times 48$ and $32^3 \times 64$
- Lattice step sizes: $a = 0.068,\ 0.085,\ 0.10$ fm
- Action is Symanzik tree-level improved with maximally twisted-mass Wilson fermions
- Non perturbative renormalization in the RI/MOM scheme [ETMC 1004.1115]
- 3-point correlators with all-to-all stochastic propagator calculation, increase accuracy
- Breit momentum frame: $\vec{p}_K = \vec{p},\ \vec{p}_\pi = -\vec{p}$

Electromagnetic form factor calculation

$$\mathcal{Q}_{EM} = \bar{s}\sigma^{\mu\nu} d$$

The EM form factor is acquired from the EM matrix element by [Becirevic et al. 2001]:

$$\left\langle \frac{\pi^0}{\sqrt{2}} \left| \mathcal{Q}_{EM} \right| K^0 \right\rangle = i \left(p_K^\mu p_\pi^\nu - p_K^\nu p_\pi^\mu \right) \frac{\sqrt{2} f_T}{m_K + m_\pi}$$

To obtain the matrix elements from the 3-point correlators, we look at the lattice times far from the pion and Kaon sources

$$C_3^{K\pi} \rightarrow \frac{\sqrt{Z_K Z_\pi}}{4 E_K E_\pi} \left\langle \pi^0 \left| \mathcal{Q}_{EM} \right| K^0 \right\rangle e^{-E_K t_x - E_\pi (t_y - t_x)}$$

and use the ratio

$$\frac{C_3^{K\pi} C_3^{\pi K}}{C_2^\pi(t_y) C_2^K(t_y)} \rightarrow \frac{\left\langle \pi^0 \left| \mathcal{Q}_{EM} \right| K^0 \right\rangle^2}{16 E_K E_\pi}$$

where t_y is a fixed point $t_y = T/2$.

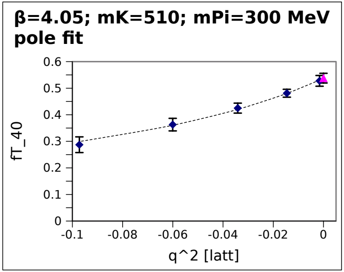
Calculations

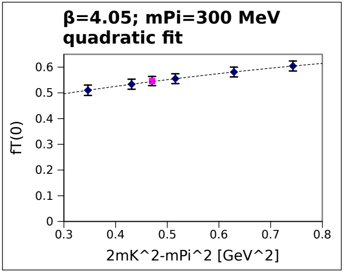
- Interpolation in momentum to $q^2 = 0$, assuming pole behaviour:

$$f_T(q^2) = \frac{f_T(0)}{1 - q^2 \lambda}$$

- Interpolation to physical strange mass:

$$(2m_K^2 - m_\pi^2)_{LATT} \rightarrow (2m_K^2 - m_\pi^2)_{PHYS} \propto (m_s)_{PHYS}$$

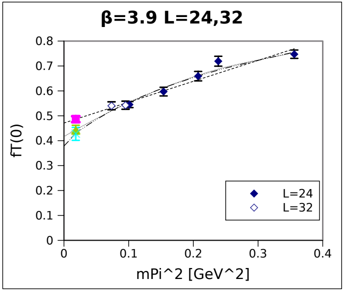


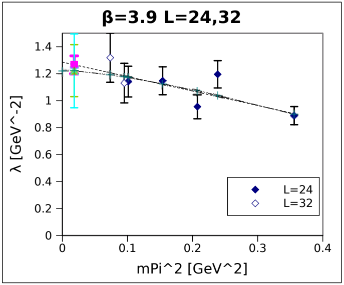


Extrapolation in masses

Extrapolation in m_π^2 to physical pion mass $m_{\pi^0} = 135$ MeV

- linear $f = Am_\pi^2 + B$
- quadratic $f = A'm_\pi^4 + B'm_\pi^2 + C'$
- log-linear $f = A''m_\pi^2 ln(m_\pi^2) + B''m_\pi^2 + C''$

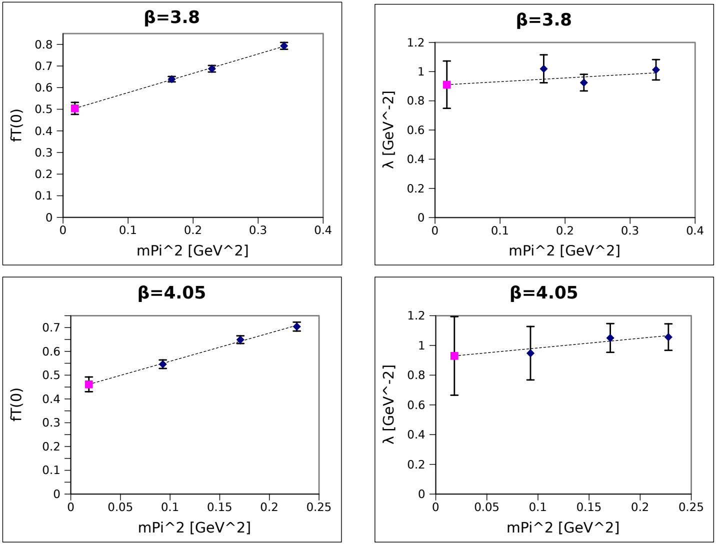




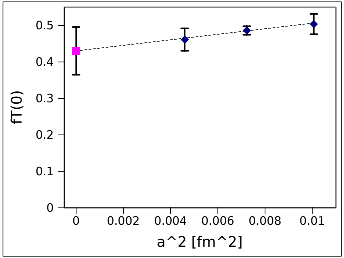
- Small finite volume effects

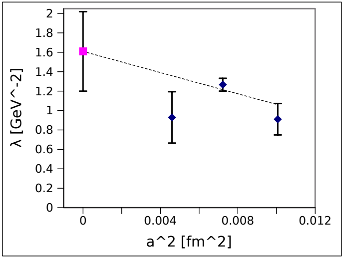
Results

Other lattice spacings:



Results





We find (Preliminary results, no systematic effects)

$$f_T(q^2 = 0) = 0.430 \pm 0.066^{stat}$$

$$\lambda = 1.61 \pm 0.41^{stat} \text{ GeV}^{-2} \text{ (slope in } q^2\text{, pole fit)}$$

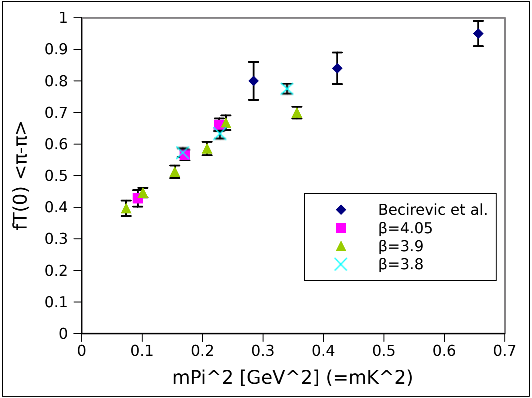
To compare with [Becirevic et al. 2001] (linear fit)

$$f_T(0) = 0.77 \pm 0.06 \pm 0.03$$

$$\lambda = 1.21 \pm 0.05 \text{ GeV}^{-2}$$

Results

Confronting results for $m_K = m_\pi$ for similar lattice sizes a



Similar behaviour, difference may be due to:

- Extrapolation from large pion masses
- Quenching effects

Work in progress

- Electro-magnetic operator $\mathcal{Q}_{EM}^{+} = \bar{s}\sigma^{\mu\nu}d$
  - Combined fit for all lattice spacings
  - Systematic errors analysis (chiral extrapolation, momentum dependence)
- Chromo-magnetic operator $\mathcal{Q}_{CM}^{+} = \bar{s}G_{\mu\nu}\sigma^{\mu\nu}d$
  - No previous lattice calculation
  - Matrix elements calculation
  - Renormalization

$$\mathcal{Q}_{CM}^{renorm} = Z_{CM}\left(\mathcal{Q}_{CM}^{bare} + \frac{c}{a}\mathcal{Q}_S\right)$$

  - additive – subtraction of mixing with scalar operator
  - multiplicative – 1-loop lattice perturbation theory [H. Panagopoulos et al.]

Conclusions

- Previously, single calculation (2001) of the EM operator
- Our calculations were performed for a large range of masses and lattice spacings
- Higher statistical accuracy achieved
- Values at $q^2 = 0$ differ, may be due to either quenching or smaller pion masses
- Slope in q^2 is consistent with previous result, but with higher preliminary statistical error
- Chromo-magnetic operator is work-in-progress

Thank you!

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