

Form factors of the $D \rightarrow \pi$ and $D \rightarrow K$ semileptonic decays

S. Di Vita*, B. Haas, V. Lubicz, F. Mescia, S. Simula and C. Tarantino

Università degli Studi Roma Tre & INFN Sezione Roma Tre

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Outline

- 1 Motivation and simulation details
- 2 Steps towards physical results
- 3 Results



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Motivation

- ▶ recent accurate experimental determinations of the semileptonic rates (Belle, BaBar, BES, Cleo III, CLEO-c, FOCUS).

Two strategies:

$$\Gamma_{exp} + \text{LQCD input} \\ \Rightarrow |V_{cq}|$$

$\Gamma_{exp} + \text{CKM unitarity}$
 $\Rightarrow$ LQCD results test,
then use for B physics

- ▶ Two *unquenched* determinations for $D \rightarrow \pi, K$ (FNAL-MILC, HPQCD), both use staggered dynamical quarks [Aubin et al. 2005, Bernard et al. 2009]



$D \rightarrow P \ell \nu_\ell$ ($P = \pi, K$) differential decay width

Approximate formula for $m_\ell \sim 0$

$$\frac{d\Gamma}{dq^2}(D \rightarrow P \ell \nu_\ell) = |V_{cq}|^2 \frac{G_F^2}{192\pi^3 m_D^3} \lambda^{3/2}(q^2) |f_+(q^2)|^2$$

Ingredients

- ▶ CKM matrix element squared
- ▶ Known, well measured factors
- ▶ Kinematic triangle function



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Form factors (scalar functions of $q^2 = (p - k)^2$, m_D , m_P)

$$\langle P(k) | \bar{q} \gamma_\mu c | D(p) \rangle = \left(p + k - q \frac{m_D^2 - m_P^2}{q^2} \right)_\mu f_+(q^2) + q_\mu \frac{m_D^2 - m_P^2}{q^2} f_0(q^2)$$

- ▶ **Vector form factor**
- ▶ **Scalar form factor** (negligible contribution $\propto m_\ell^2 \simeq 0$ if $\ell = e, \mu$)
- ▶ Kinematic constraint: $f_+(q^2 = 0) = f_0(q^2 = 0)$



ETMC simulation details

► Gauge sector $\Rightarrow$ Tree level Symanzik improved action

► Quarks sector $\Rightarrow$ Twisted Mass Wilson fermions at maximal twist $\Rightarrow$ automatic $O(a)$ improvement for physical quantities [Frezzotti and Rossi 2004]

► $N_f = 2$ dynamical degenerate light quarks

► $a \simeq \{0.102, 0.086, 0.068\} \text{ fm}$ (using f_π^{phys})

► $L^3 \times T = 24^3 \times 48$ and $32^3 \times 64$, $aL \sim 2.0 \div 2.7 \text{ fm}$

► $m_{ud}^{\text{lat}} \gtrsim m_s/6 \simeq 15 \div 20 \text{ MeV} \Rightarrow \underline{m_\pi \simeq 270 \div 500 \text{ MeV}}$

► $\underline{m_s^{\text{lat}} \simeq m_s \simeq 90 \div 120 \text{ MeV}}$, $\underline{m_c^{\text{lat}} \simeq m_c \simeq 1.2 \text{ GeV}}$

► twisted boundary conditions $\Rightarrow a\vec{p} = \frac{2\pi\vec{n}}{L} + \vec{\theta}$
[Bedaque 2004, Petronzio et al. 2004, Sachrajda and Villadoro 2005]



Extraction method (e.g. $D \rightarrow \pi$)

- ▶ reduce stat. fluctuations
- ▶ suitable double ratios of 2pts and 3pts $\Rightarrow$ no need for $\mathcal{Z}_V$ and $\mathcal{Z}_{\pi(D)}$
[Hashimoto et al. 2000, Becirevic et al. 2005]
- ▶ exploit electric charge normalization
 $\langle H(-\vec{q}) | V_4 | H(\vec{q}) \rangle = 2m_H \cdot f(q^2 = 0) = 2m_H \cdot 1$

Ratios and double ratios

$$\begin{aligned} & \frac{C_4^{DV\pi}(\vec{0}, t) C_4^{\pi VD}(\vec{0}, t)}{C_4^{\pi V\pi}(\vec{0}, t) C_4^{DVD}(\vec{0}, t)} \xrightarrow{\text{plateau}} R'_0, \\ & \frac{C_4^{\pi VD}(\vec{q}, t) C^{\pi\pi}(\vec{0}, t)}{C_4^{\pi VD}(\vec{0}, t) C^{\pi\pi}(\vec{q}, t)} \times \frac{C^{DD}(\vec{0}, (t_{\text{source}} - t))}{C^{DD}(\vec{q}, (t_{\text{source}} - t))} \xrightarrow{\text{plateau}} R'_1(q^2), \\ & \frac{C_i^{\pi VD}(\vec{q}, t)}{C_4^{\pi VD}(\vec{q}, t)} \xrightarrow{\text{plateau}} R'_2(q^2), \end{aligned}$$

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$$\begin{aligned} R'_0 &= \frac{(m_\pi + m_D)^2}{4m_\pi m_D} (f_0(q_{\max}^2))^2, \\ R'_1(q^2) &= \frac{E_D + E_\pi}{m_\pi + m_D} \frac{f_+(q^2)}{f_0(q_{\max}^2)} \left[1 + \frac{E_D - E_\pi}{E_D + E_\pi} \xi(q^2) \right], \\ R'_2(q^2) &= \frac{2|\vec{q}| \xi(q^2)}{E_D + E_\pi + (E_D - E_\pi) \xi(q^2)}. \end{aligned}$$

$$\xi(q^2) = f_-(q^2)/f_+(q^2)$$



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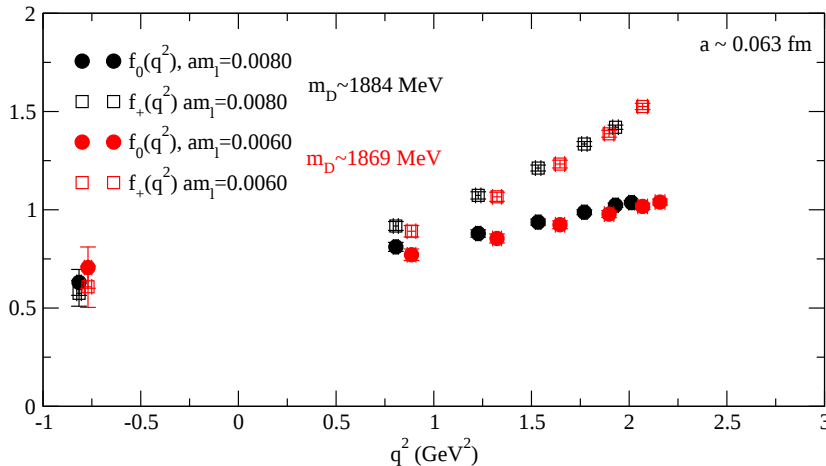
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Recipe

Simulated quark masses are not the physical ones. Moreover one wants $a \rightarrow 0$ limit.

- extract $f_{+,0}(q^2)$ at the several values of (a, m_l, m_s, m_c)



Recipe

Simulated quark masses are not the physical ones. Moreover one wants $a \rightarrow 0$ limit.

- ▶ extract $f_{+,0}(q^2)$ at the several values of (a, m_l, m_s, m_c)
- ▶ interpolate to physical charm quark mass [see F. Sanfilippo's talk]
- ▶ interpolate to physical strange quark mass by fixing the combination $2m_K^2 - m_\pi^2$ to its physical value ($\sim 476 \text{ MeV}$)
- ▶ use HMChPT to fit the data in (m_π, q^2) and extrapolate to m_π^{phys}
- ▶ parametrically include $\mathcal{O}(a^2)$ effects in the chiral formulae



HMChPT formulae (1) [Becirevic et al. 2003, 2004]

for calculations on the lattice and in HQET it is more convenient to use the decomposition

$$\langle P | \bar{q} \gamma_\mu c | D \rangle = \sqrt{m_D} [v^\mu f_v(E_P) + p_T^\mu f_p(E_P)]$$

- ▶ $v^\mu = p_D^\mu / m_D$, 4-velocity of the D meson
- ▶ p_T^μ is the component of p_P^μ orthogonal to v^μ
- ▶ $E_P = v \cdot p_P = (m_D^2 + m_P^2 - q^2) / (2m_D)$,
P meson energy in the D rest frame

$$f_0(q^2) = \frac{\sqrt{2m_D}}{m_D^2 - m_P^2} [(m_D - E_P) f_v(E_P) - p_T^2 f_p(E_P)]$$
$$f_+(q^2) = \frac{1}{m_D} [f_v(E_P) + (m_D - E_P) f_p(E_P)]$$

(note that the kinematical constraint is automatically satisfied)



HMChPT formulae (2) [Becirevic et al. 2003, 2004]

Zeroth order in $1/m_D$, LO continuum formulae (chiral limit, soft pion)

$g = g_{D^* D \pi} \simeq 0.6$ from [Becirevic et al. 2009]

$$f_p^{LO}(m, E) = \frac{f_D \sqrt{m_D}}{f_\pi} \frac{g}{E + \Delta}, \quad f_v^{LO}(m, E) = \frac{f_{D^*} \sqrt{m_D^*}}{f_\pi}$$

($\Delta = m_{D^*} - m_D$ correctly accounts for the pole, ($\Delta \simeq 145 \text{ MeV}$, from PDG). The fit formulae, including $\mathcal{O}(a^2)$ discretization effects

$$f_p(m, E) = \frac{C_0}{E + \Delta} \left(1 + \underbrace{\frac{3}{4}(1 + 3g^2)\xi \log \xi}_{\text{chiral logs}} + \underbrace{C_1(E)m^2 + C_2(E)}_{\text{analytic terms}} + C_3 a^2 \right)$$
$$f_v(m, E) = D_0 \left(1 + \underbrace{\frac{9g^2 - 5}{4}\xi \log \xi - 2E^2 F(m, E)}_{\text{chiral logs}} + \underbrace{D_1(E)m^2 + D_2(E)}_{\text{analytic terms}} + D_3 a^2 \right)$$

($\xi = m^2/(4\pi f)^2$, $f \simeq 0.121 \text{ MeV}$, [Baron et al. 2009])



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$$F(m, E) = \begin{cases} \log \xi + 2\sqrt{1 - (m/E)^2} \log \left[E/m(1 + \sqrt{1 - (m/E)^2}) \right] & E \geq m \\ \log \xi - 2\sqrt{(m/E)^2 - 1} \left[\pi/2 - \arctan \left(1/(\sqrt{(m/E)^2 - 1}) \right) \right] & E < m \end{cases}$$

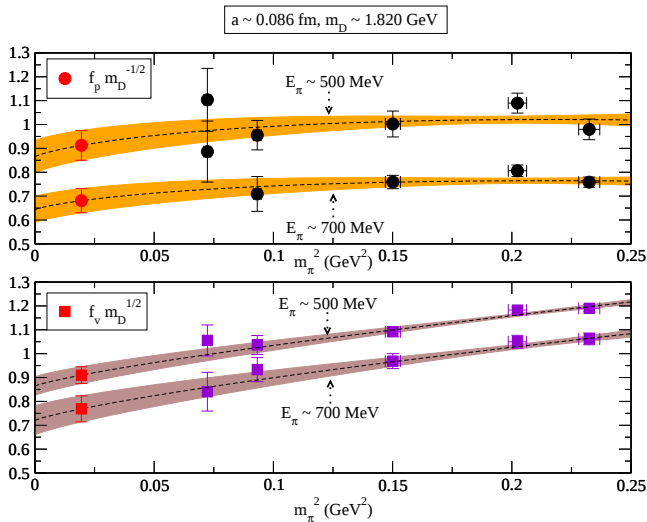


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Fit quality: m_π dependence

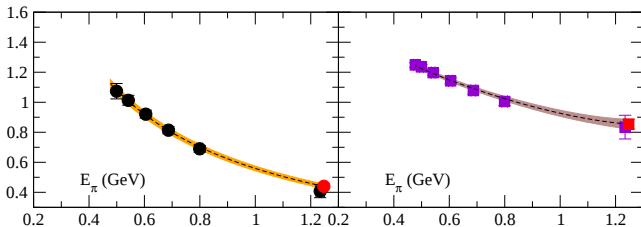
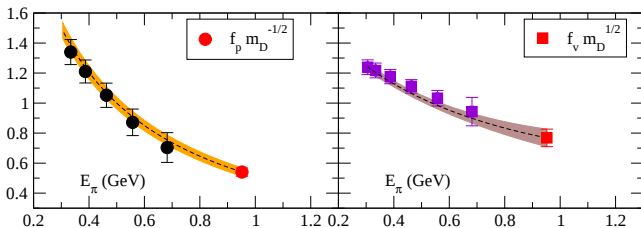


- chiral behaviour for fixed a and at a given value of E_π



Fit quality: dependence on E_π

$a \sim 0.086 \text{ fm}, m_D \sim 1.717 \text{ GeV}, m_\pi \sim 0.305 \text{ GeV}$

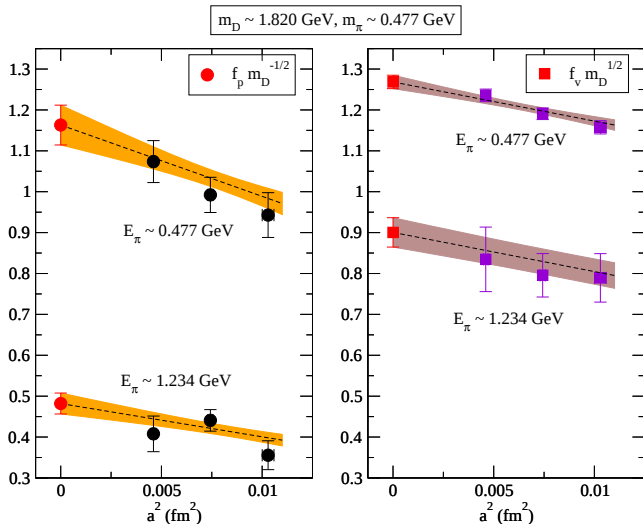


$a \sim 0.068 \text{ fm}, m_D \sim 1.820 \text{ GeV}, m_\pi \sim 0.477 \text{ GeV}$

- ▶ dependence on E_π , for fixed m_π and fixed a
- ▶ $C_1(E)$, $C_2(E)$ and $D_1(E)$, $D_2(E)$
- ▶ simple polynomial ansatz for E dependence
- ▶ pole dominates f_p
- ▶ more terms in f_v



Fit quality: discretization effects

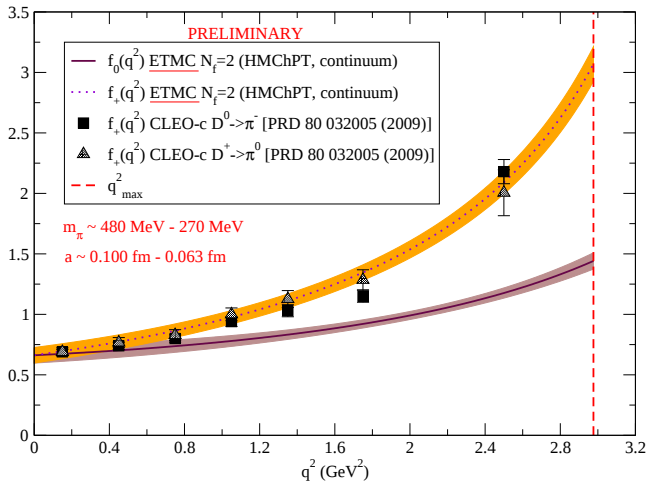


- dependence on a^2 , for fixed m_π and at different values of E_π

well described by a term linear in a^2



$N_f = 2$ form factors at the physical point: $D \rightarrow \pi$

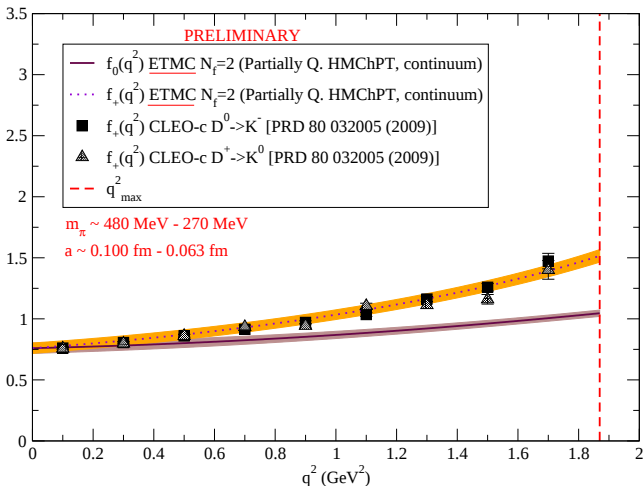


- CKM unitarity is assumed
- $|V_{cd}| \simeq \lambda = 0.2258$ [UTFIT]
- good agreement with bin per bin exp data



$N_f = 2$ form factors at the physical point: $D \rightarrow K$ (1)

$f_{+,0}(q^2)$: lattice vs CLEO-c

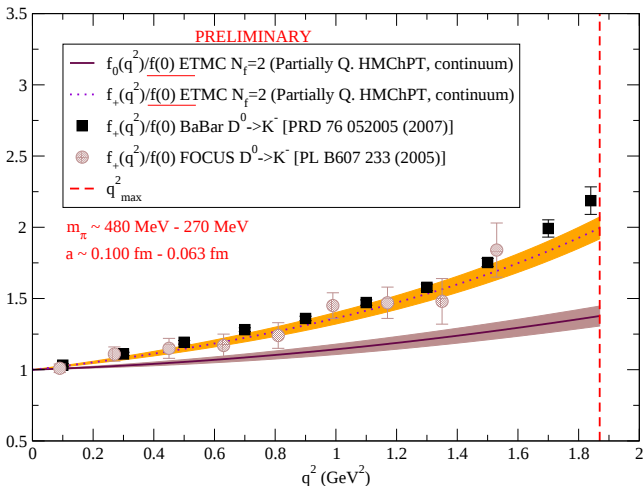


- CKM unitarity is assumed
- $|V_{cs}| \simeq 1 - \lambda^2/2 = 0.9745$ [UTFIT]
- good agreement with bin per bin exp data



$N_f = 2$ form factors at the physical point: $D \rightarrow K(2)$

$f_{+,0}(q^2)$ normalized to $f(0)$: lattice vs BaBar and FOCUS



- independent on $|V_{cs}|$
- good agreement with bin per bin exp data



Conclusions and outlook



- ▶ Obtained: $N_f = 2$ form factors for both $D \rightarrow \pi$ and $D \rightarrow K$ at the physical point
 - ▶ HMChPT for extrapolating towards physical pion mass, down to $q^2 = 0$
 - ▶ HMChPT coefficients of analytic terms depend on energy $\Rightarrow$ with simple polynomial ansatz the formulae describe the q^2 dependence
 - ▶ Maximally twisted tmLQCD $\Rightarrow$ only $\mathcal{O}(a^2)$ discr. effects in phys. obs.
 - ▶ FSE under control ($M_\pi L \gtrsim 4$ or $M_\pi L \gtrsim 3.7$)
- ▶ Outlook:
 - ▶ Momentum dependence through different parameterizations (BK , z -expansion)
 - ▶ $|V_{cd}|$ and $|V_{cs}|$
 - ▶ What about B physics?

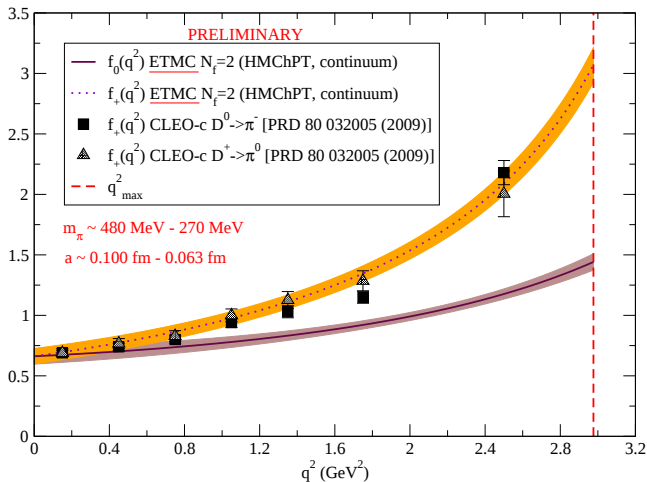




Thanks everybody!



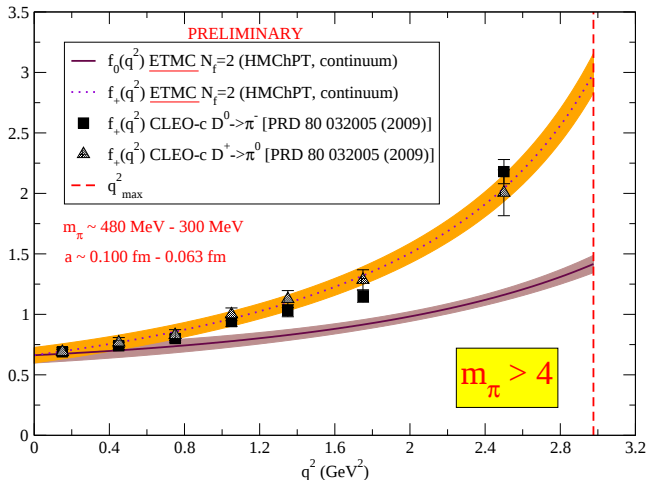
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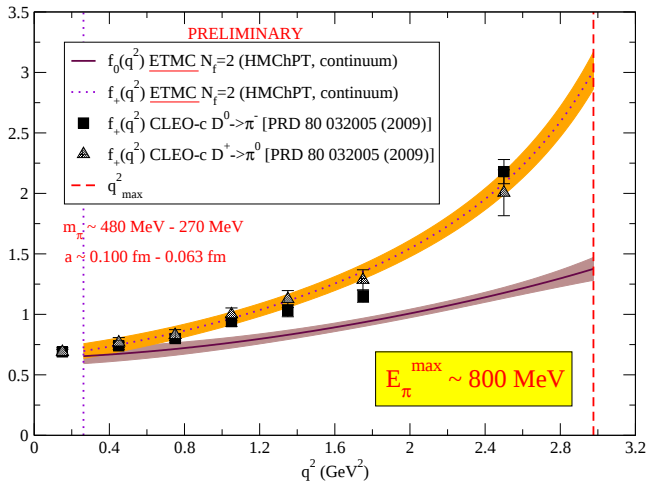
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