Heavy decay constants and KLM factors

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Twisted mass fermions

- Motivation for computing decay constants in charmonuim
- Lattice calculation of decay constants in charmonium
- Trying to understand $O((aM)^n)$ factors

Use the correlators from the ETM collaboration's n_f = 2 unquenched lattice QCD calculations at β = 3.9 and β = 4.05. Wilson twisted mass quarks and tree level improved Symanzik gauge action. Silvano Simula and Petros Dimopoulos have also worked on this analysis of decay constants in charmonuim.

Decay constants in charmonium

One way to understand charmonium production is to use the NRQCD formalism, where non-perturbative information is coded in a few matrix elements. NRQCD mostly produces a good description of experiment [a]

However consider recent result from from Belle (hep-ex/0205104)

$$\sigma[e^+e^- \rightarrow J/\psi + \eta_c]\mathcal{B} = 25.6 \pm 2.8 \pm 3.4 \ fb$$

where $\mathcal{B} < 1$. (BaBar has a similar result). Compare to the prediction from leading order NRQCD by Bodwin et al. (hep-ph/0212352).

$$\sigma[e^+e^- \rightarrow J/\psi + \eta_c]_{(NRQCD;LO)} = 3.78 \pm 1.26 \ fb$$

- It is claimed that calculations that use light cone wave functions of charm mesons agree with the Belle result (eg. hep-ph/0412335, Bondar and Chernyak)
- Bodwin (hep-ph/0509203) not clear that model light cone wave functions are to close to true quarkonuim wave functions.

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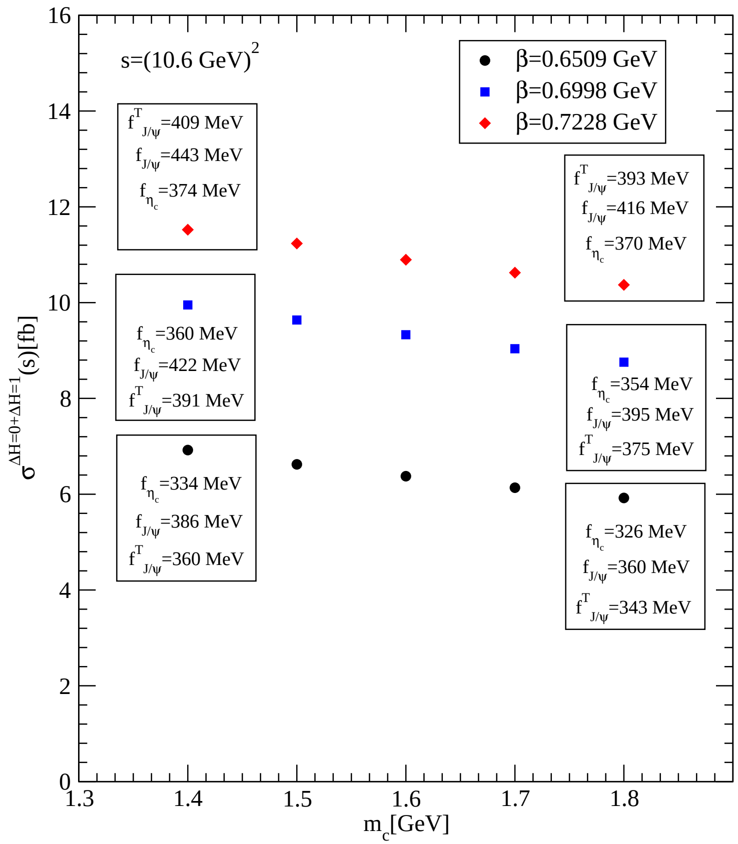
[a] see review Heavy quarkonium physics, hep-ph/0412158

From Choi and Ji (arXiv:0707.1173)



The above is from a quark model calculation. Probably also need higher order QCD corrections to get agreement with Belle.

Leptonic decay constants

In the continuum the decay constant of the J/ψ meson is defined via

$$\langle 0 \mid \overline{c}(x)\gamma_\mu c(x) \mid J/\psi\rangle = m_{J/\psi} f_{J/\psi}\epsilon_\mu$$

The transverse decay constant ($f^T_{J/\psi}(\mu)$) of the J/ψ meson is defined by

$$\langle 0 \mid \overline{c}\sigma_{\mu\nu} c \mid \rho\rangle = i f^T_{J/\psi}(\mu)(p_\mu\epsilon_\nu - p_\nu\epsilon_\mu)$$

where $\sigma_{\mu\nu} = i/2[\gamma_\mu, \gamma_\nu]$. The transverse decay constant is used in light cone sum rule calculations.
For the η_c meson will use the standard definition of the decay constant of the pseudoscalar meson.

$$f_{PS} = (2\mu)\frac{\mid \langle 0 \mid P^1(0) \mid P\rangle \mid}{M^2_{PS}}$$

As stressed by FNAL and HPQCD collaborations (PoS LATTICE2007:353,2007) the computation of the decay constant of the J/ψ meson is an important validation test for decay constants, because it is known from experiment.

Renormalisation

Use the results from the Rome-Southampton method, reported by ETMC in arXiv:0710.0975, Dimopoulos et al for light decay constants. Used updated analysis by Frezzotti at Trento meeting 2008 for charm data (1.4% (3%) increase Z_A (Z_T)).

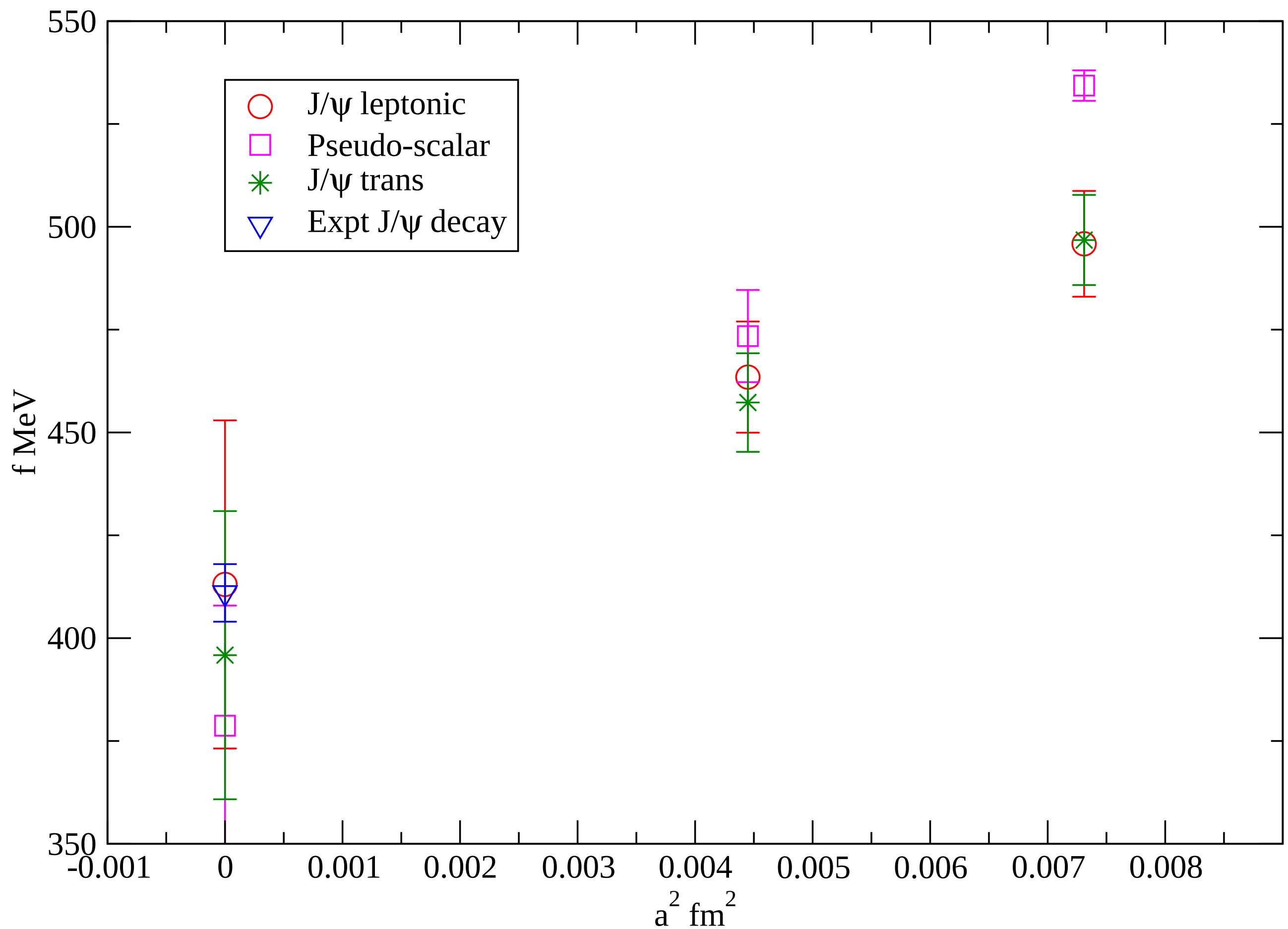
β	Z_A	Z_V	$Z_T(\mu = 1/a)$
3.90	0.76	0.65	0.75
4.10	0.77	0.67	0.79

The Z_T renormalisation factor depends on the scale. Run Z_T to 2 GeV from the scale of the inverse lattice spacing. Use method described by Becirevic et al. (hep-lat/0301020). There are updated expressions due to Gracey's three loop computation in the RI' scheme (hep-ph/0304113) that will be incorporated later.

Lattice calculation

- Use correlators generated using stochastic point sources (“one end trick”) on ApeNext at Rome.
- Use data at two lattice spacings (beta=3.9 and beta=4.05).
- Interpolate to charm with three heavy masses
- Simple chiral extrapolation in sea quark mass
- Used charged interpolating operators. We tried neutral operators and the results were similar (this is a test of $O(a^2)$ corrections).
- No disconnected diagrams included
- Used lattice spacing for β=3.9 (β=4.05) 0.0855 fm (0.0667 fm) from f_π.

Scaling of decay constants (preliminary)



Results for decay constants

Preliminary results. Needs more work on continuum extrapolation.

Decay	f MeV (preliminary)
$f_{J/\psi}$	413(40)
$f_{J/\psi;trans}$	396(35)
f_{η_c}	379(29)

- Compare to experiment $f_{J/\psi}$= 411(7) MeV (PDG, Dudek et al. hep-ph/0601137).
- "Sort of experiment result" f_{η_c} = 335(75) MeV from $B \to \eta_c K$ with factorization assumption from CLEO (hep-ex/0007012).
- Jlab group obtained (hep-ph/0601137) $f_{J/\psi}$ = 399(4) MeV f_{η_c} = 429(4)(28) MeV from quenched QCD
- Chiu and Hsieh obtained f_{η_c} = 438(5)(6) MeV from a quenched QCD calculation using the overlap fermion action (0705.2797).

Conclusions at lattice 2008

- I showed why computing the decay constants of charmonuim mesons is important for phemomenology.
- Results for the ground state decay constants in charmonium.
- Perhaps need a twisted FNAL heavy quark formalism to deal with $O((aM_Q)^n)$ $n > 1$ corrections [a]
- May also need to compute second moment of η_c. Also possible with twisted mass fermions (Baron et al. arXiv:0710.1580 for the pion).

[a] If anyone had asked me about this at the lattice conference, I would have said that it was a 2 hour job, and most of the work has been done in " Twisted Mass, Overlap and Creutz Fermions: Cut-off Effects at Tree-level of Perturbation Theory." by Cichy et al. However, two hours was an underestimate

KLM factors for Wilson ????

As the quark mass gets large $O(am_q)$ errors become important, the FNAL group and Peter Lepage [a] developed a formalism to deal with $O((am_q)^n)$ errors to all orders.

- Use the standard clover/Wilson action
- Use the kinetic mass M_2 defined from the dispersion relation by $E^2 = M_1^2 + \frac{M_1}{M_2}\vec{p}^2 + \dots$
- Use the KLM factor $Z_{KLM} = \sqrt{2\kappa e^{m_0}}$ where m_0 is defined by $m_0 = \log(\frac{1}{2\kappa} - 3)$ to multiply the quark fields

Before the majority of European lattice groups didn't like staggered fermions, they didn't like the FNAL formalism. The main criticism of the FNAL approach was that it used mass dependent re-normalization factors. (Also the original papers are difficult to understand and are almost unreadable). However, lattice groups such as UKQCD, Martinelli et al, Lubicz et al. sometimes have used the above to estimate systematic errors.

$$
\begin{aligned}
\lim_{m_q \to \infty} Z_{KLM} &= \lim_{\kappa \to 0} (1 - 6\kappa) = 1 \\
\lim_{m_q \to \infty} Z_{standard} &= \lim_{\kappa \to 0} (2\kappa) = 0
\end{aligned}
$$

The KLM factor

I originally started this study by modifying the free field study of: Massive Fermions in Lattice Gauge Theory Authors: Aida X. El-Khadra, Andreas S. Kronfeld, Paul B. Mackenzie (hep-lat/9604004). However the paper by Groote and Shigemitsu (hep-lat/0001021) explains the renormalisation condition in a better way

Define the Z_{KLM} quark renormalisation factor by considering the quark propagator at zero 3-momentum

$$
\begin{aligned}
S(t,0) &= \int_{-\pi/a}^{\pi/a} \frac{1}{2\pi} e^{ip_0 t} \overline{S}(p_0, 0) \\
&\equiv \frac{1}{Z_{KLM}} e^{-M_1 t} \frac{1+\gamma_4}{2} + \ldots.
\end{aligned}
$$

This looks like a “static quark propagator”, so sort of sensible for massive quarks.

Note the KLM factor was originally derived by Lüscher in Commun.Math.Phys.54:283,1977 (Construction Of A Selfadjoint, Strictly Positive Transfer Matrix For Euclidean Lattice Gauge Theories) (Andrea has worked through this construction for twisted mass QCD).

Twisted mass and FNAL

I can use the results by Cichy et al. (arXiv:0802.3637 [hep-lat]) for the twisted mass quark propagator in free field theory in 3-momentum and time, for $t > 0$

$$\begin{aligned} S(\vec{p}, t) &= \frac{1}{2\mathcal{Z} \sinh E_1} (1_f (\mathrm{sgn}(t) \sinh E_1 \gamma_4 - ia\mathcal{K}(\vec{p}) + [(1 - \cosh E_1) + aM(\vec{p})]) \\ &- ia\mu_q \gamma_5 \tau^3) e^{-E_1 t} \end{aligned}$$

where $\mathcal{Z}$ is calculated to be

$$\mathcal{Z} = 1 + m_0 + \frac{1}{2}\hat{p}^2$$

The mass of the state $M_1 = E_1(\vec{p} = \vec{0}, t)$

$$\cosh M_1 = 1 + \frac{a^2 m_0^2 + a^2 \mu_q^2}{2(1 + am_0)}$$

At maximal twist $m_0 = 0$ and there is an expansion of M_1 in terms of $(a\mu_q)^2$.
However the KLM factor is independent of the heavy twisted mass μ_q (a bit weird!)

$$Z_{KLM} = 1 + m_0$$

An aside – action tuning

The full FNAL formulation requires a tuning of the action. Consider adding a twisted term to the FNAL action (in the physical basis) written down by Christ and Lin (hep-lat/0608005). There are two parameters in the action r_s and ζ, after I have set the coefficient of the clover term to be zero. The standard twisted mass action is r_s = ζ =1.

$$S_F = \sum_x \overline{\psi}(x)(\gamma_0 D_0 + \zeta\gamma_i D_i + \mu_q - i\tau_3\gamma_5(-\frac{1}{2}D_0^2 - r_s\frac{1}{2}D_i^2 + m_{cr})\psi(x)$$

where m_{cr} mass is the critical mass tuned from making the PCAC mass zero. The above action is invariant under $\mathcal{P} \times \mathcal{D}_d \times (\mu_q \to -\mu_q)$ and hence should have automatic $O(a)$ improvement from the argument by Frezzotti et al. (hep-lat/0503034).

Automatic $O(a)$ improvement for this case means that r_s and ζ are an even function of the heavy mass and the clover coefficient is an odd function of the heavy mass.

The symmetry transformations.

The $\mathcal{P}$ symmetry transformation is defined by

$$\begin{aligned}
U_0(x) &\rightarrow U_0(x_P) \\
U_k(x) &\rightarrow U_k^\dagger(x_P - a\hat{k}) \\
\psi(x) &\rightarrow \gamma_0 \psi(x_P) \\
\overline{\psi}(x) &\rightarrow \overline{\psi}(x_P)\gamma_0
\end{aligned}$$

where ($x_P = (-x, t)$), and the $\mathcal{D}_d$ symmetry is defined by

$$\begin{aligned}
U_\mu(x) &\rightarrow U_\mu^\dagger(-x - a\hat{\mu}) \\
\psi(x) &\rightarrow e^{3i\pi/2}\psi(-x) \\
\overline{\psi}(x) &\rightarrow e^{3i\pi/2}\overline{\psi}(-x)
\end{aligned}$$

Conclusions from KLM factors

- The fact that the KLM factor is independent of the twisted mass is unusual. The HISQ and ASQTAD improved staggered actions actions have mass dependent KLM factors. I believe the overlap actions will also have mass dependent KLM factors, because of normalization issues with the square root.
- Perhaps, the KLM factor for twisted mass QCD is independent of the twist mass, because as Shindler notes the twisted mass and real mass “point in different directions.” (0707.4093)
- I am not sure why the scaling violations are so large for the heavy-heavy decay constants. Perhaps disconnected diagrams are more important.