Hadronic contribution to g-2

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Muon g-2

- radiative corrections to muon magnetic moment, g_μ, are given by

$$a_\mu = (g_\mu - 2)/2 = \frac{\alpha}{2\pi} + \mathcal{O}(\alpha^2)$$

- experimental measurement at BNL*

$$a_\mu^{\text{ex}} = 11\,659\,208.0\,(6.3) \times 10^{-10} \text{ [0.54 ppm]}$$

- standard model prediction† using $e^+e^- \to$ hadrons

$$a_\mu^{\text{th}} = 11\,659\,179.3\,(6.8) \times 10^{-10} \text{ [0.58 ppm]}$$

- discrepancy between theory and experiment

$$a_\mu^{\text{ex}} - a_\mu^{\text{th}} = 28.7\,(9.3) \times 10^{-10} \text{ [}3.1\,\sigma\text{]}$$

- leading order hadronic† (hlo) contribution dominates theory error

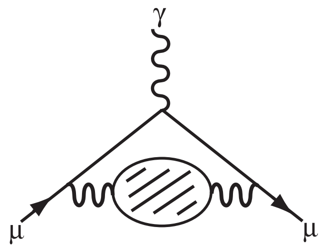
$$a_\mu^{\text{hlo}} = 692.1\,(5.6) \times 10^{-10} \text{ [60\% of theory error]}$$

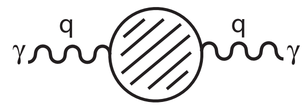
*Muon G-2, PRD73:072003, 2006

†e.g. review by Jegerlehner, Acta.Phys.Polon.B38:3021, 2007

Hadronic Vacuum Polarization

- vacuum polarization by quarks or equivalently hadrons





- vacuum polarization tensor

$$\pi_{\mu\nu}(q^2) = \int d^4x\, e^{iq\cdot(x-y)} \langle J_\mu(x) J_\nu(y) \rangle = (q_\mu q_\nu - q^2 \delta_{\mu\nu}) \pi(q^2)$$

- leading order hadronic contribution*

$$a_\mu^{\mathrm{hlo}} = \alpha^2 \int_0^\infty \frac{dq^2}{q^2} w(q^2/m_\mu^2)(\pi(q^2) - \pi(0))$$

- $w(q^2/m_\mu^2)$ is maximum at $q^2 = (\sqrt{5} - 2)m_\mu^2 \approx 0.003$ GeV^2
- lowest momentum on lattice is $q^2_{\mathrm{min}} = (2\pi/L)^2 \approx 0.05$ GeV^2

*Blum, PRL95:052001, 2003

Electromagnetic Currents

- physical charge (and flavor) currents remain unchanged under twisting

$$Q\gamma_\mu = \exp(-i\gamma_5\tau_3\theta)Q\gamma_\mu \exp(-i\gamma_5\tau_3\theta) \quad \text{for} \quad Q = 1, \tau_3$$

- we use the conserved vector current in the twisted basis

$$J_\mu^{\text{lc,ph}} \stackrel{a \to 0}{=} J_\mu^{\text{lc,tw}} \stackrel{a \to 0}{=} J_\mu^{\text{cc,tw}}$$

- conserved current has same point-split form as for Wilson fermions

$$J_{\mu x}^{\text{tw}} = \frac{1}{2}\left\{\bar{\chi}_{x+\mu}^{\text{tw}}(r+\gamma_\mu)\chi_x^{\text{tw}} - \bar{\chi}_x^{\text{tw}}(r-\gamma_\mu)\chi_{x+\mu}^{\text{tw}}\right\}$$

Flavor Breaking

- γ_5 hermiticity, $\gamma_5 D_u^\dagger \gamma_5 = D_d$, relates u and d quark loops

$$\pi^d_{\mu\nu}(x,y) = \pi^{u*}_{\mu\nu}(x,y)$$

- explicit flavor symmetry breaking is removed by retaining only real part

$$\mathrm{re}(\pi^d(q^2)) = \mathrm{re}(\pi^u(q^2))$$

- this is true for each background gauge field
- real part accurate to $\mathcal{O}(a^2)$ but imaginary part likely only $\mathcal{O}(a)$

q^2 **Dependence**

- the leading order hadronic contribution, again

$$a_\mu^{\mathrm{hlo}} = \alpha^2 \int_0^\infty \frac{dq^2}{q^2} w(q^2/m_\mu^2)(\pi(q^2) - \pi(0))$$

- polynomial fits: cubic and quartic

$$\pi(q^2) = \sum_n a_n \left(q^2\right)^n$$

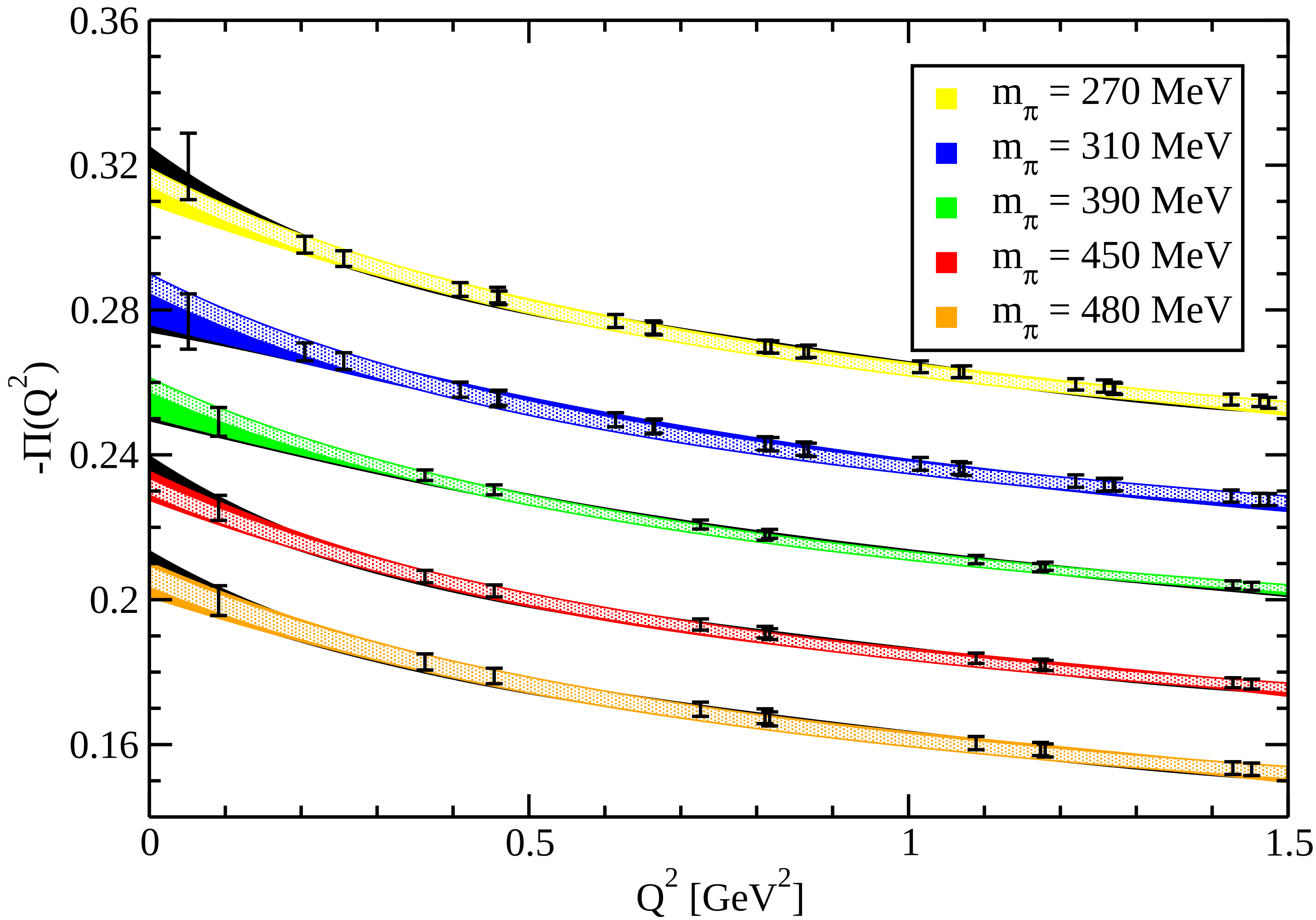
- χpt fits

$$\pi(q^2) = \frac{c f_\rho^2}{m_\rho^2 + q^2} + A$$

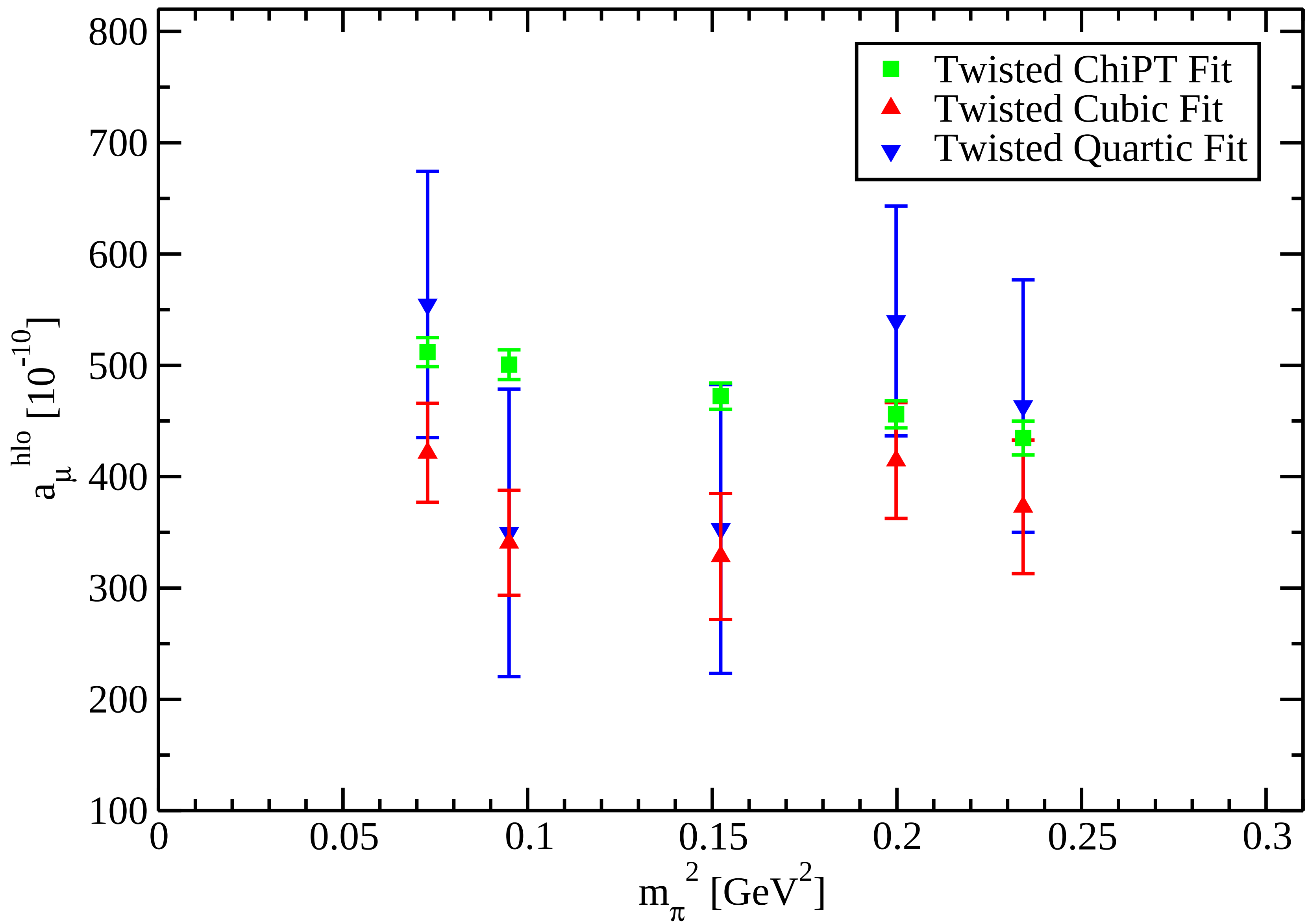
Status of Calculation

β	$a\mu$	V/a^4	a	L	m_π	$m_\pi L$	N_{vac}	stat
3.90	0.0150	$24^3 \times 48$	0.086	2.1	597	6.2	-	-
3.90	0.0100	$24^3 \times 48$	0.086	2.1	484	5.0	121	F
3.90	0.0085	$24^3 \times 48$	0.086	2.1	447	4.6	207	F
3.90	0.0064	$24^3 \times 48$	0.086	2.1	390	4.1	278	F
3.90	0.0040	$16^3 \times 32$	0.086	1.4	365	2.5	-	-
3.90	0.0040	$20^3 \times 40$	0.086	1.7	318	2.8	-	-
3.90	0.0040	$24^3 \times 48$	0.086	2.1	310	3.3	324	F
3.90	0.0040	$32^3 \times 64$	0.086	2.7	310	4.3	124	F
3.90	0.0030	$32^3 \times 64$	0.086	2.7	270	3.7	129	F
4.05	0.0120	$32^3 \times 64$	0.067	2.1	598	6.4	-	-
4.05	0.0080	$32^3 \times 64$	0.067	2.1	488	5.2	98	F
4.05	0.0060	$20^3 \times 48$	0.067	1.3	449	3.0	-	-
4.05	0.0060	$24^3 \times 48$	0.067	1.6	428	3.5	$>$80	R
4.05	0.0060	$32^3 \times 64$	0.067	2.1	420	4.6	118	F
4.05	0.0030	$20^3 \times 40$	0.067	1.3	359	2.4	-	-
4.05	0.0030	$32^3 \times 64$	0.067	2.1	310	3.3	104	F

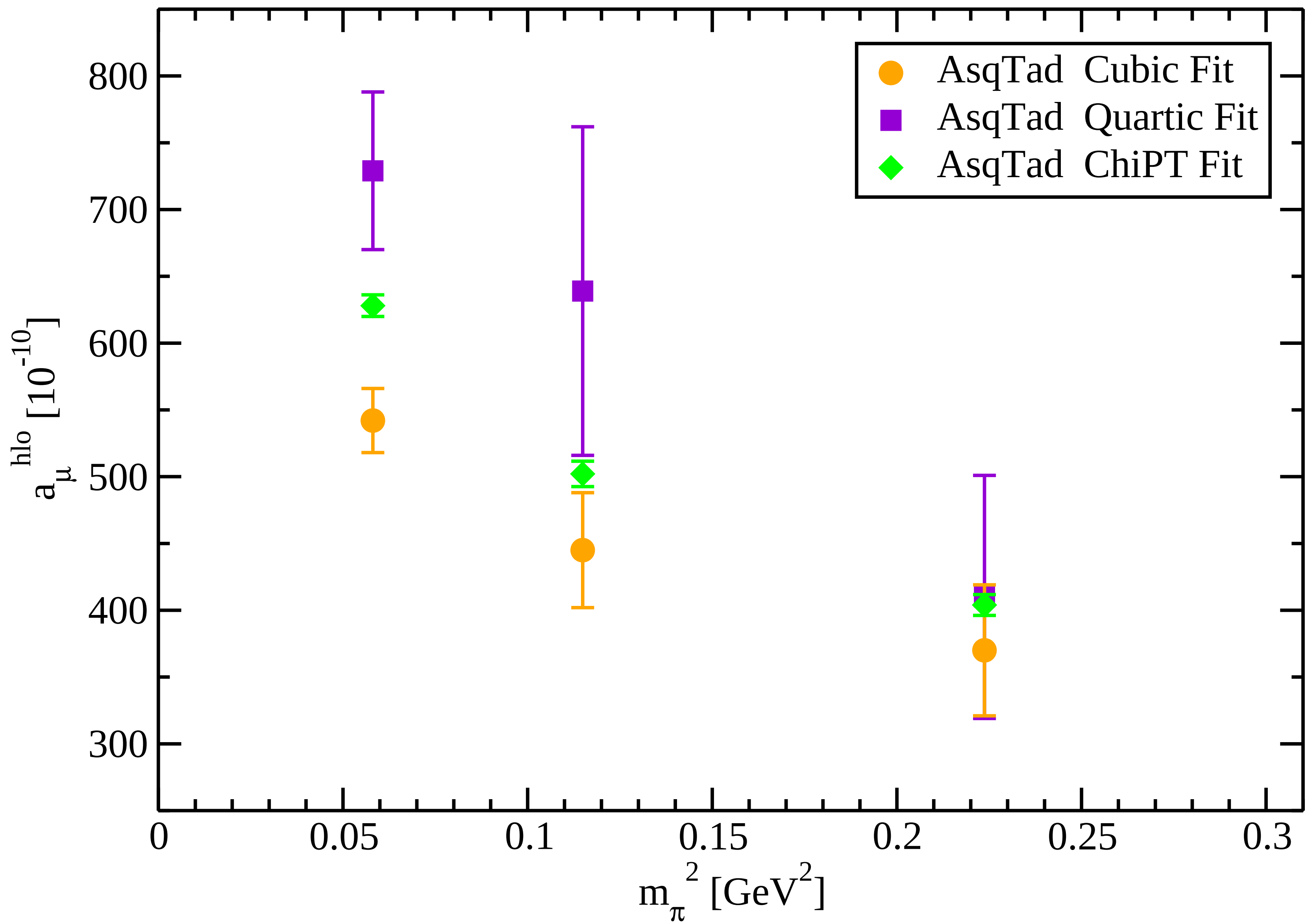
Low q^2 Extrapolation



- quartic fits are black, cubic fits are colored, χpt fits are dotted

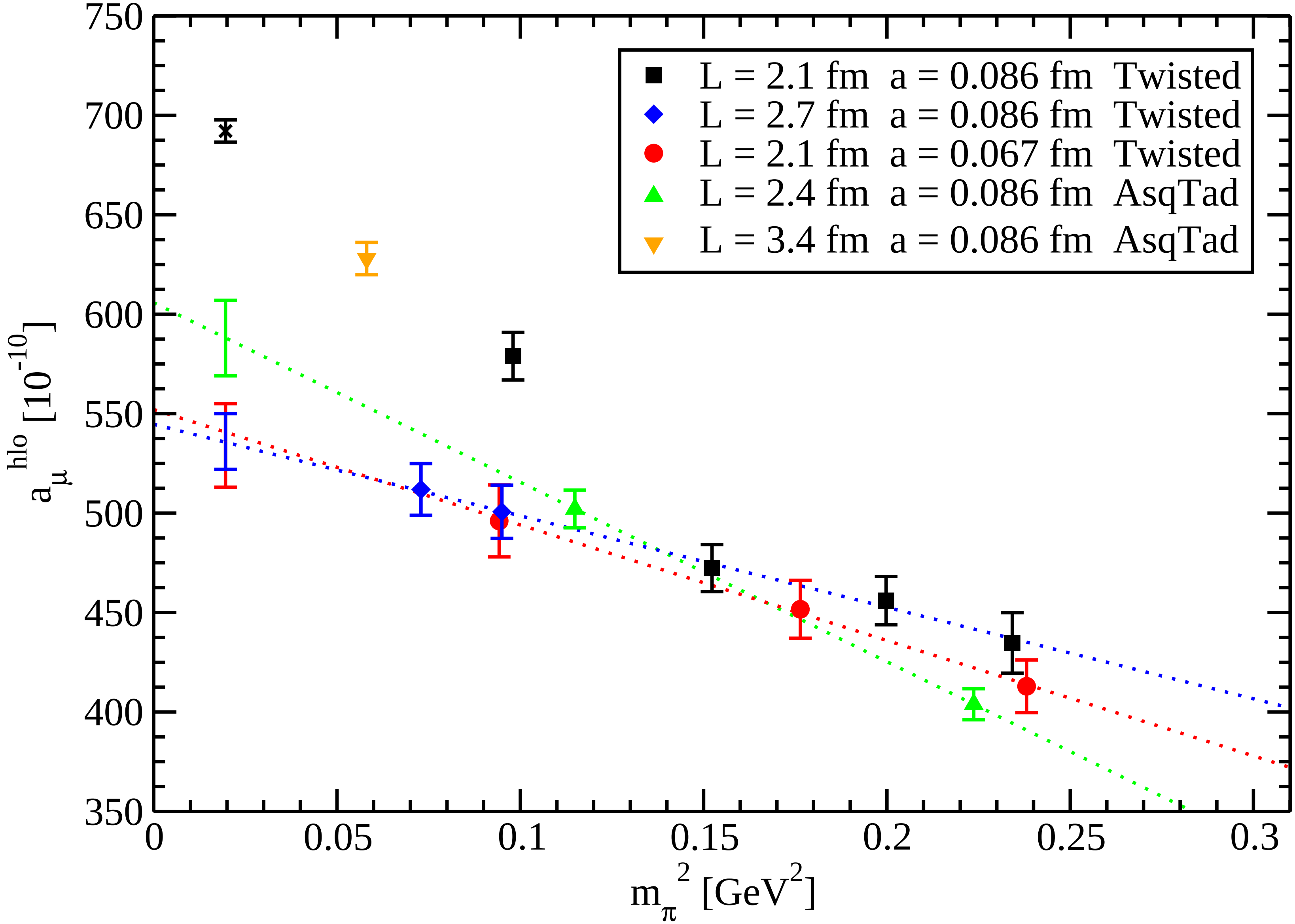


- twisted cubic, quartic and χpt results for a_μ^{hlo}



- asqtad cubic, quartic and χpt results for a_μ^{hlo}

a_μ^{hlo}



- comparison of χpt fits only

Options for Further Work

- constrain f_ρ as well as m_ρ
- calculate f_ρ and m_ρ with same propagators and conserved current
- complete the chiral fits, i.e. $A = A_0 + a^2 A_1$ independent of m_π
- NLO vacuum polarization contribution: a_μ^{hnlo}
- think about $\rho \to \pi\,\pi$ contributions
- calculate at $m_\pi = 600$ MeV at $a = 0.086$ fm and $a = 0.086$ fm
- variance reduction: all-q and single-q
- disconnected diagrams
- strange quark contributions
- 2+1+1
- light-by-light contributions