B_K from N_F=2 tmQCD:

Federico Mescia, *INFN-Frascati -> U. of Barcelona (UB)*

on behalf of

P.Dimopoulos, R. Frezzotti, G.C Rossi, A. Vladikas
V. Gimenez
V. Lubicz, S. Simula
+ ...

OUTLINE

1. HOT News on B_K
2. **Our Lattice calculation**: B_K (N_F=2)
   => a) run about to finish; b) renormalisation to be completed
3. Schedule and New applications

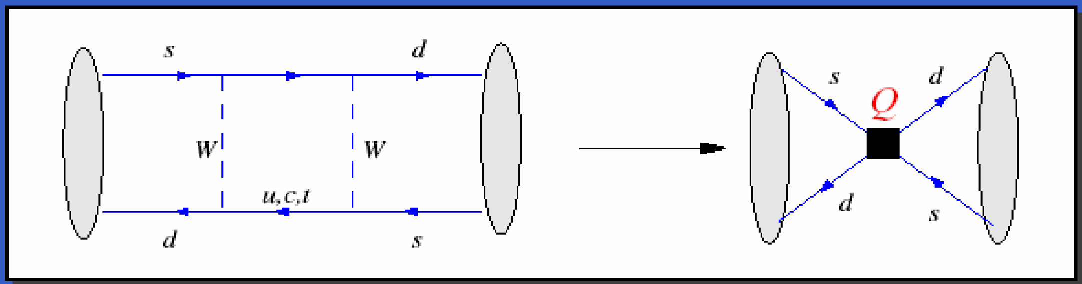
Glasgow, September 28

CP-Violation in $K - \bar{K}$ Mixing: ε_K and B_K

$$K_L \sim \underbrace{(K^0 - \bar{K}^0)}_{\text{CP}=-1} + \varepsilon_K \underbrace{(K^0 + \bar{K}^0)}_{\text{CP}=+1}$$

$\varepsilon_K \rightarrow$ indirect CP-violation

The Effective $\Delta S=2$ Hamiltonian



$$\varepsilon_K \sim \langle \bar{K}^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | K^0 \rangle = C(\mu) \cdot \langle \bar{K}^0 | \overbrace{\bar{s}\gamma_\mu(1-\gamma_5)d \; \bar{s}\gamma_\mu(1-\gamma_5)d}^{Q(\mu)} | K^0 \rangle$$

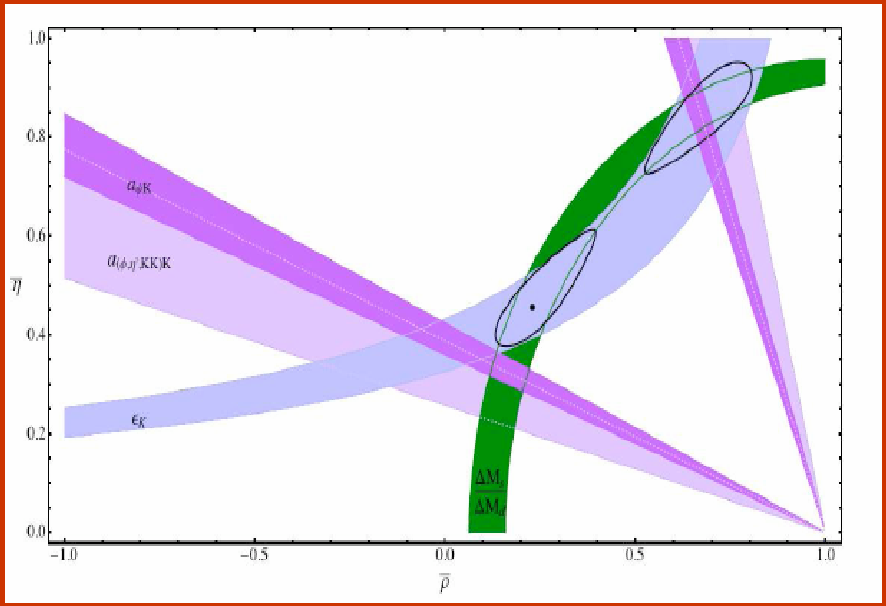
$$\varepsilon_K^{\text{exp.}} = (2.280 \pm 0.013) \times 10^{-3} e^{i\pi/4}$$

$$\langle \bar{K}^0 | Q(\mu) | K^0 \rangle = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)$$

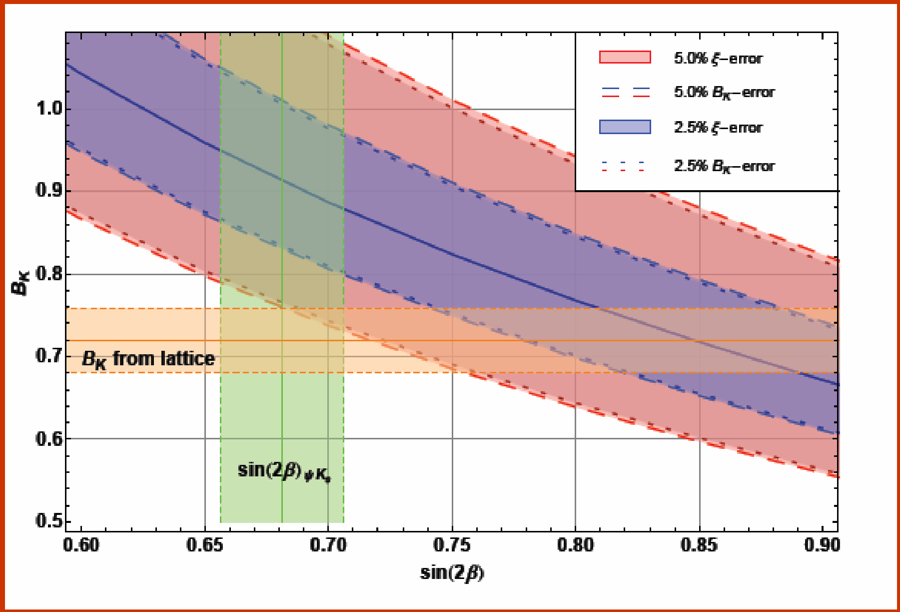
Hot News:

This year, renewed interest on B_K from phenomenology

Tensions in the Unitarity Triangle **~2 σ**



Lunghi, Soni '08



Buras, Guadagnoli '08

▶ non perturbative parameter
$B_K = 0.72 \pm 0.013 \pm 0.037$ (Antonio et al. '07)

☺ $N_F=2+1$, ☺ DWF,
☹ small volume and ☹ no continuum limit
☹ 2 sea q. masses ☹ residual mass: $m_{sea}/m_{res} \sim 1.6$

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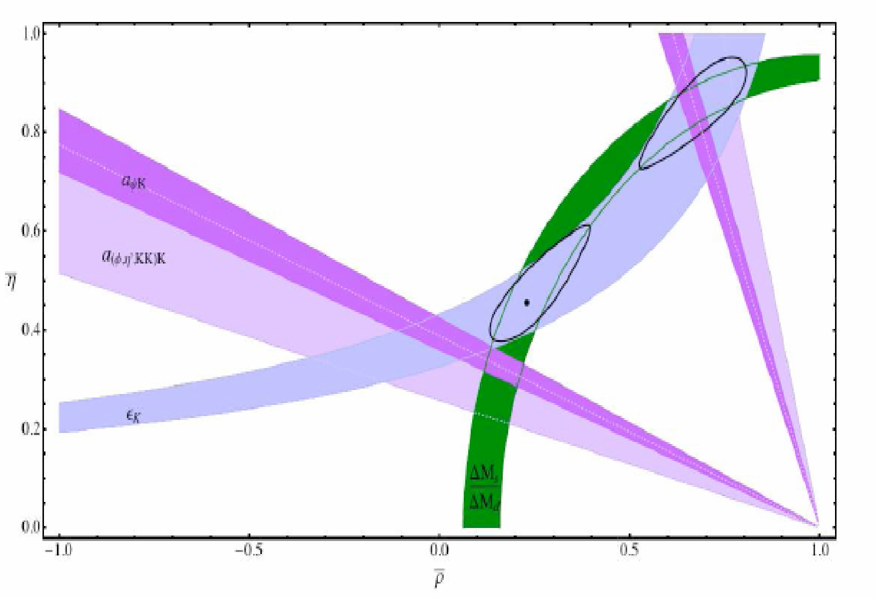
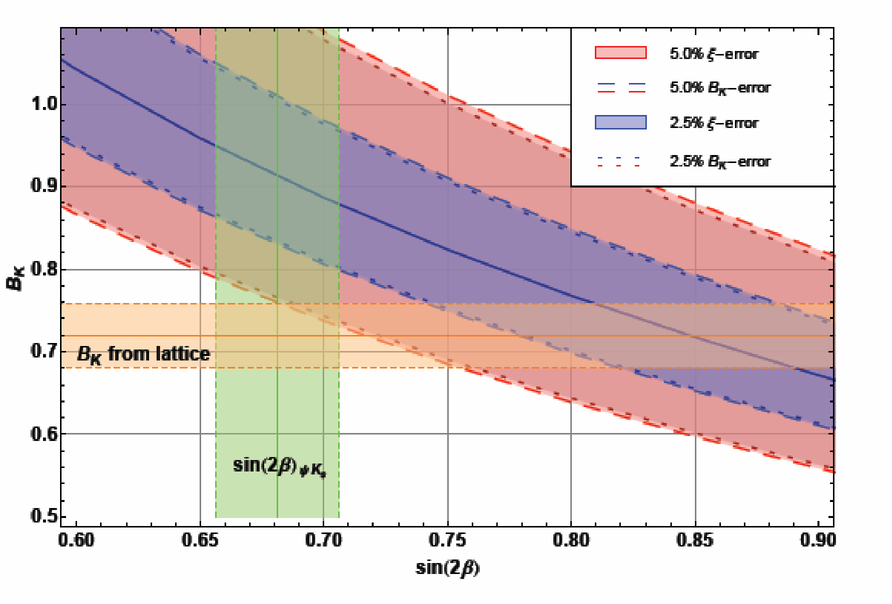


Lunghi, Soni '08



Buras, Guadagnoli '08

⇒ NP phase in B_d mixing?

⇒ Additional CP violation in K mixing?

Hot News:

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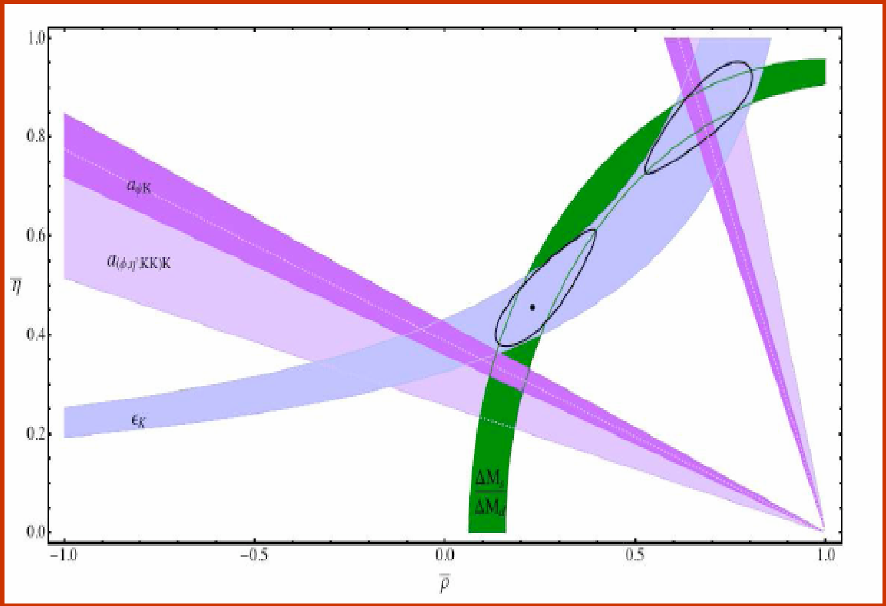

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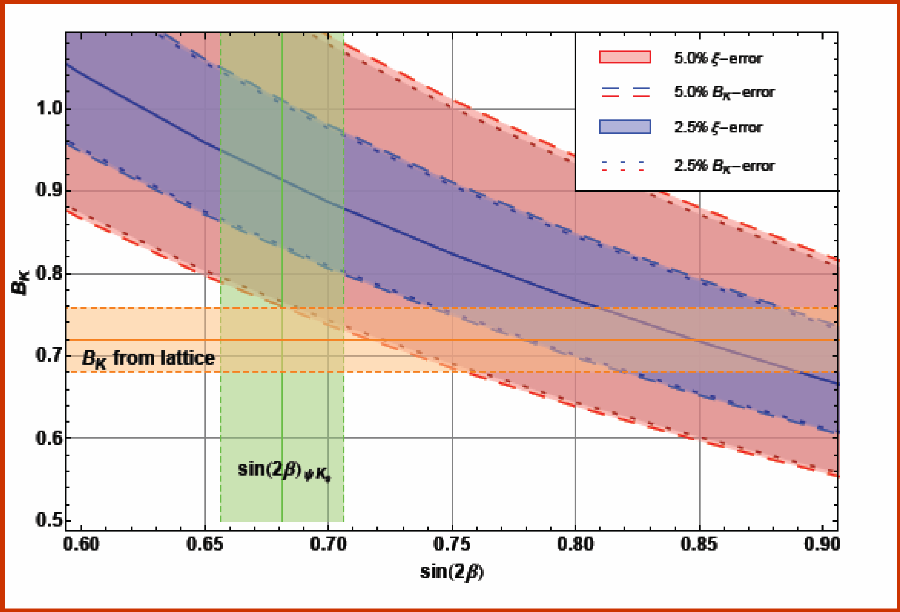


Buras, Guadagnoli '08

New Physics? It looks very premature, but

☺ Our B_K can help it and is welcome

☹ $N_F=2$, ☺ tmQCD,
☺ large volume ☹ continuum limit
☹ $O(a^2)$ and good chirality

2008 B_K summary

N_F=2+1:

☹ no continuum limit

⇒ a^{-1}=1.73 Gev,
L=2.74 fm, m_π>=330

☹ 2 sea q. masses!!!

N_F = 2: tmQCD

☺ continuum limit

=> a^{-1}=>2.3 Gev,
L=2.2/2.9 fm, m_π>=300

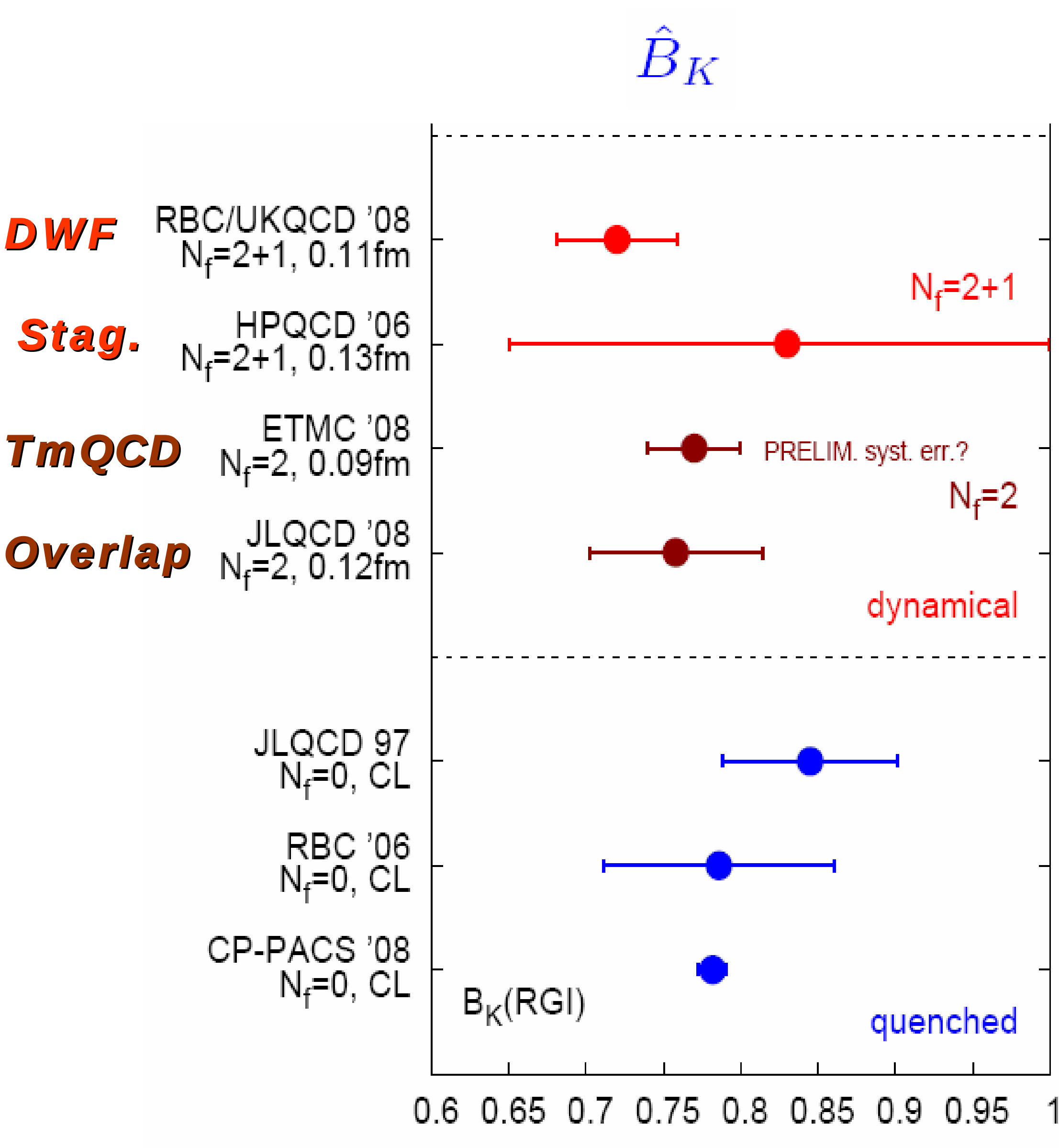
☺ sea q. masses dep.

☹ Quenching the strange

N_F = 2: Overlap

☹ no continuum limit

=> a^{-1}=1.67 Gev,
L=1.9 fm, m_π>=290



Enno E. Scholz - CKM08

OUR CALCULATION: RUN, post-lattice 2008

► $\beta = 3.90$: *higher statistics for lighter quarks*

	$a\mu_{sea}$		N_{meas}
$\beta = 3.90$	0.0040	$L^3 \times T = 24^3 \times 48$	400
	0.0064		200
	0.0085		200
	0.0100		200

$\beta = 3.90$	0.0040	$L^3 \times T = 32^3 \times 64$	$N_{meas} = 100$

► *Lighter q. mass at* $32^3 \times 64$ *is running*

$\beta = 3.90$	0.0030	$L^3 \times T = 32^3 \times 64$	$N_{meas} = 60$

► *Continuum Limit and scaling test with mixed action*

	$a\mu_{sea}$		N_{meas}
$\beta = 4.05$	0.0030	$L^3 \times T = 32^3 \times 64$	200
	0.0060		150
	0.0080		220
$\beta = 3.8$	0.0060	$L^3 \times T = 24^3 \times 48$	210
	0.0080		170
	0.00110		180

OUR CALCULATION: Mixed action approach

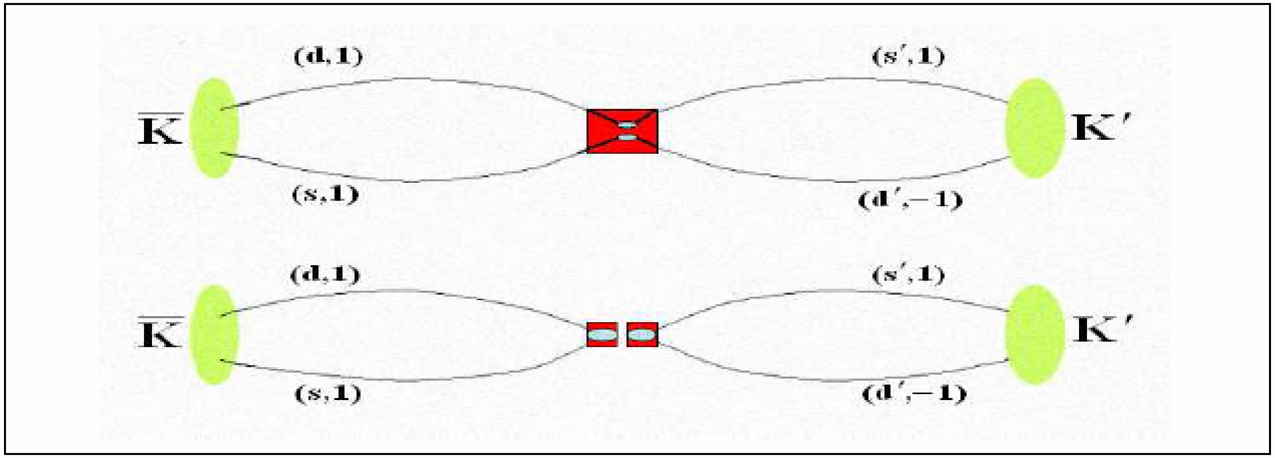
(Frezzotti-Rossi, hep-lat/ 0407002)

Calculate the three-point correlator:

$$C_{KQK}(z_0 - x_0, z_0 - y_0) = \sum_{\vec{x},\vec{y},\vec{z}} \langle (\bar{d}'\gamma_5 s')(x)\; Q^{\Delta S=2}_{VV+AA}(z)\; (\bar{d}\gamma_5 s)(y) \rangle$$

with the 4-fermion operator:

$$Q^{\Delta S=2}_{VV+AA} = 2\{ (\bar{s}\gamma_\mu d)(\bar{s}'\gamma_\mu d') + (\bar{s}\gamma_\mu\gamma_5 d)(\bar{s}'\gamma_\mu\gamma_5 d') + (\bar{s}\gamma_\mu d')(\bar{s}'\gamma_\mu d) + (\bar{s}\gamma_\mu\gamma_5 d')(\bar{s}'\gamma_\mu\gamma_5 d) \}$$



$\phi_{K'} = \bar{d}'\gamma_5 s'$	$-r_{d'} = r_{s'} = 1$	(tm-like)
$\phi_K = \bar{d}\gamma_5 s$	$r_d = r_s = 1$	(OS-like)

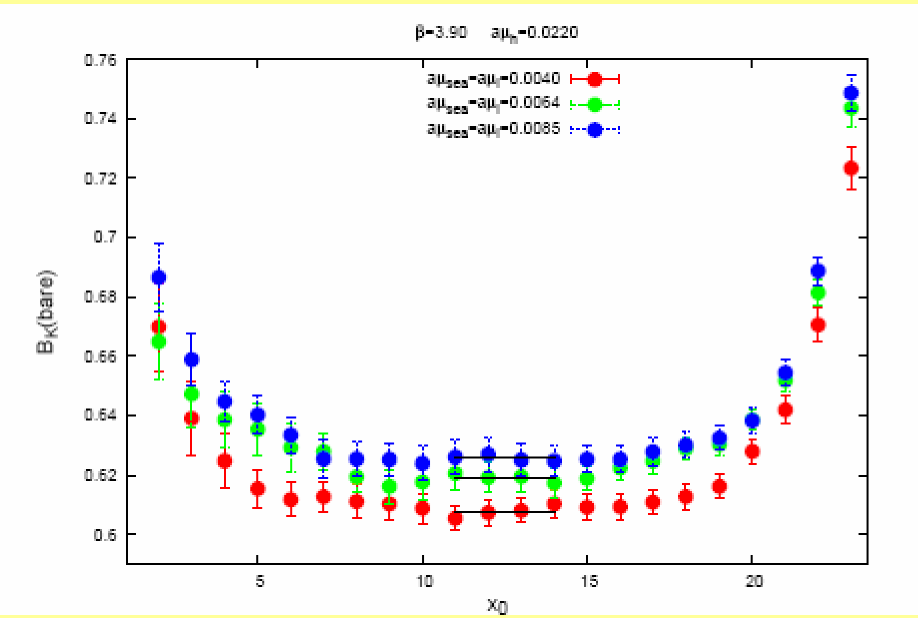
GAIN: no mixing in the renormalization of the 4-fermion operator + O(a) improvement

Petros's slide at the Trento-ETMC meeting

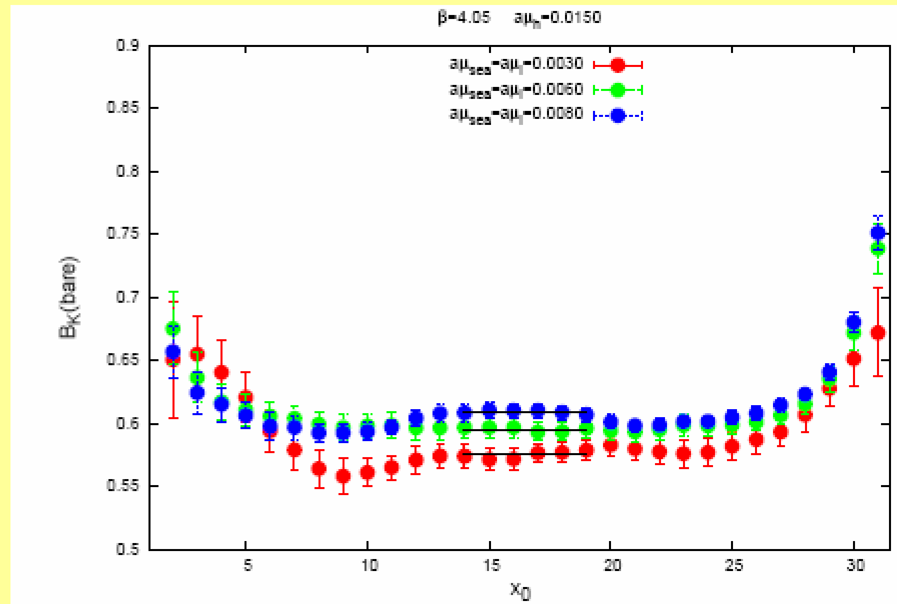
OUR CALCULATION: Quality of Plateau for B_K bare

$$R_{B_K} = \frac{8}{3} \frac{C^{(3)}_{K'QK}(t-t_L, t-t_R)}{C^{(2)}_{K'}(t-t_L) C^{(2)}_{K}(t-t_R)} \xrightarrow{t_L << t << t_R} B_K$$

$$x_0 = t_R - t_L = \frac{T}{2}$$



$\beta = 3.9$

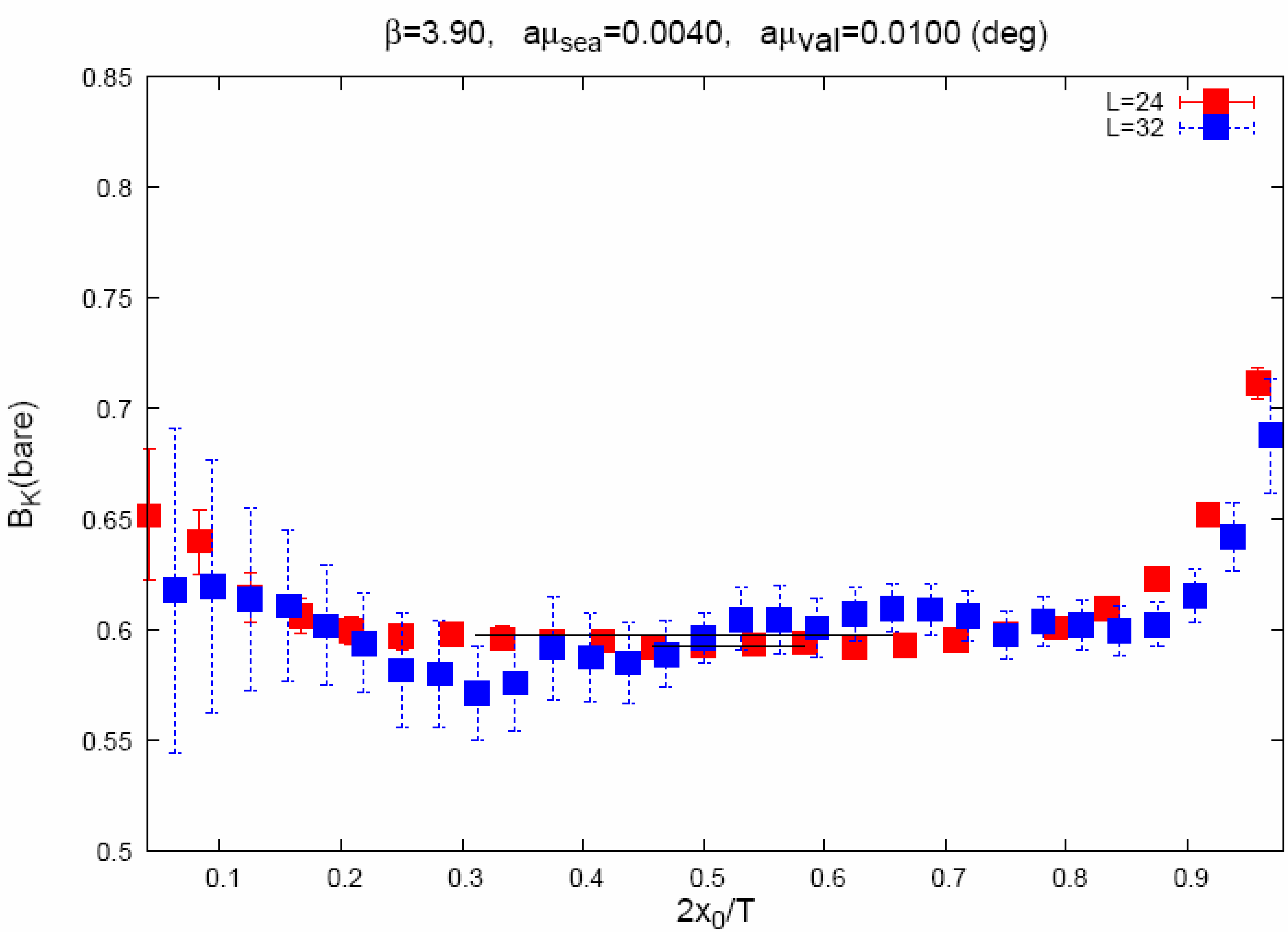


$\beta = 4.05$

$a\mu_h$ takes values around the strange quark mass, while the "light" valence quark mass is equal to the sea quark mass value.

► ***Finite volume effects:***

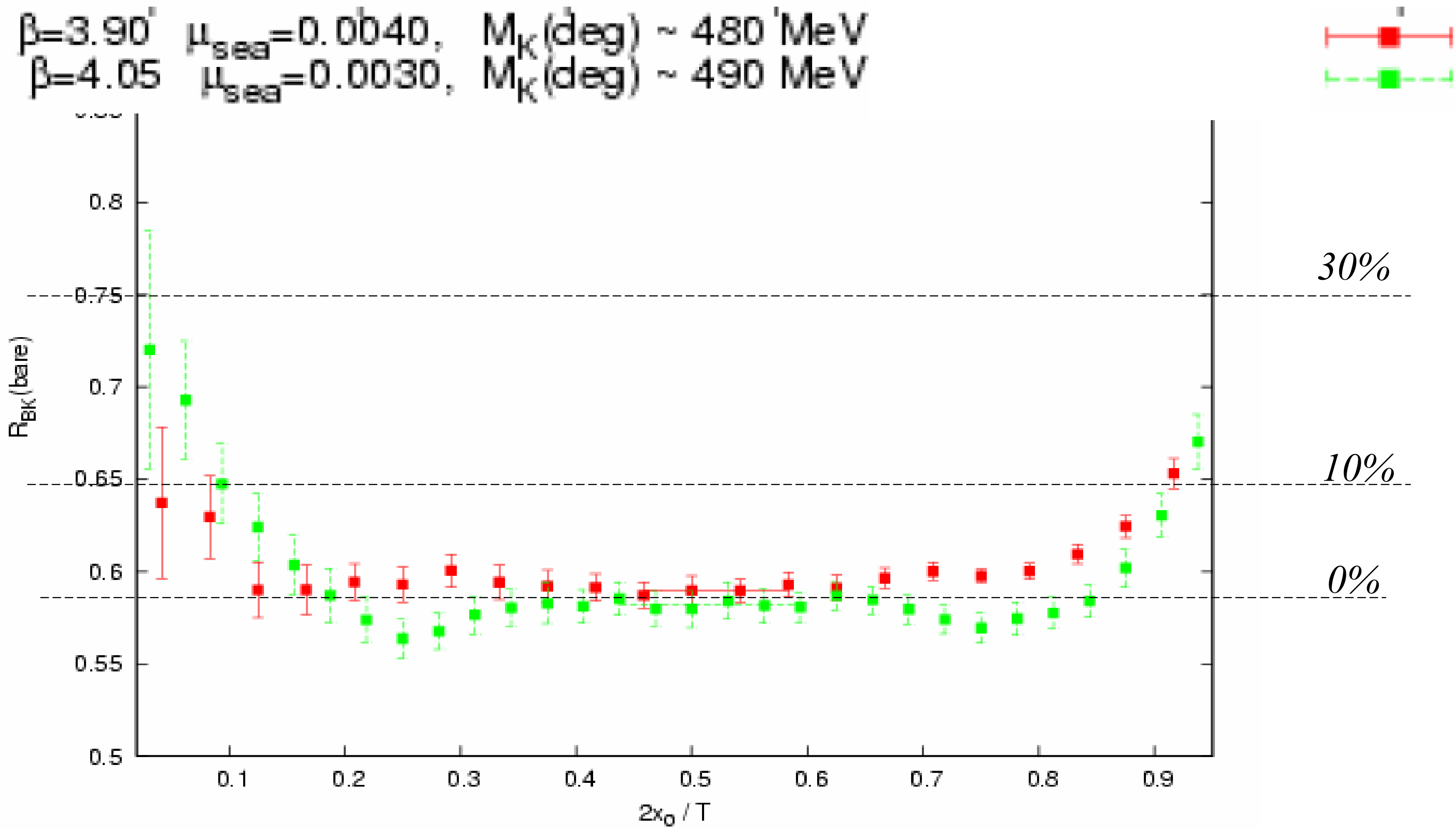
$\beta = 3.90$	0.0040	L^3 x T= 32^3x 64	N_{meas} = 100



► ***Lighter q. mass at higher volume is running***

$\beta = 3.90$	0.0030	L^3 x T= 32^3x 64	N_{meas} = 60

► **Scaling test:** *No Anomalous SCALING for our Mixed action approach*



β	3.90	4.05
$\mathcal{Z}$	0.965(38)	0.962(43)

Mind: renormalization c. by RI-MOM
=> see later for details

► **Scaling test:** *awaiting for definitive renormalization constants*

Quantitative cross-check on the scaling **=> to prove our Mixed action approach**

β	3.80	3.90	4.05
B_K(bare)	0.609(8)	0.586(06)	0.558(12)

$B_K^{bare}(a^{-1} = 2\text{GeV})$ $B_K^{bare}(a^{-1} = 2.3\text{GeV})$ $B_K^{bare}(a^{-1} = 2.9\text{GeV})$

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1. Assuming large a⁻¹, discretization errors are negligible and RG evolution is ok! namely,

$$B_K^{bare}(a^{-1}) = \text{Cost} \cdot \alpha_s^{-\gamma/\beta}(a^{-1}) \times (1 + \ldots) \dashrightarrow \text{RGE}$$

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$$\frac{B_K^{bare}(a^{-1})}{B_K^{bare}(a_{ref}^{-1})} = \frac{\text{RGE}(a^{-1})}{\text{RGE}(a_{ref}^{-1})} \Rightarrow \frac{B_K^{bare}(a^{-1})}{B_K^{bare}(a_{ref}^{-1})} \Big/ \frac{\text{RGE}(a^{-1})}{\text{RGE}(a_{ref}^{-1})} = 1$$

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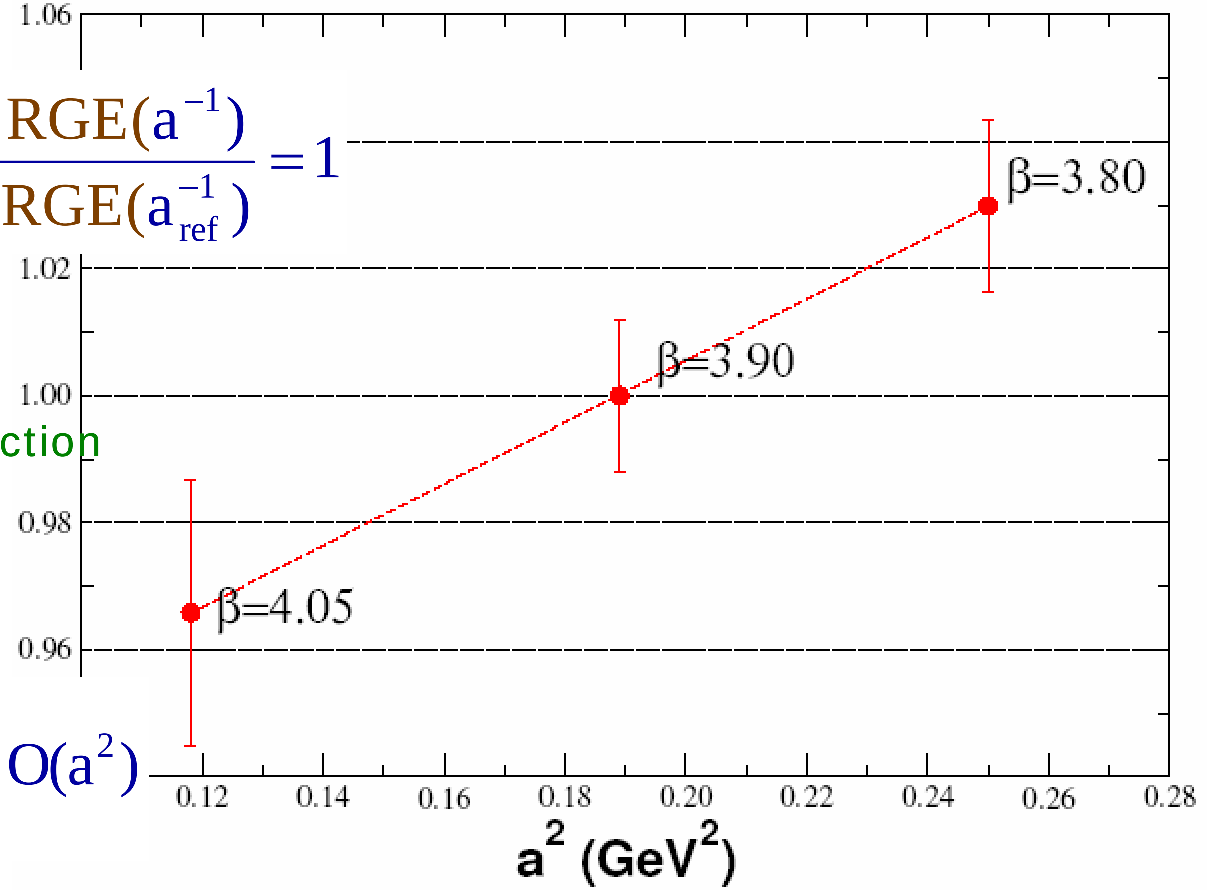
$$\frac{B_K^{bare}(a^{-1})}{B_K^{bare}(a_{ref}^{-1})} \Big/ \frac{RGE(a^{-1})}{RGE(a_{ref}^{-1})} = 1$$

O(a²) violations at percent level

=> Frezzotti and Rossi mixed action

$$\frac{B_K^{bare}(a^{-1})}{B_K^{bare}(a_{ref}^{-1})} \Big/ \frac{RGE(a^{-1})}{RGE(a_{ref}^{-1})} = 1 + O(a^2)$$

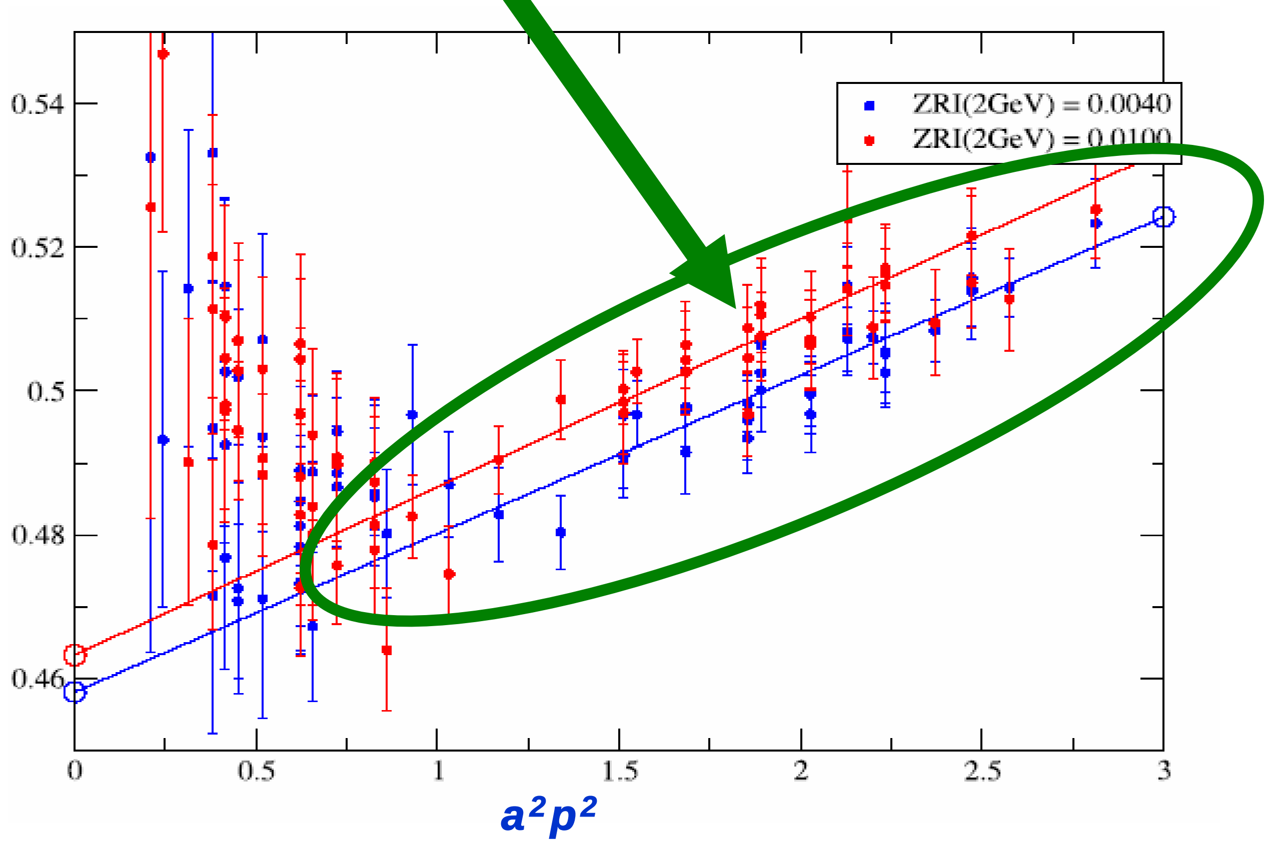


▸ **Renormalization Constants:** ***BIG working in progress.***

$$B_K(\text{ren}) = \mathcal{Z} B_K(\text{bare}) \;\; \text{where} \;\; \mathcal{Z} = Z_4/(Z_A\, Z_V)$$

A complete RI-MOM analysis will be ready in 2/3 weeks for all βs

However, potential effects from O(a^2g^2) look already sizable



- ***For the time beeping, we subtract them from a linear fit***
- ***Next, we may need help from Harris Papadopoulos et al.***

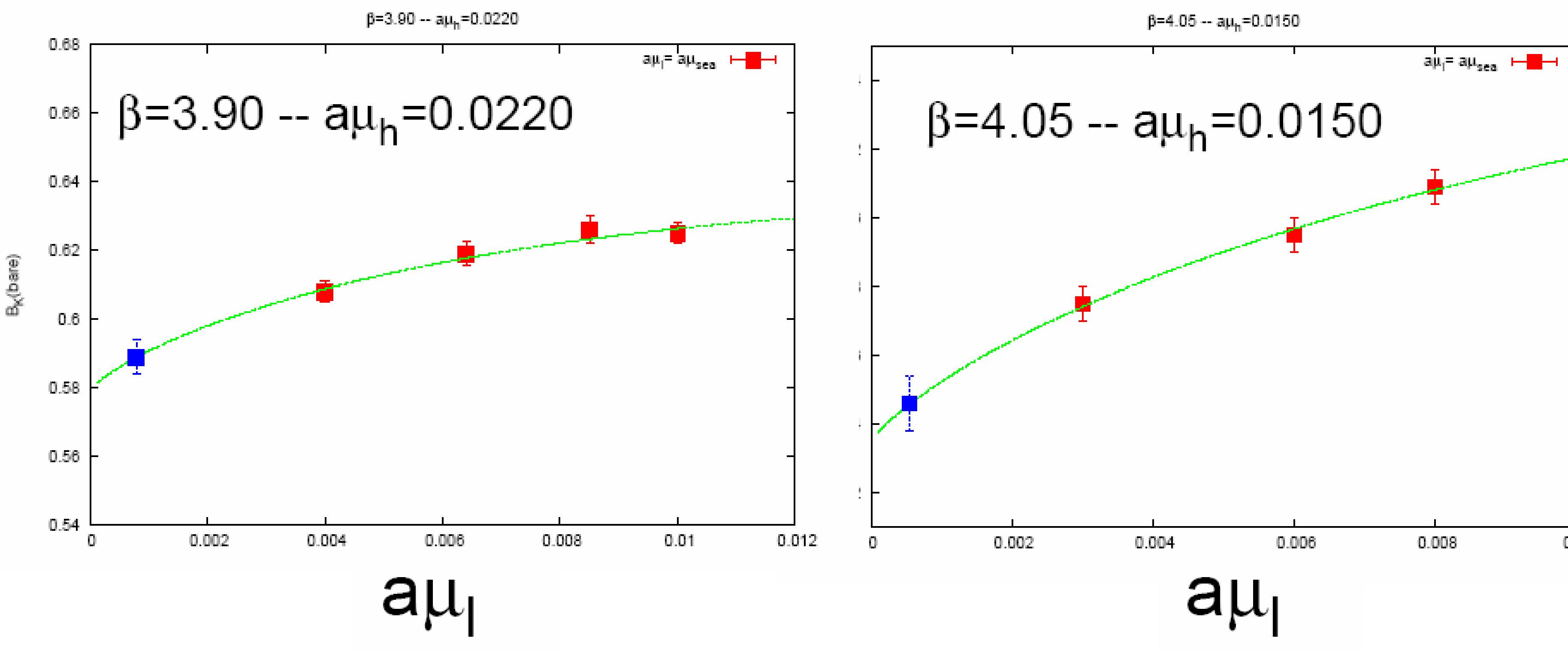
► *Chiral-fit Setup:* ***SU(2)$_L$x SU(2)$_R$*** **fit (μ_v) + static strange quarks (μ_h)**

Unitarity case

$$B_K^{bare}(\mu_h, \mu_v = \mu_{sea}) = B_\chi^{bare}(\mu_h)\left[1 + \left(P_v(\mu_h) + P_{sea}(\mu_h)\right)\mu_{sea} - \frac{2B_0}{32\pi^2 f_0^2}\mu_{sea}\log\mu_{sea}\right]$$

Sharpe & Zhang (1996)

B_K(bare)

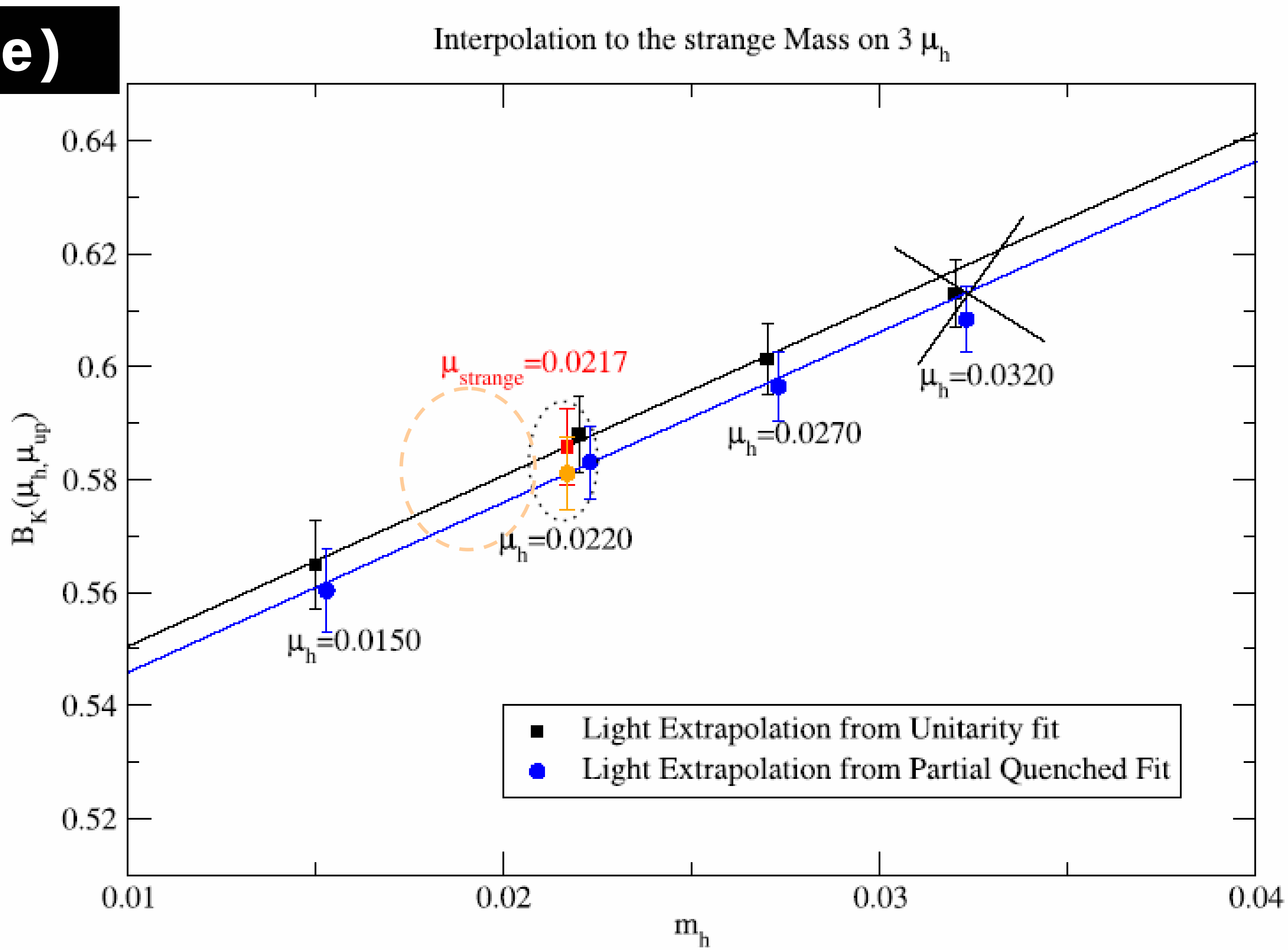


$\delta B_K/B_K \sim 2\%$

► *Chiral-fit Setup:* ***SU(2)$_L$ x SU(2)$_R$*** **fit (μ_v) + static strange quarks (μ_h)**

B_K(bare)

β=3.90 ·

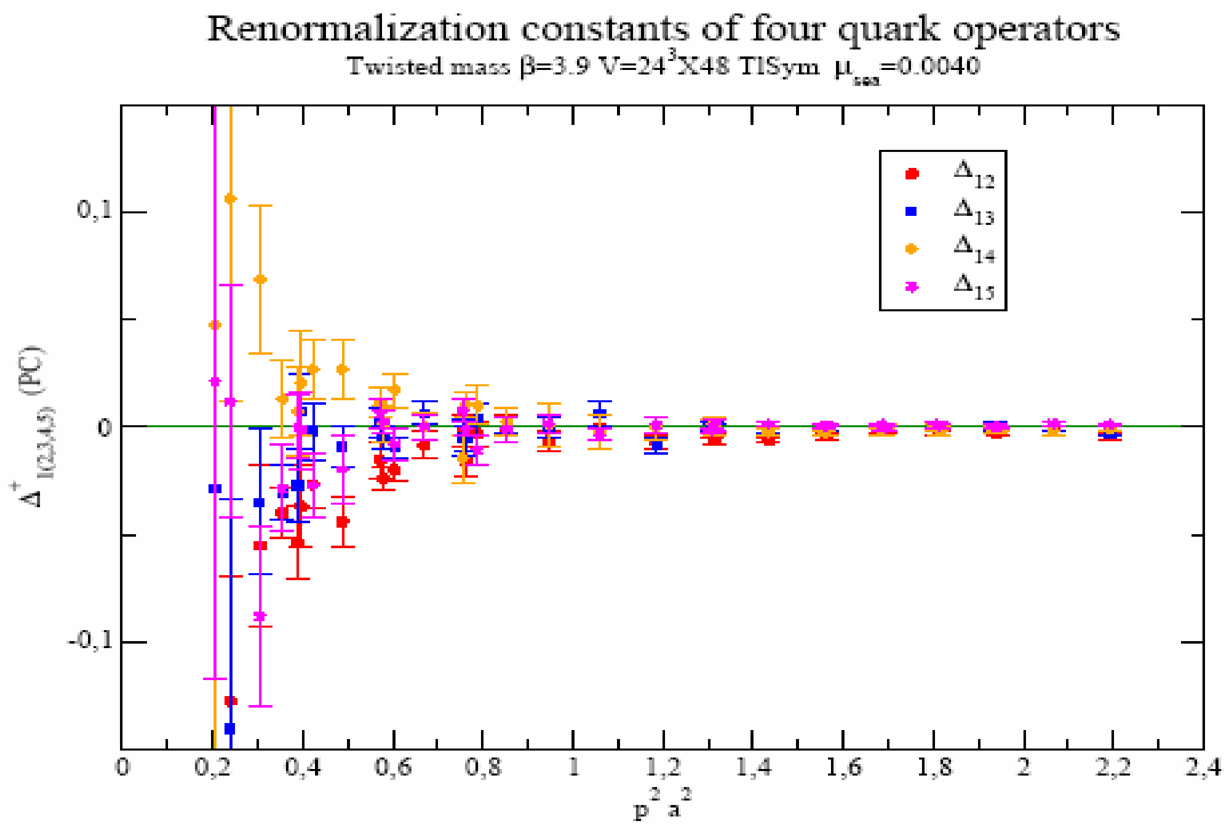


1. $\delta B_K/B_K \sim 2\%$
2. **Uni and Pq fits OK**

► *I schedule one month to finish*

β	3.90	4.05	3.80
$\mathcal{Z}$	0.965(38)	0.962(43)	–
B_K(bare)	0.586(06)	0.558(12)	0.609(8)
$B_K(\overline{\text{MS}};\ 2\ \text{GeV})$	0.565(6)[22]	0.537(12)[23]	–

$$B_K(\text{ren}) = \mathcal{Z} B_K(\text{bare}) \;\; \text{where} \;\; \mathcal{Z} = Z_4/(Z_A\, Z_V)$$



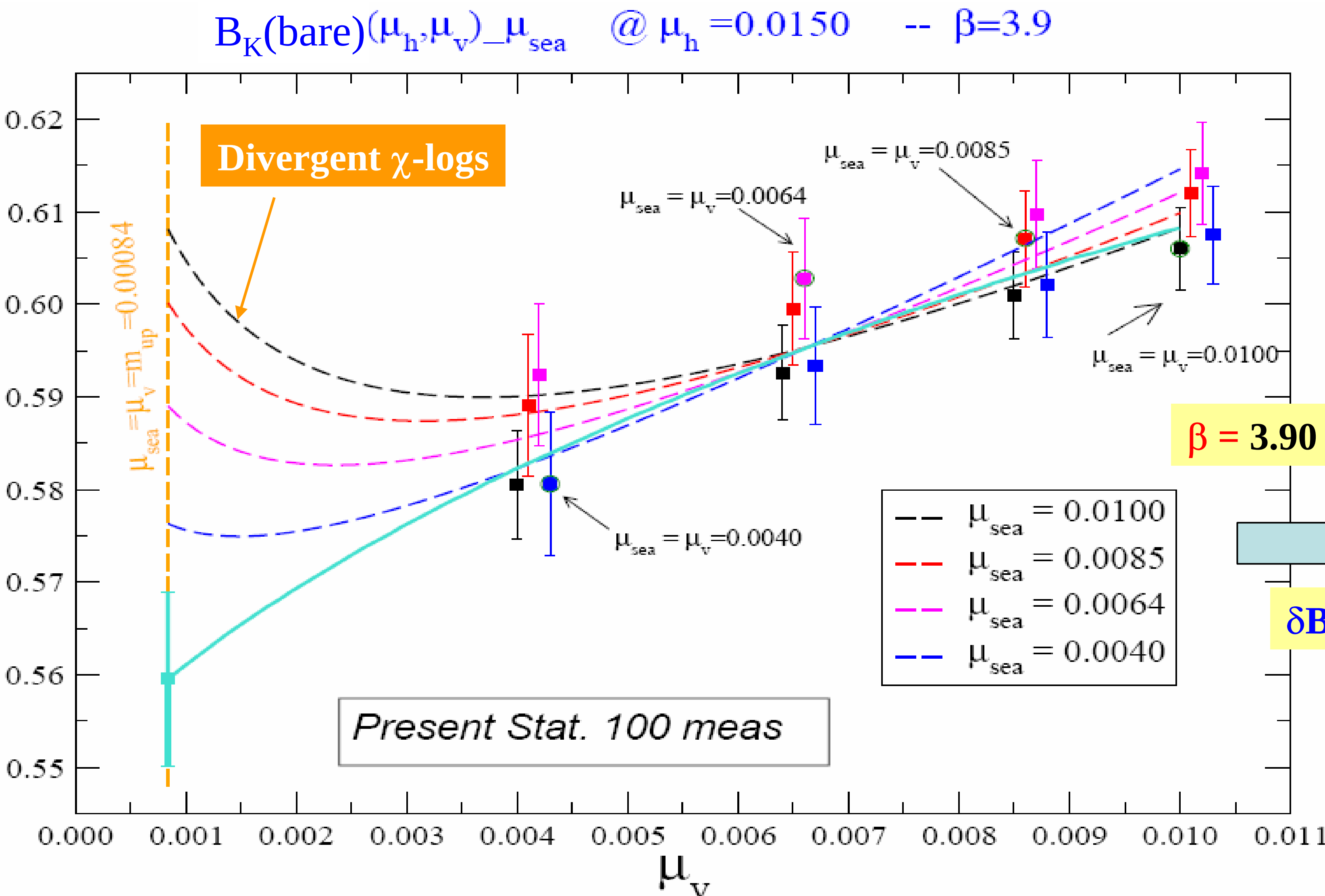
The mixing coefficients with other four-fermion operators with "wrong chirality".

coefficients are compatible with zero

► *Chiral-fit Setup:* **$SU(2)_L$ x $SU(2)_R$ fit (μ_v) + static strange quarks (μ_h)**

Partial-Quenched case

$$B_K^{bare}(\mu_h, \mu_v : \mu_{sea}) = B_\chi^{bare}(\mu_h)\left[1 + P_v(\mu_h)\mu_v + P_{sea}(\mu_h)\mu_{sea} - \frac{2B_0}{32\pi^2 f_0^2}\mu_v \log \mu_{sea}\right]$$



β = 3.90 at 100 meas

$\delta B_K/B_K \sim 2\%$