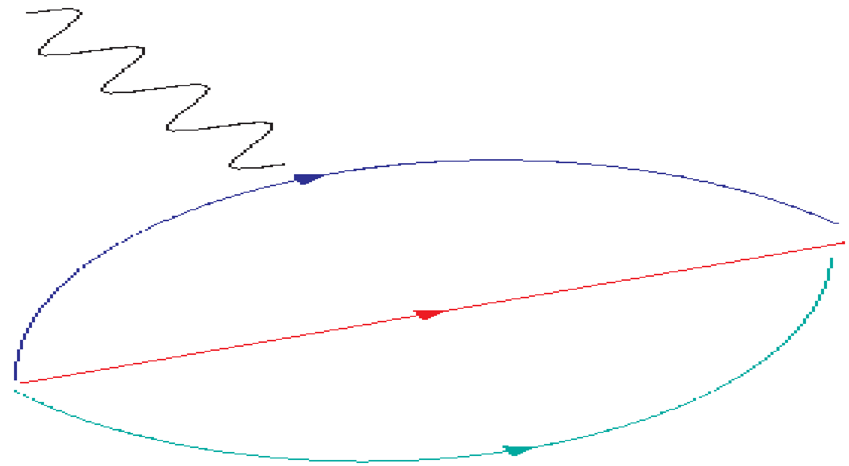

3-point function

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R.Baron

Collaboration with Berlin and Cyprus

THREE-POINT FUNCTIONS

Which observable can we obtain ?

Informations about the structure of the baryons

- parton distribution function

- Nucleon g_A

- moments of the generalized parton distribution function

....

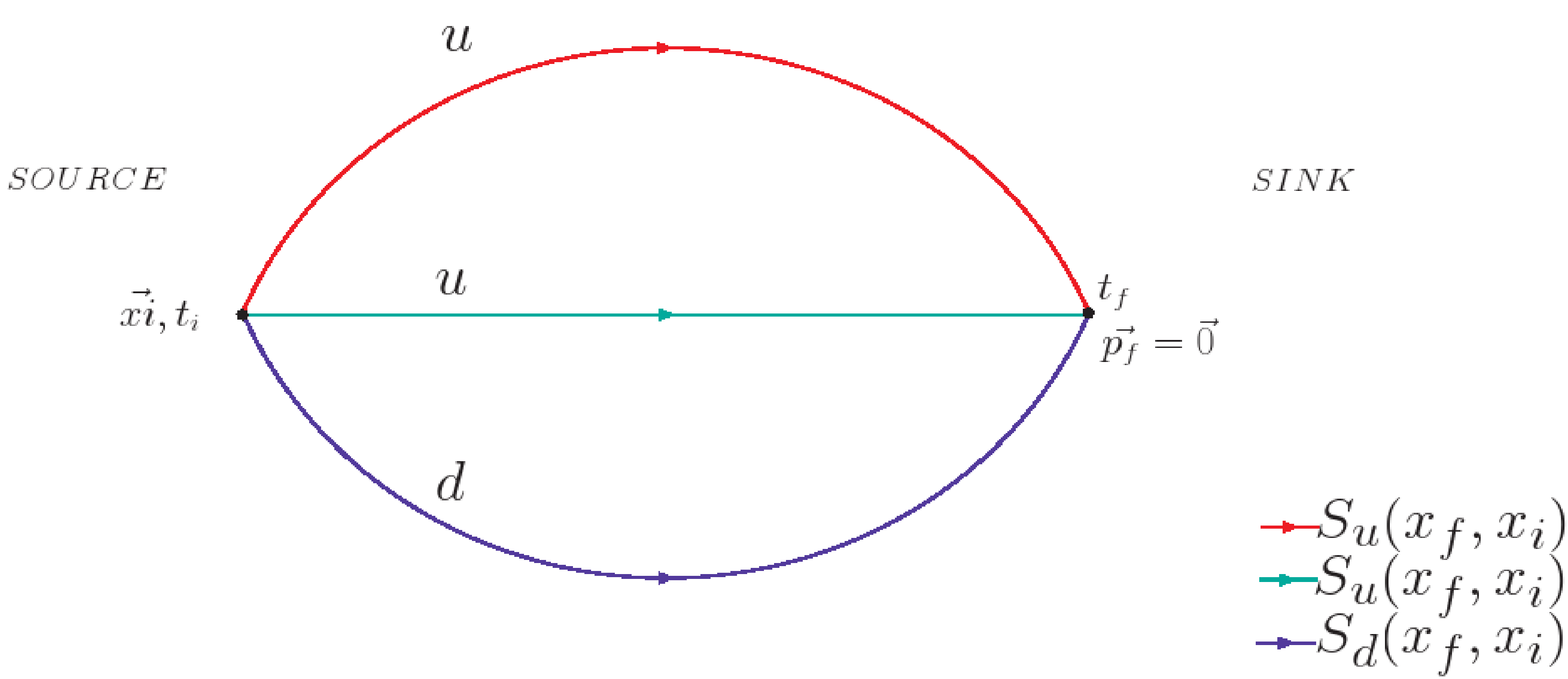
How ?

Proton two point function

Interpolating field of the proton :

$$P_\alpha(x) = \epsilon_{abc}(u_a(x)^T C\gamma_5 d_b(x))u_c(x)$$

$$\langle P_{\alpha_f}(t_f, \vec{p_f} = \vec{0})\overline{P_{\alpha_i}}(x_i)\rangle = \prod_{spin,color} S_{u1}(x_f, x_i)S_d(x_f, x_i)S_{u2}(x_f, x_i)$$



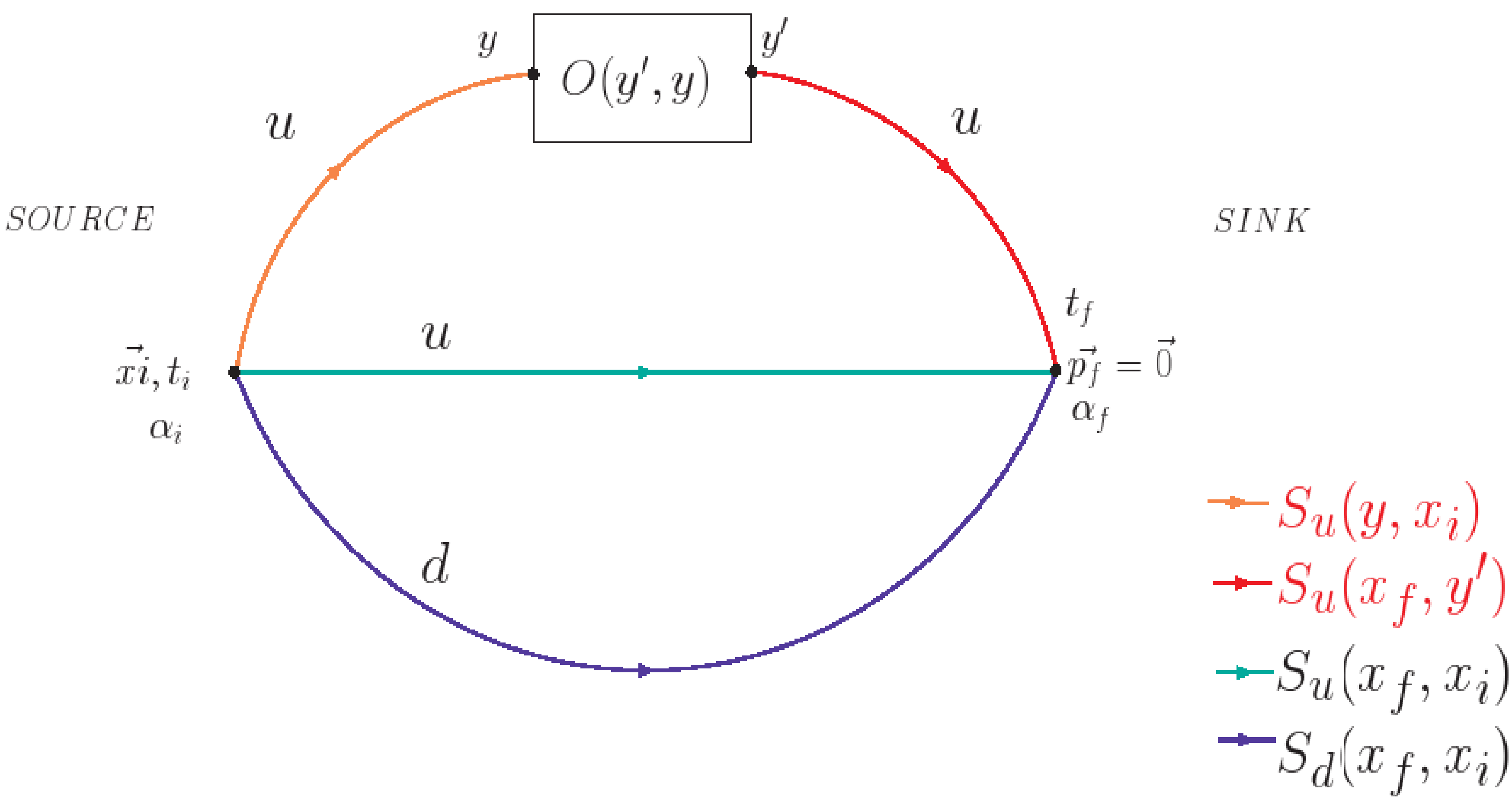
3-point function with an operator acting on a quark u

- Interpolating field of the proton :

$$P_\alpha(x) = \epsilon_{abc}(u_a(x)^T C\gamma_5 d_b(x))u_c(x)$$

- Observables: operator $O(y', y)$

$$\langle P_{\alpha_f}(t_f, \vec{p_f} = \vec{0}) O(y', y) \overline{P_{\alpha_i}}(x_i)\rangle = \prod_{spin,color} S_{u1}(x_f, y')O(y', y)S_{u1}(y, x_i)S_d(x_f, x_i)S_{u2}(x_f, x_i)$$



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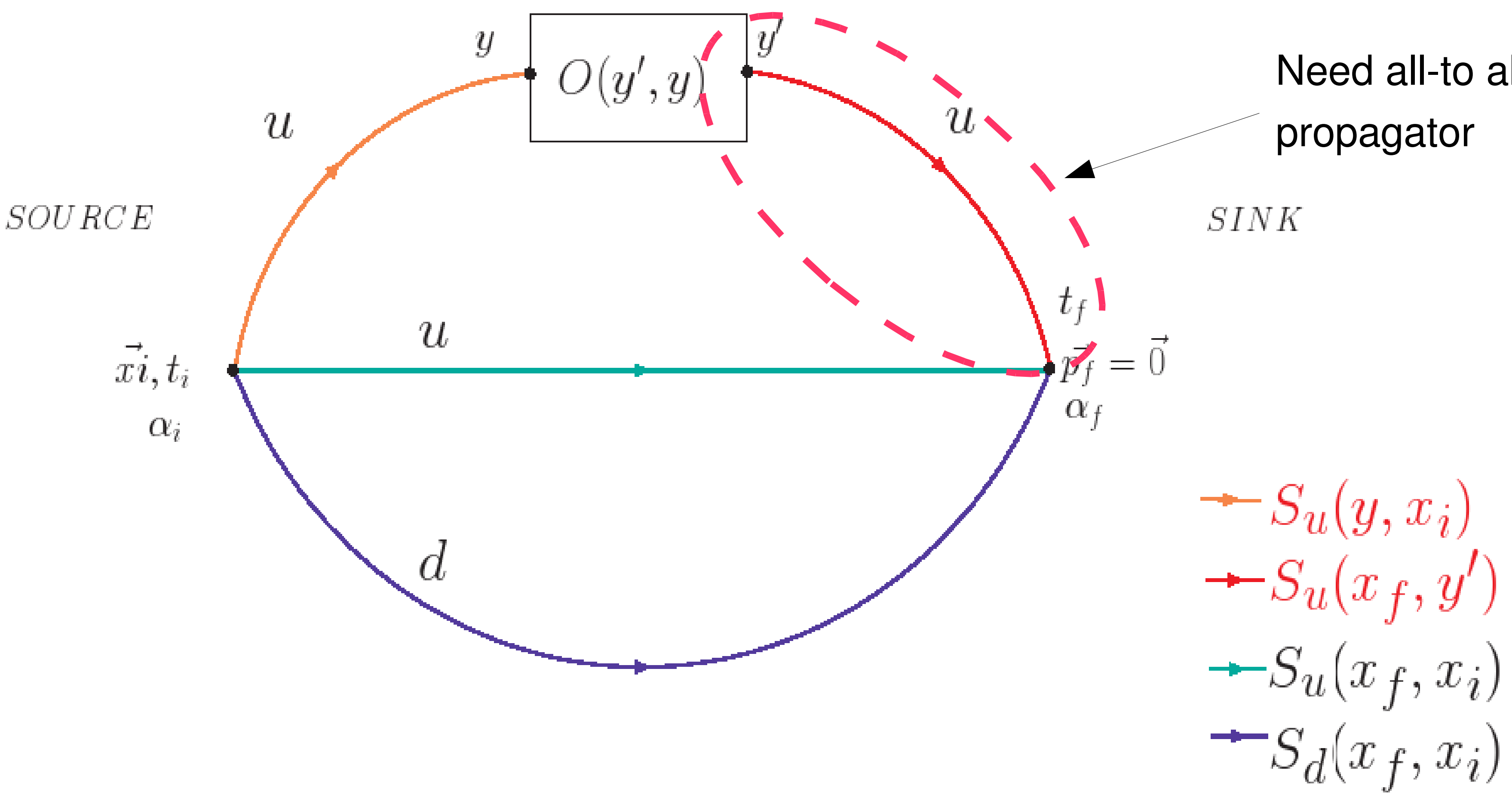
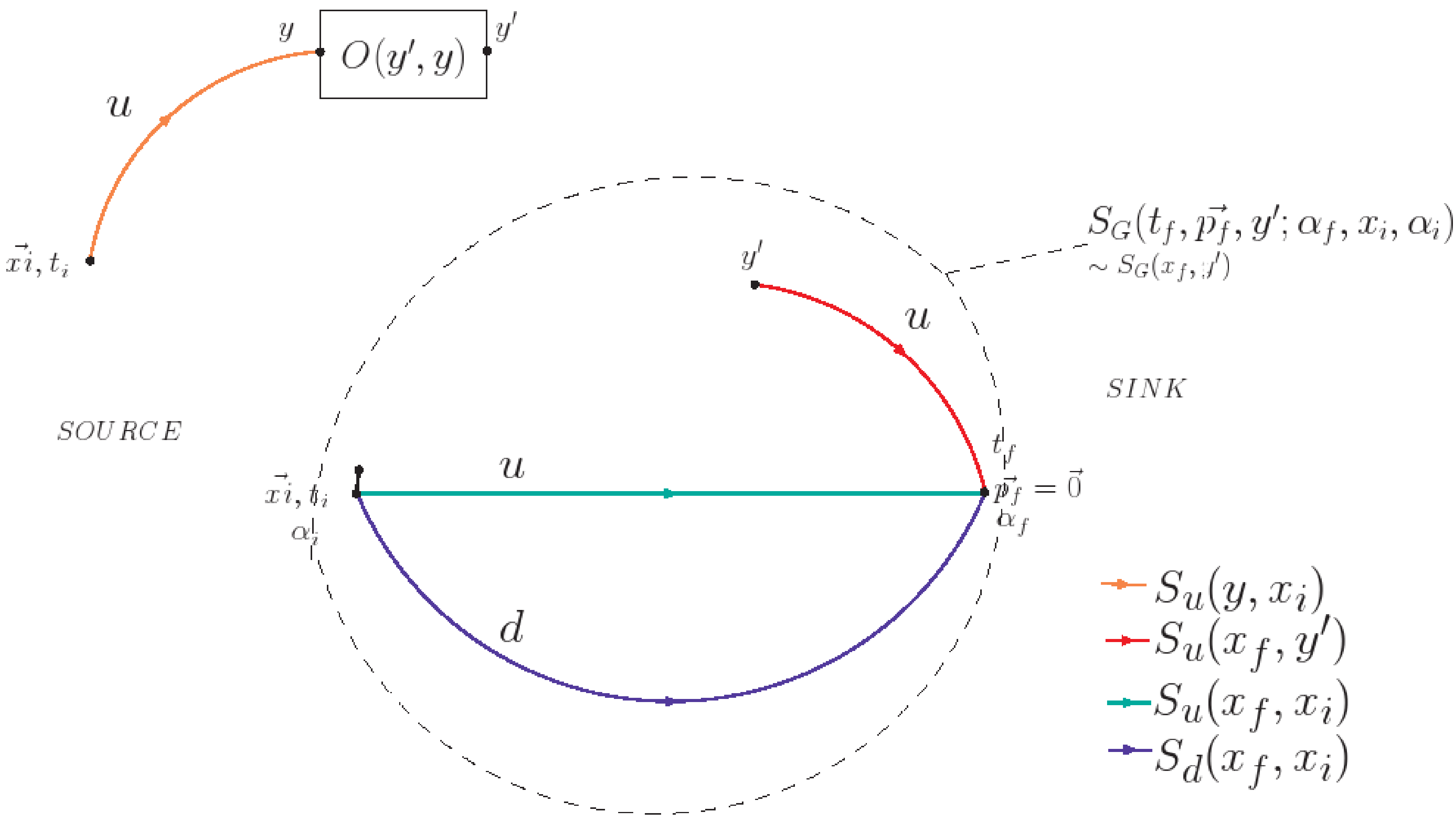


Generalized propagator

$$\langle P_{\alpha_f}(t_f, \vec{p_f} = \vec{0}) O(y', y) \overline{P_{\alpha_i}}(x_i) \rangle = \prod_{spin,color} S_G(x_f, y') \quad O(y', y) S_{u1}(y, x_i)$$



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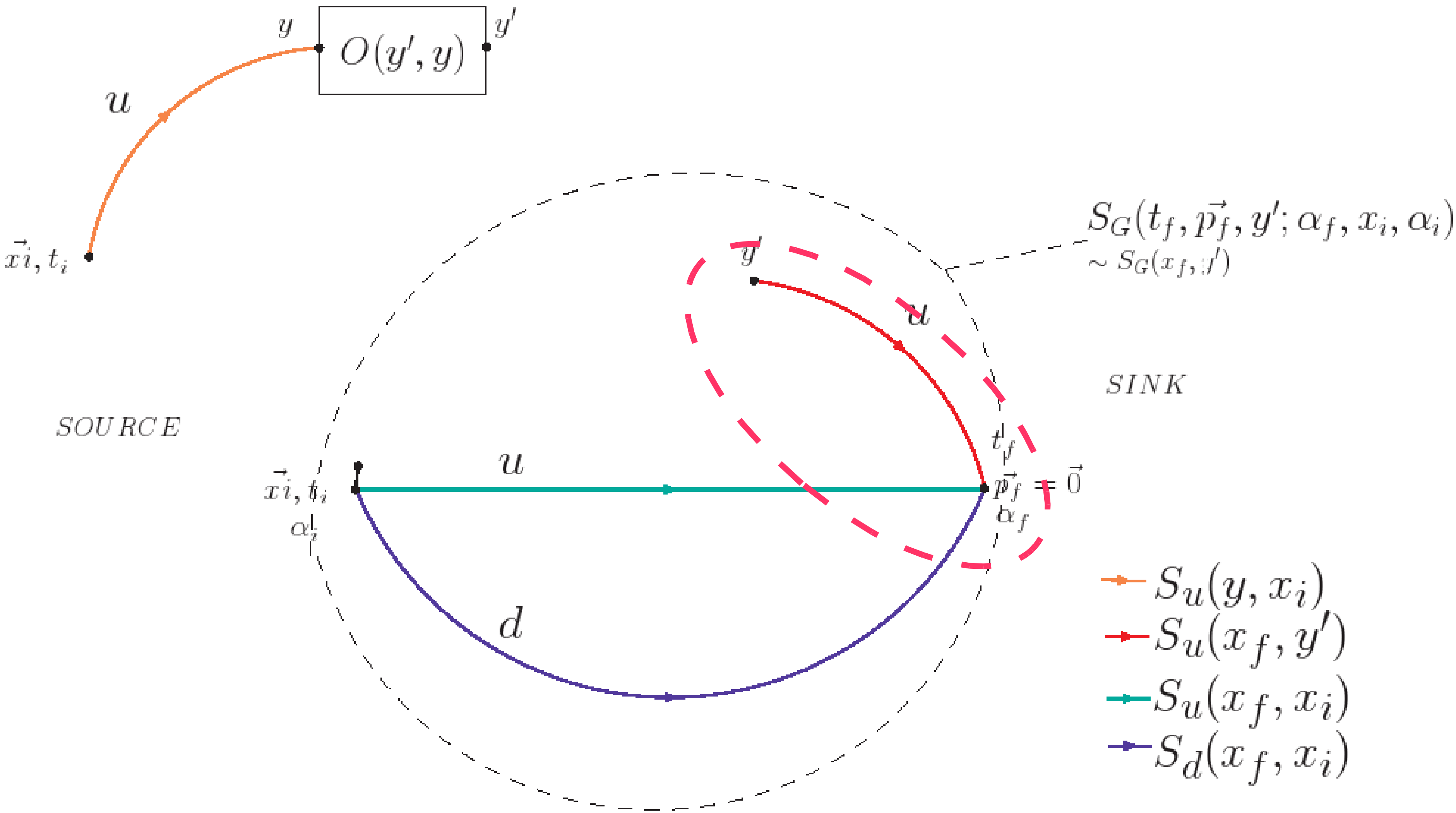
$$\langle P_{\alpha_f}(t_f, \vec{p_f} = \vec{0}) O(y', y) \bar{P}_{\alpha_i}(x_i) \rangle = \prod_{spin,color} S_G(x_f, y') \quad O(y', y) S_{u1}(y, x_i)$$

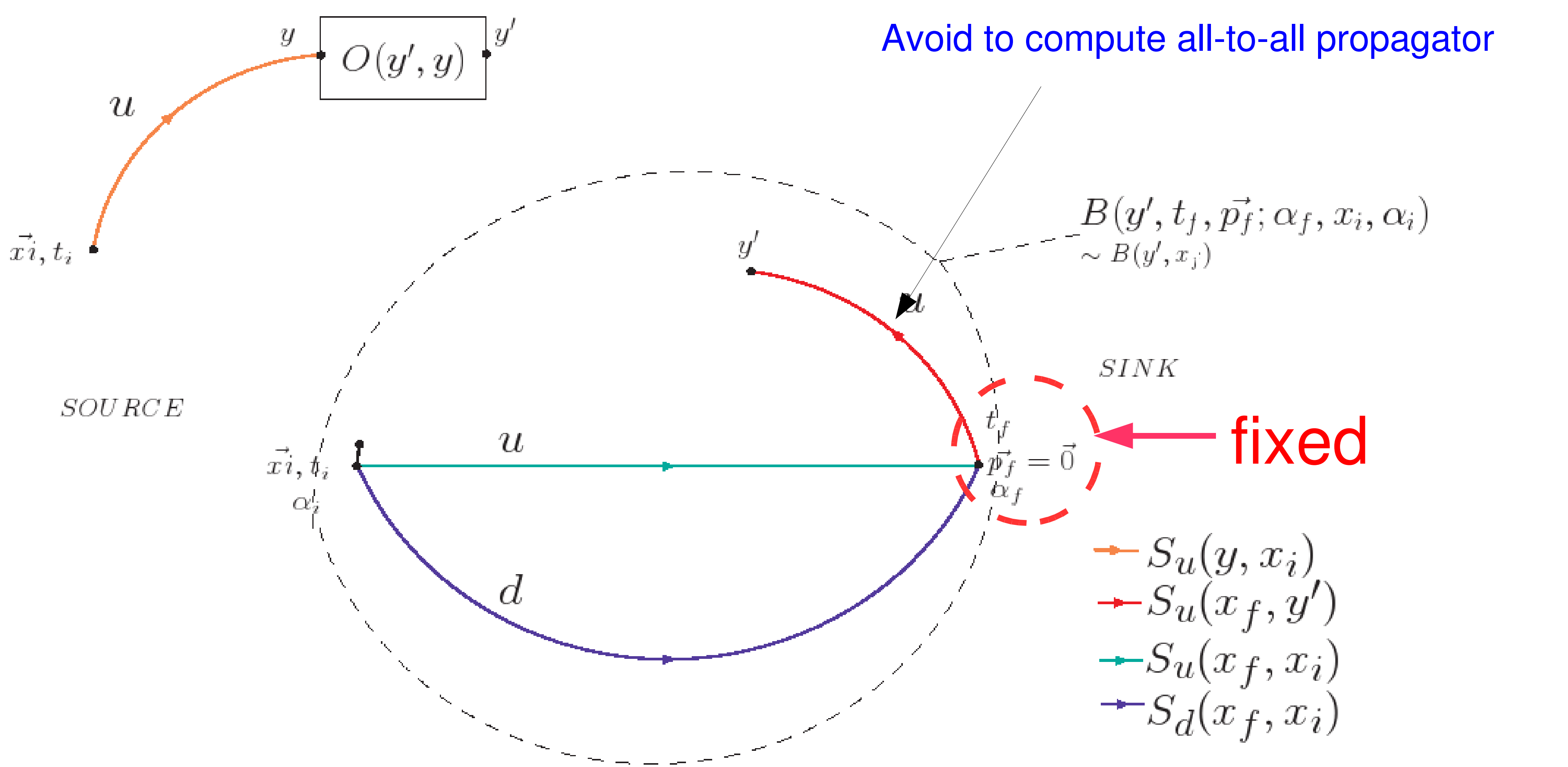


Backward propagator

$$\text{Using}: \; S_u(x_p, y') = \gamma_5 S_d^\dagger(y', x_p)\gamma_5$$

$$\longrightarrow B(y', t_f, \vec{p_f}; \alpha_f, x_i, \alpha_i) = \gamma_5 S_G^\dagger(t_f, \vec{p_f}, y'; \alpha_f, x_i, \alpha_i)$$

$$\langle P_{\alpha_f}(t_f, \vec{p_f} = \vec{0}) O(y', y) \bar{P}_{\alpha_i}(x_i)\rangle = \prod_{spin,color} B^\dagger(y', x_f)\gamma_5 \quad O(y', y) S_{u1}(y, x_i)$$



Source of the generalized propagator

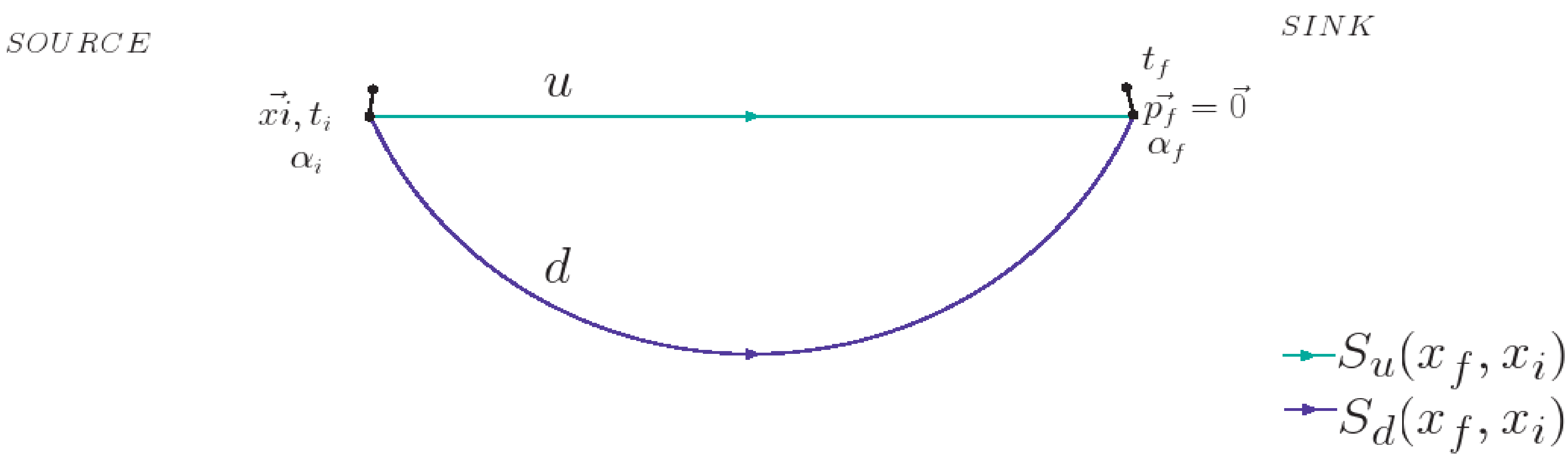
$$D_d(z,y')B(y',t_f,\vec{p_f}=\vec{0}) = \gamma_5 \prod_{spin,color}\Big(S_d(x_f,x_i)S_{u2}(x_f,x_i)\Big)^\dagger$$

Dirac operator of the d quark

Term source of the backward propagator

One source for each :

- t_f
- $\vec{p_f}$
- $\alpha_i \ , \ \alpha_f$

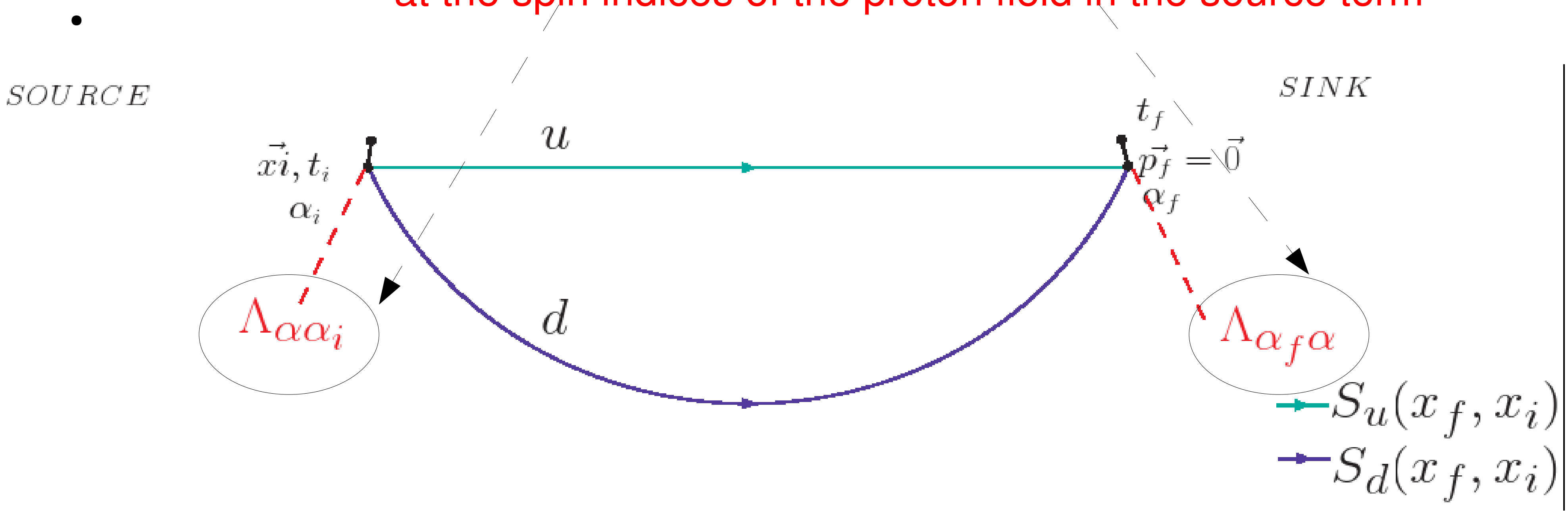


Source of the generalized propagator

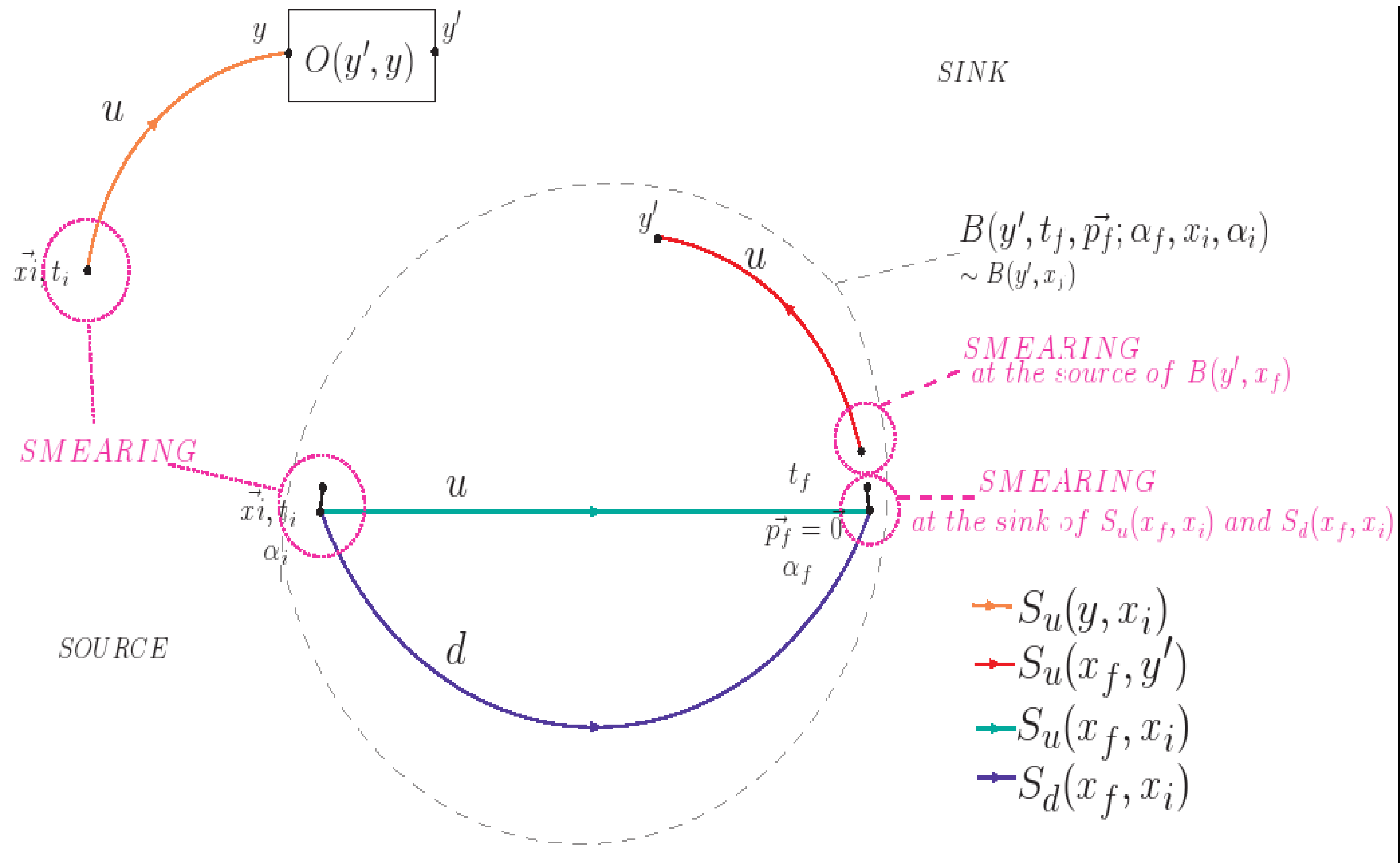
$$D_d(z,y')B(y',t_f,\vec{p_f}=\vec{0}) = \gamma_5 \prod_{spin,color}\Big(S_d(x_f,x_i)S_{u2}(x_f,x_i)\Big)^\dagger$$

One source for each :

- t_f
- $\vec{p_f}$
- $\alpha_i \;,\; \alpha_f \longrightarrow$ Trace with projecteur on parity (and/or) spin at the spin indices of the proton field in the source term



Smearing



y
$O(y', y)$
y'
SINK
u
$\vec{xi}, t_i$
y'
u
$B(y', t_f, \vec{p_f}; \alpha_f, x_i, \alpha_i)$
$\sim B(y', x_f)$
SMEARING
at the source of $B(y', x_f)$
SMEARING
u
t_f
SMEARING
at the sink of $S_u(x_f, x_i)$ and $S_d(x_f, x_i)$
$\vec{xi}, t_i$
$\vec{p_f} = \vec{0}$
α_i
α_f
SOURCE
d
$S_u(y, x_i)$
$S_u(x_f, y')$
$S_u(x_f, x_i)$
$S_d(x_f, x_i)$

Twisted Mass at full twist

- Through the relation

$$S_u(x_p, y') = \gamma_5 S_d^\dagger(y', x_p)\gamma_5$$

- Projector on the positive parity (sum over the polarisations) with TM rotation:

  - $\Lambda_{\alpha\alpha_i} = \frac{1+\gamma_0}{2} \quad exp(\pm i\gamma_5\frac{\pi}{4})$
  - $\Lambda_{\alpha_f\alpha} = exp(\pm i\gamma_5\frac{\pi}{4}) \quad \frac{1+\gamma_0}{2}$

  Projectors: $\frac{1+\gamma_0}{2}$ is an easy choice to compare with 2-pt functions but should be, as a second step, optimized depending on the chosen observable.

- Operators with TM rotation: $O^{phys} = exp(\pm i\gamma_5\frac{\pi}{4})O^{tm}exp(\pm i\gamma_5\frac{\pi}{4})$

Setup for tests

- $\beta = 3.9$, $L = 24^3$, $T = 48$, $\kappa = 0.160856$, $\mu = 0.0085$

  antiperiodic boundaries conditions

- time of the sink : $\begin{cases} t_f = 12 \\ t_f = 16 \end{cases}$

- $\vec{p_f} = \vec{0}$

- Projector on the positive parity (sum over the polarisations) with TM rotation:
  - $\Lambda_{\alpha\alpha_i} = \frac{1+\gamma_0}{2} \quad exp(\pm i\gamma_5 \frac{\pi}{4})$
  - $\Lambda_{\alpha_f\alpha} = exp(\pm i\gamma_5 \frac{\pi}{4}) \quad \frac{1+\gamma_0}{2}$

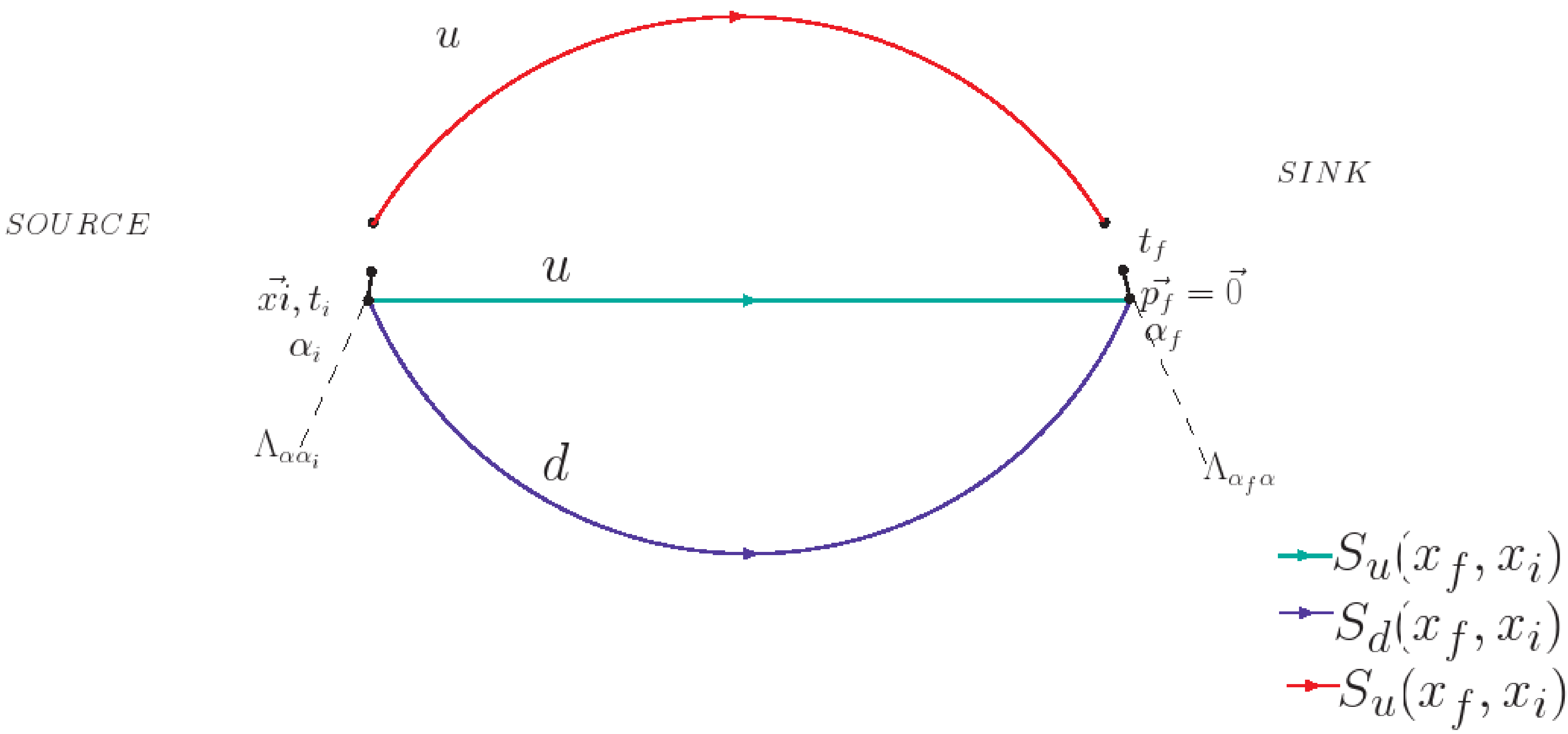
- Operators with TM rotation: $O^{phys} = exp(\pm i\gamma_5 \frac{\pi}{4}) O^{tm} exp(\pm i\gamma_5 \frac{\pi}{4})$

- Only connected diagramms

- Smearing only at the source of propagators

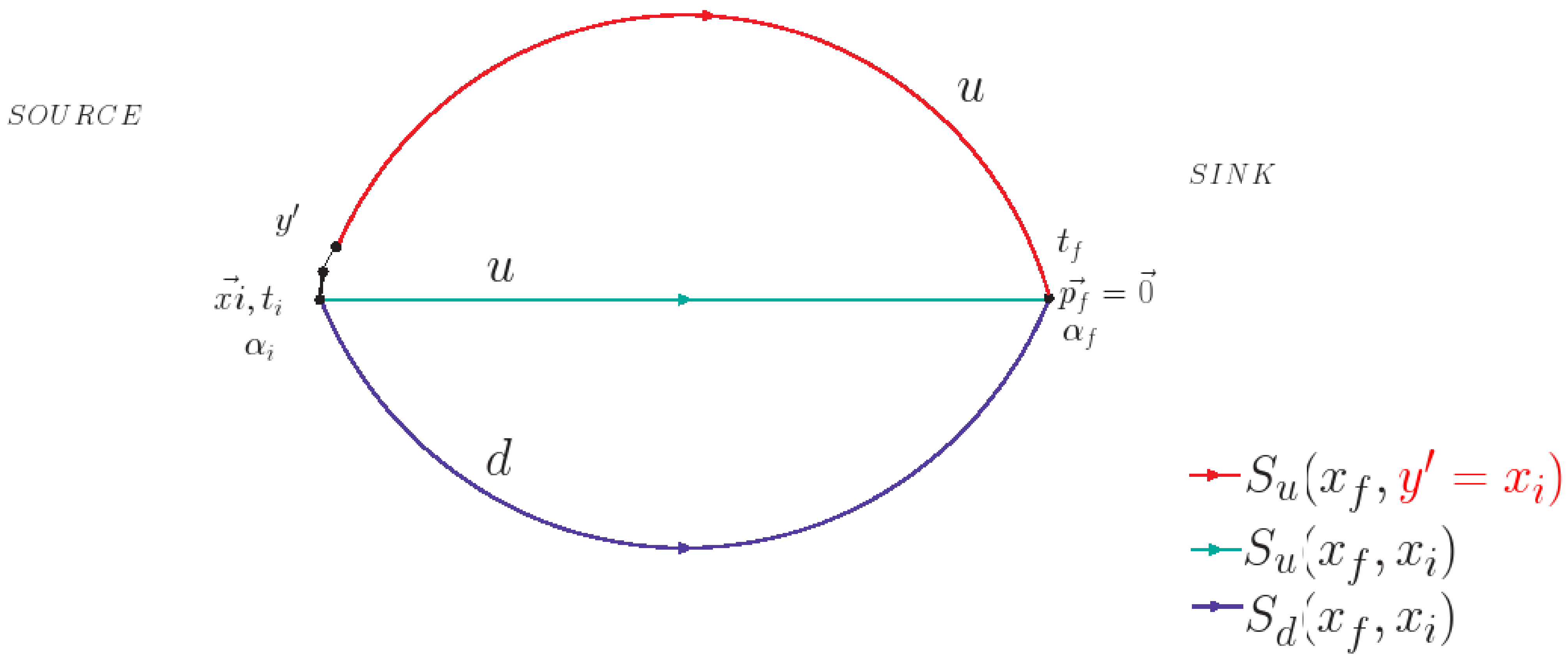
Test of the source with the 2-points function

$$\langle P_{\alpha_f}(t_f, \vec{p_f} = \vec{0})\bar{P}_{\alpha_i}(x_i)\rangle = \prod_{spin,color} \Big(S_d(x_f, x_i)S_{u2}(x_f, x_i)\Big) * S_u(x_f, x_i)$$

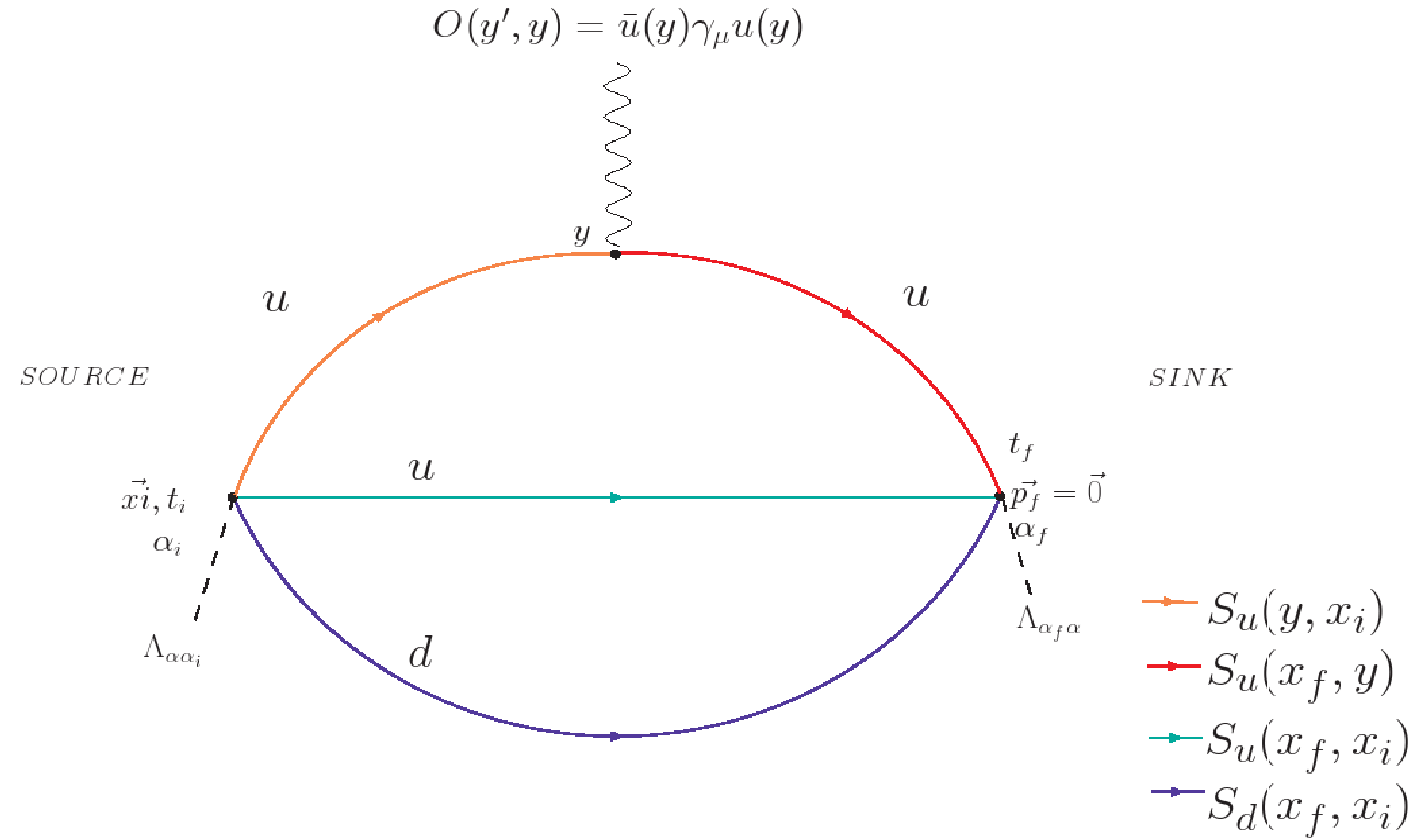


Test of the generalized propagator with the 2-points function

$$\langle P_{\alpha_f}(t_f,\vec{p_f}=\vec{0})\overline{P}_{\alpha_i}(x_i)\rangle = Tr\left(B^\dagger(y'=x_i,t_f,\vec{p_f}=\vec{0};\alpha_f,x_i,\alpha_i)\gamma_5\right)$$



Study of the local vectoriel current



Charge extraction

Only the current with a quark u computed

No isospin combinaison possible

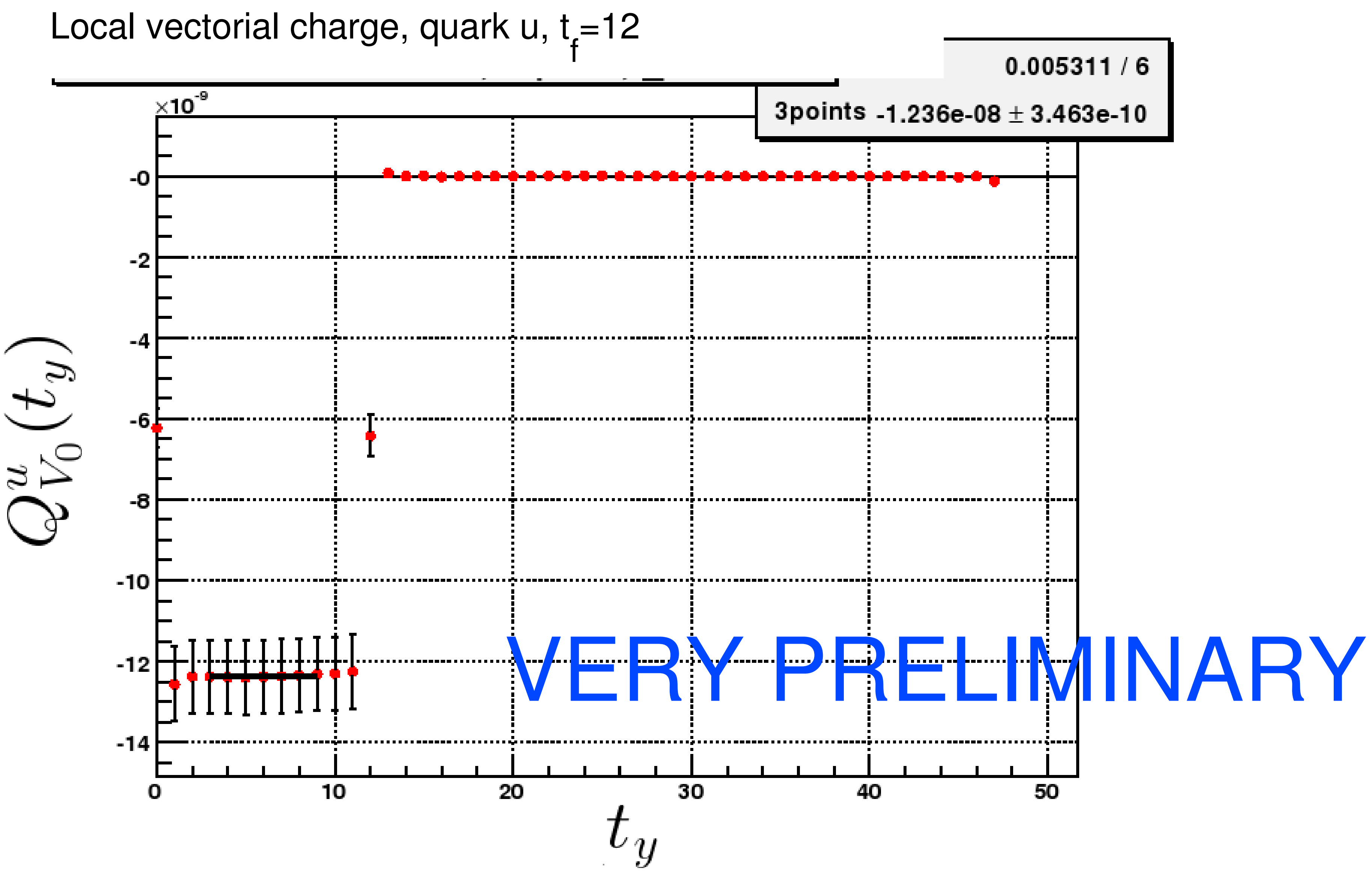
$$V_0^u(y) = \bar{u}(y)\gamma_0 u(y)$$

$$Q_{V_0}^u(t_y) = \int V_0^u(y) d^3 y_i$$

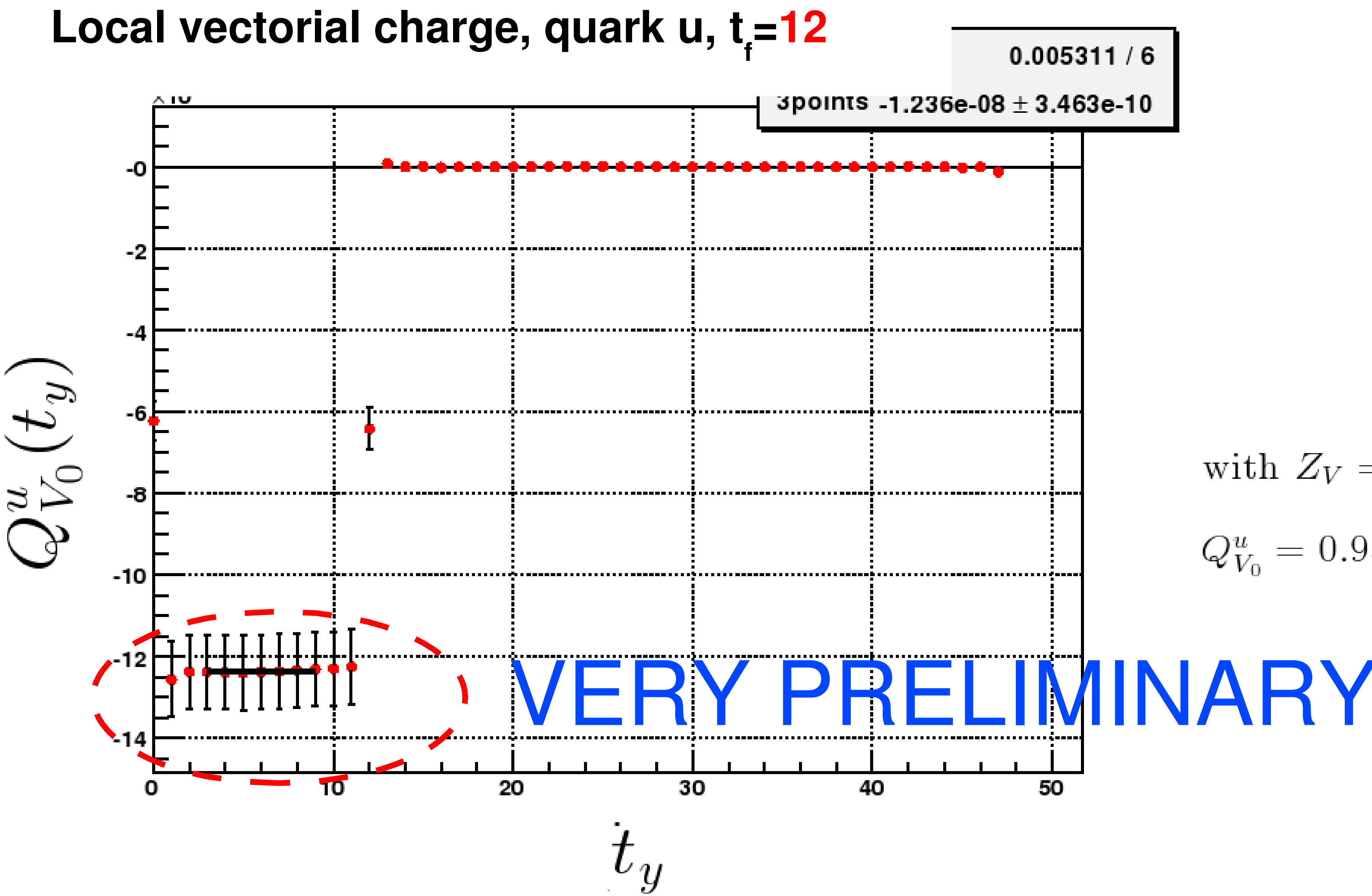
$$\langle P_{\alpha_f}(t_f, \vec{p_f} = \vec{0}) Q(t_y) P_{\alpha_i}(x_i) \rangle = \frac{1}{Z_V} \; Q(t_y) \; \langle P_{\alpha_f}(t_f, \vec{p_f} = \vec{0}) P_{\alpha_i}(x_i) \rangle$$

Constant -> Plateau

Local vectorial charge



Local vectorial charge

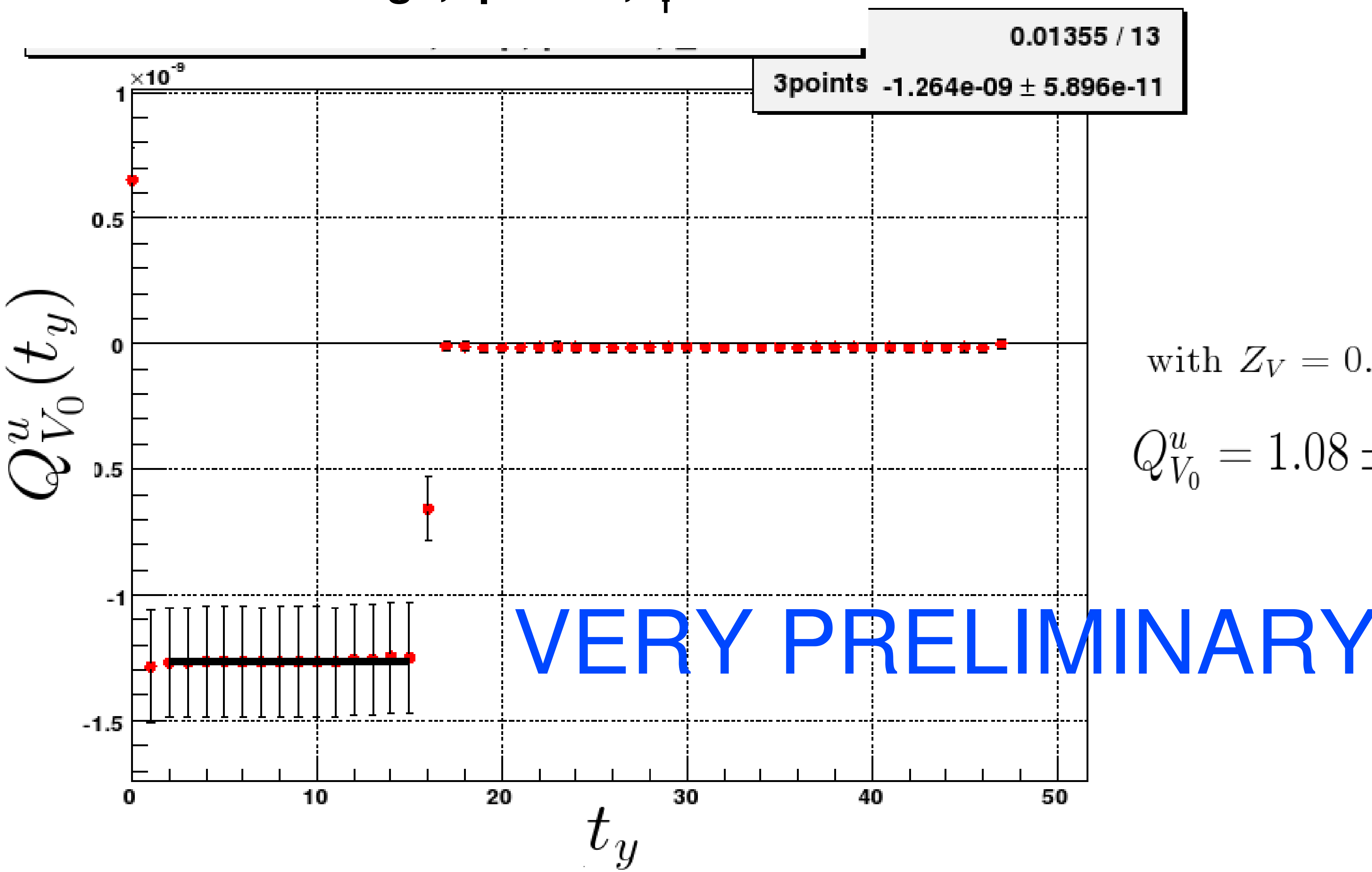


with $Z_V = 0.61$

$Q^u_{V_0} = 0.95 \pm 15\%$

Local vectorial charge

Local vectorial charge, quark u, t_f=16

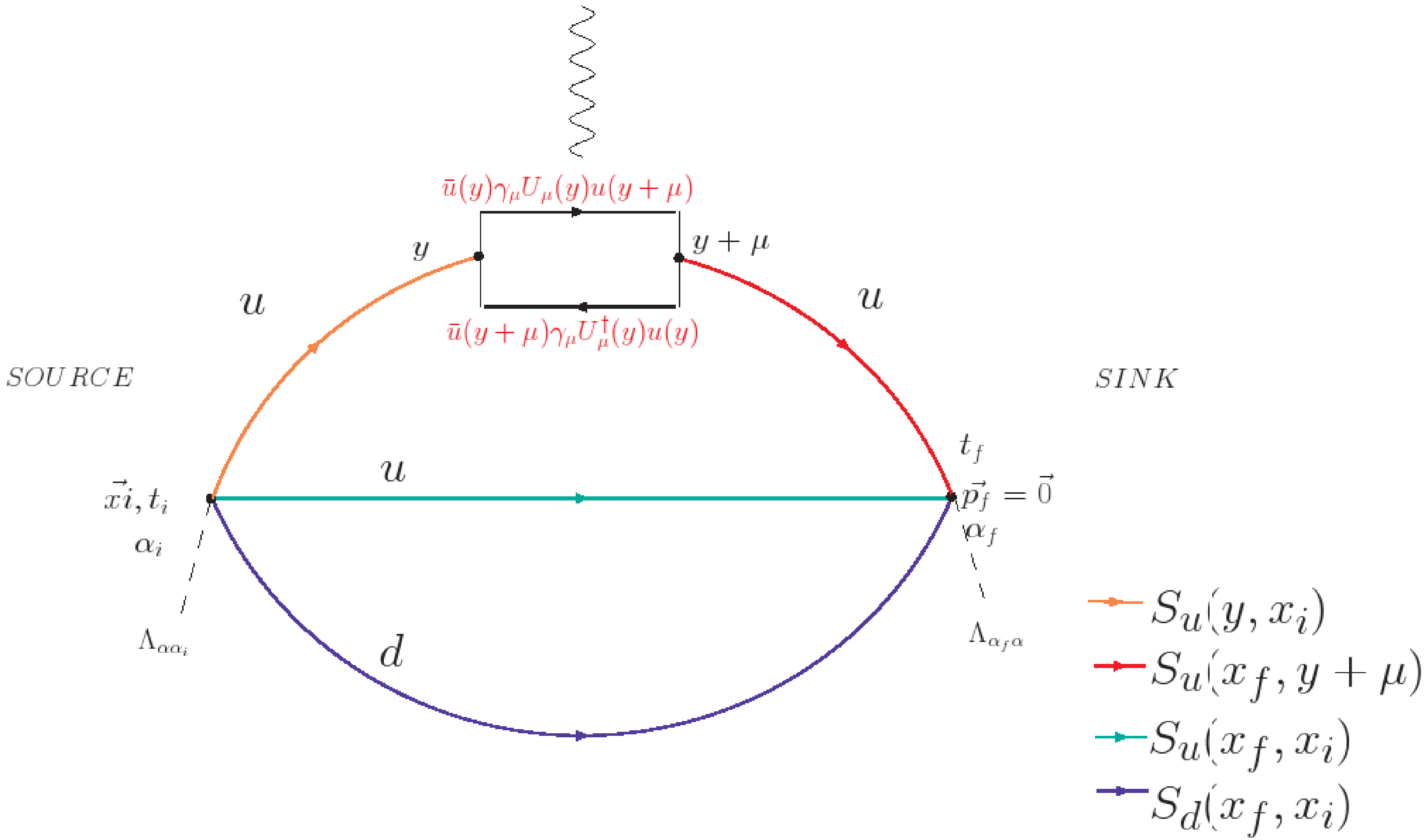


with $Z_V = 0.61$

$$Q^u_{V_0} = 1.08 \pm 15\%$$

Study of the lattice conserved vectorial current

$$O(y+\mu, y) = \bar{u}(y)(\gamma_\mu - r)U_\mu(y)u(y+\mu) + \bar{u}(y+\mu)(\gamma_\mu + r)U^\dagger_\mu(y)u(y)$$

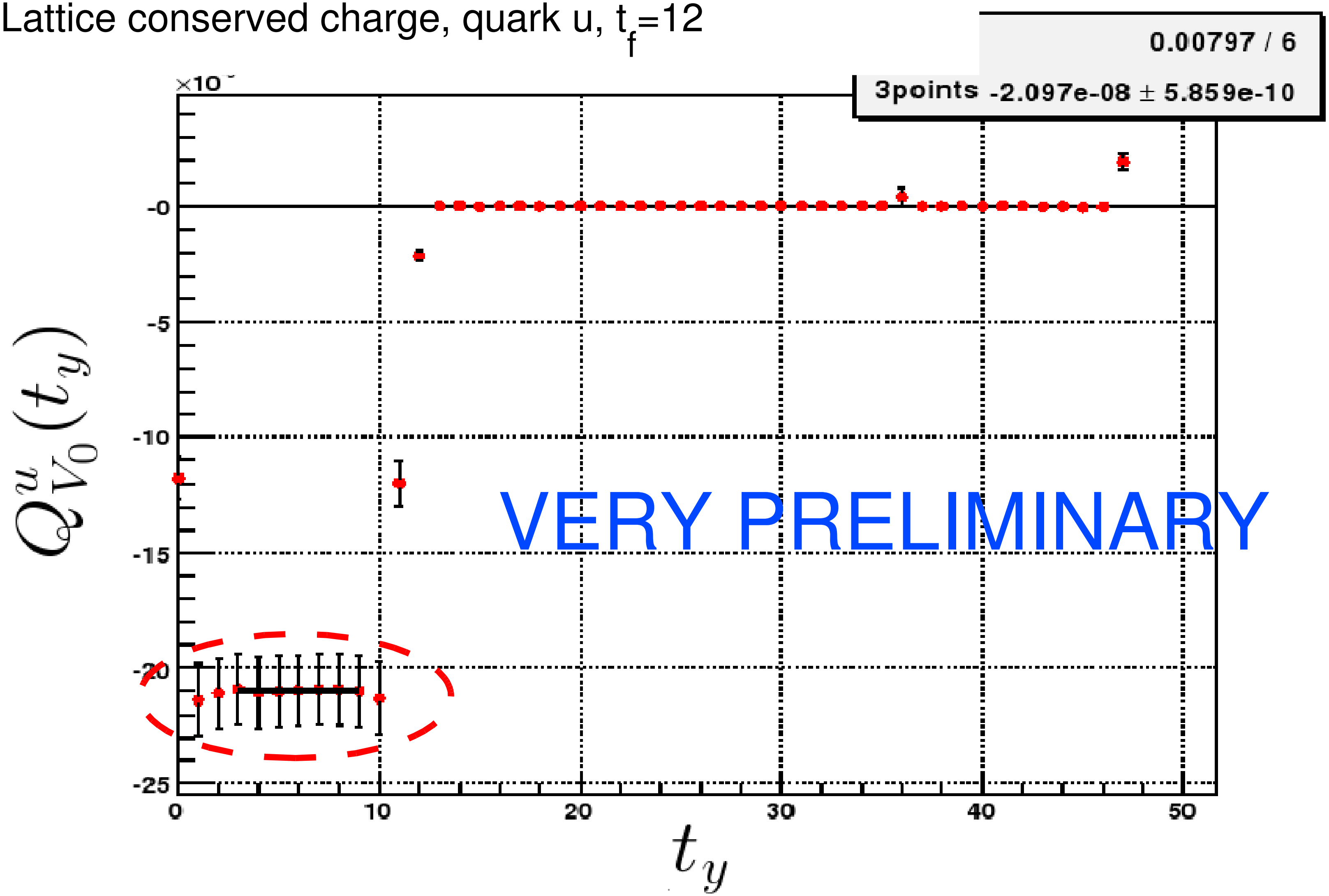


Lattice conserved vectoriel charge

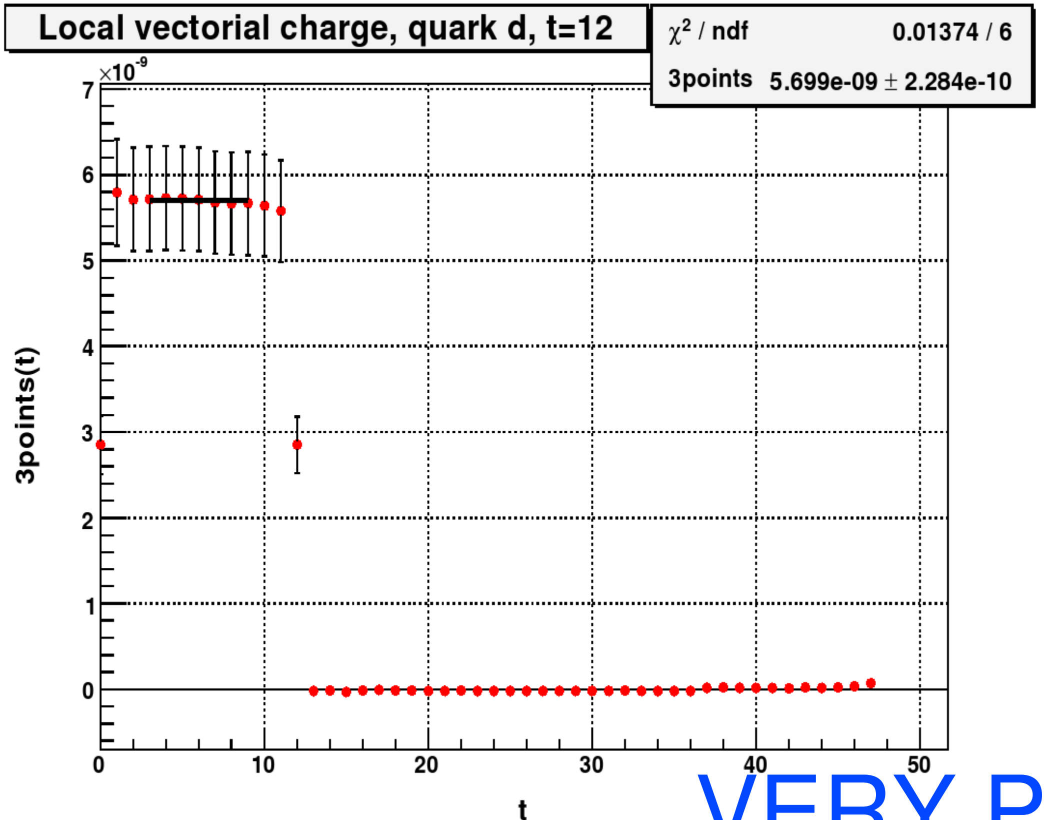
$$
\begin{aligned}
V_0(y) &= \bar{u}(y)(\gamma_0 - r)U_0(y)u(y+\mu_0) \\
&+ \bar{u}(y+\mu_0)(\gamma_0 + r)U_0^\dagger(y)u(y)
\end{aligned}
$$

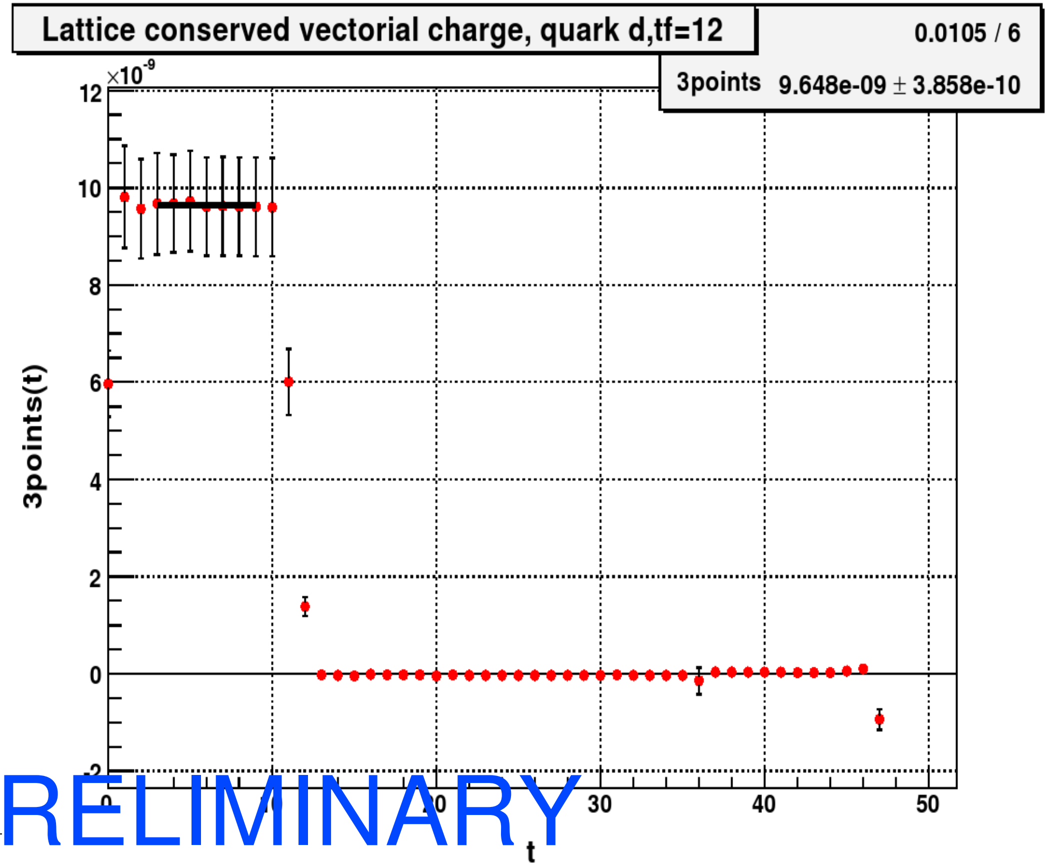
$$
Q^u_{V_0}(t_y) = \int V_0^u(y)d^3y_i
$$

Lattice conserved charge, quark u, t_f=12



OVERVIEW OF THE LOCAL VECTORIEL CHARGE ON D QUARK





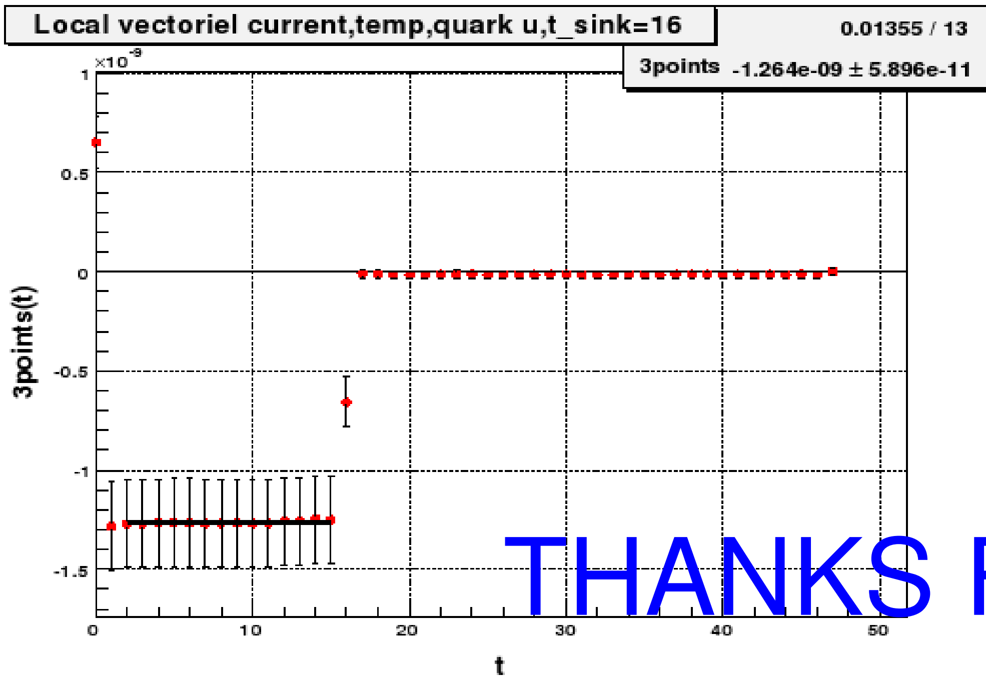
VERY PRELIMINARY

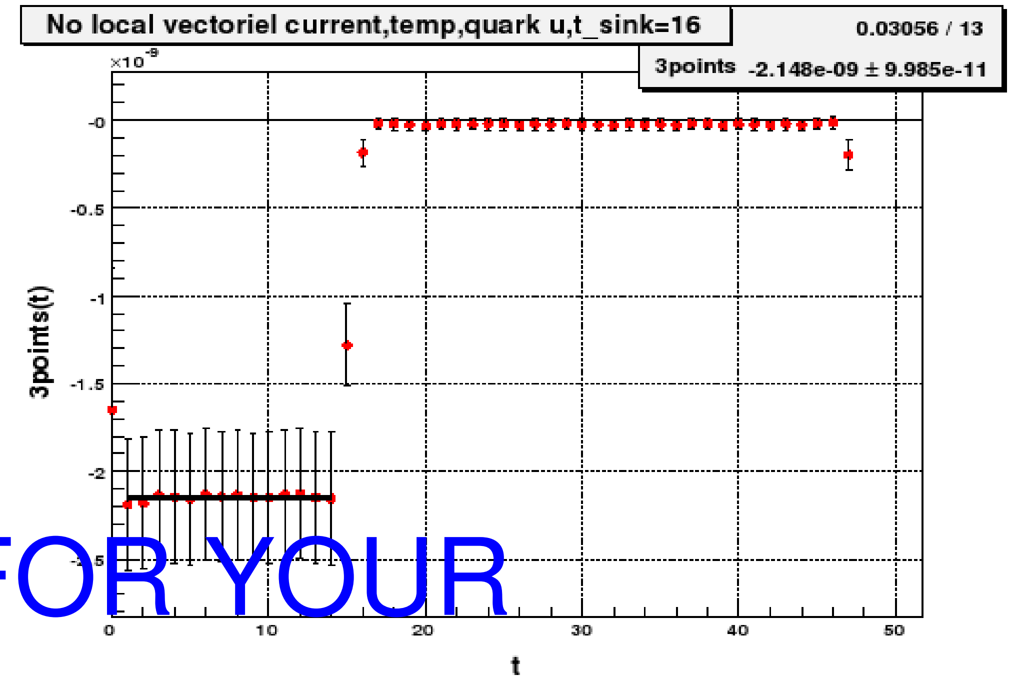
38 configurations

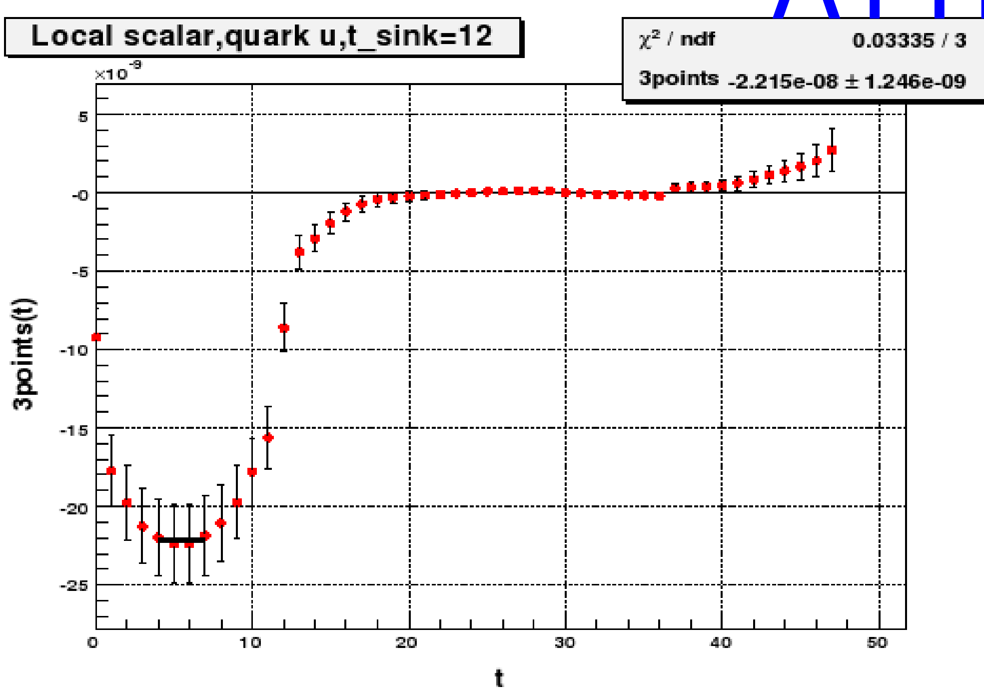
To do list

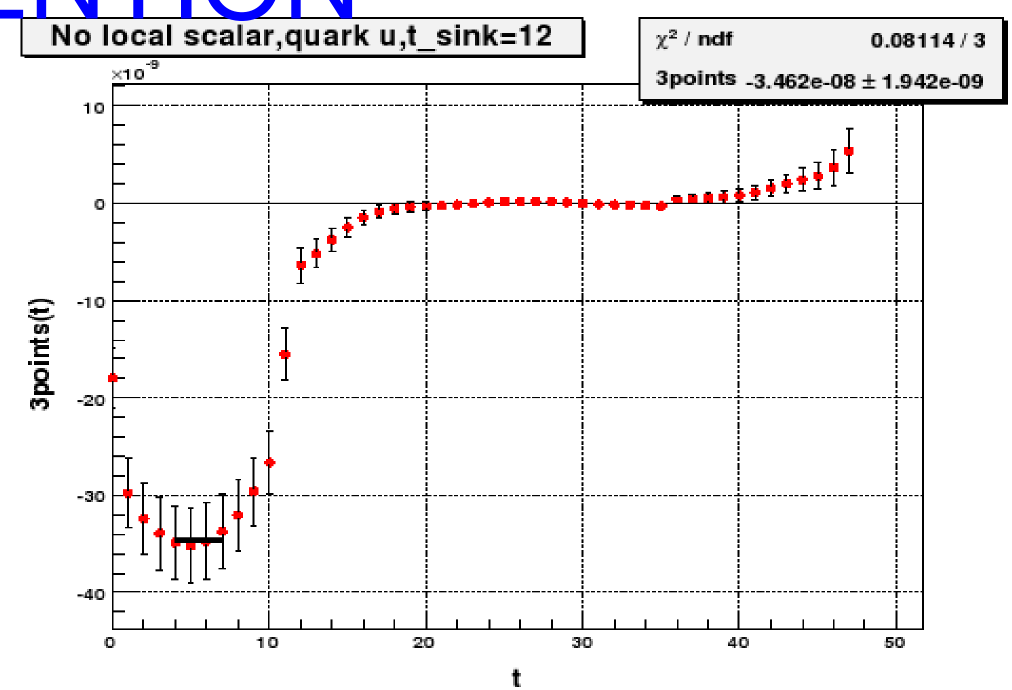
- Check the preliminary results (Z_V ...)
- Choose sink time
- Test with polarized proton
- Compute generalized propagator for others beta and mu
- Compute renormalisation constant $\longrightarrow$ Zhaofeng
- Add twisted boundaries conditions to get observables at small q^2
(and doing all the process again for each q^2)
- Contractions for the moments of gpdf $\longrightarrow$ Dru

THANKS FOR YOUR ATTENTION









3 point function with an operator acting on a quark u

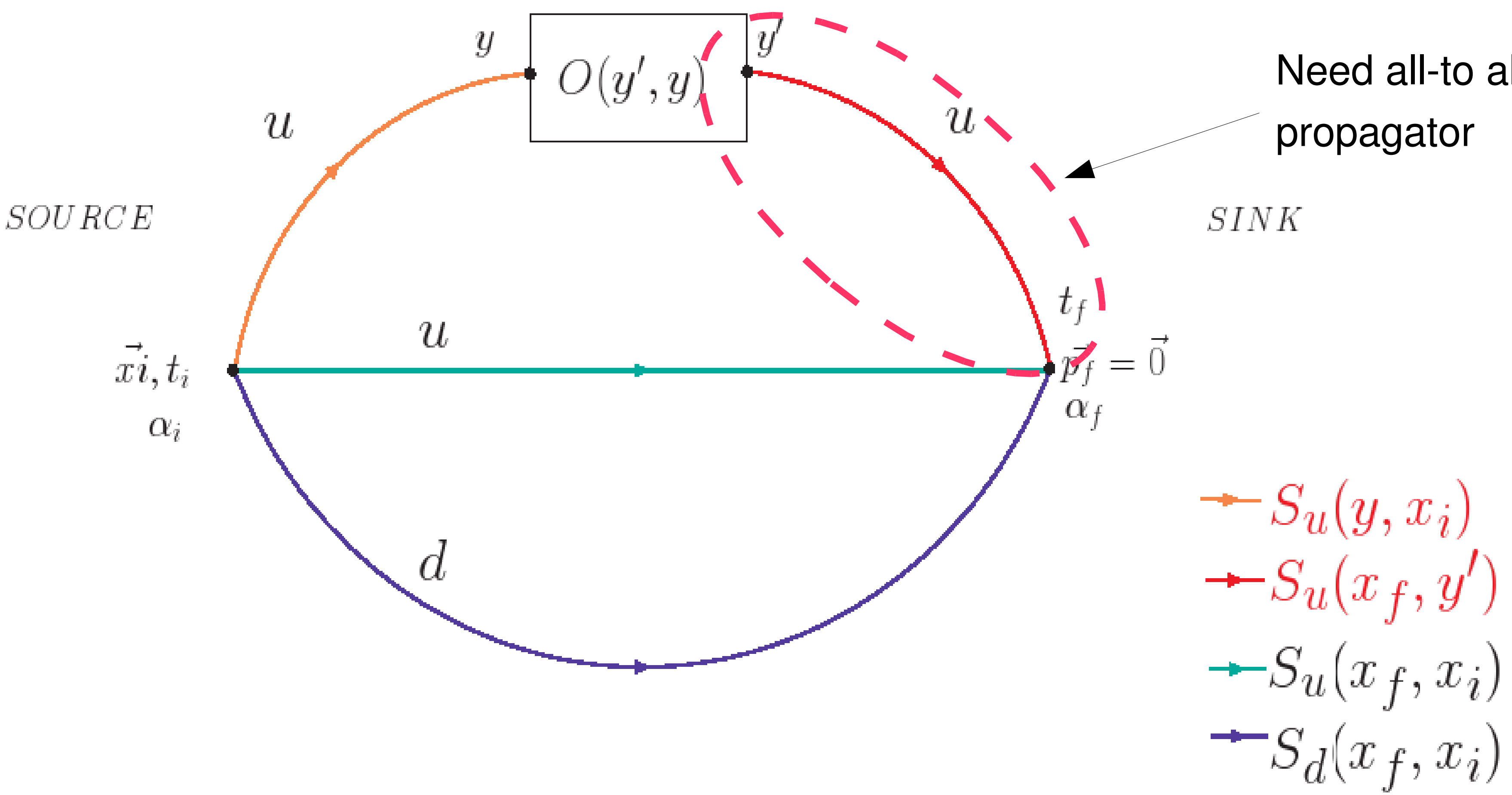
Interpolating field of the proton :

$$P_\alpha(x) = \epsilon_{abc}(u_a(x)^T C\gamma_5 d_b(x))u_c(x)$$

Observables :

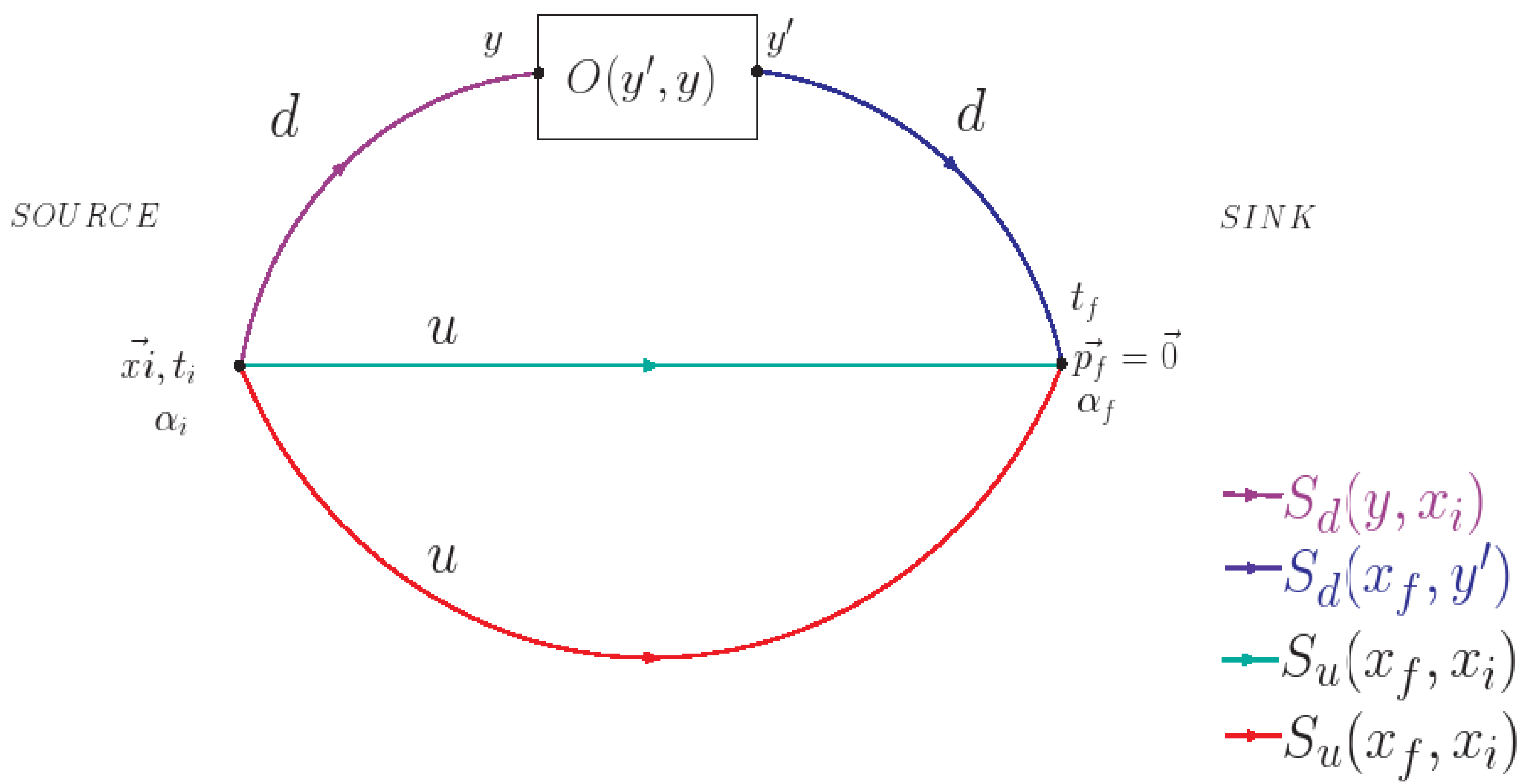
operator O(y',y)

$$\langle P_{\alpha_f}(t_f, \vec{p_f} = \vec{0}) O(y', y) P_{\alpha_i}(x_i) \rangle = \prod_{spin,color} S_{u1}(x_f, y') O(y', y) S_{u1}(y, x_i) S_d(x_f, x_i) S_{u2}(x_f, x_i)$$



3 points function with an operator acting on a quark d

$$\langle P_{\alpha_f}(t_f, \vec{p_f} = \vec{0}) O(y', y) P_{\alpha_i}(x_i) \rangle = \prod_{spin,color} S_{u1}(x_f, x_i) S_d(x_f, y') O(y', y) S_d(y, x_i) S_{u2}(x_f, x_i)$$



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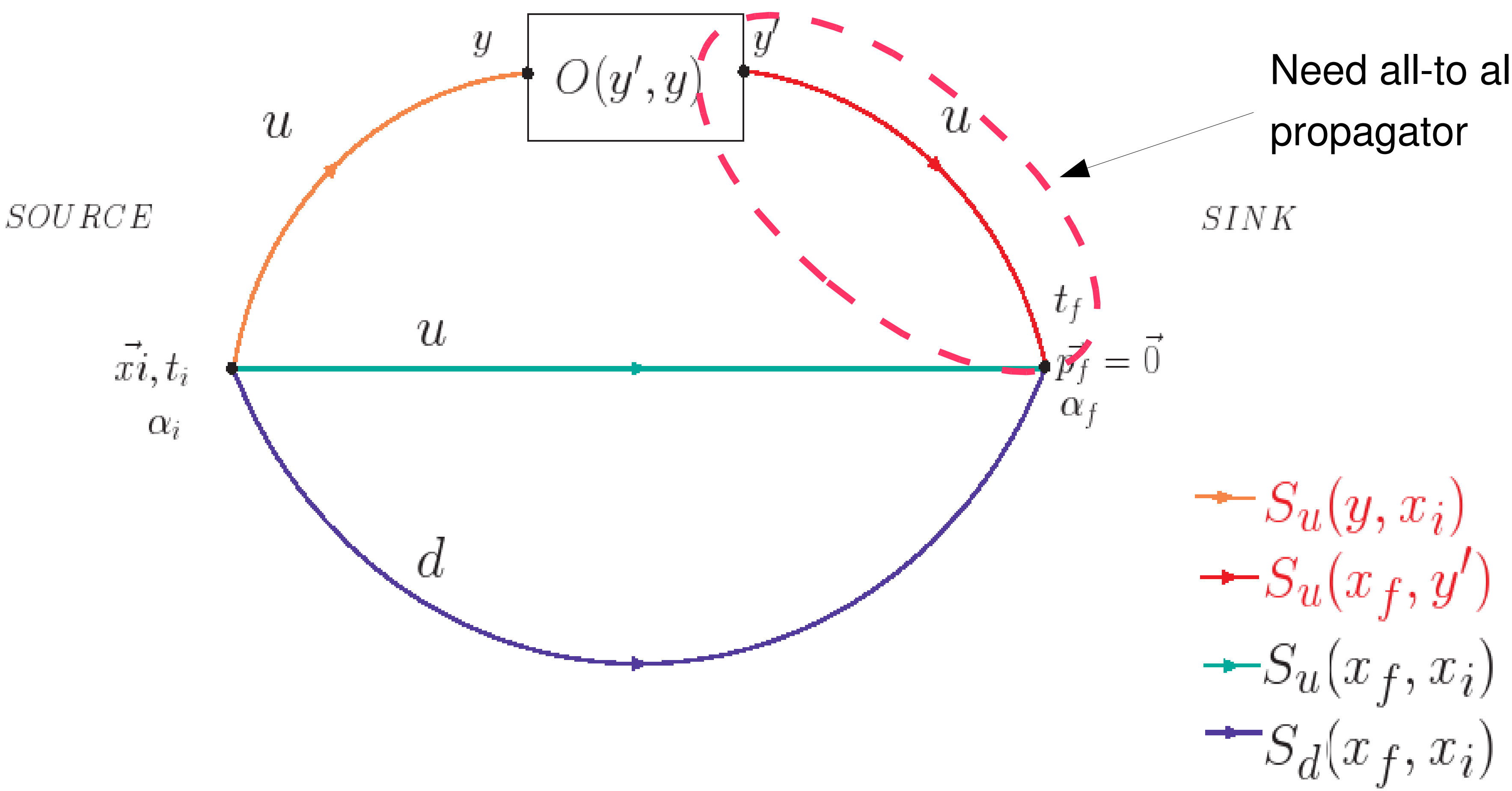

Generalized propagator

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