Observable-dependent test of maximal twist

Petros Dimopoulos **Roberto Frezzotti**

***a presentation based on the notes prepared by*:**
P. Dimopoulos, R. Frezzotti, G. Herdoiza and G.C. Rossi

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- Safe observable-dependent estimates of the systematic effects due to the uncertainty on κ_{cr}.

- Implementation on the observables F_{ps}, M^2_{ps} and m_N

- Similar methods may be applied to other observables

Assume simulations that have been done at some β with : $m_0 - M_{cr}^{opt} \neq 0$

The action in **twisted mass basis** reads:

$$S_F = \bar{\chi}[\gamma \cdot \tilde{\nabla} + W_{cr} + (m_0 - M_{cr}^{opt}) + i\mu\gamma_5\tau^3]\chi$$

with : $W_{cr} = -ar\frac{1}{2}\nabla_\nu^*\nabla_\nu + M_{cr}^{opt}$ (the optimal critical Wilson term)

The renormalised quark mass is given:

$$m_R = Z_{S^0}^{-1}(m_0 - M_{cr}^{opt}) = Z_A Z_P^{-1} m_{PCAC}\,, \qquad \mu_R = Z_P^{-1}\mu\,.$$

The action in **physical basis** is:

$$S_F = \bar{\psi}[\gamma \cdot \tilde{\nabla} + e^{-i\omega\gamma_5\tau^3} W_{cr} + M]\psi$$

with: $M = \sqrt{(m_0 - M_{cr}^{opt})^2 + \mu^2}\,,$ $\qquad \omega = \arctan[\mu/(m_0 - M_{cr}^{opt})]$

and fields given by: $\psi = e^{i\omega\gamma_5\tau^3/2}\chi,\ \ \overline{\psi} = \overline{\chi}\, e^{i\omega\gamma_5\tau^3/2}$ *(angle ω not known)*

The twisted angle **α** is estimated in terms of the renormalised quark masses:

$$\alpha \equiv \arctan(\mu_R/m_R)\,, \qquad \theta \equiv \pi/2 - \alpha$$

and **θ** is the deviation from π/2.

The *renormalised* quark mass is given by:

$$M_R = \sqrt{m_R^2 + \mu_R^2} = m_R / \sin(\theta) = \mu_R / \cos(\theta)$$

The **effective Symanzik action** which corresponds for the case of $\theta \neq 0$ is :

$$\mathcal{L}_{\rm Sym} = \mathcal{L}_4^{YM} + \bar\psi(D + M_{\rm R})\psi + a\mathcal{L}_5 + {\rm O}(a^2)\,,$$
$$\mathcal{L}_5 = b_1[\sin(\theta)\bar\psi i\sigma\cdot F\psi + \cos(\theta)\bar\psi\gamma_5\tau^3\sigma\cdot F\psi] + b_4 M_{\rm R}\sin(\theta)\mathcal{L}_4^{YM} +$$
$$+[b_5(M_{\rm R}\sin(\theta))^2 + b_7(M_{\rm R}\cos(\theta))^2][\sin(\theta)\bar\psi\psi - \cos(\theta)\bar\psi i\gamma_5\tau^3\psi]\,.$$

Implications

► Data have to be interpreted in terms of the renormalised quark mass:

$$M_R = \mu_R / \sin(\alpha) = \mu_R / \cos(\theta) \neq \mu_R$$

► Operators allowed to rotate *must* rotate, e.g.

$$[A_\mu^1]_R \to \cos(\theta)[A_\mu^1]_R + \sin(\theta)[V_\mu^2]_R + O(a)$$

} ***"Analytical" estimates***

► Existence of ***O(αθ)*** **discretization errors** in hadron masses and matrix elements induced by $\boxed{\sin(\theta)\bar\psi i\sigma\cdot F\psi}$ of L_5 .

→ ***Estimates from the fits***

The maximal twist quality test

For all the values of β, we imagine to work at the same value of the twisted renormalised quark mass:

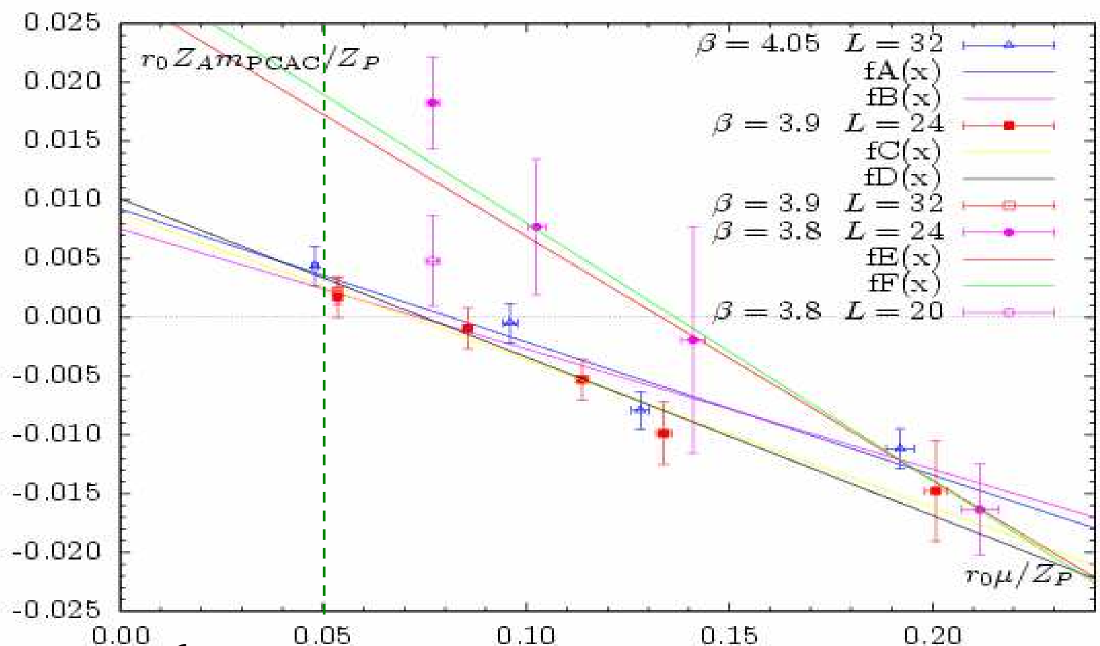
$$\mu_{\mathrm{R,low}} r_0 = 0.05$$

We are off $m_{PCAC}=0$ by:

$$\Delta_{m,\mathrm{low}} = \sqrt{[m^2_{\mathrm{PCAC}} + \sigma^2(m_{\mathrm{PCAC}})]|_{\mu_{\mathrm{R,low}}}}$$

The deviation of the twisted angle from $\pi/2$ is estimated by:

$$\theta_{\mathrm{low}} \equiv \arctan[Z_A \Delta_{m,\mathrm{low}}/\mu]\,, \qquad \theta_{\mathrm{low}} = \theta_{\mathrm{low}}(a\mu, \beta)$$



(*Extrapolation at the reference point* $\mu_{\mathrm{R,low}} r_0 = 0.05$)

For our simulations:

$$\beta = 3.8\,,\ a\mu_{\rm low} = 0.0039 \quad \Rightarrow \quad a\Delta_{m,\rm low} = 0.00183$$
$$\beta = 3.9\,,\ a\mu_{\rm low} = 0.0037 \quad \Rightarrow \quad a\Delta_{m,\rm low} = 0.00027$$
$$\beta = 4.05\,,\ a\mu_{\rm low} = 0.0031 \quad \Rightarrow \quad a\Delta_{m,\rm low} = 0.00031$$

β	$Z_P^{RI'}(2.3\ \mathrm{GeV})$	Z_A	$am_{\rm PCAC}\vert_{\mu_{\rm low}}$	r_0/a
3.80	0.348(7)	0.75(1)	0.00180(33)	4.46(3)
3.90	0.390(5)	0.76(1)	0.00026(09)	5.22(2)
4.05	0.413(7)	0.77(1)	0.00029(11)	6.61(3)

$$\theta_{low} = \theta_{low}\,(a\mu\,,\ \beta)$$

ensembles	$\beta = 3.80$	$\beta = 3.90$	$\beta = 4.05$
$\mu = \mu_{\rm low}$	0.338	0.055	0.077
B_1, C_1		0.051	0.079
A_1, B_2,	0.225	0.032	
A_2 B_3, C_2	0.170	0.024	0.040
A_3 B_4, C_3	0.124	0.021	0.030
A_4 B_5, C_4	0.083	0.014	0.020
B_6, C_5		0.051	0.040

Test Condition : Statistical error ≥ Systematic uncertainties

$$\sigma_{m_{\rm PS}^2 r_0^2} \gtrsim \left\{ \left| \left.\frac{\partial m_{\rm PS}^2}{\partial M_{\rm R}}\right|^{\rm cont} \delta M_{\rm R} \right| + \left| u\theta_{\rm low}\overline{C}_m r_0^{-2} \right| \right\} r_0^2$$

$$\sigma_{f_{\rm PS} r_0} \gtrsim \left\{ \left| \left.\frac{\partial f_{\rm PS}}{\partial M_{\rm R}}\right|^{\rm cont} \delta M_{\rm R} + (\cos(\theta_{\rm low})^{-1} - 1) f_{\rm PS} \right| + \left| u\theta_{\rm low}\overline{C}_f r_0^{-1} \right| \right\} r_0$$

$$\sigma_{m_{\rm N} r_0} \gtrsim \left\{ \left| \left.\frac{\partial m_{\rm N}}{\partial M_{\rm R}}\right|^{\rm cont} \delta M_{\rm R} \right| + \left| u\theta_{\rm low}\overline{C}_N r_0^{-1} \right| \right\} r_0$$

(where $u = a / r_0$)

- **Statistical error**: $\sigma_Y = \sigma_Y^{\rm raw} + \left| \frac{\partial Y}{\partial X} \right| \sigma_X$ (linear sum due to correlations)

- **Systematic uncertainties**:

  - For: $Y = f_{PS}, m_{PS}^2, m_N$

$$\left.\frac{\partial Y}{\partial M_{\rm R}}\right|^{\rm cont} \delta M_{\rm R} = \left.\frac{\partial Y}{\partial \mu_{\rm R}}\right|^{\rm cont} \left.\frac{\partial \mu_{\rm R}}{\partial M_{\rm R}}\right|^{\rm cont} \delta M_{\rm R} = \left.\frac{\partial Y}{\partial \mu_{\rm R}}\right|^{\rm cont} \mu_{\rm R}(1 - \cos(\theta_{\rm low})^{-1})|^{\beta}$$

with: $\delta M_{\rm R} = \mu_{\rm R}(\cos(\theta_{\rm low})^{-1} - 1)|^{\beta} \sim \theta_{\rm low}^2$

  - $(\cos(\theta_{\rm low})^{-1} - 1) f_{\rm PS}$ comes from the rotation of the operator
  - "$u\theta_{low}\overline{C}_{m,f,N}$" quantifies the *O(aθ)* corrections

Estimates of $\overline{C}_Y$

Do a Combined chiral fit including θ and O(aθ) corrections
(μ_R independent variable):

$$f_{\rm PS} r_0 = r_0 f_0 \Big[1 - 2\xi \log\left(\tfrac{\chi_\mu}{\Lambda_4^2}\right) + u^2 D_f^0\Big]\ K_f^{\rm CDH}(\sqrt{\chi_\mu}L) + \\ + \left\{ \frac{\partial f_{\rm PS}}{\partial M_{\rm R}}\bigg|^{\rm cont} \delta M_{\rm R} - (\cos(\tilde\theta_{\rm low})^{-1} - 1) f_{\rm PS} + u\tilde\theta_{\rm low} C_f r_0^{-1} \right\} r_0$$

$$r_0^2 m_{\rm PS}^2 = \chi_\mu r_0^2 \Big[1 + \xi \log\left(\tfrac{\chi_\mu}{\Lambda_3^2}\right) + u^2 D_m^0\Big]\ \left(K_m^{\rm CDH}(\sqrt{\chi_\mu}L)\right)^2 + \\ + \left\{ \frac{\partial m_{\rm PS}^2}{\partial M_{\rm R}}\bigg|^{\rm cont} \delta M_{\rm R} + u\tilde\theta_{\rm low} C_m r_0^{-2} \right\} r_0^2 \,,$$

$$m_{\rm N} r_0 = r_0 M_N - \tfrac{4c_1}{r_0} \chi_\mu r_0^2 - \tfrac{6 g_A^2}{32\pi f_0^2 r_0^2} (\chi_\mu r_0^2)^{3/2} + u^2 D_N^0 r_0 M_N + \\ + \left\{ \frac{\partial m_{\rm N}}{\partial M_{\rm R}}\bigg|^{\rm cont} \delta M_{\rm R} + u\tilde\theta_{\rm low} C_N r_0^{-1} \right\} r_0 \,,$$

[♠]

$$\xi \equiv \frac{2B_R\mu_R}{(4\pi f_0)^2} \,, \qquad \chi_\mu \equiv 2B_R\mu_R$$

$$\tilde\theta_{\rm low} \equiv \arctan[Z_A m_{\rm PCAC}|_{\mu_{\rm low}}/\mu] \,, \qquad \tilde\theta_{\rm low} = \tilde\theta_{\rm low}(a\mu, \beta)$$

For $$\tilde{\theta}_{\rm low} \equiv \arctan[Z_A m_{\rm PCAC}|_{\mu_{\rm low}}/\mu]\,, \qquad \tilde{\theta}_{\rm low} = \tilde{\theta}_{\rm low}(a\mu, \beta)$$

ensembles	$\beta = 3.80$	$\beta = 3.90$	$\beta = 4.05$
$\mu = \mu_{\rm low}$	0.338	0.055	0.077
B_1, C_1		0.049(17)	0.075(29)
A_1, B_2,	0.221(39)	0.031(11)	
A_2 B_3, C_2	0.167(30)	0.023(08)	0.038(14)
A_3 B_4, C_3	0.122(22)	0.020(07)	0.028(11)
A_4 B_5, C_4	0.082(15)	0.013(05)	0.019(07)
B_6, C_5		0.049(17)	0.038(14)

Final combined fit of "safe" data

Use the data that pass the Maximal Twist test do the **combined fit:**

$$r_0 f_{\rm PS}(r_0\mu_R) = r_0 f_0 \left[1 - 2\xi \log\left(\frac{\chi_\mu}{\Lambda_4^2}\right) + u^2 D_f^0\right] \; K_f^{\rm CDH}(\sqrt{\chi_\mu}L)$$

$$(r_0 m_{\rm PS})^2(r_0\mu_R) = \chi_\mu r_0^2 \left[1 + \xi \log\left(\frac{\chi_\mu}{\Lambda_3^2}\right) + u^2 D_m^0\right] \; K_m^{\rm CDH}(\sqrt{\chi_\mu}L)^2$$

$$r_0 m_N(r_0\mu_R) = r_0 M_N - \frac{4c_1}{r_0}\chi_\mu r_0^2 - \frac{6g_A^2}{32\pi f_0^2 r_0^2}(\chi_\mu r_0^2)^{3/2} + u^2 D_N^0 r_0 M_N$$

$$\xi \equiv \frac{2B_R\mu_R}{(4\pi f_0)^2}\,, \qquad \chi_\mu \equiv 2B_R\mu_R$$

[♠♠]

… and obtain the LEC's

Combined Fits for $\beta = 3.90$ and 4.05

- Use Eq. [♠♠] with D's=0
- *Bootstrap method* for the error analysis: create bootstrap samples from the raw data for m^2_{ps} and F_{ps}; m_N samples are generated according a gaussian distribution (*following C. Urbach*); Pick up values Z_P from a gaussian distribution to form μ_R . Apply the ***Grid search method.***
- *Combined-concatenated chiral fits*: chain application of the fits on m^2_{ps} , F_{ps} and m_N until convergence is succeeded.
- add (linearly) in the end of the procedure the error of r_0/a

$$\beta = 3.90 : \quad \mu = 0.0040\,,\, 0.0040\,(L=32), 0.0064\,,\, 0.0085\,,\, 0.0100$$

$$\beta = 4.05 : \quad \mu = 0.0030\,,\, 0.0060\,,\, 0.0060\,(L=24), 0.0080$$

$$r_0 f_0, 2r_0 B_0 \text{ (renormalised)}, r_0\Lambda_3, r_0\Lambda_4, r_0 M_N, c_1, g_A, R_1$$

($R_1=[r0/a]_{3.90} / [r0/a]_{4.05}$)

Combined fits for $\beta = 3.90$ and 4.05

	C. Urbach	Bootstrap method	Combined-Concatenated fits
$2r_0B_0$	10.06(20)	10.30(24)	10.12(25)
r_0f_0	0.275(2)	0.273(5)	0.274(4)
r_0M_N	2.01(13)	2.04(12)	2.05(18)
$r_0\Lambda_3$	1.75(7)	1.98(8)	1.87(18)
$r_0\Lambda_4$	3.14(4)	3.13(9)	3.13(7)
c_1	-1.13(26)	-1.05(17)	-1.04(36)
g_A	1.13(17)	1.06(15)	1.06(26)
R_1	0.782(3)	0.785(6)	0.790(5)
χ^2/dof	18.5/19	0.30	0.77

Combined Fits for β = 3. 90 and 4. 05 + O(a²) terms

[Use Eq. [♠♠]]

Combined fits for $\beta = 3.90$ and 4.05 plus $O(a^2)$ terms

	C. Urbach	Bootstrap method
$2r_0B_0$	10.44(70)	10.80(51)
r_0f_0	0.265(6)	0.264(9)
r_0M_N	1.96(16)	2.05(10)
$r_0\Lambda_3$	1.76(7)	2.12(15)
$r_0\Lambda_4$	3.06(6)	3.06(11)
c_1	-1.02(30)	-0.98(14))
g_A	1.04(24)	0.98(12)
R_1	0.790(3)	0.792(6))
D_m	-1(2)	-1.5(1.4)
D_f	0.7(8)	0.6(5)
D_n	1(1)	0.1(2)
χ^2/dof	15.3/16	0.94

Combined Fits for β = 3. 90, 4. 05 and 3.80 + O(aθ) terms

[Use Eq. [♠] and the bootstrap method]

$\beta = 3.90: \quad \mu = 0.0040\,, 0.0040\,(L=32), 0.0064\,, 0.0085\,, 0.0100$

$\beta = 4.05: \quad \mu = 0.0030\,, 0.0060\,, 0.0060\,(L=24), 0.0080$

$\beta = 3.80: \quad \mu = 0.0060\,, 0.0080\,, 0.0110$

Combined fits for $\beta = 3.90$, 4.05 and 3.80 plus $O(a^2)$ and $O(a\theta_{low})$ terms

	only $O(a\theta_{low})$ terms	$O(a\theta_{low})$ with $O(a^2)$ fixed	without $O(a^2)$ and $O(a\theta_{low})$
$2r_0B_0$	10.18(39)	10.80(50)	9.50(39)
r_0f_0	0.275(6)	0.268(4)	0.273(6)
$r_0\Lambda_3$	2.13(18)	2.33(14)	1.74(12)
$r_0\Lambda_4$	3.18(11)	3.04(10)	3.40(8)
C_f	-0.14(3)	-0.27(6)	—
C_m	-1.06(30)	-0.44(27)	—
C_π	-0.24(7)	—	—
χ^2/dof	0.4	0.3	1.2

		$\beta = 3.80$ (M_{ps}^2 in units of a-lattice spacing)			
$a\mu$	σ_1^{stat}	σ_2^{stat}		σ_M^{syst}	σ_{fit}^{syst}
0.0060	3.3×10^{-4}	5.6×10^{-4}	N	6.2×10^{-4}	2.6×10^{-3}
0.0080	3.3×10^{-4}	7.4×10^{-4}	N	4.5×10^{-4}	2.0×10^{-3}
0.0110	2.4×10^{-4}	1.1×10^{-3}	?	3.4×10^{-4}	1.5×10^{-3}
0.0165	3.0×10^{-4}	1.7×10^{-3}	Y	2.5×10^{-4}	1.0×10^{-3}

♦ ♦ ♦

			$\beta = 3.80$ (F_{ps} in units of a-lattice spacing)			
$a\mu$	σ_1^{stat}	σ_2^{stat}		σ_M^{syst}	σ_{fit}^{syst}	σ_{op}^{syst}
0.0060	8.0×10^{-4}	2.5×10^{-4}	N	2.7×10^{-4}	1.4×10^{-3}	1.8×10^{-3}
0.0080	4.0×10^{-4}	2.8×10^{-4}	N	1.7×10^{-4}	1.1×10^{-3}	1.2×10^{-3}
0.0110	3.0×10^{-4}	2.9×10^{-4}	?	0.9×10^{-4}	8.0×10^{-4}	7.0×10^{-4}
0.0165	2.0×10^{-4}	2.6×10^{-4}	?Y	0.5×10^{-4}	5.0×10^{-4}	3.0×10^{-4}

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		$\beta = 3.80$ (m_N in units of a-lattice spacing)			
$a\mu$	σ_1^{stat}	σ_2^{stat}		σ_M^{syst}	σ_{fit}^{syst}
0.0060	9.3×10^{-3}	2.1×10^{-3}	Y	2.2×10^{-3}	2.6×10^{-3}
0.0080	8.7×10^{-3}	2.3×10^{-3}	Y	1.4×10^{-3}	1.9×10^{-3}
0.0110	9.0×10^{-3}	2.3×10^{-3}	Y	0.7×10^{-3}	1.4×10^{-3}
0.0165	5.7×10^{-3}	2.2×10^{-3}	Y	0.2×10^{-3}	1.0×10^{-3}

			$\beta = 3.90$ (M_{ps}^2 in units of a-lattice spacing)		
$a\mu$	σ_1^{stat}	σ_2^{stat}		σ_M^{syst}	σ_{fit}^{syst}
0.0040	1.9×10^{-4}	2.2×10^{-4}	Y	0.2×10^{-4}	3.7×10^{-4}
0.0064	1.4×10^{-4}	3.4×10^{-4}	Y	0.1×10^{-4}	2.3×10^{-4}
0.0085	1.9×10^{-4}	4.5×10^{-4}	Y	0.9×10^{-5}	1.7×10^{-4}
0.0100	2.1×10^{-4}	5.4×10^{-4}	Y	0.8×10^{-5}	1.5×10^{-4}
0.0040(L=32)	0.5×10^{-4}	2.2×10^{-4}	?Y	0.2×10^{-4}	3.7×10^{-4}

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			$\beta = 3.90$ (F_{ps} in units of a-lattice spacing)			
$a\mu$	σ_1^{stat}	σ_2^{stat}		σ_M^{syst}	σ_{fit}^{syst}	σ_{op}^{syst}
0.0040	4.0×10^{-4}	1.2×10^{-4}	Y	0.1×10^{-4}	2.5×10^{-4}	0.8×10^{-4}
0.0064	4.0×10^{-4}	1.4×10^{-4}	Y	0.5×10^{-5}	1.6×10^{-4}	0.3×10^{-4}
0.0085	2.0×10^{-4}	1.5×10^{-4}	Y	0.3×10^{-5}	1.2×10^{-4}	0.2×10^{-4}
0.0110	2.0×10^{-4}	1.6×10^{-4}	Y	0.2×10^{-5}	1.0×10^{-4}	0.2×10^{-4}
0.0040(L=32)	2.0×10^{-4}	1.2×10^{-4}	?Y	0.1×10^{-4}	2.5×10^{-4}	0.8×10^{-4}

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			$\beta = 3.90$ (m_N in units of a-lattice spacing)		
$a\mu$	σ_1^{stat}	σ_2^{stat}		σ_M^{syst}	σ_{fit}^{syst}
0.0040	4.9×10^{-3}	0.9×10^{-3}	Y	8.8×10^{-5}	4.2×10^{-4}
0.0064	4.2×10^{-3}	1.2×10^{-3}	Y	4.4×10^{-5}	2.8×10^{-4}
0.0085	5.9×10^{-3}	1.3×10^{-3}	Y	2.6×10^{-5}	2.1×10^{-4}
0.0100	4.6×10^{-3}	1.3×10^{-3}	Y	2.0×10^{-5}	1.8×10^{-4}
0.0040(L=32)	4.8×10^{-3}	0.9×10^{-3}	Y	8.8×10^{-5}	4.2×10^{-4}

$\beta = 4.05$

(M_{ps}^2 in units of a-lattice spacing)

$a\mu$	σ_1^{stat}	σ_2^{stat}		σ_M^{syst}	σ_{fit}^{syst}
0.0030	1.2×10^{-4}	2.0×10^{-4}	Y	4.2×10^{-5}	2.8×10^{-4}
0.0060	1.7×10^{-4}	4.0×10^{-4}	Y	2.1×10^{-5}	1.4×10^{-4}
0.0080	1.7×10^{-4}	5.4×10^{-4}	Y	1.6×10^{-5}	1.0×10^{-4}

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$\beta = 4.05$

(F_{ps} in units of a-lattice spacing)

$a\mu$	σ_1^{stat}	σ_2^{stat}		σ_M^{syst}	σ_{fit}^{syst}	σ_{op}^{syst}
0.0030	3.5×10^{-4}	1.2×10^{-4}	Y	2.5×10^{-5}	2.4×10^{-4}	1.4×10^{-4}
0.0060	4.0×10^{-4}	1.6×10^{-4}	Y	0.8×10^{-5}	1.2×10^{-4}	0.4×10^{-4}
0.0080	5.0×10^{-4}	1.6×10^{-4}	Y	0.5×10^{-5}	0.9×10^{-4}	0.2×10^{-4}

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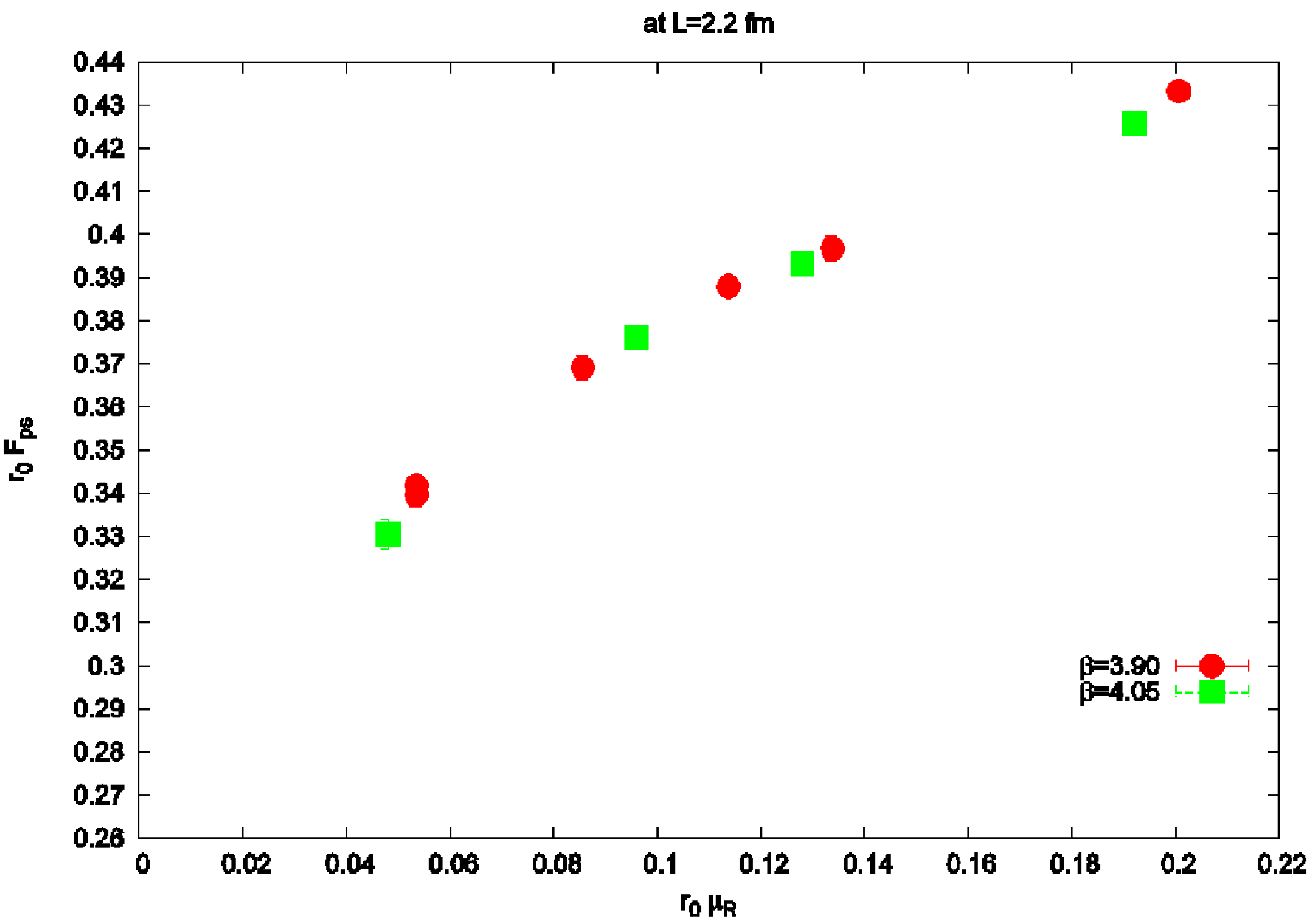
$\beta = 4.05$

(m_N in units of a-lattice spacing)

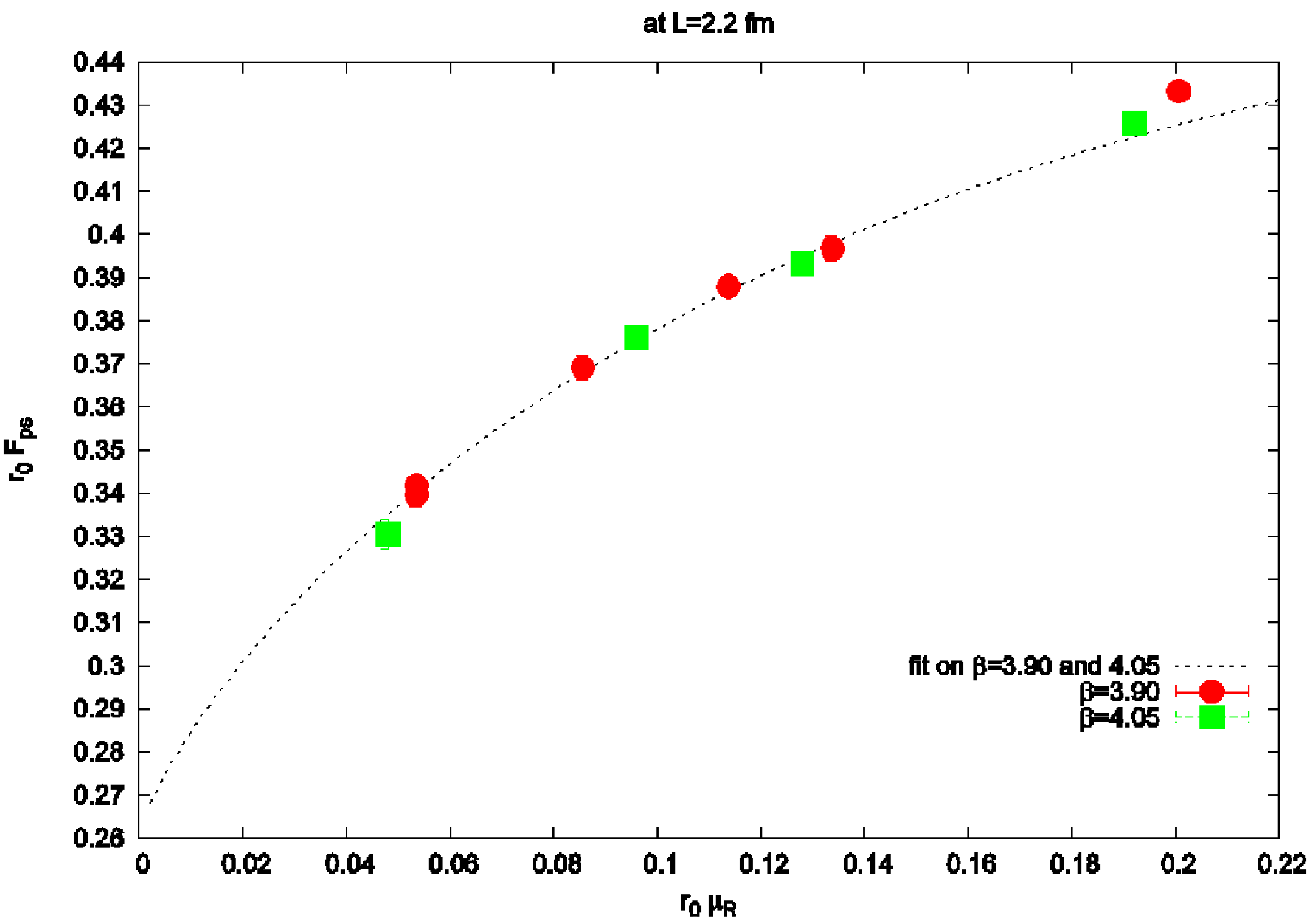
$a\mu$	σ_1^{stat}	σ_2^{stat}		σ_M^{syst}	σ_{fit}^{syst}
0.0030	8.4×10^{-3}	9.2×10^{-4}	Y	1.9×10^{-4}	4.1×10^{-4}
0.0060	5.8×10^{-3}	1.3×10^{-4}	Y	0.7×10^{-4}	2.2×10^{-4}
0.0080	3.6×10^{-3}	1.3×10^{-4}	Y	0.4×10^{-4}	1.1×10^{-4}

Let’s see if the systematic corrections are reasonable…

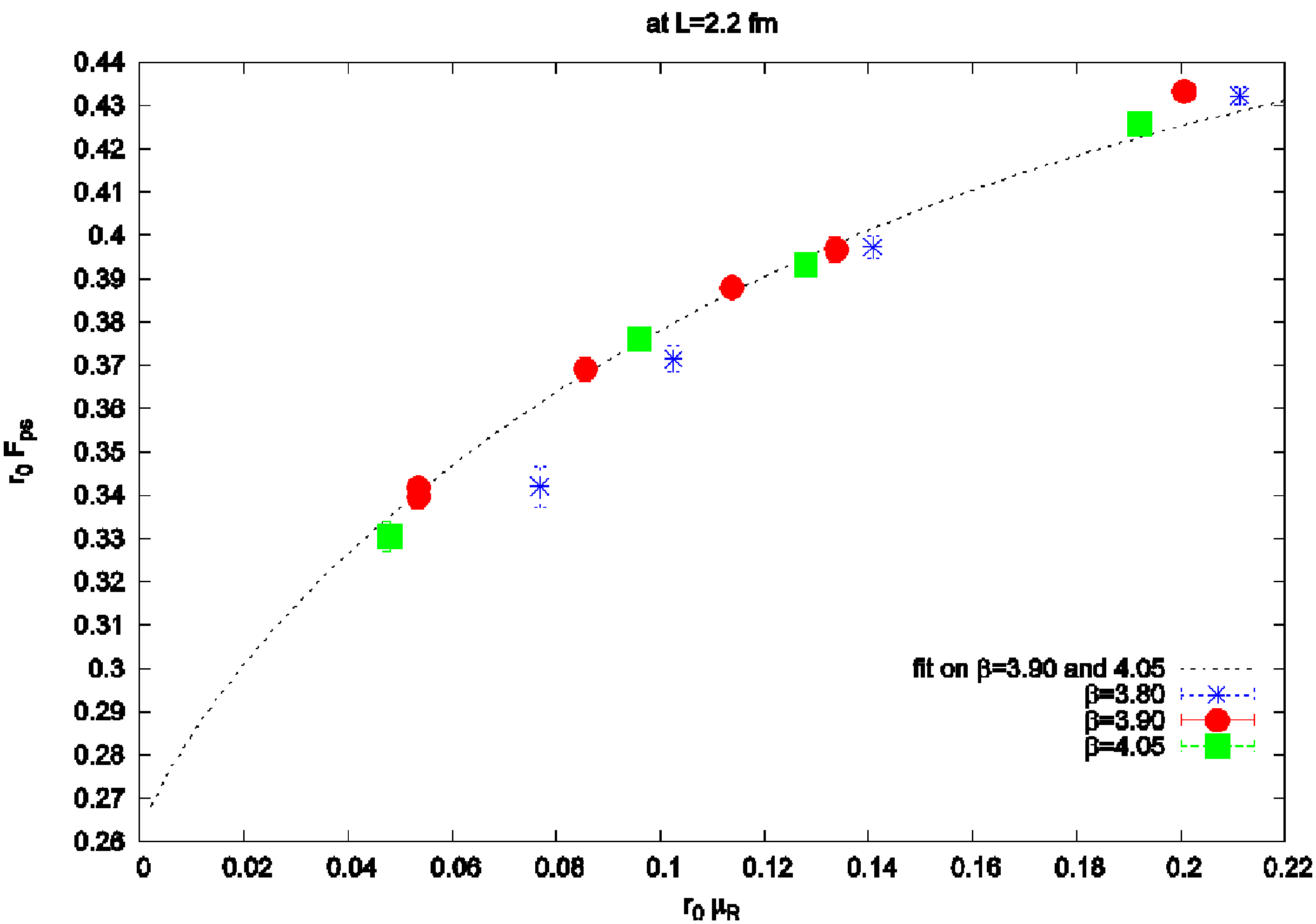
r_0F_{ps} vs. $r_0\mu_R$



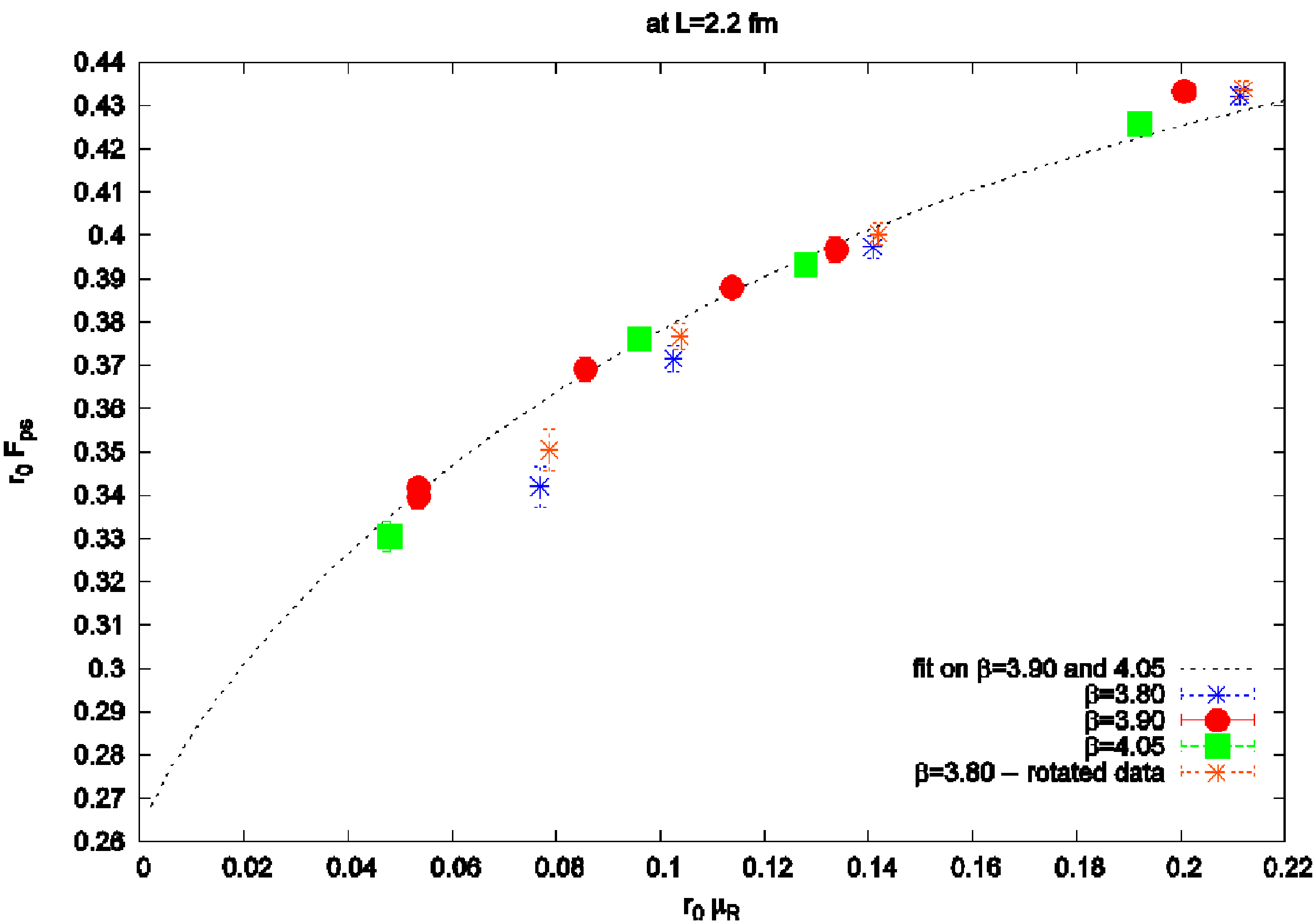
$r_0 F_{ps}$ vs. $r_0 \mu_R$



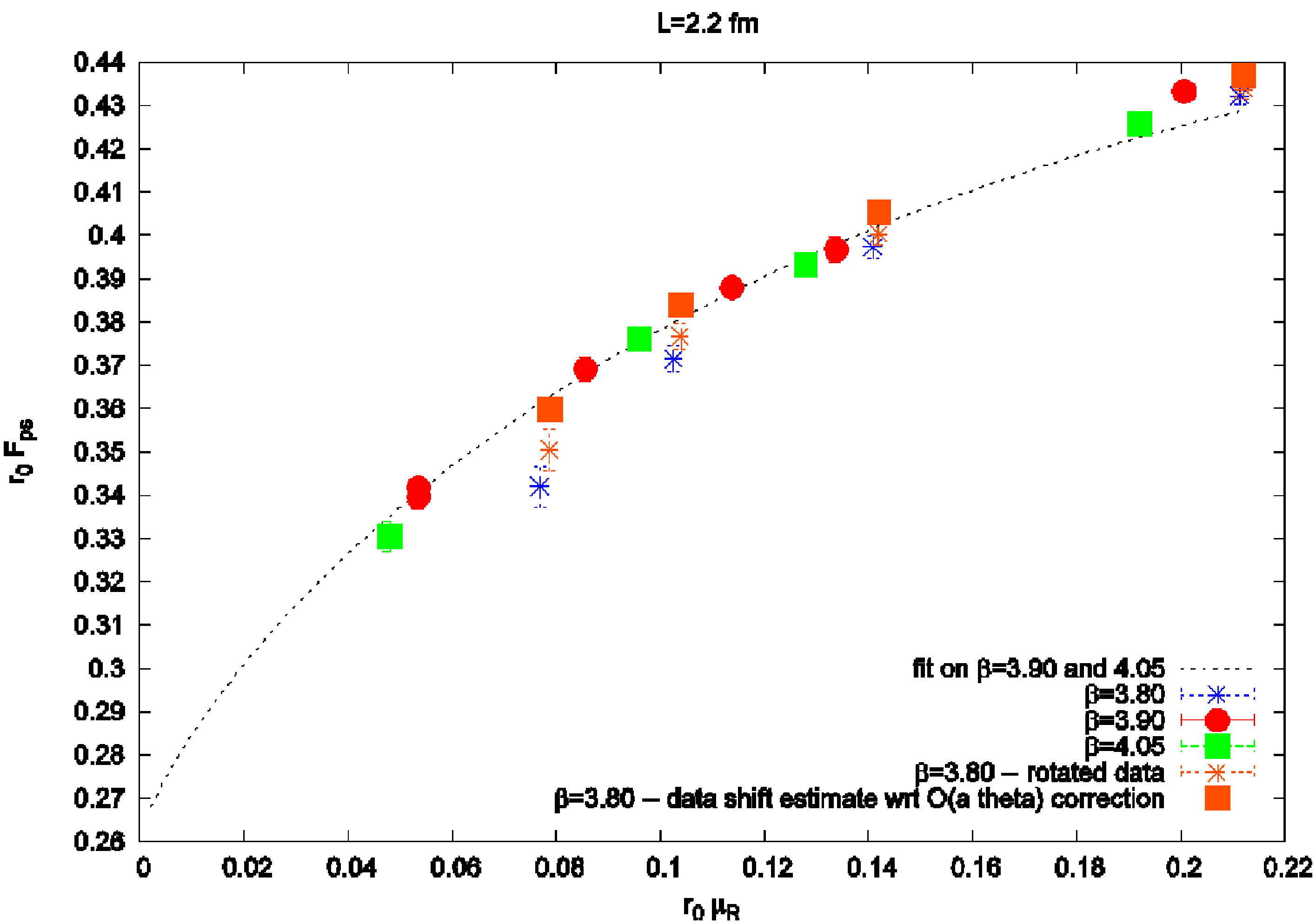
r_0F_{ps} vs. $r_0\mu_R$



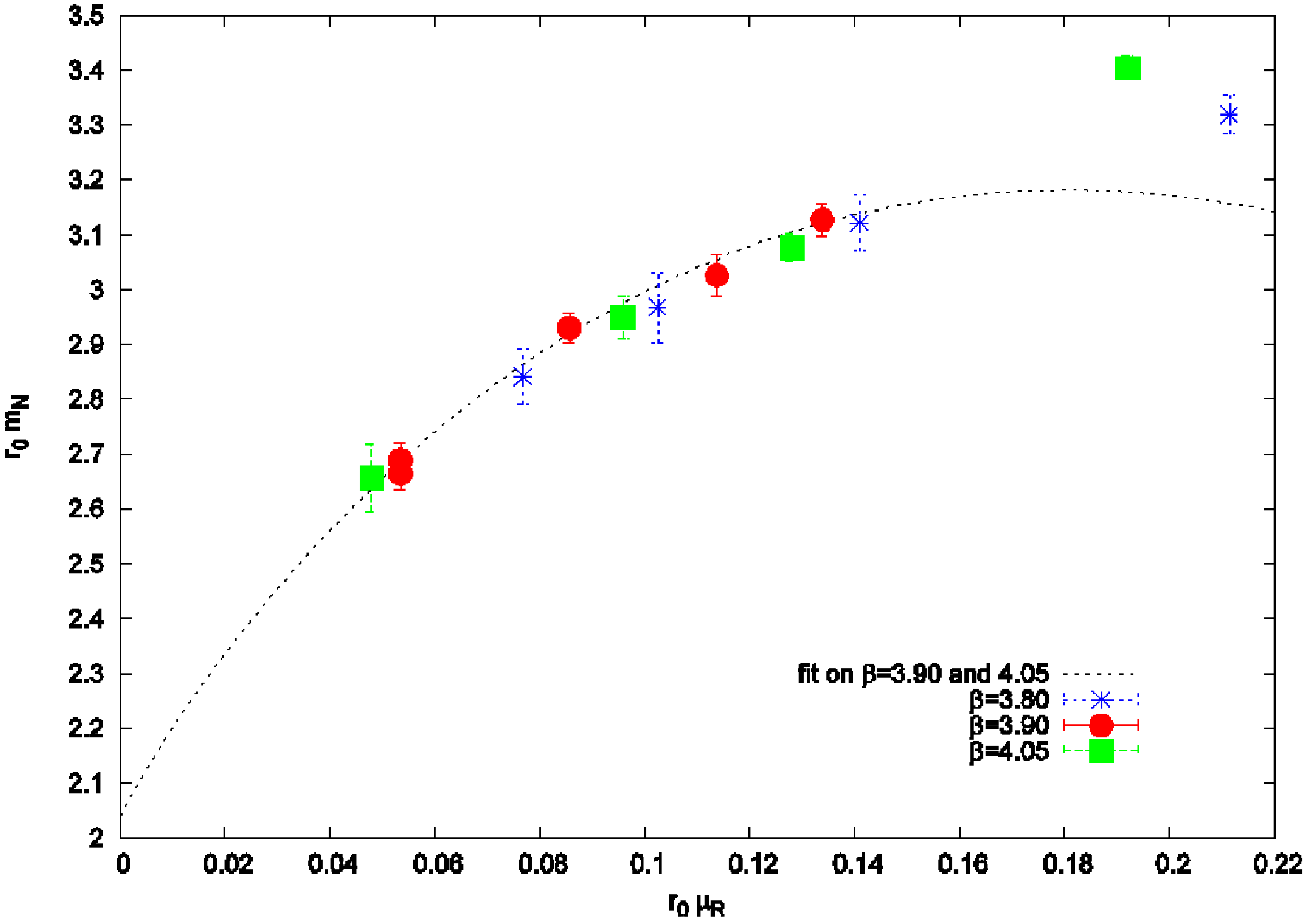
$r_0 F_{ps}$ vs. $r_0 \mu_R$



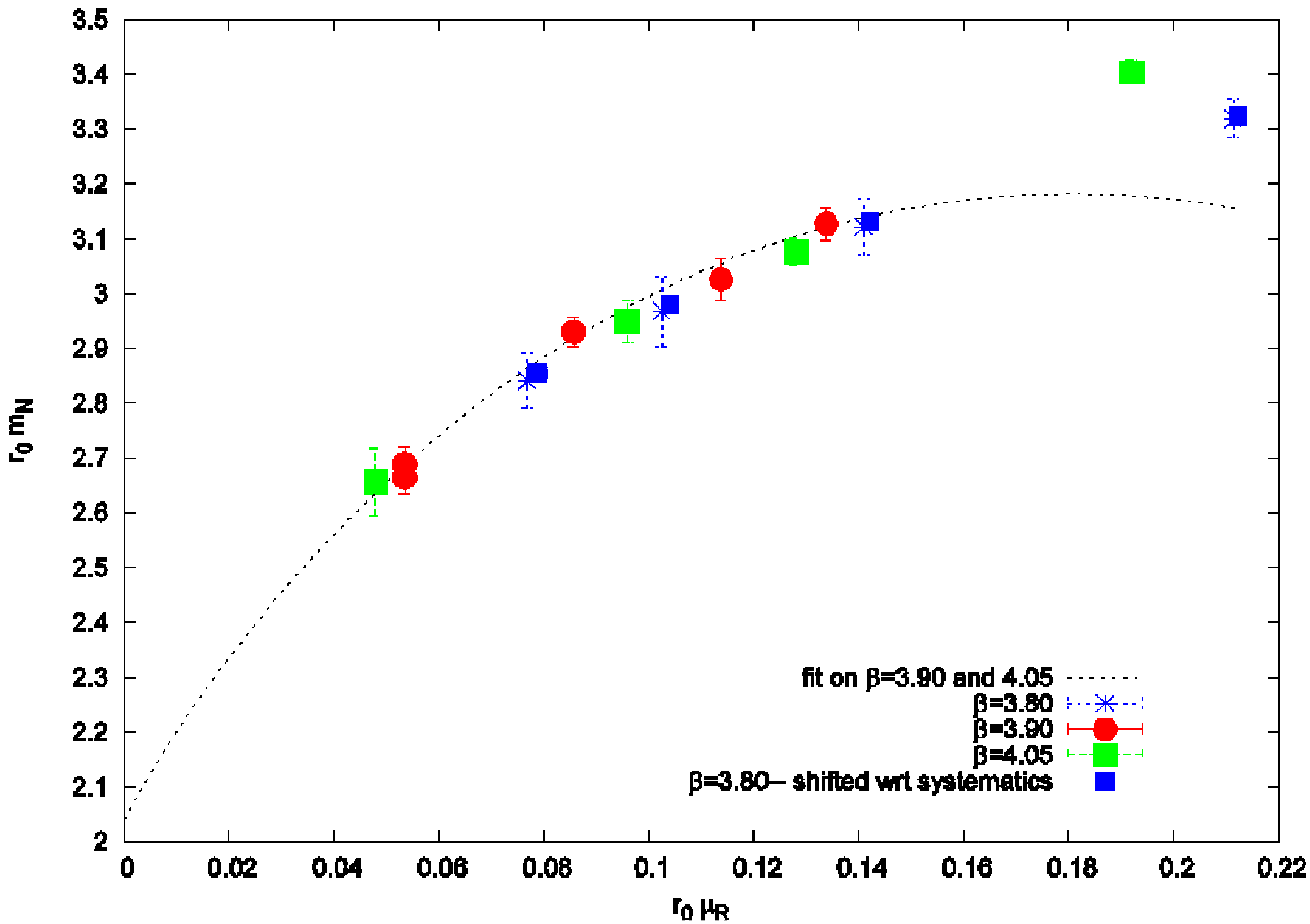
$r_0 F_{ps}$ vs. $r_0 \mu_R$



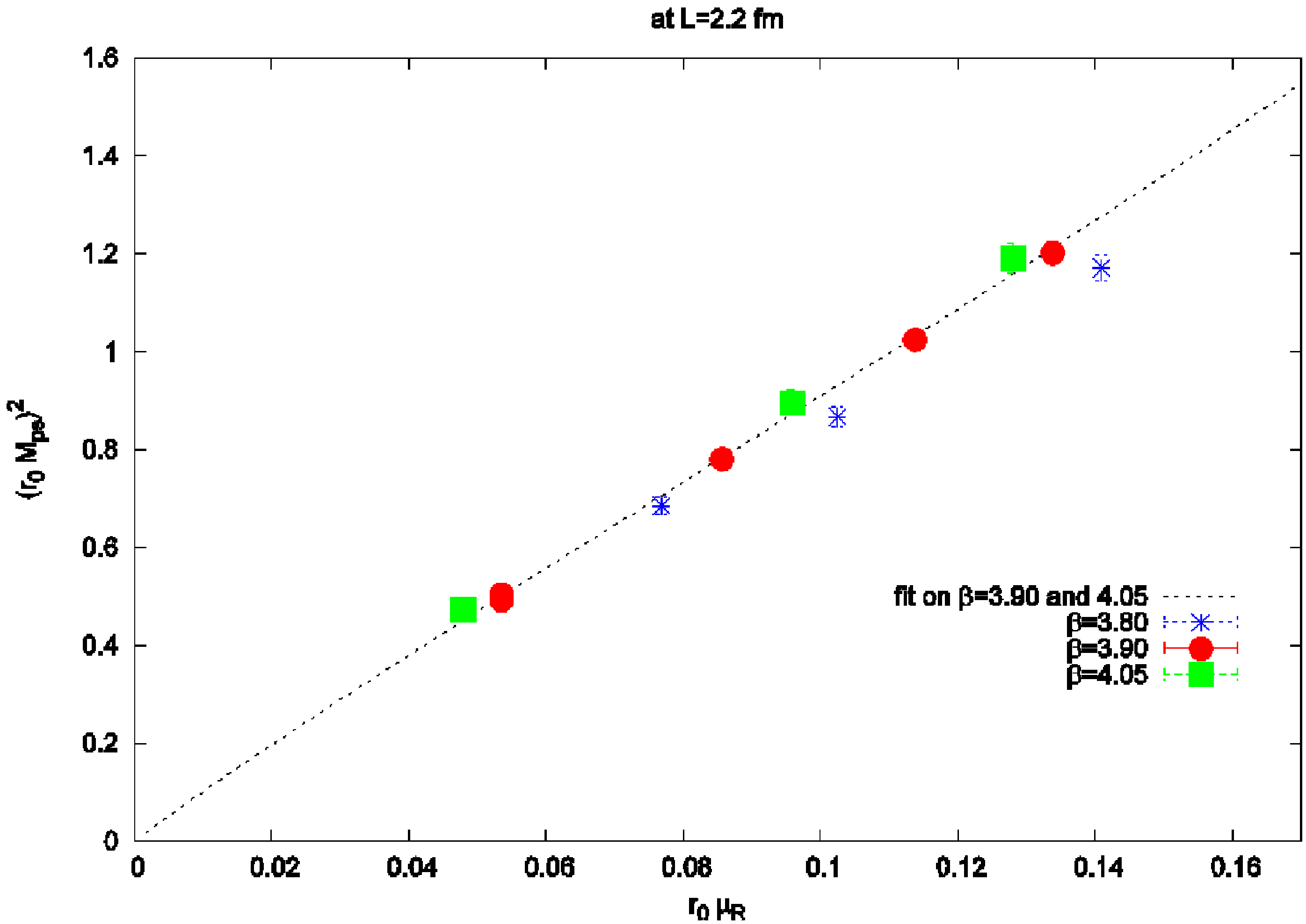
$r_0 m_N$ vs. $r_0 \mu_R$



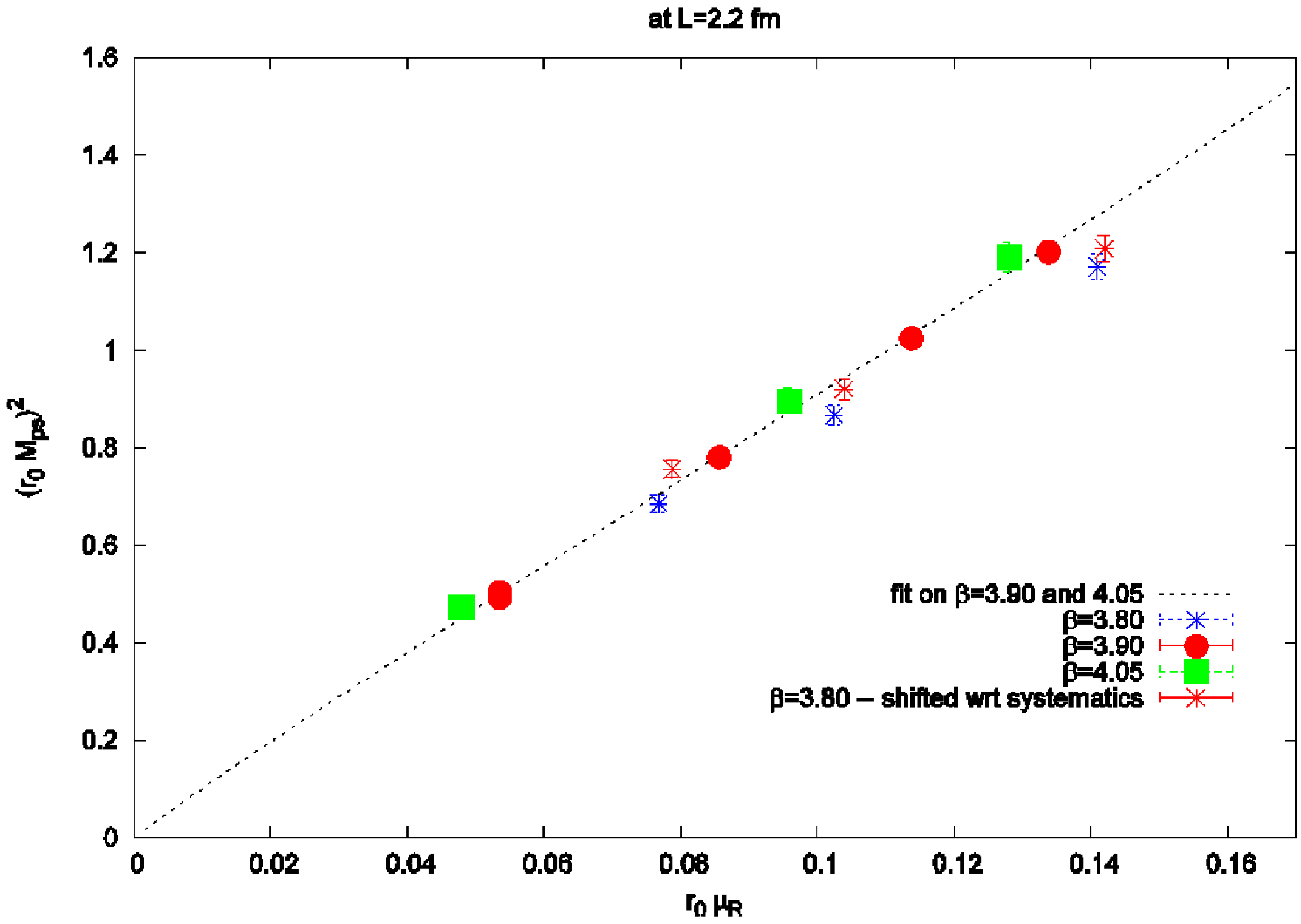
$r_0 m_N$ vs. $r_0 \mu_R$



$r_0 m^2_{PS}$ vs. $r_0 \mu_R$



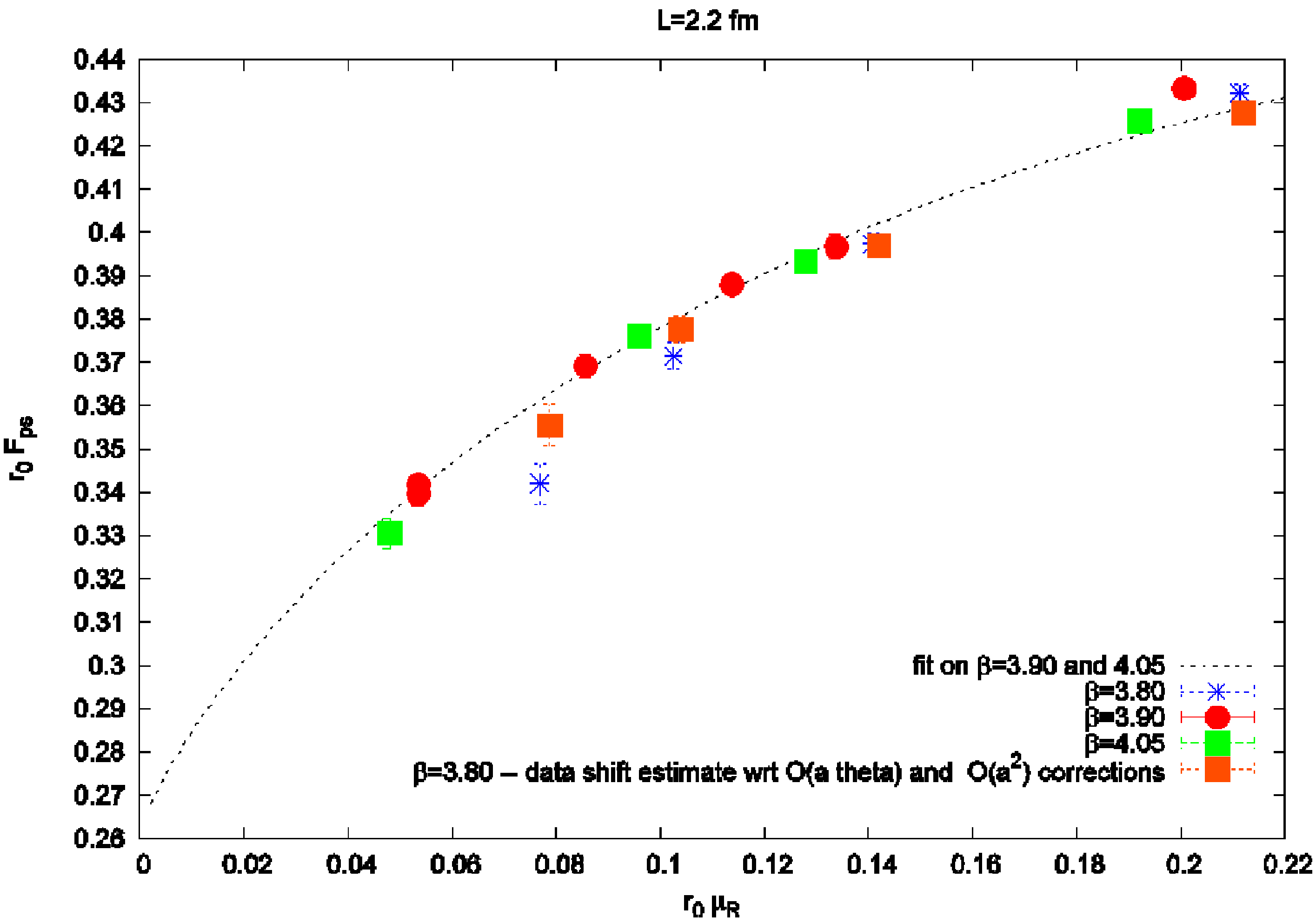
$r_0 m^2_{PS}$ vs. $r_0 \mu_R$



Let's include, besides O($a\theta$), also O(a^2) corrections

and implement the combined chiral fits' method …

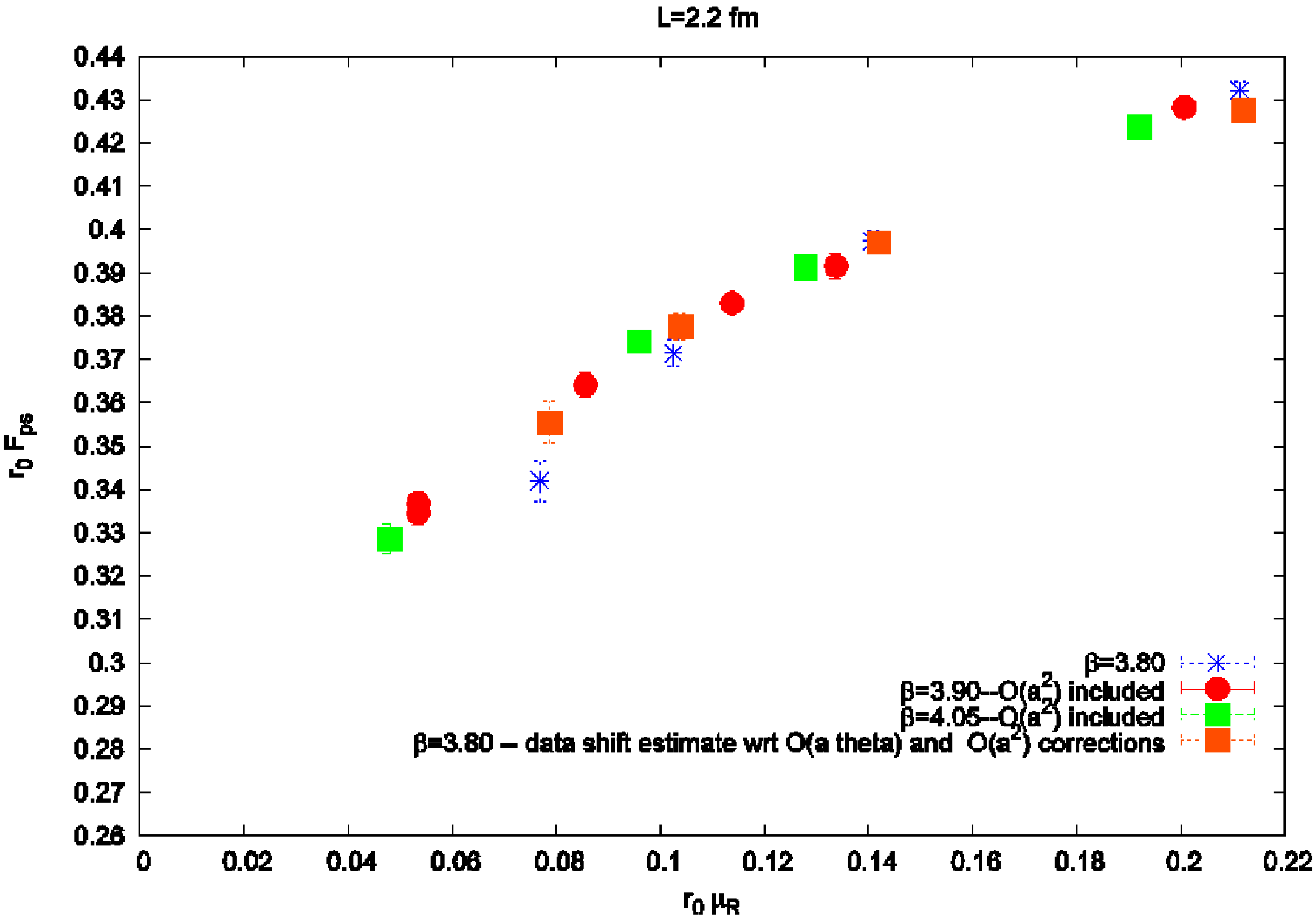
r_0F_{ps} vs. $r_0\mu_R$
O(aθ) and O(a^2) corrections included in β=3.80 data



r_0F_{ps} vs. $r_0\mu_R$

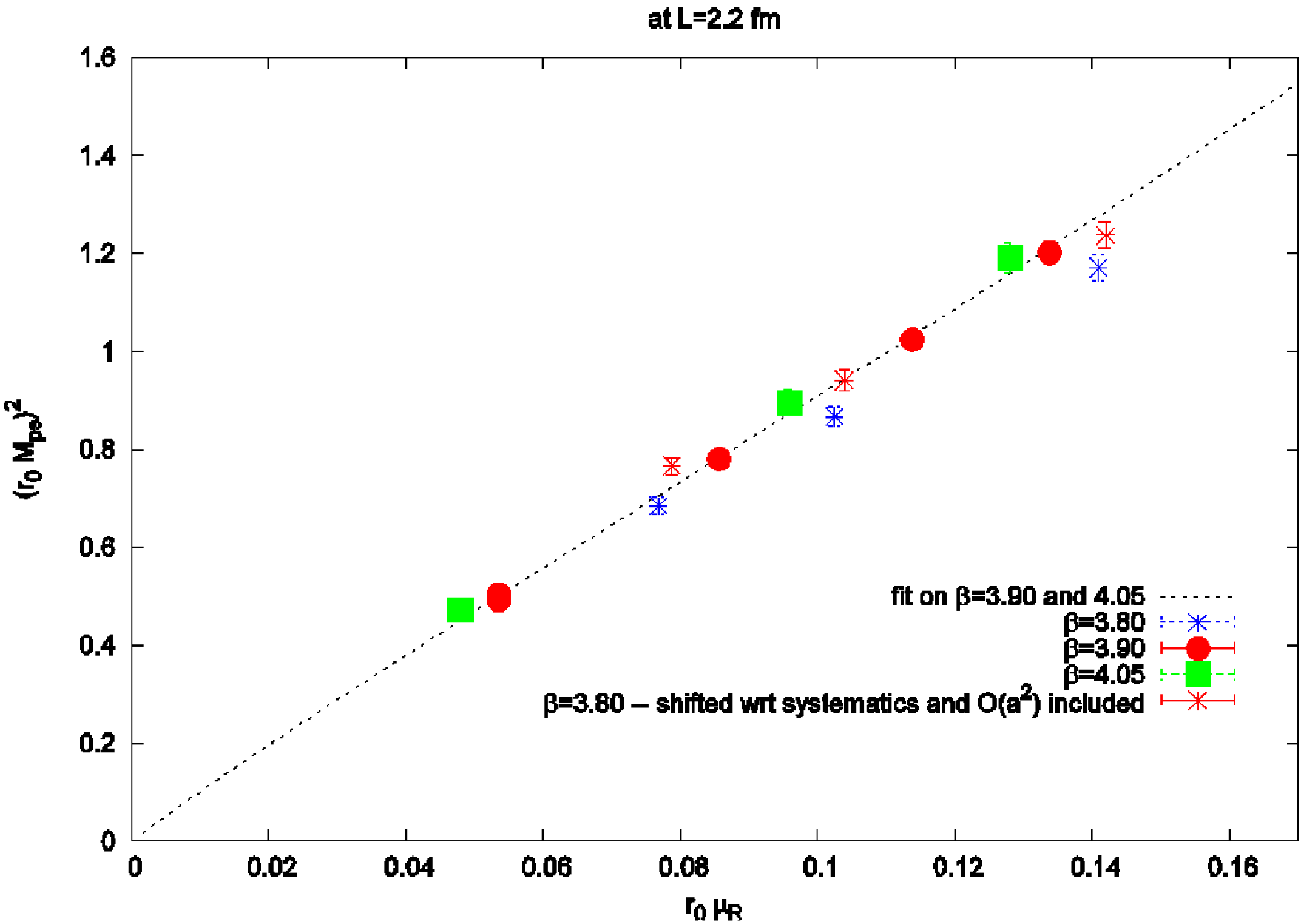
$O(a\theta)$ and $O(a^2)$ corrections included in $\beta=3.80$ data

$O(a^2)$ corrections included in $\beta=3.90$ and 4.05 data



$r_0 m^2_{PS}$ vs. $r_0 \mu_R$

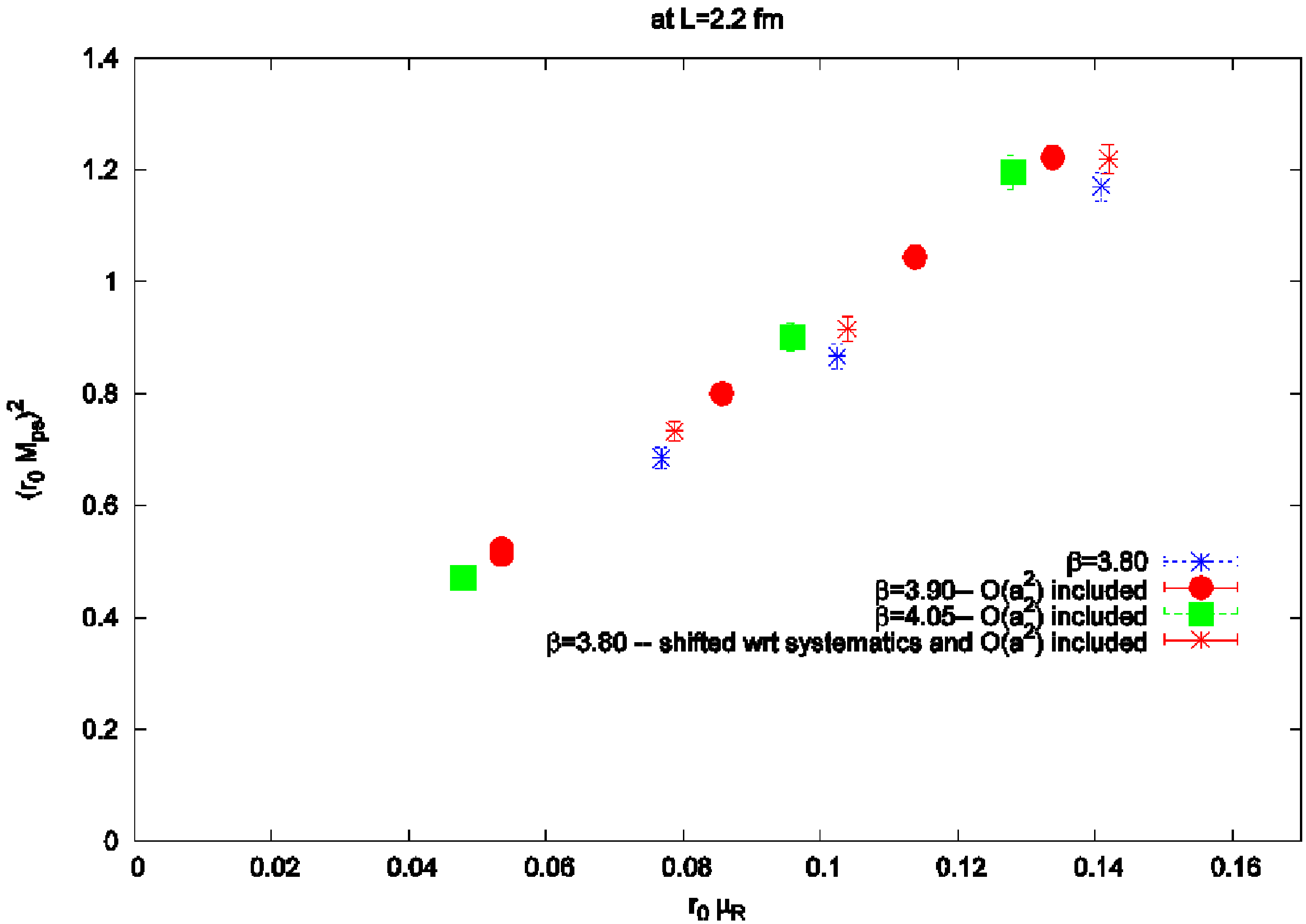
O($a\theta$) and O(a^2) corrections included in β=3.80 data



$r_0 m^2_{PS}$ vs. $r_0 \mu_R$

O(aθ) and $O(a^2)$ corrections included in β=3.80 data

$O(a^2)$ corrections included in β=3.90 and 4.05 data



$r_0 m^2_{PS}$ vs. $r_0 \mu_R$
$O(a^2)$ corrections included in β=3.80 data