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INFN - Roma Tre

ETMC meeting

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Determination of V_{us} from $K_{\ell 3}$ decays

$$\langle \pi(p_\pi) | \hat{V}_\mu | K(p_K) \rangle = f_+(q^2)(p_K + p_\pi)_\mu + f_-(q^2)(p_\pi - p_K)_\mu$$

$$f_0(q^2) = f_+(q^2) + f_-(q^2)\, q^2 / (M_K^2 - M_\pi^2)$$

scalar f.f. vector f.f.: $f_+(0) = f_0(0)$

* from experiment: $\Gamma(K \to \pi \ell \bar{\nu}_\ell) \longrightarrow |V_{us}| f_+(0) = 0.21664\ (48)$ [0.22%] [FlaviAnet '07]
* to match the experimental error one needs $\delta[f_+(0) - 1] \sim 5\%$ [factor ~ 5 larger than in $K_{\ell 2}$ decays]
* ChPT expansion of the vector form factor at zero momentum transfer, $f_+(0)$

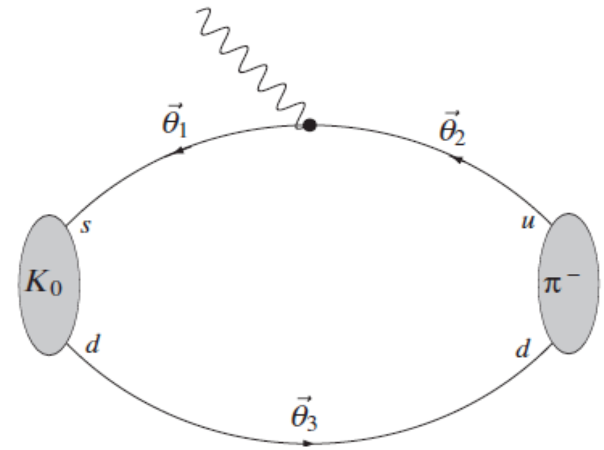
$$f_+(0) = 1 + f_2 + f_4 + O(p^8) \xrightarrow[AG]{} 1 + O\left[(m_s - m_\ell)^2\right]$$

* NLO f_2 is independent of LECs, calculable in terms of M_K, M_π and f_π: $f_2^{phys} = -0.023$
* NNLO f_4 depends on $O(p^6)$ LECs [Post and Schilcher ('01), Bijnens and Talavera ('03)]

***** f_4 may be obtained from the slope and curvature of $f_0(q^2)$, but present data are not accurate enough *****

- from quark model [Leutwyler and Roos '84]: $f_4 = -0.016 \pm 0.008 \Rightarrow f_+(0) = 0.961 \pm 0.008$ [used by PDG]
- from NNLO ChPT + 1/Nc [Cirigliano et al. '06]: $f_4 = 0.007 \pm 0.012 \Rightarrow f_+(0) = 0.984 \pm 0.012$
- from NNLO ChPT + disp. rel. [Jamin et al. '04]: $f_4 = -0.003 \pm 0.011 \Rightarrow f_+(0) = 0.974 \pm 0.011$

semileptonic form factors on the lattice



$$\vec{p}_K = \frac{2\pi}{L}\left(\vec{\theta}_3 - \vec{\theta}_1\right), \qquad \vec{p}_\pi = \frac{2\pi}{L}\left(\vec{\theta}_3 - \vec{\theta}_2\right),$$

$$\vec{q} = \vec{p}_K - \vec{p}_\pi = \frac{2\pi}{L}\left(\vec{\theta}_2 - \vec{\theta}_1\right)$$

need of 2-point and 3-point correlators
(more expensive than f_K / f_π)

2-point correlators: $$C_{\pi(K)}\left(t;\vec{p}\right) \equiv \sum_{x,z} \left\langle O_{\pi(K)}\left(\vec{x},t_x\right) O^\dagger_{\pi(K)}\left(\vec{z},t_z\right)\right\rangle \cdot \delta_{t,t_x-t_z} e^{-i\vec{p}\cdot\left(\vec{x}-\vec{z}\right)}$$

$$\xrightarrow[t\to\infty]{} \frac{Z_{\pi(K)}}{2E_{\pi(K)}\left(\vec{p}\right)} e^{-E_{\pi(K)}\left(\vec{p}\right)\cdot t}$$

3-point correlators: $$C^{K\pi}_\mu\left(t,t';\vec{p}_K,\vec{p}_\pi\right) \equiv \sum_{x,y,z} \left\langle O_\pi\left(\vec{y},t_y\right)\hat{V}_\mu\left(\vec{x},t_x\right) O^\dagger_K\left(\vec{z},t_z\right)\right\rangle \cdot \delta_{t,t_x-t_z}\delta_{t',t_y-t_z} \cdot e^{-i\vec{p}_K\cdot\left(\vec{x}-\vec{z}\right)} e^{i\vec{p}_\pi\cdot\left(\vec{x}-\vec{y}\right)}$$

$$\xrightarrow[\substack{t\to\infty \\ t'-t\to\infty}]{} \frac{\sqrt{Z_K Z_\pi}}{4E_K\left(\vec{p}_K\right)E_\pi\left(\vec{p}_\pi\right)} \left\langle \pi\left(p_\pi\right)\middle|\hat{V}_\mu\middle|K\left(p_K\right)\right\rangle e^{-E_K\left(\vec{p}_K\right)\cdot t} e^{-E_\pi\left(\vec{p}_\pi\right)\cdot\left(t'-t\right)}$$

*** suitable ratios of 3-point / 2-point correlators** $\longrightarrow$ $\left\langle \pi\left(p_\pi\right)\middle|\hat{V}_\mu\middle|K\left(p_K\right)\right\rangle$

* local interpolating PS fields and vector current: $O_K = \bar{d}\gamma_5 s, \quad O_\pi = \bar{d}\gamma_5 u, \quad \hat{V}_\mu = Z_V\ \bar{s}\gamma_\mu u$

lattice methodology [started by SPQ$_{cd}$R coll. '04]

1. **high-precision evaluation** (better than percent level) of the scalar form factor f_0 at $q^2_{max} = (M_K - M_\pi)^2$, obtained using a suitable ratio of 3-point correlation functions

2. **extrapolation to $q^2 = 0$** to get $f_+(0) = f_0(0)$ (determination of the scalar slope λ_0)

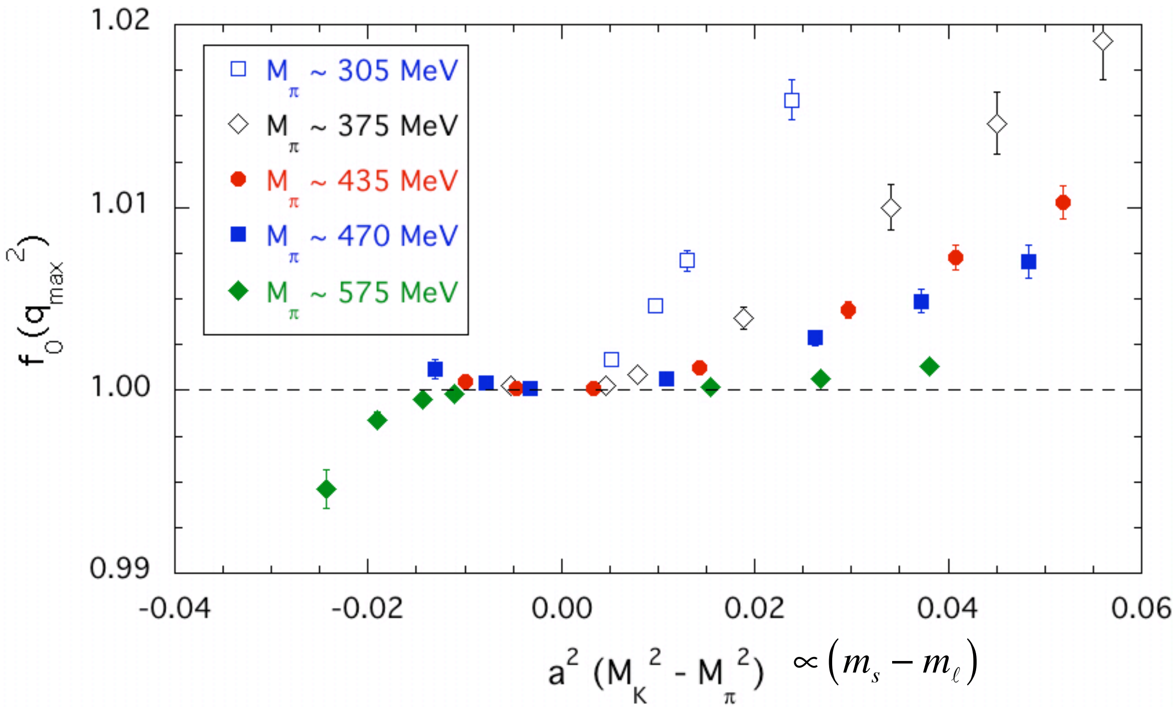
3. **subtraction of the leading chiral logs** [study of $\Delta f = f_+(0) - 1 - f_2$] and **extrapolation to physical masses [SU(3) ChPT]**

4. **extrapolation of $f_+(0)$ to the physical point using SU(2) ChPT**

ETMC calculations

β	a (fm)	$am_\ell = am_{sea}$	V • T	M_π (MeV)	$M_\pi L$	gauge confs
3.9	~ 0.088	0.0030	$32^3 \cdot 64$	~ 260	~ 3.7	240
		0.0040	$32^3 \cdot 64$	~ 300	~ 4.3	240
		0.0040	$24^3 \cdot 48$	~ 305	~ 3.3	480
		0.0064	$24^3 \cdot 48$	~ 375	~ 4.0	240
		0.0085	$24^3 \cdot 48$	~ 435	~ 4.7	240
		0.0100	$24^3 \cdot 48$	~ 470	~ 5.0	240
		0.0150	$24^3 \cdot 48$	~ 575	~ 6.2	240
4.05	~ 0.070	0.0080	$32^3 \cdot 64$	~ 470	~ 5.3	240

$\beta = 3.9$: $a\, m_s = \{0.015, 0.022, 0.027, 0.032\}$, $a\, m_s^{phys} \sim 0.021$



$$\frac{\langle \pi | V^0 | K \rangle \langle K | V^0 | \pi \rangle}{\langle \pi | V^0 | \pi \rangle \langle K | V^0 | K \rangle} = \frac{\left(M_K + M_\pi\right)^2}{4 M_K M_\pi} \left[f_0\left(q^2_{\max}\right) \right]^2$$

$< 0.1\%$ at small SU(3) breakings

still < 0.2 % at larger SU(3) breakings

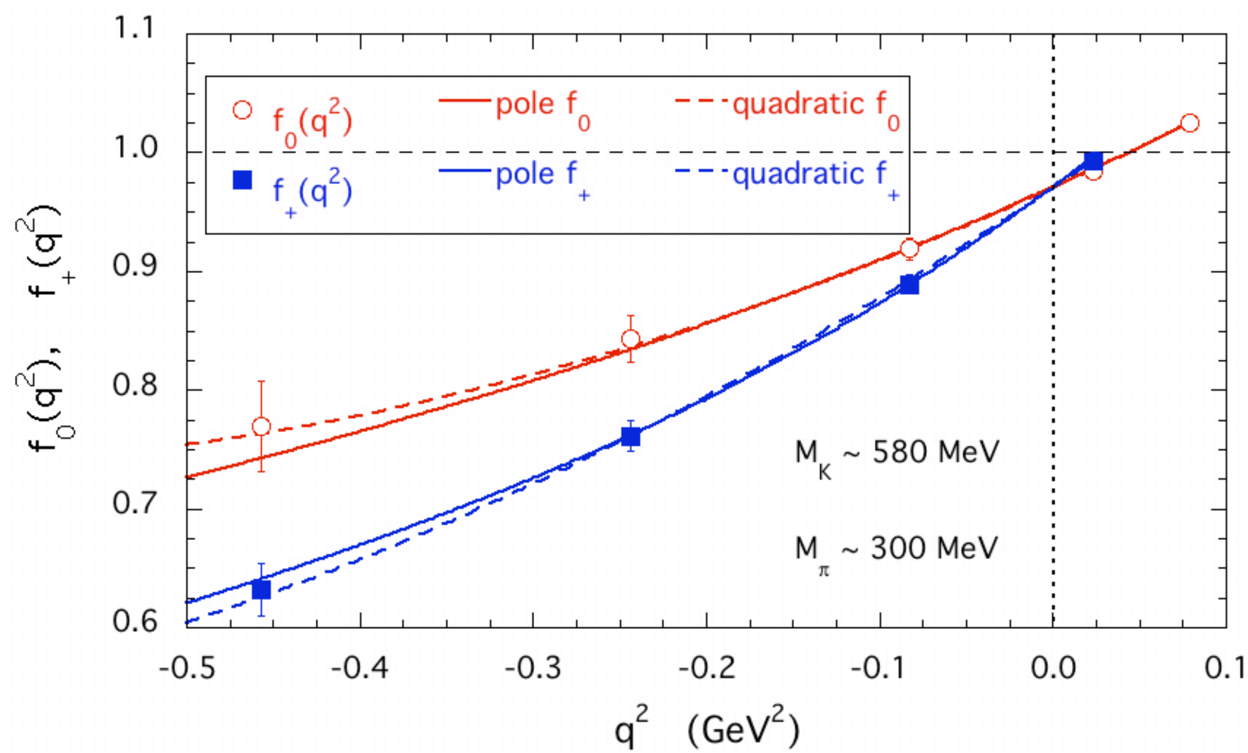
pole fit

$$f_{0,+}\left(q^2\right) = f_+(0) \big/ \left(1 - s_{0,+}\, q^2\right)$$

quadratic fit

$$f_{0,+}\left(q^2\right) = f_+(0)\left(1 + s_{0,+}\, q^2 + c_{0,+} q^4\right)$$

extraction of $f_+(0; M_K, M_\pi)$



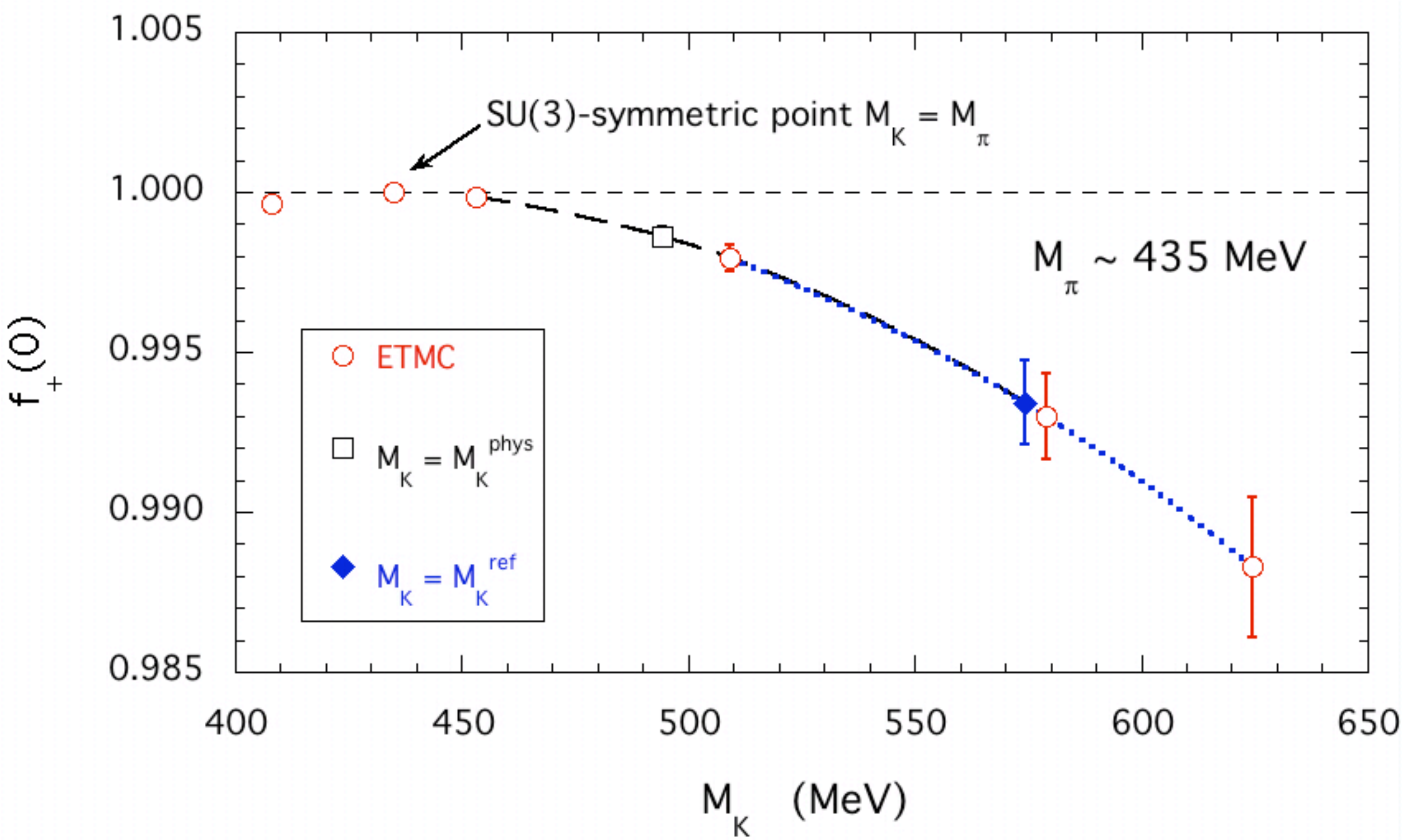
interpolation to fixed kaon mass

A) $M_K = M_K^{phys}$ $\Rightarrow$ $m_s(m_\ell) \leq m_s^{phys}$ ⟵ better for SU(3) ChPT

B) $M_K = M_K^{ref}$: $M_\eta^2 \sim 2M_K^2 - M_\pi^2 =$ phys. value $\Rightarrow$ $m_s(m_\ell) \sim m_s^{phys}$

$$\left[M_K^{ref}\right]^2 = \left[M_K^{phys}\right]^2 + \frac{1}{2}\left\{M_\pi^2 - \left[M_\pi^{phys}\right]^2\right\}$$

necessary for SU(2) ChPT



interpolation

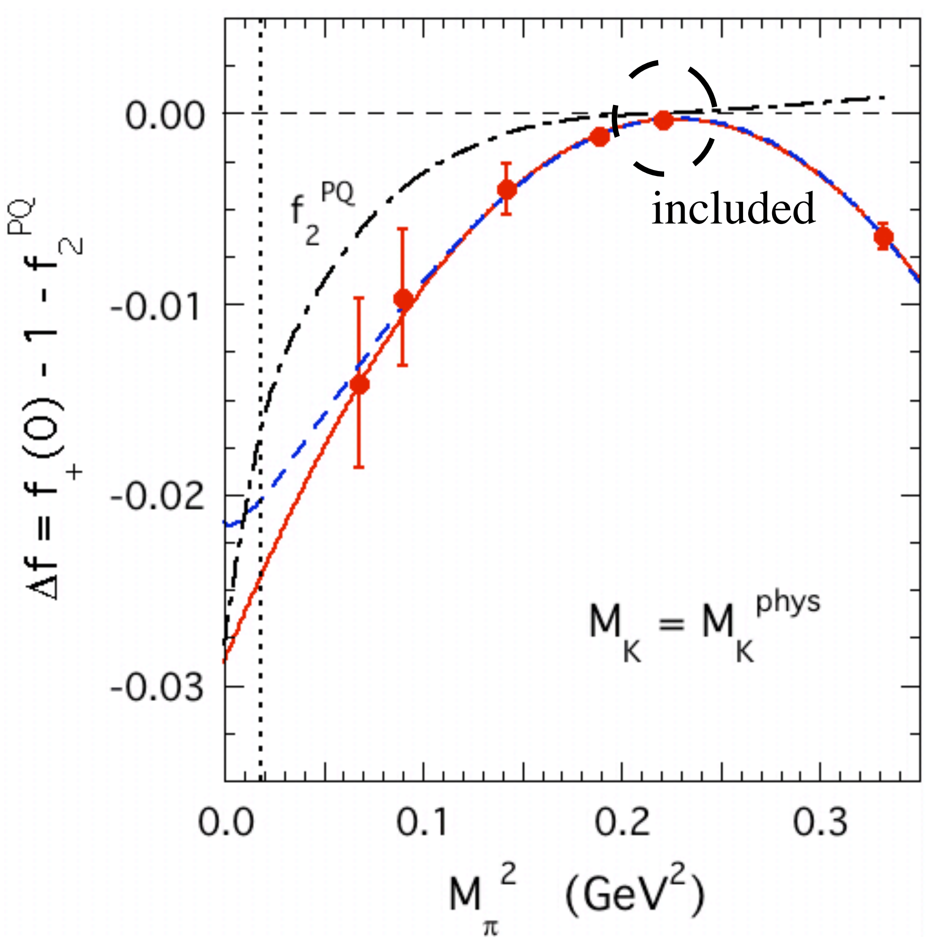
well under control

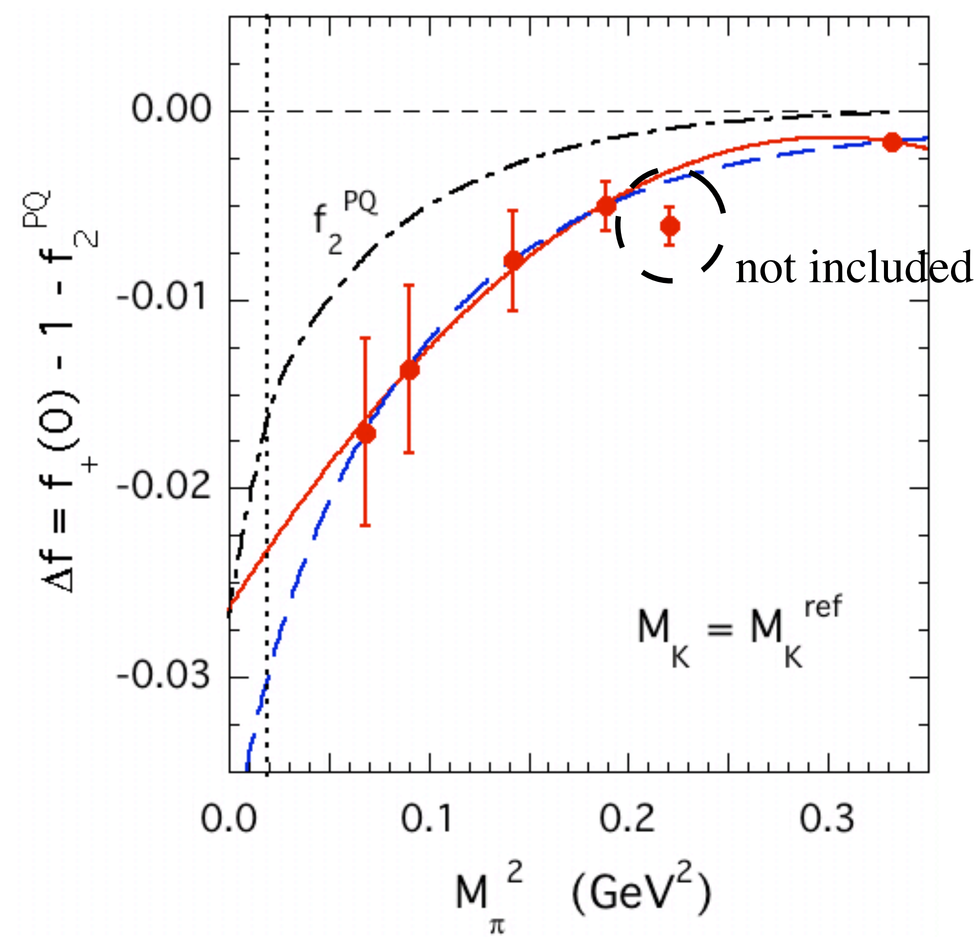
(no model dependence)

SU(3) ChPT

[Becirevic et al. '06]

$$f_2^{PQ} = -\frac{2M_K^2 + M_\pi^2}{32\pi^2 f_\pi^2} + \frac{M_K^2\left(4M_K^2 - M_\pi^2\right)\log\left(2 - M_\pi^2/M_K^2\right)}{64\pi^2 f_\pi^2\left(M_K^2 - M_\pi^2\right)} - \frac{3M_K^2 M_\pi^2 \log\left(M_\pi^2/M_K^2\right)}{64\pi^2 f_\pi^2\left(M_K^2 - M_\pi^2\right)} \longrightarrow -\frac{\left(M_K^2 - M_\pi^2\right)^3}{96\pi^2 f_\pi^2 M_K^4} + O\left[\left(M_K^2 - M_\pi^2\right)^4\right]$$





$$\Delta f^{phys} = -0.0231 \pm 0.0030_{stat.} \pm 0.0029_{syst.}$$

$$\Delta f^{phys} = -0.0278 \pm 0.0062_{stat.} \pm 0.0043_{syst.}$$

——— $A_0 + A_1 M_\pi^2 + A_2 M_\pi^4$

– – – $A_0 + A_1 M_\pi^2 + A_2 M_\pi^4 + A_3 M_\pi^2 \log\left(M_\pi^2\right)$

* dominant chiral logs from f_2^{PQ}

* smooth extrapolation for Δf

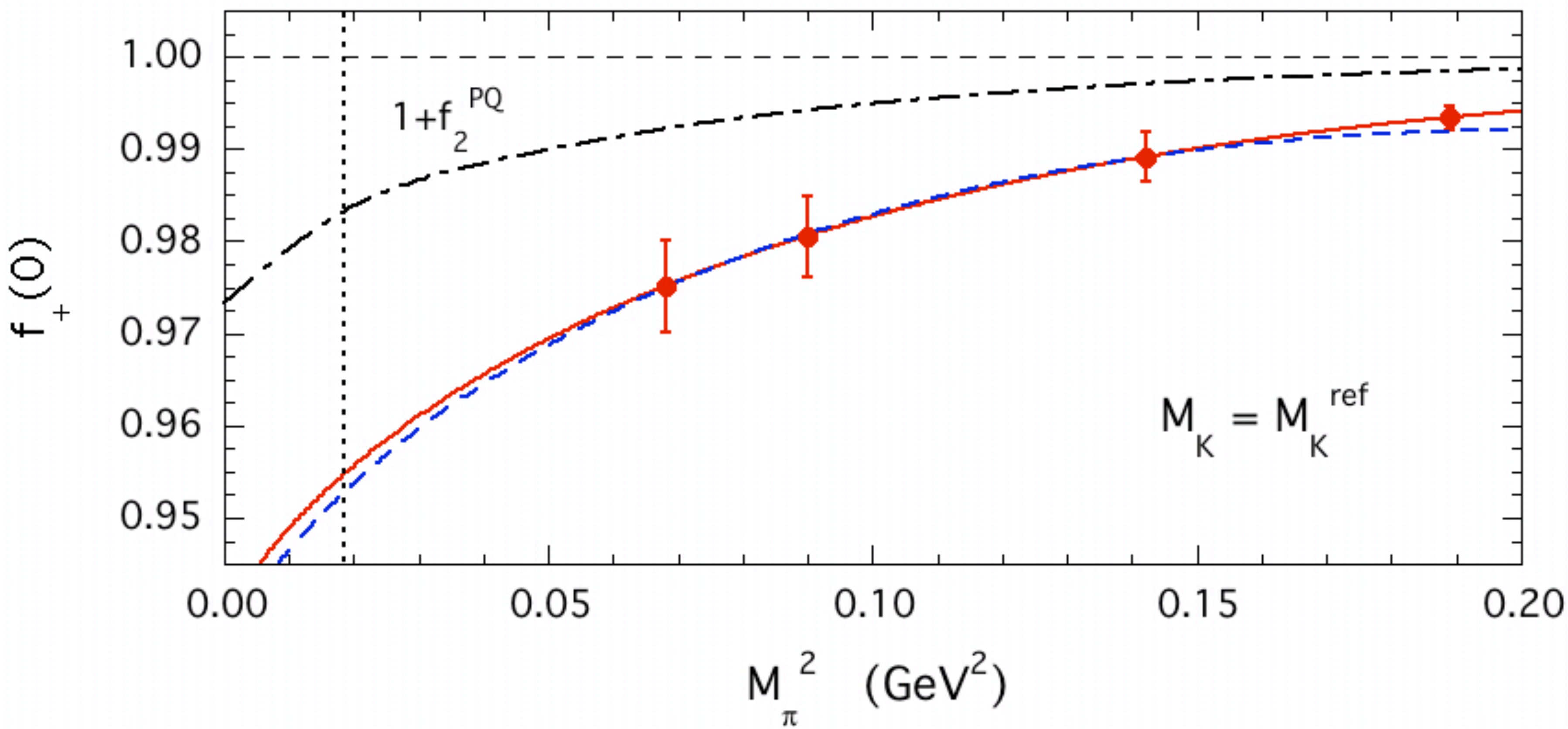
SU(2) ChPT @ NLO

[Flynn & Sachrajda '08]

$$f_+(0) = F_+ - \frac{3}{4}\frac{M_\pi^2}{16\pi^2 f_\pi^2}\log\left(\frac{M_\pi^2}{\mu^2}\right) + c_+(\mu) M_\pi^2 + O\left(M_\pi^4\right)$$

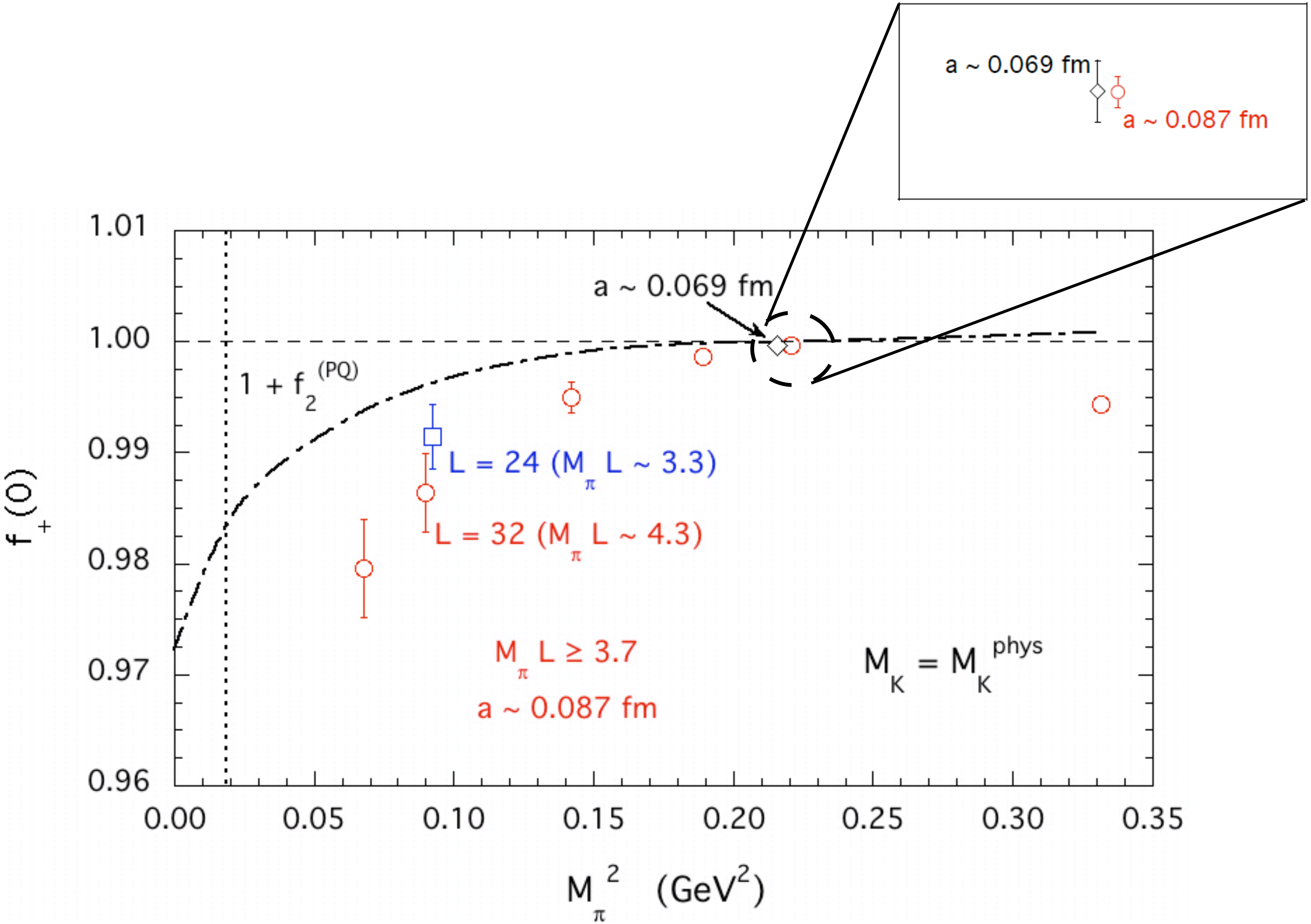
A) $m_s >> m_\ell$

B) F_+ and c_+ are LEC's depending on m_s ⇨ $M_K = M_K^{ref}$ ($\geq M_K^{phys}$)



- - - - SU(2) NLO (lowest three points: M_π < 400 MeV, m_s / m_ℓ > 3.3)

——— SU(2) NLO + quadratic term ∝ M_π^4 (four points: m_s / m_ℓ > 2.5)



- small discretization effects
- small residual FSE when M_π L ≥ 4

SU(3) ChPT: $f_+^{PQ}(0) = 0.9601 \pm 0.0030_{stat.} \pm 0.0029_{syst.}$

SU(2) ChPT: $f_+^{PQ}(0) = 0.9528 \pm 0.0068_{stat.} \pm 0.0017_{syst.}$

include q^2-dependence and chiral extrapolation

* weighted average: $f_+^{PQ}(0) = 0.9589 \pm 0.0027_{stat.} \pm 0.0034_{syst.}$

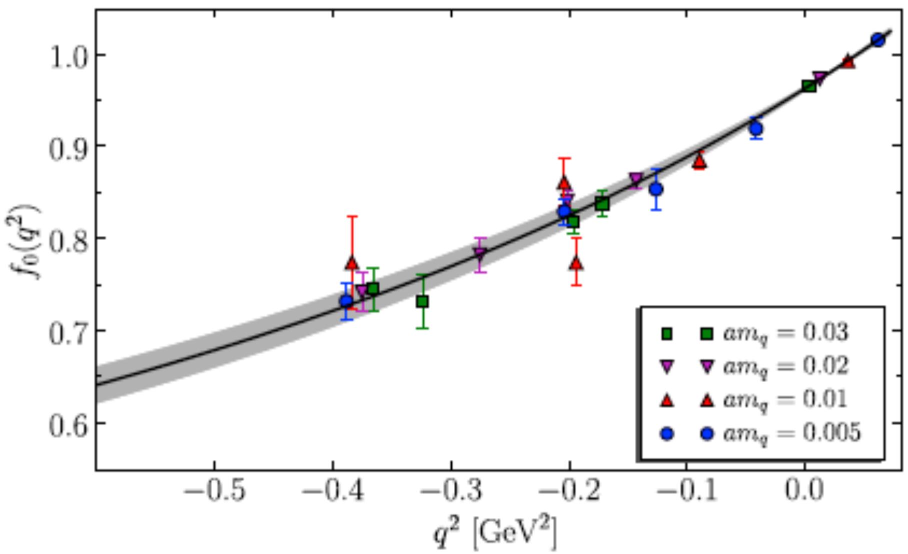
* add the difference between f_2 and f_2^{PQ}: -0.0226 + 0.0168 = -0.0058

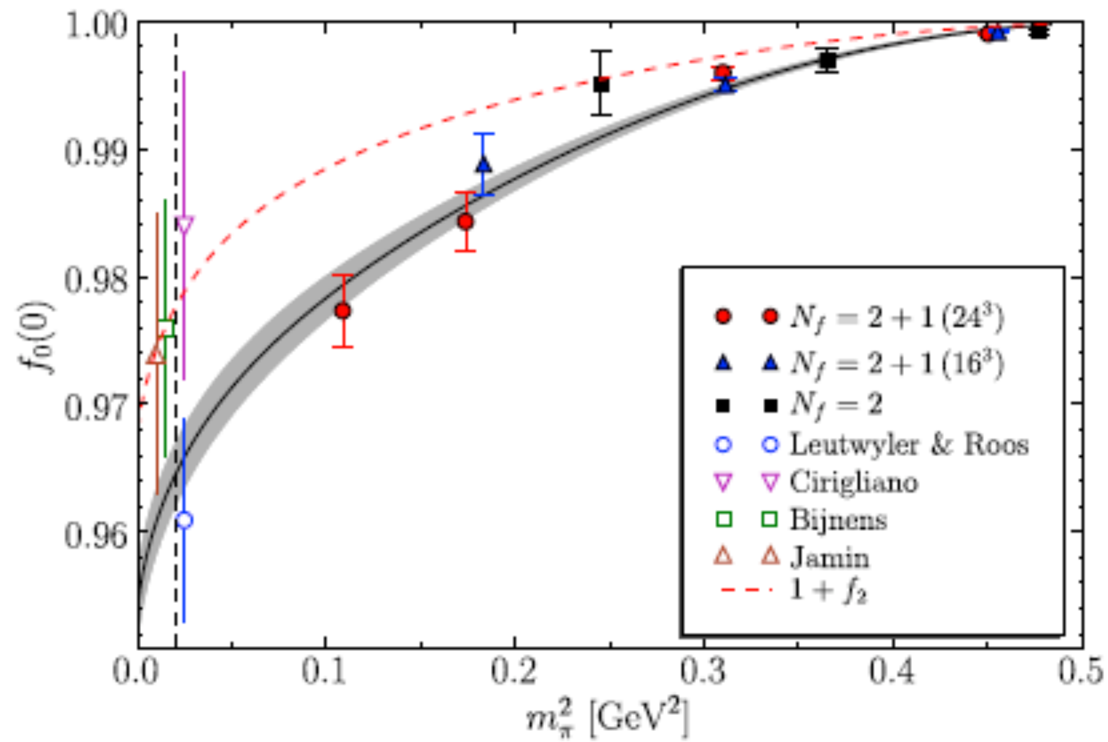
error budget :

* statistical error: 0.0027
* finite size effect: 0.0015 (~ 1/2 stat. err.)
* discretization effects: 0.0015 (~ 1/2 stat. err.)
* q^2-dependence and chiral extrapolation: 0.0034
* quenching of m_s: 0.0030 (~ 1/2 [f_2 - f_2^{PQ}])

$$f_+(0) = 0.9531 \pm 0.0027_{stat.} \pm 0.0050_{syst.} = 0.9531 \pm 0.0057$$

RBC/UKQCD '08





- $N_f = 2 + 1$
- Iwasaki gauge action [$a = 0.114(2)$ fm]
- Domain Wall Fermions [$am_{res} = 0.00315$]
- volumes: $(1.83\text{ fm})^3$ and $(2.74\text{ fm})^3$
- $M_\pi \geq 330$ MeV
- $m_s \sim 1.15\, m_s^{phys}$

combined chiral and q^2 fits

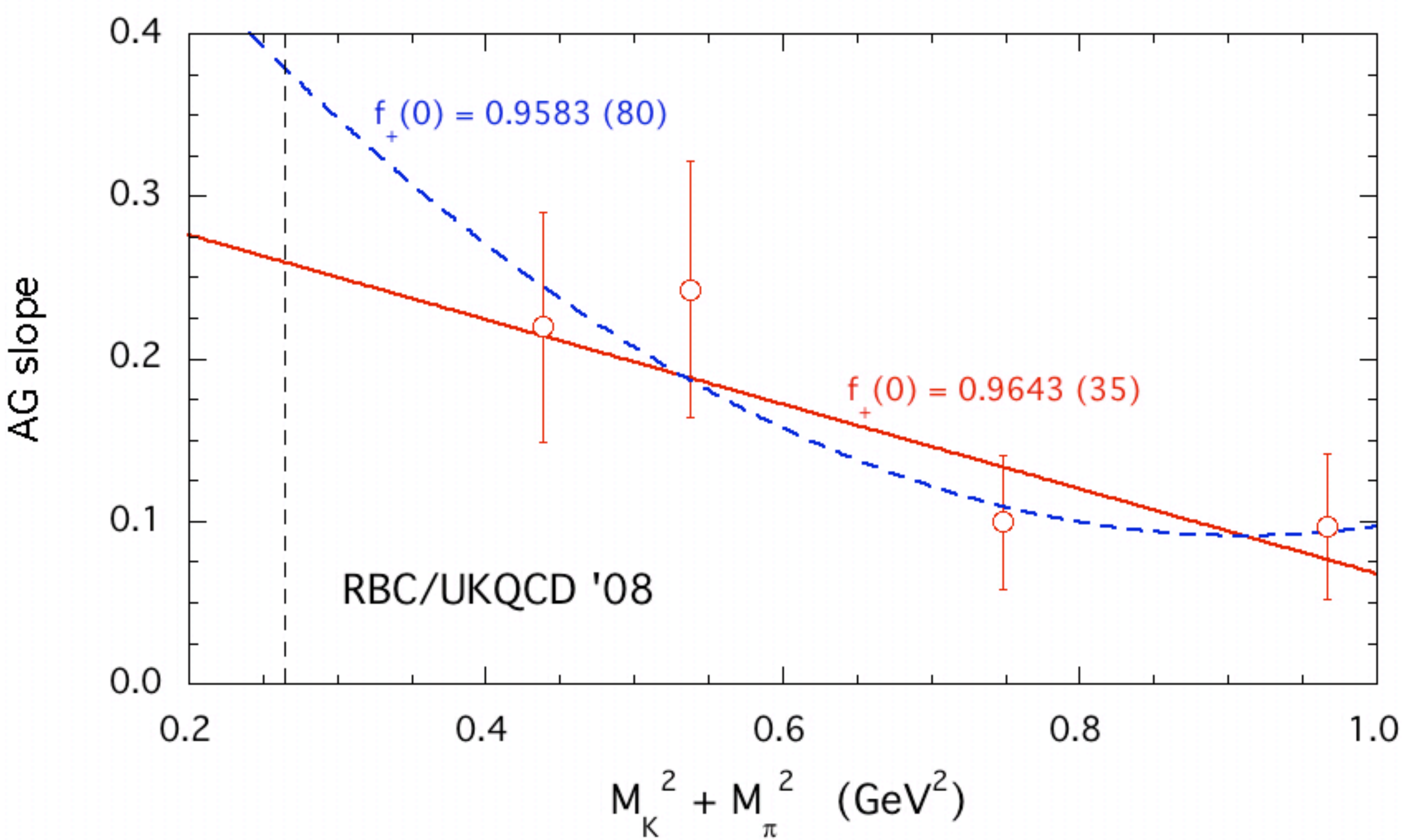
$$f_0(q^2, m_\pi^2, m_K^2) = \frac{1 + f_2 + (m_K^2 - m_\pi^2)^2 (A_0 + A_1(m_K^2 + m_\pi^2))}{1 - q^2/(M_0 + M_1(m_K^2 + m_\pi^2))^2},$$

$$f_+(0) = 0.9644\,(33)_{stat.}\,(34)_{extrap.}\,(14)_{discr.} = 0.9644\,(49)$$

4% of [$f_+(0)$ - 1]

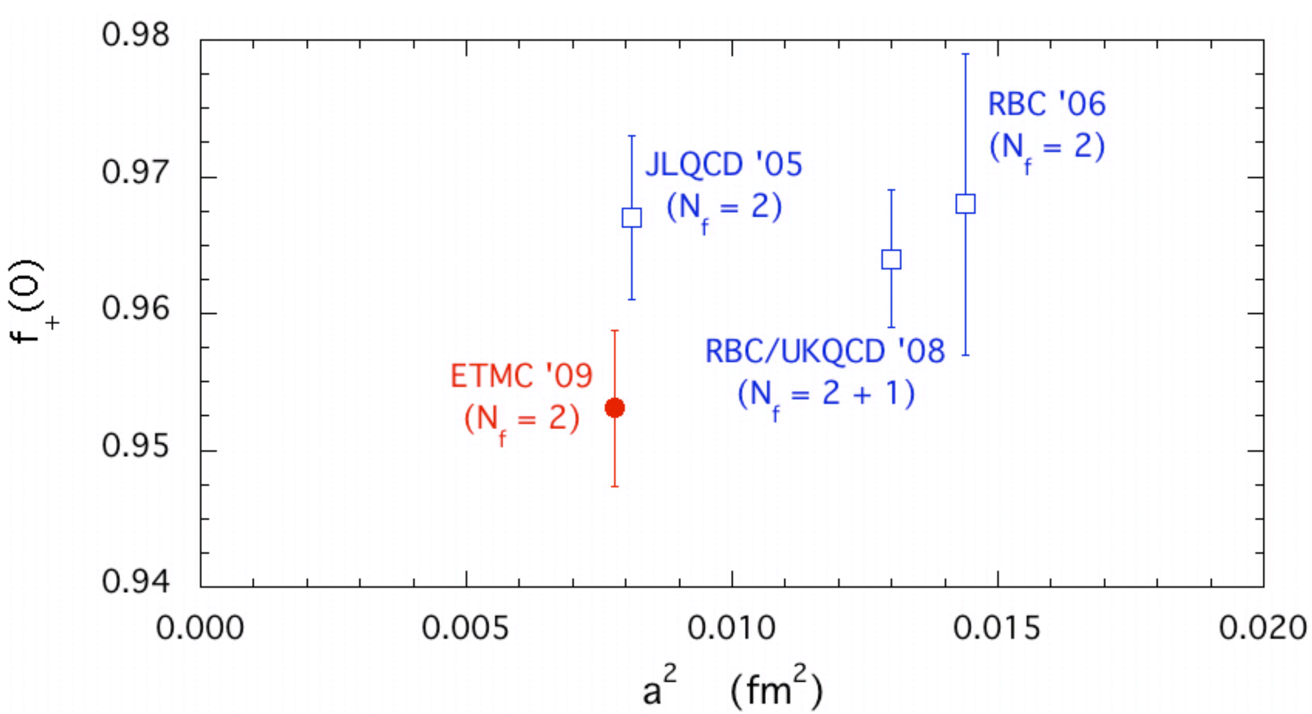
- main concerns: 1) coarse lattice spacing
  2) simplified chiral and q^2 fits

$$\text{AG slope} \equiv \frac{f_+(0) - 1 - f_2}{\left(M_K^2 - M_\pi^2\right)^2} \approx A_0 + A_1\left(M_K^2 + M_\pi^2\right) \quad \text{(RBC/UKQCD '08)}$$

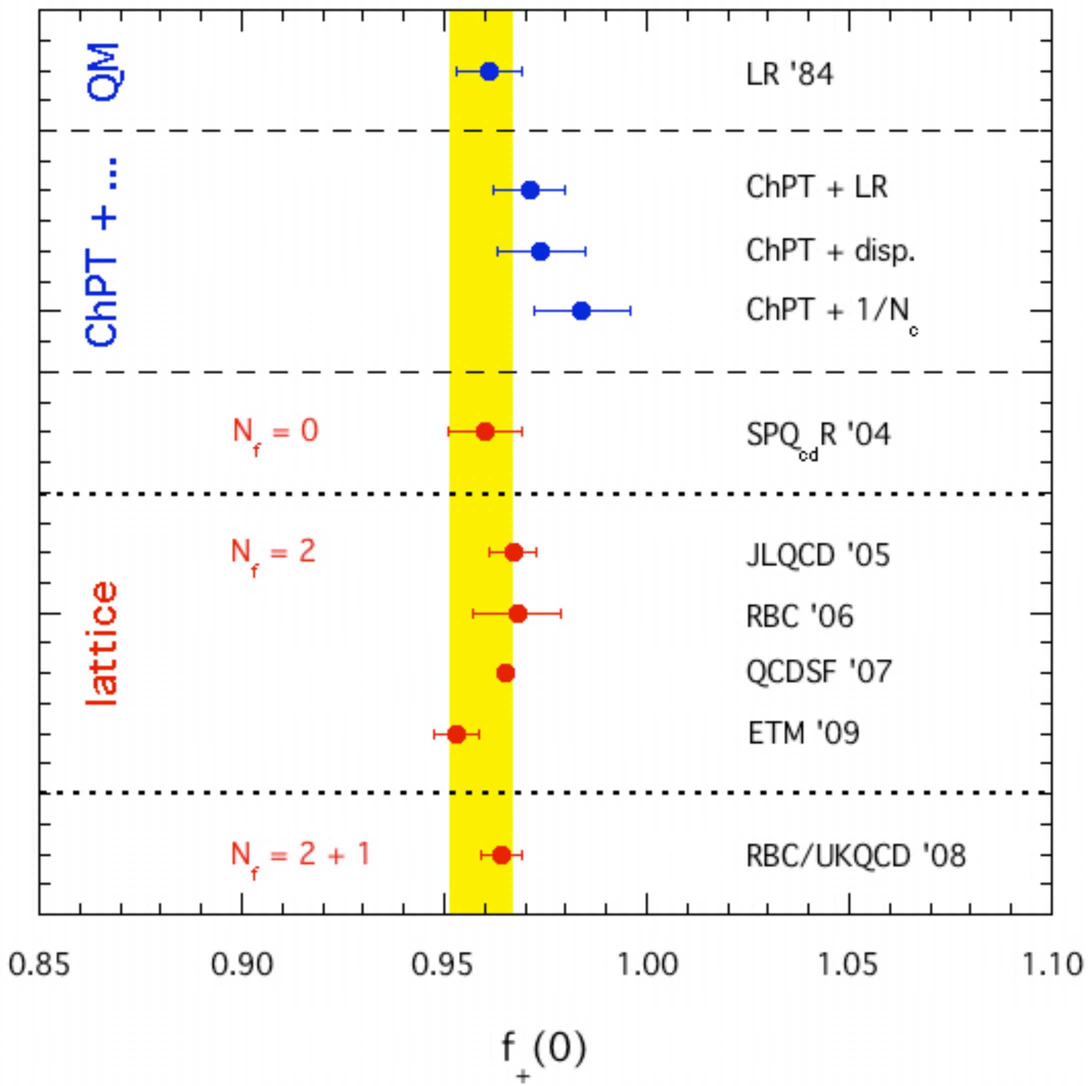


$f_+(0)$ from lattice

Collaboration	N_f	fermions	a (fm)	M_π L	M_π (MeV)	$f_+(0)$
RBC/UKQCD '08	2+1	DWF	0.11	≥ 4	≥ 330	0.9644 (33) (37)
ETM '09	2	tmW	0.09	≥ 4	≥ 260	0.9531 (27) (50)
QCDSF '07	2	NP-SW	0.08	≥ 6	≥ 590	0.9647 (15) (?)
RBC '06	2	DWF	0.12	≥ 6	≥ 490	0.968 (9) (6)
JLQCD '05	2	NP-SW	0.09	≥ 5	≥ 550	0.967 (6)
$SPQ_{cd}R$ '04	0	NP-SW	0.08	≥ 5	≥ 500	0.960 (5) (7)



general concern: a systematic study of the scaling property of $f_+(0)$ is still lacking



$$f_+(0) = 0.964 \pm 0.005$$

[Lellouch @ Lattice '08]

my lattice average

$f_+(0) = 0.959 \pm 0.008$ (~ 0.8%)

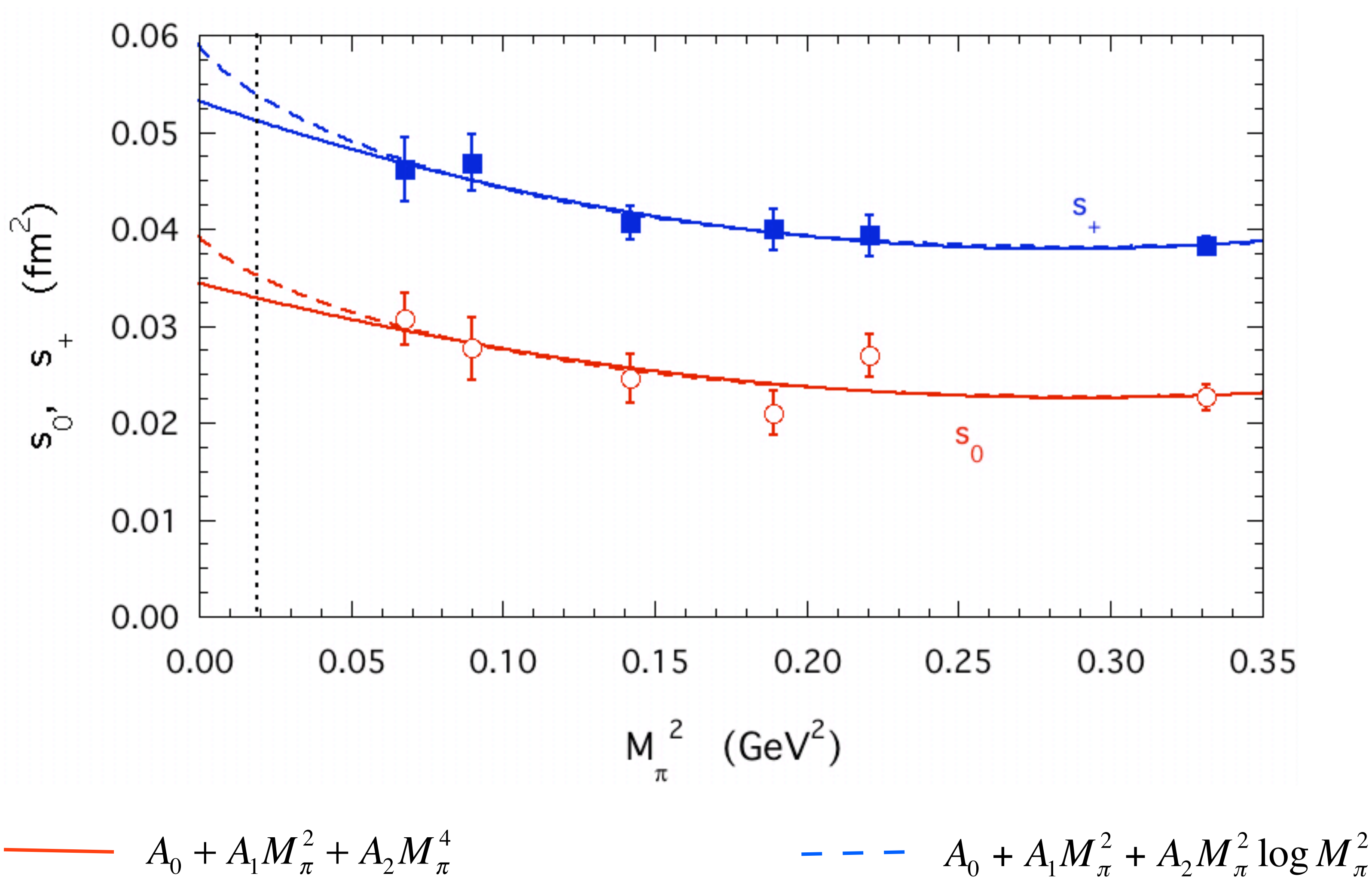
$|V_{us}| = 0.2259\ (19)$ (~ 0.8%)

V_{us} from $K_{\ell 3}$ decays: $|V_{us}| = 0.2273\ (14)$ (~ 0.6%)

ETMC '09

V_{us} from $K_{\ell 2}$ decays: $|V_{us}| = 0.2239\ (40)$ (~ 1.7%)

slopes of $f_+(q^2)$ and $f_0(q^2)$



$$\lambda_+ = s_+ M_\pi^2 = \left(24.5 \pm 2.4_{stat.} \pm 2.3_{syst.}\right)\cdot 10^{-3}$$

$$\lambda_0 = s_0 M_\pi^2 = \left(16.0 \pm 2.2_{stat.} \pm 4.5_{syst.}\right)\cdot 10^{-3}$$

$$\lambda_+ = \left(25.2 \pm 0.9\right)\cdot 10^{-3}$$

$$\lambda_0 = \left(13.4 \pm 1.2\right)\cdot 10^{-3}$$

[FlaviAnet ’07]

exp.: KTeV, KLOE, ISTRA+ and NA48