RCs of the bilinear operators: Results

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- RCs from the RI-MOM method

- Scale Independent RCs (Z_A, Z_V, Z_P/Z_S) from alternative methods

- N_f = 2+1+1: a plan for the calculation of the RCs

Statistics

$\beta = 3.80$		N_{meas}
μ_{sea}	0.0060	
μ_{val}(RI-MOM)	0.0060, 0.0080, 0.0110, 0.0165, 0.0200	240
μ_{val}(	0.0060, 0.0080, 0.0110, 0.0165, 0.0200	231
μ_{sea}	0.0080, 0.0110, 0.0165	
μ_{val}(RI-MOM)	0.0060, 0.0080, 0.0110, 0.0165, 0.0200	240
μ_{val}(alternative calc.)	0.0060, 0.0080, 0.0110, 0.0165, 0.0200	400

$\beta = 3.90$		N_{meas}
μ_{sea}	0.0040, 0.0064, 0.0085, 0.0100, 0.0150	
μ_{val}(RI-MOM)	0.0040, 0.0064, 0.0085, 0.0100, 0.0150	240
μ_{val}(alternative calc.)	0.0040, 0.0064, 0.0085, 0.0100, 0.0150	240

$\beta = 4.05$		N_{meas}
μ_{sea}	0.0030	
μ_{val}(RI-MOM)	0.0030, 0.0060, 0.0080, 0.0120	240
μ_{val}(alternative calc.)	0.0030, 0.0060, 0.0080, 0.0120	152
μ_{sea}	0.0060, 0.0080	
μ_{val}(RI-MOM)	0.0030, 0.0060, 0.0080, 0.0120	160
μ_{val}(alternative calc.)	0.0030, 0.0060, 0.0080, 0.0120	130
μ_{sea}	0.0120	
μ_{val}(alternative calc.)	0.0030, 0.0060, 0.0080, 0.0120	130

excluded data
from the final analysis

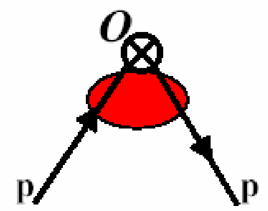
RI-MOM method

Define the Green function:

$$G_\Gamma^{ud}(p,p') = a^8 \sum_{x,y} \langle u(x)(\bar{u}\Gamma d)_0 \bar{d}(y) \rangle e^{-ip\cdot x + ip'\cdot y} \quad \textit{for} \quad \Gamma = S,P,V,A,T \quad \leftrightarrow \quad I, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu}$$

Quark propagator : $S_q(p) = \sum_x \langle q(x)\bar{q}(0) \rangle e^{-ip\cdot x}, \quad q = u,d$

Evaluate: $\Gamma_\Gamma^{ud}(p,p') = \mathrm{Tr}\left[S_u(p)^{-1} G_\Gamma^{ud}(p,p') S_d(p')^{-1} P_\Gamma \right]$



at fixed (Landau) gauge

Impose the condition: $Z_\Gamma Z_q^{-1} \Gamma_\Gamma^{ud}(p,p) \Big|_{p^2=k^2}^{\mu_q \to 0} = 1$

with: $Z_q \dfrac{i}{12} \mathrm{Tr}\left[\dfrac{(\gamma \cdot p)\, S(p)^{-1}}{p^2} \right]_{p^2=k^2}^{\mu_q \to 0} = 1$

"$\mu_q \to 0$" : **take valence & sea chiral limit**

$(\Lambda_{QCD} << k << \pi / a)$

Martinelli, Pittori, Sachrajda, Testa, Vladikas, NPB 1995
Gimenez, Giusti, Rapuano, Talevi, NPB 1998

O(a) improvement

- Asymptotic O(a)-improvement at large p^2 and $\mu_q \to 0$

  Becirevic, Gimenez, Lubicz, Martinelli, Papinutto, Reyes, JHEP 2004

- O(a)-improvement for any p^2 thanks to the symmetries of the action at maximal twist (*see Roberto's talk at Trento Workshop 2008*)

- Increase the statistics by taking the average:

$$\mathbf{Z_\Gamma = (Z_\Gamma^{ud} + Z_\Gamma^{du})/2}$$
$$\mathbf{Z_q = (Z_q^{u} + Z_q^{d})/2}$$

- For the scale dependent RC: $\mathbf{Z(\mu) = C(\mu)\; Z^{RGI}}$

  where: $\mathbf{C(\mu) = e^{\int^{\alpha(\mu)} d\alpha\, \gamma(\alpha)/\beta(\alpha)}}$ (N^2LO for Z_T, N^3LO for Z_S, Z_P)

Gracey, NPB 2003
Chetyrkin & Retey, NPB 2000

The Goldstone pole:

- The pseudoscalar vertex couples to the PGB pole:

$$\Gamma_P \approx A(p^2) + B(p^2)\frac{\langle \bar{\psi}\psi \rangle}{m_q p^2} + \cdots$$

Cudell, Yaouanc, Pittori, PLB 1999

- For the pole subtraction use two methods:
  (a) Fit the pole term and subtract;
  (b) Calculate the "subtracted" vertex:

$$\Gamma_P^{SUB} = \frac{m_1 \Gamma_P(m_1) - m_2 \Gamma_P(m_2)}{m_1 - m_2} \approx \Gamma_P(m_1) + m_1 \frac{\partial \Gamma_P}{\partial m_1} = A + \cdots$$

Giusti, Vladikas, PLB 2000

In our analysis both methods give compatible results.

- Scalar vertex (twist): the PGB pole is suppressed by a factor $O(a^2)$

Subtract $O(a^2g^2)$ perturbative terms from RI-MOM vertices:

$\mathcal{O}(a^2)$ corrections to the propagator and bilinears of clover fermions with Symanzik improved gluons

M. Constantinou , V. Giménez , V. Lubicz , D. Palao , H. Panagopoulos , F. Stylianou

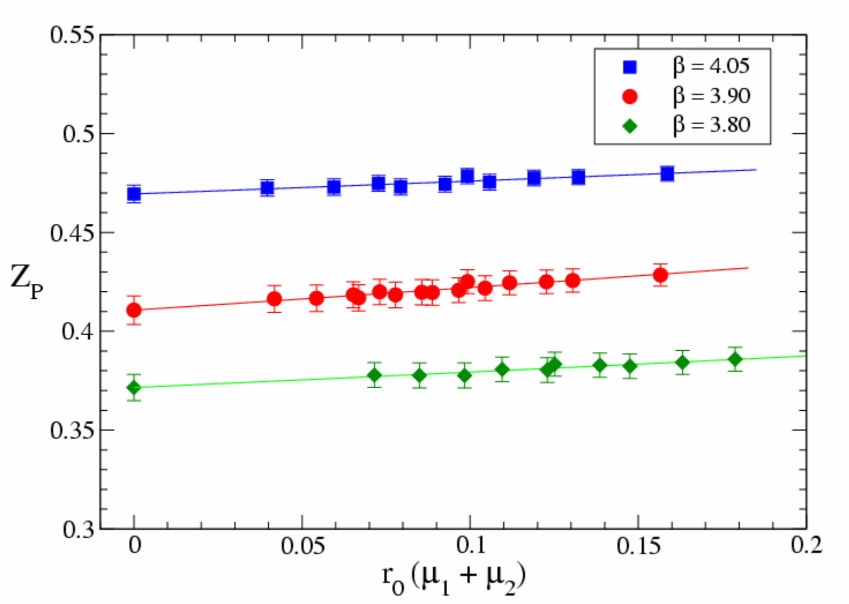
Internal report, 2008

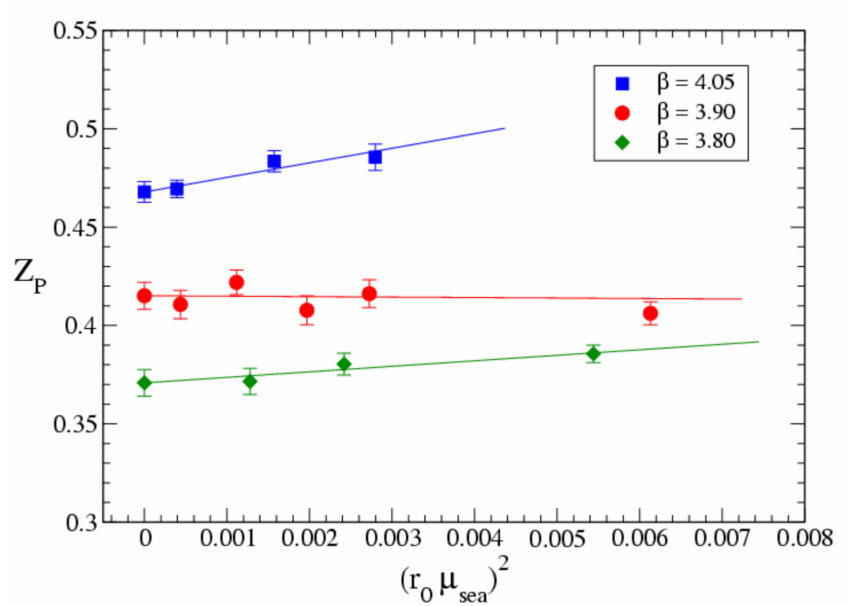
Numerical application of the $O(a^2g^2)$ subtractions using two choices for g^2:

(i) $g_0^2 = 6 / \beta$
(ii) $g^2(\text{boosted}) = g_0^2 / \langle U_{plaq} \rangle$

- *In the following we show results obtained with (ii).*
- *Moreover, it will be seen that the results for the RCs, before and after the subtractions, are compatible within the errors.*

Z_P : valence & sea chiral limit





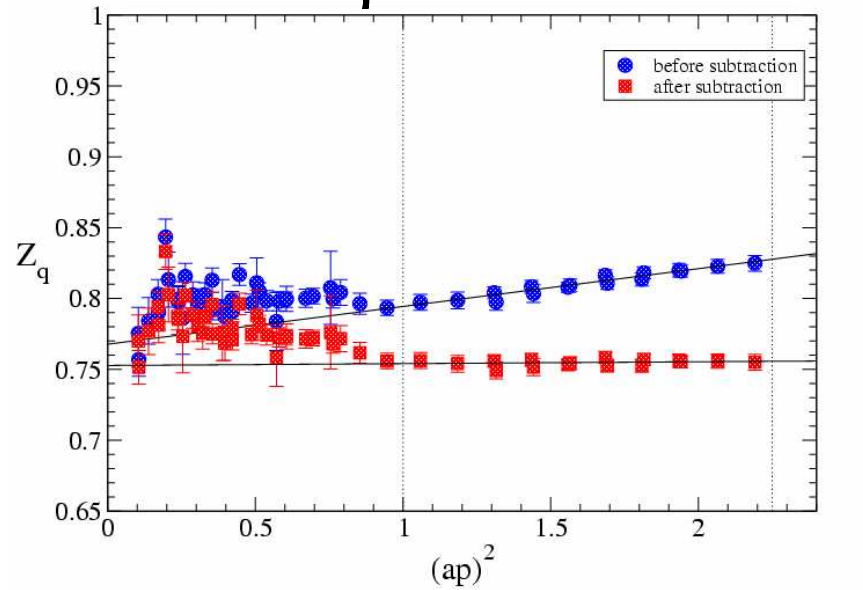
Valence chiral limit at
$a\mu_{sea} = a\mu_{sea}^{min}$
for each β value

Sea chiral limit

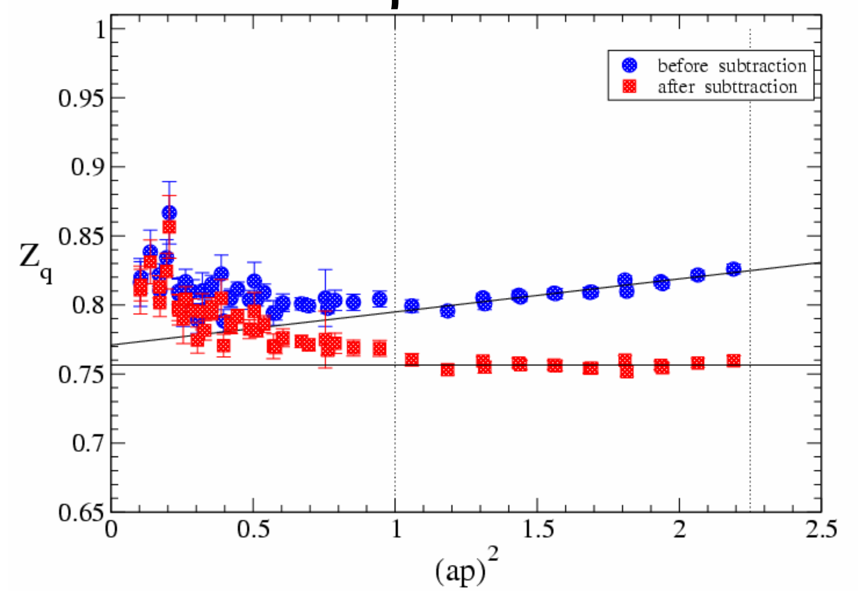
(*after subtracting the $O(a^2g^2)$ terms*)

$Z_q\ (1/a)_\beta$

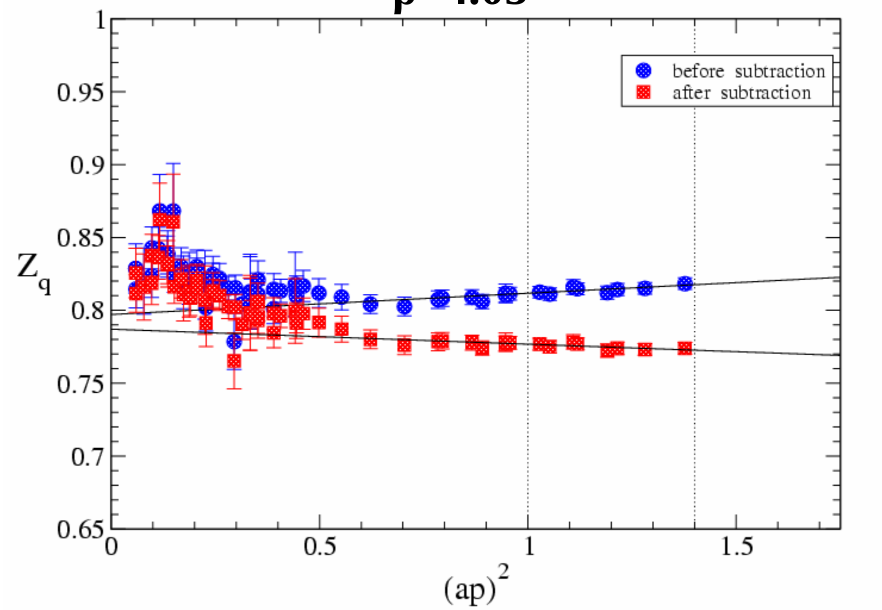
β=3.80



β=3.90

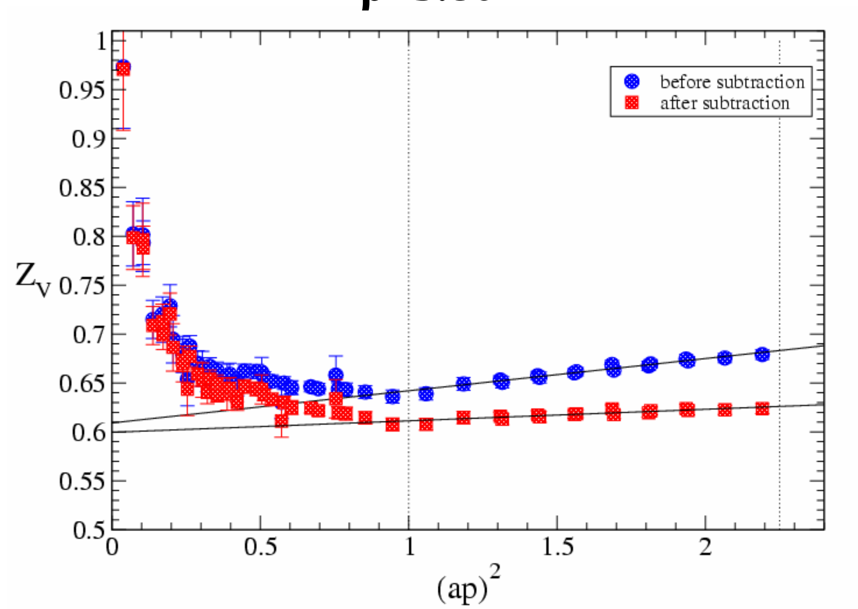


β=4.05

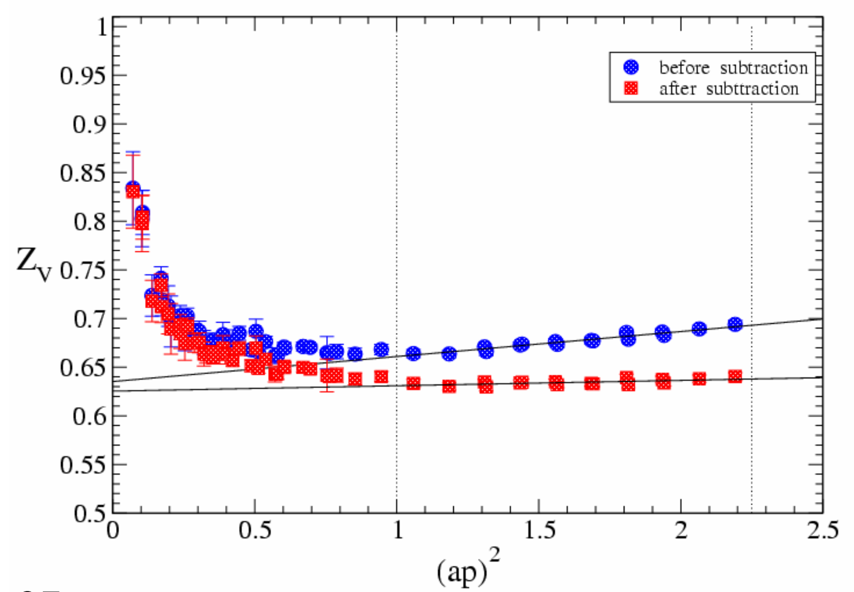


Z_V

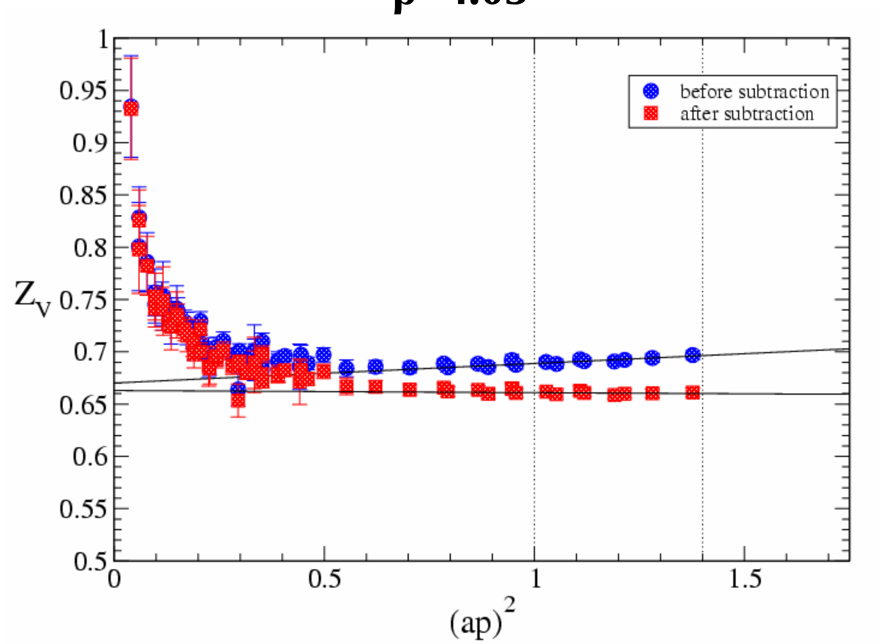
β=3.80



β=3.90

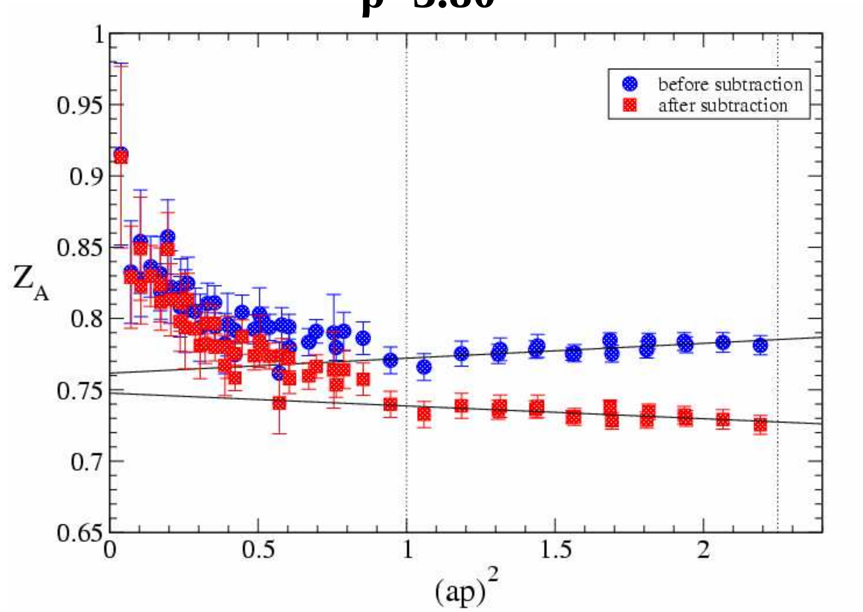


β=4.05

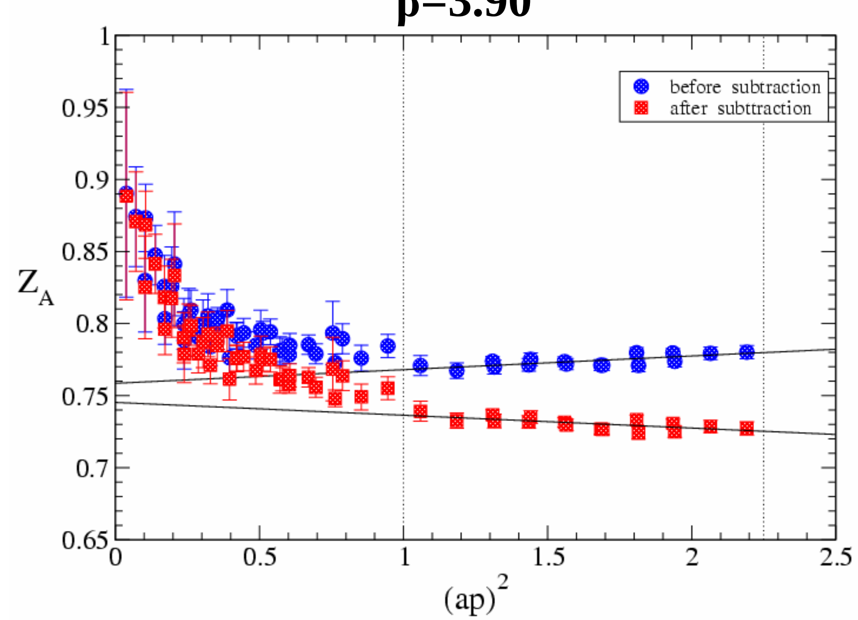


Z_A

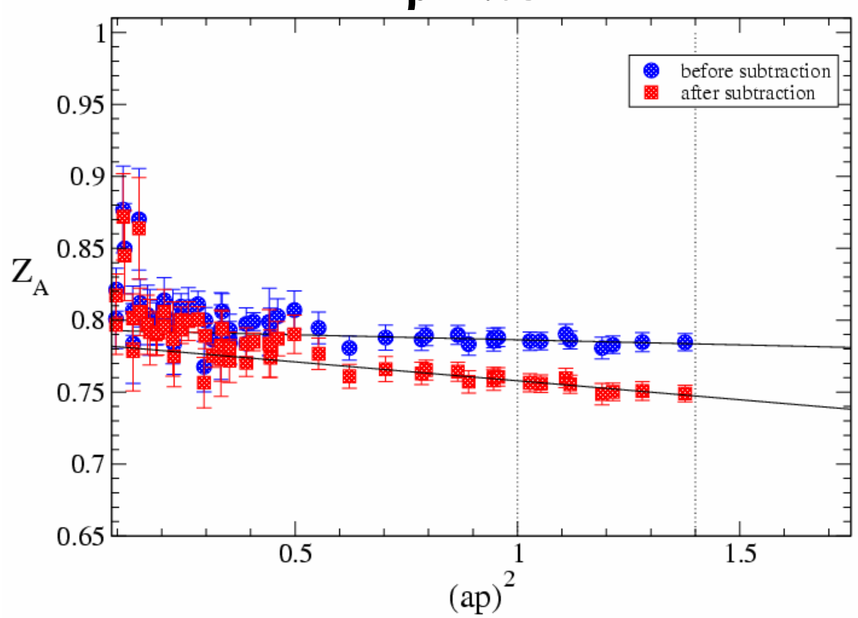
β=3.80



β=3.90

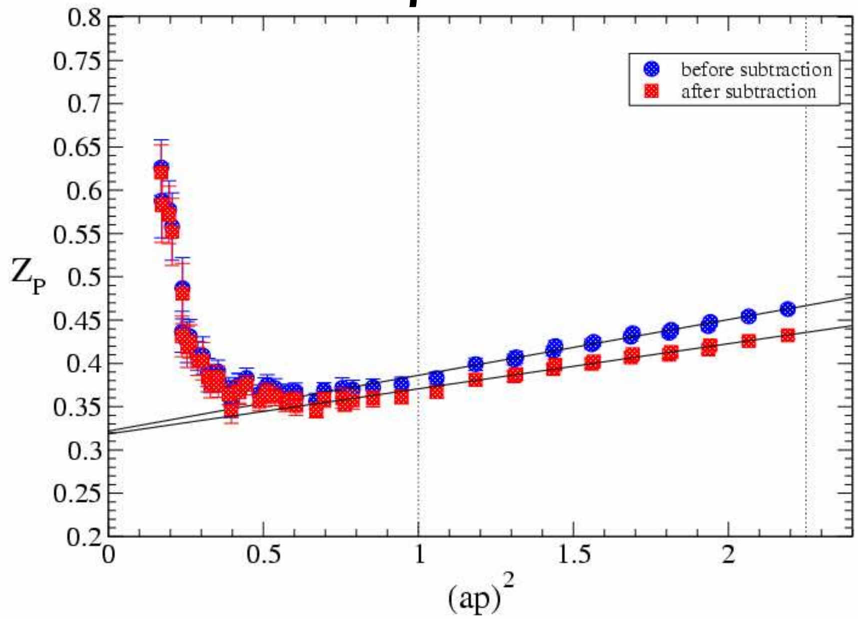


β=4.05

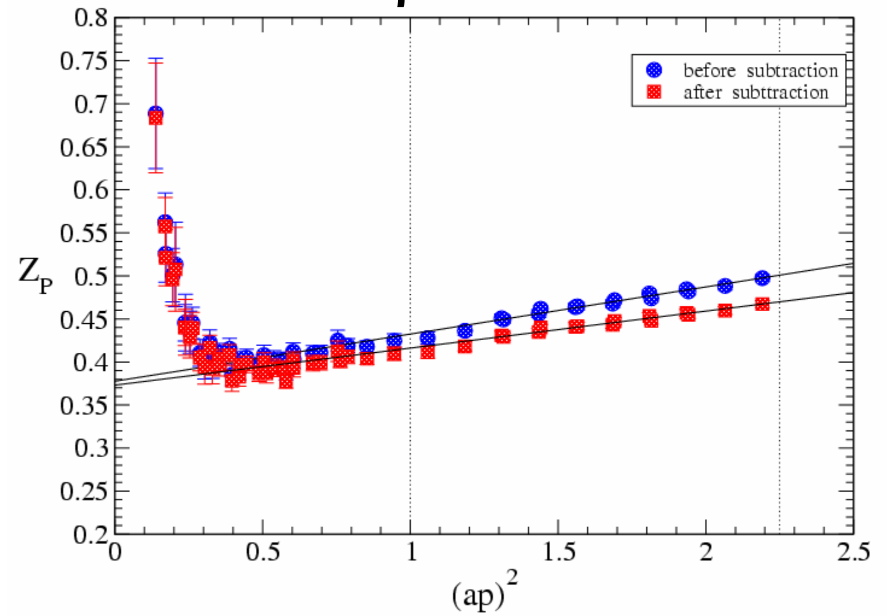


$Z_P\,(1/a)_\beta$

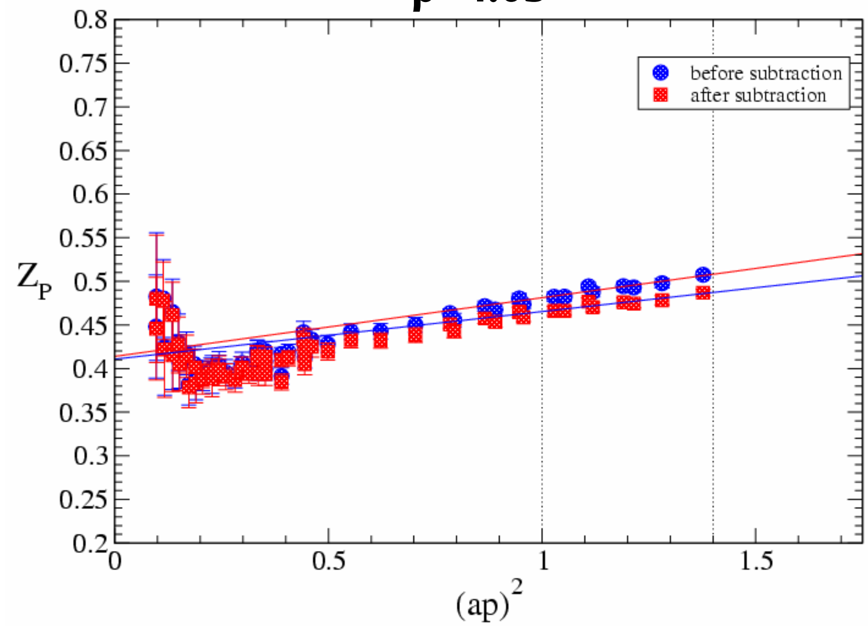
β=3.80



β=3.90

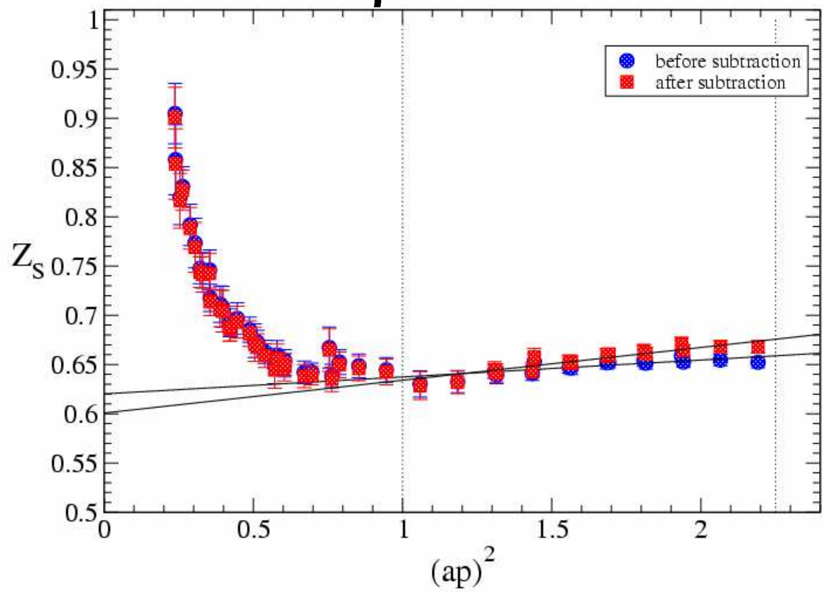


β=4.05

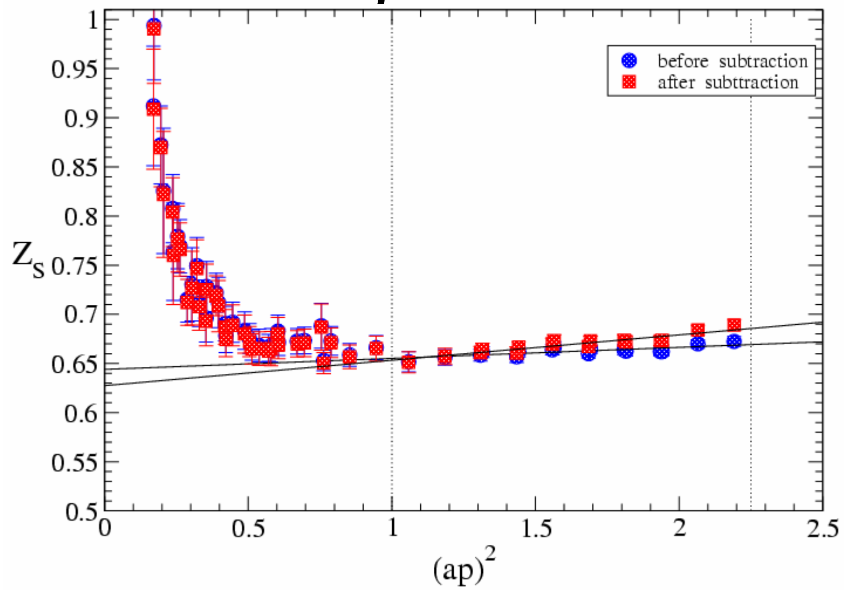


$Z_S\,(1/a)_\beta$

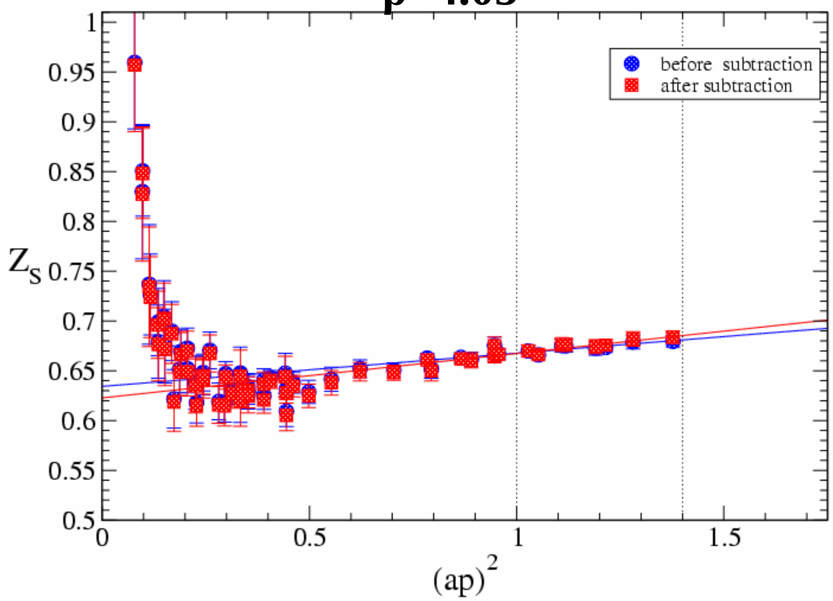
β=3.80



β=3.90

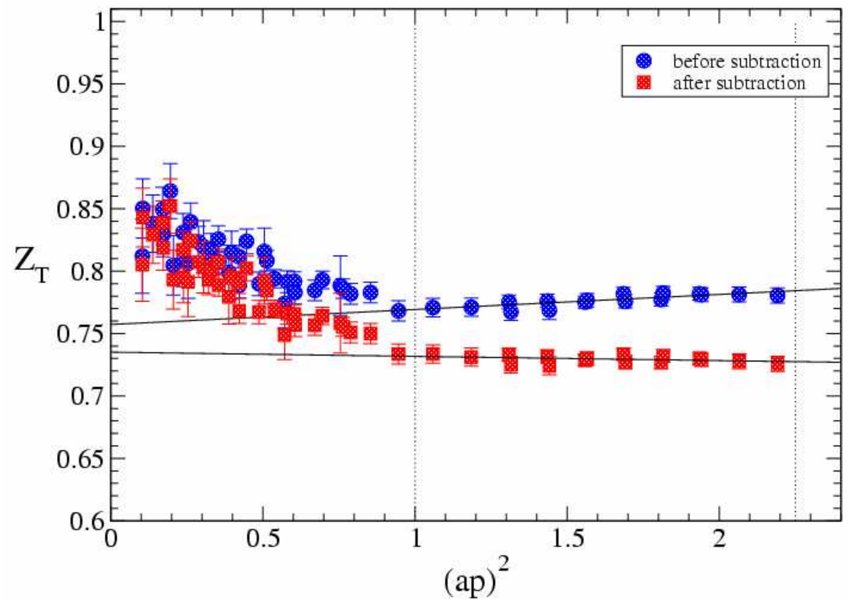


β=4.05

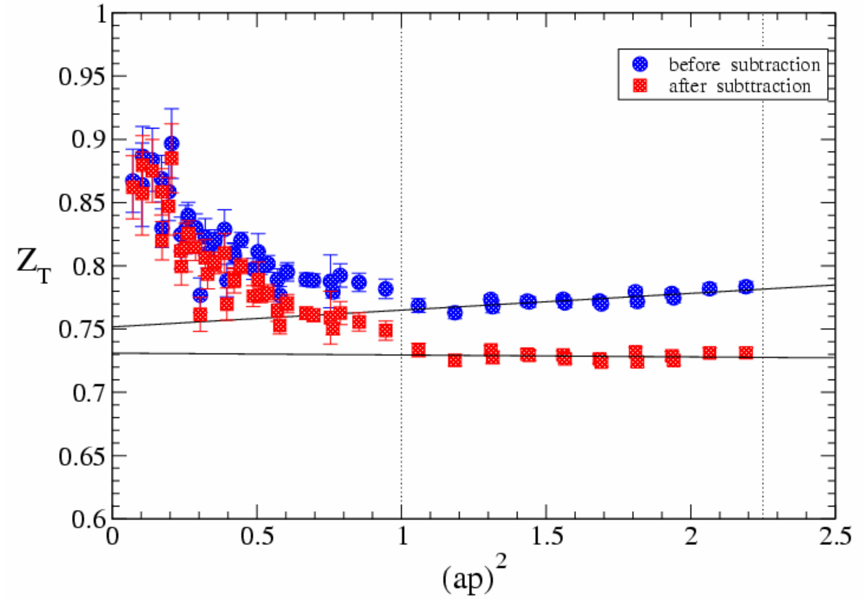


$Z_T\,(1/a)_\beta$

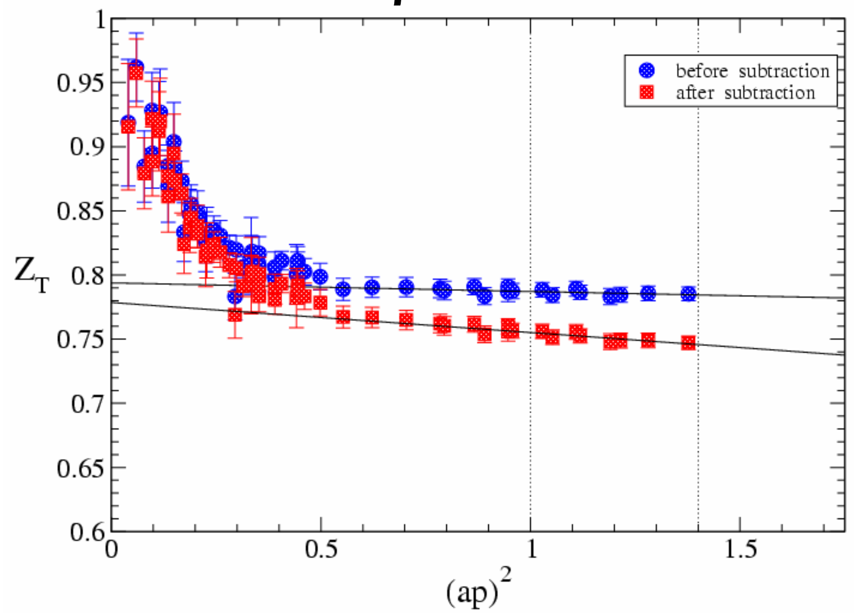
β=3.80



β=3.90

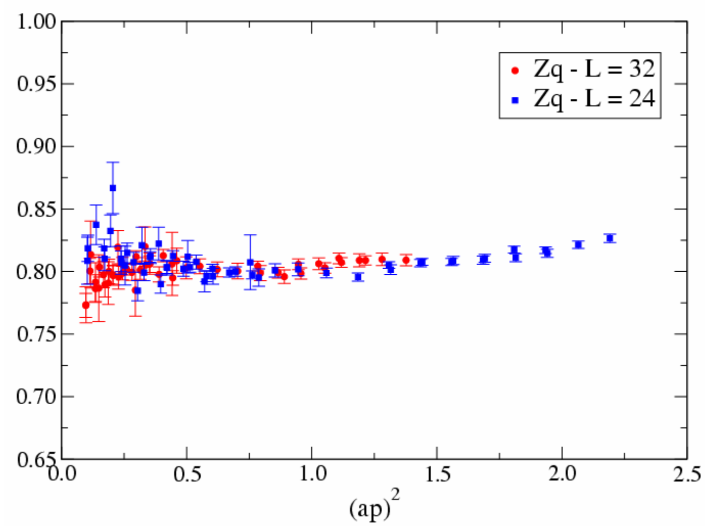


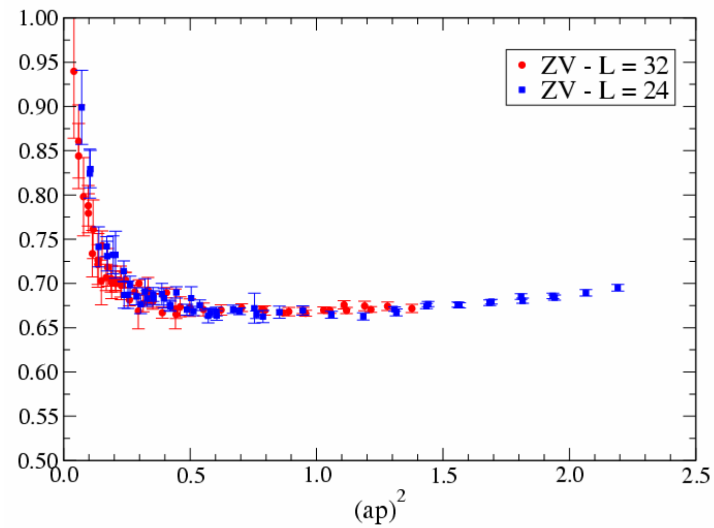
β=4.05

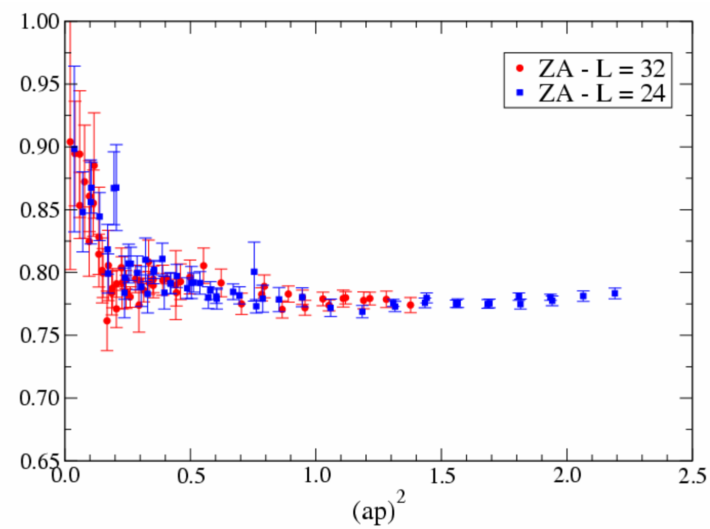


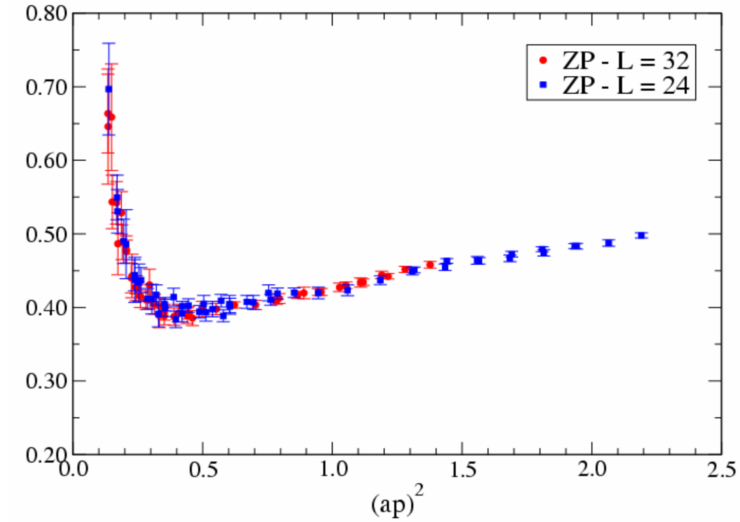
Check for finite volume effects

$\beta = 3.90$ @ $a\mu_{sea}=0.0040$









Scale Independent RCs: alternative methods

Consider the valence action in the physical basis :

$$S_{val} = a^4 \sum_x \bar{\psi}(x)(\gamma\tilde{\nabla} - i\gamma_5\, r\, W_{cr} + \mu_q)\psi(x)$$

with:

$$W_{cr} = -\frac{a}{2}\sum_\mu \nabla^*_\mu \nabla_\mu + M_{cr}(r = 1)$$

$$\mu_q = \mathrm{diag}(\mu_u \;\; \mu_d) \qquad r = \mathrm{diag}(r_u \;\; r_d)$$

$r_u = r_d = \pm 1$ ⟶ *OS case*

$r_u = -r_d = \pm 1$ ⟶ *tm case*

Make a chiral rotation back to the twisted basis:

tm case

$$(u,d) = \exp[i(\gamma_5 \tau_3 \pi/4)](u', d')$$

OS case

$$(u,d) = \exp[i(\gamma_5 \pi/4)](u', d')$$

Then for a renormalised bilinear operator we have:

$$Z_{O_{\tilde{\Gamma}}} \langle O_{\tilde{\Gamma}} \rangle^{tm} = Z_{O_{\tilde{\tilde{\Gamma}}}} \langle O_{\tilde{\tilde{\Gamma}}} \rangle^{OS} + O(a^2)$$

OS case	tm case
$(A_R)_{\mu,ud} = Z_A A_{\mu,ud} = Z_A A'_{\mu,ud}$	$(A_R)_{\mu,ud} = Z_V A_{\mu,ud} = -iZ_V V'_{\mu,ud}$
$(V_R)_{\mu,ud} = Z_V V_{\mu,ud} = Z_V V'_{\mu,ud}$	$(V_R)_{\mu,ud} = Z_A V_{\mu,ud} = -iZ_A A'_{\mu,ud}$
$(P_R)_{ud} = Z_S P_{ud} = iZ_S S'_{ud}$	$(P_R)_{ud} = Z_P P_{ud} = Z_P P'_{ud}$

Z_P / Z_S

Consider the matrix element, $g_\pi = < 0|P|\pi >$
in the two regularisations

For the renormalised values we set the condition:

$$[g_{\pi\pm}]^{cont} \; = \; Z_P\,[g'_{\pi\pm}]^{tm} + O(a^2) \; = \; Z_S\,[g'_\pi]^{OS} + O(a^2)$$

from which, at the chiral limit, we obtain the value of **Z_P / Z_S**

Z_A

Consider the pseudoscalar decay constant in the two regularisations

$$f_{\pi\pm}^{tm} = 2\mu_q g_\pi / m_\pi^2$$ (no renormalisation constant is needed)

$$f_\pi^{OS} = \frac{\langle 0|A_0|\pi\rangle}{m_\pi}$$ (Z_A is needed for the renormalisation)

We use the equality of renormalised quantities up to $O(a^2)$ terms:

$$[f_{\pi\pm}]^{cont} \; = \; f_{\pi\pm}^{tm} + O(a^2) \; = \; Z_A f_\pi^{OS} + O(a^2)$$

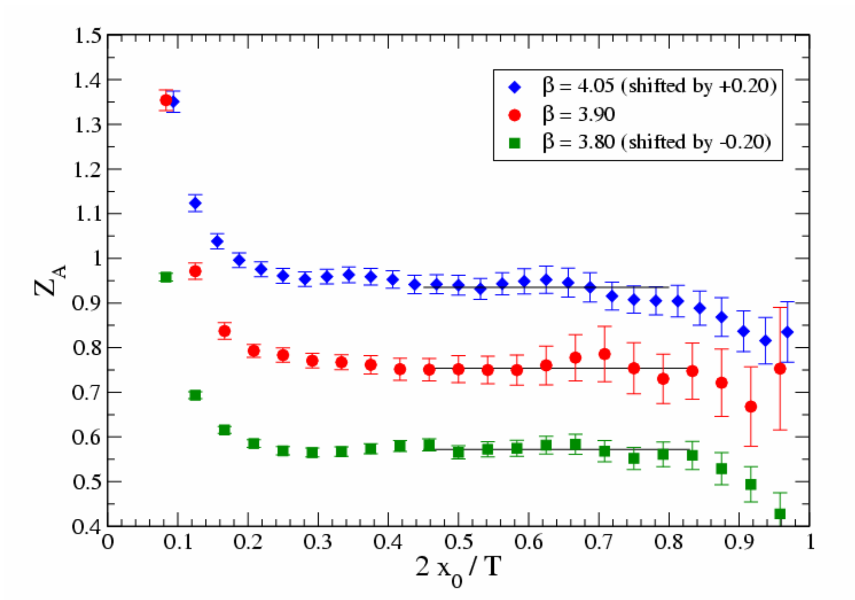
and in the chiral limit we obtain **Z_A**

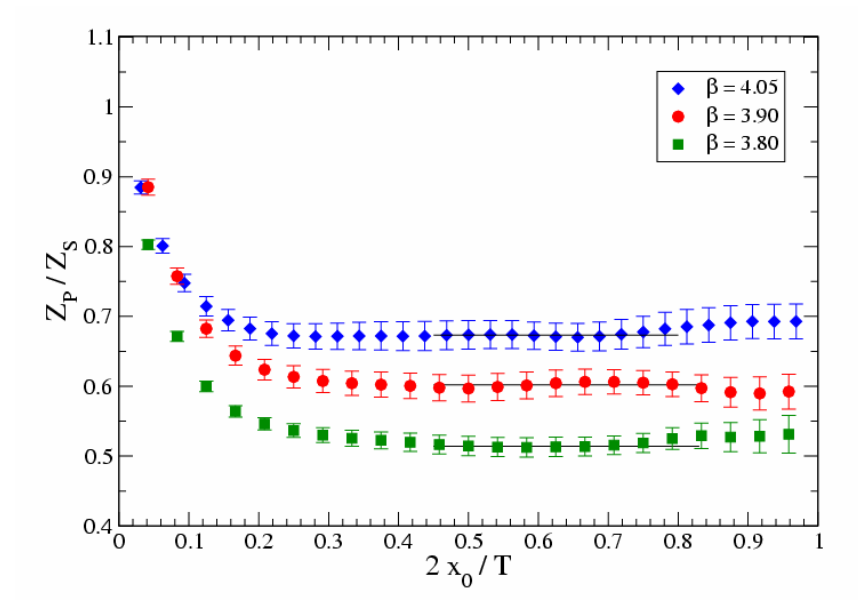
Z_V

Use of only tm quarks and WI yields **Z_V** :

$$Z_V = \frac{(\mu_1 + \mu_2) C_{PP}(x_0)}{\tilde{\partial} C_{A_0 P}(x_0)}\Big|_{\chi-\mathrm{limit}}$$

Quality of plateaux



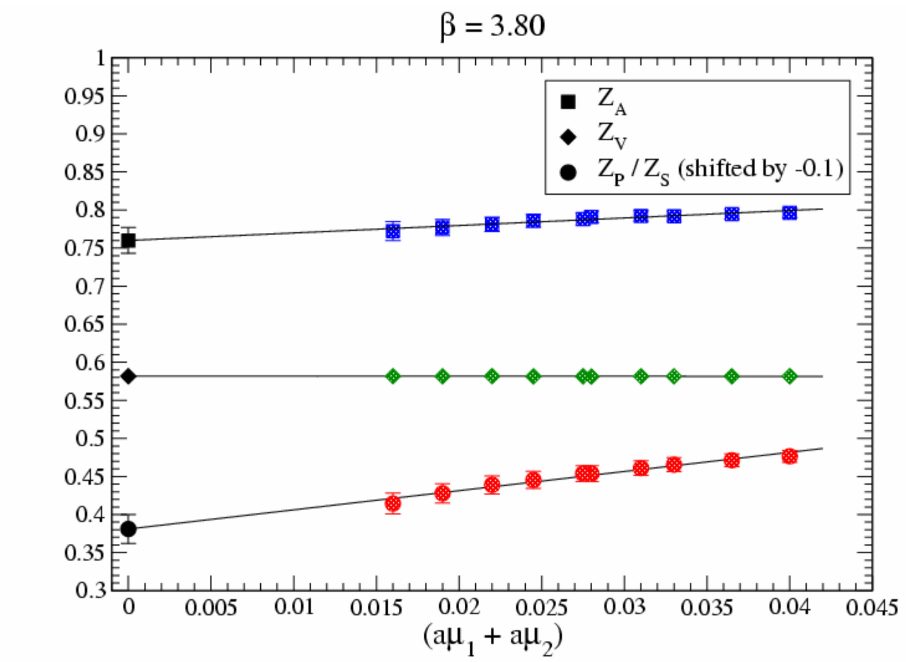


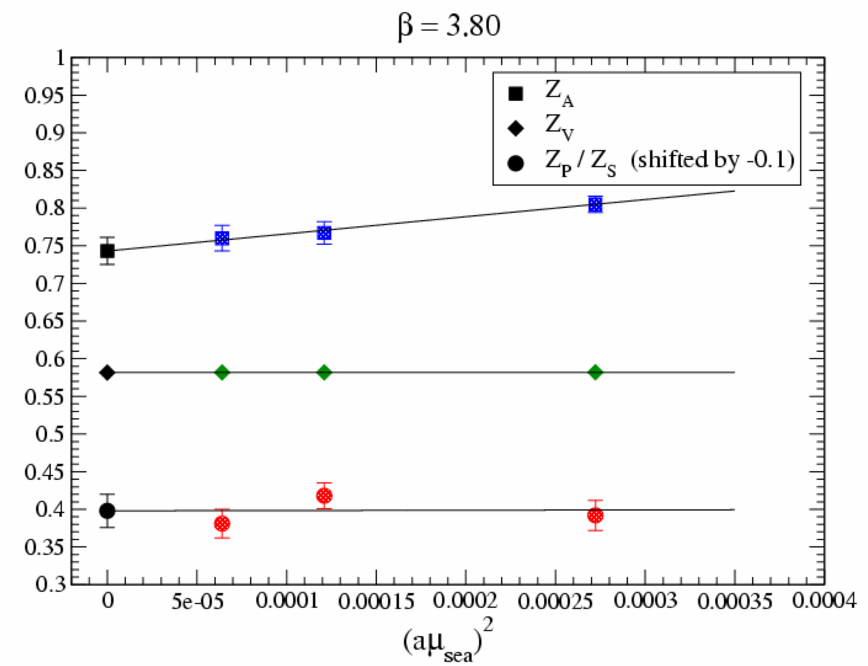
$$\left(a\mu_{sea}^{min} = a\mu_1 = a\mu_2\right)$$

at the minimum value of the sea quark mass for each value of β

Valence & Sea chiral limit

β = 3.80



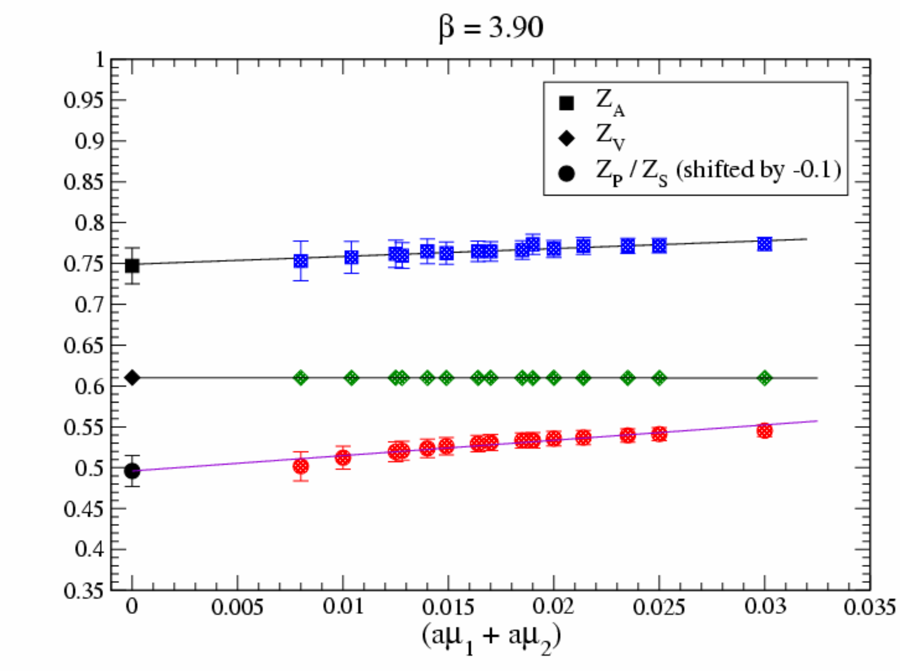


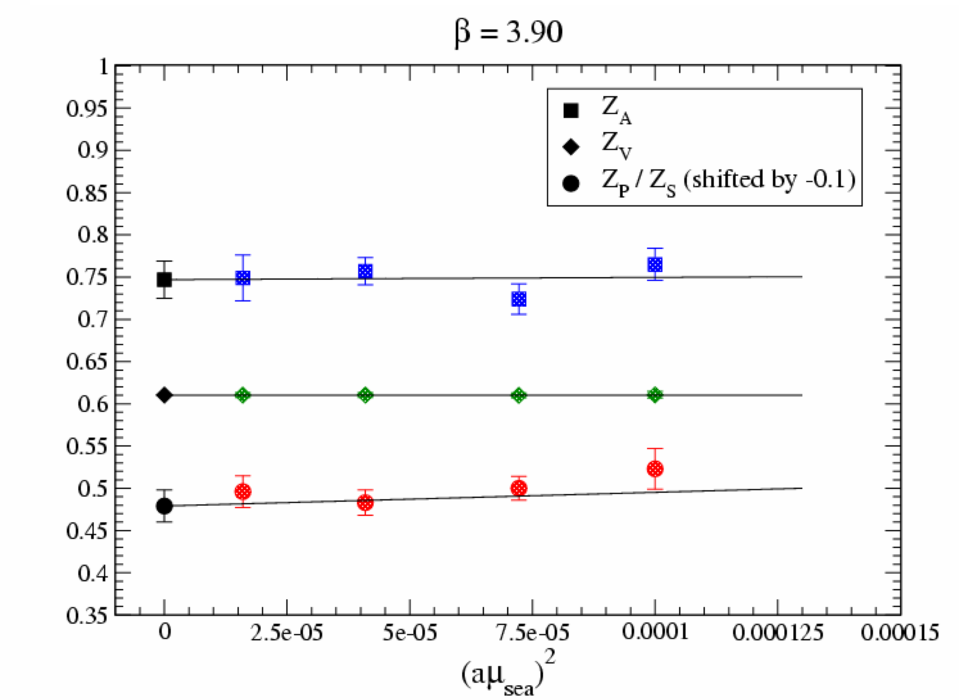
Valence chiral limit at $a\mu_{sea} = a\mu_{sea}^{min}$

Sea chiral limit

Valence & Sea chiral limit

$\beta = 3.90$



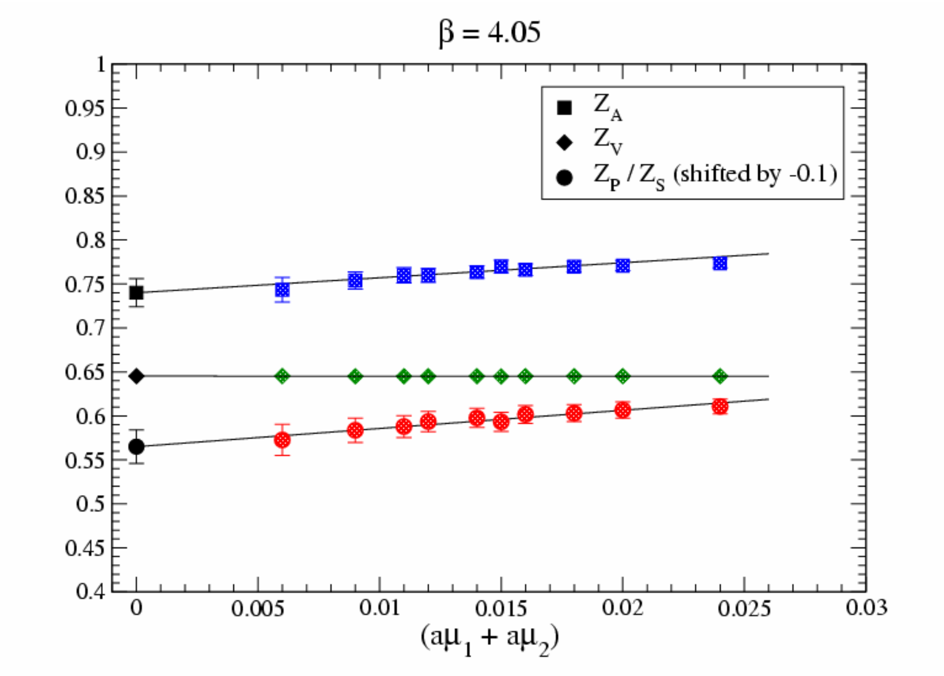


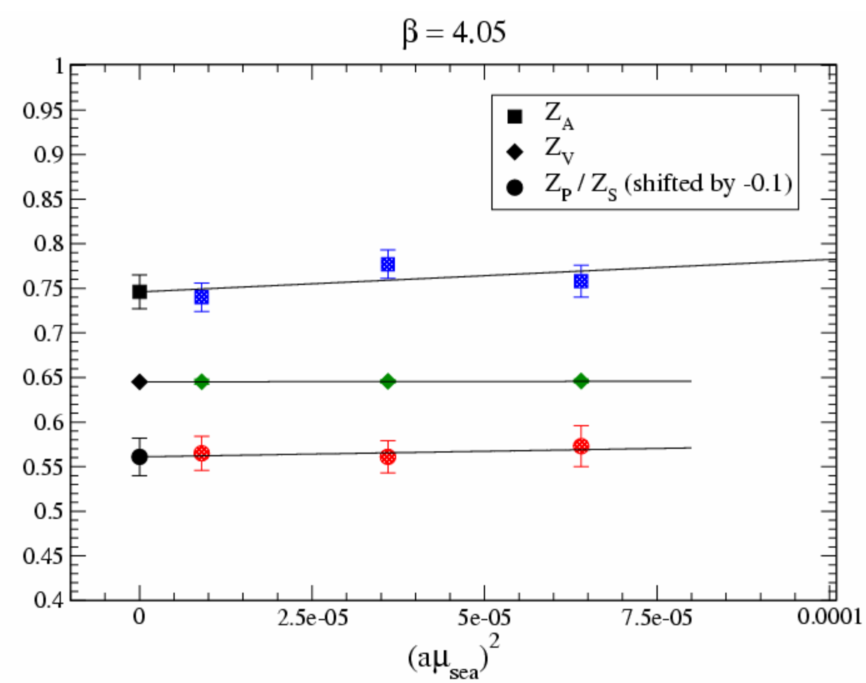
Valence chiral limit at $a\mu_{sea} = a\mu_{sea}^{min}$

Sea chiral limit

Valence & Sea chiral limit

$\beta = 4.05$

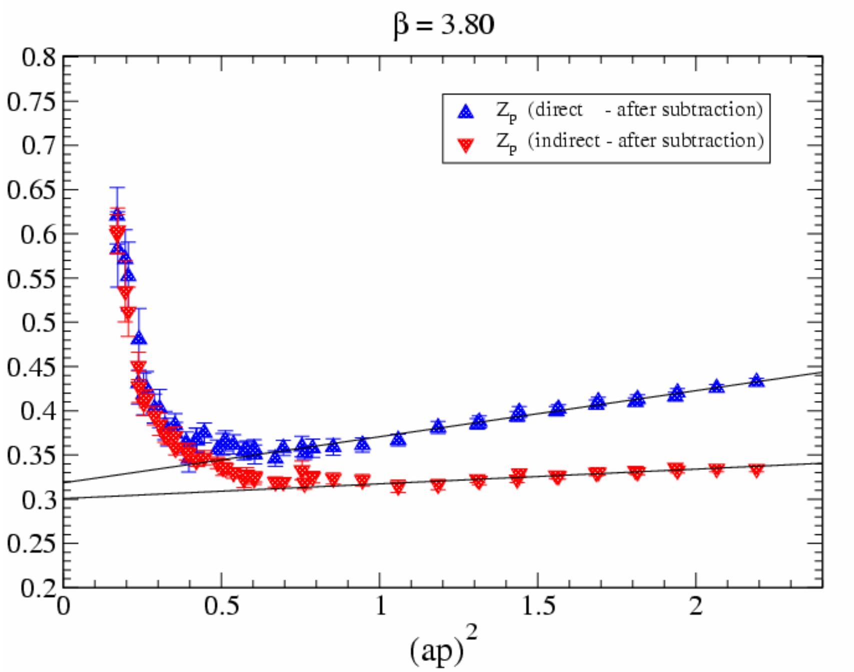


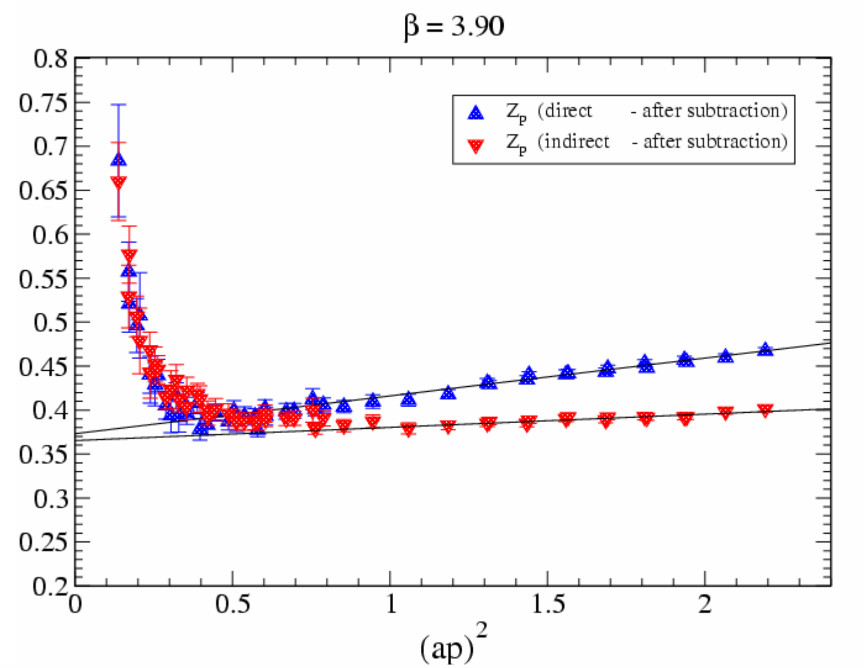


Valence chiral limit at $a\mu_{sea} = a\mu_{sea}^{min}$

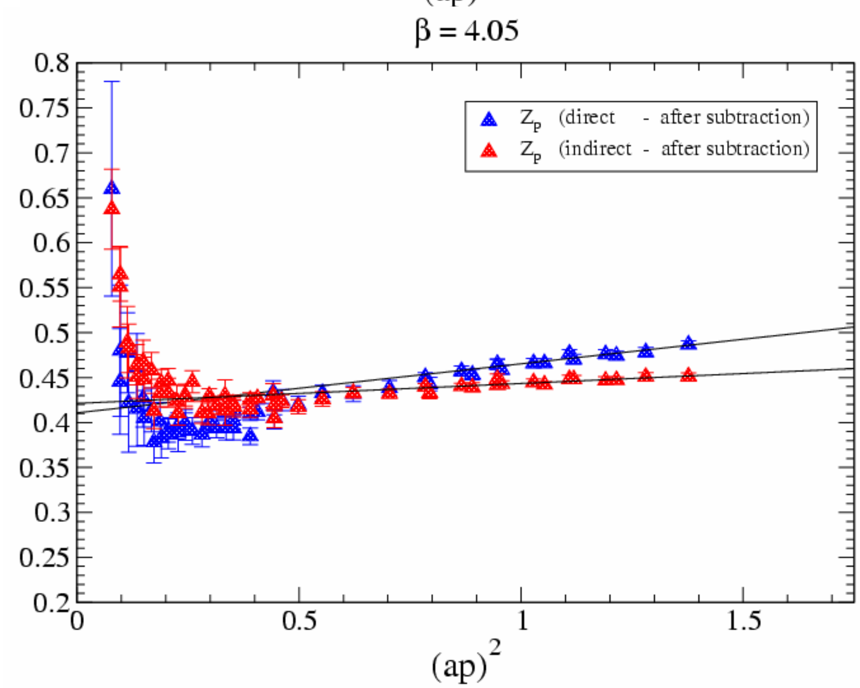
Sea chiral limit

Z_P –RI-MOM vs. Z_P -indirect





$$Z_P^{\text{indirect}}(1/a) = \left(\frac{Z_P}{Z_S}\right)_{OS/tm} Z_S^{\text{RI-MOM}}(1/a)$$



Compatible results within the errors!

RESULTS

	β	lin. fit in p^2 w/o sub.	lin. fit in p^2 with sub.	"p^2-window" with sub.	alternative method
Z_V	3.80	0.609(8) [33(4) $\times 10^{-3}$]	0.600(8) [12(4) $\times 10^{-3}$]	0.621(4)	0.5816(2)
	3.90	0.635(7) [26(4) $\times 10^{-3}$]	0.625(7) [6(4) $\times 10^{-3}$]	0.633(3)	0.6103(3)
	4.05	0.670(9) [19(7) $\times 10^{-3}$]	0.663(9) [-2(7) $\times 10^{-3}$]	0.663(3)	0.6451(3)
Z_A	3.80	0.762(11) [11(6) $\times 10^{-3}$]	0.748(11) [-9(6) $\times 10^{-3}$]	0.732(5)	0.747(22)
	3.90	0.759(11) [10(6) $\times 10^{-3}$]	0.745(11) [-9(6) $\times 10^{-3}$]	0.734(4)	0.743(18)
	4.05	0.793(11) [-7(8) $\times 10^{-3}$]	0.784(11) [-26(8) $\times 10^{-3}$]	0.762(6)	0.746(18)
Z_P/Z_S	3.80	0.538(17) [75(8) $\times 10^{-3}$]	0.555(17) [41(8) $\times 10^{-3}$]		0.498(22)
	3.90	0.592(18) [70(8) $\times 10^{-3}$]	0.603(18) [37(8) $\times 10^{-3}$]		0.579(19)
	4.05	0.648(20) [70(16) $\times 10^{-3}$]	0.654(20) [41(16) $\times 10^{-3}$]		0.661(21)
$Z_S(1/a)$	3.80	0.620(15) [17(7) $\times 10^{-3}$]	0.601(16) [33(8) $\times 10^{-3}$]	0.661(6)	
	3.90	0.644(11) [12(6) $\times 10^{-3}$]	0.627(12) [26(6) $\times 10^{-3}$]	0.663(5)	
	4.05	0.634(14) [33(12) $\times 10^{-3}$]	0.623(15) [45(13) $\times 10^{-3}$]	0.662(5)	
$Z_P(1/a)$	3.80	0.322(10) [64(4) $\times 10^{-3}$]	0.318(10) [52(4) $\times 10^{-3}$]		0.300(14)
	3.90	0.378(08) [55(4) $\times 10^{-3}$]	0.373(08) [43(4) $\times 10^{-3}$]		0.363(13)
	4.05	0.414(11) [67(7) $\times 10^{-3}$]	0.411(11) [55(7) $\times 10^{-3}$]		0.412(13)
$Z_T(1/a)$	3.80	0.757(10) [12(5) $\times 10^{-3}$]	0.735(10) [-3(5) $\times 10^{-3}$]	0.730(5)	
	3.90	0.752(09) [13(5) $\times 10^{-3}$]	0.731(09) [-2(5) $\times 10^{-3}$]	0.730(3)	
	4.05	0.794(13) [-7(9) $\times 10^{-3}$]	0.779(12) [-23(9) $\times 10^{-3}$]	0.759(6)	
$Z_q(1/a)$	3.80	0.768(07) [27(4) $\times 10^{-3}$]	0.753(07) [1(4) $\times 10^{-3}$]	0.755(4)	
	3.90	0.771(07) [24(4) $\times 10^{-3}$]	0.757(07) [0(4) $\times 10^{-3}$]	0.757(3)	
	4.05	0.797(12) [15(9) $\times 10^{-3}$]	0.787(12) [-10(9) $\times 10^{-3}$]	0.777(5)	

Z_P Indirect

We provide the final results from the RI-MOM calculation using a linear fit in (a^2p^2) ***after subtracting*** the estimated $O(a^2g^2)$ contributions. The reasons for this choice (instead of the "p^2-window" method) are:

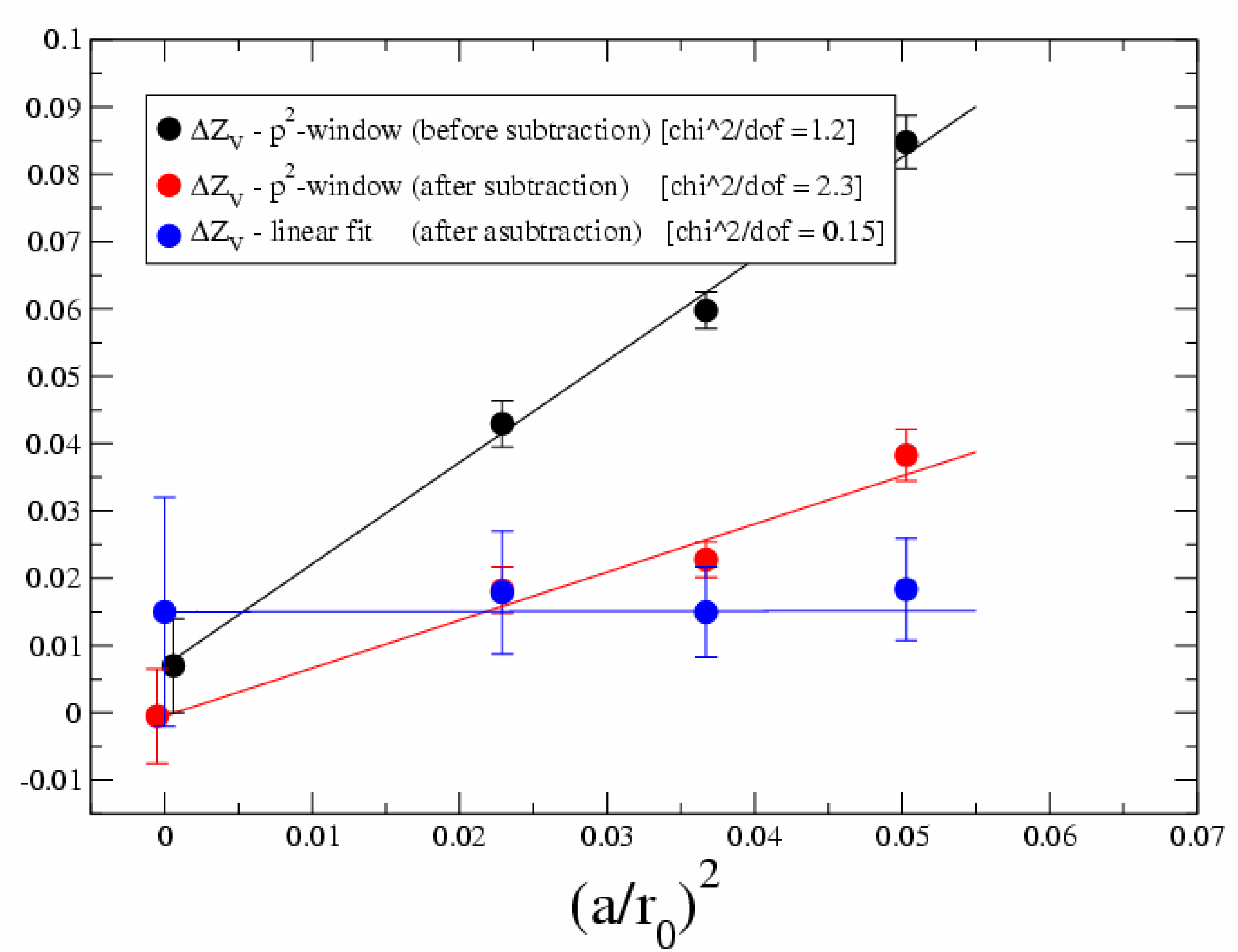
(a) the small difference in the values of Z_V between RI-MOM and the WI method.

(b) the fact that, with this choice, the final results are practically the same either using the g^2(boosted) or the g_0^2 values in the calculation of the $O(a^2g^2)$ terms.

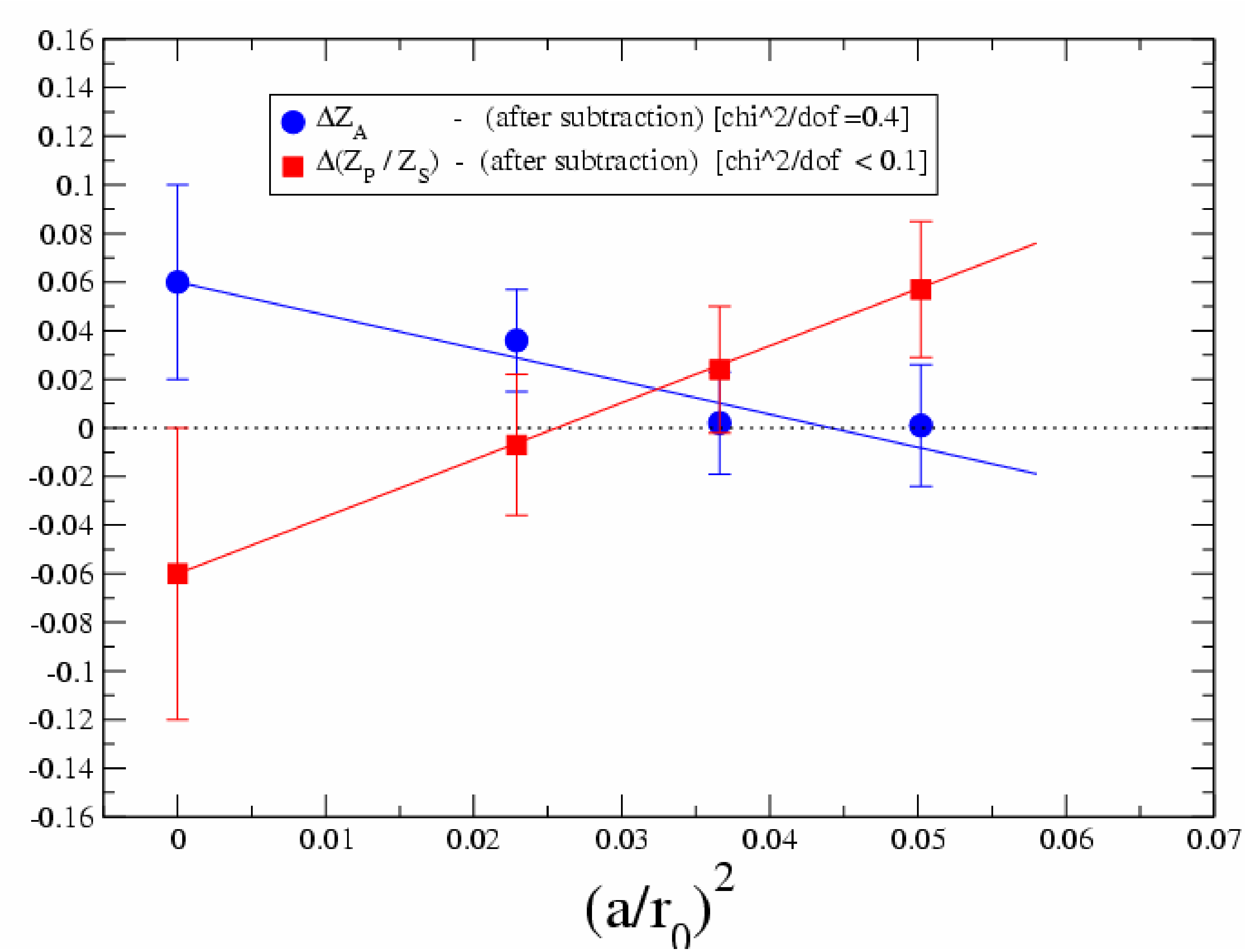
FINAL RI-MOM RESULTS

	β	lin. fit in p^2 w/o sub.	lin. fit in p^2 with sub.	"p^2-window" with sub.	alternative method
Z_V	3.80	0.609(8) [33(4) $\times 10^{-3}$]	0.600(8) [12(4) $\times 10^{-3}$]	0.621(4)	0.5816(2)
	3.90	0.635(7) [26(4) $\times 10^{-3}$]	0.625(7) [8(4) $\times 10^{-3}$]	0.633(3)	0.6103(3)
	4.05	0.670(9) [19(7) $\times 10^{-3}$]	0.663(9) [-2(7) $\times 10^{-3}$]	0.663(3)	0.6451(3)
Z_A	3.80	0.762(11) [11(6) $\times 10^{-3}$]	0.748(11) [-9(6) $\times 10^{-3}$]	0.732(5)	0.747(22)
	3.90	0.759(11) [10(6) $\times 10^{-3}$]	0.745(11) [-9(6) $\times 10^{-3}$]	0.734(4)	0.743(18)
	4.05	0.793(11) [-7(8) $\times 10^{-3}$]	0.784(11) [-26(8) $\times 10^{-3}$]	0.762(6)	0.746(18)
Z_P/Z_S	3.80	0.538(17) [75(8) $\times 10^{-3}$]	0.555(17) [41(8) $\times 10^{-3}$]		0.498(22)
	3.90	0.592(18) [70(8) $\times 10^{-3}$]	0.603(18) [37(8) $\times 10^{-3}$]		0.579(19)
	4.05	0.648(20) [70(16) $\times 10^{-3}$]	0.654(20) [41(16) $\times 10^{-3}$]		0.661(21)
$Z_S(1/a)$	3.80	0.620(15) [17(7) $\times 10^{-3}$]	0.601(16) [33(8) $\times 10^{-3}$]	0.661(6)	
	3.90	0.644(11) [12(6) $\times 10^{-3}$]	0.627(12) [26(6) $\times 10^{-3}$]	0.663(5)	
	4.05	0.634(14) [33(12) $\times 10^{-3}$]	0.623(15) [45(13) $\times 10^{-3}$]	0.662(5)	
$Z_P(1/a)$	3.80	0.322(10) [64(4) $\times 10^{-3}$]	0.318(10) [52(4) $\times 10^{-3}$]		0.300(14)
	3.90	0.378(08) [55(4) $\times 10^{-3}$]	0.373(08) [43(4) $\times 10^{-3}$]		0.363(13)
	4.05	0.414(11) [67(7) $\times 10^{-3}$]	0.411(11) [55(7) $\times 10^{-3}$]		0.412(13)
$Z_T(1/a)$	3.80	0.757(10) [12(5) $\times 10^{-3}$]	0.735(10) [-3(5) $\times 10^{-3}$]	0.730(5)	
	3.90	0.752(09) [13(5) $\times 10^{-3}$]	0.731(09) [-2(5) $\times 10^{-3}$]	0.730(3)	
	4.05	0.794(13) [-7(9) $\times 10^{-3}$]	0.779(12) [-23(9) $\times 10^{-3}$]	0.759(6)	
$Z_q(1/a)$	3.80	0.768(07) [27(4) $\times 10^{-3}$]	0.753(07) [1(4) $\times 10^{-3}$]	0.755(4)	
	3.90	0.771(07) [24(4) $\times 10^{-3}$]	0.757(07) [0(4) $\times 10^{-3}$]	0.757(3)	
	4.05	0.797(12) [15(9) $\times 10^{-3}$]	0.787(12) [-10(9) $\times 10^{-3}$]	0.777(5)	

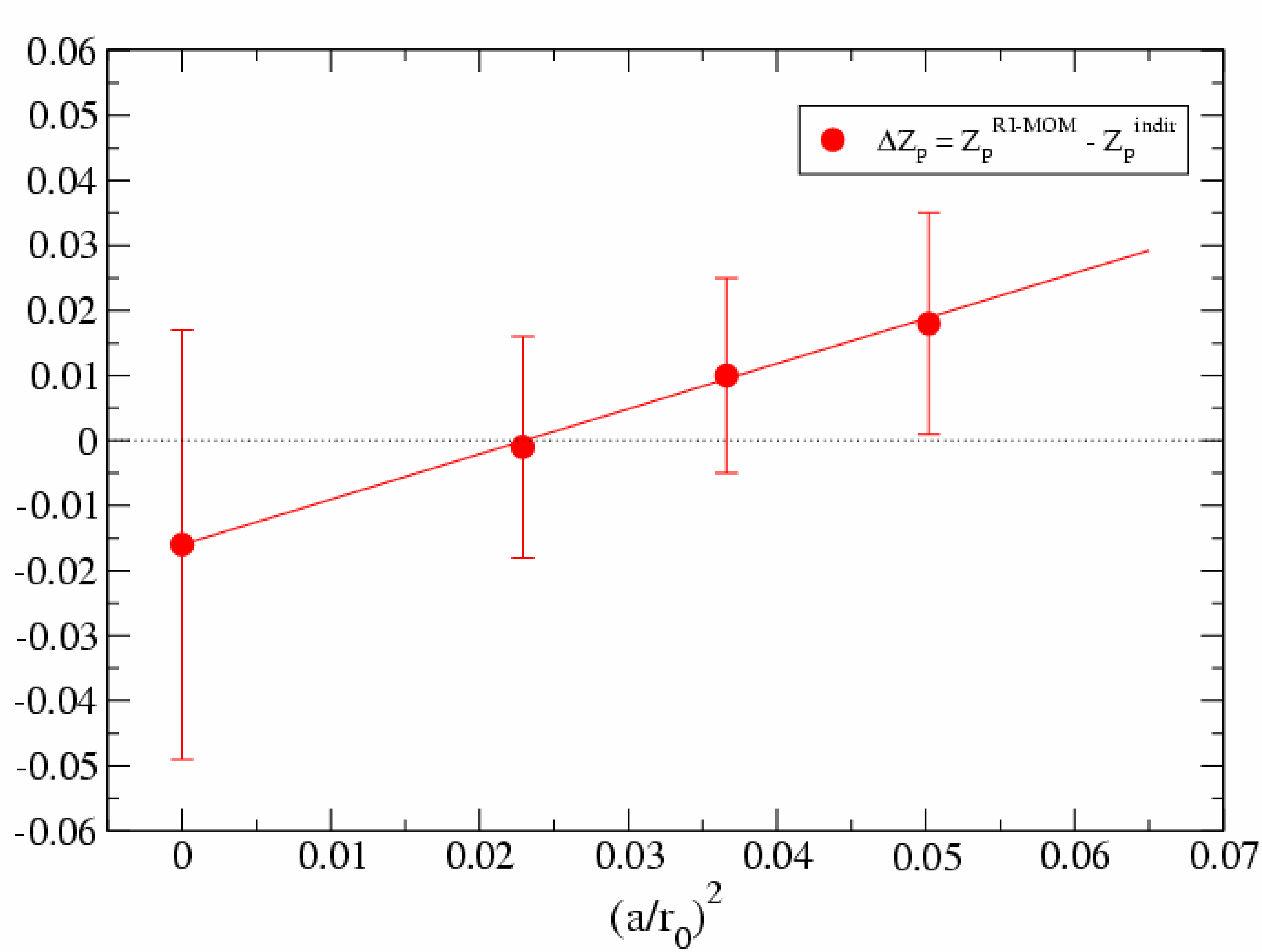
$\Delta Z_V = Z_V^{RI\text{-}MOM} - Z_V^{WI}$



ΔZ_A & $\Delta(Z_P/Z_S)$



$\Delta Z_P = Z_P^{RI\text{-}MOM} - Z_P^{Indirect}$



$N_f = 2+1+1$: RCs for the bilinear operators

- New (special) production runs with $N_f=4$.
- Relax the accuracy of am_{pcac} ~ 0.005-0.010; from the $N_f=0$ experience we have

  $dZ/d(am)_{pcac} < 1$ (***Becirevic, Gimenez, Lubicz, Martinelli, Papinutto, Reyes, JHEP 2004***)

  Hence we have good reasons to expect the same in the unquenched case. Therefore, the final error of the Z's, which is of order of 1%, will not be substantially affected.
- Plans for the near future are for the calculation of the RCs at**:**

  β = 1.90

  L=24, **T**=48

  (the choice of the temporal extension reflects the need to be sure about the pion state isolation and therefore get a safer estimate for am_{pcac})

  $a\mu_{sea}$ = 0.0060, 0.0080, 0.0100, [or 0.0120]

  $a\mu_{val}$ = 0.0060, 0.0080, 0.0100, 0.0120, 0.0140, 0.0160 …

Next run at…

Thermalisation is already under way. This **$a\mu_{sea}$** value will serve for a useful comparison, since for it we already have (massive-scheme) estimates of the RCs calculated in the $N_f=2+1+1$ theory.