Chirally rotated Schrödinger Functional: first checks at tree-level of PT

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- **Twisted mass lattice QCD simulations for $N_f = 2 + 1 + 1$:**
  - want mass independent non-perturbative renormalisation
  - need massless renormalisation scheme
  - keep automatic $\mathrm{O}(a)$ improvement
    (R. Frezzotti and G.C. Rossi, hep-lat/0306014)

The SF is the (gauge invariant) euclidean partition function:

(M. Lüscher et al., hep-lat/9207009), (S. Sint, hep-lat/9312079)

$$\mathcal{Z}[\bar{\rho}', \rho', C'; \bar{\rho}, \rho, C] = \int D[U]D[\bar{\psi}]D[\psi]\, e^{-S[U,\bar{\psi},\psi]}$$

- finite space-time volume: $V = L^3 \times T$
  - ★ can use $\frac{1}{L}$ as renormalisation scale
  - ★ finite size techniques: non-perturbative running with L
- boundary conditions:
  - ★ periodic in spatial directions
  - ★ Dirichlet in time direction
    $\Longrightarrow$ non-zero bound in the spectrum of the Dirac operator $(1/2T)$
- lattice regulator (cutoff dependence a):
  - ★ non-zero bound $\Longrightarrow$ allows lattice simulations at the chiral point
    $\Longrightarrow$ non-perturbative and massless renormalisation of QCD
  - ★ Independently on lattice action: $\mathrm{O}(a)$ effects from the boundaries

- **Wilson-type fermions and Schrödinger Functional b.c:**

  Incompatible with bulk **automatic** $\mathrm{O}(a)$ **improvement**

  - need bulk improvement counterterms even for:
    - *twisted mass Wilson fermions at maximal twist*

- **Let's try with a chiral rotation of the standard SF formulation**

  (S. Sint, hep-lat/0511034)

- Standard SF boundary conditions for fermion fields:

  (S. Sint, hep-lat/9312079), (M. Lüscher, hep-lat/0603029)

$$P_+\psi(x)|_{x_0=0} = 0 \qquad P_-\psi(x)|_{x_0=T} = 0$$

$$P_\pm = \frac{1}{2}\left(\mathbb{1} \pm \gamma_0\right)$$

- Chiral rotation of the the quark fields (with maximal twist):

$$\psi(x) \to e^{i\frac{\alpha}{2}\gamma_5\tau^3}\,\psi(x) \qquad \bar{\psi}(x) \to \bar{\psi}(x)\,e^{i\frac{\alpha}{2}\gamma_5\tau^3} \quad ; \quad \alpha = \pi/2$$

- Chirally rotated SF boundary conditions

  (S. Sint, hep-lat/0511034), (S. Sint, talk at Lattice 2008)

$$Q_+\psi(x)|_{x_0=0} = 0 \qquad Q_-\psi(x)|_{x_0=T} = 0$$

$$Q_\pm = \frac{1}{2}\left(\mathbb{1} \pm i\gamma_0\gamma_5\tau^3\right)$$

- Fermion lattice action of a theory with boundaries:
  - need to define the lattice action near the desired time boundaries
  - use orbifold techniques (Y. Taniguchi, hep-lat/0412024)

- Get: desired boundary conditions at tree-level up to $\mathrm{O}(a)$ effects

(S. Sint, private comunication)

$$Q_+(1 - \frac{1}{2}a\partial_0^*)\psi(x)|_{x_0=0} = 0 \qquad Q_-(1 + \frac{1}{2}a\partial_0)\psi(x)|_{x_0=T} = 0$$

Relevant (d=3) boundary operator

- $\gamma_5\tau^1$-odd: breaking of flavour and parity symmetries
- bulk $\mathrm{O}(a)$ effects $\Leftrightarrow$ loose bulk automatic $\mathrm{O}(a)$ improvement
- add $d=3$ finite boundary counterterm with coefficient z_f

$$\delta S = (z_f - 1)a^3 \sum_{\vec{x}} \left(\bar{\psi}\psi|_{x_0=0} + \bar{\psi}\psi|_{x_0=T}\right)$$

- need to **tune** z_f to restore the symmeries

  $\Longrightarrow$ bulk automatic $\mathrm{O}(a)$ improvement
- **non-perturbative tuning**
  - massless scheme: $m_0 \to m_c$
  - bulk improvement: $z_f \to z_f^*$

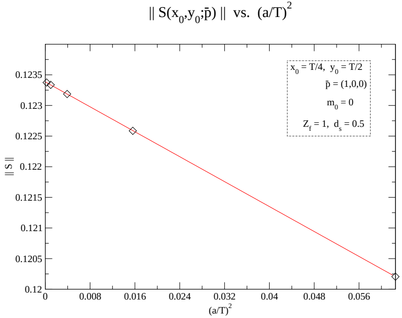
- **Irrelevant (d=4) boundary operator**
  - boundary $\mathrm{O}(a)$ effects
  - add $d = 4$ boundary counterterm with coefficient d_s

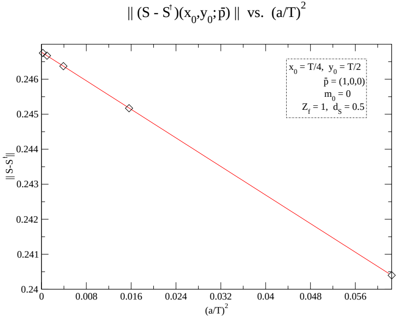
$$\delta S = a(d_s - 1)a^3 \sum_{\vec{x}} \left(\bar{\psi}\gamma_k D_k \psi|_{x_0=0} + \bar{\psi}\gamma_k D_k \psi|_{x_0=T} \right)$$

  - tune d_s to cancel the $\mathrm{O}(a)$ boundary effects
  - **perturbative tuning**
    - ★ first need to find the tree-level value: d_s^0 $(d_s = d_s^0 + d_s^1 \mathrm{O}(g_0^2))$

- **Theory with χSF b.c. is automatic $\mathbf{O}(a)$ improved at tree-level if:**
  - $m_0 = 0$
  - $\mathcal{Z}_f = 1$
  - $d_s^0 = \frac{1}{2}$

- Example:





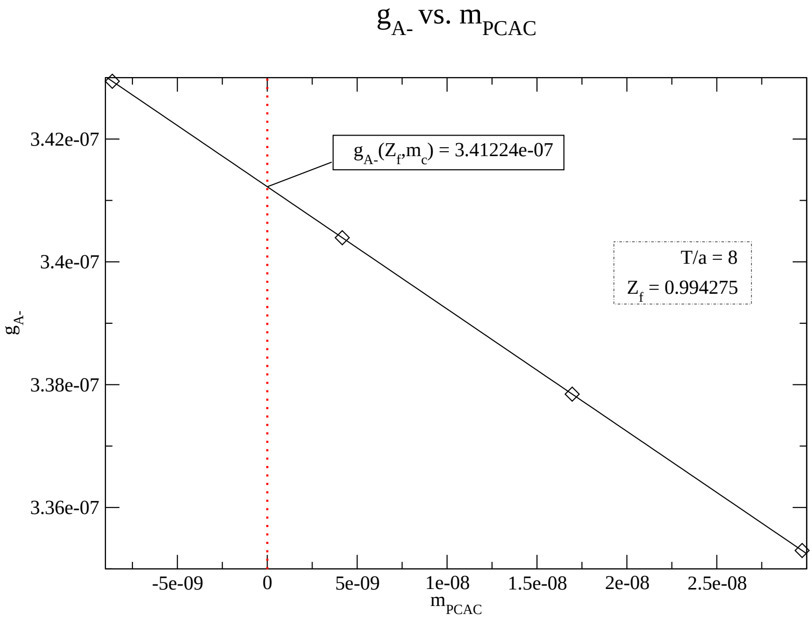
- Definitions:

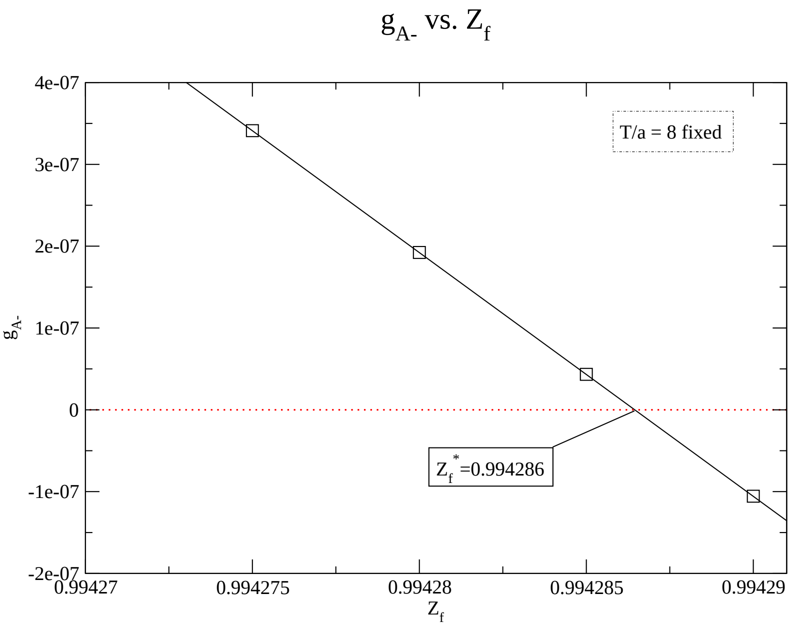
$$g_A^{ab}(x_0)_\pm = - < A_0^a(x)\mathcal{Q}_\pm^b > \qquad g_P^{ab}(x_0)_\pm = - < P^a(x)\mathcal{Q}_\pm^b >$$

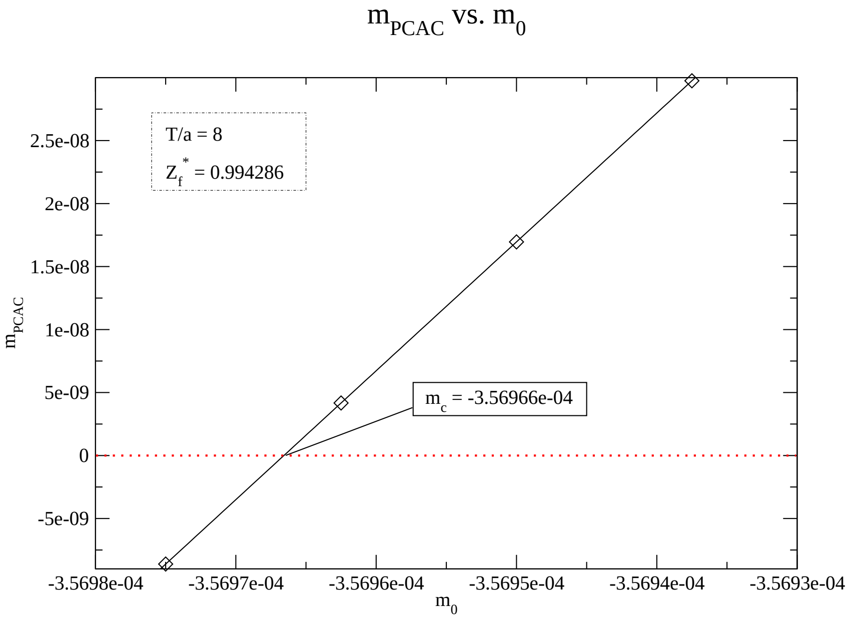
$$\mathcal{Q}_\pm^a = a^6 \sum_{\vec{y},\vec{z}} \bar{\zeta}(\vec{y})\gamma_5 \frac{1}{2}\tau^a Q_\pm \zeta(\vec{z})\, e^{i\vec{p}(\vec{y}-\vec{z})}$$

$$\zeta(\vec{x}) = \psi(x)|_{x_0=0} \qquad \bar{\zeta}(\vec{x}) = \bar{\psi}(x)|_{x_0=0}$$

- Set: $d_s = 1/2$
- Tuning of m_0: $m_{PCAC,-} \equiv \dfrac{\partial_0 g_A(T/2)_-}{2g_P(T/2)_-} = 0$
- Tuning of Z_f: $g_A(T/2)_- = 0$


g_A- vs. m_PCAC
g_A-(Z_f,m_c) = 3.41224e-07
T/a = 8
Z_f = 0.994275
3.42e-07
3.4e-07
3.38e-07
3.36e-07
g_A-
-5e-09
0
5e-09
1e-08
1.5e-08
2e-08
2.5e-08
m_PCAC


g_A- vs. Z_f
T/a = 8 fixed
Z_f* =0.994286
4e-07
3e-07
2e-07
1e-07
0
-1e-07
-2e-07
g_A-
0.99427
0.994275
0.99428
0.994285
0.99429
Z_f


m_PCAC vs. m_0
T/a = 8
Z_f^* = 0.994286
m_c = -3.56966e-04
2.5e-08
2e-08
1.5e-08
1e-08
5e-09
0
-5e-09
m_PCAC
-3.5698e-04
-3.5697e-04
-3.5696e-04
-3.5695e-04
-3.5694e-04
-3.5693e-04
m_0

- From the study of the cutoff effects:
  - At tree-level the theory is automatic $\mathrm{O}(a)$ improved if
    - ★ $m_0 = 0$
    - ★ $\mathcal{Z}_f = 1$
    - ★ $d_s = 1/2$
- From the study of the tuning:
  - Fix T/a, choose Z_f, take 3-4 values of m_0:
    plot g_{A-} vs. m_{PCAC}, interpolate to $m_{PCAC} = 0 \Rightarrow g_{A-}(Z_f, m_{PCAC} = 0)$
  - Repeat for other 2-3 values of Z_f
  - Plot g_{A-} vs. Z_f: interpolate to $g_{A-} = 0 \Rightarrow Z_f^*(T/a)$
  - Take 3-4 values of m_0 and calculate m_{PCAC} for the Z_f^*:
    plot m_{PCAC} vs. m_0 and interpolate to $m_{PCAC} = 0 \Rightarrow m_c$

- **Also done:**
  - implemented Iwasaki action with SF b.c.
  - implemented definition of the running coupling
  - tree-level code as a starting point for perturbation theory
- **On going work:**
  - Modifying existing HMC to have:
    - ★ quenched option (independently on b.c.)
    - ★ HMC with SF
- **Short-time goal:**
  - first quenched studies
  - lattice perturbation theory