Scaling and χPT fits at $N_\mathrm{f} = 2$
&
Preminimary scaling and mixed action at $N_\mathrm{f} = 2 + 1 + 1$

F. Farchioni, G. Herdoiza, C. Urbach *et al.*

ETM Collaboration

ETMC meeting
Autrans, 18-20 April 2009

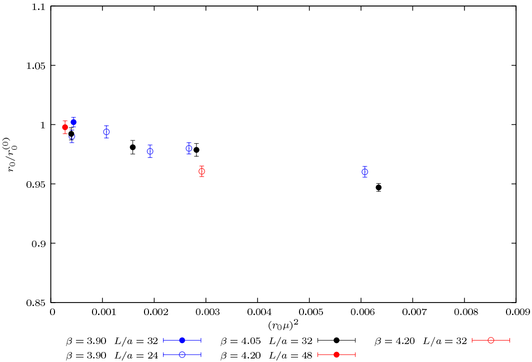
Outline

- $N_\mathrm{f} = 2$
  - scaling
  - chiral fits : updated analysis by Carsten
- $N_\mathrm{f} = 2 + 1 + 1$
  - scaling
  - mixed action : analysis by Federico

$N_f = 2$

Ensemble	β	a (fm)	$L^3 \cdot T$	L (fm)	$a\mu$	m_{PS} (MeV)
"D_1"	4.20	~ 0.053	$48^3 \cdot 96$	2.5	0.0020	~ 280
"D_2"			$32^3 \cdot 64$	1.7	0.0065	~ 490
"D_3"			$24^3 \cdot 48$	1.3	0.0020	~ 350
C_1	4.05	~ 0.066	$32^3 \cdot 64$	2.2	0.0030	~ 300
C_2					0.0060	~ 420
C_3					0.0080	~ 480
C_4					0.0120	~ 600
C_5			$24^3 \cdot 48$	1.6	0.0060	~ 420
C_6			$20^3 \cdot 48$	1.3	0.0060	~ 420
B_1	3.90	~ 0.084	$24^3 \cdot 48$	2.1	0.0040	~ 300
B_2					0.0064	~ 380
B_3					0.0085	~ 440
B_4					0.0100	~ 480
B_5					0.0150	~ 590
B_7			$32^3 \cdot 64$	2.8	0.0030	~ 265
B_6					0.0040	~ 300
A_1	3.80	~ 0.100	$24^3 \cdot 48$	2.4	0.0060	~ 360
A_2					0.0080	~ 410
A_3					0.0110	~ 480
A_4					0.0165	~ 580
A_5			$20^3 \cdot 48$	2.0	0.0060	~ 360

static inter-quark force : r_0 vs. μ^2



Note : $Z_P = 1$ in figure (not in fit)

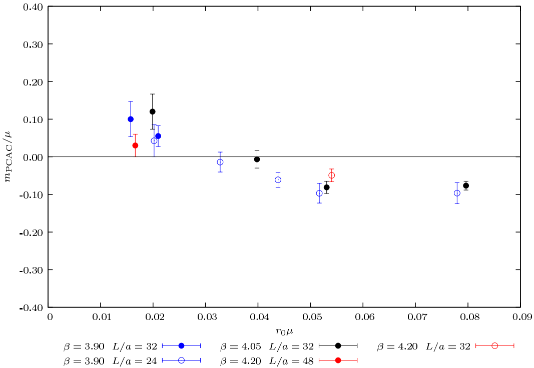
$\Rightarrow$ at $\mu \to 0$:

[PRELIMINARY]

$\beta = 4.20 : \ r_0/a = 8.31(6)$
$\beta = 4.05 : \ r_0/a = 6.64(2)$
$\beta = 3.90 : \ r_0/a = 5.25(2)$
$\beta = 3.80 : \ r_0/a = 4.43(2)$

Tuning to maximal twist

$$m_R \propto m_{\rm PCAC} = \frac{\sum_{\mathbf{x}} \langle \partial_0 A_0^a(\mathbf{x},t) P^a(0)\rangle}{2\sum_{\mathbf{x}} \langle P^a(\mathbf{x},t) P^a(0)\rangle} = 0 \qquad \text{at } \mu = \mu_{\rm LOW}$$



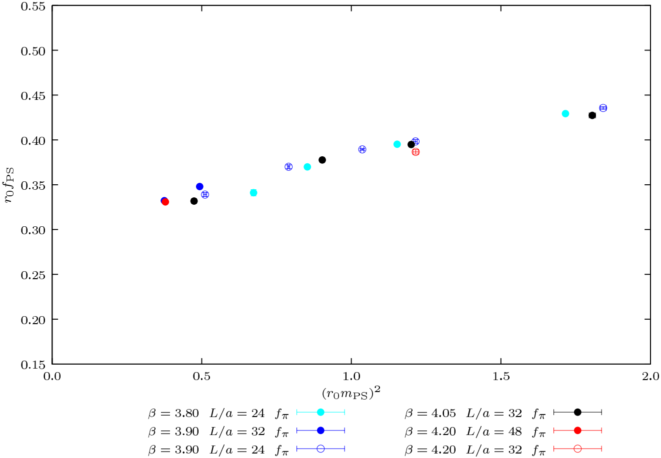
Note : $Z_P = Z_A = 1$ in figure

$$\mu > a^2 \Lambda_{\rm QCD}^3 \qquad\qquad m_{\rm PCAC} < a\mu\Lambda_{\rm QCD}$$

Scaling of $f_{\rm PS}$

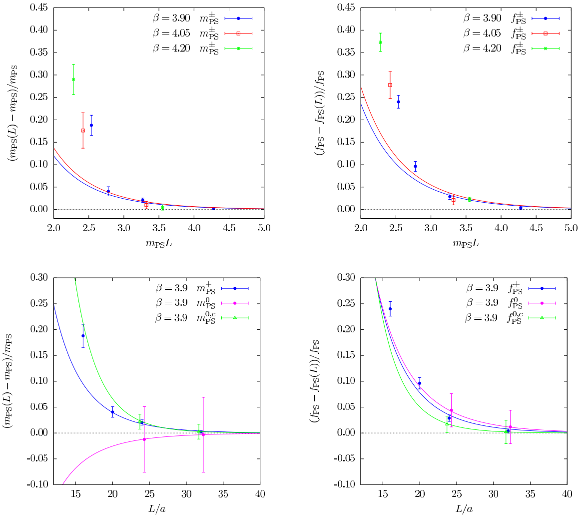
$\beta = 4.20,\ 4.05,\ 3.90,\ 3.80$

$$f_{\rm PS} = \frac{2\mu}{m_{\rm PS}^2} |\langle 0|P^1(0)|\pi^{\pm}\rangle|$$



Finite size effects

- relative deviation : $R_O = (O_\infty - O_L)/O_\infty$



Finite size effects

Comparison of lattice data at several volumes to :

- NLO χPT : GL [Gasser, Leutwyler, 1987, 1988]
- resummed Lüscher formula : CDH [Colangelo, Dürr, Haefeli, 2005]
- relative deviation : $R_O = (O_\infty - O_L)/O_\infty$

	a (fm)	$m_{\rm PS}L_1 \to m_{\rm PS}L_2$	meas.(%) $(L_1 \to L_2)$	GL (%) $(L_1 \to \infty)$	CDH (%) $(L_1 \to \infty)$
$m_{\rm PS}$	0.09	$3.3 \to 4.3$	-1.8	-0.6	-1.2
$f_{\rm PS}$	0.09	$3.3 \to 4.3$	$+2.6$	$+2.6$	$+2.6$
$m_{\rm PS}$	0.07	$3.0 \to 4.6$	-6.1	-1.9	-6.3
$f_{\rm PS}$	0.07	$3.0 \to 4.6$	$+10.7$	$+7.0$	$+9.0$
$m_{\rm PS}$	0.07	$3.5 \to 4.6$	-1.1	-0.8	-2.1
$f_{\rm PS}$	0.07	$3.5 \to 4.6$	$+1.9$	$+3.3$	$+3.5$

- non negligeable FSE since relative stat. error : $\sim$ 1% on $m_{\rm PS}$ and $f_{\rm PS}$
- CDH describes data in general better than GL but needs more parameters

Chiral perturbation theory : $f_\pi,\ m_\pi$

- Use of χPT to describe the dependence on :
  - the quark mass μ
  - finite spatial size L

- Simultaneous fit to $N_f = 2\ \ \chi$PT

$$m^2_{\rm PS}(L) = \chi_\mu \left[1 + \frac{1}{2}\xi\tilde{g}_1(\lambda)\right]^2 \left[1 + \xi\ln(\chi_\mu/\Lambda_3^2) + T_m^{\rm NNLO} + a^2 D_m\right]$$

$$f_{\rm PS}(L) = f_0\ [1 - 2\xi\tilde{g}_1(\lambda)] \left[1 - 2\xi\ln(\chi_\mu/\Lambda_4^2) + T_f^{\rm NNLO} + a^2 D_f\right]$$

$$r_0/a(\mu_q) = r_0/a(\mu_q = 0) + D_{r_0}(a\mu_q)^2$$

where $\chi_\mu = 2\widehat{B}_0\mu_R\,, \qquad \xi = \chi_\mu/(4\pi f_0)^2\,, \qquad \lambda = \chi_\mu L\,, \qquad f_0 = \sqrt{2}F_0$

- data : $af_{\rm PS},\ \ am_{\rm PS},\ r_0/a$ and $Z_{\rm P}$
- parameters : $r_0 f_0,\ r_0 B_0,\ r_0\Lambda_3,\ r_0\Lambda_4,\ \{r_0/a(\mu = 0)\}_\beta,\ \{Z_{\rm P}\}_\beta,\ D_{m_{\rm PS}},\ D_{f_{\rm PS}}$
- Finite size corrections : (CDH : Colangelo *et al.*, 2005)
- Mass dependence : NLO and NNLO (extra parameters : $r_0\Lambda_{1,2}, k_M, k_F$)
- Include $O(a^2)$ terms in the fits

χPT combined fits

Updated analysis by Carsten

- set of fits
  - over different ensembles : A_i, B_j, C_k $\rightsquigarrow$ 16 data-sets
  - different fit ansatz : NLO, NNLO, $O(a^2)$ $\rightsquigarrow$ 4 fit ansatz

- analysis
  - keep only the physically relevant data-sets/fits
  - use χ^2 distribution to quantify the goodness of the fit (CL) : weighted average
  - the distribution of fits $\rightsquigarrow$ systematic error

χPT combined fits

- data-sets

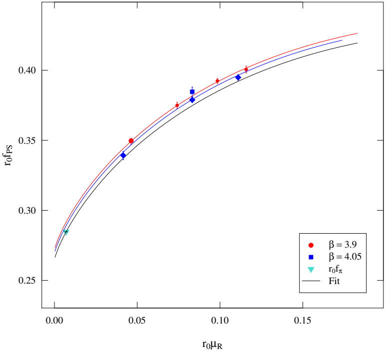
set	A_2	A_3	A_4	B_1	B_2	B_3	B_4	B_5	B_6	B_7	C_1	C_2	C_3	C_4	C_5
1				x	x	x	x		x		x	x	x		x
2				x	x	x	x	x	x		x	x	x	x	x
3				x	x	x	x		x	x	x	x	x		x
4				x	x	x	x	x	x	x	x	x	x	x	x
5				x	x	x			x	x	x	x			x
6					x	x	x		x		x	x	x		
7					x	x	x	x	x		x	x	x	x	
8					x	x	x		x	x	x	x	x		
9	x	x		x	x	x	x		x		x	x	x		x
10	x	x	x	x	x	x	x	x	x		x	x	x	x	x
11	x	x		x	x	x	x		x	x	x	x	x		x
12	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
13	x			x	x	x			x	x	x	x			x
14	x	x			x	x	x		x		x	x	x		
15	x	x	x		x	x	x	x	x		x	x	x	x	
16	x	x			x	x	x		x	x	x	x	x		

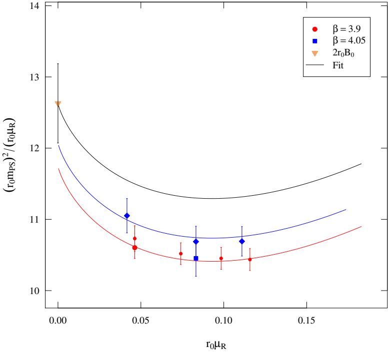
- fit ansatz :
  1. Fit A: NLO continuum χPT, $T_{m,f}^{\mathrm{NNLO}} \equiv 0$, $D_{m_{\mathrm{PS}},f_{\mathrm{PS}}} \equiv 0$, priors for $r_0\Lambda_{1,2}$
  2. Fit B: NLO continuum χPT, $T_{m,f}^{\mathrm{NNLO}} \equiv 0$, $D_{m_{\mathrm{PS}},f_{\mathrm{PS}}}$ fitted, priors for $r_0\Lambda_{1,2}$
  3. Fit C: NNLO continuum χPT, $D_{m_{\mathrm{PS}},f_{\mathrm{PS}}} \equiv 0$, priors for $r_0\Lambda_{1,2}$ and $k_{M,F}$
  4. Fit D: NNLO continuum χPT, $D_{m_{\mathrm{PS}},f_{\mathrm{PS}}}$ fitted, priors for $r_0\Lambda_{1,2}$ and $k_{M,F}$

χPT fits

fit B & data-set 1

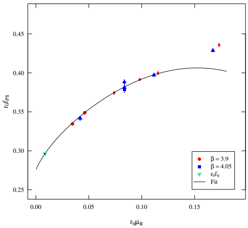
$\chi^2/dof = 19.6/17$

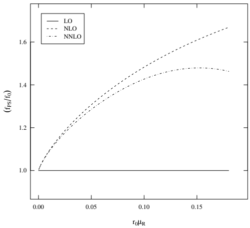




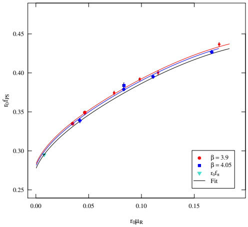
χPT fits

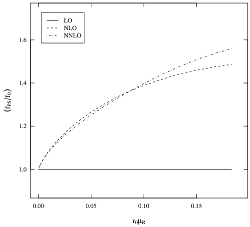
fit C & data-set 1 $\chi^2/dof = 23.7/19$





fit D & data-set 4 $\chi^2/dof = 30.9/23$





χPT fits : NLO and NNLO

- fit of f_{PS} and m_{PS}
- mass dependence : continuum NLO and NNLO
- volume dependence : CDH

combining $a = 0.07, 0.09$ fm
higher masses ($m_{\mathrm{PS}} \sim 600$ MeV) not included

[Colangelo et al., 2005]

	NLO	NNLO
$\bar{l}_3$	3.38(7)	3.52(65)
$\bar{l}_4$	4.62(3)	5.02(30)
$\widehat{B}_0$ [GeV]	2.55(4)	2.51(8)
f_0 [MeV]	121.62(7)	121.15(56)
r_0 [fm]	0.449(3)	0.446(6)
$\bar{l}_1$	$-0.55(58)$	$-0.60(59)$
$\bar{l}_2$	4.31(10)	4.33(10)
k_M	–	$-0.55(99)$
k_F	–	1.7(1.1)
χ^2/dof	30.9/21	23.8/21

Input some knowledge on $\bar{l}_{1,2}$, k_M and k_F in the fit:
$\bar{l}_1 = -0.4 \pm 0.6 \qquad \bar{l}_2 = 4.3 \pm 0.1 \qquad k_M = k_F = 0 \pm 10$

χPT fits : discretization effects

$$r_0 f_{\mathrm{PS}} = r_0 f_0 \left[1 - 2\xi \log(\chi_\mu/\Lambda_4^2) + (a/r_0)^2 D_f\right] K_f^{\mathrm{CDH}}(L)$$

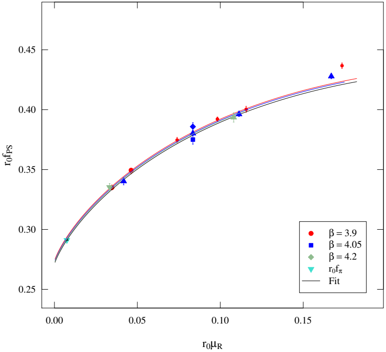
- fit of f_{PS} and m_{PS} combining $a_1 = 0.05$, $a_2 = 0.07$, $a_3 = 0.09$ fm [PRELIMINARY]
- mass dependence : NLO higher masses ($m_{\mathrm{PS}} \sim 600$ MeV) not included
- volume dependence : CDH

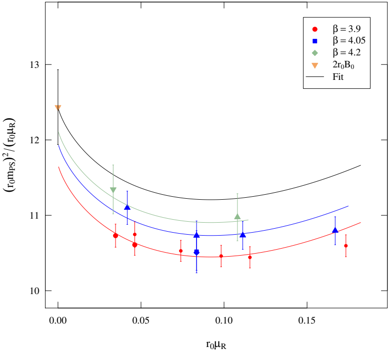
	$D_{m,f} = 0$	fit $D_{m,f}$	fit $D_{m,f}$
a_i	$a_{2,3}$	$a_{2,3}$	$a_{1,2,3}$
$\bar{l}_3$	3.38(7)	3.51(7)	3.47(6)
$\bar{l}_4$	4.62(3)	4.63(3)	4.59(3)
$\widehat{B}_0$ [GeV]	2.55(4)	2.89(14)	2.79(12)
f_0 [MeV]	121.62(7)	121.58(7)	121.65(6)
r_0 [fm]	0.449(3)	0.429(9)	0.439(6)
χ^2/dof	30.8/21	23.2/19	26.7/23

- values of $D_{m,f}$: $D_m = -1.08(95)$; $D_f = 0.70(56)$
- $Z_P(\beta = 4.20)$ is fitted with prior $0.5 \pm 0.1 \rightsquigarrow Z_P = 0.495(13)$

χPT fits : with $\beta = 4.20$

$\chi^2/dof = 26.7/23$





Results : LEC, m_q, $\langle \bar{q}q \rangle$, ...

Estimate systematic effects

- discretization
- NLO/NNLO
- FSE

[PRELIMINARY]

$\bar{l}_3$	3.49(19)
$\bar{l}_4$	4.57(15)
$\widehat{B}_0$ [GeV]	2.77(19)
f_0 [MeV]	121.8(5)
$(-\langle \bar{q}q \rangle)^{1/3}$ [MeV]	274(6)
$m_{u,d}$ [MeV]	3.37(23)
r_0 [fm]	0.433(14)

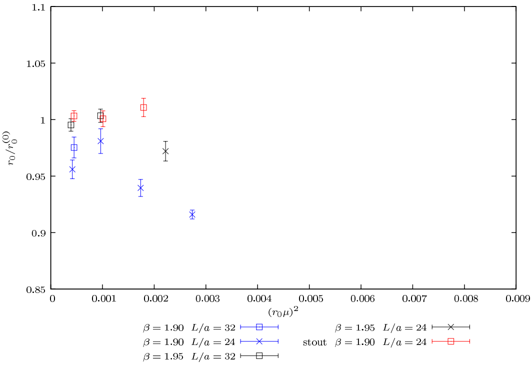
B_0, $\langle \bar{q}q \rangle$ and $m_{u,d}$ are given in $\overline{\mathrm{MS}}$ at 2 GeV

$N_f = 2 + 1 + 1$

β	$L^3 \cdot T$	$a\mu_l$	$a\mu_\sigma$	$a\mu_\delta$
1.90	$32^3 \cdot 64$	0.0030	0.150	0.190
		0.0040		
		0.0050		
	$24^3 \cdot 48$	0.0040		
		0.0060		
		0.0080		
		0.0100		
	$20^3 \cdot 48$	0.0040		
	$24^3 \cdot 48$	0.0100	0.150	0.197
1.95	$32^3 \cdot 64$	0.0035	0.135	0.170
		0.0055	0.135	0.170
		0.0075	0.135	0.170
	$24^3 \cdot 48$	0.0085	0.135	0.170
stout 1.90	$24^3 \cdot 48$	0.0040	0.170	0.185
		0.0060		
		0.0080		

Masses: $m_\pi \in [280; 600]\ \mathrm{MeV}$ $\quad m_K \gtrsim m_K^{\mathrm{exp.}}$ $\quad m_c \gtrsim 10 m_s$
$\beta = 1.90:\ a \approx 0.082\ \mathrm{fm}$; $\quad m_{\mathrm{PS}} \times L \gtrsim 3.5$; $\quad L \approx 2.0$ and $2.6\ \mathrm{fm}$

static inter-quark force : r_0 vs. μ^2

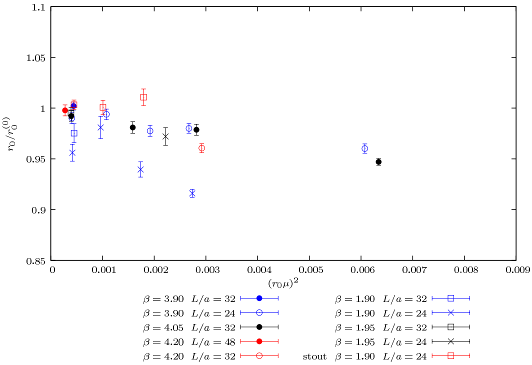


Note : $Z_P = 1$ in figure

$\Rightarrow$ at $\mu \longrightarrow 0$:

$\beta = 1.95$: $r_0/a \approx 5.64$
$\beta = 1.90$: $r_0/a \approx 5.31$
stout $\beta = 1.90$: $r_0/a \approx 5.30$

static inter-quark force : r_0 vs. μ^2



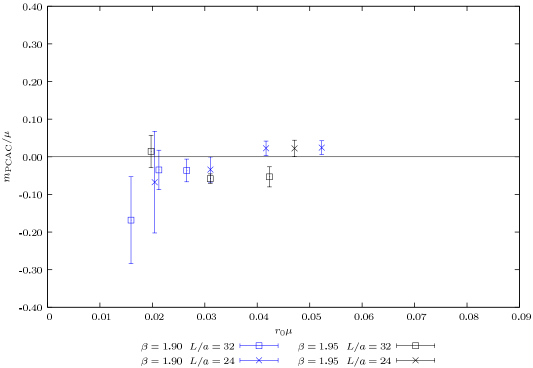
Note : $Z_P = 1$ in figure

$\Rightarrow$ at $\mu \to 0$:

$\beta = 1.95$: $r_0/a \approx 5.64$
$\beta = 1.90$: $r_0/a \approx 5.31$
stout $\beta = 1.90$: $r_0/a \approx 5.30$

Tuning to maximal twist

$$m_R \propto m_{\rm PCAC} = \frac{\sum_{\mathbf{x}} \langle \partial_0 A_0^a(\mathbf{x}, t) P^a(0) \rangle}{2 \sum_{\mathbf{x}} \langle P^a(\mathbf{x}, t) P^a(0) \rangle} = 0 \qquad \text{at} \;\; \mu = \mu_{\rm LOW}$$

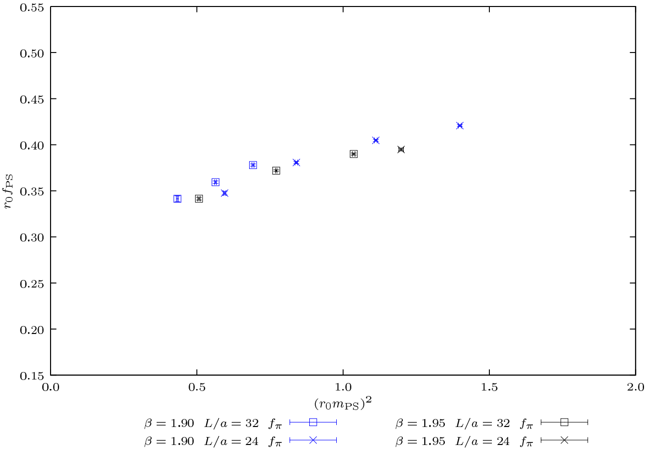


Note : $Z_P = Z_A = 1$ in figure

$\mu > a^2 \Lambda_{\rm QCD}^3$ $\qquad$ $m_{\rm PCAC} < a \mu \Lambda_{\rm QCD}$

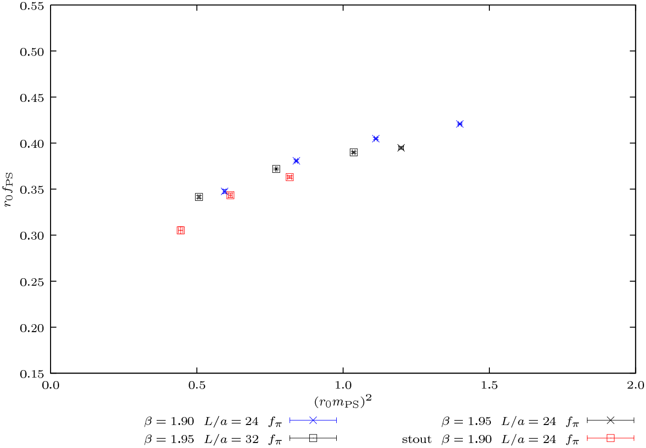
Scaling of $f_{\rm PS}$

$$f_{\rm PS} = \frac{2\mu}{m_{\rm PS}^2} |\langle 0|P^1(0)|\pi^\pm\rangle|$$



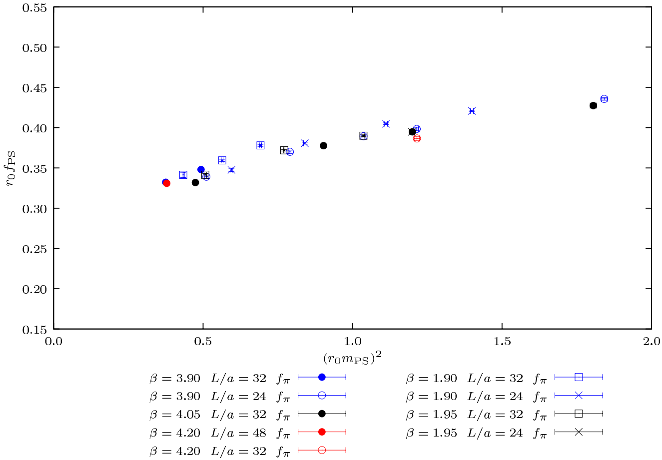
Scaling of f_{PS}

$$f_{\mathrm{PS}} = \frac{2\mu}{m_{\mathrm{PS}}^2} |\langle 0|P^1(0)|\pi^{\pm}\rangle|$$



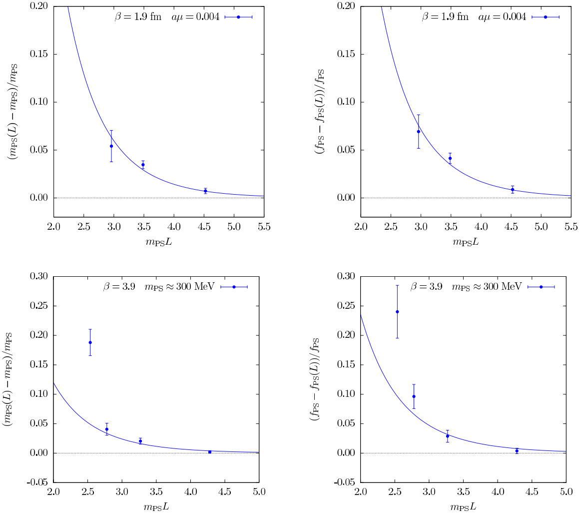
Scaling of $f_{\rm PS}$

$$f_{\rm PS} = \frac{2\mu}{m_{\rm PS}^2} |\langle 0|P^1(0)|\pi^\pm\rangle|$$

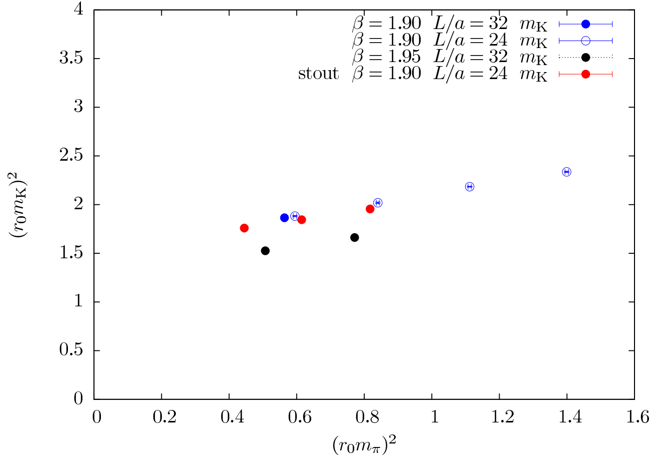


Finite size effects

- relative deviation : $R_O = (O_\infty - O_L)/O_\infty$



Kaon mass



Note: using $r_0 = 0.44$ fm : $r_0 m_K \approx 1.1$

Action (unitary setup)

Combination of :

- Twisted mass lattice action for the light mass degenerate (u-d)-doublet : $N_\mathrm{f} = 2$

$$S_\mathrm{tm} = a^4 \sum_x \{\bar{\chi}_\ell(x)\,[D_W[U] + m_{0\ell} + i\mu_\ell\gamma_5\tau_3]\,\chi_\ell(x)\} \;,$$

- Heavy mass non-degenerate (c-s)-doublet term in the action : $N_\mathrm{f} = 1 + 1$

$$\bar{\chi}_h(x)\,[\,D_W[U] + m_{0h} + i\mu_\sigma\gamma_5\tau_1 + \mu_\delta\tau_3\,]\,\chi_h(x)$$

- Mixing in currents: the field transformations imply for the renormalized (physical) bilinears :

$$\hat{P}_{K,x} = \cos\frac{\omega_l}{2}\cos\frac{\omega_h}{2}Z_P P_{K,x} - \sin\frac{\omega_l}{2}\sin\frac{\omega_h}{2}Z_P P_{\breve{D},x} + i\sin\frac{\omega_l}{2}\cos\frac{\omega_h}{2}Z_S S_{\breve{K},x} + i\cos\frac{\omega_l}{2}\sin\frac{\omega_h}{2}Z_S S_{D,x}$$

  where the Kaon- and D-meson-doublet are $K \equiv (K^+ \;\; K^0)$ and $D \equiv (D^0 \;\; D^-)$, respectively

  $K^+ = (\bar{s}u)$, $K^0 = (\bar{s}d)$, ...

- Non-degenerate quark masses :

$$\hat{m}_{c,s} = 1/Z_\mathrm{P}\;\mu_\sigma \pm 1/Z_\mathrm{s}\;\mu_\delta$$

Mixed Action

Valence sector : Notes and analysis by Federico F.

- Twisted mass lattice action for the mass non-degenerate (u-s)-doublet

$$S_{\mathrm{tm}} = a^4 \sum_x \left\{ \bar{\chi}_\ell(x) \left[D_W[U] + m_{0\ell} + \begin{pmatrix} i\mu_\ell\gamma_5 & 0 \\ 0 & -i\mu_s\gamma_5 \end{pmatrix} \right] \chi_\ell(x) \right\} ,$$

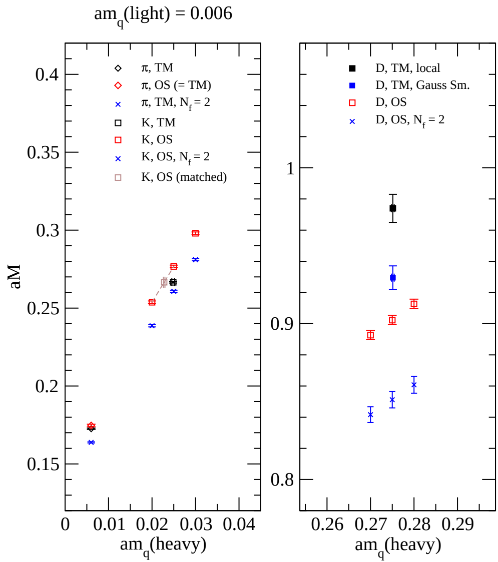
- No flavour mixing
- Matching unitary (U) and mixed action (MA)

$$m_K|_{\mathrm{MA}} = m_K|_{\mathrm{U}} \quad \Rightarrow \quad \begin{cases} \hat{m}_s|_{\mathrm{MA}} \equiv 1/Z_{\mathrm{P}} m_s \;=\; \hat{m}_s|_{\mathrm{U}} \equiv 1/Z_{\mathrm{P}}\,\mu_\sigma \pm 1/Z_{\mathrm{s}}\,\mu_\delta \\ \hat{m}_\ell|_{\mathrm{MA}} \equiv \hat{m}_\ell|_{\mathrm{MA}} \;\rightsquigarrow\; m_\pi|_{\mathrm{MA}} = m_\pi|_U \end{cases}$$

- Estimate of Z_P/Z_S through the matching

Matching : m_K

- $\beta = 1.90$
- $a\mu_{\ell,sea} = 0.0060$
- 500 meas., point-like sources



f_K

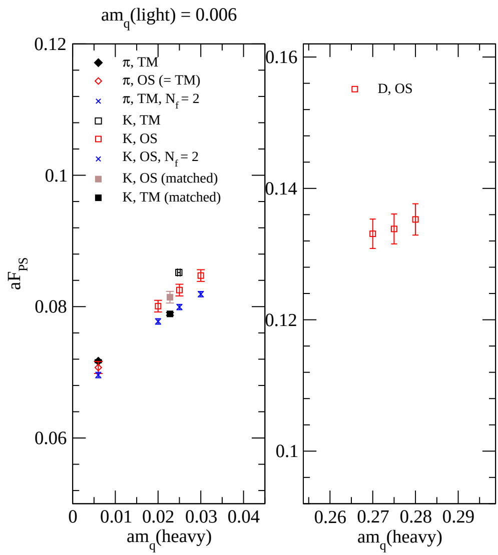
Unitary :

$$f_{\rm PS} = \frac{\mu_\ell + \mu_\sigma - \frac{Z_P}{Z_S}\mu_\delta}{2m_{\rm K}^2} \times$$

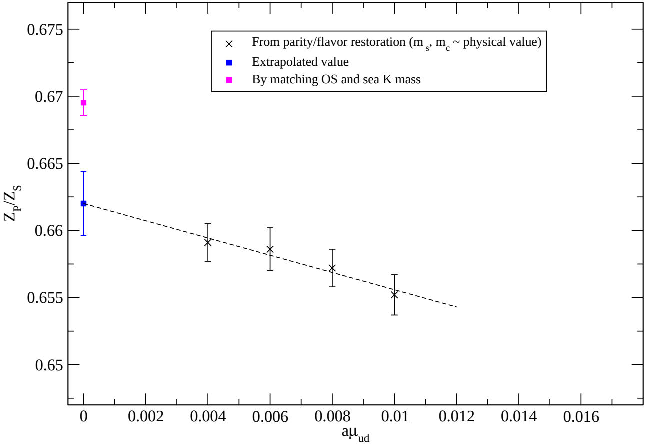
$$|\langle 0|(P_{K^+} - P_{D^0}) + i\frac{Z_P}{Z_S}(S_{K^+} - S_{D^0})|K^+|\rangle|$$

Mixed action :

$$f_{\rm K} = \frac{\mu_\ell + \mu_s}{m_{\rm K}^2}|\langle 0|P_{(\mu_\ell+\mu_s)}|K\rangle|$$



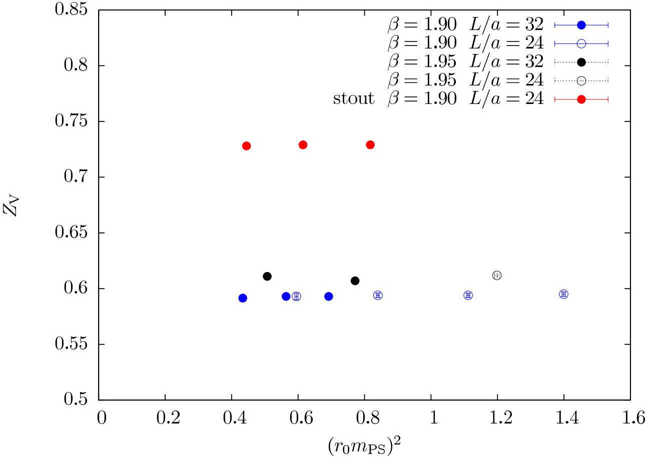
Renormalisation : Z_P/Z_S



- RIMOM at $\beta = 1.9,\ a\mu = 0.0080,\ a\mu_\sigma = 0.15,\ a\mu_\delta = 0.19:\ Z_P/Z_S = 0.589(17)$
- Note : $\beta = 3.9$: $Z_P/Z_S(OS) = 0.579(19)$; $Z_P/Z_S(RIMOM) = 0.603(18)$

Renormalisation : Z_V

$a\mu_\sigma \neq 0$ and $a\mu_\delta \neq 0$



- RIMOM at $\beta = 1.9,\ a\mu = 0.0080,\ a\mu_\sigma = 0.15,\ a\mu_\delta = 0.19:\ Z_V = 0.623(5)$
- Note : $\beta = 3.9$: $Z_V(WI) = 0.6103(3)$; $Z_V(RIMOM) = 0.625(7)$

Conclusions

To be concluded . . .