Lattice and (Super)B Physics

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Outline

- Prologue
  - Super-B factory

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- Plan of action
  - a call for lattice study groups and collaborations

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- Where can the lattice be of help
  - (light and) heavy flavour physics

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◘ Where can the lattice be of help

- (light and) heavy flavour physics

◘ How can we control systematics

- various Wilson fermion/glue options
- lattice spacing
- pion mass

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Where can the lattice be of help

- (light and) heavy flavour physics

How can we control systematics

- various Wilson fermion/glue options
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Conclusions

◘ Prologue

- Super-B factory

♠ The idea of constructing a Super-B factory in the area between Tor Vergata Campus and Frascati INFN Lab’s has been accepted by Italian Authorities

♠ A number of scientific Institutions and individual countries around the world have expressed their interest in this enterprise, among which France, Spain, Russia and USA

♠ RECFA working group at CERN has been put up to study the physics potential and technical feasibility of the proposed high luminosity Super-B facility

♠ Some first amount of money has been allocated to the project for R&D and dedicated computing facilities by local (regional) Authorities

Plan of action

- a call for lattice study groups and collaborations

- Establishing a strategy

  put up lattice study groups
  promote collaborations

  identify research projects for BSM physics
  determine CPU requirements

- Specifically

  identify hadronic matrix elements of interest
  streamline data analysis

  control systematics to the necessary level of accuracy
  devise feasibility studies and simulation projects

Where can the lattice be of help

- e.g. (light and) heavy flavour physics

Hadronic masses

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f_π

f_K B_K

---

f_D f_{Ds}

f_B f_{Bs} B_B B_{Bs}

$N_f = 2$

$N_f = 3$

$N_f = 4$

How can we control systematics

RF & GCR

- various Wilson fermion/glue options
- lattice spacing
- pion mass

“mass” term

- Choice of glue action & c_{sw}
- Wilson fermions

W - pair	$r_1 = r_2$	$\omega = 0$	1×1
tw - pair	$r_1 = -r_2$	$\omega = \pi/2$	$\gamma_5 \times \tau_3$
OS - pair	$r_1 = r_2$	$\omega = \pi/2$	$\gamma_5 \times 1$

Lattice action, sea and/or valence

- “Three” sufficiently fine lattice spacings
- to accommodate 200-250 MeV pions

Lattice parameters

NOTES

1) Determinant for Wilson and tw is P-even and isospin blind
2) Simulation stability worsens as $\mu_q \rightarrow 0$ and/or N_f increases

• Comparing W, tw, OS-val fermions

O(a) improvement	W	need improv. coeff's	
	tw	for free	
	OS	for free	
Isospin symmetry	W	OK	important for Meson→ππ and FSI's
	tw	KO	
	OS	OK	
Chiral sym. & mixing	W	KO	
	tw	~OK	but…
	OS	~OK	
Unitarity $\hat{m}_{ud} = Z_P^{-1}\mu_l\,,\ \ \hat{m}_{\pm} = Z_P^{-1}(\,\mu_h \pm Z_P\,/\,Z_S\ \varepsilon_h\,)$	W	OK (on their own sea)	
	tw	OK (on their own sea)	
	OS	KO (mass matching → O(a^2))	
Large quark mass	a blend of the above + FSS a la ToV → renormalizability and O(a) improv.		

- The structure of “mass” terms

$$\begin{array}{l|lll}
\text{W} & \bar{\chi}(c_{SW}\sigma\cdot F\chi + M_{cr})\chi + \boxed{\mu_q\bar{\chi}\chi} & \rightarrow \bar{\psi}(c_{SW}\sigma\cdot F + M_{cr})\psi & + \boxed{\mu_q\bar{\psi}\psi} \\
\text{tw} & \bar{\chi}(c_{SW}\sigma\cdot F\chi + M_{cr})\chi + \boxed{i\mu_q\bar{\chi}\gamma_5\tau^3\chi} & \rightarrow \bar{\psi}(c_{SW}\sigma\cdot F + M_{cr})i\gamma_5\tau^3\psi & + \boxed{\mu_q\bar{\psi}\psi} \\
\text{OS} & \bar{\chi}(c_{SW}\sigma\cdot F\chi + M_{cr})\chi + \boxed{i\mu_q\bar{\chi}\gamma_5\chi} & \rightarrow \bar{\psi}(c_{SW}\sigma\cdot F + M_{cr})i\gamma_5\psi & + \boxed{\mu_q\bar{\psi}\psi}
\end{array}$$

twisted basis → *physical basis* →

- The currents

Conserved currents

$$\begin{array}{l|ll}
\text{W} & V_\mu^{1,2,3} & 1\text{- }ps \\
\text{tw} & A_\mu^{1,2},\ V_\mu^3 & 1\text{- }ps \\
\text{OS} & V_\mu^{1,2,3} & 1\text{- }ps
\end{array}\left.\vphantom{\begin{array}{l}a\\a\\a\end{array}}\right\} Z_V = 1$$

physical basis

Axial currents for f_π

$$\begin{array}{llll}
\bar{\psi}\gamma_\mu\gamma_5\tau^b\psi & Z_A \neq 1 & \leftarrow & \bar{\chi}\gamma_\mu\gamma_5\tau^b\chi \\
(A_\mu^\pm)_{1-ps} & Z_V = 1 & \leftarrow & \bar{\chi}\gamma_\mu\tau^\pm\chi \\
\bar{\psi}\gamma_\mu\gamma_5\tau^b\psi & Z_A \neq 1 & \leftarrow & \bar{\chi}\gamma_\mu\gamma_5\tau^b\chi
\end{array}$$

twisted basis

Fixing M_{cr} and c_{SW}

Wilson

$$\mu_q = 0, \quad a << x_0 << T, \quad \hat{\mu}_q = Z_A Z_P^{-1} m^W_{PCAC} = Z_m(M_0 - M^W_{cr})$$

$$\frac{\partial m^W_{PCAC}(x_0, M_0)}{\partial x_0} = 0 \quad \Rightarrow \quad c_{SW}(g^2) + [O(a)]^W$$

$$\frac{<\partial_\mu A^b_\mu(x_0) P^b(0)>}{<P^b(x_0) P^b(0)>} \equiv 2 m^W_{PCAC}(\cancel{x_0}, M_0) \to 0 \quad \Rightarrow \quad M^W_{cr} = M^{m'al}_{cr}(g^2) + [O(a^2)]^W$$

tw

$$\mu_q \neq 0, \quad a << x_0 << T, \qquad \hat{\mu}_q = Z_P^{-1} \mu_q$$

$$\frac{\partial m^{tw}_{PCAC}(x_0, M_0)}{\partial x_0} = 0 \quad \Rightarrow \quad c_{SW}(g^2) + [O(a)]^{tw}$$

Optimal critical mass

$$\frac{<\partial_\mu V^b_\mu(x_0) P^b(0)>}{<P^b(x_0) P^b(0)>} \equiv 2 m^{tw}_{PCAC}(\cancel{x_0}, M_0) = 0 \quad \Rightarrow \quad M^{opt}_{cr} = \underbrace{M^{m'al}_{cr}(g^2) + [O(a^3)]^{tw}}_{}$$

Note - $M_{cr}^{m'al}$ and c_{SW} take the same values for Wilson and tw

- Fixing M_{cr} at $c_{SW}=0$ in tw

$$\left.\frac{<\partial_\mu V_\mu^2(x_0)P^1(0)>}{<P^1(x_0)P^1(0)>}\right|_{\pi^1} \equiv 2m_{PCAC}^{tw}(M_0)=0 \;\rightarrow\; M_{cr}^{opt} = \overbrace{M_{cr}^{m'al}(g^2)+[O(a)]^{tw}}^{\boxed{\text{Optimal critical mass}}}$$

$$<\Omega|L_5^{tw}|\pi^3> = <\Omega|b_5^{tw}\overline{\psi}i\gamma_5\tau^3\sigma\cdot F\psi + \delta_1^{tw}\Lambda^2\overline{\psi}i\gamma_5\tau^3\psi|\pi^3> = 0+O(\mu_q)$$

- What can we do for OS-val fermions?
  - $c_{sw}=0$

$$|<\partial_\mu A_\mu^1(x_0)P^1(0)> -2\mu_q <P^1(x_0)P^1(0)>|$$

minimize

$$|m_{\pi\,OS}^2 f_\pi G_{\pi\,OS} - 2\mu_q G_{\pi\,OS}^2| = O(a^2) \qquad G_{\pi\,OS} =<\Omega|P^1|\pi^1>$$

  - $c_{sw}\neq 0$, i.e. at its appropriate (non-perturbative) value

$$L_5^{OS} = \underset{\searrow\, 0}{b_5^{OS}}\overline{\psi}i\gamma_5\sigma\cdot F\psi + \underset{\searrow\, 0}{\delta_1^{OS}}\Lambda^2\overline{\psi}i\gamma_5\psi + O(\mu_q) = 0$$

• Computing f_M, m_M, $<\bar{M}|O_{VV+AA}|M>$ ($M = \pi,D,B$)

$f_M=<\Omega|A_0|M>$

W
- Isospin OK, need
- Z_A, c_A (and b_A for large μ_q)
- $O(a^2)$ corr's not too small

tw
- Isospin KO, no need
- for Z_A, c_A and b_A
- $f_{\pi 3}$ and $f_{\pi\pm}$ are fine

OS
- Isospin OK, need
- $Z_A \neq 1$, not c_A and b_A
- f_π seem to be fine

m_π

W

$$m_\pi^2 \equiv 2Bm_{PCAC} = 2B(M_0 - M_{cr}^W) =$$

$$= 2B(M_0 - M_{cr}^{opt} + O(a^2)),\ c_{SW} \neq 0$$

tw

$$m_{\pi^\pm}^2 = 2B\mu_q + O(a^2\mu_q, a^4)|_{M_{cr}^{opt}}$$

$$m_{\pi^3}^2 = 2B\mu_q - O(a^2)|_{M_{cr}^{opt}}$$

OS

$$m_\pi^2 = 2B\mu_q + O(a^2)|_{M_{cr}}$$

- $N_f=0$ $c_{SW} = 0$ → large χLF, Regina
- $N_f=0$ $c_{SW} \neq 0$ → small α-coll
- $N_f=2$ $c_{SW} = 0$ → large ETMC
- $N_f=2$ $c_{SW} \neq 0$ → to be tried

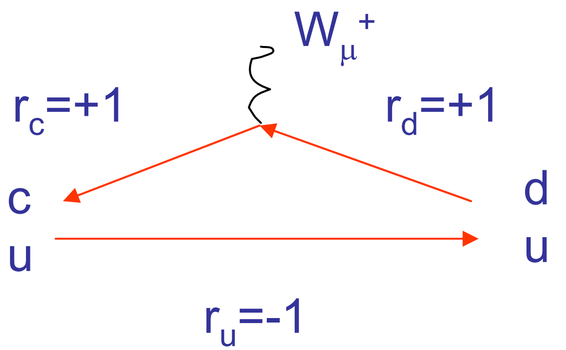
Use, e.g. CLS $N_f=2$ sea on OS-val quark pairs

- $<\overline{M}|O_{VV+AA}|M>$

$$\begin{array}{ccccc} <\overline{M}_{12} & | & O^{FR}_{VV+AA} & | & M_{34}> \\ \begin{pmatrix} r_1=-r_2 \\ tw \end{pmatrix} & & \left(-r_1=r_2=r_3=r_4\right) & & \begin{pmatrix} r_1=r_2 \\ OS \end{pmatrix} \end{array}$$

- No mixing
- O(a) improvement
- O(a^2) unitarity violations
- $m_{M_{12}} - m_{\overline{M}_{34}}$ small (?) @ $c_{SW} \neq 0$
- $m_{M_{12}} - m_{\overline{M}_{34}}$ large, B_K fine @ $c_{SW} = 0$

- Form factor, e.g. $<D|V_\mu|\pi>$



- In order to have a “unitary” pion we better take it twisted $r_d = -r_u$, so that $m^2_{\pi^\pm}|_{M_{cr}^{opt}} = 2B\mu_q + O(a^2\mu_q, a^4)$

- We take $r_d = r_c = 1$. The current is OS. We either employ the 1-ps current (Z_V=1), or the well-known Z_V value.

- With the above r-choices also D is OK

• Understanding O(a^2) effects in PS meson masses

In the Symanzik language L_6 and L_5L_5 matter

- L_6 - In the massless theory → 8 (sets of) operators potentially responsible for O(a^2) pion mass (splitting)

$$\underline{S^0S^0,\ \ P^bP^b}\,,\ b=1{,}2{,}3\ \ -\ \ \underline{P^0P^0,\ \ S^bS^b}\,,\ b=1{,}2{,}3$$

$$\underline{V^b_\mu V^b_\mu,\ A^b_\mu A^b_\mu}\,,\ b=1{,}2{,}3\ \ -\ \underline{T^0_{\mu\nu}T^0_{\mu\nu},\ T^b_{\mu\nu}T^b_{\mu\nu}}\,,\ b=1{,}2{,}3$$

Note

1) Operators gets reshuffled moving
   - from W to tw (by a $i\gamma_5\tau^3$ rotation)
   - from W to OS (by a $i\gamma_5$ rotation)

   but coefficients in front stay the same

2) Because of χ-symmetry breaking, coefficients in front of operators belonging to the same χ-multiplet are not (necessarily) equal

Question

What are the operators that give the largest contribution to $<\pi|L_6|\pi>$?

An “order of magnitude” estimate can be obtained by a combined use of

- • Perturbation Theory - PT
- •• Soft Pion Theorems - SPT’s
- ••• Vacuum State Aproximation - VSA

• To leading order in PT (i.e. α_s^2) only

$$\boxed{S^0S^0},\ T^0_{\mu\nu}T^0_{\mu\nu},\ V^0_\mu V^0_\mu$$

four-quark operators are generated (next order is $\alpha_s^3/8N_c$)

•• SPT’s yield

VSA

χ-condensate^2

W $$<\pi^b|S^0S^0|\pi^b> = \frac{2}{f_\pi^2}\left[-<\Omega|P^bP^b|\Omega> + <\Omega|S^0S^0|\Omega>\right] \approx \frac{2}{f_\pi^2}\Sigma_\chi^2[1]$$

tw $$<\pi^3|P^3P^3|\pi^3> = \frac{2}{f_\pi^2}\left[<\Omega|P^bP^b|\Omega> - <\Omega|S^0S^0|\Omega>\right] \approx -\frac{2}{f_\pi^2}\Sigma_\chi^2[1]$$

OS $$<\pi^b|P^0P^0|\pi^b> = \frac{2}{f_\pi^2}\left[<\Omega|P^0P^0|\Omega> - <\Omega|S^bS^b|\Omega>\right] \approx \frac{2}{f_\pi^2}\Sigma_\chi^2[1-1] = 0$$

••• VSA gives the last ≈ relation and $<\pi|V^0_\mu V^0_\mu,\ T^0_{\mu\nu}T^0_{\mu\nu}|\pi> \approx 0$

- $L_5 L_5 \rightarrow \Delta_{55} = -\frac{1}{2}<\pi|L_5 L_5|\pi>$

  - $$\Delta_{55}^{OS} = -\frac{1}{m_\eta^2} < \Omega | L_5^{OS} | \eta >< \pi\eta | L_5^{OS} | \pi > - f^2 \frac{< \pi | L_5^{OS} | \pi\eta >< \pi\eta | L_5^{OS} | \pi >}{E_{\eta\pi}^2 - m_\pi^2} + $$
    $$+ \frac{< \pi | L_5^{OS} | \sigma >< \sigma | L_5^{OS} | \pi >}{m_\sigma^2 - m_\pi^2} + \cdots \quad \textit{may not be small}$$

  - $$\Delta_{55}^{tw} = -\frac{1}{m_\pi^2} < \Omega | L_5^{tw} | \pi^3 >< \pi^3\pi^3 | L_5^{tw} | \pi^3 > - f^2 \frac{< \pi^3 | L_5^{tw} | \pi\pi >< \pi\pi | L_5^{tw} | \pi^3 >}{E_{\pi\pi}^2 - m_\pi^2} + $$
    $$+ \frac{< \pi^3 | L_5^{tw} | \sigma >< \sigma | L_5^{tw} | \pi^3 >}{m_\sigma^2 - m_\pi^2} + \cdots \quad \textit{small @ } M_{cr}^{opt}$$

- Tentative conclusions

  OS - $L_5 L_5$ is relevant, but not L_6

  This is perhaps why $c_{SW} \neq 0$ seems to help

  tw - L_6 is relevant, but not $L_5 L_5$

  W - similar to "-tw"

- Phenomenologically one finds

$$\text{OS}\quad \begin{cases} m_{\pi^{OS}}^2 \;>\; m_{\pi^\pm}^2 & @\, N_f = 0,\ c_{SW} \neq 0 \\ m_{\pi^{OS}}^2 >> m_{\pi^\pm}^2 & @\, N_f = 0,\ c_{SW} = 0 \\ m_{\pi^{OS}}^2 >> m_{\pi^\pm}^2 & @\, N_f = 2,\ c_{SW} = 0 \end{cases}$$

$$\text{tw}\quad \begin{cases} m_{\pi^0}^2 \;>\; m_{\pi^\pm}^2 & @\, N_f = 0,\ c_{SW} = 0 \\ m_{\pi^0}^2 \;<\; m_{\pi^\pm}^2 & @\, N_f = 2,\ c_{SW} = 0 \end{cases}$$

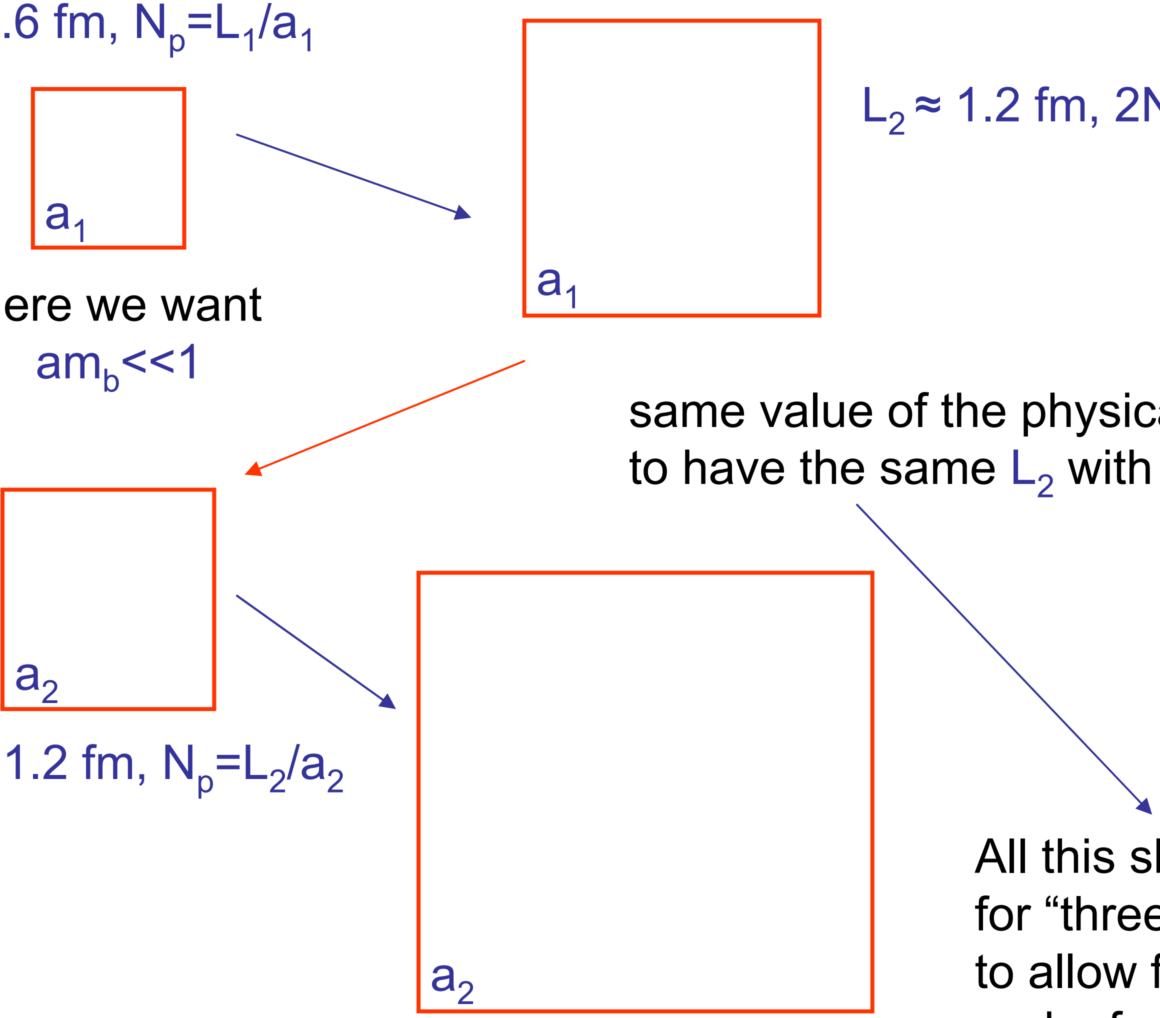
- Numerically

$$\text{OS}\quad a^2\left[m_{\pi^{OS}}^2 - m_{\pi^\pm}^2\right] = c_{OS} a^4 \Lambda^4, \qquad c_{OS} \approx 80\text{-}90 \qquad @\, N_f = 2,\ c_{SW} = 0$$

$$\text{tw}\quad a^2\left[m_{\pi^0}^2 - m_{\pi^\pm}^2\right] = -c_{tw} a^4 \Lambda^4, \qquad c_{tw} \approx 25\text{-}30 \qquad @\, N_f = 2,\ c_{SW} = 0$$

Finite Size Scaling for B-physics

- Matching physics



$L_1 \approx 0.6$ fm, $N_p=L_1/a_1$

here we want

$am_b<<1$

$L_2 \approx 1.2$ fm, $2N_p$

same value of the physical quantity $f\pi(L)L=f(\Lambda L)$
to have the same L_2 with N_p points

$L_2 \approx 1.2$ fm, $N_p=L_2/a_2$

$L_3 \approx 2.4$ fm, $2N_p$

All this should be repeated
for “three” sets of lattice spacings
to allow for $a/L \to 0$ limit
and a few light masses

- For the physical quantity f_M one finally uses the formula

$$f_M(L_1)\left(\frac{f_M(L_2)}{f_M(L_1)}\right)\left(\frac{f_M(L_3)}{f_M(L_2)}\right) = f_M(L_3) \approx f_M(L_\infty)$$

m_b $\quad$ m_b $\quad$ $(...m/2...)$

chiral extrapolation
continuum limit

- There is still a problem with the large b mass on L_2 a possibility is to
  - put on L_2 an intermediate m with
    $m_b L_1 = m L_2 \rightarrow m = m_b/2$
  - then extrapolate last ratio $1/m \rightarrow 1/m_b$

- How much does it cost?
  - config's: 3 latt spac's x 4 latt's x 2 light sea m's = 24
  - correl's: $3 \times (m_b^{(1)}+m_b^{(2)}) \times (m_b+m_b+3m+3m) \times 4 \times 2$
    (a $\quad L_1 \quad L_1 \quad L_2 \quad L'_2 \quad L_3 \quad$ l-v l-s)

- More conservative alternative (already feasible by ETMC)
  - stretch f_D to larger masses
  - fit f_B by interpolating through its static value

Conclusions

Super-B factory

- a challenge to the Lattice community

- an opportunity for
  - joining efforts
  - enlarging competences
  - sharing configurations and algorithms

- a great chance for Lattice to have an impact on Particle Physics