ω-ρ; pion Form factor.

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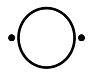
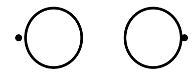
ω - ρ mass difference

Flavour singlet vector meson mass matrix:

m connected  x disconnected 

$$\begin{matrix} m_{nn} + 2x_{nn} & \sqrt{2}x_{ns} \\ \sqrt{2}x_{ns} & m_{ss} + x_{ss} \end{matrix}$$

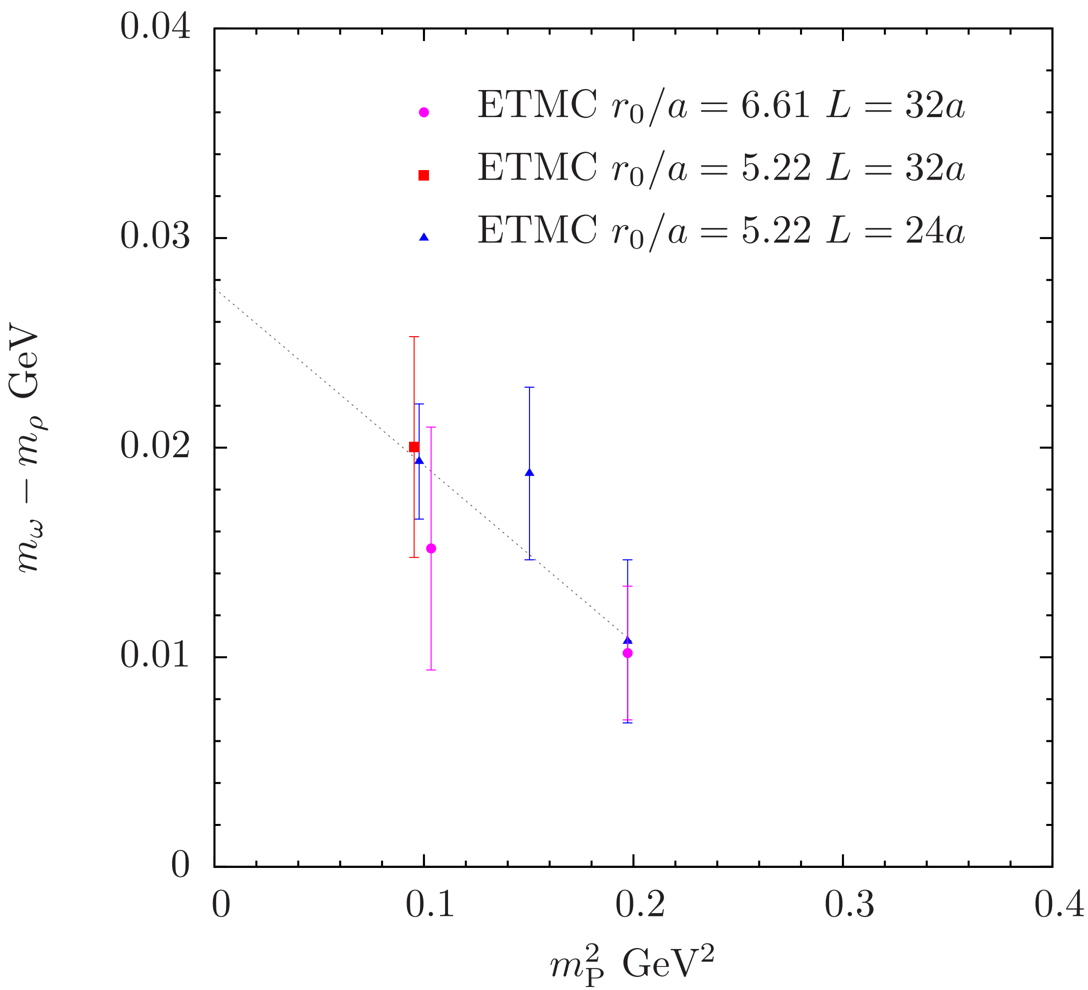
$n = (\bar{u}u + \bar{d}d)/\sqrt{2}$

$m_{nn} = m_\rho$; $m_{ss} = 2m_{ns} - m_{nn} = 2m_{K^*} - m_\rho \to 1.012$ GeV.

Approximately $m(\omega) = m(\rho) + 2x_{nn}$ and $m(\phi) = m_{ss} + x_{ss}$.

PDG: $m(\omega) - m(\rho) = 7$ MeV but ρ is very wide

ω - ρ mass difference

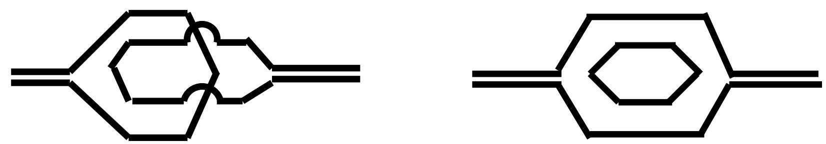


Craig; Chris; Carsten.

Models for vector meson masses

Models for quark-mass dependence:

Disconnected diagram can come from 2-body intermediate state:



$V \to VP$ and $V \to PP$ contribute

	Vπ	$\pi\pi$
ω	3	0
ρ	1	1
m_P dep:	m_P^3	$m_P^4 \log m_P$

Gives input to models of quark mass dependence of ρ.

(needed for paper on extrapolation to physical ρ mass - to appear)

ω - ρ mixing

u-d mass difference breaks isospin invariance.

Correlator matrix ($\rho = (uu - dd)/\sqrt{2}$; $\omega = (uu + dd)/\sqrt{2}$)

$$C(\rho,\rho) = (C_{uu} + C_{dd})/2 + (D_{u|u} - D_{u|d} - D_{d|u} + D_{d|d})/2$$

$$C(\rho,\omega) = (C_{uu} - C_{dd})/2 + (D_{u|u} + D_{u|d} - D_{d|u} - D_{d|d})/2$$

$$= (C_{uu} - C_{dd})/2 + (D_{u|u} - D_{d|d})/2$$

$$C(\omega,\omega) = (C_{uu} + C_{dd})/2 + (D_{u|u} + D_{u|d} + D_{d|u} + D_{d|d})/2$$

where we have used $D_{u|d} = D_{d|u}$.

Thus the ω to ρ cross-correlator is given by a difference of correlators with u and d quarks.

ω - ρ mixing

The ω-ρ mass matrix mixing element is given by
$T_{\omega\rho} = (\frac{dm_\rho}{dm_q} + \frac{dm_\omega}{dm_q})\ \frac{m_u - m_d}{4}$ with $m_q = (m_u + m_d)/2$.
$\frac{dm_\rho}{dm_q} + \frac{dm_\omega}{dm_q} = 2\frac{dm_\rho}{dm_q} + \frac{d(m_\omega - m_\rho)}{dm_q}$
We measure second term and find that lattice QCD naturally produces effects of the correct size to explain the observed ω-ρ mixing.

ω - ρ summary

First lattice determination of ω-ρ mass difference.

Small effect but non-zero.

(Also gives non-strange component of ϕ and strange component of ω)

Full lattice determination of QCD contribution from quark mass difference (u-d) to ω-ρ mixing.

Pion form factor - Chiral PT

The pion form factor $F(q^2)$ is analytic with a cut $q^2 > 4m_\pi^2$.

$$F(q^2) = 1 + \frac{q^2}{\pi} \int_0^\infty 4dk^2 \frac{\mathrm{Im}F(k)}{s(s - q^2 - i\epsilon)}$$

with LO discontinuity $\mathrm{Im}F(k) = k^3/(6\pi E f^2) \quad s = E^2 = 4(m_\pi^2 + k^2)$
This integral is logarithmically divergent and introducing a cut-off at $s = \Lambda^2$ we obtain

$$F(q^2) = 1 + \left(q^2 \log(\frac{z\Lambda^2}{m_\pi^2}) + 4m_\pi^2 \mathcal{H}(\frac{q^2}{4m_\pi^2}) \right) \frac{1}{48\pi^2 f^2}$$

The factor of $z\Lambda^2$ in the logarithm can be related to the usual LEC L_6 and renormalisation scale μ in the field theory treatment. With that identification, the dispersion treatment gives an identical result to the conventional field theoretic evaluation at NLO.

Pion form factor - Chiral PT

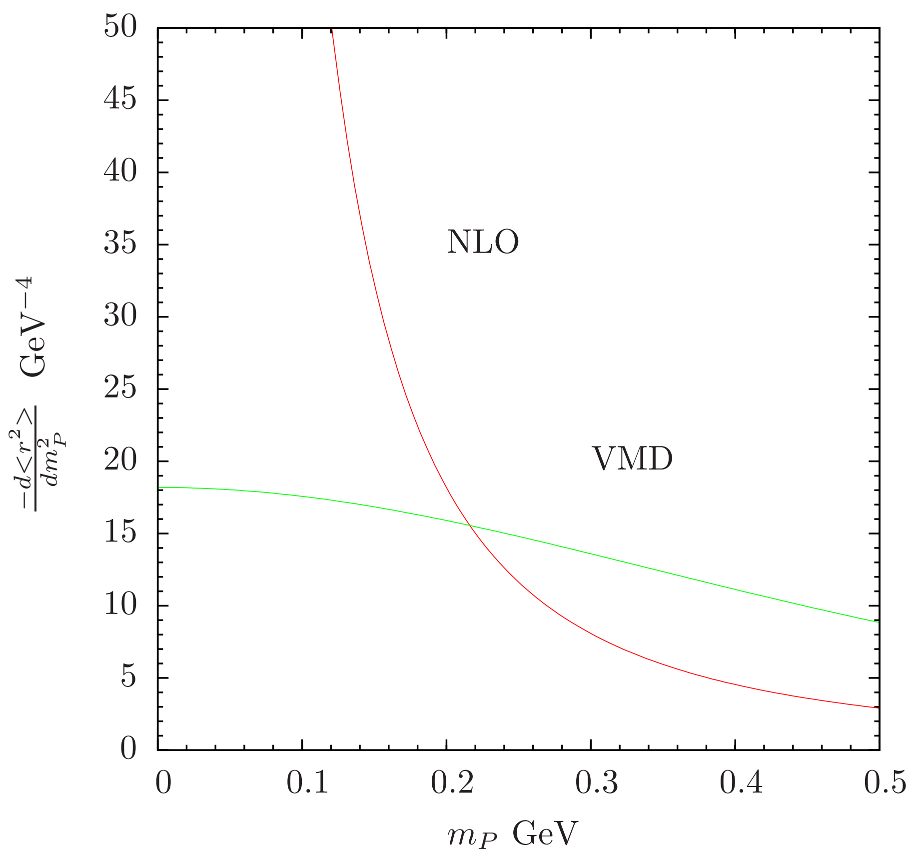
Extrapolate the pion charge squared-radius $< r^2 >$ from lattice results at $m_\pi > 300$ MeV to the physical pion at $m_\pi \approx 140$ MeV.
$< r^2 >= 6dF(q^2)/dq^2$ at $q^2 = 0$. Then DR gives convergent expression:

$$\frac{d < r^2 >}{dm_\pi^2} = -\frac{240}{\pi} \int_0^\infty dk^2 \frac{\mathrm{Im}F(k)}{s^3}$$

Integrand is positive and peaked at $k^2 = 3m_\pi^2/4$ ($s \approx 7m_\pi^2$) whereas $m_\rho^2 \approx 30m_\pi^2$ for physical pion. So ρ contribution is far away (compared to NLO ChPT) and can be absorbed in LEC L_6.
What about for heavier pions?

Pion form factor - Chiral PT

Assuming $m_V^2 = m_\rho^2 + m_P^2 - m_\pi^2$ then V peak is at peak of NLO expression when $m_V^2 = 7m_P^2$. This corresponds to $m_P \approx 0.3$ GeV.
So only for $m_P < 0.3$ GeV can the vector meson contribution be "ignored".



Estimate of NLO and VDM contributions

Pion form factor - Finite size effects

Even when the ρ meson mass is above the effective $\pi\pi$ threshold, (which is not yet the case in lattice studies) there can be significant finite size effects to the ChPT result.

Using ChPT at NLO in a finite (pbc) box of size L Borasoy, Bunton.

$$\mathcal{R} = \frac{d}{dm_\pi^2} \frac{6(F(q^2) - 1)}{q^2}$$

evaluated (from V_4) for (minimum) momentum $q = 4\pi/L$ in Breit frame.

$\mathcal{R}_L/\mathcal{R}_\infty - 1$ > 10% for $m_\pi L < 4.5$ > 1% for $m_\pi L < 6.8$.

This is a much stronger finite size effect than that appropriate to m_π or f_π.

These criteria are similar to the requirement that the ρ lies above the effective $\pi\ \pi$ threshold at $E = 2\sqrt{m_\pi^2 + (2\pi/L)^2}$ which should then be close to the infinite volume value, so $m_\pi L < 2\pi$.