HEM
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On the Resummation of small x Contributions to Unpolarized and Polarized Non–Singlet and Singlet Structure Functions

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DESY

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1. Introduction

$x \to 0$:

SINGULARITIES IN THE N-PLANE

$\sim \left(\frac{\alpha}{N-1}\right)^k$ $\sim N \left(\frac{\alpha}{N^2}\right)^\ell$

UNPOL. SINGLET

QCD

NON SINGLET (POL & UNPOL)

POL. SINGLET

QCD, QED

DO THEY IMPLY LARGE CORRECTIONS FOR NS &/OR S STRUCTURE FUNCTIONS BEYOND NLO ?

$\longrightarrow$ NON-PERTURBATIVE INPUT AT Q_0^2 :

$\sim x^{\alpha_i} \ldots$

- WHAT ARE THE EFFECTS ON THE EVOLUTION ?
- FOR A SERIES OF CASES FERMION OR MOMENTUM CONSERVATION HOLDS $\longrightarrow$ $\exists$ SUBLEADING TERMS ! (HOW 'SUB'-LEADING THEY ARE ?)
- WHAT IS CHANGED BEYOND THE KNOWN NLO CONTRIBUTIONS ?
- PREDICTIONS FOR 3-LOOP SPLITTING FUNCTIONS

2. Evolution in fixed order perturbative QED and QCD

THE EVOLUTION EQS:

$$\frac{\partial q_{NS}(x,Q^2)}{\partial \log Q^2} = P_{NS}^{\pm}(x,\alpha) \otimes q_{NS}(x,Q^2)$$

$$\frac{\partial}{\partial \log Q^2}\begin{pmatrix}\Sigma(x,Q^2)\\ G(x,Q^2)\end{pmatrix} = \mathbb{P}^S(x,\alpha) \otimes \begin{pmatrix}\Sigma(x,Q^2)\\ G(x,Q^2)\end{pmatrix}$$

RGE FOR THE COUPLING CONSTANT:

$$\frac{da}{d\log Q^2} = -\sum_{k=0}^{\infty} \beta_k a^{2+k}, \quad a = \frac{\alpha}{4\pi}$$

$$P^{\pm}(x,\alpha) = \sum_{\ell=0}^{\infty} a^{\ell+1} P_\ell^{\pm}(x); \quad \mathbb{P}^S(x,\alpha) = \sum_{\ell=0}^{\infty} a^{\ell+1}\mathbb{P}_\ell^S(x)$$

$$\int_0^1 dz\, P_\ell^-(z) = 0, \ \forall \ell \quad ; \quad \int_0^1 dz\, z \sum_{p'} P_{p'p;\ell}^{S,unp}(z) = 0$$

F-number conservation   EM-conservation

$$F_i^{\pm}(x,Q^2) = c_i^{\pm}(x,Q^2) \otimes q_i^{\pm}(x,Q^2)$$

$$F_i^S(x,Q^2) = c_i^{\Sigma}(x,Q^2) \otimes \Sigma(x,Q^2) + c_i^G(x,Q^2) \otimes G(x,Q^2)$$

$$c_i(x,Q^2) = \delta(1-x)\,\delta_q + \sum_{l=1}^{\infty} a^\ell c_{i\ell}(x)$$

NLO: keep only the terms up to α^2 (α) in the splitting functions P (coefficient functions c), and β_0, β_1 in $da/d\ln Q^2$

NNLO: $P_2(x)$ unknown so far.

SINGULAR TERMS @ $x \to 0$:

UNPOLARIZED	NS :	$\sim N \left(\frac{a}{N^2}\right)^\ell$	$P_i^\pm$	$c_i^\pm$
UNPOLARIZED	S :	$\sim \left(\frac{a}{N-1}\right)^\ell$	$P^S_i, c_i^{\Sigma,G}$	
POLARIZED	NS :	$\sim N \left(\frac{a}{N^2}\right)^\ell$	$P_i^\pm$	$c_i^\pm$
POLARIZED	S :	$\sim N \left(\frac{a}{N^2}\right)^\ell$	P_i^S	$c_i^{\Sigma,G}$

SUBLEADING IN $\overline{MS}$ UP TO $O(a^2)$.

$$N \left(\frac{a}{N^2}\right)^\ell \longleftrightarrow a \frac{1}{(2\ell-2)!}\left(a \ln^2 x\right)^{\ell-1}$$

$$\left(\frac{a}{N-1}\right)^k \longleftrightarrow a \frac{1}{(k-1)!}\frac{1}{x}\left(a \ln \frac{1}{x}\right)^{k-1}$$

$\longrightarrow$ SUM RULES AS F-NUMBER & ENERGY MOMENTUM CONSERVATION ENFORCE SUBLEADING TERMS ! (ON-DIAGONAL).

INSPECT ALSO THE LO & NLO ANOMALOUS DIMENSIONS FOR LESS SINGULAR TERMS IN $1/N$ OR $1/(N-1)$ KNOWN SO FAR : (e.g. g_1^{SING})

3. Resummation of the dominant terms for $x \to 0$

NS RESUMMED KERNELS:

KIRSCHNER, LIPATOV '83

$$\Gamma^{+,QCD}_{x\to 0}(N,a) = -N\left\{1-\sqrt{1-\frac{8aC_F}{N^2}}\right\}$$

$$\Gamma^{-,QCD}_{x\to 0}(N,a) = -N\left\{1-\sqrt{1-\frac{8aC_F}{N^2}\left[1-\frac{8aC_F}{N}\frac{d}{dN}\ln\left(e^{z^2/4}D_p(z)\right)\right]}\right\}$$

JB, A. VOGT '96

$p = \frac{1}{2N_c^2}$

$z = N/\sqrt{2N_c a}$

$$\Gamma^{+,QED}_{x\to 0}(N,a) = -N\left\{1-\sqrt{1-\frac{8a}{N^2}}\right\}$$

$$\Gamma^{-,QED}_{x\to 0}(N,a) = -N\left\{1-\sqrt{1+\frac{8a}{N^2}\left[1-\sqrt{1-\frac{8a}{N^2}}\right]}\right\}$$

$F_2^{ep} - F_2^{en} \quad \propto \quad \Gamma^{+,QCD}$

$xF_3^{\nu N} \quad \propto \quad \Gamma^{-,QCD}$

$g_{5,NS}^{\gamma Z} \quad \propto \quad \Gamma^{+,QCD}$

$g_{1,NS}^{i} \quad \propto \quad \Gamma^{-,QED}$

THE NLO ANOM. DIMS. AGREE WITH THE ACCORDING TERMS IN THE ABOVE RESUMMATIONS IN THEIR 'MOST SINGULAR' TERMS.

3 LOOP ANOM. DIM: $|_{\rm SING.}$

$$P^{+,QED}_{2,x\to 0,\overline{MS}}(x,a) = \frac{2}{3}a^3\ln^4 x$$

$$P^{-,QED}_{2,x\to 0,\overline{MS}}(x,a) = -\frac{10}{3}a^3\ln^4 x \equiv K^{-,QED}_{2,x\to 0,\,\rm OMS}$$

$$P^{+,QCD}_{2,x\to 0,\overline{MS}}(x,a) = \frac{2}{3}C_F^3 a^3\ln^4 x$$

$$P^{-,QCD}_{2,x\to 0,\overline{MS}}(x,a) = \left(-\frac{10}{3}C_F^3 + 4C_F^2C_G - C_FC_G^2\right)a^3\ln^4 x$$

SINGLET RESUMMATION:

- DETAILS ARE WELL-KNOWN — LIPATOV et al. LO; CATANI, HAUTMANN NLO$_q$
  → UNPOLARIZED CASE

- POLARIZED CASE: BARTELS, ERMOLAEV, RYSKIN

$$\mathbb{F}_0(N,a) = 16\pi^2 \frac{a}{N}\mathbb{M}_0 - 8\frac{a}{N^2}\mathbb{F}_8(N,a)\mathbb{G}_0 + \frac{1}{8\pi^2}\frac{1}{N}\mathbb{F}_0^2(N,a)$$

$$\mathbb{F}_8(N,a) = 16\pi^2 \frac{a}{N}\mathbb{M}_8 + 2\frac{a}{N}C_A\frac{d}{dN}\mathbb{F}_8(a,N) + \frac{1}{8\pi^2 N}\mathbb{F}_8^2(N,a)$$

$$\mathbb{M}_0 = \begin{pmatrix} C_F & -2T_f N_f \\ 2C_F & 4C_A \end{pmatrix} \qquad \mathbb{M}_8 = \begin{pmatrix} C_F - C_A/2 & -T_f N_f \\ C_A & 2C_A \end{pmatrix}$$

$$\mathbb{G}_0 = \begin{pmatrix} C_F & 0 \\ 0 & C_A \end{pmatrix}.$$

- SOLVE FOR THE ANOMALOUS DIM. MATRIX.

$$\Gamma_{S,pol}(N,a) = -\frac{1}{4\pi^2}\mathbb{F}_0(N,a)$$

→ U-MATRIX IN EVOLUTION, (S).

- agrees with the 'singular' parts of $\mathbb{P}_0^{S,pol}, \mathbb{P}_1^{S,pol}$ ($\overline{MS}$)

- again $\mathbb{P}_{2,x\to 0}^{S,pol}(x)_{\overline{MS}}$ can be derived (c_2 behaviour)

$$\mathbb{P}_S^2 = \begin{cases} P^2_{qq,x\to 0}(N) = \frac{16}{N^5}C_F\left[-5C_F^2 - 8C_F T_F N_f - 6C_A T_f N_f + 6C_A C_F - \frac{3}{2}C_A^2\right] \\ P^2_{qg,x\to 0}(N) = \frac{16}{N^5}T_f N_f\left[2C_F^2 + 8C_F T_f N_f - 6C_A C_F - 15C_A^2\right] \\ P^2_{gq,x\to 0}(N) = \frac{16}{N^5}C_F\left[-2C_F^2 - 8C_F T_f N_f + 6C_A C_F + 15C_A^2\right] \\ P^2_{gg,x\to 0}(N) = \frac{16}{N^5}\left[-4C_F^2 T_f N_f - 24C_A C_F T_f + 2C_A^2 T_f + 28C_A^3\right] \end{cases}$$

4. Numerical results

- UNPOLARIZED
  - NS • REALISTIC INPUTS (JB, A. VOGT PLB)
  - S • MOM. SR, F-SR. (JB, A. VOGT, S. RIEMERSMA DESY 96-096)

- POLARIZED
  - NS • COMPARE FOR SOME INPUTS (JB, A. VOGT PLB & DESY 96-041)
  - S • ΔG ! (JB, A. VOGT DESY 96-050) F-SR.

- QED : ISR (NS) @ HERA. JB, A. VOGT, S. RIEMERSMA

- HOW IMPORTANT (NOT YET KNOWN) SUBLEADING TERMS CAN BECOME ?

SUM RULES:

A: $\Gamma(N,a) \rightarrow \Gamma(N,a) - \Gamma(1,a)$

B: $\Gamma(N,a) \rightarrow \Gamma(N,a)(1-N)$

C: $\Gamma(N,a) \rightarrow \Gamma(N,a)(1-2N+N^2)$

D: $\Gamma(N,a) \rightarrow \Gamma(N,a)(1-2N+N^3)$

$\Gamma \equiv \Gamma^-, \Gamma^S$

FSR, MSR

4.1. UNPOLARIZED NS

Γ^{+}_{QCD} :

BLÜMLEIN, VOGT
PHYS. LETT. B370 (1996) 149
& DESY 96-041

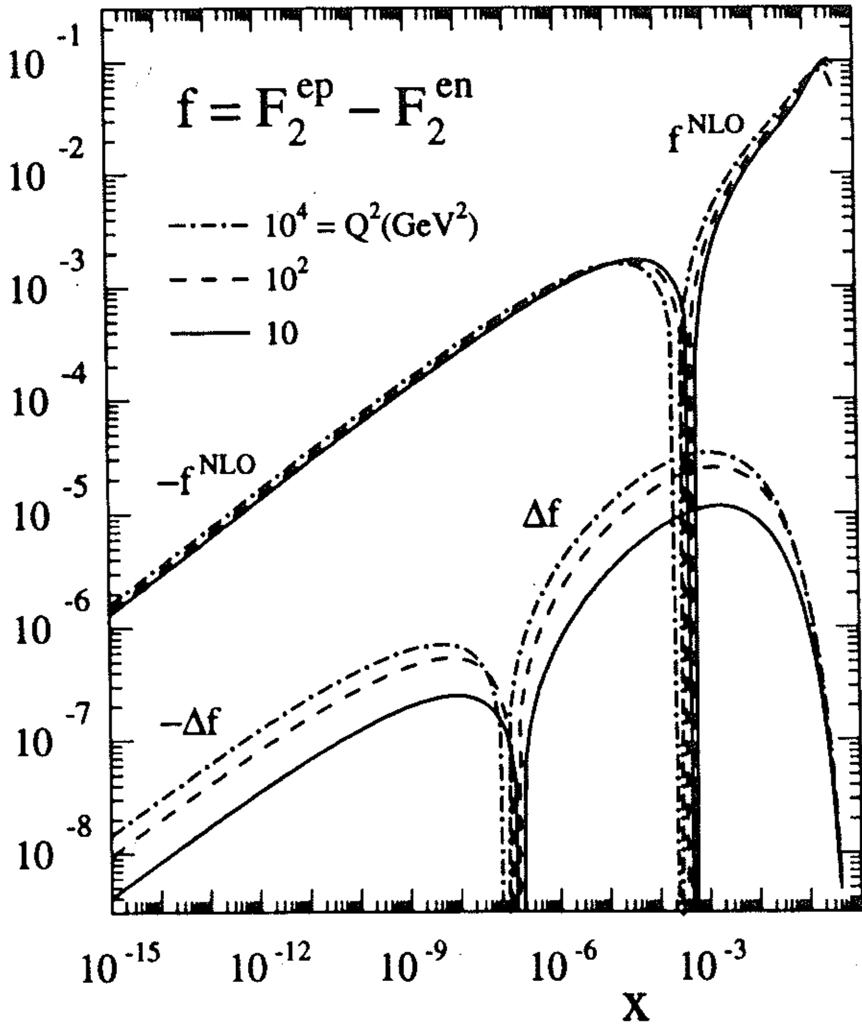
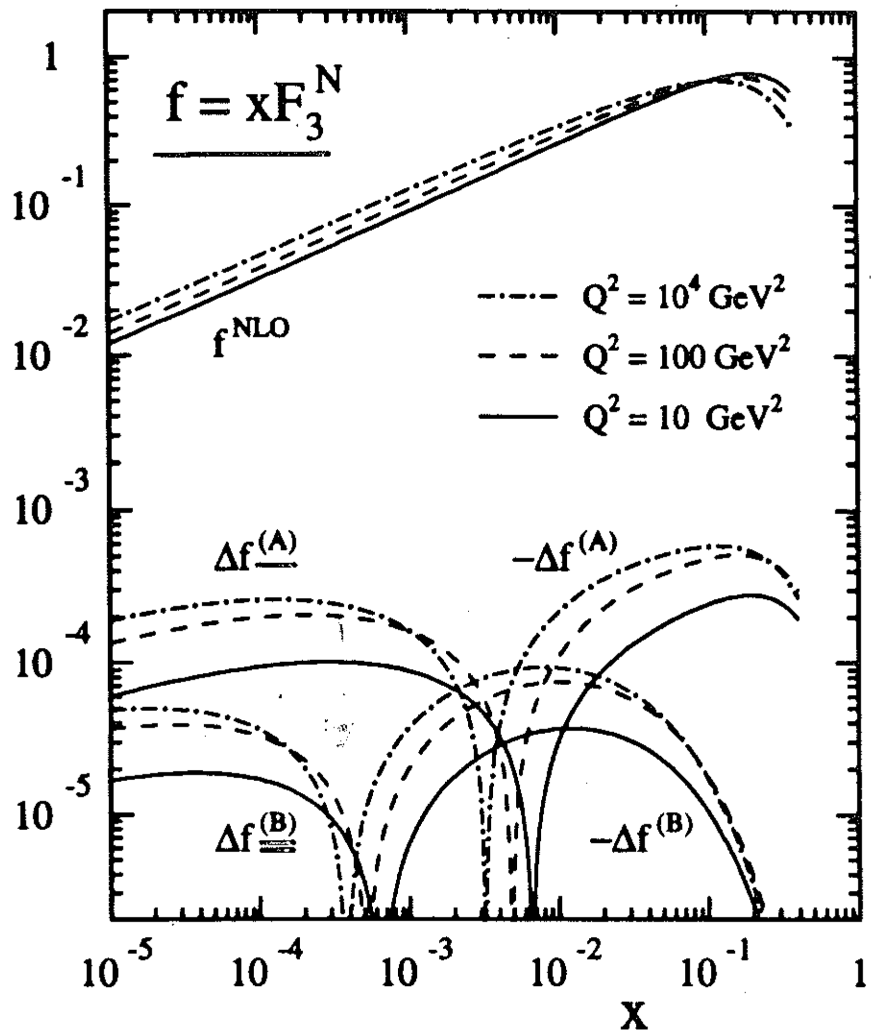


Figure 1: The small-x Q^2–evolution of the unpolarized non–singlet structure function combination $F_2^{ep} - F_2^{en}$ in NLO and the absolute corrections to these results due to the resummed kernel derived from ref. [3]. The initial distributions at $Q_0^2 = 4$ GeV^2 have been adopted from [16].

$\bar{\Gamma}_{QCD}$:



– TYPE

CORR.
<0 (1%)

FACTOR 3!

F-NUMBER.

Figure 1: The small-x Q^2–evolution of the non–singlet structure function $xF_3^N \equiv \frac{1}{2}(xF_3^{\nu N}+xF_3^{\bar{\nu} N})$ for an isoscalar target N in NLO and the corrections to these results due to the resummed kernels derived from ref. [7]. 'A' and 'B' denote the two prescriptions for implementing the fermion number conservation discussed in the text.

4.2. Unpolarized Singlet

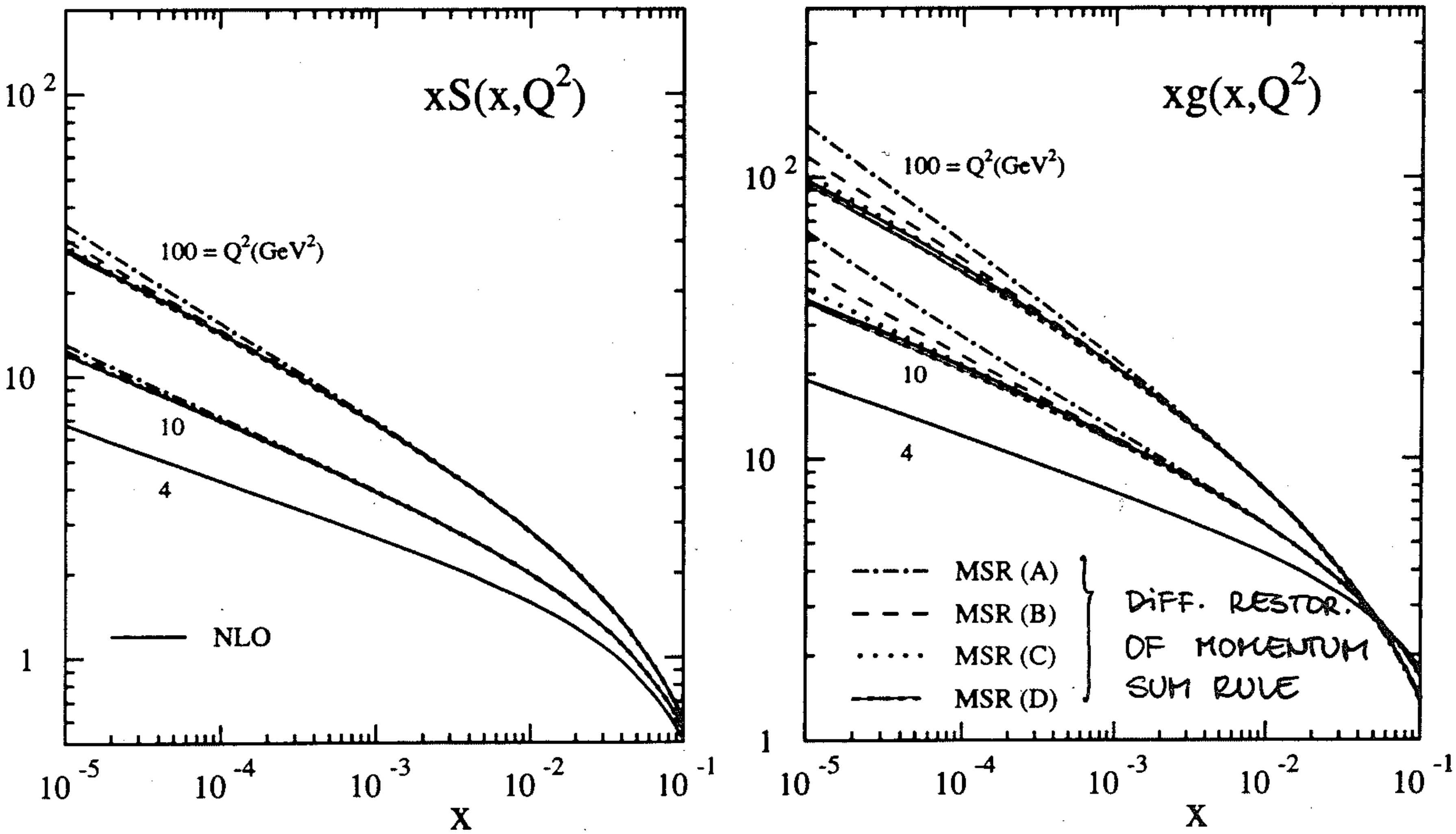
- NUMERICAL UPDATE OF EARLIER INVESTIGATIONS e.g. ELLIS, HAUTMANN, WEBBER;
  - F_2 rises @ $Q_0^2 = 4\,GeV^2$ already

⟶ STUDY OF SUBLEADING TERMS IN MORE DETAIL (WHAT COULD HAPPEN ?)

⟶ SORTING OUT OF <u>ONLY</u> SINGULAR TERMS IN HO.

NLO+
LiP. SERIES ONLY $\left(\frac{\alpha}{N-1}\right)^{\ell}$

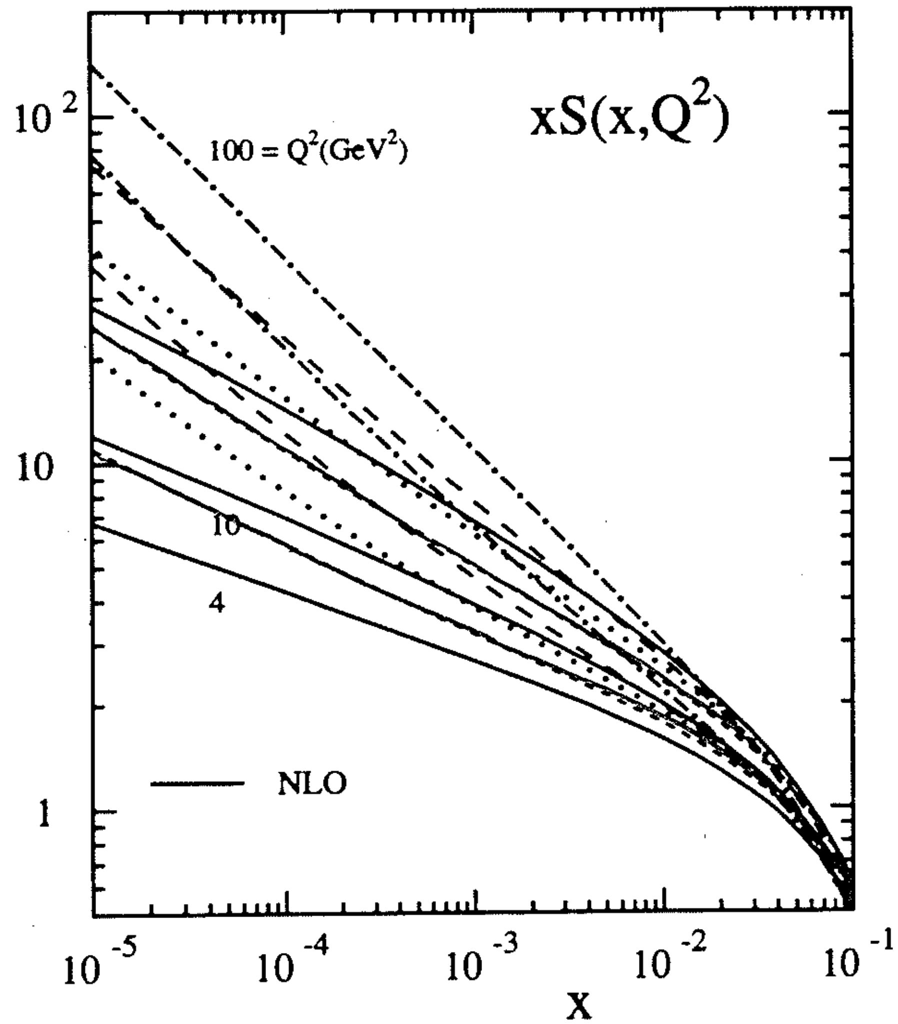
Toy input at $Q_0^2 = 4\ \mathrm{GeV}^2$, f=4, NLO (DIS) + Lx

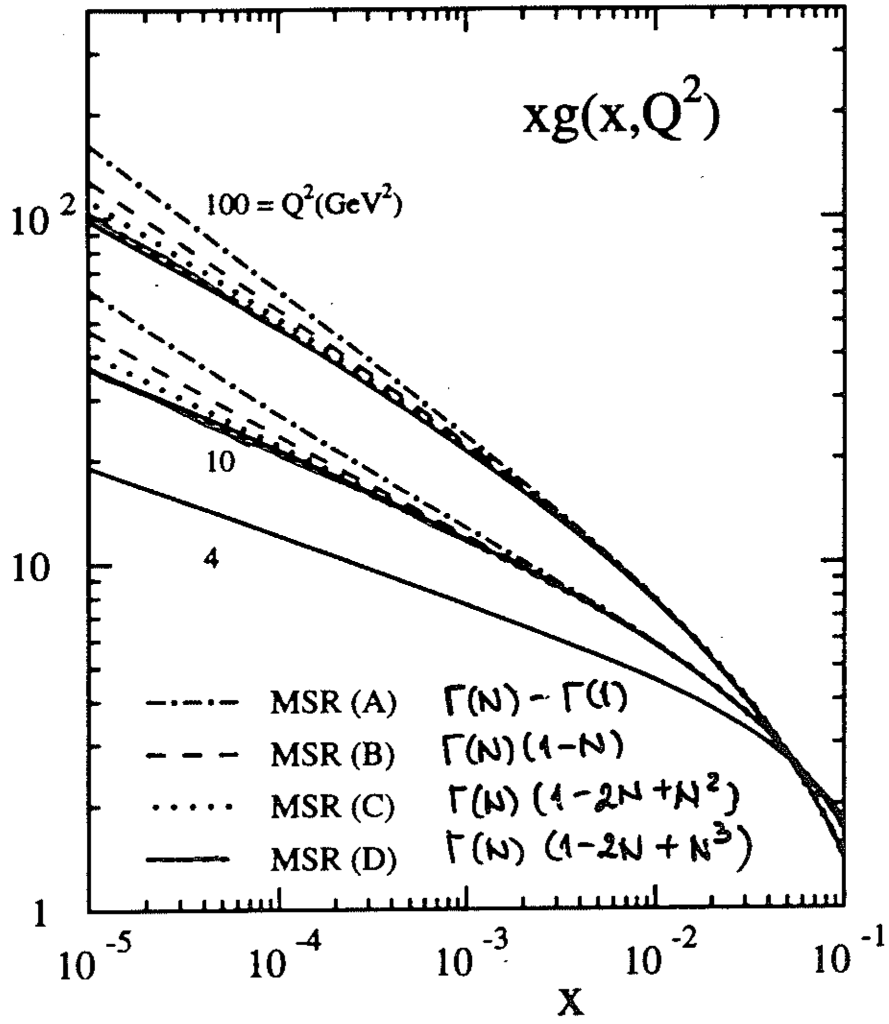


BLÜMLEIN, VOGT, RIEMERSMA.

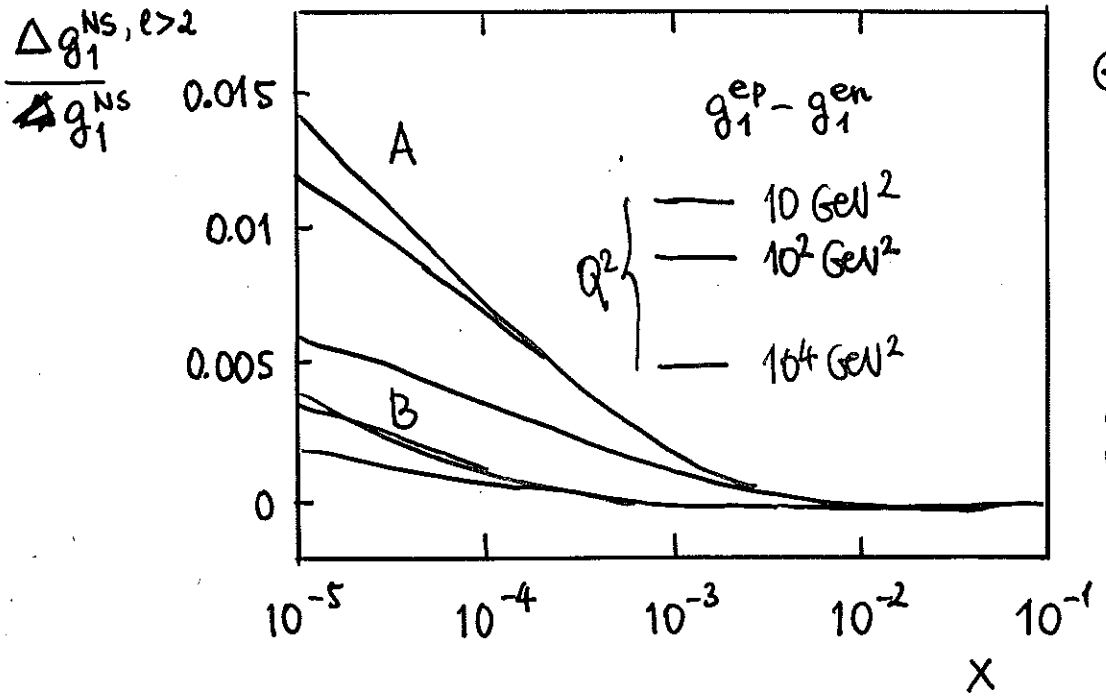
NLO + LIP + NLO SING$_q$ + $\left(\frac{\alpha}{N-1}\right)^\ell$ + $\alpha\left(\frac{\alpha}{N-1}\right)^\ell_q$

Toy input at $Q_0^2 = 4$ GeV^2, f=4, NLO (DIS) + NLx





4.3. POLARIZED NS - DISTRIBUTION



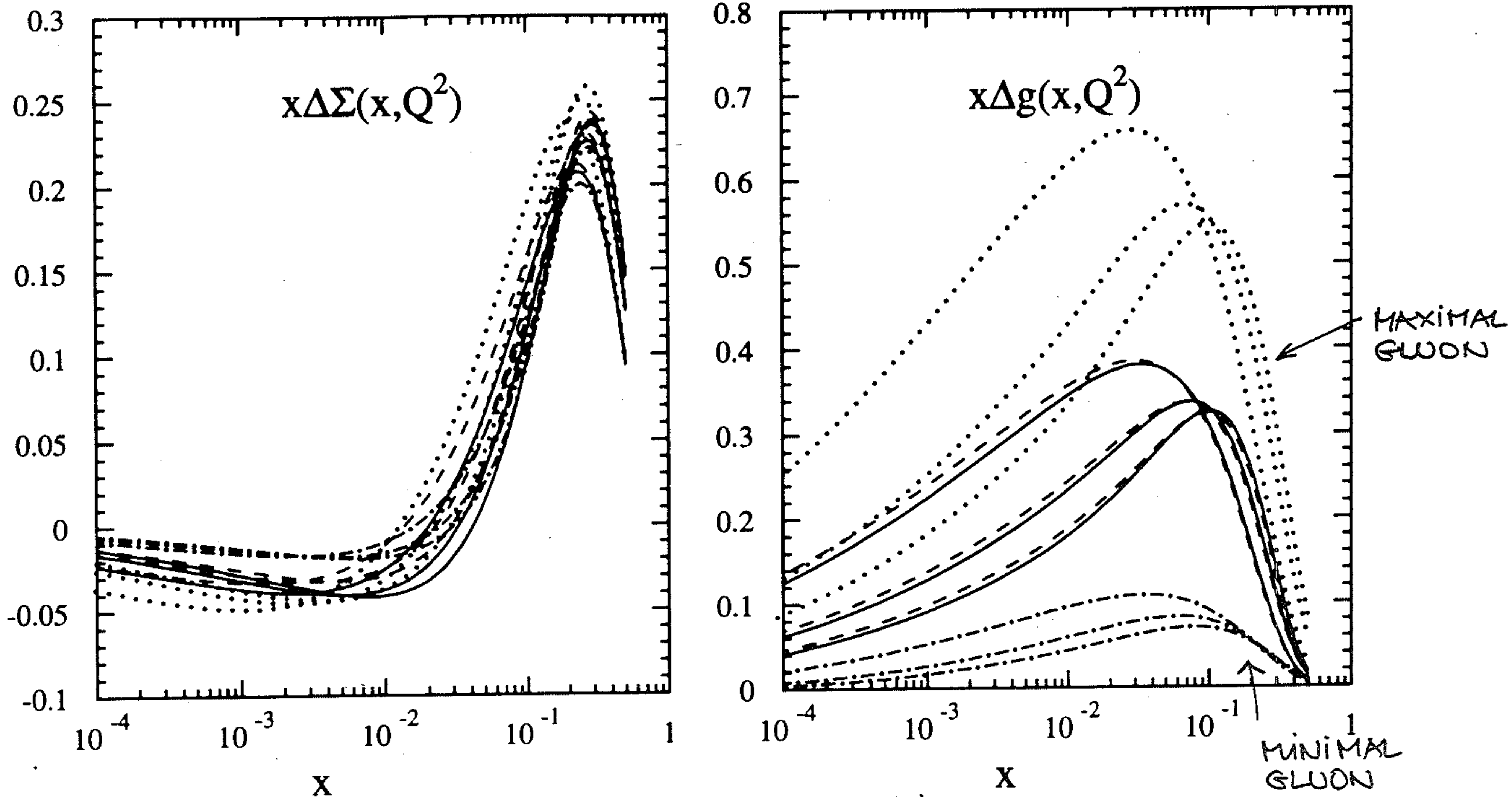
GRSV '95

A: $\Gamma(N,a) - \Gamma(1,a)$
↓ $\cdot \approx 1/3$.
B: $\Gamma(N,a) \cdot (1-N)$

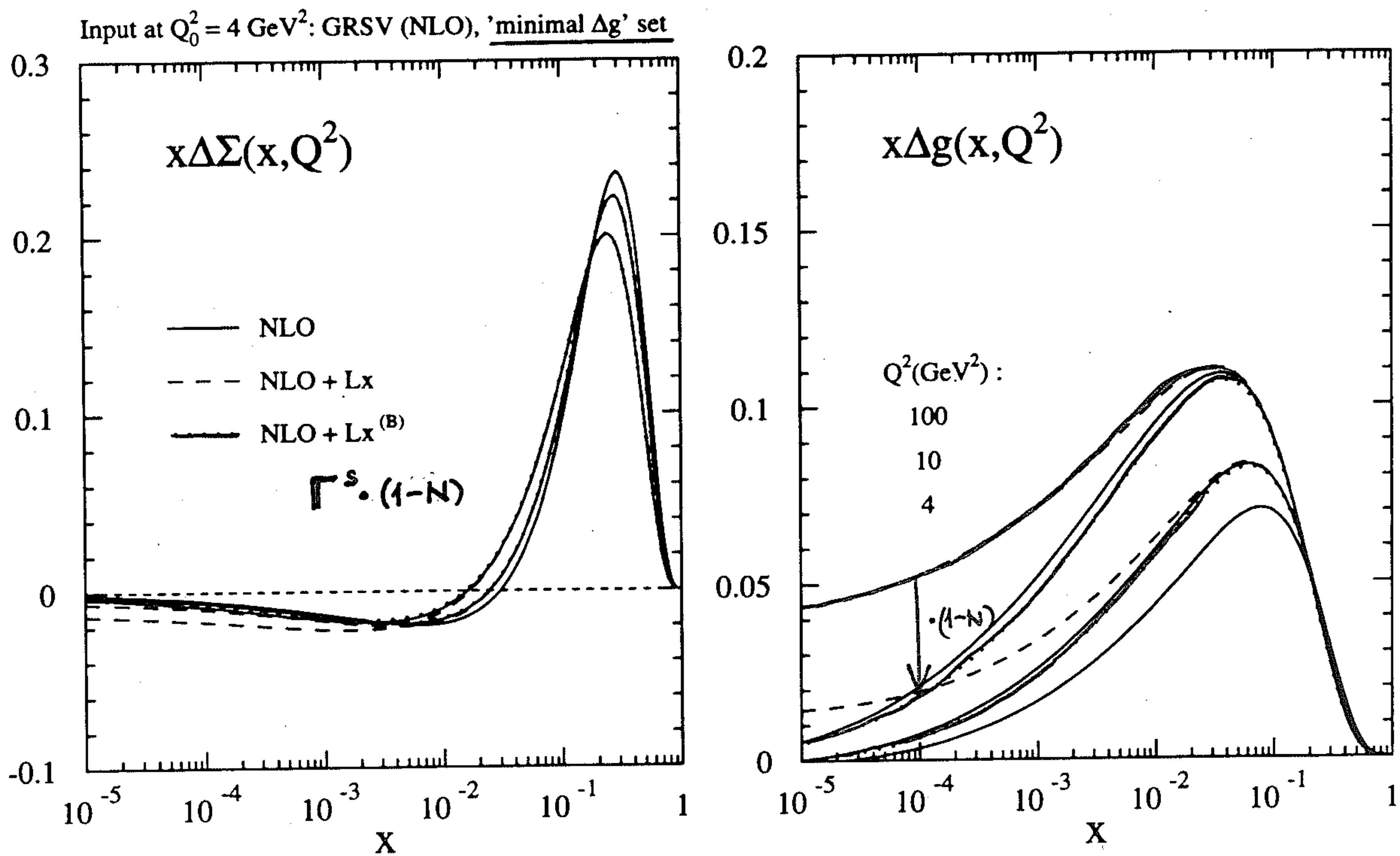
(earlier parametrizations yield a somewhat larger correction.

→ However, similar uncertainty for F-number conservation!)

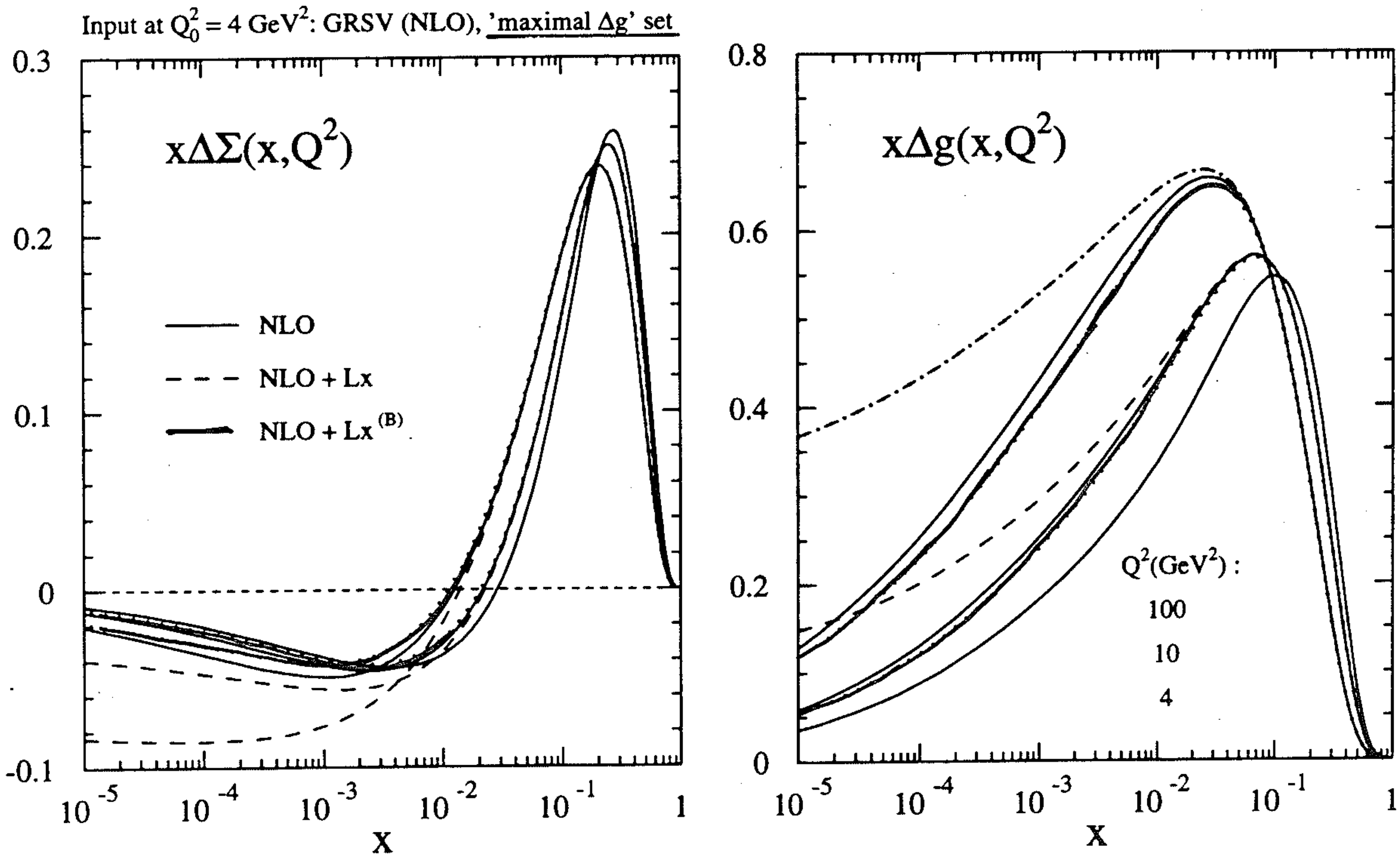
4.4. POLARIZED SINGLET



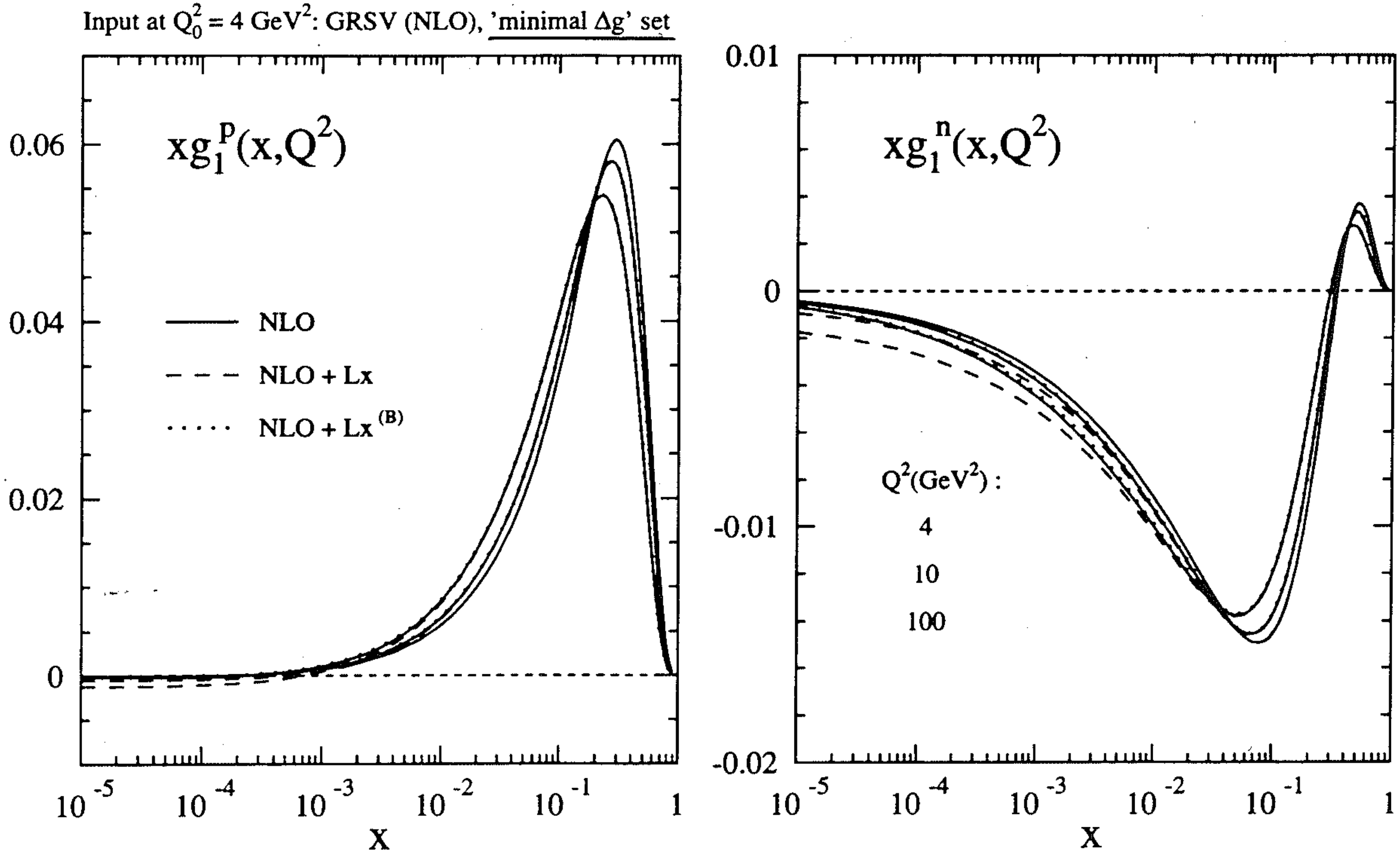
GRSV, DIFFERENT SETS.
(VAL 3 GLUONS, STD)
& THEIR SCALING VIOLATIONS



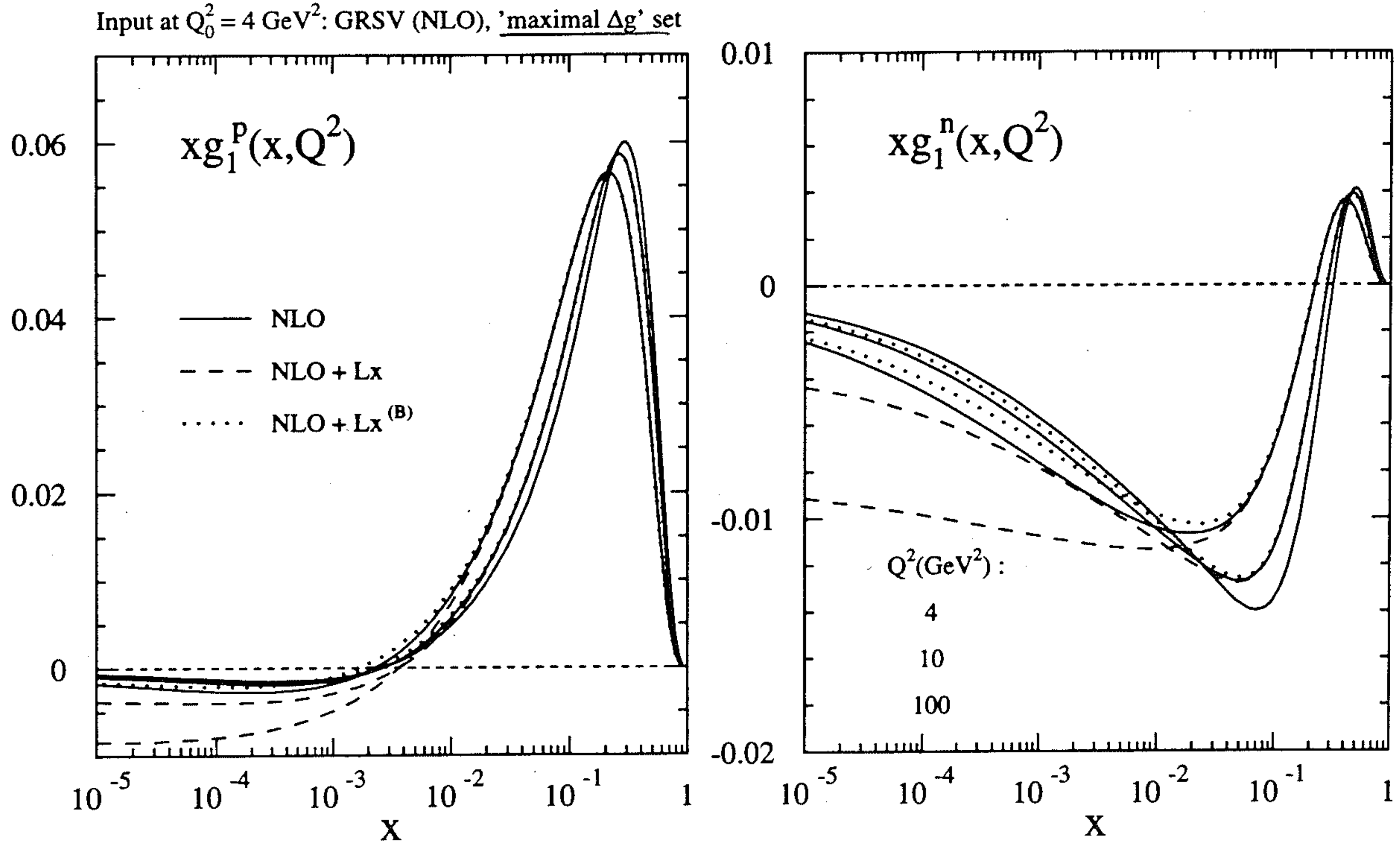
Input at $Q_0^2 = 4$ GeV^2: GRSV (NLO), 'minimal Δg' set
xΔΣ(x,Q²)
NLO
NLO + Lx
NLO + Lx (B)
Γ^s · (1-N)
xΔg(x,Q²)
Q²(GeV²) :
100
10
4
·(1-N)
x



Input at $Q_0^2 = 4$ GeV^2: GRSV (NLO), 'maximal Δg' set
$x\Delta\Sigma(x,Q^2)$
NLO
NLO + Lx
NLO + Lx$^{(B)}$
0.3
0.2
0.1
0
-0.1
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x
$x\Delta g(x,Q^2)$
Q^2(GeV^2) :
100
10
4
0.8
0.6
0.4
0.2
0
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x



Input at $Q_0^2 = 4$ GeV^2: GRSV (NLO), 'minimal Δg' set
$xg_1^p(x,Q^2)$
NLO
NLO + Lx
NLO + Lx$^{(B)}$
0.06
0.04
0.02
0
10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1
x
$xg_1^n(x,Q^2)$
Q^2(GeV^2) :
4
10
100
0.01
0
-0.01
-0.02
x



Input at $Q_0^2 = 4$ GeV^2: GRSV (NLO), 'maximal Δg' set
$xg_1^p(x,Q^2)$
NLO
NLO + Lx
NLO + Lx$^{(B)}$
0.06
0.04
0.02
0
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x
$xg_1^n(x,Q^2)$
Q^2(GeV^2) :
4
10
100
0.01
0
-0.01
-0.02

5. Conclusions

1) THE SMALL x RESUMMATIONS $\left(\frac{\alpha}{N-1}\right)^k$, $\alpha\left(\frac{\alpha}{N-1}\right)^k$, $N\left(\frac{\alpha}{N^2}\right)^k$ AGREE WITH THE ACCORDING RESULTS OF FIXED ORDER PT IN ALL KNOWN ORDERS (NLO).

2) PREDICTIONS FOR THE NNLO SPLITTING FUNCTIONS FOR THE $\alpha(\alpha \ln^2 x)^k$ TERM IN 3-LOOP ORDER CAN BE MADE DUE TO THE KNOWN BEHAVIOUR OF THE COEFFICIENT FUNCTIONS (POL. S; UNPOL. NS, QED NS) ($\overline{MS}$).

3) DUE TO THE VIOLATION OF THE GL-RELATION IN NLO NO PREDICTION CAN BE MADE FOR $q^2 > 0$.

4) THE CORRECTIONS DUE TO THE $\alpha(\alpha \ln^2 x)^k$ TERMS IS OF $< O(1\%)$ FOR ALL QCD NS STRUCTURE FCTS. (xF_3, F_2^{NS}, g_{1NS}) IN THE KIN RANGE TO BE REACHED @ HERA e.g.

5) AT HIGH y AND SMALL x A RATHER LARGE QED CORRECTION IS IMPLIED (STILL UP TO 10%), HERA RANGE.

6) FERMION NUMBER CONSERVATION (OR 4MOMENTUM CONSERVATION) MAY IMPLY DRASTIC CHANGES IN THE TERMS BEYOND NLO.

$\Gamma \to \Gamma(N) - \Gamma(1)$; $\Gamma(N)(1-N)$; $\Gamma(N)(1-2N+N^2)$
$\Gamma(N)(1-2N+N^3)$

I.E. THE $\exists$ 'SUB' LEADING TERMS ARE AS IMPORTANT. $\longrightarrow$ 3 LOOP CALCULATIONS...