$O(\alpha)$ and $O(\alpha^2)$ QED Radiative Corrections to Deep Inelastic $ep$ Scattering

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DESY – Zeuthen

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2. Application of the Renormalization Group
3. Different Variables
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1. Introduction

Aim of deep inelastic scattering experiments:

- Measurement of Born-level structure functions $F_i = F_i(x, Q^2)$
- QCD analysis

Born cross sections:

$\frac{d^3 \sigma^{\text{MC}}}{dx dy} = \frac{2 \pi \alpha^2}{x Q^2} S_x \left\{ Y_+ F_2(x, Q^2) - y^2 F_2(x, Q^2) \right\}$

Radiative corrections:

- EW-loops:

  e\gamma q

  1\text{loop}

  e\gamma q

  2\text{loop}

  e\gamma q

  Boxes; no effective vertex!"
**QED CORRECTIONS:**

\[ \begin{align*}
&2 \rightarrow 2 \\
&\text{\textbullet} \quad \gamma^* \quad \text{\textbullet} \\
&\text{\textbullet} \quad \gamma_{\text{Born}} \\
&2 \rightarrow 3 \\
&\mathcal{O}(\alpha) \\
&\vdots \\
&2 \rightarrow 2+n \\
&\mathcal{O}(\alpha^n)
\end{align*} \]

**V**S **BORN**: \( 2 \rightarrow 2 \)

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**CALCULATE K-FACTORS**

\[ \delta(x,y) = \frac{\frac{d^2 \sigma}{dxdy} (\text{Born+RC})}{\frac{d^2 \sigma}{dxdy} (\text{Born})} \]

---

MOST OF THE IRRADIATED PHOTONS CANNOT BE TAGGED OR EVEN FULLY MEASURED.
• **Problem:** What are \( x \) & \( y \) for \( d^2 \sigma / d^2 Q^2 \)?

Many ways to **define** \( x \) & \( y \)!

**Experimental Choice:**

• *Different kinematical regions at HERA (e.g.) require different choices.*
• *Compare at least two variable's sets!*

• *1st order corrections are found to be large in some ranges of the phase space.*

→ How large are the 2nd order corrections?

• **Full calculations:** time consuming

→ \( x \cdot \text{number of different variable sets!} \)

→ Do dominant corrections suffice?
2. Application of the Renormalization Group

**Factorization of Logarithmic Contributions:**

\[
\frac{d\sigma^{\text{rad}}}{dx dy} = \int_0^1 dz \ G(z, p^2, Q^2, m_e^2) \ \hat{d}\sigma^{\text{rad}}
\]

**Splitting Functions**

\[
\mathcal{G}(z, p^2, Q^2, m_e^2) = \Gamma_{\alpha}(z, \frac{p^2}{m_e^2}, \alpha(p^2)) \otimes \Gamma_{\beta}(z, \frac{p^2}{m_e^2}, \alpha(p^2)) \otimes \tilde{c}_{\rho}(z, \frac{Q^2}{p^2}, \alpha(p^2))
\]

\[
\mu^2: \text{Factorization Scale & Renormalization Scale.}
\]

\[
\frac{d}{d\mu^2} \ \mathcal{G}(z, p^2, Q^2, m_e^2) = 0
\]

\[
\Gamma_{ij} = \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} a_{nk} \log^n \left( \frac{m_e^2}{p^2} \right)
\]

\[
\tilde{c}_{ij} = \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} b_{nk} \log^n \left( \frac{Q^2}{p^2} \right)
\]

\[a_{nk} \text{ & } b_{nk} \text{ are related!}\]


\[
\Gamma_{ij}^{(m)} = \int_0^1 \! \! \int_0^1 \! \! z^{i-1} \delta z^{j-1} \Gamma_{ij} \ (z)
\]

\[
\bar{\sigma}_{ij}^{(m)} = \int_0^1 \! \! \int_0^1 \! \! z^{i-1} \delta z^{j-1} \bar{\sigma}_{ij} \ (z)
\]

\[
\left[ \left( p \frac{\partial}{\partial p} + \beta(q) \frac{\partial}{\partial q} \right) \delta_{ae} + \gamma_{ae}^{(m)}(q) \right] \gamma_{i}^{(m)} \left( \frac{\mu^2}{m_c^2}, g(q) \right) = 0
\]

\[
\left[ \left( p \frac{\partial}{\partial p} + \beta(q) \frac{\partial}{\partial q} \right) \delta_{ke} \delta_{kk} - \gamma_{ke}^{(m)} \delta_{kk} - \gamma_{kk}^{(m)} \delta_{ke} \right] \bar{\sigma}_{kk} \left( \frac{\theta^2}{\mu^2}, g(q) \right) = 0.
\]

**In fact, one obtains:**

\[
\left[ p \frac{\partial}{\partial p} + \beta(q) \frac{\partial}{\partial q} \right] \frac{d\sigma^{ead}}{dx dy} = 0.
\]

\[
\beta(q) = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2} + \cdots \quad \beta_0 = -\frac{4}{3}, \quad \beta_1 = -4 \quad (e)
\]

\[
\gamma_{ae}^{(m)}(q) = \left( \gamma_{ee}^{(m)}(q), \gamma_{xe}^{(m)}(q) \right)
\]

\[
\gamma_{ij}^{(m)}(q) = \gamma_{0ij}^{(m)} \frac{g^2}{16\pi^2} + \delta_{ij}^{(m)} \left( \frac{g^2}{16\pi^2} \right)^2 + \cdots
\]
1) LLA: \[ \text{Inspect } \Gamma_{ij}, \delta_{ij}. \]

Only contributions containing \[ \gamma_{ij}^m \]
\[ \propto \alpha^n (\rho^2 - \rho^2) \log^m \left( \frac{Q^2}{m^2} \right). \]

I.e. no 'nontrivial' contributions due to the Wilson coefficients

\[ \rightarrow \text{Solve evolution eqn. with running } \alpha_{\text{QED}} \text{ to some order}. \]

2) Non leading logs:

E.g. \[ \alpha^2 \log \left( \frac{Q^4}{m^2} \right). \]

Requires the knowledge of the complete \( \mathcal{O}(\alpha) \) calculation (same kin. variables).

\[ \rightarrow \text{Way to determine nontrivial parts in the Wilson coefficients}. \]
3. Different Variables

- Integration over the $PS^{(2+n)} \setminus PS^{(2)}$ phase space ($n \gamma$ hard bremsstrahlung $f\bar{f}, e^+e^-$ pair radiation).

**CHOICES:**

- (Classical): Lepton measurement: $Q_e^2, y_e$
- Double Angel Method
- Jet measurement NC $Q_J^2, y_J$
- Jet measurement CC $Q_J^2, y_J$
  - CC: Only way
- Mixed variables $Q_e^2, y_J$

All k-factors turn out to be different functions of $x$ and $y$. 
Table 1: The shifted variables for different types of cross section measurement

<table>
<thead>
<tr>
<th></th>
<th>$\hat{s}$</th>
<th>$Q^2$</th>
<th>$\hat{y}$</th>
<th>$J(x,y,z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>lepton measurement</td>
<td>$zs$</td>
<td>$Q^2 z$</td>
<td>$(z+y-1)/z$</td>
<td>$y/(z+y-1)$</td>
</tr>
<tr>
<td>jet measurement</td>
<td>$zs$</td>
<td>$Q^2(1-y)/(1-y/z)$</td>
<td>$y/z$</td>
<td>$(1-y)/(z-y)$</td>
</tr>
<tr>
<td>mixed variables</td>
<td>$zs$</td>
<td>$Q^2 z$</td>
<td>$y/z$</td>
<td>$1$</td>
</tr>
<tr>
<td>double angle method</td>
<td>$zs$</td>
<td>$Q^2 z^2$</td>
<td>$y$</td>
<td>$z$</td>
</tr>
<tr>
<td>$y_{JB}$ and $\theta_c$</td>
<td>$zs$</td>
<td>$Q^2 z(z-y)/(1-y)$</td>
<td>$y/z$</td>
<td>$(z-y)/(1-y)$</td>
</tr>
</tbody>
</table>

- $z_0$:
  
  **LEPTON MEASUREMENT**
  
  $\hat{x}(z_0) = 1$
  
  **JET MEASUREMENT**
  
  $z_0 = y$
  
  $\{ \delta(x \rightarrow 0, \theta^2 \rightarrow 0) \}$
  
  $z_0 = 0$. (for $z \rightarrow z_0$)

  **BUT:**

  $2E_e = E'_e(1 - \cos \theta_e) + E_j(1 - \cos \theta_j) \geq \Box$

  **FORTUNATELY:** $z_0 = \frac{\sqrt{4}}{2E_e}$.

$\Rightarrow$ **ZEUS:** THIS HELPS ONLY IN THE CASE OF THE DOUBLE ANGEL METHOD!
4. The Corrections up to $O(\alpha^2 L^2)$

**Contributions:**

1. Bremsstrahlung: Diagrams a, b
2. Electron Pair Production: Diagram c
3. Fermion Pair Production: $f = e, \mu, \tau, u, d, s, c, b$

\[ e \rightarrow \gamma \rightarrow e \quad a \quad b \]

\[ e \rightarrow e^+ e^- \quad c \quad d \]

\[ \text{NS} \quad \text{NS} \quad \text{S} \quad \text{RUNNING} \]

\[ \alpha_{\text{RED}} \]

- 'Radiator': 'Absorb' virtual corrections (physical gauge, ladder's, QCD techniques)
- Derivation of individual contributions to initial and final state radiation is possible.

- Leading log's: $O\left(\frac{\alpha}{2\pi} L^n\right)$
- Subleading log's: $\left(\frac{\alpha}{2\pi}\right)^n L^{n-1}$, $n > 1$.

RGE - method.
Figure 1: Diagrams contributing to the radiative corrections up to $O(\alpha^2 L^2)$.

Figure 2: $e^- \rightarrow e^+$ transition probability for different values of $Q^2$. Full line: $Q^2 = 10 \text{ GeV}^2$, dashed line: $Q^2 = 100 \text{ GeV}^2$, and dotted line: $Q^2 = 1000 \text{ GeV}^2$. 

\[ P(z, Q^2; e^- \rightarrow e^+) \]
\[ \log_{10}(z) \]
\[ e^-e^+ \text{ Conversion Probability} \]
THE CONTRIBUTIONS:

\[
\begin{aligned}
\frac{d^2\sigma^{(2)}}{dxdy} &= \frac{d\sigma^{(0)}}{dxdy} + \frac{d^2\sigma^{(1)}}{dxdy} + \frac{d^2\sigma^{(0)}}{dxdy} \\
\frac{d^2\sigma^{(1)}}{dxdy} &= \frac{\alpha}{2\pi} \ln \left( \frac{Q^2}{m_e^2} \right) \int_0^1 dz \ p_{ee}^{(1)}(z) \left\{ \theta(z-z_0) \ \mathcal{H}(x,y,z) \ \frac{d\sigma^{(0)}}{dxdy} \right\} - \frac{d^2\sigma^{(0)}}{dxdy} \\
\frac{d^2\sigma^{(2)}}{dxdy} &= \left( \frac{\alpha}{2\pi} \right)^2 \ln^2 \left( \frac{Q^2}{m_e^2} \right) \int_0^1 dz \ p_{ee}^{(2)}(z) \left\{ \theta(z-z_0) \ \mathcal{H}(x,y,z) \ \frac{d^2\sigma^{(0)}}{dxdy} \right\} - \frac{d^2\sigma^{(0)}}{dxdy} \\
&+ \left( \frac{\alpha}{2\pi} \right)^2 \int_0^1 dz \ln^2 \left( \frac{Q^2}{m_e^2} \right) p_{ee}^{(1)}(z) + \sum \ln^2 \left( \frac{Q^2}{m_i^2} \right) p_{ee,i}^{(1)}(z) \\
&\times \mathcal{H}(x,y,z) \ \frac{d\sigma^{(0)}}{dxdy} \\
\mathcal{H}(x,y,z) &= \frac{\sigma}{\theta(x) \theta(y)} \\
\end{aligned}
\]

RESCALING: SEE TABLE ABOVE.
SPLITTING FUNCTIONS:

\( z < 1 \):

\[ P_{ee}(z) = \frac{1 + z^2}{1 - z} \]

\( O(\alpha) \):

\[ P_{ee}^{(2,1)}(z) = \frac{1}{2} \left[ P_{ee}(z) \otimes P_{ee}(z) \right] \]
\[ = \frac{1 + z^2}{1 - z} \left[ 2 \ln(1 - z) - \ln z + \frac{3}{2} \right] + \frac{1}{2} (1 + z^2) \ln z - (1 - z) \]

\( O(\alpha^2) \):

\[ P_{ee}^{(4,2)}(z) = \frac{1}{2} \left[ P_{ee}(z) \otimes P_{ee}(z) \right] \]
\[ = (1 + z^2) \ln z + \frac{1}{2} (1 - z) + \frac{3}{2} \frac{4}{z} (1 - z) \]
\[ P_{ee}^{(2,3)}(z) = N_c(4) e_f^2 \frac{1}{3} P_{ee}(z) \Theta(1 - z - \frac{2m_4}{E_c}) \]

OTHER CONTRIBUTIONS: (UNIVERSAL, LARGE)

\[ \sum_f^0 x \delta_{\text{vac-pot}}(Q^2) \]
SOFT EXPONENTIATION:

SOLVE: LO-GRIBOV-LIPATOv eq. (NS) for $z \to 1$

$$D_{NS}(z, Q^2) = (1-z)^{-1} \exp \left[ \frac{1}{2} \zeta \left( \frac{1}{2} - 2\gamma_E \right) \right] \frac{\Gamma(1 + \zeta)}{\Gamma(1)}$$

(8)

with

$$\zeta = -3 \ln \left[ 1 - (\alpha/3\pi) \ln(Q^2/m_c^2) \right]$$

(9)

(RUNNING $\alpha_S$ED !)

$p_{>2, \text{soft}}^2(z, Q^2) = p_{NS}(z, Q^2) - \frac{\alpha}{2\pi} \ln \left( \frac{Q^2}{m_c^2} \right) \frac{2}{1-z} \left[ 1 + \frac{\alpha}{2\pi} \ln \left( \frac{Q^2}{m_c^2} \right) \left[ \frac{11}{6} + 2\ln(1-z) \right] \right]$

(10)

and

$$\frac{d^2\sigma^\text{ (>2,soft)}}{dz dy} = \int_0^1 dz P_{>2}^\text{ ( >2) } (z) \left\{ \theta(z - z_0) \mathcal{J}(x, y, z) \frac{d^2\sigma^{(0)}}{dz dy} |_{x=x, y=y, z=z} - \frac{d^2\sigma^{(0)}}{dz dy} \right\}$$

(11)

\[\rightarrow \text{NOTE: NO 'UNIQUE' EXPONENTIATION EXISTS!}\]
**Collinear Situations**

- $e$: initial state electron
- $e^\prime$: final state electron
- $q, \bar{q}$: initial state quark (antiquark)
- $q^\prime, \bar{q}^\prime$: final state quark (antiquark)

- Compton peak:
  \[ q^2(y^*) \leq \frac{\mu^2}{4} \]

**Leading Log Corrections**

Relevance of all these contributions in the $k$-factors.

- ISR lepton radiation remains (beam-hole).
Final State Electron Radiation

\[ \text{Collinear Situation} \quad \theta_{\text{rad}} \approx \theta_{e'} \]

Calorimetric measurement of the final state, final resolution

\( \pm \) Integrate radiated \( \gamma \) momenta & fermion momenta

\( \uparrow \) Measure \( e'' \) kinematics

KLN Theorem for rad. final state!

No logarithmic contributions!

\[ \rightarrow \text{Leptonic Final State} \]

\[ \rightarrow \text{Quark Final State} \]
QED Corrections: Radiation from Quarks

**Consider ISR now.**

- Every photon emission can be substituted by a gluon emission!
- $\alpha_s \gg \alpha$!
- QED correction to scaling violations

**In more detail:** $(m_q \to 0)$

\[
\tilde{P}_{ff}(z) = \frac{g_s}{2\pi} \frac{1+ z^2}{1- z} \left[ 1 + \frac{\alpha_s}{4 \pi} \frac{1}{\alpha_s C_F} \right]
\]

& similar other terms.

**Modify evolution equations!**

$\Lambda (1\%)$ effects.
The Compton Peak

\[ 1q^2 \leq M_p^2 \]

J.B, G. Levman, H. Spiesberger '93

THE SIGNATURE:

- \( \varphi : \sim \text{BACK TO BACK} \)
- \( \sim \text{BOOSTED} \ (e, \gamma) \text{PAIR AT FINITE } P_1 \)
- LITTLE HADRONIC ACTIVITY (SEEN ?)
  \( W^2 \sim M_p^2 \ldots \text{few } GeV^2 \)

**NO REAL DIS SIGNATURE!**

ADVANTAGE: LEAVE IT OUT FOR DIS RC'S
  (LEPTONIC VARIABLES)
  \( \sigma_{\text{NC,LEPT}} \text{ DIMINISHES!} \)

USE \( \sim \text{WINDOW!} \) TO \( F_{2L} \ (x \to 0, \, Q^2 \to 0) \)

NONPERTURBATIVE RANGE!
WHY DO WE WANT TO KNOW $F_1(x \to 0, Q^2 \to 0)$ AT ALL?

- WE DO NOT KNOW HOW TO PREDICT IT WITHIN QCD (SMALL $x$!).

1°: PRACTICAL REASON:

(OTHER) DIS RC'S: E.G. INITIAL STATE RADIATION

$$\frac{d\sigma}{dx \, dQ^2} \to \frac{d\sigma^{(0)}}{dx \, dQ^2} \left( \hat{Q}^2 \to zQ^2 \right)$$

↑ SMALL AT SMALL $x$!

INPUT: STANDARD RC'S.

FIG

- WE WANT TO SEE THE TRANSITION IN

$\sigma(\gamma^*p)$ FROM $Q^2 > 0$ TO $Q^2 \to 0$.

DIS $\longrightarrow$ PHOTOPRODUCTION

ALSO AT SMALL $x$. 
$F_2(x, Q^2)$ at low $Q^2$
Stein damping factor

$F_2^{LO}(x,Q^2)$

$x=0.0001$

$x=0.001$

$x=0.01$

$x=0.1$

$x=0.5$

$Q^2 \text{ GeV}^2$
Badelek/Kwiecinski damping

IVAR=5

$x=0.0001$
$x=0.001$
$x=0.01$
$x=0.1$
$x=0.5$

$F_{2}^{LO}(x, Q^{2})$

$Q^{2}$ GeV$^{2}$
THE CROSS SECTION:

\[ \frac{d^2\sigma}{dx_e dy_e} = \frac{\alpha^3}{x_e s} \frac{1 + (1 - y_e)^2}{1 - y_e} \int \frac{d\tau}{\tau} \int \frac{d^2Q_h^2}{Q_h^2} \frac{dz}{z} \cdot \left\{ \frac{1 + (1 - z)^2}{z^2} F_2 \left( \frac{x_e}{z}, Q_h^2 \right) - F_L \left( \frac{x_e}{z}, Q_h^2 \right) \right\} \]

no y-separation possible!

\[ \frac{d\sigma}{dx_e dy_e dz} \rightarrow \text{difficult to measure.} \]

\[ \frac{d^2\sigma}{dx_e dy_e} = \int \frac{dz}{z} \ D_{\gamma/p} (x_e, Q_h^2) \ \frac{d^2\hat{\sigma} (e\gamma \to e\gamma)}{d\hat{x} d\hat{y}_e} \Bigg|_{\hat{z} = \hat{s} \atop \hat{x} = \hat{x}_e/\hat{z}} \]

\[ \frac{d^2\hat{\sigma}}{d\hat{x} d\hat{y}_e} (e\gamma \to e\gamma) = \frac{2\pi d^2}{5} \ \frac{1 + (1 - y_e)^2}{1 - y_e} \delta(1 - \hat{x}) \]

TRANSV. PHOTONS : CALLEU-GROSS REL.: (APPR.)

\[ F_L = 0 \]

\[ D_{\gamma/p} (x_e, Q_h^2) = \frac{\alpha_e^2}{2\pi} \sum_{t \neq \bar{t}} \int \frac{dQ_h^2}{Q_h^2} \int \frac{dz}{z} \frac{d\sigma_{\gamma/p} (x_e, Q_h^2)}{dz} \]

\[ \times \sum_{t \neq \bar{t}} \frac{e_t^2}{e_0^2} \ \frac{x_e}{z} \ \frac{Q_t (x_e/\hat{z}, Q_h^2)}{F_2 (\hat{x}, Q_h^2)} \]

\[ Q_h^2 \sim O(M_W^2) \text{ or smaller!} \]
\[ P_{q_1 \bar{q}_1}(x) = \frac{1 + (1-x)^2}{x} \]

\[ \rightarrow \text{ONE CAN UNFOLD:} \]

\[ D_{q_1/\text{proton}}(x, q^2) \propto D_{q_1/\text{quark}}(x, q^2). \]

\[ \rightarrow \text{THIS IS A COMBINATION OF } F_2 \text{ AND } F_L \text{ AT SMALL } x \text{ AND } q^2. \]
Figure 1: Differential Compton cross section Eq. (8) as a function of $y$, for $x_1 = 10^{-4}$ (dotted line), $10^{-3}$ (dashed line), and $10^{-2}$ (full line).
Figure 2: The difference in azimuth of the photon and electron for accepted Compton events.

Figure 3: The hadronic mass distribution $W^2$ for accepted Compton events.
Figure 4: A scatter plot of $x_t = Q_t^2/2p \cdot (1 - l')$ and $x_M = M_x^2/s$.
Figure 5: The expected photon density after the extraction procedure described in the text. The errors represent the statistical errors for an integrated luminosity of 100 pb$^{-1}$. The solid histogram is the prediction of Eq. (13) for HMRSB [21]. The units of the vertical scale are arbitrary.
5. Numerical Results

PDF's: MRS D\(^{-}\), similar results
- GRV
- CTEQ2
- MRSA-new (June '94 after MRSH, MRSA).

\(O(\alpha L), O(\alpha^2 L^2)\).

Consider:
- ISR - leptons
  - for integrated calorimeter measurement
  - quarks: \(O(\alpha_s^2/\alpha)\) corr. to scaling violations

- \(O(\alpha)\): comparison with full \(O(\alpha)\) calculation.

\(\rightarrow\) Low \(Q^2\) behaviour of struct. fct.
\(\leftrightarrow\) RC's. measurement required!
Lepton Measurement

\[ \delta_N^{(1)}(x,y) \]

- \( Q^2 \) lept, yeup

- \( x = 10^{-2} \) dashed line
- \( x = 10^{-3} \) dotted line
- \( x = 10^{-4} \)
- \( x = 0.1 \)
- \( x = 0.3 \)
- \( x = 0.9 \)
Lepton Measurement

2nd order QED + soft exp.

$S^{(2)}_{NC}(x,y)$
Figure 3: Leptonic initial state radiative corrections $\delta_{NC}(x, y) = (d\sigma^{(2+>2,\text{jet})}/dzdy)/(d\sigma^0/dzdy)$ in LLA for $e^- p$ deep inelastic scattering in the case of jet measurement for $\sqrt{s} = 314$ GeV, $A = 0$, and $Q^2 \geq 5$ GeV$^2$. $O(\alpha^2)$ corrections: full lines: $x = 0.01$ and $x = 0.9$; dash–dotted line: $x = 10^{-4}$.

Contributions due to $e^- \rightarrow e^+$ conversion eq. (13), $\delta_{NC-e^+}^e(x, y) = (d\sigma^{(3,e^-\rightarrow e^+)}/dzdy)/(d\sigma^0/dzdy)$: upper dotted line: $z = 0.01$, lower dotted line: $z = 0.9$. Both graphs are scaled by $\times 100$; dashed line: $z = 10^{-4}$, scaled by $\times 500$. 
Figure 4: $\delta_{CC}(x, y) = (d\sigma_{CC}^{(2+>1,soft)}/dxdy)/(d\sigma_{CC}^{0}/dxdy)$ for deep inelastic $e^- p$ scattering in the case of jet measurement. Full lines: $O(\alpha^2)$ corrections for $z = 0.9, 0.01$ and $10^{-4}$. $\delta_{CC}^{e^{-}\rightarrow e^+}(x, y)$: dotted lines: $z = 10^{-2}$ and $z = 10^{-4}$. Both graphs are scaled by $\times 100$. Dashed line: $z = 0.9$ scaled by $\times 1000$. The other parameters are the same as in figure 3.
Jet Measurement
mixed variables

\[ \delta_{NC}(x,y) \]

- \( x = 0.5 \)
- \( x = 0.01 \)
- \( x = 0.0001 \)
Figure 5: \( \delta_{NC}(x, y) \) for the case of mixed variables. Full lines: \( O(\alpha^2) \) corrections for \( z = 0.5, z = 0.01 \) and \( z = 10^{-4} \). The latter graph is scaled by \( \times 20 \). \( \delta_{NC}^{-z^{+}}(x, y) \): upper dotted line: \( z = 0.5 \), lower dotted line: \( z = 0.01 \), dash-dotted line: \( z = 10^{-4} \) scaled by \( \times 100 \). The other parameters are the same as in figure 3.
Figure 6: $\delta_{NC}(x,y)$ for the case of the double angle method for $A = 35\text{ GeV}$. Full lines: $\delta_{NC}^{(1+2+\gamma+\gamma\gamma)_{2\gamma}}(x,y)$, dashed lines: $\delta_{NC}^{(1)}(x,y)$. Dotted lines: $\delta_{NC}^{e^-e^+}(x,y)$ scaled by $\times 100$; upper line: $x = 0.5$, middle line: $x = 0.01$, lower line: $x = 0.0001$. The other parameters are the same as in figure 3.
A DANGEROUS CASE: \( \theta_e \& y_f \)

**RESCALING:**

\[
\hat{Q}^2 = Q^2 \frac{z-y}{1-y}
\]
\[
\hat{x} = x \frac{z}{1-y} (z-y)
\]
\[
z_0 = y
\]

**ZEUSS:**

\[
z_0 = \max \left\{ \frac{35 \text{ GeV}}{2E_e}, y \right\}
\]

\[\delta_{nc}(x,y) \text{ JUMPS! AT } y \geq \frac{\sqrt{y}}{2E_e}, \sqrt{y} = 35 \text{ GeV.} \]

\[
\frac{\sigma'(Q^2, x \to 0)}{\sigma(Q^2, x)} ! \quad \text{NO CONTROL ON INPUT AT ALL !}
\]

---

**UNFORTUNATE CHOICE OF VARIABLES.**
Figure 7: $\delta_{NC}(x, y)$ for the measurement based on $\theta_e$ and $y_{JB}$ for $A = 35\text{ GeV}$. Full lines: $\delta_{NC}^{(1, 2, +, 2, 20, 200)}(x, y)$, dotted lines: $\delta_{NC}^{(1)}(x, y)$. Dashed lines: $\delta_{NC}^{x ightarrow e^+}(x, y)$ scaled by $\times 20$; upper line: $z = 0.5$, middle line: $z = 10^{-2}$, lower line: $z = 10^{-4}$. The other parameters are the same as in figure 3.
Figure 1: Comparison of complete $O(\alpha)$ and LLA calculation for leptonic variables
Figure 2: Comparison of complete $O(\alpha)$ and LLA calculation for jet variables
Figure 3: Comparison of complete $O(\alpha)$ and LLA calculation for mixed variables.
HECTOR — a program to calculate QED and electroweak corrections to $ep$ and $l^{\pm}N$ deep inelastic NC and CC scattering

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ABSTRACT

A description of the Fortran program HECTOR for a variety of semi-analytical calculations of radiative QED, QCD, and electroweak corrections to the double-differential cross sections of NC and CC deep-inelastic charged lepton–proton (or –deuteron) scattering is presented. HECTOR originates from the substantially improved and extended earlier programs HELIOS and TERAD91. It is mainly intended for the calculations at HERA or other $ep$–colliders, but may be also used for similar processes like muon–proton scattering in fixed–target experiments. The QED corrections may be calculated in several different sets of variables: leptonic, hadronic, mixed, Jaquet-Blondel, double angle etc. Besides the leading-logarithmic approximation up to order $O(\alpha^2)$, the exact $O(\alpha)$ corrections and soft-photon exponentiation are taken into account. The photoproduction region is also covered.

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HELIOS

TERAD 91

JB. + BARDIN et al.

UPGRADES & IMPROVEMENTS
QED, QCD, new variables,...

HADRON ELECTRON CODE TO CALCULATE 1ST AND HIGHER ORDER RADIATIVE CORRECTIONS.

HECTOR

ENGRAVING BY J. FLAXMAN 1790ies:

POLYDAMAS ADVISES HECTOR TO MAKE THE ASSAULT ON TO THE CAMP OF THE GREEKS ON FOOT:
6. Conclusions

1) Leading and nonleading logarithmic contributions to the radiative corrections to deep inelastic scattering can be determined studying the RGE-behaviour for the differential cross sections.

2) The LLA terms result from the evolution equations only. NLA terms in higher order req. the knowledge of complete lower order calculations.

3) K-factors are very different for different choices of Born level variables.

4) LLA terms to $O(xL^2)$ have been studied quantitatively for 6 choices used at HERA currently.

5) LO terms can be factorized in terms of radiating fermion lines.

6) The Compton term has been analysed; it is rather to be considered as an exclusive channel yielding a window to structure functions at small $Q^2$ & small $x$ than a contribution to the RC's.
7) Not every choice of Born level variables leads to stable RC's ($\theta_e, y_J$).
8) Leptonic variables require a complete solution of the evolution eqns. $\rightarrow$ high $y$.

9) The double angel method is most convenient from the point of view of the RC's: they are flat in $y$ for $x \approx 0.50$. And they are small. $\rightarrow$ effective Born level measurement.

10) $O(x_L)$ complete & $O(x_L)$ results are very close to each other numerically.