1993 St. Petersburg Winter School

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January 25 – February 5 1993

Structure Functions and Parton Distributions at HERA
STRUCTURE FUNCTIONS
AND
PARTON DISTRIBUTIONS
AT HERA

(AND OTHER HE-FACILITIES)

ST. PETERSBURG
JAN 25 FEB 5 1993
DEEP INELASTIC SCATTERING -
BASIC ISSUES

\[ l = e^\pm, \nu, \nu^\pm, \bar{\nu}, \bar{\nu}_e \]
\[ l' = e^\pm, \mu^\pm, \nu, \nu_e \]
\[ B = W^\pm, Z^0, \gamma \]

\[ N = p, n \]
\[ P \]
\[ \text{current jet.} \]
\[ \text{(remnant) diquark jet} \]
\[ \text{spectator jet} \]

\[ q_i \]

**VARIABLES:**

\[ Q^2 = -(l - l')^2 = -(p_{q_i} - p_{q_f})^2 \geq 0 \]
\[ x_B = \frac{Q^2}{2p_q} \]
\[ y_B = \frac{q \cdot p}{l \cdot P} \]

\[ s = (P + l)^2 \]
THE BORN CROSS SECTIONS

CHARGED LEPTONS:

NC:

\[
\frac{d^2\sigma}{dx dQ^2} = 2\pi \alpha^2 \frac{M_{NN}}{(S - H^2)^2} \frac{1}{Q^4} L_{\mu\nu} W_{\mu\nu}
\]

\(e^+N \rightarrow e^+X\) \((\mu^+N \rightarrow \mu^+X)\)

pure photon exchange:

\(L_{\mu\nu} = 2 \left[ l_\mu l_\nu' + l'_\mu l_\nu - g_{\mu\nu} l.l' \right]\)

\(W_{\mu\nu} = \frac{1}{4\pi} \sum_n \langle P | J_{\mu}^{em}(0) | n \rangle \langle n | J_{\nu}^{em}(0) | P \rangle (2\pi)^4 \delta^{(4)}(P + q - P_n)\)

\(W_{\mu\nu} = \left( -g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2} \right) W_1 (x_1, Q^2) + \frac{1}{M^2} \left[ (P_{\mu} - \frac{Pq}{q^2} q_{\mu}) (P_{\nu} - \frac{Pq}{q^2} q_{\nu}) \right] \cdot W_2 (x_1, Q^2)\)

\[F_2 (x_1, Q^2) = x \left( -g_{\mu\nu} + \frac{12x^2}{Q^2} P_{\mu} P_{\nu} \right) W_{\mu\nu}\]

\[F_L (x_1, Q^2) = \frac{8x^3}{Q^2} P_\mu P_\nu W_{\mu\nu}\]

\((O (\alpha_s^2))\)

2 STRUCTUREFCT.

\((O (\alpha_s^0))\) 1 STRUCT. FCT.

\[
\frac{d^2\sigma}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4} Y_+ F_2 (x_1, Q^2)
\]

\[Y_+ = 1 \pm (1\% y)^2\]
INCLUSION OF BEAM POLARIZATION

& EXCHANGE:

\[
\frac{d^2 \sigma^{\pm}}{d\omega dQ^2} = \frac{2\pi \alpha^2}{x Q^2} \left\{ Y_{+} \times F_2^{\pm}(x, Q^2) + Y_{x} \times F_3^{\pm}(x, Q^2) \right\}
\]

\[
F_2^{\pm}(x, Q^2) = F_2(x, Q^2) + \kappa_{\pm}(Q^2) (-\nu = \lambda a) \times G_2(x, Q^2)
\]

\[
+ \kappa_{\pm}^2(Q^2) (v^2 + a^2 \pm 2\lambda a) H_2(x, Q^2)
\]

\[
\times F_3^{\pm}(x, Q^2) = \kappa_{\pm}(Q^2) (\pm a + \lambda v) \times G_3(x, Q^2)
\]

\[
+ \kappa_{\pm}^2(Q^2) (\mp 2\nu + \lambda (v^2 + a^2)) \times H_3(x, Q^2)
\]

5 Structurefct. (without longitudinal)

+ 3 longitudinal Structurefct.

\[
\kappa_{\pm}(Q^2) = \frac{1}{4 \sin^2 \theta_w \cos^2 \theta_w} \frac{Q^2}{Q^2 + H_\pm^2}
\]

\[
a = a_e = -\frac{1}{2}
\]

\[
v = v_e = -\frac{1}{2} + 2 \sin^2 \theta_w
\]
\[ e^\pm (\nu^\pm)N \rightarrow \bar{\nu}_{e'}e' \]

\[
\frac{d^2\sigma^\pm}{dx dQ^2} = \frac{2\pi \alpha^2}{x Q^2} K_W(Q^2) \left( \frac{1 \pm \lambda}{2} \right) \left\{ \gamma^+ W_2^{\pm}(x, Q^2) \pm \gamma^- \bar{W}_3^{\pm} \right\}
\]

\[
K_W(Q^2) = \frac{Q^2}{Q^2 + M_W^2} \cdot \frac{1}{4 \sin^2 \theta_W}
\]

4 Structure fct.

+ 2 long. Structure fct.

Born: \( e^+p \quad \text{14 Structure functions!} \)

\( e^+d \quad \text{—} \)

(Composed out of: \( u, d, s, c, b \)
\( \bar{u}, \bar{d}, \bar{s}, c=\bar{c}, b=\bar{b} \) & \( g \)
\( \leq 10 \) parton densities)
\[
\frac{d^2\sigma}{dx\,dQ^2} = \frac{G_F^2 M_N^4}{4\pi^2 Q^4 (M_N^2 + Q^2)^2} \cdot \frac{1}{2} Y_+ F_2(x, Q^2) + Y_- F_3(x, Q^2)
\]

Note, however, that the kinematics in

\[\bar{\nu}N \rightarrow \bar{\nu}'X\]

is difficult to be determined precisely in fixed target experiments.
$\bar{\nu}_e N \rightarrow p^{\pm}(e^{\pm})X$

$$\frac{d\sigma_{\nu\bar{\nu}}}{dx dQ^2} = \frac{G_F^2 M_W^4}{4\pi x Q^4 (Q^2 + M_W^2)^2} \left[ Y_1 W_2^\nu \alpha_1 (Q^2) \pm Y_2 W_3^\nu \alpha_3 (Q^2) \right]$$

$$\frac{G_F^2 M_W^4}{4\pi} = \frac{2\pi \alpha^2}{16 \sin^4 \theta_W}.$$
KINEMATICS, LUMINOSITIES & EVENT RATES

CONSIDER & COMPARE

- HERA
- LEP x LHC
- UNK @ UNK fixed target (3 TeVp)

NOW
> 2000 AD (?)
(WOULD BE INTERESTING.)

HERA:
\[ s = 4 \cdot 30 \cdot 820 \text{ GeV}^2 \]
\[ = 4 \cdot 10 \cdot 300 \text{ GeV}^2 \]
\[ L_{\text{yr}} = 100 \text{ pb}^{-1} \text{ (future)} \]
\[ e^+ p, e^+d \text{ (future possible)} \]

LEP x LHC:
\[ s = 4 \cdot 8000 \cdot 100 \text{ GeV}^2 \text{ or } = 4 \cdot 2000 \cdot 50 \text{ GeV}^2 \]
\[ L_{\text{typical}} = 100 \text{ pb}^{-1} \text{ or } = 1 \text{ fb}^{-1} \]
\[ e^+ p, e^+ d \]

UNK 3 TeV:
\[ \langle E_\nu \rangle = 400...700 \text{ GeV} \]
depending on the magnetic system
\[ \phi_\nu = 2.5 \cdot 10^{-3}...10^{-4} \text{ m}^{-2} \text{ p}^{-1} \]
\[ 10^{14} \text{ p per 120 sec (\text{'cycle period'})} \]
Fig. 7. Comparison of the $s$-$Q^2$ ranges at SPS, UNK and HERA. Below the --- line: SPS range; shaded area: UNK range; the dashed line bounds the range accessible by the $e$-measurement, the dash-dotted line the range for the jet measurement at HERA for $s = 10^4$ GeV$^2$. 

LHC & LEP

UNK 

D FACILITY
Figure 10: Summary of the \( \{z, Q^2\} \) region explored.
Fig. 2. Distributions of the neutrino energy for the ideal focussed WBB's, Fig. 1 (p-target)
Eventrates

$p/d$ Facility
In UNK $^7$ Beam.

Figure 2: Integrated DIS cross-sections versus (a) a lower cut-off in $Q^2$ and (b) an upper cut-off in $x$. Charged and neutral current interactions are compared for different $e^- p$
QED RADIATIVE CORRECTIONS

- LARGE CONTRIBUTIONS
- DEPENDING ON THE CHOICE OF OUTER VARIABLES

\[ \text{DOMINATING: } \left( \frac{\alpha}{2\pi} \log \frac{Q^2}{m_e^2} \right) \times \left[ \left( \frac{\alpha}{2\pi} \log \frac{Q^2}{m_e^2} \right)^2 \right] \text{ terms} \]

A PEDAGOGICAL VIEW (WHICH FULLY SUFFICES)

LEADING LOGS.

RECIPE:

- ANALYZE ALL DIAGRAMS \( O(\alpha^3, \alpha^4, \ldots) \) FOR COLLINEAR SITUATIONS OF MASSLESS PARTICLES.
- WRITE FOR THE COLLINEAR TRANSITION THE ACCORDING SPLITTING FUNCTION (E.G. BREMSSSTRAHLUNG ETC.)
- NORMALIZE THE AMPLITUDES TO CONSERVE PROBABILITY, E.G.

\[ e \rightarrow e^- \quad P_{ee} (x) = \delta(1-x) + P_{ee}^{(\gamma)} (x) \frac{\alpha}{2\pi} + \ldots \]

\[ \int dx \ P_{ee} (x) = 1. \]
**DIAGRAMS:**

**NC:** ep→eX

**INITIAL STATE LEPTON**

**FINAL STATE LEPTON**

**RADIATION**

(ISL)  (FSL)

(ISQ)  (FSQ)

**COMPTON**

**SIMILAR:** $O(\alpha^2)$

**THERE ALSO**
BREMSSTRAHLUNG \( O \left( \frac{Q^2}{M^2} \right) \)

\[
\frac{d\sigma}{dxdy} = \frac{\alpha}{2\pi} \ln \left( \frac{Q^2}{m_e^2} \right) \int d\zeta \frac{1+\zeta^2}{1-\zeta} \int \frac{\theta(\zeta-z_0)}{dxdy} \frac{d\sigma_0}{dxdy} \frac{d\zeta}{dxdy} \frac{d\zeta}{dxdy}
\]

\( \mathcal{F} = \left| \begin{array}{ccc} \frac{\partial X}{\partial \bar{z}} & \frac{\partial \bar{y}}{\partial \bar{y}} \\ \frac{\partial \bar{z}}{\partial \bar{y}} & \frac{\partial \bar{y}}{\partial \bar{z}} \\ \frac{\partial X}{\partial \bar{z}} & \frac{\partial \bar{y}}{\partial \bar{z}} \end{array} \right| \)

INITIAL \( \hat{y} = \frac{2+y-1}{x} \)

\( \hat{Q}^2 = Q^2 x \)

\( \hat{z} = s x \)

\( \hat{\chi}(z_0) = 1 \)

FINAL \( \hat{y} = \frac{2+y-1}{z} \)

\( \hat{Q}^2 = Q^2 / x \)

\( \hat{z} = s z \)

\( \hat{\chi}(z_0) = 1 \)

Fig. 1 a–c. Comparison of the neutral current leading log radiative corrections [full lines] with the results of [3b] (dashed lines) for lepton bremsstrahlung at \( \sqrt{s} = 314 \) GeV for e⁻⁻p scattering. The ratios \( \hat{\sigma}_{\text{em}}(\bar{z}) \) denote the contribution \( \sigma(x) \) for a y-exchange, b y-Z interference, e Z-exchange normalized to the corresponding term in the F m \( \frac{d\omega}{dxdy} \) for a
\[ \frac{d^2\sigma^{ep}}{dy\,dQ^2} = \frac{d^2\sigma_0^{ep}}{dy\,dQ^2} \left( 1 + \delta^{ep}(y, Q^2) \right) \]

\( \mathcal{O}(\alpha) \) QED.
Fig. 2. $\delta_c = \frac{d^2\sigma^c}{dxdy/d^2\sigma_0(y)/dxdy}$ due to $\gamma^*e \rightarrow ye$ scattering as a function of $x$ and $y$ (10). The logarithmic term in (5) is $\sim \ln(\Omega^2/\Lambda^2)$ with $\Lambda = 200$ MeV.

$$\frac{d^2\sigma^c}{dxdy} = \frac{\alpha^3}{sx} \sum_f \frac{\mu_f(Q^2)}{\Lambda^2} \int \frac{d^2z}{z^3} \frac{1}{x} \left[ q_f(x, Q^2) + \bar{q}_f(x, Q^2) \right] \frac{z^2 + (x-z)^2}{x(1-y)} \cdot \gamma_+$$

Figure 2: Distribution of the Compton events versus the energy (a) and the polar angle (b) of the electron in the center of mass. The distribution of $W^2$ (c) and the difference in the electron's angles (d) are also shown.

**THE COMPTON PEAK**
Fig. 3. Comparison of the charged current leading log radiative corrections (full lines) with the results of [6] (dashed lines) due to lepton bremsstrahlung for $e^-p$ scattering. $\delta^e_C(W)$ denotes the contribution $O(\alpha)$ normalized to the Born cross section (4).

Figure 1: Comparison of the $R_C$s to $e^-p$ using the lepton or jet measurement: a) neutral current; b) charged current; c) Effect of the non-perturbative behaviour of quark distributions at low $Q^2$ (full line). The dashed line illustrates the extrapolation using the distributions [7].
Figure 2: $O(\alpha)$ leading log QED corrections to deep inelastic scattering using jet measurement at $\sqrt{s} = 314$ GeV in dependence of $x$ and $y$. a) neutral current scattering; b) charged current scattering.

**MIXED VARIABLES**

\[ \hat{y} = y/\tau, \quad \hat{s} = s/\tau \]
\[ \hat{Q}^2 = Q^2 \frac{1-y}{1-y/\tau} \]
\[ \hat{x} (\hat{\tau}^0) = 1 \]
\[ \delta_{FS} = 0 \]
BREMSSTRAHLUNG OFF QUARKS:

$ q; \quad \Rightarrow \quad g \quad \chi \quad x \quad g$

$ q; \quad \Rightarrow \quad g \quad \chi \quad x \quad g$

$KLN \rightarrow \equiv 0$

$q_i$: $P_{qq}$ - evolution of $q_i$ in INITIAL OR FINAL STATE!

SOLVE AP - EQU. WITH:

\[
P_{ff}(x) \Rightarrow \left(1 + \frac{3\alpha}{4\alpha_s}\right) P_{ff}(x)
\]

$\Rightarrow 0(\%)$ CORRECTION TO SCALING VIOLATIONS OF STRUCTURE FUNCTIONS.
Figure 7: QED leptonic corrections to $d\sigma/dm.dm$ in per cent. Full lines are complete $O(\alpha)$ results from TERAD91, the dashed lines represent results from the leading logarithmic approximation obtained from HELIOS.
WAYS TO EXTRACT STRUCTURE FUNCTIONS

CHARGED LEPTON - N DIS'S

\[ \sigma_{NC}^{\pm} = \frac{d\sigma_{NC}^{\pm}}{dx dq^2}, \quad \sigma_{CC}^{\pm} = \frac{d\sigma_{CC}^{\pm}}{dx dq^2} \]

**F_2(x_1q^2):**

\[ \sigma_{NC}^{\pm} \left( k_Z(q^2) \ll 1 \right) \]

i.e.

\[ \frac{Q^2}{Q^2 + M_Z^2} \ll 1, \quad Q^2 \ll M_Z^2 \]

\[ (Q^2 \leq 700 \text{ GeV}^2) \]

\[ F_2(x_1q^2) = \frac{x Q^4}{2 \pi \alpha^2} \frac{1}{Y_+} \cdot \sigma_{NC}^{\pm}(k_1q^2) \]

APPROACHING EVEN HIGHER \( Q^2 \):

\[ B_+(\lambda) = \frac{1}{2} \left[ \sigma_{NC}^{+}(\lambda) + \sigma_{NC}^{-}(-\lambda) \right] \frac{1}{Y_+} = F_2 + \left( -v + \lambda a \right) G_2 K_{43} \]

\[ + (v^2 + q^2 - 2vq\lambda) h_2 K_{2}^2 \]

\[ v/a = \lambda \approx 0 \]
\( \bar{\sigma}_{NC}(e^+) \) \( \bar{\sigma}_{em} \) 200 pb\(^{-1} \) \( \sqrt{s} = 314 \) GeV

\( F_2^{em} \)

\( x = 0.014 \)

\( x = 0.024 \)

\( x = 0.043 \)

\( x = 0.076 \)

\( x = 0.12 \)

\( x = 0.17 \)

\( x = 0.22 \)

\( x = 0.27 \)

\( x = 0.35 \)

\( x = 0.45 \)

\( x = 0.69 \)

\( Q^2 \) [GeV\(^2\)]
$\tilde{\sigma}_{NC}(e^+) \quad \frac{1}{100} \text{ pb}^{-1} \quad \sqrt{s}=314 \text{ GeV}$

$F^\text{em}_2$

1: $\bar{x}=1.7 \times 10^{-4}$
2: $\bar{x}=3.4 \times 10^{-4}$
3: $\bar{x}=7.1 \times 10^{-4}$
4: $\bar{x}=1.3 \times 10^{-3}$
5: $\bar{x}=3.3 \times 10^{-3}$
6: $\bar{x}=7.0 \times 10^{-3}$

$Q^2 \ [\text{GeV}^2]$
Figure 2: Expected HERA measurement of $F_2(x, Q^2)$ at lower $Q^2$ for a luminosity of 10 $pb^{-1}$ at 10 $\times$ 300 (a) and at higher $Q^2$ for 1000 $pb^{-1}$ at 45 $\times$ 1140 GeV$^2$ (b). Only statistical errors are shown. The solid points are obtainable with electron detection only ($y \geq 0.1$). The HERA data where simulated using the parametization $B^-$ of KMRS. The $x$ values above $x = 0.1$ are 0.14, 0.18, 0.22, 0.27, 0.35, 0.45, 0.55, 0.65. Below they are $(0.16, 0.24, 0.34, 0.5, 0.7) \cdot 10^{-n}$, $n = 1, 2, 3, 4$. 
Figure 6: $Q^2$ dependence of the scaled differential NC $e^+p$ cross-section at LEP+LHC for (a) $x > 10^{-3}$ and (b) $10^{-5} < x < 10^{-2}$. The full curves correspond to $\hat{\sigma}_{NC}(e^+)$, also represented by the MC data, while the dotted curves represent $F_1^{em}$, i.e. pure photon exchange, and show the pure QCD scaling violations. The full (open) MC data symbols are with (without) the restriction to the experimentally acceptable phase space region shown in Fig. 5.
\[ x G_3(x, Q^2) : \text{PROJECT OUT THE } \gamma\tau^- \text{ INTERFERENCE TERM.} \]

\[
B_-(\lambda) = \frac{1}{2} \frac{1}{Y^-(\rho, \lambda = 0)} \frac{1}{k_\tau(Q^0)} \left[ \sigma^+(\lambda) - \sigma^-(\lambda) \right]
\]

\[ = x G_3(x, Q^2) + k_\tau(-2\rho + \lambda(Q^2 + m^2)) \times H_3 \]

\[ \rightarrow \text{MEASUREMENT AT HIGH } Q^2! \]

\[ \text{LEP1 x LHC : } L = 1000 \text{ pb}^{-1} \]
\[ \text{LEP2 x LHC : } L = 100 \text{ pb}^{-1} \]

\[ \text{HERA} \]

\[ \text{LEP x LHC} \]
DEUTERON STRUCTURE FUNCTIONS

\[ e^+p \rightarrow e^+d \]

**NC:**
\[ e^+p \]

**CC:**
\[ W_2^{en} = \frac{1}{\gamma^2} \left[ \frac{\sigma^{cc}^+}{\lambda^+} + \frac{\sigma^{cc}^-}{\lambda^-} \right] \]
\[ W_3^{en} = \frac{1}{\gamma^2} \left[ \frac{\sigma^{cc}^+}{\lambda^+} - \frac{\sigma^{cc}^-}{\lambda^-} \right] \]

\[ e^+e^- \text{ requir.} \]
\[ L-\text{splits.} \]

\[ N_p (\bar{N}_e) d \]

**NC:**
\[ N_p \]
\[ \text{very difficult to measure in 2-dimensions (x, Q^2).} \]

**CC:**
\[ W_2^d = \frac{2\pi \times (M_W^2 + Q^2)^2}{G^2_F Y_+} \left\{ \sigma^{\nu d} + \sigma^{\bar{\nu} d} \right\} - \frac{2x}{Y_+} (s+b-c) \]
\[ W_3^d = \frac{2\pi \times (M_W^2 + Q^2)^2}{G^2_F Y_-} \left\{ \sigma^{\nu d} - \sigma^{\bar{\nu} d} \right\} \]
Figure 3: Statistical precision for a measurement of $xG_3(x, Q^2)$ (a) and of $xG_3(x)$ averaged over $Q^2$ in the accessible kinematical range (b) for $\mathcal{L} = 1 \text{ fb}^{-1}$.

Figure 4: Statistical precision for a measurement with deuterons of $P_2^{en}$ and $W_2^{en}$, for $\mathcal{L} = 100 \text{ pb}^{-1}$. 
\[ W_2^d = \frac{1}{2} \left[ W_2^{ud} + W_2^{\bar{u}\bar{d}} \right] \]

\[ xW_3^d = \frac{1}{2} \left[ xW_3^{ud} + xW_3^{\bar{u}\bar{d}} \right] \]

**Fig. 8.** Statistical preparation of \( W_3 \) in \( W^+ \)-WBB's, Eq. (5.2)

\[ U_{NK} - WBB \]

\[ \omega_{CC} \cong \frac{1}{2} \left( \chi^2 + \frac{k^2}{2} \right) \]

\( \gamma \chi W_3 \leq W_2 \)

treat as correction.

\[ W_2^- \text{, CC, ep} \]

\[ 100 \text{ pb}^{-1} \]
PARTON MODEL AND FLAVOUR CONTENTS
OF STRUCTURE FUNCTIONS

CHARGED LEPTON (BORN) STRUCTURE FCT.:

\[ F_2(x, Q^2) = \sum_q e_Q^2 \left[ q(x, Q^2) + \bar{q}(x, Q^2) \right] \]

\[ G_2(x, Q^2) = \sum_q 2e_Q v_q \left[ q(x, Q^2) + \bar{q}(x, Q^2) \right] \]

\[ H_2(x, Q^2) = \sum_q (v_q^2 + a_q^2) \left[ q(x, Q^2) + \bar{q}(x, Q^2) \right] \]

\[ xG_3(x, Q^2) = 2x \sum_q e_Q a_Q \left[ q(x, Q^2) - \bar{q}(x, Q^2) \right] \]

\[ xH_3(x, Q^2) = 2x \sum_q v_Q a_Q \left[ q(x, Q^2) - \bar{q}(x, Q^2) \right] \]

\[ W_2^+(x, Q^2) = 2x \sum_i [u_i(x, Q^2) + \bar{u}_i(x, Q^2)] \]

\[ W_2^-(x, Q^2) = 2x \sum_i [d_i(x, Q^2) + \bar{d}_i(x, Q^2)] \]

\[ xW_3^+(x, Q^2) = 2x \sum_i [u_i(x, Q^2) - \bar{d}_i(x, Q^2)] \]

\[ xW_3^-(x, Q^2) = 2x \sum_i [d_i(x, Q^2) - \bar{u}_i(x, Q^2)] \]

\[ \bar{u}_i = (\bar{u}, \bar{c}, \bar{t}) \]

\[ \bar{d}_i = (\bar{d}, \bar{s}, \bar{b}) \]
NEUTRINO (BORN) STRUCTURE FACTORS

\[ F_2^\nu(x_1 Q^2) = 2x \left[ a_{21} \sum_i (u_i + \bar{u}_i) + a_{22} \sum_i (d_i + \bar{d}_i) \right] \]

\[ = F_2^\nu(x_1 Q^2) \]

\[ x F_3^\nu(x_1 Q^2) = 2x \left[ a_{31} \sum_i (u_i - \bar{u}_i) + a_{32} \sum_i (d_i - \bar{d}_i) \right] \]

\[ = -x F_3^\nu(x_1 Q^2) \]

\[ a_{21} = \frac{1}{4} - e_u \sin^2 \theta_W + 2 e_u^2 \sin^4 \theta_W \]

\[ a_{22} = \frac{1}{4} + e_d \sin^2 \theta_W + 2 e_d^2 \sin^4 \theta_W \]

\[ a_{31} = \frac{1}{4} - e_u \sin^2 \theta_W \]

\[ a_{32} = \frac{1}{4} + e_d \sin^2 \theta_W \]

\[ W_2^\nu(x_1 Q^2) = 2x \sum_i (d_i + \bar{u}_i) \]

\[ x W_3^\nu(x_1 Q^2) = 2x \sum_i (d_i - \bar{u}_i) \]

\[ W_2^\nu(x_1 Q^2) = 2x \sum_i (u_i + \bar{d}_i) \]

\[ x W_3^\nu(x_1 Q^2) = 2x \sum_i (u_i - \bar{d}_i) \]
DEUTERONS & ISOVECTOR NUCLEI

d's at colliders: \( s \rightarrow s/2 \)

quark contents:

\[
\begin{align*}
\overline{u}_q, \overline{d}_q &= \overline{u}, \overline{d} \rightarrow \frac{1}{2} (u+d)
\end{align*}
\]

\( \rightarrow \) previous formulae modify accordingly.

EXAMPLES:

\[
\begin{align*}
F_2^{en} &= \frac{5}{18} x (u_v + d_v) + \frac{10}{9} x u_s + \frac{2}{9} x s + \frac{8}{9} x c + \frac{2}{9} x b \\
G_2^{en} &= \frac{1}{2} x (u_v + d_v) = \frac{1}{2} V \\
W_2^{e\pm d} &= x (u_v + d_v) + 4 x u_s + 2 x s + 2 x c + 2 x b = \sum \\
W_3^{e\pm d} &= x (u_v + d_v) \pm 2 x (s - c)
\end{align*}
\]
\[ W_{2}^{\nu d} = \sum_{i} x \left[ q_{i} (\alpha_{i} \alpha') + \bar{q}_{i} (\alpha_{i} \bar{\alpha'}) \right] = \sum \]

\[ \frac{1}{2} \left[ x W_{3}^{\nu d} + x W_{3}^{\bar{\nu} d} \right] = \frac{p_{f}}{p_{t}} \times W_{3}^{d} = \times (u_{\nu} + d_{\nu}) = \gamma \]
WAYS TO UNFOLD PARTON DENSITIES

\[ e^+ p \]

4 CROSS SECTIONS
\[ \sigma_{NC}^\pm, \sigma_{CC}^\pm \]

\[ \rightarrow \] 4 COMBINATIONS OF PARTON DENSITIES

LINEAR MAPPING:
\[ \vec{U} = \sum_i \vec{u}_i ; \quad \vec{D} = \sum_i \vec{d}_i \]

\[
\begin{pmatrix}
\frac{U}{U} \\
\frac{U}{D}
\end{pmatrix}
= (A_{ij})
\begin{pmatrix}
\sigma_{NC}^- \\
\sigma_{NC}^+ \\
\sigma_{CC}^+ \\
\sigma_{CC}^-
\end{pmatrix}
\]

\[ \text{det}_4 \left( A_{ij} \right) \sim \left\{ K_x (Q^2) \left[ 1 - (1-y)^4 \right] \right\}^{-1} \]

\[ (A_{ij}) \text{ becomes singular both for: } Q^2 \ll \Lambda^2_x \]
\[ y \ll 1 \]

CONSIDER e.g. (WITH ASSUMPTIONS ON SEA-QUARKS)

\[
\begin{pmatrix}
xu_N \\
x d_N \\
x s
\end{pmatrix}
= (B_{ij})
\begin{pmatrix}
\sigma_{NC}^- \\
\sigma_{NC}^+ \\
\sigma_{CC}^+ \\
\sigma_{CC}^-
\end{pmatrix}
\]
\[ f_{\pm} = \frac{1}{2} (\gamma_+ + \gamma_-) k_W^2 \]

\[ L = 100 \text{ pb}^{-1} \]
$e^+p \, \& \, e^+d$

$$\overline{Q} = \frac{1}{2} \left( W_{2e}^{en} - x W_{3e}^{en} \right)$$

$$x (s-c) = \frac{5}{18} W_2^{en} - F_2^{en}$$

$L = 50 \text{ pb}^{-1/\text{beam}}$
\[ x(u_y - d_y) = \frac{4\pi x}{G_F^2} \left( \frac{M_{W^2}^2 + Q^2}{M_W^2} \right)^2 \frac{1}{\gamma_+ + \gamma_-} \left[ \frac{1}{2} \sigma \nu d - \sigma^\nu_{\nu p} \right] \]

**Fig. 12.** Statistical precision of a measurement of $x(u_y - d_y)$ using Eq. (7.1)

\[ \sum_i x_q = \frac{Q}{2} = \frac{2\pi x}{G_F^2} \left( \frac{M_{W^2}^2 + Q^2}{M_W^2} \right)^2 \left[ \sigma \nu d - \sigma^\nu_{\nu p} (1-y) \right] \frac{1}{\gamma_+ + \gamma_-} - \frac{x(s+b+c)}{\gamma_+} \]

**Fig. 15.** Statistical precision of a measurement of the antiquark distribution Eq. (7.3)
HEAVY FLAVOURS

\( c, b, t \)

\[ \begin{align*}
  \text{e}^- & \rightarrow l' = e^-; \nu_e \\
  \gamma^0 (Z^0); W^- & \rightarrow Q' (m') \\
  g & \rightarrow Q (m) \\
  p & \rightarrow \text{crossed}
\end{align*} \]

Fig. 1

GLÜCK, REYA, GODBOLE

NC

CC

\( x = 0.01 \)

\( m_1 \text{ (GeV)} \)

\( x = 0.01 \)

\( Q^2 \text{ (GeV}^2) \)

Fig. 3a
Figure 1: The valence $u$-quark distribution at $Q^2 = M_W^2$ [4].

Figure 3: $F_2^{ep}$ structure function predictions.
Fig. 4. $F_2(x,Q^2 = 9\text{GeV}^2)$ as measured from NMC and BCDMS, compared with the extrapolation of the earlier KMRS and with the new MRS (labelled $D_0$) parametrization of parton densities.
Fig. 1. Comparison of first- and second-order evolved parton distributions. Plotted are $x$ times the probability distributions. Parton species and $Q$-values are as labeled. Initial distributions at $Q = 4.0$ GeV are taken from EHLQ set 1.
Fig. 3. Comparison of DIS-scheme and MS-bar scheme parton distributions. Plotted are $x$ times the probability distributions. Parton species and $Q$-values are as labeled. Initial distributions at $Q = 4.0$ GeV are taken from EHLQ set 1.
ACCESS TO THE GLUON: \( F_L, \sigma_{J/\psi}, \sigma_{c\bar{c}} \ldots \)

SO FAR: ACCESS TO QUARKS ONLY.

\[
\begin{align*}
&\rightarrow \not{\ell} \ell^0, \gamma \\
&g, \bar{g}
\end{align*}
\]

1) \( W^\pm, Z^0, u, d, s \) + comp. \( \bar{u}, \bar{d}, \bar{s} \)

2) NC, CC \( c, b \) \((c)(c)\) \( \bar{c}, \bar{b} \) \((s)(b)\)

GLUONS

3) \( J/\psi, \gamma \)

4) SCALING VIOLATIONS: next paragraph.
\( \mathcal{O}(\alpha_s) : \)

\[
F_L(x, Q^2) = \frac{\alpha_s}{2\pi} \left\{ \frac{8}{3} \int_0^1 \frac{dy}{y} \left( \frac{x}{y} \right)^2 F_2(y, Q^2) \ight\} + 2 \sum_{q, \bar{q}} e_q^2 \int_0^1 \frac{dy}{y} \left( \frac{x}{y} \right)^{1-\frac{1}{2}} y G(y, Q^2) \]

**ROBERTS :**

\[
x G(x, Q^2) \approx \frac{3}{5} \times 5.85 \left\{ \frac{3\pi}{4\alpha_s} F_L(0.4x, Q^2) - \frac{1}{2} F_2(0.8x, Q^2) \right\}
\]
LO QCD

**HERA**

![Graph](image1)

**UNK**

![Graph](image2)

*Fig. 11. Possible measurement of $R(x)$ using Eq. (6.1)*

**'y'-Dependence (Invariance)**

![Graph](image3)

**LEP x LHC**

![Graph](image4)
\[ xG(x, Q^2) \simeq 1.77 \frac{3\pi}{2\alpha_s(Q^2)} F_L(0.4x, Q^2) \]

**FIG. 8a**

\[ Q^2 = 50 \text{ GeV}^2 \]

Figure 4: Statistical precision of a possible measurement of \( xG(x) \)
$$R^{(q)}(x, Q^2) = \frac{F_L^{(q)}(x, Q^2)}{(1 + \frac{4H_0^2 x^2}{Q^2}) F_2^{(q)}(x, Q^2) - F_L^{(q)}(x, Q^2) + O(\alpha_s^2)}$$
Figure 16: The gluon density reconstructed from inelastic $J/\psi$ production for the input function of KMRS. The statistical error bars correspond to an integrated luminosity of 100 pb$^{-1}$. The curves show the input gluon density.

Figure 16

Reconstructed gluon densities from inclusive $D^{*+}$ production. The curves show the input gluon functions taken from Morfin and Tung [36]. The error bars include statistical errors for an integrated luminosity of 6 pb$^{-1}$.
QCD TESTS: $\alpha_s$, $\Lambda$ & THE GLUON DENSITY

EVOLUTION EQUATIONS: GLAP

NON-SINGLET DENSITIES:

$$\Delta_{ij}^{NS} = q_i(x_1, Q^2) - q_j(x_1, Q^2); \quad i, j \text{ arbitrary}$$

SINGLET - DENSITIES:

$$\sum = \sum_i [q_i(x_1, Q^2) + \bar{q}_i(x_1, Q^2)]$$

or flavour by flavour:

$$\sum_f = q_f(x_1, Q^2) + \bar{q}_f(x_1, Q^2)$$

$$G(x_1, Q^2) \quad \text{or} \quad \frac{1}{N_f} G(x_1, Q^2)$$

LO DIAGRAMS

COLLINEAR POLES

\[ \text{Diagram Images} \]

\[ \text{Diagram Images} \]
**STRONG ORDERING IN $k_\perp$:**

\[ Q^2 \gg k^2_{1\perp} \gg \ldots \gg k^2_{n\perp} \gg \Lambda^2, \quad p^2 = m_q^2 \]

1. **A SINGLE GLUON EMISSION:** \( q \rightarrow q \)

\[
\int \mathcal{A} q(x, Q^2) = \int \frac{1}{2} \frac{g^2(k^2)}{m^2} P(x) \frac{dk^2}{k^2} \otimes q(x_1, Q^2)
\]

\[
P(x) \otimes R(x) = \int_0^1 dx_1 \int_0^1 dx_2 \delta(x-x_1x_2) P(x_1) R(x_2)
\]

\[
P(x) = \frac{1}{3\pi^2} \frac{1+x^2}{1-x} = p_{qq} \cdot \frac{1}{3\pi^2}, \text{NS for } q \rightarrow q.
\]

\[
\frac{\partial q(x, Q^2)}{\partial \log Q^2} = \frac{1}{6\pi^2} \frac{\alpha_s(Q^2)}{4\pi} P_{qq}(x) \otimes q(x_1, Q^2)
\]

\[
C_F \cdot \frac{1}{2\pi} \quad \text{QCD: } C_F = \frac{4}{3}
\]

\[
\frac{d q_{\text{NS}}(x_1, Q^2)}{d \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} C_F \cdot P_{qq}(x) \otimes q_{\text{NS}}(x_1, Q^2)
\]

**EVOLUTION OPERATORS:**

\[
q_{\text{NS}}(x_1, Q^2) = E_{\text{NS}}(x_1, Q^2; \Lambda^2) \otimes q_{\text{NS}}(x_1, Q_0^2)
\]
\[
\frac{\partial E_{NS}(x, Q^2)}{\partial \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} C_F \left[ P_{qq}(x) \otimes E_{NS}(x, Q^2) \right]
\]

**LO Splitting Functions:**

\[
P_{qq}(x) = C_F \left[ \frac{1 + x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right]
\]

\[
P_{gq}(x) = C_F \frac{1 + (1-x)^2}{x}
\]

\[
P_{gg}(x) = T_R \left[ x^2 + (1-x)^4 \right]
\]

\[
P_{qq}(x) = 2N_C \left[ x(1-x) + \frac{1-x}{x} + \frac{x}{(1-x)_+} \right] + \delta(1-x) \cdot \frac{1}{2} \beta_0
\]

\[C_F = 4/3, \quad T_R = 1/2, \quad N_C = 3, \quad \beta_0 = M/3 \cdot N_C - 4/3 N_{FTR}.
\]

\[
\frac{\partial}{\partial \log Q^2} \begin{pmatrix} E_{FF} & E_{FG} \\ E_{GF} & E_{GG} \end{pmatrix} = \begin{pmatrix} P_{qq} & N_f P_{qg} \\ P_{eq} & P_{gg} \end{pmatrix} \begin{pmatrix} E_{FF} & E_{FG} \\ E_{GF} & E_{GG} \end{pmatrix}
\]

**Singlet**

**Similar Structure in NTLO:**

\[
P_{ij}(x, Q^2) = P_{ij}^{(0)}(x) + \frac{\alpha_s}{2\pi} P_{ij}^{(1)}(x) + \ldots
\]
\[ \frac{d f^a(x_i Q^2)}{d \ln(Q^2)} = P(x_i \frac{\alpha_s(Q^2)}{2\pi})_{ab} \otimes f_b(x_i Q^2) \]

\[ P(x_i \frac{\alpha_s}{2\pi})_{ab} = \frac{\alpha_s}{2\pi} \left\{ P_{ab}^0(x) + \frac{\alpha_s}{2\pi} P_{ab}^1(x) + \ldots \right\} \]

\[ x \ll 1 \]

**1st Order**

- **FF**
  \[ C_F \frac{1+x^2}{1-x} \]
  \[ \frac{1}{x} \quad 2N_f \quad T_R \quad C_F \quad \frac{20}{9} \]

- **FG**
  \[ 2N_f \quad T_R \quad [x^2 + (1-x)^2] \]
  \[ \frac{1}{x} \quad 2N_f \quad T_R \quad C_F \quad \frac{20}{9} \]

- **GF**
  \[ C_F \frac{1}{x} \quad [1 + (1-x)^2] \]
  \[ \frac{1}{x} \quad 2N_f \quad T_R \quad \left( -\frac{20}{9} \right) + C_F \quad C_F \]

- **GG**
  \[ 2 \quad C_F \quad \left[ \frac{1}{x} + \frac{1}{1-x} -2+x-x^2 \right] \]
  \[ \frac{1}{x} \quad 2N_f \quad T_R \quad \left( -\frac{23}{9} \right) \quad C_F + \frac{2}{3} \quad C_F \]
$e^\pm p$ (d)

$\sqrt{s} = 314$ GeV

- $\Lambda = 50$ MeV
- $\Lambda = 200$ MeV
- $\Lambda = 500$ MeV

Too large errors to allow a QCD analysis.

$F_{2}^{em}$

$\sqrt{s} = 314$ GeV

- total
- valence
- sea
- glue

Gluel dominted!
\( e^p: \text{GENERAL SITUATION} \)

\[
\begin{align*}
B_+ (x_1 Q^2) &= C_\Sigma (Q^2) \Sigma (x_1 Q^2) + C_\Delta (Q^2) \Delta (x_1 Q^2) \\
&= C_\Sigma (Q^2) \left[ E_{qq} (x_1 Q^2) \otimes \Sigma (x_1 Q_0^2) \\
&\quad + E_{gq} (x_1 Q^2) \otimes G(x_1 Q_0^2) \right] \\
&\quad + C_\Delta (Q^2) E_{NS} (x_1 Q^2) \otimes \Delta (x_1 Q^2) \\
Q^2/H_0^2 \to 0: &\quad C_\Sigma \to \frac{5}{18}, \quad C_\Delta \to \frac{1}{6}
\end{align*}
\]

\( \text{VALENCE RANGE:} \)

\[
\begin{align*}
B_+^{\text{VAL}} (x_1 Q^2) &= C_\Sigma (Q^2) \otimes E_{qq} (x_1 Q^2) \otimes \Sigma^{\text{val}} (x_1 Q_0^2) \\
&\quad + C_\Delta (Q^2) \otimes E_{NS} (x_1 Q^2) \otimes \Delta^{\text{val}} (x_1 Q_0^2) \\
\text{for} &\quad C_\Sigma \approx 5/18, \quad C_\Delta \approx 1/6 \text{ & LO: } E_{qq} = E_{NS}
\end{align*}
\]

\[
B_+^{\text{VAL}} (x_1 Q^2) = E_{NS} (x_1 Q^2) \otimes \left[ \frac{5}{18} \Sigma^{\text{val}} (x_1 Q_0^2) + \frac{1}{6} \Delta^{\text{val}} (x_1 Q_0^2) \right]
\]
QCD - ANALYSIS:

νN-SCATTERING

ONLY $\nu e^+ N \rightarrow e^+(n\pi) X$ REACTIONS MAY BE MEASURED TO THE REQUIRED PRECISION FOR A QCD TEST.

i) NON-SINGLET ANALYSIS:

OBSERVABLE:  \[
x W^d_3(x_1 Q^2) = \frac{1}{2} \left[ x W_3^{ud}(x_1 Q^2) + x W_3^{d\bar{d}}(x_1 Q^2) \right].
\]

\[
\frac{\partial x W^d_3(x_1 Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} P_{NS}(x) \otimes x W^d_3(x_1 Q^2)
\]

\[
\chi^2 := \sum_{\text{bins}} \left[ \frac{x W_3^{\text{exp}}(x_1 Q^2) - E_{NS}(\Lambda_{1}, x_1 Q^2, Q^2_0) \otimes x W_3(x_1 Q^2_0)}{\delta W_3^{\text{exp}}(x_1 Q^2)} \right]^2
\]

with:

\[
x W_3(x_1 Q^2) = E_{NS}(x_1 Q^2) \otimes x W_3(x_1 Q^2_0)
\]

\[
\Lambda_{QCD} = \Lambda_{QCD} \quad \text{NO CORREL. TO } x G(x_1 Q^2).
\]
2) **COMBINED SINGLET & NON-SINGLET ANALYSIS**:

**OBSERVABLES**:

\[
\begin{align*}
\times W_3^d &= \Sigma \\
W_2^d &= \Sigma \\
\overline{Q} &= \Sigma x_i \overline{q}_i
\end{align*}
\]

\[
\begin{align*}
\times W_3(x_1 \alpha_2^2) &= E_{NS}(x_1 \alpha_2^2) \otimes V \\
W_2(x_1 \alpha_2^2) &= E_{FF}(x_1 \alpha_2^2) \otimes (V + S) + E_{FG}(x_1 \alpha_2^2) \otimes G \\
\overline{Q}(x_1 \alpha_2^2) &= (E_{FF} - E_{NS})(x_1 \alpha_2^2) \otimes V \\
&\quad + E_{FF}(x_1 \alpha_2^2) \otimes S + E_{FG}(x_1 \alpha_2^2) \otimes G \\
V_i(S, G) &= V(x_1 \alpha_2^2) (S(x_1 \alpha_2^2), G(x_1 \alpha_2^2))
\end{align*}
\]
Figure 6: (a) Average slope \( \frac{\partial F_2^p}{\partial \ln Q^2} \) versus \( x \) using the KMRS distribution \( B^- \) in the valence range. The inner error bars represent the statistical error for \( 100 \, pb^{-1} \) with a systematical error of 5% superimposed. (b) Dependence of \( \delta \Lambda_{\text{stat}} \) for \( x \geq 0.25 \) \( 100 \, pb^{-1} \) and \( \sqrt{s} = 110 \, GeV \) on the minimum jet angle. Dependence of \( \delta \Lambda_{\text{stat}} \) on the minimum \( Q^2 \) (c) and \( x \) (d) used in the QCD fit for the combined data sets at \( \sqrt{s} = 110 \) and 314 GeV for \( \mathcal{L} = 100 \, pb^{-1} \) each.
NECESSITY OF CROSS CALIBRATION OF CALORIMETERS

Figure 1: $\delta_{sys} = (d^2 \sigma_{ee}/dxdy)/(d^2 \sigma_{had}/dxdy)$ for displacements of $\epsilon_{ee} = -1\%$ with
$$\frac{d^2 \sigma}{dxdy}(x) = \frac{d^2 \sigma}{dxdy}(1 + \epsilon).$$

<table>
<thead>
<tr>
<th>electromagnetic calorimeter</th>
<th>hadronic calorimeter</th>
<th>$\mathcal{L}$ in $pb^{-1}$</th>
<th>$\sqrt{s} = 314 , GeV$</th>
<th>$\sqrt{s} = 190 , GeV$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEMC</td>
<td>CB</td>
<td>10</td>
<td>0.0049</td>
<td>0.0050</td>
</tr>
<tr>
<td></td>
<td>CB</td>
<td>10</td>
<td>0.0053</td>
<td>0.0058</td>
</tr>
<tr>
<td>CB</td>
<td>CB</td>
<td>10</td>
<td>0.0128</td>
<td>0.0130</td>
</tr>
<tr>
<td>CB</td>
<td>CB</td>
<td>10</td>
<td>0.0128</td>
<td>0.0130</td>
</tr>
<tr>
<td>FB/OF</td>
<td>CB</td>
<td>100</td>
<td>0.0158</td>
<td>0.0130</td>
</tr>
<tr>
<td>BEMC</td>
<td>all</td>
<td>10</td>
<td>0.0025</td>
<td>0.0026</td>
</tr>
<tr>
<td>BBE</td>
<td>all</td>
<td>10</td>
<td>0.0073</td>
<td>0.0085</td>
</tr>
<tr>
<td>CB</td>
<td>all</td>
<td>10</td>
<td>0.0031</td>
<td>0.0028</td>
</tr>
<tr>
<td>OF and IF</td>
<td>all</td>
<td>100</td>
<td>0.0258</td>
<td>0.0324</td>
</tr>
</tbody>
</table>

Table 2: Accuracies of $\epsilon_{ee}$ and $\epsilon_{ij}$ using $d^2 \sigma_{ee}/dxdy$.

$\delta \epsilon_{ee}$ & $\delta \epsilon_{en}$ could have a systematic impact on $\Delta \lambda = \pm 50\ldots150$ km!
Figure 8: Possible determination of $xG(x, Q_0^2)$ in a QCD fit for $x < 0.1$, see text. The upper error band corresponds to the choice $\alpha = -0.5$ and the lower band to $\alpha = 0$, see eq. 15. The inner error denotes the statistical error for $L = 100$ pb$^{-1}$ for both the low and high $s$ option.
Figure 7: Dependence of $\alpha_s$ on $Q^2$ from a combined fit using two samples of $\sqrt{s} = 314$ GeV and $\sqrt{s} = 110$ GeV with $L = 100$ pb$^{-1}$ each. The upper point corresponds to a nonsinglet fit for $\theta > 5\degree$ and $x > 0.25$. The lower point at $Q^2 \sim 50$ GeV$^2$ corresponds to a fit in the range $x < 0.25$.

Fig. 16. The dependence of the strong coupling constant $\alpha_s$ on $Q^2$ for measurements at UNK and HERA. UNK: $\Delta$ eW, HERA: (cf. [6]) eG fixed
- $x > 0.01$ $z = 12000$ GeV
- $x > 0.25$ $z = 12000$ GeV
- $x > 0.01$ $z = 98000$ GeV
- $x > 0.25$ $z = 98000$ GeV

LEP x LHC

$\sqrt{s}$ [GeV]
- 314
- 1300

$\alpha_s (Q^2)$
- $\Lambda$ [MeV]
  - 300
  - 200
  - 100
### Table 4. Processes and Observables from which significant determinations of $\alpha_s$ are derived.

<table>
<thead>
<tr>
<th>Process</th>
<th>Observable</th>
<th>Theory</th>
<th>Caveats</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^-$</td>
<td>hadronic event shapes, jet production rates, energy correlations</td>
<td>NLO and resummed NLO</td>
<td>hadronization corrections</td>
</tr>
<tr>
<td></td>
<td>$R_Z = \frac{\Gamma(\gamma^* \rightarrow \text{hadrons})}{\Gamma(\gamma^* \rightarrow \text{leptons})}$</td>
<td>NNLO</td>
<td>small QCD corrections</td>
</tr>
<tr>
<td></td>
<td>$R_\tau = \frac{Br(\tau \rightarrow \text{hadrons})}{Br(\tau \rightarrow \text{eV})}$</td>
<td>NNLO</td>
<td>nonperturbative corrections</td>
</tr>
<tr>
<td></td>
<td>scaling violations in $\frac{d\sigma}{dx}$ spectra</td>
<td>NLO</td>
<td>only through MC models</td>
</tr>
<tr>
<td>DIS</td>
<td>$\frac{d\ln F_2(x,Q^2)}{d\ln Q^2}$; $\cdots$ ; $J/\Psi$; ...</td>
<td>NLO</td>
<td>relativistic corrections</td>
</tr>
<tr>
<td>p$\bar{p}$</td>
<td>$p\bar{p} \rightarrow W + \text{jets}$</td>
<td>NLO</td>
<td>higher twist; $g(x,Q^2)$</td>
</tr>
<tr>
<td></td>
<td>$p\bar{p} \rightarrow b\bar{b}X$</td>
<td>NLO</td>
<td>statistics; $k$-factors</td>
</tr>
<tr>
<td>c$\bar{c}$ states</td>
<td>mass difference of 1s and 1p charmonium states</td>
<td>lattice gauge theory</td>
<td>quenched approximation</td>
</tr>
</tbody>
</table>
Fig. 31. Summary of measurements of $\alpha_s(M_{Z^0})$. 
THE ONSET OF SHADOWING
AT SMALL X

- SCREENING : RESTORING UNITARITY
  'PARTON RECOMBINATION'

GLR
& SUBSEQUENT WORK

⇒ QUANTIFICATION.

⇒ STRATEGY TO SEE THESE EFFECTS
IN $F_2^{em}(x_1 Q^2)$ ⇔ $\Lambda, x \in G(x_1 Q_0^2)$!
\[ \frac{d x q_s(x, Q^2)}{d \ln Q^2} = \frac{\alpha_s}{2\pi} \left[ P_{qg} \otimes xG \quad P_{qq} \otimes xq_s \right] \]

\[ - \frac{27 \alpha_s^2}{160 R^2 Q^2} \left( x G \alpha_s Q^4 \right)^2 \]

\[ + \frac{\alpha_s}{16 Q^2} \theta(x_0 - x) \int_{x_0}^{x_0} \frac{dx'}{x'} \gamma \left( \frac{x}{x'} \right) x G_{H} x' \alpha_s^2 \]

\[ \gamma(y) = -2y + 15y^2 - 30y^3 + 18y^4 \]

\[ \frac{d x G_H(x, Q^2)}{d \ln Q^2} = -\frac{81 \alpha_s^2}{16 R^2} \theta(x_0 - x) \int_{x_0}^{x_0} \frac{dx'}{x'} \left[ x' G(x', Q^4) \right]^2 \]

\[ \frac{d x G(x, Q^2)}{d \ln Q^2} = \frac{\alpha_s}{2\pi} \left[ P_{qg} \otimes xG + P_{qq} \otimes xq \right] \]

\[ - \frac{81 \alpha_s}{16 R^2 Q^2} \theta(x_0 - x) \int_{x_0}^{x_0} \frac{dx'}{x'} \left[ x' G(x', Q^4) \right]^2 \]

**USED IN: KMRS**

**MODIFICATIONS CURRENTLY WORKED OUT!**
TRAJECTORIES STARTING BELOW

ABOVE THE 'CRITICAL' LINE
\( \frac{a_F^2}{a_G^2} \)

- \( \text{GRV LO} \)
- \( \text{GRV NTLO} \)
- \( \text{KMRS } B^- \)
- \( \text{KMRS } B^-, R = 5 \text{ GeV}^{-1} \)
- \( \text{KMRS } B^-, R = 2 \text{ GeV}^{-1} \)
- \( \text{KMRS } B^0 \)

\( \mathcal{L} = 100 \text{ pb}^{-1} \)

1\% SYST. ERROR

SUPERIMPOSED

\( \text{SOFT SCREENING} \)

\( \text{HOT SPOT SCREENING} \)

[\sim \text{AP range}]
FIRST RESULTS FROM HERA:
DIS AT ZEUS & H1

ZEUS

LUMINOSITIES: \( L = 2.1 \text{ nb}^{-1} \)

H1

NEW DATA \( \rightarrow \times 10 \)

\( E_e = 26.7 \text{ GeV} \)

\( E_p = 820 \text{ GeV} \)

\( 5 \leq Q^2 \leq 50 \text{ GeV}^2 \) dominantly, \( Q_{\text{max}}^2 > 800 \text{ GeV}^2 \)

\( 10^{-4} \leq x \leq 10^{-2} \)

27.11.92, H1: \( Q^2 = 2600 \text{ GeV}^2 \)
$Q^2 = 2600 \text{GeV}^2 \quad y = 0.16 \quad x = 0.18$
Experiment ZEUS
Experimental Hall HERA South
Experimentierhalle HERA-S2f

Dimensions: 11.6 x 10.8 x 20.0 m
Aussmaß: 11.6 x 10.8 x 20.0 m

Total weight: 3600 tons
Gesamtgewicht: 3600 t
Figure 3: Electron energy spectrum of the DIS events compared with a Monte Carlo simulation [13,14] of the H1 detector using the parametrisation MRSD0 [15]. The simulated spectrum is normalised to the measured integrated luminosity of 1.3 nb$^{-1}$.
SUMMARY

1) DIS experiments in the future will extend our insight in the proton/nucleon structure going to:
   - Smaller $x$
   - Higher $Q^2$
   - Using the whole flavour variety

2) Current study: HERA: $x \gtrsim 10^{-4}$
   $Q^2 \approx 10^4 \text{GeV}^2$

   Future possibilities:
   - LEP1,2 × LHC
   - $\nu$ beams in the TeV fixed target range

3) QED & EW radiative corrections are under full control.

4) HO QCD corrections are still to be worked out.
   Low $x$: yet a status nascendi for theory (QCD)?!

5) Various SF can be measured, allowing to unfold quark densities

6) $xG(x, Q^2)$ may be derived (with some assumptions: $\psi$, $\phi$, $\psi^\prime$)
   From: scaling violations, $F_L$, $G_\psi$, $G_{\psi^\prime}$, $\sigma_\psi$. 
7) $\alpha_s(Q^2) \& \Lambda$ may be measured. Running of $\alpha_s$ may be established at the statistical level in long term.

8) Shadowing may be inferred in $F_2^{\text{em}}$ combined with a careful QCD analysis.
Further Reading

1 Reports and Textbooks


2 Recent Conference Proceedings


