INTEGRAL RELATIONS BETWEEN
POLARIZED STRUCTURE FUNCTIONS

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1) INTRODUCTION

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     STRUCTURALLY DIFFERENT?

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   - A REMARK ON ELT

5) CONCLUSIONS

JB, A. Tkabladze, NPB '99 hep-ph/9912181
1. Introduction

How many structure functions are determining the DIS cross section?

→ Exp.
→ Thy.

Lorentz structure:

\[ W_{\mu\nu} = \left( -g_{\mu\nu} + \frac{q_{\mu} q_{\nu}}{q^2} \right) F_1 + \frac{\hat{p}_\mu \hat{p}_\nu}{p.q} F_2 \]

\[ -i \varepsilon_{\mu\nu\sigma\rho} \frac{q_{\rho} p_{\sigma}}{2p.q} F_3 \]

\[ + i \varepsilon_{\mu\nu\sigma\rho} \frac{q_{\rho} s_{\sigma}}{p.q} g_4 + i \varepsilon_{\nu\rho\sigma\delta} \frac{q^8 (p.q s^\delta - s q^\delta)}{(p.q)^2} g_5 \]

\[ \left\{ \frac{(\hat{p}_\nu s_{\nu} + s_{\nu} \hat{p}_\nu - s.q \frac{\hat{p}_\rho \hat{p}_\nu}{p.q}}{2} \right\} g_3 \]

\[ + s.q \frac{\hat{p}_\rho \hat{p}_\nu}{(p.q)^2} g_4 + \left( -g_{\mu\nu} + \frac{q_{\mu} q_{\nu}}{q^2} \right) \frac{s.q}{p.q} g_5 \]

+ CURRENT NONCONSERV. TERMS

\[ \hat{R}_\mu = R_\mu - \frac{R.q}{q^2} q_\mu \]

S = NUCLEON SPIN.
CONSIDER $m_q \to 0$: CURRENT CONSERVATION ALSO FOR WEAK TERMS.

1ST CONTRIBUTIONS: QUARKONIC OPERATORS.

NUMBER OF STRUCTURE FUNCTIONS:

A) UNPOLARIZED:

$|\gamma|^2 \quad 2$

$|\gamma + \bar{\gamma}|^2, |W|^2 \quad 3$ (AND MORE FLAVOR + PROPAG. STRUCTURE)

B) POLARIZED:

$|\gamma|^2 \quad 2$

$|\gamma + \bar{\gamma}|^2, |W|^2 \quad 5$

↑

LORENTZ STRUCTURE.
- WHAT IMPLIES QCD (SO FAR LO)?

- RELATIONS BETWEEN STRUCTURE FUNCTIONS
  -→ MINIMAL REPRESENTATIONS

- PREDICTION FOR EXPERIMENT
  ←→ EXPERIMENTAL TESTGROUND FOR THEORY!

- WHAT ARE THE CONTRIBUTIONS AT:
  - TWIST 2
  - TWIST 3?
2. LIGHT CONE EXPANSION

FORWARD COMPTON AMPLITUDE:

\[ T_{\mu \nu}^i = T \left( J_{\mu}^{a^+} (x) J_{\nu}^{i2} (0) \right) \]

\[ T_{\mu \nu}^{NC} \equiv T_{\mu \nu}^{\text{cc},+} \]

\[ T_{\mu \nu}^{\text{cc}, \pm} = T_{\mu \nu}^{W^-} \pm T_{\mu \nu}^{W^+} \]

\[ S(x) \rightarrow \frac{2i x^2}{(2\pi)^2 (x^2 - i0)^2} \]

POLARIZED CASE:

O TWIST DECOMPOSITION

LO:

- TWIST 2
- TWIST 3

TWIST 2: \( \text{PARTON MODEL} \) \( (\text{COVARIANT}) \) = LOE

\[ \text{JACKSON, ROBERTS, ROSS} \]

\[ \text{BLÜHLEIN, KOCHELY} \]

TWIST 3 ?
TECHNICAL STEPS:

1) LIGHT CONE EXPANSION (LCE)

2) DISPERSION RELATIONS

3) LOCAL LCE

4) RELATIONS BETWEEN STRUCTURE FUNCTIONS
   
   EQUATING THE NONPERTURBATIVE OPERATOR MATRIX ELEMENTS
   
   \( a_n^\pm, d_n^\pm \) resp.
3. Relations between structure functions:

Twist 2

5 Structure Functions

2. Basic Operators @ leading order (LO)

3. Relations between the twist 2 parts of polarized structure functions

Operator: (symmetric)

\[ \Theta_{\pm}^{\mu_1 \mu_2 \cdots \mu_n} = \frac{1}{n+1} \left[ \Theta_{\pm}^{\mu_1 \mu_2 \cdots \mu_n} + \cdots + \Theta_{\pm}^{\mu_n \mu_1 \cdots \mu_{n-1}} \right] \]

\[ (g_V^q)^2 + (g_A^q)^2 \] (couplings) \[ g_1, g_2 \]

\[ 2g_V^q g_A^q \] \[ g_3, g_4, g_5 \]
3 RELATIONS @ TWIIST 2

\( g_4^i (x, Q^2) = 2x g_5^i (x, Q^2) \quad \text{Dicus '72} \)

\( g_2^i (x, Q^2) = -g_1^i (x, Q^2) + \int \frac{dy}{y} g_1^i (y, Q^2) \quad \text{Wandzura - Wilczek '77} \)

\( g_3^i (x, Q^2) = 2x \int \frac{dy}{y^2} g_4^i (x, Q^2) \)

\( = 4x^2 \int \frac{dy}{y^2} g_5^i (x, Q^2) \quad \text{JB, Kochelev '96} \)

\[ \rightarrow \ \text{STABLE, NON-DESTRUCTIVE} \quad M \rightarrow 0 \ \text{LIMIT} \]

\( \text{(NUCLEON MASS)} \)

Basis: \( g_1^i (x, Q^2), \quad g_5^i (x, Q^2) \)

\( M^2/Q^2 \ll 1 : \quad \text{JB, A. Tkabladze DESY 98-181} \)

\( \text{NPB IN PR.} \)

- \text{CORRECTION TERM TO DICUS RELATION}
- \text{COMMUTATION OF TARGET MASS RESUMMATION}
  & \text{K WW-RELATION, BK-RELATION RESP.}
  \uparrow
  \text{ALSO INCLUDING QUARK HAMSES.}
WHY ARE THE WANDZURA-WILCZEK AND CALLEN-GROSS RELATION STRUCTURALLY DIFFERENT?

CONSIDER THE COMPTON AMPLITUDE FOR NON-FORWARD SCATTERING.

\[ T_{\gamma \nu} (q, p_{+}, p_{-}) \rightarrow T_{\gamma \nu}^{\text{form.}} (p, q) \]

\[ \text{like: p}_{-} \rightarrow p \]

JB, B. Geyer, D. Dobaschik
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\[ T_{\gamma \nu} (q, p_{+}, p_{-}) = T_{\gamma \nu}^{\text{symm}} + T_{\gamma \nu}^{\text{asymm}} \]

\[ T_{\gamma \nu}^{\text{symm}} = (q \nu - \frac{P_{\nu} q_{\nu} + P_{\gamma} q_{\gamma}}{p \cdot q}) \int \mathcal{D} \mathbf{z} \]

\[ \left\{ \left( \frac{1}{\xi + z_{+} - i\epsilon} - \frac{1}{\xi - z_{+} - i\epsilon} \right) \right\} - i \frac{1}{z_{+} \left( \frac{1}{(\xi + z_{+} + i\epsilon)^{2}} + \frac{1}{(\xi - z_{+} - i\epsilon)^{2}} \right) } G(z_{\nu}, \xi) \]

\[ + \frac{1}{2} \int \mathcal{D} \mathbf{t} \hat{g}(t) \]

\[ \int \mathcal{D} t_{+} G(t_{+}, z_{-}) = \int \frac{dt_{+}}{t_{+}} \hat{g}(t) \]

\[ + \frac{1}{2} \int \mathcal{D} \mathbf{t} \hat{g}(t) \]

\[ \uparrow \text{partonic interpretation.} \]
\[ \int_{-1}^{1} \frac{dz_+}{\sqrt{(\xi + z_+ - i\varepsilon)^2}} = \frac{1}{\sqrt{z_+}} \left( \frac{d^2}{dz_+^2} \hat{g}^{(2)}(z) \right) \]

I.E.: THE 2ND TERM CONTAINS INTEGRAL CONTRIBUTIONS AS WELL, WHICH CANCEL IN THE FORWARD CASE. INTEGRAL TERMS ARE NORMALLY THERE, BUT MAY CANCEL IN SPECIAL SITUATIONS.

\[ T_{\mu\nu} = i\varepsilon_{\mu\nu} \frac{q \times p}{(p-q)^2} q.S \times \]

\[ \int_{-1}^{1} \frac{dz_+}{\sqrt{(\xi + z_+ - i\varepsilon)^2}} + \frac{1}{\sqrt{z_+}} \left( \frac{d^2}{dz_+^2} \hat{g}^{(2)}(z) - g^{(2)}(z) \right) \]

\[ -i\varepsilon_{\mu\nu} \frac{q \times \delta}{p.q} \int_{-1}^{1} \frac{dz_+}{\sqrt{(\xi + z_+ - i\varepsilon)^2}} + \frac{1}{\sqrt{z_+}} \left( \frac{d^2}{dz_+^2} \hat{g}^{(2)}(z) \right) \]
\[ W_{\mu\nu} = \frac{1}{2\pi} \partial_{\mu} \partial_{\nu} T_{\mu\nu} \]

\[ \frac{1}{2\pi} \partial_{\mu} \frac{1}{\xi^{\pm} \pm \epsilon} = \frac{1}{2} \delta(\xi^{\pm} \pm \epsilon) \]

\[ F_2(x, Q^2) = 2x F_1(x, Q^2) \]

\[ \frac{\partial}{\partial x} F_2(x, Q^2) = -\frac{1}{x} \frac{1}{x} \int \frac{dx}{z} \frac{F_1(z, Q^2)}{z} \]
4. RELATIONS BETWEEN STRUCTURE FUNCTIONS:

TWIST 3

5 STRUCTURE FUNCTIONS

\[ \leftrightarrow \ 2 \ \text{BASIC OPERATORS @ LEADING ORDER (ME)} \]

\[ \implies \ 3 \ \text{RELATIONS BETWEEN THE TWIST 3 PARTS OF POLARIZED STRUCTURE FUNCTIONS} \]

OPERATOR:

\[ \Theta_R^{\pm \beta \nu_1...\nu_n} = \Theta^{\pm \beta \nu_1...\nu_n} - \Theta_S^{\pm \beta \nu_1...\nu_n} \]

TWO OPERATOR MATRIX ELEMENTS

\[ \propto (g_V^q)^2 + (g_A^q)^2 \quad g_1, g_2 \]

\[ \propto 2 g_V^q g_A^q \quad g_3, g_4, g_5 \]
CRUCIAL POINT:

- $\frac{M^2}{Q^2}$ CANNOT BE NEGLECTED!
- IF SO, THEY HAVE TO BE RESUMMED TO ALL ORDERS TO AVOID UNPHYSICAL (NON-EXISTENT) SINGULARITIES FOR $x \to 1$

(JB, A. TRABIDзе '98)

\[ \text{LCE WITH TARGET MASSES.} \]

\[ \text{ALL POLARIZED STRUCTURE FUNCTIONS RECEIVE TWIST 3 TERMS!} \]

EARLIER TREATMENT ($S_1$, JB, KOCHELEV
$S_\parallel$ VARIOUS AUTHORS)

\[ \text{HAD TO BE EXTENDED.} \]
\[
\frac{q_{1}^{c=3}(x, Q^2)}{q_{1}^{c=3}(x, Q^2)} = \sum_{q} \frac{1}{2} \left( (q_0) + (q_3) \right) \frac{4M^2x^2}{Q^2} \left[ \frac{D^{\pm q}(\xi)}{[1 + 4M^2x^2/Q^2]} \right]^{3/2}
\]
\[
- \frac{3}{(1 + 4M^2x^2/Q^2)^2} \int \frac{d\xi_1}{\xi} \frac{D^{\pm q}(\xi_1)}{\xi_1}
\]
\[
+ \frac{(2 - 4M^2x^2/Q^2)}{(1 + 4M^2x^2/Q^2)^{3/2}} \int \int \frac{d\xi_1}{\xi_1} \frac{d\xi_2}{\xi_2} \frac{D^{\pm q}(\xi_1)}{\xi_1} \frac{D^{\pm q}(\xi_2)}{\xi_2}
\]

\[
\frac{q_{2}^{c=3}(x, Q^2)}{q_{2}^{c=3}(x, Q^2)} = \sum_{q} \frac{1}{2} \left( (q_0) + (q_3) \right) \left[ \frac{D^{\pm q}(\xi)}{[1 + 4M^2x^2/Q^2]} \right]^{3/2}
\]
\[
- \frac{1 - 8M^2x^2/Q^2}{(1 + 4M^2x^2/Q^2)^2} \int \frac{d\xi_1}{\xi} \frac{D^{\pm q}(\xi_1)}{\xi_1}
\]
\[
- \frac{12M^2x^2/Q^2}{(1 + 4M^2x^2/Q^2)^{3/2}} \int \frac{d\xi_1}{\xi} \frac{d\xi_2}{\xi} \frac{D^{\pm q}(\xi_1)}{\xi_1} \frac{D^{\pm q}(\xi_2)}{\xi_2}
\]

**Exact Relations:**

\[
\frac{q_{1}^{c=3}(x, Q^2)}{q_{1}^{c=3}(x, Q^2)} = \frac{4M^2x^2}{Q^2} \left[ g_{2}^{c=3}(x, Q^2) - \int \frac{dy}{y} g_{2}^{c=3}(y, Q^2) \right]
\]

**Q^2 dependence through:** $M^2/Q^2 \rightarrow \text{see above.}$
THE NEW RELATIONS:

\[ g_{1, \tau = 3}^{i}(x, Q^2) = \frac{4M^2x^2}{Q^2} \left[ g_{2, \tau = 3}^{i}(x, Q^2) \right. \]
\[ \left. - 2 \int_{x}^{1} \frac{dy}{y} g_{2, \tau = 3}^{i}(x, Q^2) \right] \]
\[ = \frac{4M^2x^2}{Q^2} g_{3, \tau = 3}^{i}(x, Q^2) = g_{4, \tau = 3}^{i}(x, Q^2) \left( 1 + \frac{4M^2x^2}{Q^2} \right) \]
\[ + 3 \int_{x}^{1} \frac{dy}{y} g_{4, \tau = 3}^{i}(y, Q^2) \]
\[ 2x g_{5, \tau = 3}^{i}(x, Q^2) = - \int_{x}^{1} \frac{dy}{y} g_{4, \tau = 3}^{i}(y, Q^2) \]

**Twist 2:** \( g_{1, \tau = 2}^{i}, g_{4, \tau = 2} \)
**Determines:** \( g_{2,3,5; \tau = 2} \)

**Twist 3:** \( g_{2,4; \tau = 3} \)
**Determines:** \( g_{1,3,5; \tau = 3} \)

\( M^2/Q^2 \rightarrow 0 \) **Break down of the above relations.**

**Twist 3 lives in the domain** \( M^2/Q^2 \neq 1 \), however!

**In the above relations, all terms were resummed exactly.**
\[ g_{A1T=3}^{1x1^2}(x, Q^2) = \frac{4M^2x^2}{Q^2} \left[ g_{A2T=3}^{1x1^2}(x, Q^2) \right. \]
\[- 2 \int \frac{dy}{y} g_{A2T=3}^{1x1^2}(x', Q^2) \left. \right] \]

Can be tested in precision measurements of 
\[ g_{em}^{e.m.}(x, Q^2). \]

→ CEBAF, SLAC \( g_2 \)-measurement.
\[ \int_0^1 dx \times \left[ g_1^{\text{val}}(x, Q^2) + 2 g_2^{\text{val}}(x, Q^2) \right] = 0 \]

(TWIST 3 REL.)

INTRODUCE TARGET MASSES (TW2, TW3)

\[ \int_0^1 dx \times \left[ g_1^{\text{val}}(x, Q^2) + 2 g_2^{\text{val}}(x, Q^2) \right] = \sum_{q} \frac{e_q^2}{2m_q} \left( \frac{1}{M} \int_0^1 dx \frac{h_{1q}(x) - \overline{h}_{1q}(x)}{\left[ 1 - \frac{M^2 x^2}{Q^2} \right]^2} \right) \]

IFF: \( Q^2 > M^2 \) \( \forall \) THE INTEGRAL CONVERGES.
ONE MAY THEN PERFORM \( m_q \to 0 \)!

\( \forall \) ELT SR, OTHERWISE NOT.
WHAT HAS TO BE DONE IN THE SLAC $g_2$
EXPERIMENTS?

$$\frac{d^2\sigma(\pm S_L)}{dx dy} = \mp 4\pi \frac{\alpha^2}{Q^2} \left\{ (2 - y - \frac{2xyM^2}{S}) \left[ g_1^{(2)} + g_1^{(3)} \right] + \frac{4xM^2}{S} \left[ g_2^{(2)} + g_2^{(3)} \right] \right\}$$

$$\equiv \mathcal{L}_L \left( g_1^{(2)}, g_2^{(3)} \right)$$

$$\frac{d^3\sigma(\pm S_T)}{dx dy d\phi} = \mp 4 \frac{\alpha^2}{Q^2} \sqrt{\frac{M^2}{S}} \sqrt{xy} \left[ 1 - y - \frac{xyM^2}{S} \right] \cos(\alpha - \phi)$$

$$\left\{ \left[ g_1^{(2)} + g_1^{(3)} \right] - \frac{2}{3} \left[ g_2^{(2)} + g_2^{(3)} \right] \right\}$$

$$\equiv \mathcal{L}_T \left( g_1^{(2)}, g_2^{(3)} \right)$$

WITH:

$$g_2^{(1)}(x) = -g_1^{(1)}(x) + \int_{x}^{1} \frac{dz}{z} g_1^{(2)}(z)$$

$$g_1^{(3)}(x) = \frac{4M^2x^2}{Q^2} \left[ g_2^{(1)}(x) - 2 \int_{x}^{1} \frac{dz}{z} g_2^{(2)}(z) \right]$$

**MEASURE:**

- $g_1^{(2)}(x, Q^2, M^2)$
- $g_2^{(3)}(x, Q^2, M^2)$

*(DETERMINED ITERATIVELY).*
5. CONCLUSIONS

1) In LO in QCD the five polarized structure functions receive \textcolor{red}{Twist 2} \textbf{and} \textcolor{red}{Twist 3} contributions.

2) The Renormalization Group requ. the twist decomposition.

3) All structure functions have to be represented in the twist decomposition.

4) The operator and flavor structure (current projection) results into
   \textcolor{blue}{2 indep. Twist 2 OME's}
   \textcolor{blue}{2 indep. Twist 3 OME's}

5) \textcolor{red}{3 twist 2} \textbf{and} \textcolor{red}{3 twist 3} relations between the structure functions are implied.

6) For the twist 3 terms \textcolor{blue}{4 H^2/\xi^2 terms} are needed.
   The ETL relations remain intact for \textcolor{blue}{Q^2 > H^2}.

7) The relation for \textcolor{blue}{g_{1,T=3}^{1/2}} \leftrightarrow \textcolor{blue}{G[g_{2,T=3}^{1/2}]} can be measured in the future!