QED Corrections to Polarized DIS with HECTOR

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DESY

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2. **Born Cross Section**
3. $O(\alpha)$ **Corrections**
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Based on : J. Blümlein et al. Comp. Phys. Commun. **94** (1996) 128, Nucl. Phys. **B 506** (1997) 295

INTRODUCTION

HECTOR - SEMIANALYTIC QED CORRS.

DIS : POLARIZED, UNPOLARIZED

→ BORN : NC : $\gamma + Z$
(POL) S_L & $S_\perp$.

→ SOME CUTS

→ FAST CODE

→ LLA AVAILABLE

→ 'UNIVERSAL' CORRECTIONS
FOR MORE PROCESSES

2 The Born Cross Section

$$d\sigma_{\mathrm{Born}} = \frac{2\alpha^2}{\sqrt{\lambda_S}}\frac{1}{Q^4}\left[L^{\mu\nu}W_{\mu\nu}\right]\frac{d\vec{k}_2}{k_2^0} = \frac{2\pi\alpha^2}{\lambda_S}\frac{S^2 y}{Q^4}\left[L^{\mu\nu}W_{\mu\nu}\right]dxdy. \quad (9)$$

x and y are the Bjorken variables

$$x = \frac{Q^2}{2p.q}, \qquad y = \frac{p.q}{p.k_1}. \quad (10)$$

The calculation is performed for incoming longitudinally polarized leptons. We used the spin density matrix

$$\rho(k_1) = \sum_s u^s(k_1)\bar{u}^s(k_1) = \frac{1}{2}\left(1 - \gamma_5\hat{\xi}_l\right)(\hat{k_1} + m). \quad (11)$$

The 4–vector of the lepton polarization is given by

$$\xi_l = \frac{\lambda_l}{m}\left(k_1 - \frac{2m^2}{S}p\right)\frac{S}{\sqrt{\lambda_S}}, \quad (12)$$

with

$$\xi_l^2 = -\lambda_l^2, \qquad \text{and} \qquad \xi_l.k_1 = 0. \quad (13)$$

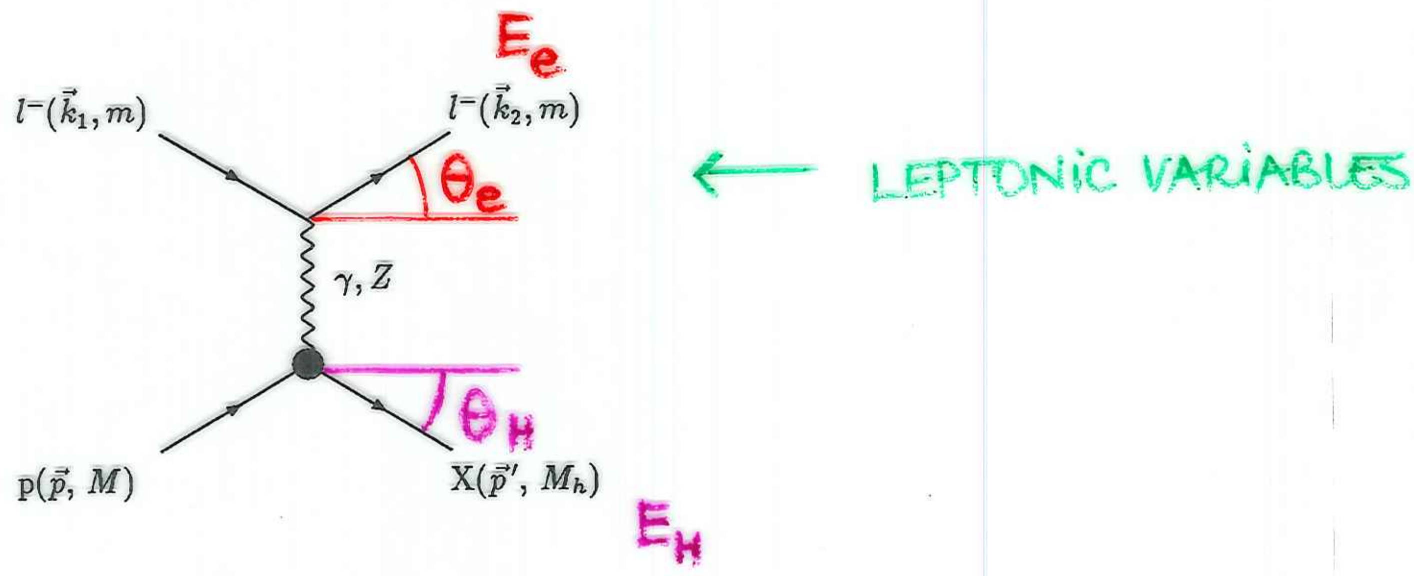


Figure 1: Born diagram for neutral current deep-inelastic lepton–proton scattering.

HADRONIC TENSOR

$$\begin{aligned} W_{\mu\nu} &= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\mathcal{F}_1(x,Q^2) + \frac{\widehat{p_\mu}\widehat{p_\nu}}{p.q}\mathcal{F}_2(x,Q^2) - i\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda p^\sigma}{2p.q}\mathcal{F}_3(x,Q^2) \\ &+ i\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda s^\sigma}{p.q}\mathcal{G}_1(x,Q^2) + i\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda(p.qs^\sigma - s.qp^\sigma)}{(p.q)^2}\mathcal{G}_2(x,Q^2) \\ &+ \left[\frac{\widehat{p_\mu}\widehat{s_\nu} + \widehat{s_\mu}\widehat{p_\nu}}{2} - s.q\frac{\widehat{p_\mu}\widehat{p_\nu}}{p.q}\right]\frac{1}{p.q}\mathcal{G}_3(x,Q^2) \\ &+ s.q\frac{\widehat{p_\mu}\widehat{p_\nu}}{(p.q)^2}\mathcal{G}_4(x,Q^2) + \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\frac{s.q}{p.q}\mathcal{G}_5(x,Q^2), \end{aligned} \quad (19)$$

SF

where

$$\widehat{p_\mu} \equiv p_\mu - \frac{p.q}{q^2}q_\mu, \qquad \widehat{s_\mu} \equiv s_\mu - \frac{s.q}{q^2}q_\mu. \quad (20)$$

s denotes the polarization 4–vector of the nucleon. In the nucleon rest frame it is given by

$$s \equiv M(0, \vec{n}_\lambda). \quad (21)$$

$$\mathcal{F}_{1,2}(x,Q^2) \equiv Q_l^2 F_{1,2}^{\gamma\gamma}(x,Q^2) + 2|Q_l^2|\,(v_l - p_l\lambda_l a_l)\,\chi(Q^2) F_{1,2}^{\gamma Z}(x,Q^2)$$

$$+ \left(v_l^2 + a_l^2 - 2p_l\lambda_l v_l a_l\right)\chi^2(Q^2) F_{1,2}^{ZZ}(x,Q^2),$$

$$\mathcal{F}_3(x,Q^2) \equiv 2|Q_l|\,(p_l a_l - \lambda_l v_l)\,\chi(Q^2) F_3^{\gamma Z}(x,Q^2)$$
$$+ \left[2p_l v_l a_l - \lambda_l\left(v_l^2 + a_l^2\right)\right]\chi^2(Q^2) F_3^{ZZ}(x,Q^2),$$

$$\mathcal{G}_{1,2}(x,Q^2) \equiv -Q_l^2\lambda_l g_{1,2}^{\gamma\gamma}(x,Q^2) + 2|Q_l|\,(p_l a_l - \lambda_l v_l)\,\chi(Q^2) g_{1,2}^{\gamma Z}(x,Q^2)$$
$$+ \left[2p_l v_l a_l - \lambda_l\left(v_l^2 + a_l^2\right)\right]\chi^2(Q^2) g_{1,2}^{ZZ}(x,Q^2),$$

$$\mathcal{G}_{3,4,5}(x,Q^2) \equiv 2|Q_l|\,(v_l - p_l\lambda_l a_l)\,\chi(Q^2) g_{3,4,5}^{\gamma Z}(x,Q^2)$$

$$+ \left(v_l^2 + a_l^2 - 2p_l\lambda_l v_l a_l\right)\chi^2(Q^2) g_{3,4,5}^{ZZ}(x,Q^2).$$

$$F_1^{J_1J_2}(x,Q^2) = \sum_q \alpha^q_{J_1J_2}\left[q(x,Q^2)+\bar{q}(x,Q^2)\right],$$

$$F_2^{J_1J_2}(x,Q^2) = 2xF_1^{J_1J_2}(x,Q^2),$$

$$F_3^{J_1J_2}(x,Q^2) = \sum_q \beta^q_{J_1J_2}\left[q(x,Q^2)-\bar{q}(x,Q^2)\right],$$

$$\begin{aligned}
g_1^{J_1J_2}(x,Q^2) &= \frac{1}{2}\sum_q \alpha^q_{J_1J_2}\left[\Delta q(x,Q^2)+\Delta\bar{q}(x,Q^2)\right],\\
g_2^{J_1J_2}(x,Q^2) &= -g_1^{J_1J_2}(x,Q^2)+\int_x^1 \frac{dy}{y} g_1^{J_1J_2}(y,Q^2), \quad \text{WW}\\
g_3^{J_1J_2}(x,Q^2) &= 4x\int_x^1 \frac{dy}{y} g_5^{J_1J_2}(y,Q^2), \quad \text{BK}\\
g_4^{J_1J_2}(x,Q^2) &= 2xg_5^{J_1J_2}(x,Q^2), \quad \text{D}\\
g_5^{J_1J_2}(x,Q^2) &= \sum_q \beta^q_{J_1J_2}\left[\Delta q(x,Q^2)-\Delta\bar{q}(x,Q^2)\right],
\end{aligned}$$

$$\begin{aligned}
\alpha^q_{J_1J_2} &= \alpha^q_{\gamma\gamma,\gamma Z,ZZ} = \left[e_q^2,\ \ 2e_qv_q,\ \ v_q^2+a_q^2\right],\\
\beta^q_{J_1J_2} &= \beta^q_{\gamma\gamma,\gamma Z,ZZ} = \left[0,\ \ 2e_qa_q,\ \ 2v_qa_q\right],
\end{aligned}$$

ıplings are

$$\begin{aligned}
e_u &= +\frac{2}{3}, & e_d &= -\frac{1}{3},\\
v_u &= \frac{1}{2}-\frac{4}{3}\sin^2\theta_W, & v_d &= -\frac{1}{2}+\frac{2}{3}\sin^2\theta_W,\\
a_u &= \frac{1}{2}, & a_d &= -\frac{1}{2}.
\end{aligned}$$

$$\frac{d^2\sigma_{\text{Born}}}{dxdy} = \frac{d^2\sigma^{\text{unpol}}_{\text{Born}}}{dxdy}+\frac{d^2\sigma^{\text{pol}}_{\text{Born}}}{dxdy},$$

with

$$\frac{d^2\sigma^{\rm unpol}_{\rm Born}}{dxdy} = \frac{2\pi\alpha^2}{Q^4} S \sum_{i=1}^{3} S_i^U(x,y)\mathcal{F}_i(x,Q^2), \tag{38}$$

and

$$\frac{d^2\sigma^{\rm pol}_{\rm Born}}{dxdy} = \frac{2\pi\alpha^2}{Q^4}\lambda_N^p f^p S \sum_{i=1}^{5} S_{gi}^p(x,y)\mathcal{G}_i(x,Q^2). \tag{39}$$

λ_N^p denotes the degree of nucleon polarization. For unpolarized deep-inelastic scattering only the first term, $d^2\sigma^{\rm unpol}$, contributes. Eq. (39) applies both to the case of longitudinal (L) and transversal (T) nucleon polarization, where

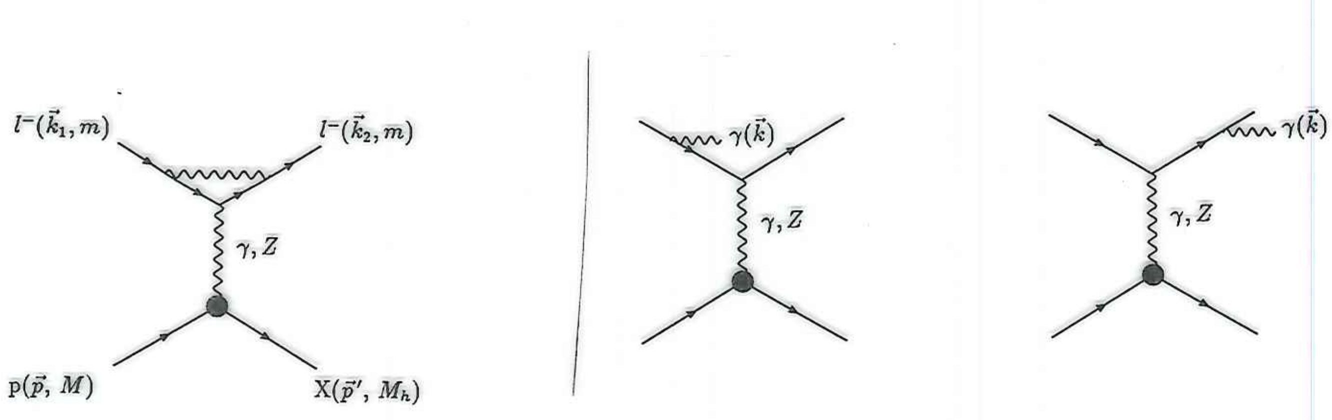
$$f^L = 1, \tag{40}$$

$$f^T = \cos\varphi \frac{d\varphi}{2\pi}\sqrt{\frac{4M^2x}{Sy}\left(1-y-\frac{M^2xy}{S}\right)} \equiv \cos\varphi\frac{d\varphi}{2\pi}\frac{1-y}{y}\sin\theta_2. \tag{41}$$

$$\vec{n}^L = \lambda_N^L\frac{\vec{k}_1}{|\vec{k}_1|},$$

$$\vec{n}^T = \lambda_N^T\vec{n}_\perp, \quad \text{with} \quad \vec{n}_\perp\vec{k}_1 = 0.$$

The $O(\alpha)$ Leptonic Correction



$$\frac{d^2\sigma_{\rm rad}^{\rm QED,1}}{dx_l dy_l} \equiv \frac{\alpha}{\pi}\delta_{\rm VR}\frac{d^2\sigma_{\rm Born}}{dx_l dy_l} + \frac{d^2\sigma_{\rm Brems}}{dx_l dy_l} \equiv \frac{d^2\sigma_{\rm rad}^{\rm unpol}}{dx_l dy_l} + \frac{d^2\sigma_{\rm rad}^{\rm pol}}{dx_l dy_l}.$$

$$x_l \equiv \frac{Q_l^2}{S y_l}, \quad y_l \equiv \frac{p.(k_1 - k_2)}{p.k_1}, \quad \text{and} \quad Q_l^2 \equiv -(k_1 - k_2)^2.$$

$$x_h \equiv \frac{Q_h^2}{S y_h}, \quad y_h \equiv \frac{p.(p' - p)}{p.k_1}, \quad \text{and} \quad Q_h^2 \equiv -(p' - p)^2 .$$

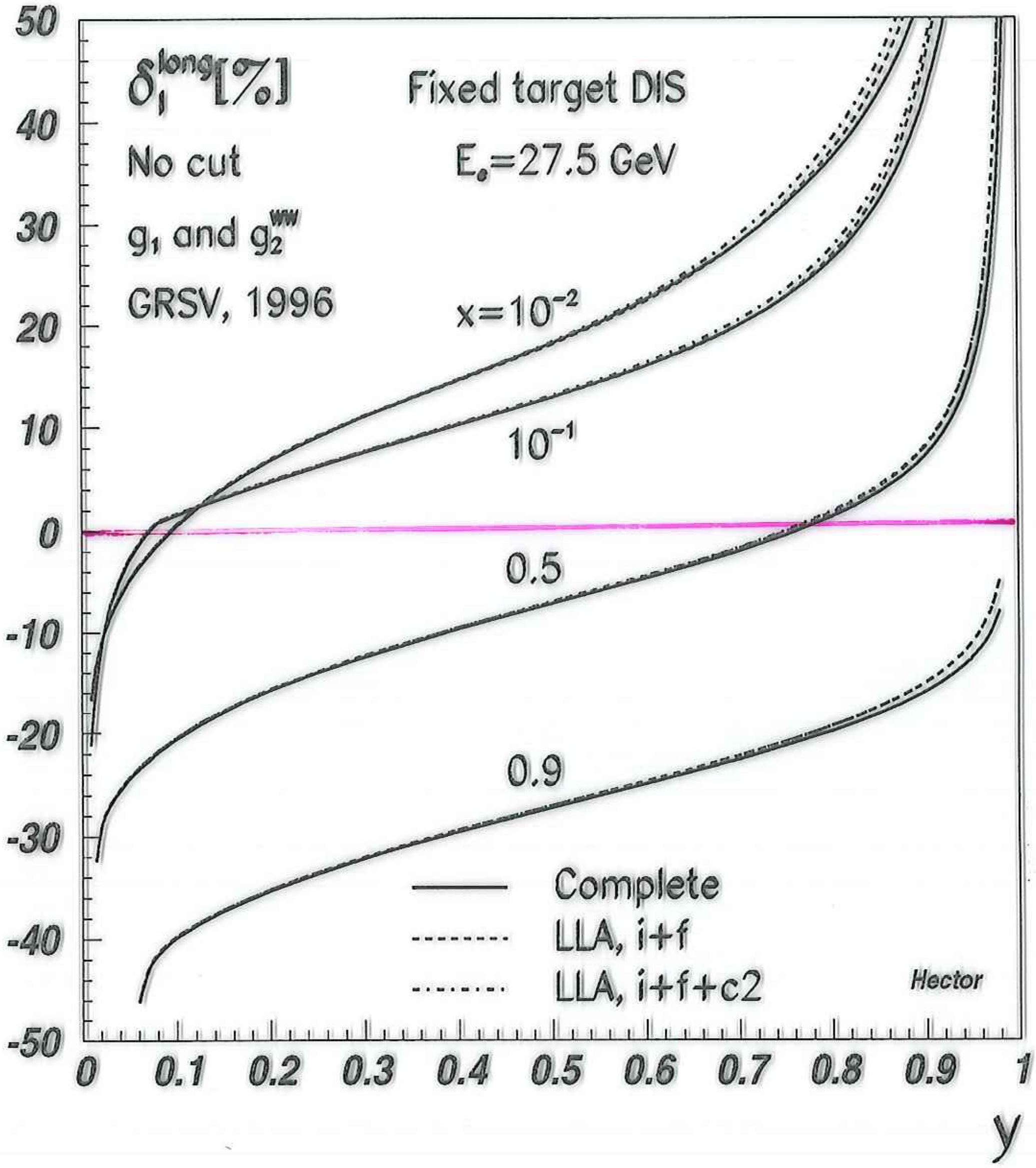


Figure 5 : $O(\alpha)$ leptonic QED correction, eq. (47), to the polarized part of the differential deep-inelastic scattering cross section for longitudinally polarized protons at $\sqrt{S} = 7.4$ GeV. Full lines : complete corrections; dashed lines : initial and final-state Bremsstrahlung contributions in LLA; dash-dotted lines : complete LLA contributions, eq. (94).

The Leading Log Approximation

$$\frac{d^2\sigma^k_{i,f}}{dxdy} = \frac{\alpha}{2\pi}\left(\ln\frac{Q^2}{m^2}-1\right)\int_0^1 dz\frac{1+z^2}{1-z}\left\{\theta(z-z_0)\,\mathcal{J}\,\frac{d^2\sigma^k_{\rm Born}}{dxdy}\bigg|^{i,f}_{x=\hat{x},y=\hat{y},S=\hat{S}} - \frac{d^2\sigma^k_{\rm Born}}{dxdy}\right\},$$

$$\hat{S} = zS, \quad \hat{y} = \frac{y+z-1}{z}, \quad \hat{Q}^2 = zQ^2, \quad \hat{x} = \frac{\hat{Q}^2}{\hat{y}\hat{S}},$$

$$\hat{S} = S, \quad \hat{y} = \frac{y+z-1}{z}, \quad \hat{Q}^2 = \frac{Q^2}{z}, \quad \hat{x} = \frac{\hat{Q}^2}{\hat{y}\hat{S}},$$

ISR FSR C

$$\frac{d^2\sigma^{\rm LLA}_{\rm rad}}{dxdy} = \frac{d^2\sigma_i}{dxdy} + \frac{d^2\sigma_f}{dxdy} + \frac{d^2\sigma_{\rm Comp}}{dxdy}.$$

$$\mathcal{J} \equiv \mathcal{J}(x,y,Q^2) = \left|\frac{\partial(\hat{x},\hat{y})}{\partial(x,y)}\right| \qquad \hat{x}(z_0) \le 1, \qquad \hat{y}(z_0) \le 1, \qquad P_{ff}(z) = \left(\frac{1+z^2}{1-z}\right)_+$$

$$z_0^i = \frac{1-y}{1-yx}, \qquad z_0^f = 1-y+xy\,.$$

UNPOL.

$$\frac{d^2\sigma^U_{\rm Comp}}{dx_l dy_l} = \frac{\alpha^3}{S x_l^2}\frac{Y_+}{y_{l_1}}\int_{x_l}^{1} dz \int_{(Q_h^2)^{\rm min}}^{(Q_h^2)^{\rm max}} \frac{dQ_h^2}{Q_h^2}\left[\frac{Z_+}{z}F_2^{\gamma\gamma}(x_h,Q_h^2) - zF_L^{\gamma\gamma}(x_h,Q_h^2)\right],$$

POL.

$$\frac{d^2\sigma^L_{\rm Comp}}{dx_l dy_l} = \left(-2\lambda_l\lambda_N^L\right)\frac{\alpha^3}{S x_l^2}\frac{Y_-}{y_{l_1}}\int_{x_l}^{1} dz \int_{(Q_h^2)^{\rm min}}^{(Q_h^2)^{\rm max}} \frac{dQ_h^2}{Q_h^2}\frac{Z_-}{z}x_h g_1^{\gamma\gamma}(x_h,Q_h^2),$$

$$\frac{d^2\sigma^T_{\rm Comp}}{dx_l dy_l} = \left(-2\lambda_l\lambda_N^T\right)\frac{\alpha^3}{S x_l^2}\cos\varphi\frac{d\varphi}{2\pi}\frac{y_l}{y_{l_1}^2}\sqrt{\frac{4M^2x_l}{Sy_l}\left(y_{l_1} - \frac{M^2x_ly_l}{S}\right)}$$
$$\times \int_{x_l}^{1}\frac{dz}{z}\int_{(Q_h^2)^{\rm min}}^{(Q_h^2)^{\rm max}}\frac{dQ_h^2}{Q_h^2}\left\{(Y_- - y_l z)\, z x_h g_1^{\gamma\gamma}(x_h,Q_h^2) + 2\left[Y_+(1-z) + y_{l_1}\right] x_h g_2^{\gamma\gamma}(x_h,Q_h^2)\right\}$$

$$Y_\pm = 1 \pm (1-y_l)^2, \qquad Z_\pm = 1 \pm (1-z)^2.$$

$$z = \frac{x_l}{x_h}.$$

$$P^L_{\gamma f}(z) = \frac{Z_-}{z} = \frac{1-(1-z)^2}{z} \qquad P^U_{\gamma f}(z) = \frac{Z_+}{z} = \frac{1+(1-z)^2}{z} \qquad c_L^q(z) = z$$

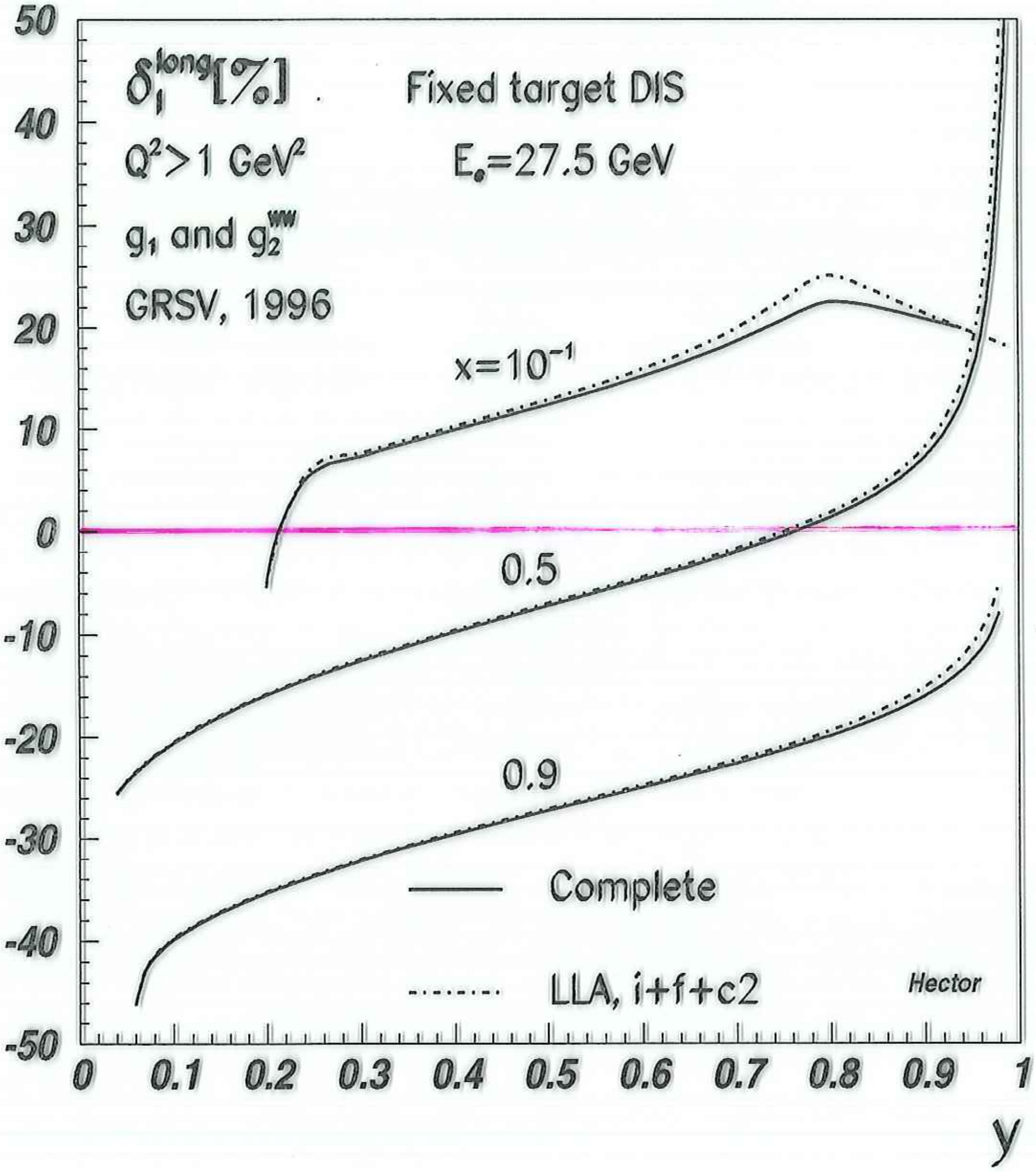


Figure 6 : The same as in figure 5, but for a Q^2-cut of $Q_h^2 > 1$ GeV. Full lines : complete corrections; dash–dotted lines : complete LLA corrections.

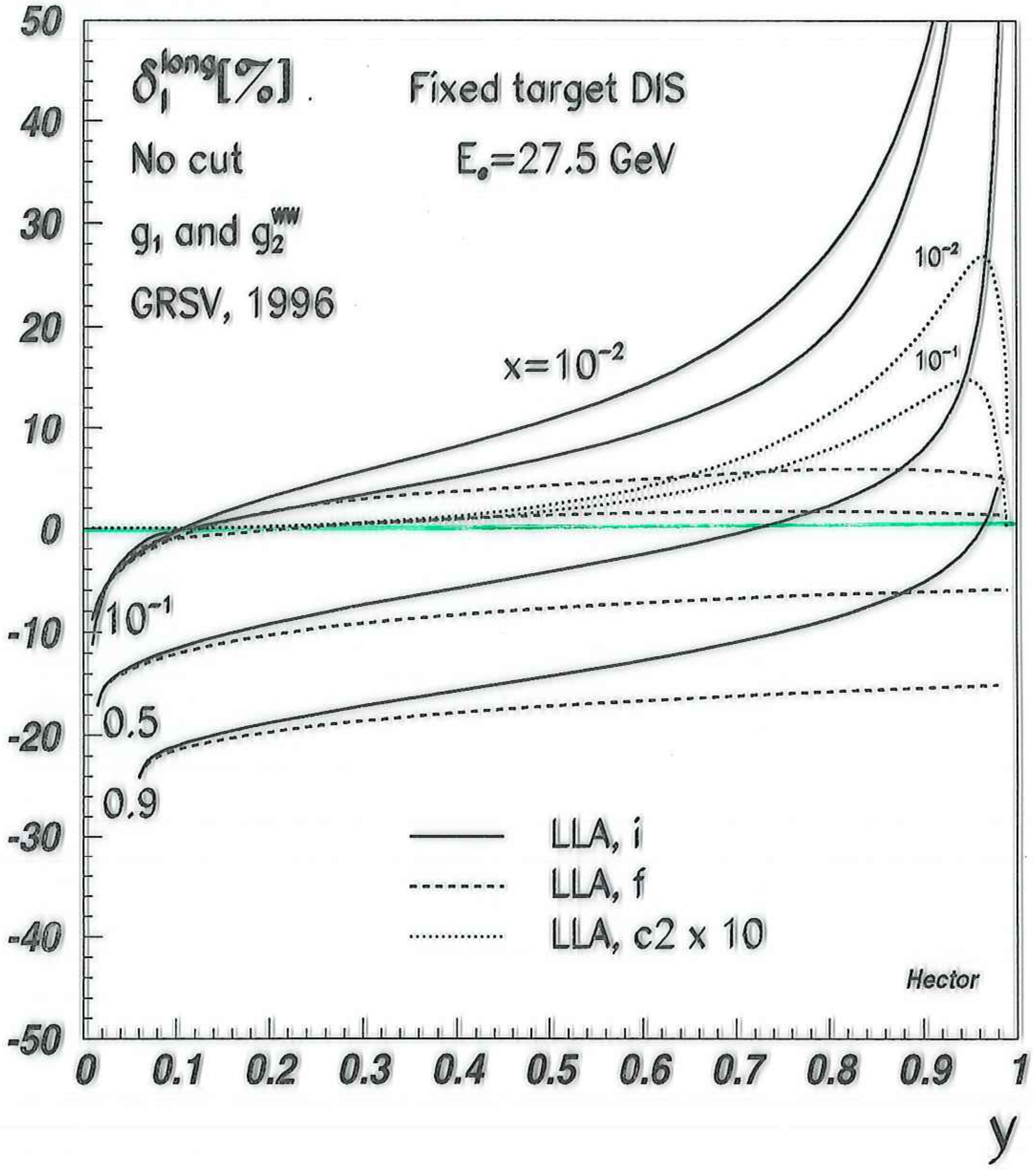


Figure 7 : Comparison of the different contributions to the $O(\alpha)$ leptonic QED corrections in LLA for longitudinally polarized protons at $\sqrt{S} = 7.4$ GeV. Full lines : initial state radiation; dashed lines : final state radiation; dotted lines : Compton contribution, eq. (101), scaled by a factor 10.

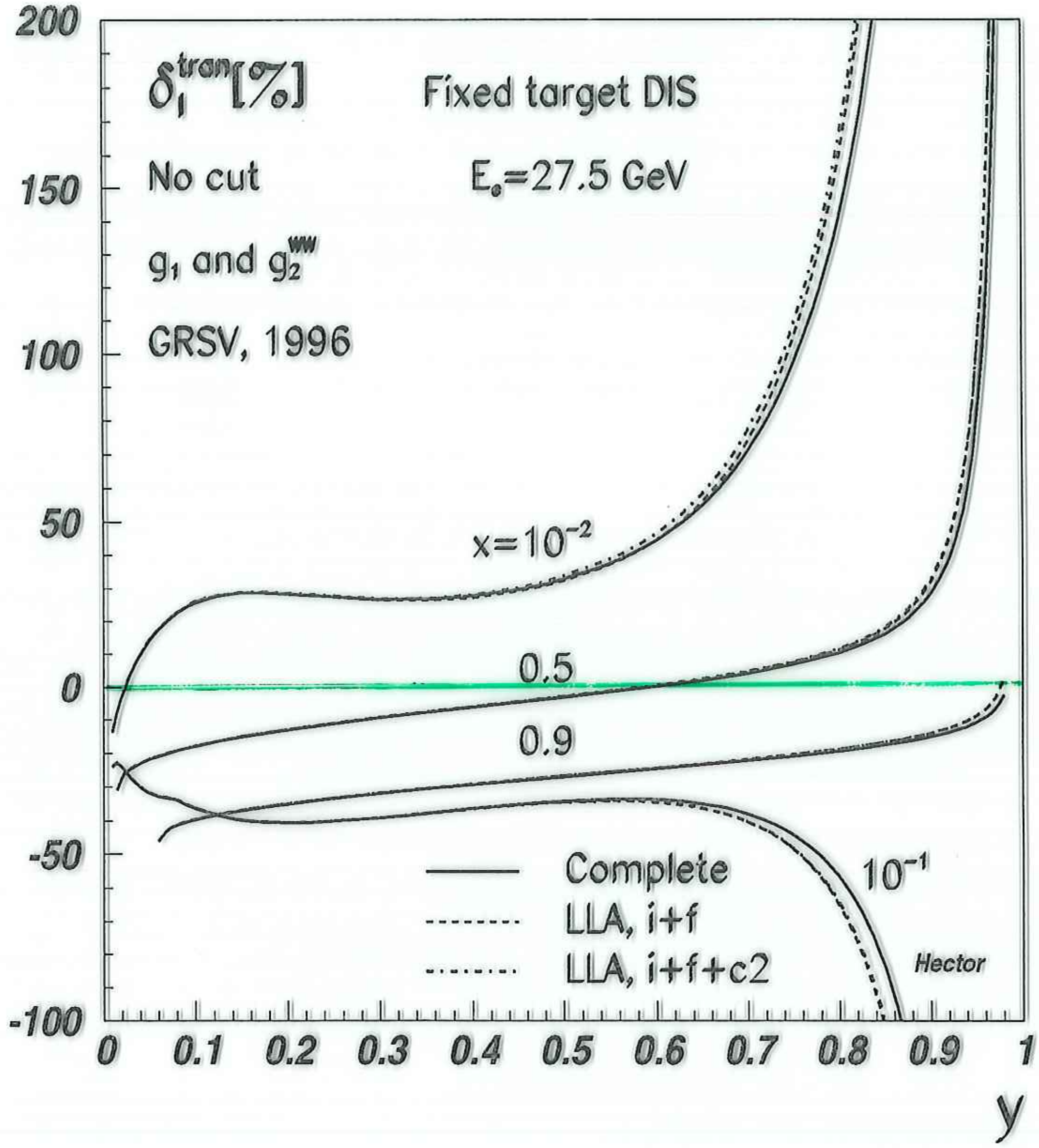


Figure 8 : $O(\alpha)$ leptonic QED correction, eq. (47), to the polarized part of the differential deep-inelastic scattering cross section for transversely polarized protons at $\sqrt{S} = 7.4$ GeV. Full lines : complete corrections; dashed lines : initial and final-state Bremsstrahlung contributions in LLA; dash-dotted lines : complete LLA contributions, eq. (94).

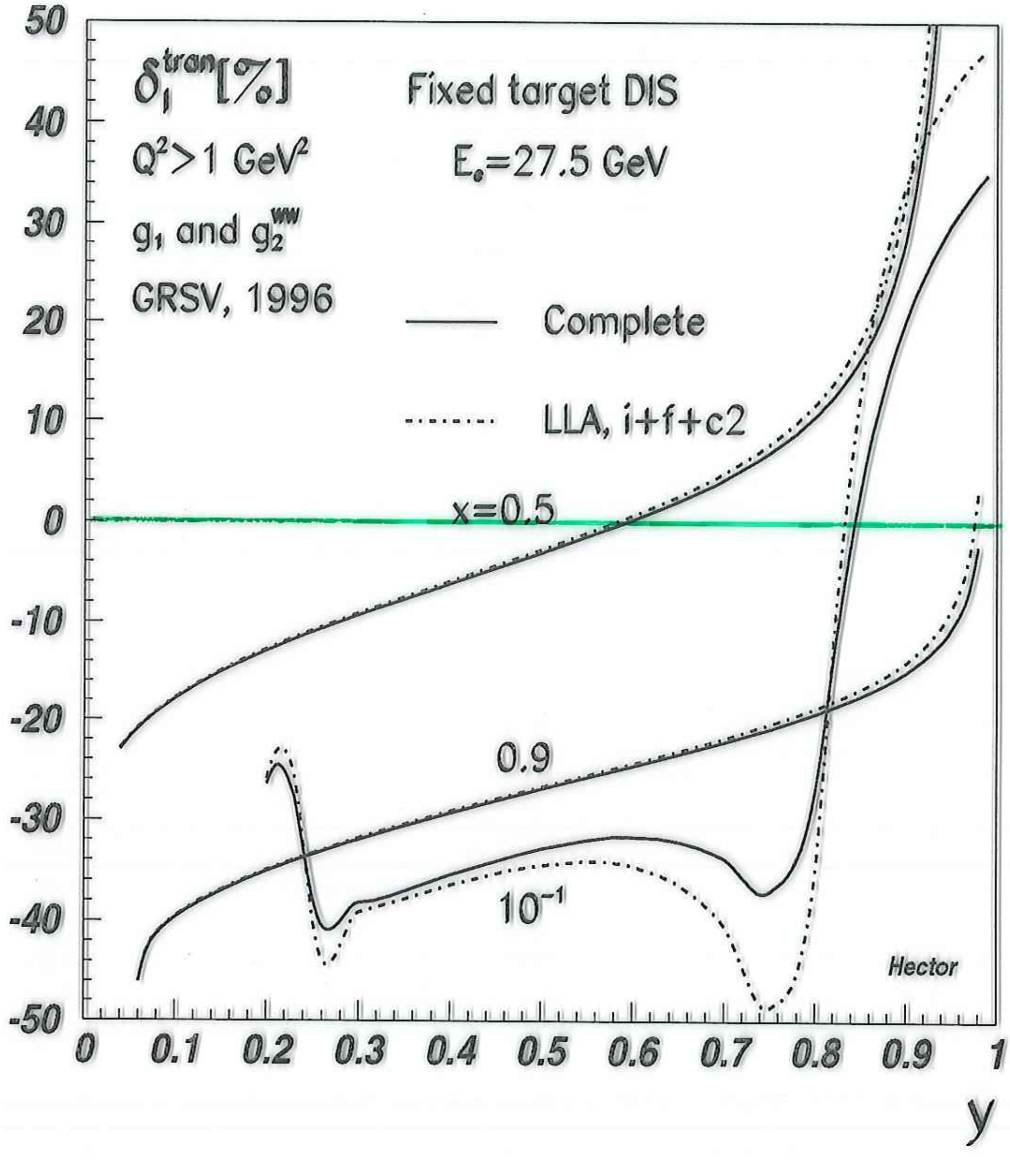


Figure 9 : The same as in figure 8 applying a Q^2-cut of $Q_h^2 > 1$ GeV. Full lines : complete corrections; dash–dotted lines : complete LLA corrections.

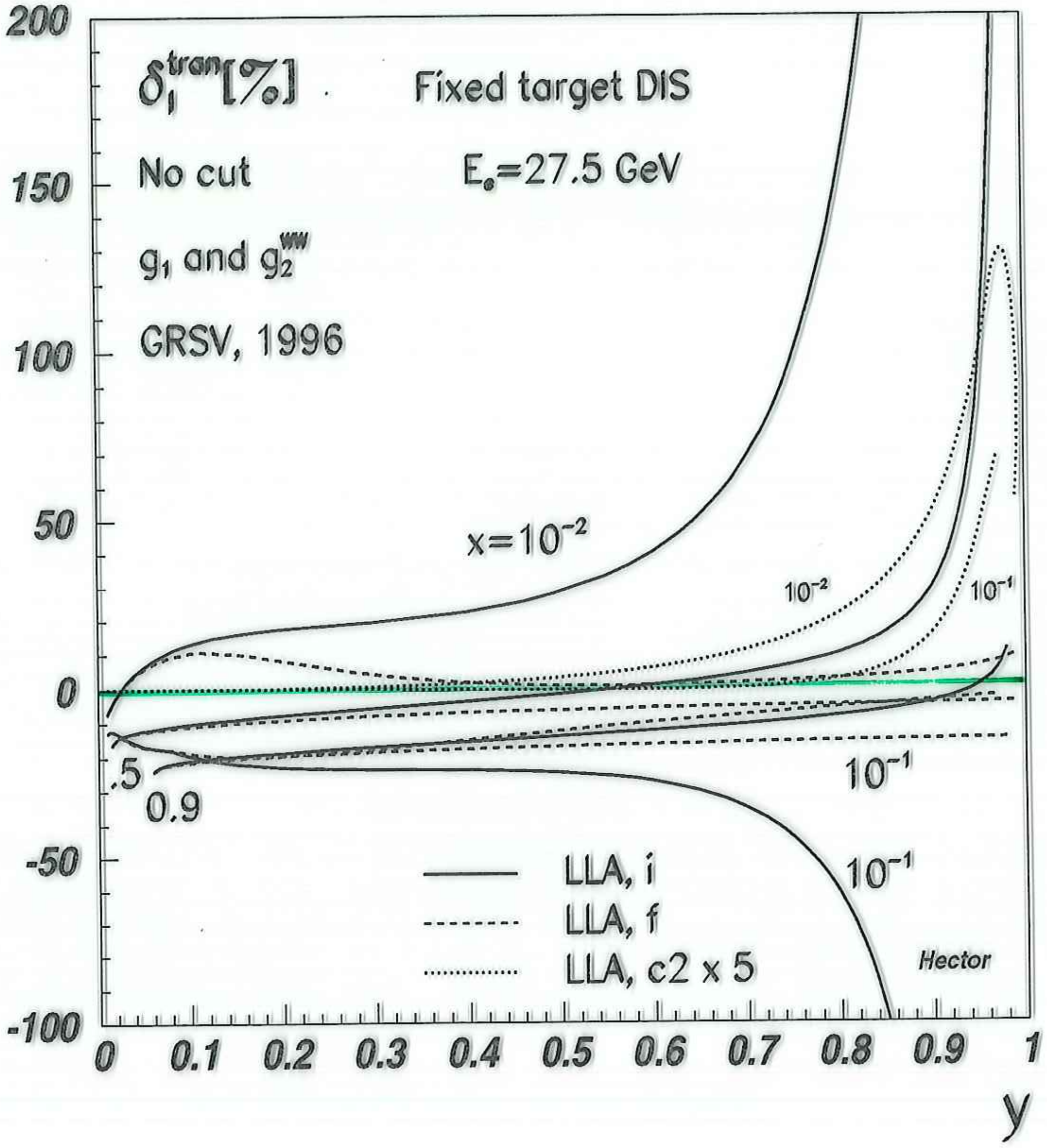


Figure 10 : Comparison of the different contributions to the $O(\alpha)$ leptonic QED corrections in LLA for transversely polarized protons at $\sqrt{S} = 7.4$ GeV. Full lines : initial state radiation; dashed lines : final state radiation; dotted lines : Compton contribution, eq. (101), scaled by a factor 5.

CONCLUSIONS

1) HECTOR IS A FAST & PRECISE SEMIANALYTIC CODE FOR DIS.

2) • POL & UNPOL : NOT ONLY $\sigma^{\uparrow} - \sigma^{\downarrow}$ BUT ALSO ASYMMETRIES.

3) MANY USER OPTIONS ⟶ EXPERIMENT

4) OUTLOOK:
   - NEW KINEMATIC VARIABLES IF NEEDED
   - HIGHER ORDERS

5) $S_{\perp}$: THE TOPOLOGY OF THE CROSS SECTIONS NEEDS VERY ACCURATE RC's.

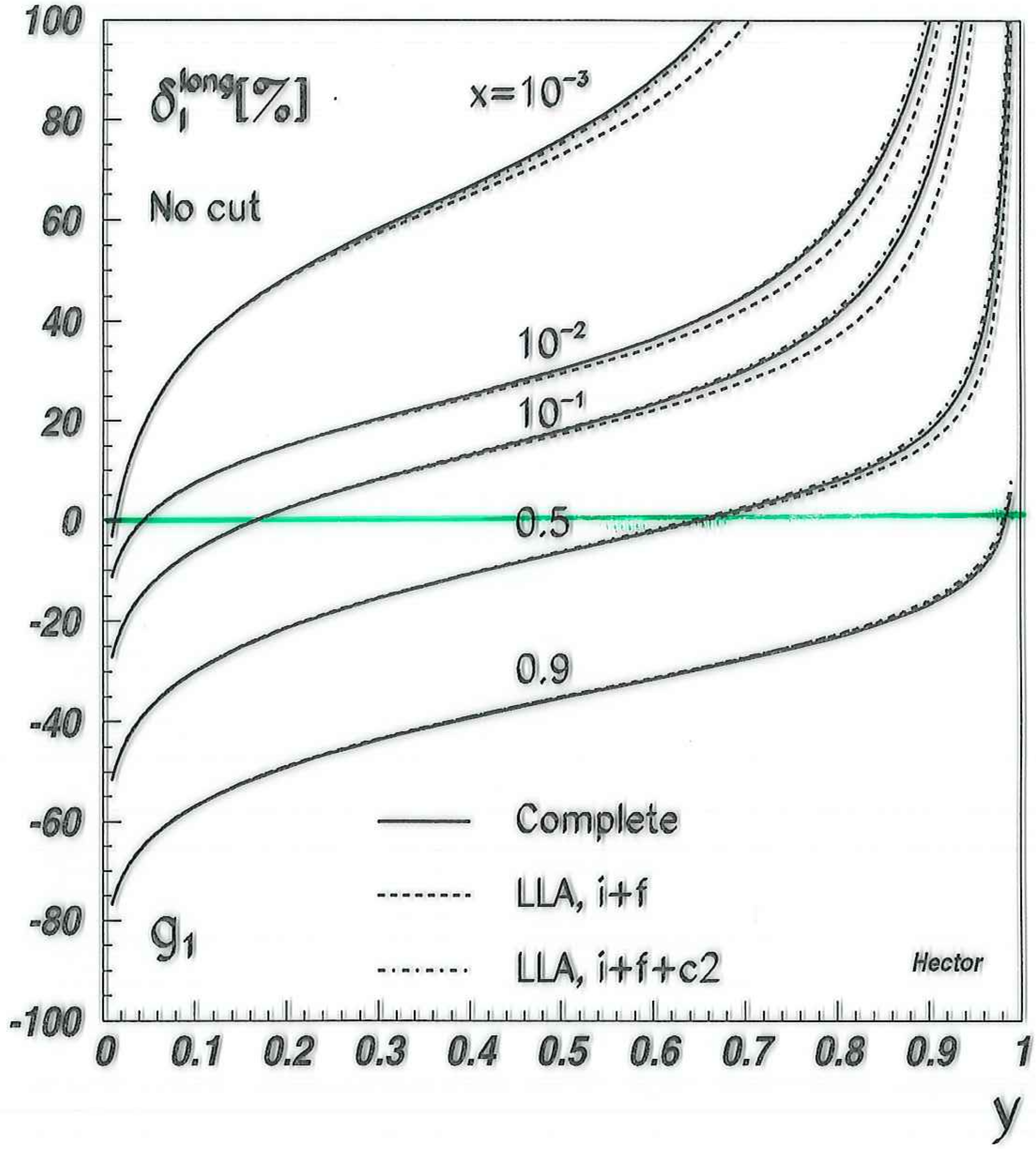


Figure 11 : $O(\alpha)$ leptonic QED correction, eq. (47), to the polarized part of the differential deep-inelastic scattering cross section for longitudinally polarized protons at $\sqrt{S} = 314$ GeV. Full lines : complete corrections; dashed lines : LLA terms, eq. (94).

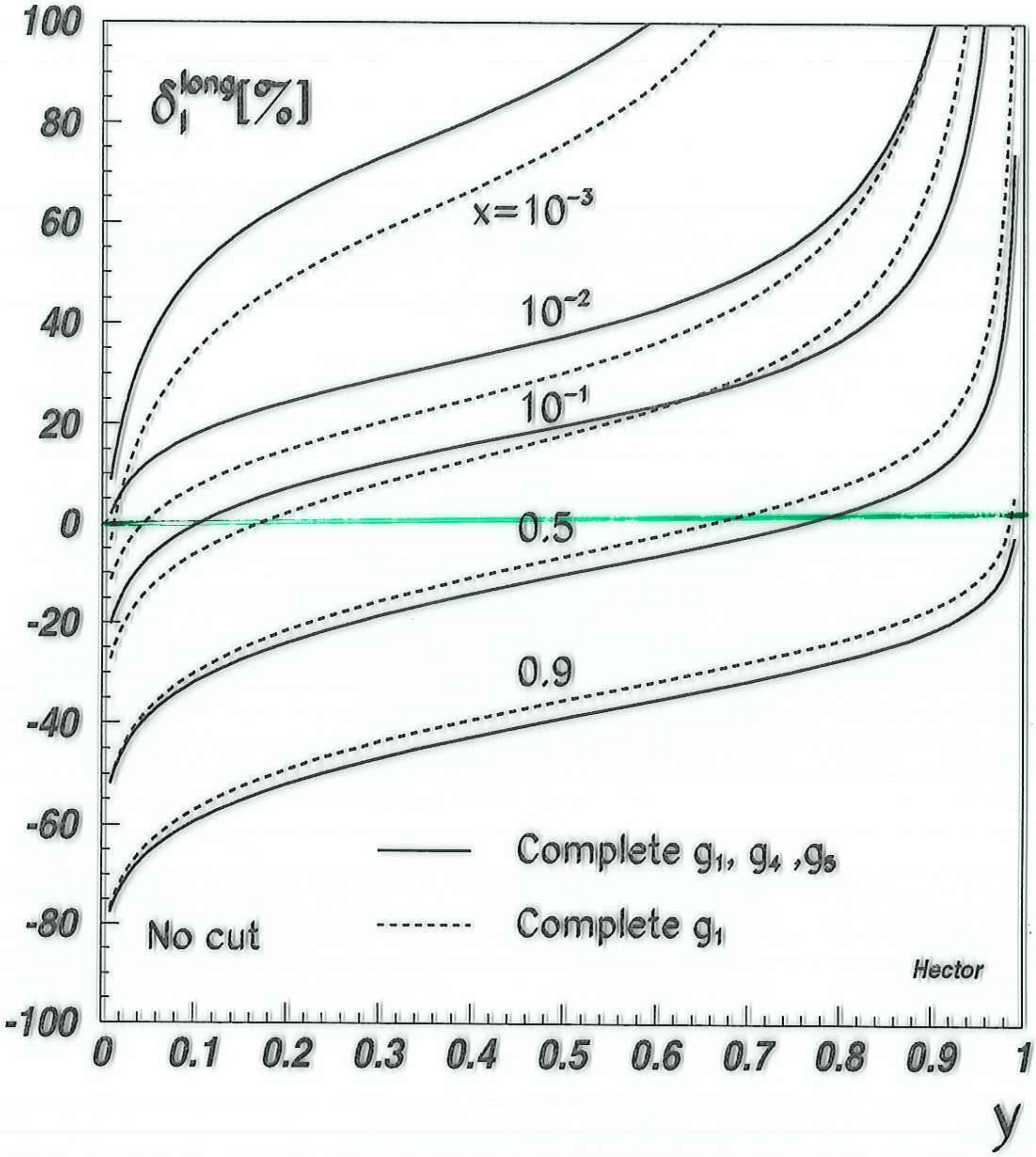


Figure 12 : $O(\alpha)$ leptonic QED correction, eq. (47), to the polarized part of the differential deep-inelastic scattering cross section for longitudinally polarized protons at $\sqrt{S} = 314$ GeV. Dashed lines : $\delta_1^{\rm long}$ for only the structure function g_1; full lines : complete correction. The contributions due to the structure functions g_2 and g_3 are of $O(M^2/S)$ and are not included.