# ZUR BESCHREIBUNG DER ELEKTRISCHEN LEITFÄHIGKEIT NICHTIDEALER QUANTEN-PLASMEN MIT DER METHODE DER GREEN-FUNKTIONEN

LEITFÄHIGKEITSTHEORIE

LRT

KIN. GLEICHUNGEN

$$\frac{1}{i^N}\langle T\{\psi_{a_1}(1)\ldots\psi_{a_N}(N)\,\psi^+_{a_N}(N')\ldots\psi_{a_1}(1')\}\rangle_{GC}$$

**PLASMAZUSTAND**

- HÖHERE MOMENTE
- HYDRODYNAMISCHER ZUSTAND
- ≈ GC-ENSEMBLE

↓

ZUBAREV-MCLENNAN ZUSTÄNDE

**RAUM-ZEITLICHE PLASMASTRUKTUR**

$(\vec{k},\omega)$

$k \to 0,\ \omega \neq 0$

KETTENBRUCHREGULARISIERUNG

$k = 0\ ,\ \omega = 0$

**TEILCHENDYNAMIK**

- STATISCHE / DYNAM. WECHSELWIRKUNG
- 2-, 3-, …, N-TEIL[illegible] MULTIPLE STREU[illegible]
- 2-LOOP-[illegible]R.

U. Greifswald, 3.3.83

J. BLÜMLEIN

# 1. STÓRUNGSTHEORIE DER GREENSCHEN FUNKTIONEN:

$G_N^{CAUS}(1,\dots,N;1'\dots N')$

FUNKTIONALABLEITUNGS-FORMALISMUS

$G_N^{CAUS} = G_N^{CAUS}\left(G_{M<N}^{CAUS},\ T_{M'},\ V^S\right)$

usw.

## WESENTLICHE PROZESSE FÜR TRANSPORT PHÄNOMENE:

$$\sigma = i\sum_{ab}\frac{e_a e_b \hbar^2}{m_a}\lim_{\omega\to 0}\left\{\lim_{k\to 0}\frac{1}{k}\int\frac{d\vec{p}}{(2\pi)^3}\, k\left(p-\frac{k}{2}\right)G_{ab}^{ret}(p,k,\omega)\right)$$

KUBO
YAMADA
PLAKIDA
ZUBAREV

$$\sigma = \frac{8\sqrt{2}}{3\pi^{3/2}}\int_0^\infty dx\, x^7\, f(x)\, \nu^{-1}(x)$$

$$\nu(x^2) \cong \int\frac{dk'\,d\omega'}{(2\pi)^4}\, W(k',p,\omega',\omega=0)\, A_a(p-k,\omega')\,(1-\cos_{p,p-k})$$

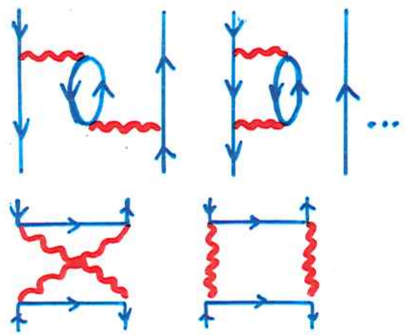
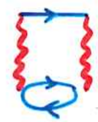
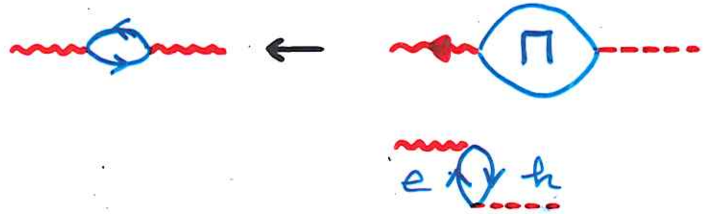
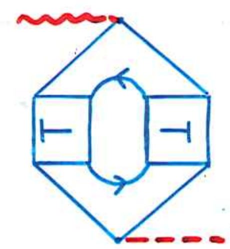
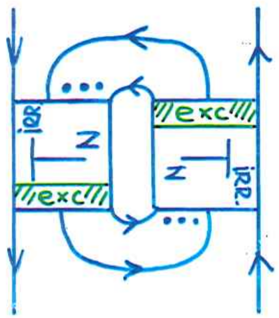
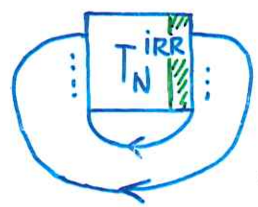
$$\times\left[2\pi n_i e^4 m_e/(p^3\hbar^2)\right]^{-1}, \quad x^2 = \beta\varepsilon_p^e$$

KADANOFF
BAYM
FUJITA
ČÁPEK

$$\bar{\Sigma}_a(p,\omega) = \int\frac{dk'\,d\omega'}{(2\pi)^4}\, G_a(p-k',\omega')\, W_a(k',p,\omega',\omega)$$

$$G_2^{ret}(\omega) = \lim_{\eta\to 0} G_2^{CAUS}(\omega+i\eta)^{\text{Cont.}}\ ;\quad \bar{\Sigma}_a \longleftarrow G_1^{CAUS.}\ \text{(DYSON-GL.)}$$

# WICHTIGE ELEMENTARPROZESSE:

| $G_2^{ab}$ | $\overline{\Sigma}_a$ | |
|---|---|---|
|  |  | **I. BORN'SCHE NÄHERUNG (STATISCH)** |
| |  | **DYN. WW. RPA** |
| |  + etc. | **DYN. WW. 'OPT.' POTENTIAL** |
|  |  | $N \geq 2$ |

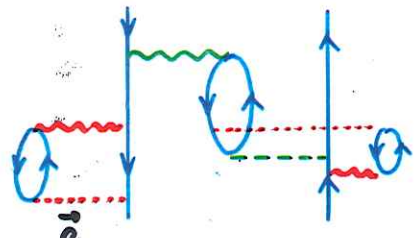
$$T_N = V_N + V_N G_N^0 T_N$$

$$V_N = \sum_{j>i} V_{ij} \quad , \quad G_N^0 = (z - H_N^0)^{-1}$$

$$T_N = T_N^{REDUZ.} + T_N^{IRR.}$$

$N = 2$ **LIPPMANN-SCHWINGER-GL.**

$N \geq 3$ **VERALLG. FADDEEV-GL.**




**TWO LOOP CORRECTION:**

$\vec{q} \to 0 \qquad \sigma := \hat{\sigma} \cdot (1 + O(n^{1/2}))$

## 2. CHARAKTERISTIK DES PLASMAZUSTANDES

KUBO-THEORIE
BZW. KIN. GL. $\simeq$ GCE.

$$W_{GC} = Z^{-1} e^{\beta(H - \sum_a \mu_a N_a)}$$

HYDRODYN. ZUSTAND

$\vec{p}(\vec{x},t), \quad n_a(\vec{x},t), \quad H(\vec{x},t)$

$$W_{ZML} = Z^{-1} \exp\{-\beta[H_s - \sum_a \mu_a N_a + \Delta H]\}$$

MOMENTE

STRÖME

$$\Delta H = -\sum_m \int dx\, F_m(x,t)\, P_m(x) + \lim_{\varepsilon \to 0^+} \int_{-\infty}^{0} dt' e^{\varepsilon t'} \cdot [\nabla F_m(x,t+t')\, j_m(x,t') + \partial_{t'} F_m(x,t+t') \cdot P_m(x,t')]$$

LAGRANGE-PARAMETER

BESTIMMUNG VON $F_m$, $\nabla F_m$ AUS QUASISTATIONARITÄTS-BEDINGUNGEN

$$\frac{d}{dt}\,\mathrm{tr}\{W P_m\} = 0 \qquad \frac{d}{dt}\{W \int_{-\infty}^{0} dt\, e^{\varepsilon t'} j_m(t')\} = 0$$

LRF - BESTIMMUNG DER STROMERWARTUNGSWERTE

LINEAR-FELD-LIMES: VERALLG. SEEBECK-KOEFF.

$$\langle \vec{j}^* \rangle = \sum_m \{ \nabla F_m \int_{-\infty}^{0} dt' e^{\varepsilon t'} \langle j^* j_m(t') \rangle_{GC} - F_m \langle j^* P_m \rangle_{GC} \}$$

SEI: $(\varphi_n)_{n=1}^{2N} = (F_j|_{j=1}^{N}, \nabla F_j|_{j=1}^{N})$ SO IST

$$A_{ij}(k)\varphi^j(k,\omega) = H_i(k,\omega) \quad ; \quad H_i \sim E, \nabla n, \nabla T \text{ etc.}$$

$H_i(k,\omega)$ - RETARDIERTE 2-TEILCHEN GF.

VON INTERESSE IST DIE REGULARISIERBARKEIT VON

$H_i$, $\varphi_j$ FÜR $\omega \to 0$.

STRUKTUREN: $\sim \quad K(\omega) = \sum_{\nu=1}^{\infty} \frac{a_\nu}{\omega^\nu}$ (*)

MAN ZEIGT:

$$a_\nu \sim \mathrm{tr}\{W [a_p^+(0) a_p(0), \frac{d^n}{dt^n}(\frac{\partial b_\ell^+(t)}{\partial \ell} b_\ell(t))]\}|_{t=0}$$

$$= O(V^{\nu+m-1}) \;,\; m \geq 0. \qquad , \beta V \sim e^2/(k_B T)$$

GESUCHT: UMORDNUNG VON (*), DIE KOPPLUNGSPARAMETERORDNUNG ERHÄLT.

DURCH $\mathcal{f}$- & $\mathcal{F}$-KETTENBRUCH VERLETZT ($\mathbb{1}^N$-KETTENBRUCH)

$$G = \sum_{i=1}^{\infty} G_i = \sum_{i=1}^{j} G_i + G_j^c \quad , \quad G_i - \frac{a_i}{\omega^i}$$

$$G = \cfrac{G_1}{1 - \cfrac{G_1^c}{G_1} \cfrac{1}{1 + \cfrac{G_1^c}{G_1}}}$$

$$\frac{G_n^c}{G_n} = \frac{G_{n+1}}{G_n} \cfrac{1}{1 - \cfrac{G_{n+1}^c}{G_{n+1}} \cfrac{1}{1 + \cfrac{G_{n+1}^c}{G_{n+1}}}}$$

KETTENBRUCH
VOM TYP
$2^N$

$$\sigma \sim \frac{A[A \ln A + i\Omega]}{(A \ln A)^2 + \Omega^2} \left\{ 1 - \frac{A \ln A \, [A/\ln A + i\Omega]}{(A/\ln A)^2 + \Omega^2} \right\} \qquad (*)$$

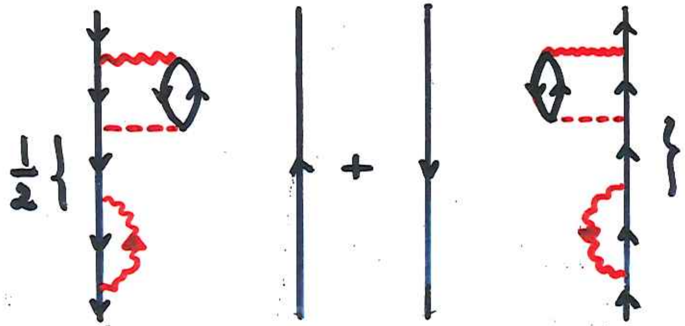
$A = \beta \omega_{pe}$ ; $\Omega - \hbar\omega/\omega_{pe}$ , $\omega_{pe} - \hbar \left[ 4\pi \sum_a e_a^2 n_a / m_a \right]^{1/2}$

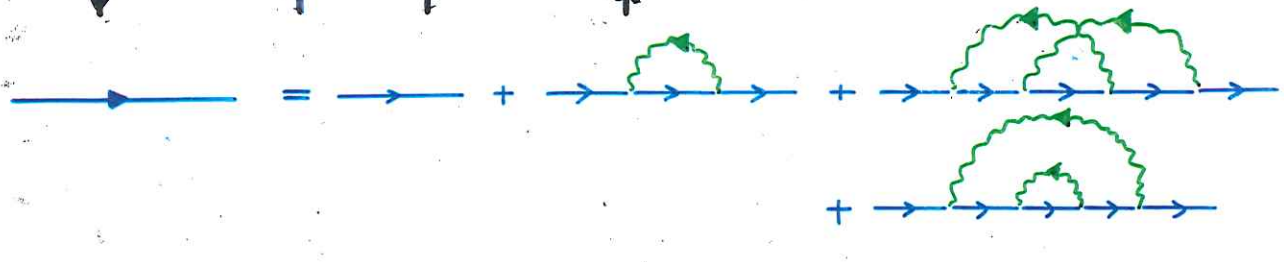
(i) $\Omega \gg A \ln A$ $\qquad \sigma \sim iA/\Omega$

(ii) $A \ln A \gg \Omega \gg A/\ln A$ $\qquad \sigma \sim 1/\ln A$

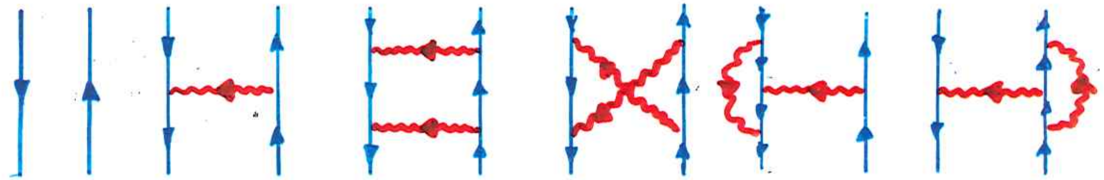
(iii) $A/\ln A \gtrsim \Omega$ $\qquad \sigma \mathrel{\hat{=}} (*)$ UND HÖHEREN NÄHERUNGEN

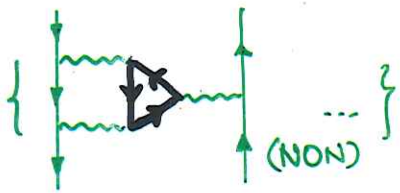
VERSCHIEDENE STOSSZEITEN BEDINGEN
VERSCHIEDENE STOSSREGIME

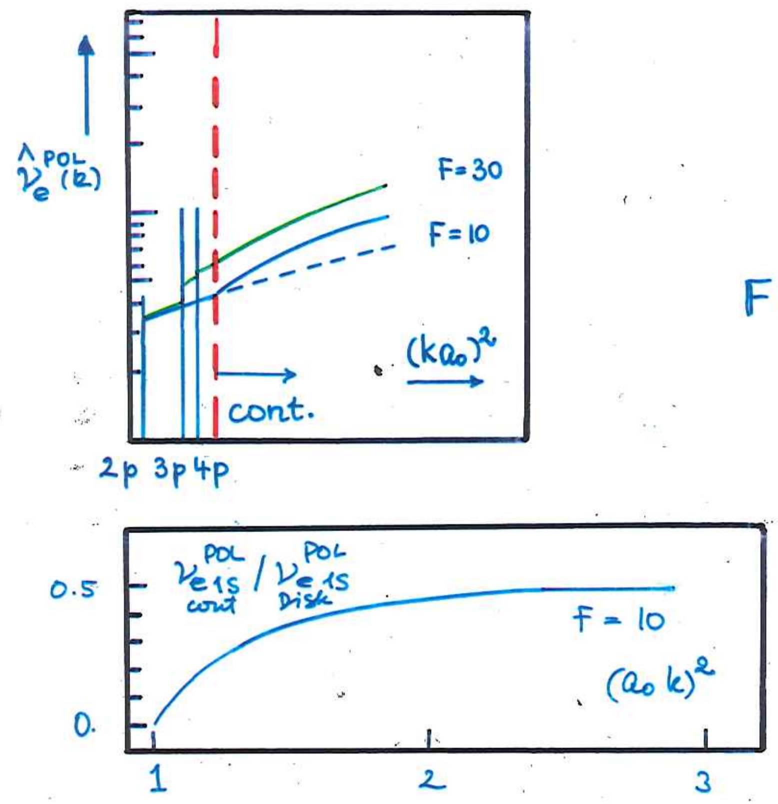

1/2
+


=
+
+
+
+


=
$\sum_c$
c




...
(NON)




$$F = 2/(a_0 \varkappa_D^*)$$

$F_m, \nabla F_m \sim E$

BERECHNUNG VON:

$$\langle j_{el} \rangle = \sum_m \left\{ \nabla F_m \int_{-\infty}^{0} dt' e^{\varepsilon t'} \langle j_{el}\, j_m \rangle_{GC} - F_m \langle j_{el}\, P_m \rangle_{GC} \right\} \quad (**)$$

BEI
$$\partial_t P_m + \nabla j_m = 0 \qquad (\mathrm{I})$$

UND LÖSUNG DES ABSCHIRMPROBLEMS DURCH (I, II)

$$\left[ i\partial_{t_1} + \nabla_1^2/(2m_a) \right] \frac{1}{i} \langle T(\psi_a(1)\psi_a^+(1')) \rangle = \delta(1,1') \qquad (\mathrm{II})$$
$$- i^{-1} \int d2 \sum_b V_{ab}(12) \langle T(\psi_a(1)\psi_b(2)\psi_b^+(2^+)\psi_a^+(1')) \rangle_{t_1=t_2}$$

FÜR:

(α) ELEKTRONENGAS ÜBER POSITIVEM UNTERGRUND

(β) ELEKTRON - ION - PLASMA $(m_e \ll m_i)$

ERGEBNISSE:

(α) (**) MODIFIZIERT DEN KUBO-TERM DURCH HYDRODYNAMISCHE MOMENTE NICHT: **DER RELAXATIONSZEITANSATZ IST STABIL FÜR e-i-STREUUNG,** FORDERUNG DER ERHALTUNG KINETISCHER BZW. GESAMTENERGIE SIND ÄQUIVALENT.

(β) (**) MODIFIZIERT DEN KUBO-TERM NUR, WENN $(P_1, P_2, P_3) = (n, \vec{p}, E)$ IN

$\sigma * 1.1484$ FÜR $E_{KIN}$ - ERHALTUNG $\quad \frac{\delta\sigma}{\sigma} = 0.04299$

$\sigma * 1.2$ FÜR $E_{TOT}$ - ERHATUNG

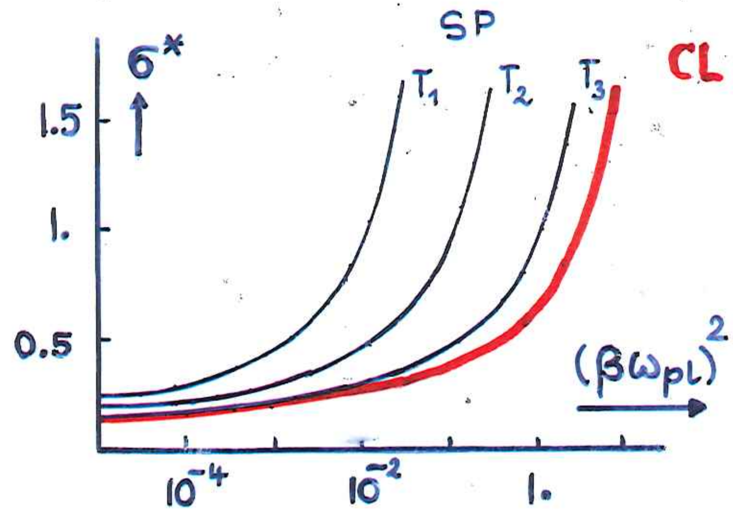
- DIE BERÜCKSICHTIGUNG DES STRESS-TENSORS IST VON UNTERGEORDNETER BEDEUTUNG.

- BISHERIGER NACHTEIL: e-e-STREUUNG IST NICHT AUTOMATISCH BESCHREIBBAR.

  AUSWEG: KADANOFF-BAYM-GLEICHUNG IN ČÁPEK-FUJITA'SCHER FORM.

# 3. EFFEKTE DER TEILCHENDYNAMIK

## 1. EIN-SCHLEIFEN-NÄHERUNG: (STATISCH, $m_e \ll m_i$)




$T_1 = 2800$ K
$T_2 = 28000$ K
$T_3 = 280000$ K

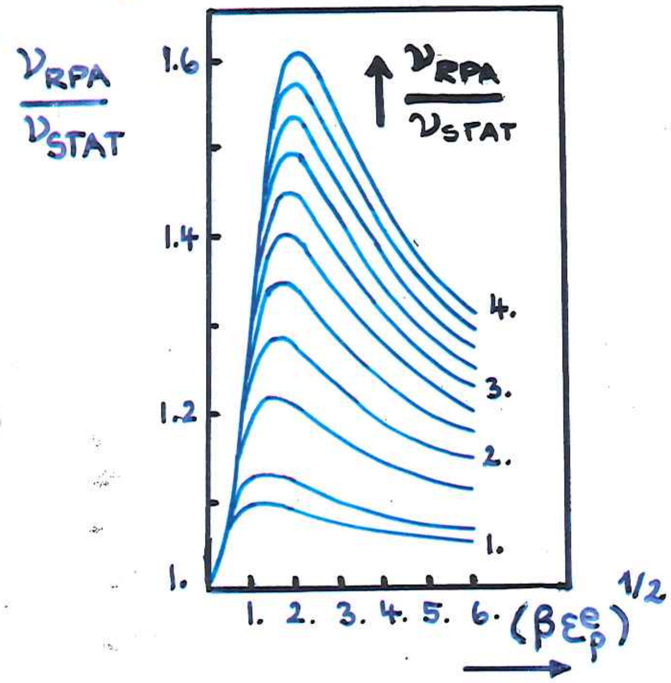
$$\sigma^*_{SP} = \frac{3\sqrt{2}}{4\sqrt{\pi}}\left[\ln\left(1 + 9/\mu^2\right)\right]^{-1}$$

$$\sigma^*_{CL} = \frac{8\sqrt{2}}{3\pi^{3/2}}\int_0^\infty dx\, \frac{x^7 e^{-x^2}}{\nu_{CL}(x)}$$

$$\nu_{CL}(x) = \ln\left(1 + \frac{4x^2}{\beta\varepsilon^e_{\varkappa_D}}\right)$$

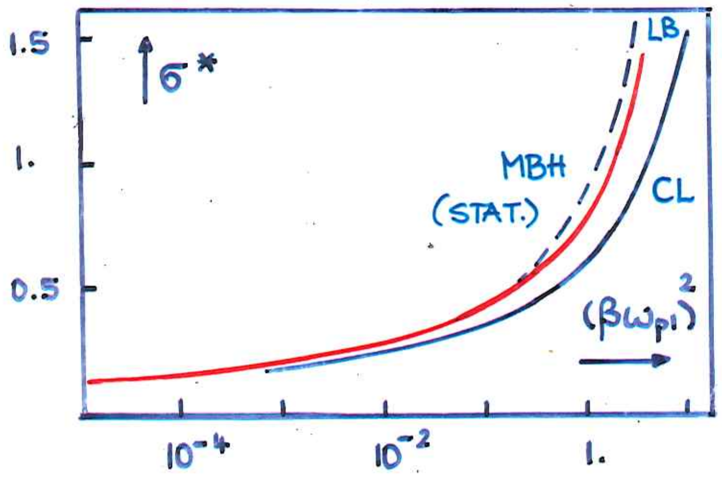
$$(\beta\omega_{pe})^2 = \beta\varepsilon^e_{\varkappa_D} = \frac{k_B T}{4\,\mathrm{ryd}}\,\mu^2$$

## 2. RPA - DYNAMISCHER PHOTONPROPAGATOR $(m_e \ll m_i)$




$$\frac{\nu_{RPA}}{\nu_{STAT}} = 2\pi \int \frac{dq\,dl}{(2\pi)^6} \left| \frac{4\pi e^2}{q^2 \varepsilon'_{RPA}(q,0)} \right|^2 \times f_i(l+q)$$
$$\times \left[1 - f_i(l)\right] \frac{qk}{pk}\, \delta(\varepsilon_p^e - \varepsilon_{p+q}^e)$$
$$\times \left\{ \frac{2\pi n_i e^4 m_e}{\hbar^2 p^3} \left[ \ln\left(1 + \frac{4\varepsilon_p^e}{\varepsilon_{\varkappa_D}^e}\right) - \frac{4p^2}{4p^2 + \varkappa_D^2} \right] \right\}^{-1}$$

$$\varepsilon_{RPA}(q,0) = 1 + \sum_a \frac{4\pi n_a e_a^2}{q^2 k_B T}\, {}_1F_1\left(1, \frac{3}{2}, -\frac{\hbar^2 q^2}{8 m_a k_B T}\right)$$




$$\sigma_1^* = \frac{8\sqrt{2}}{3\pi^{3/2}} \int_0^\infty \frac{dx\, x^7 e^{-x^2}}{\nu_{RPA}(x)}$$

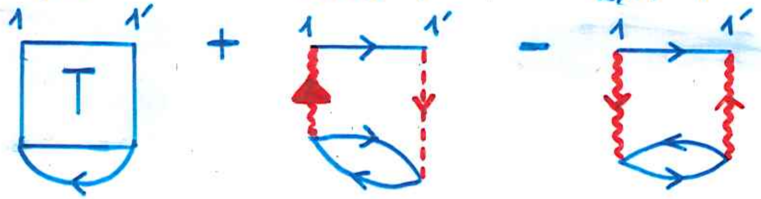
$$x = (\beta \varepsilon_p^e)^{1/2}$$

$\sigma_{LB}^*$

# 3. MULTIPLE ZWEITEILCHENSTREUUNG

$$\sigma_2^T(q,\varkappa_D) = \sigma_{2,B}^T(q,\varkappa_D) + \sigma_{2,LB}^T(q,\varkappa_D) - \sigma_{2,FP}^T(q,\varkappa_D)$$

$\Sigma_a(11') \sim$



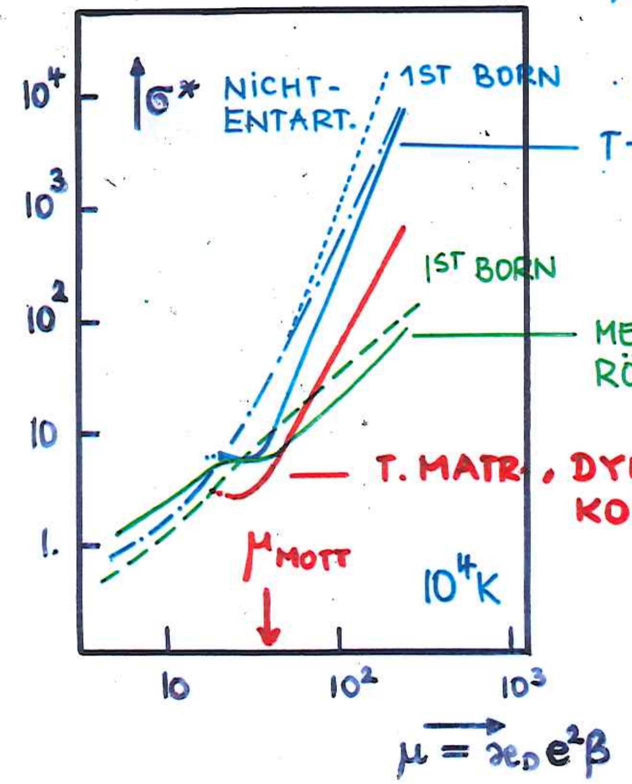

$$F = 2/(a_0\varkappa_D)$$

$$\sigma^* = \frac{8\sqrt{2}}{3\pi^{3/2}}\left[F_{1/2}(\alpha_e)\right]^{-1}\int_0^\infty dx\, f_e(x)\left[1-f_e(x)\right] x^7 \nu^{-1}(x) \quad , \quad x = (\beta\varepsilon_p^e)^{1/2}, \; Y = \frac{x}{2}\sqrt{\beta\, ryd}$$

$$\nu(x) = 8Y^2 \sum_{\ell=0}^{\infty} (\ell+1)\sin^2\left[\delta_\ell^D(Y,F) - \delta_{\ell+1}^D(Y,F)\right] + 4F^2Y^2/(4F^2Y^2+1)$$

NÄHERUNG: KLIMONTOVICH-POT. $F\{(V_q^D V_q^C)^{1/2}\}$

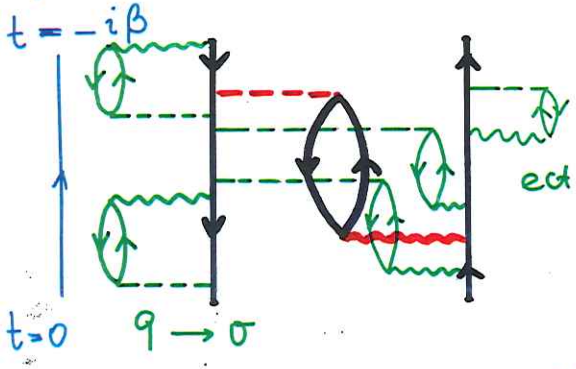
$\delta_\ell^D$ MEYER, BARTOLI 1981

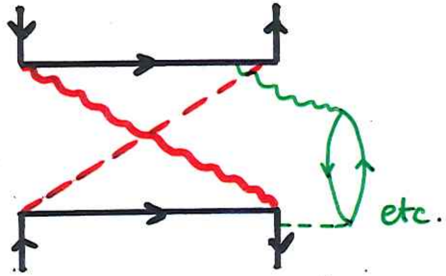



$$\varkappa_D = a_0^{-1}\left[\frac{2\left[F_{1/2}(\alpha_e) + F_{1/2}(\alpha_e)\right]}{(\pi\beta\, ryd)^{1/2}}\right]^{1/2}$$

## 4. N-TEILCHEN STREUUNG, $N \geq 3$

(i) 2-SCHLEIFEN-KORREKTUR UND $\sigma^* \sim O(1/(\ln n^{1/2}))$

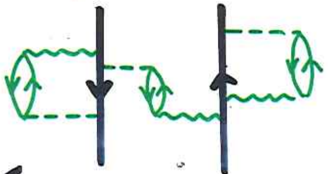



$O[(e^2)^4]$

$q \rightarrow \sigma$

(1.) MODIFIKATION DER WICHTUNGSFUNKTION $g(x)$



$g(x) \sim f_e(\beta \varepsilon_p^e) \longrightarrow f_e(\beta \varepsilon_p^e)(1 - \frac{\mu}{2})$

$\beta = 1/k_B T$

ENTSPRICHT THERMODYNAMISCH DEM DEBYE-HÜCKEL GRENZGESETZ FÜR DAS CHEMISCHE POTENTIAL.

(2.) MODIFIKATION DER STOSSFREQUENZ $\nu(x)$

ZUSAMMENFASSUNG VON ca. 1100 DIAGRAMMEN

: 4 UMLÄUFE VON  $\nu(x) \rightarrow \nu(x)(1 - 3/2\,\mu)$

$$\sigma^* \rightarrow \sigma^* \cdot \frac{1-\mu/2}{1-3/2\,\mu} \quad , \mu \ll 1$$

$$\gtrsim \sigma^* (1+\mu)$$

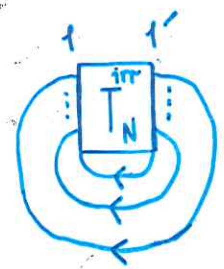
## (ii) MUTIPLE N-TEILCHENSTREUUNG

$$\nu_a^{[N]}(p,\omega) = \int \frac{dk\, d\Omega}{(2\pi)^4}\, W_a^{[N]}(p,k,\omega,\Omega)\, A_a(p-k,\Omega)\,(1-\cos_{p,p-k})$$

$$\Sigma_{a_1}^{\gtrless [N]\,(0)}(p_1,\omega_1) = \int \frac{dk\, d\Omega}{(2\pi)^4}\, W_a^N(p,k,\omega,\Omega)\, G_{a_1}^{\gtrless(0)}(p-k,\Omega)$$

$$\Sigma_{a_1}^{\gtrless [N]\,(0)}(p_1,\omega_1) = -\sum_{a_2\ldots a_N} \int \frac{dp_1}{(2\pi)^3}\cdots\frac{dp_N}{(2\pi)^3} \sum\!\!\!\!\!\!\int_E \psi_{E\,a_1\ldots a_N}^{R[N]}(p_1\ldots p_N)\,\psi_{E\,a_1\ldots a_N}^{R[N]}(p_1\ldots p_N)$$

$$\left[\cancel{f_{[N]}^{a_1\ldots a_N}(E)} - f_{[N-1]}^{a_2\ldots a_N}(\varepsilon_{p_2}^{a_2}+\ldots+\varepsilon_{p_N}^{a_N})\right]$$




$$\begin{Bmatrix} 1 - f_{a_1}(E-\varepsilon_{p_2}^{a_2}\ldots-\varepsilon_{p_N}^{a_N}) \\ f_{a_1}(\varepsilon_{p_2}^{a_2}+\ldots+\varepsilon_{p_N}^{a_N}) \end{Bmatrix} \delta(E-\varepsilon_{p_2}^{a_2}\ldots-\varepsilon_{p_N}^{a_N})$$

$$\{[1-f_{a_{N-1}}(p_{N-1})-f_{a_{N-1}}(p_N)]$$

$$\vdots$$

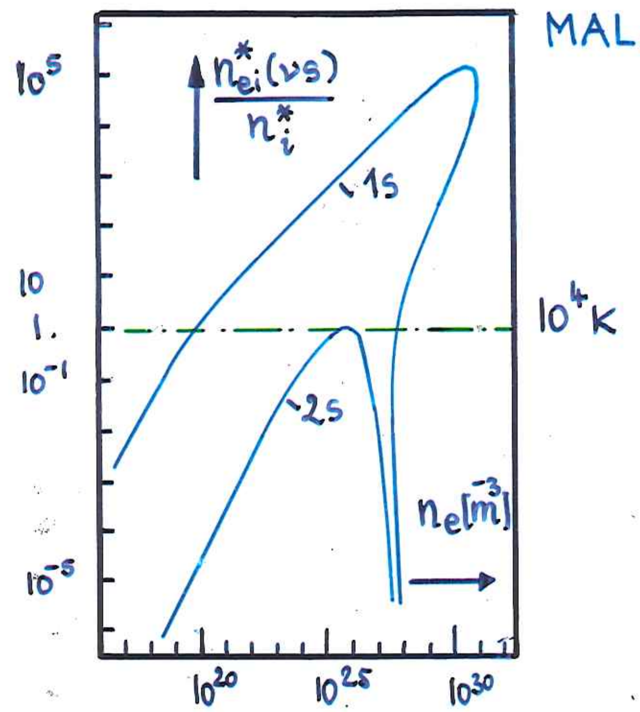
$$\times[1-f_{a_2}(p_2)-f_{a_2}[\varepsilon_{p_3}^{a_3}+\ldots+\varepsilon_{p_N}^{a_N}]]\}$$

$$\sum\!\!\!\!\!\!\int_E \longrightarrow \int_{E^{CONT}} dE \quad , \quad \textbf{SEI: } 0 \in E^{CONT.}$$

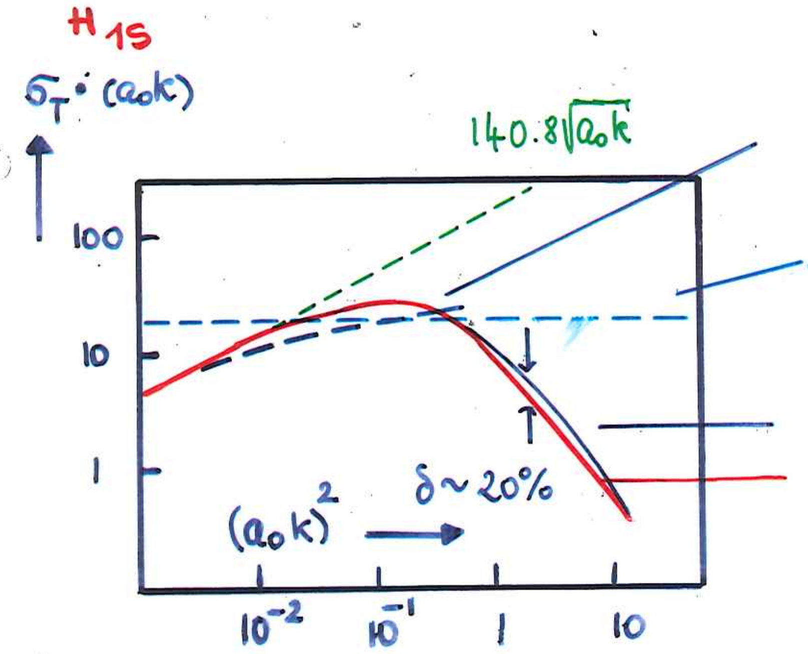
$$\boxed{\Im m\langle p_1\ldots p_N|T_{KK}|p_1\ldots p_N\rangle = -\pi\,\psi_{\substack{a_1\ldots a_N\\K}}^{R[N]}(p_1\ldots p_N)\,\psi_{\substack{a_1\ldots a_N\\K}}^{R[N]}(p_1\ldots p_N)}$$

$$= -\pi \sum\!\!\!\!\!\!\int_{K'} \int \frac{d\bar p_1}{(2\pi)^3}\cdots\frac{d\bar p_N}{(2\pi)^3}\,\delta\Big(E_K - E_{K'} - \sum_{i=1}^{N}(\varepsilon_{p_i}^{a_i}-\varepsilon_{\bar p_i}^{a_i})\Big)$$

$$|\langle p_1\ldots p_N|T_{KK'}|\bar p_1\ldots\bar p_N\rangle|^2$$




MAL.- KRAEFT, KILIMANN, KREMP
1975



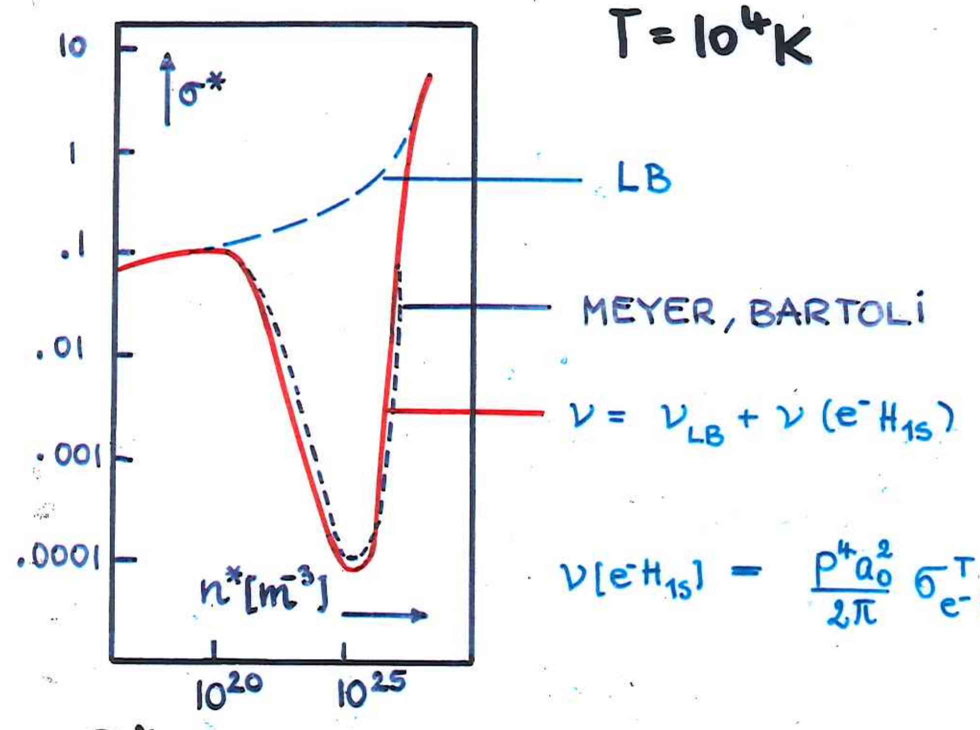

SCHWARTZE 1961 etc. ($\delta_{(3)}$)
BLAGOSKLONSKAYA 1969

MASSEY, MOISEWITSCH 1950
ERGINSOY 1950

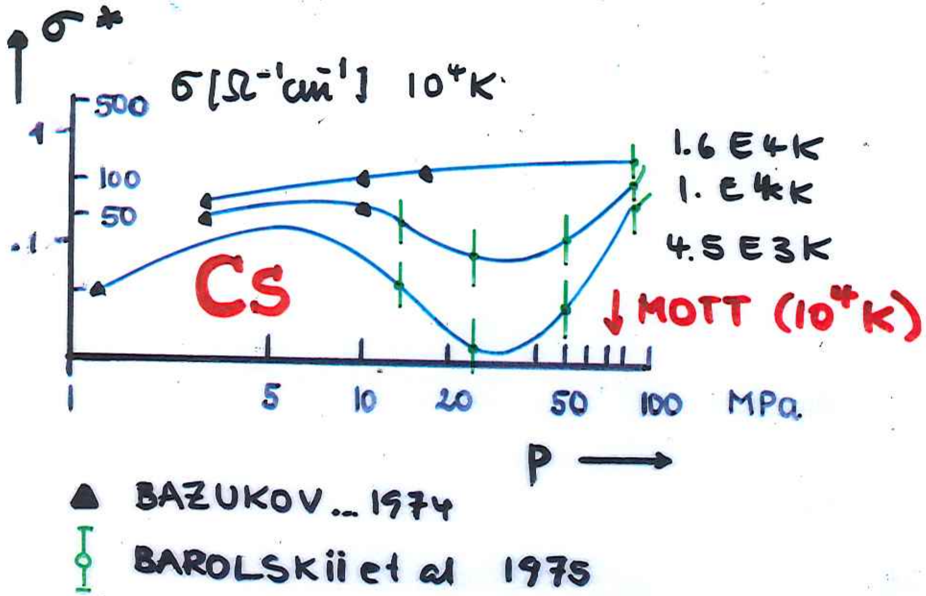
CALLAWAY, WOOTEN 1974
MEYER, BARTOLI 1981

FON et al. 1978
DIESE ARBEIT

# $T = 10^4 K$




$$\nu[e^-H_{1s}] = \frac{p^4 a_0^2}{2\pi}\, \sigma^T_{e^-H_{1s}}(p)\, \frac{n^*_{ep}(1s)}{n_p}$$



# ZUR CLUSTERENTWICKLUNG DER ZUSTANDSGLEICHUNG:

$$p - p_0 = \sum_{\alpha_1} \frac{1}{2\Omega_0} \int_0^1 \frac{d\lambda}{\lambda} \int d1\, d1' \, \Sigma^{\lambda}_{\alpha_1}(1'1^+) G_{\alpha_1}(1'1^+)$$

$$\underset{T_N}{\simeq} \sum_{\alpha_1\alpha_2} \frac{1}{2\Omega_0} \int_0^1 \frac{d\lambda}{\lambda} \int d1\, d2\, d1' d2' \, \lambda \, \phi_{\alpha_1\alpha_2}(12, 1'2') \, G_{\alpha_1}(1'1^{++}) \, G_{\alpha_2}(2'2^+)$$

$$\phi_{\alpha_1\alpha_2}(12,1'2') = \int d1'' d2'' \, T^{(2)}_{\alpha_1\alpha_2}(12, 1''2'') \det_2 \| \alpha_i' \alpha_j'' \, \delta(i''j') \|_{2,2}$$

$$+ \sum_{n=3}^{\infty} \int d1'' \dots dn'' \int d3 \dots dn \sum_{\alpha_3 \dots \alpha_n} G_{\alpha_3}(33') \dots G_{\alpha_N}(n,n')$$

$$\times T^{R[M]}_{\alpha_1 \dots \alpha_M}(1,3\dots, n,2; 1''3'' \dots n''2'')$$

$$\times \det_n \| \delta_{\alpha_i' \alpha_j''} \, \delta(i''j') \|_{n,n}$$

$$\boxed{T^R_N} = \boxed{R_N} - \text{ALLE REDUZ. SUBCLUSTER}$$

$$\frac{1}{z - H_N} - \sum_{R \in \mathcal{R}} \frac{1}{z - H^R}$$

$\longrightarrow$ separiere BOUND-STATE $-N-$ BEITRAG: $\quad n\Lambda^3 \ll 1$

$$p_N \sim \int \frac{dp_1\, dp_{1'}}{(2\pi)^6} \dots \frac{dp_N\, dp_{N'}}{(2\pi)^6} \sum_{E_N}\!\!\!\!\!\!\!\int \sum_{\alpha_1 \dots \alpha_N} \det_N \| \delta_{\alpha_i' \alpha_j} \delta_{p_i' p_j} \|_{N,N}$$

$$\cdot \, e^{\beta \sum_{\nu=1}^{N} \mu_\nu} \, \psi^{\alpha_1 \dots \alpha_N}_{E_N}(p_1 \dots p_N) \, \psi^{\alpha_1 \dots \alpha_N *}_{E_N}(p_1 \dots p_N) \, e^{-\beta E_N}$$

# CLUSTERBEITRÄGE ZUR DIELEKTRISCHEN FUNKTION

$$\varepsilon_N^{(b)}(q,\omega) = \sum_{\alpha_1\dots\alpha_N} \frac{4\pi e_{\alpha_i}}{q^2} \frac{1}{N-1} \int \frac{dp_1}{(2\pi)^3} \cdots \frac{dp_N}{(2\pi)^3} \frac{dp_1'}{(2\pi)^3} \cdots \frac{dp_N'}{(2\pi)^3} \sum\!\!\!\!\!\!\int_{E_N'} \sum_{E_N}$$

$$\delta\Big(\sum_{i=1}^N (p_i' - p_i)\Big) \psi_{E_N}^{R\alpha_1\dots\alpha_N}(p_1 \dots p_N)\, \psi_{E_N}^{R*}(p_1' \dots p_N')$$

$$\psi_{E_N'}^{R\alpha_1\dots\alpha_N}(p_1, \dots p_i - q \dots p_N) \cdot \Big\{ \sum_{j=1}^N e_{\alpha_j} (1 - \delta_{\alpha_i \alpha_j})$$

$$\cdot\, \psi_{E_N'}^{R\alpha_1\dots\alpha_N *}(p_1', \dots p_j' - q \dots p_N') \Big\}$$

$$\Big[f_{[N+1]}^{\alpha_1\dots\alpha_N}(E_N') - f_{[N+1]}^{\alpha_1\dots\alpha_N}(E_N)\Big] (E_N' - E + \omega + i0)^{-1}$$

$$\omega_{pl}^{2\,[N]} = -\lim_{\omega\to\infty} \omega^2 \varepsilon_N^{(b)}(q \to 0, \omega)$$

$$= \frac{1}{N-1} \sum_{\nu \neq \nu' = 1}^N \sum\!\!\!\!\!\!\int_{E_N'} \sum_{E_N}^{(b)} e_{\alpha_\nu} e_{\alpha_{\nu'}} \left| \langle E_n | z_{\alpha_\nu} - z_{\alpha_{\nu'}} | E_{n'} \rangle \right|^2$$

$$4\pi \cdot 2 \cdot (E_N - E_{N'})\, n^*_{E_N, \alpha_1 \dots \alpha_N}$$

$$= \frac{1}{N-1} \sum_{\nu \neq \nu' = 1}^N \sum_{E_N}^{(b)} \frac{e_{\alpha_\nu} e_{\alpha_{\nu'}}}{m_{\alpha_\nu \alpha_{\nu'}}} 4\pi\, n^*_{E_N, \alpha_1 \dots \alpha_N}$$