OXFORD
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ON $F_2(x, Q^2)$ AT LOW Q^2 IN QCD

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1. HISTORY AND CURRENT DATA
2. HIGHER TWIST CORRECTIONS TO QCD EVOLUTION EQUATIONS
3. A MINUS SIGN (IN QCD)
4. FIRST NUMERICAL RESULTS.

1. HISTORY AND CURRENT DATA

- TWIST EXPANSION
  LCE

WILSON 1969

BRANDT, PREPARATA
FRISHMAN ~1970

- TWIST : $d - n \cdot s$

GROSS
TREIMAN 1971

- LO ANOMALOUS DIM.
  TWIST 2

GROSS, WILCZEK
GEORGI, POLITZER
1973,7

⟶ UV SINGULARITY = MASS SINGULARITY

TWIST 2

ALTARELLI, PARIS
197

- CAN THE GROTH OF PARTON DENSITIES BE DAMPED?

1ST SIMPLE INTUITIVE POMERON-BASED MODEL:

GRIBOV, LEVIN,
RYSKIN 1981

A. MÜLLER, QIU 1

DLA ⟶
AGK CUTTING RULES

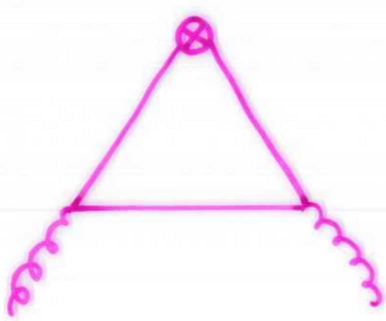
ABRAMOWS
GRIBOV
KANCHELI,

BARTELS RYSKIN

2. HIGHER TWIST CORRECTIONS TO EVOLUTION EQUATIONS

THE IDEA OF GRL WAS TO STUDY HIGHER TWIST EFFECTS ONTO LOWER TWIST (2) QUANTITIES. WE WOULD LIKE TO UNDERSTAND THAT IN PLAIN QCD, OTHERWISE UNINTERESTING. A CONNECTION BETWEEN OEV OF DIFFERENT TWIST DOES ONLY EXIST IFF THE THEORY BEARS A MASS SCALE!

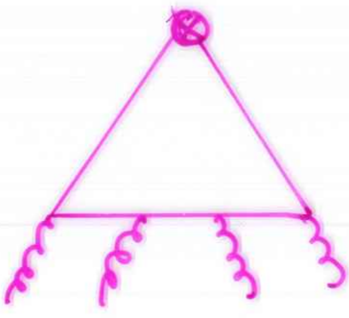
GRAPHS $G \to q$



$2 \to 2$

$$\frac{dp_\perp^2}{p_\perp^2}$$

$$d \log(p_\perp^2/m^2)$$



$2 \to 4$

$$\frac{dp_\perp^2}{p_\perp^4}$$

$$\frac{dp_\perp^2}{p_\perp^4} = \frac{m^2}{p_\perp^2}\, d\log\left(\frac{p_\perp^2}{m^2}\right)$$

VANISHES AS $m^2 \to 0$

NO CONTRIBUTION TO ANOMALOUS DIM'S!

WE CONSIDER ONLY THE $2 \to 4$ TRANSITIONS..

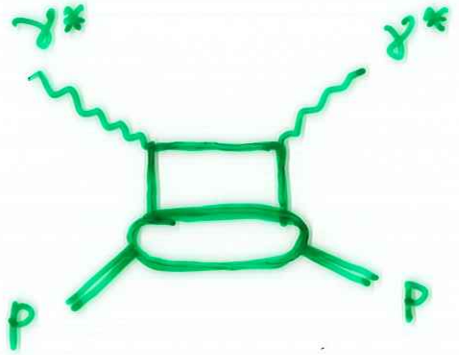
G_1, Σ_1 evolution as driven by $G_2(x_1, x_2)$.

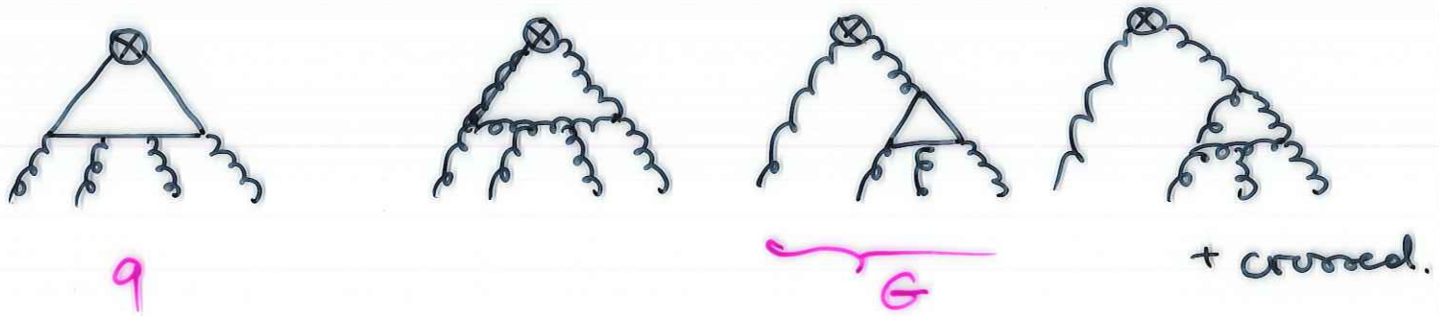
rather than the scaling violations of $G_2(x_1, x_2)$.

GOAL: COEFFICIENT FUNCTION

DIAGRAMS :

CALCULATE THE COMPTON AMPLITUDE:





WITH ALL POSSIBLE CUTS.

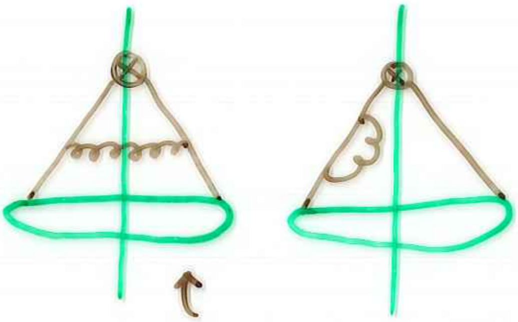
METHOD : TIME ORDERED PERTURBATION THEORY

(S. WEINBERG, 1966)

PICK THE $\frac{dp_\perp^2}{p_\perp^4}$ TERMS.

EXAMPLE

TWIST 2: AP- EQS. $\frac{dp_\perp^2}{p_\perp^2}$.



$q_1(x)$ - SINGLE PARTICLE DISTRIBUTIONS

SPLIT THE FORWARD COMPTON AMPLITUDE INTO:

- NON- PERT. INPUT DISTRIBUTION
- "SPLITTING" FUNCTION.

IS THIS POSSIBLE ?

FOR THE TERMS $\frac{dp_\perp^2}{p_\perp^4}$: YES.

NEW INPUT DENSITY:

$$G_{(2)}(x_1, x_2; x_1', x_2'; \mu^2)$$ (AND OTHERS)

NOT RELATED TO $G_{(1)}(x_1; x_1'; \mu^2)$!

(IN GENERAL.)

$C^{4\to 2}_{G\to q}$ SIMPLIFIES IF ONE PUTS:

$$x_1 = x_2, \quad x_1' = x_2'$$

(ALTHOUGH THE GENERAL EXPRESSIONS ARE AVAILABLE.)

$$C^{4\to 2}_{G\to q}(x_1, x_1; x_1', x_1') = C^{4\to 2}_{G\to q}(x_1, x)$$

$$= \frac{1}{96}\,\frac{(2x_1 - x)^2}{x_1^5}\Big[4\,(4x^2 - 6x x_1 + 5x_1^2) - \frac{1}{2}\,(4x^2 - 6x x_1 + 4x_1^2)\Big]$$

THE MELLIN POLES ARE SITUATED AT:

$N = 0, -1, \ldots, -4$

WITH SIMILAR STRENGTH.

HOWEVER THERE ARE MORE GRAPHS.

ENERGY-MOMENTUM IS PRESERVED (CORRESPONDING GRAPH IN THE GLUONIC PART.)

$P^{GG\to q\bar{q}}$ AND $P^{GG\to GG}$ SIMPLIFY IF ONE PUTS $x_1 = x_2$, $x_1' = x_2'$ (ALTHOUGH THE GENERAL EXPRESSIONS ARE AVAILABLE.)

$$P^{GG\to q\bar{q}}(x_1, \cancel{x}) = \frac{1}{96}\,\frac{(2x_1 - x)^2}{x_1^5}\Big[4\,(4x^2 - 6xx_1 + 5x_1^2) - \frac{1}{2}\,(4x^2 - 6xx_1 + 4x_1^2)\Big]$$

$$P^{GG\to GG}(x_1, x) = \frac{27}{128}\,\frac{(2x_1 - x)}{x_1^5\,x} \cdot \left(99\,x_1^4 - 130\,x_1^3 x + 132\,x_1^2 x^2 - 64\,x_1 x^3 + 16\,x^4\right)$$

THE MELLIN TRANSFORM IN x LEADS TO POLES AT:

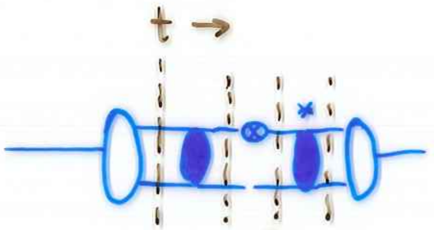
Q: $N = 0, -1, \ldots, -4$

G: $N = 1, 0, -1, \ldots -3$ SIMILAR STRENGTH.

HOWEVER THERE ARE MORE GRAPHS.

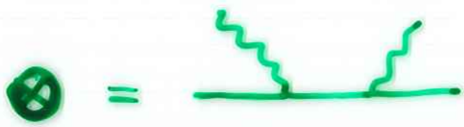
THE CALCULATION:

EXAMPLE : $\quad p^{GG \to q\bar{q}}(x_1, x_2)$



$+ \ldots$

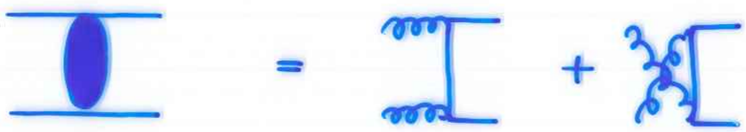
$t^2 + u^2 + tu + ut$



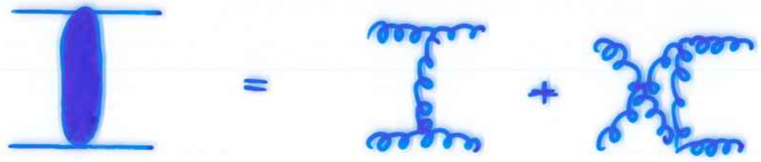
PHOTON VERTEX

$s^2, su, st \ldots$ DO NOT CONTRIBUTE

TOPT : THE ABOVE TIME LINES TELL US WHICH ENERGY DENOMINATORS TO WRITE DOWN.



$p^{GG \to q\bar{q}}$



$p^{GG \to GG}$

WE CALCULATE FOR <u>ALL</u> x.

CONTRIBUTION TO THE COEFFICIENT FCT.

$$\frac{dq_1}{d\log Q^2} = \frac{\alpha_s}{2\pi}\left[P^{2\to2}_{qq}\otimes q_1 + P^{2\to2}_{qG}\otimes G_1\right]$$

$$+\left(\frac{\alpha_s}{2\pi}\right)^2 \hat{C}^{4\to2}_{G\to q}\,\hat{\otimes}\, G_2$$

$$\hat{C}^{4\to2}_{G\to q} = \frac{Q_0^2}{Q^2}\cdot C^{4\to2}_{G\to q}\left(x_1, x_2 ; x_1', x_2', \mu^2\right)$$

$$q_1 \longrightarrow \qquad 2xF_1 = x\sum_{i=1}^{N_f}\left(q_{i1}+\bar{q}_{i1}\right).$$

HOW TO DESCRIBE IT ?

KUTI, WEISSKOPF 1971

$$D_{(n)}(x_1, \dots, x_n) = A(1 - x_1 - x_2 - \dots x_n)^{\alpha}$$

TODAY :

$$D_n \sim (x_1 \dots x_n)^{\alpha} (1 - \sum_i x_i)^{\beta}$$

$$D_1 \sim x_1^{\alpha}(1 - x_1)^{\beta}$$

$$D_1^2 \sim x_1^{\alpha} x_2^{\alpha} \; (1 - x_1 - x_2)^{\beta} + \dots$$

$$D_1^2 \not\sim D_2$$

$$D_2(x_1, x_2) = \chi(x_1, x_2) \; D_1(x_1) D_1(x_2)$$

CAN BE LARGE OR SMALL.

REMEMBER : IN 1991 NOBODY KNEW
HOW $G_1(x)$ BEHAVES AT $x \sim 10^{-4}$!

ONLY EXPERIMENT (AND MUCH LATER MAY BE LGT) MAY HELP.

WE KNOW :

$$G_{(2)}(x_1, x_2) = G_{(2)}(x_2, x_1),$$

NOTHING ELSE.

IS $G_{(2)} > 0$?

ALSO THIS WE DO NOT KNOW YET.

$$\boxed{\text{FINAL RELATION FOR: } \frac{d\Sigma}{d\log Q^2}}$$

$$\frac{d\,x\overset{(-)}{q}}{d\log Q^2} = \frac{\alpha_s}{2\pi}\left[x P_{qq}\otimes q + x P_{qG}\otimes G_1\right]$$

ANTISCREENING $\longrightarrow$

$$+\left(\frac{\alpha_s}{2\pi}\right)^2 \frac{1}{Q^2}\int_{x/2}^{1/2} dx_1 \left(\frac{x}{x_1}\right) \mathbb{G}_{(2)}(x_1)\cdot P^{GG\to q\bar{q}}(x_1, x)$$

SCREENING $\longrightarrow$

$$-2\left(\frac{\alpha_s}{2\pi}\right)^2 \frac{1}{Q^2}\int_{x}^{1/2} dx_1 \left(\frac{x}{x_1}\right) \mathbb{G}_{(2)}(x_1)\, P^{GG\to q\bar{q}}(x_1, x).$$

$$\mathbb{G}_{(2)}(x_1) = \frac{9}{8}\,\frac{1}{R^2}\,G_{(2)}(x_1)$$

AS A MODEL WE USE: $G_{(2)}(x) = x_1^2 G_1^2(x_1)$.

WE PUT NO ! MEANING IN THE SIZE OF R.

G_2 IS A NEW FUNCTION.

$$P^{GG\to q\bar{q}} = \frac{1}{96}\,\frac{(2x_1-x)^2}{x_1^5}\Big[4\,(4x^2-6x x_1+5x_1^2) - \frac{1}{2}(4x^2-6x x_1+4x_1^2)\Big].$$

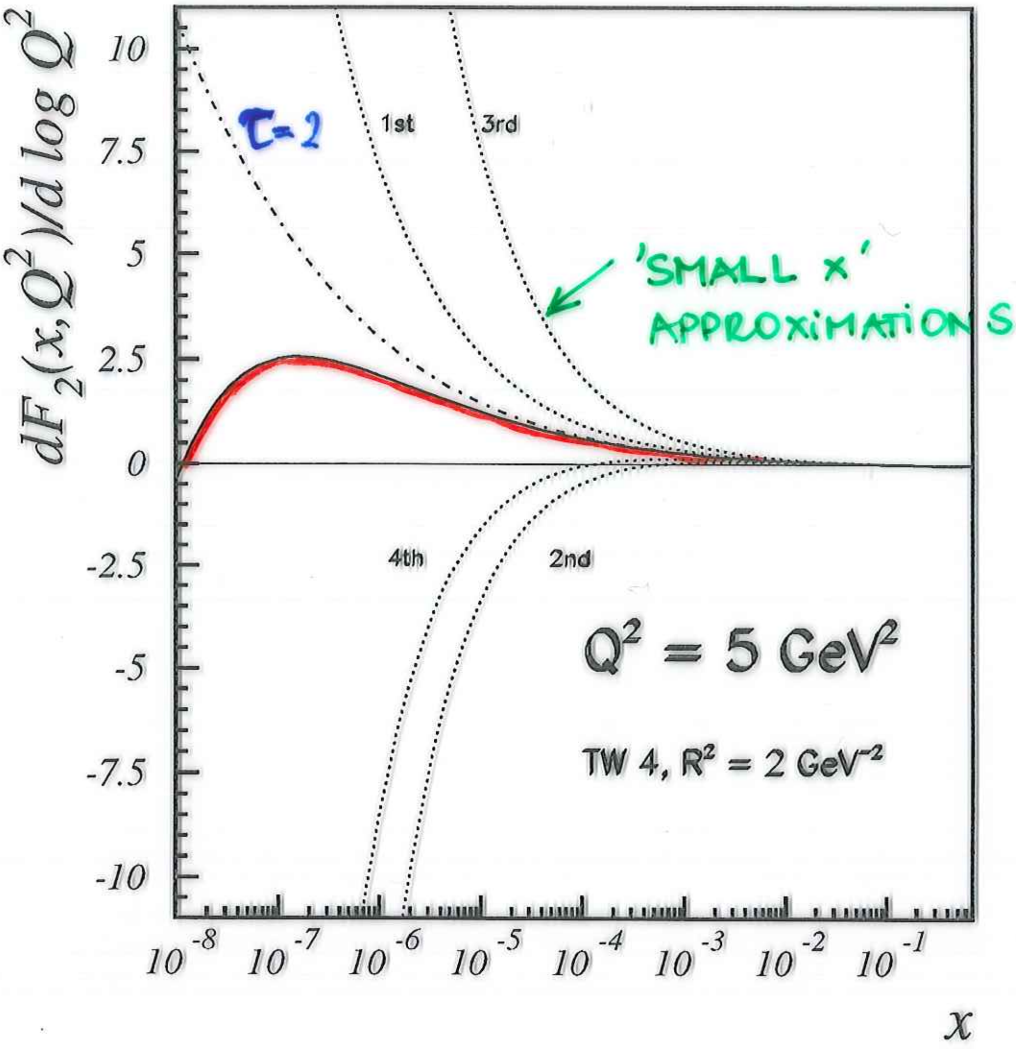


Figure 2: Comparison of the slope $dF_2(x,Q^2)/d\log Q^2$ at $Q^2 = 5\,\mathrm{GeV}^2$ and twist–4 mass scale $R^2 = 2\,\mathrm{GeV}^2$, Eq. (??) (full line) with the corresponding results obtained approximating the coefficient function Eq. (??) by the sequence of contributing powers. 1st: z^0, 2nd: z^2 etc. (dotted lines).

SCREENING CORRECTIONS:

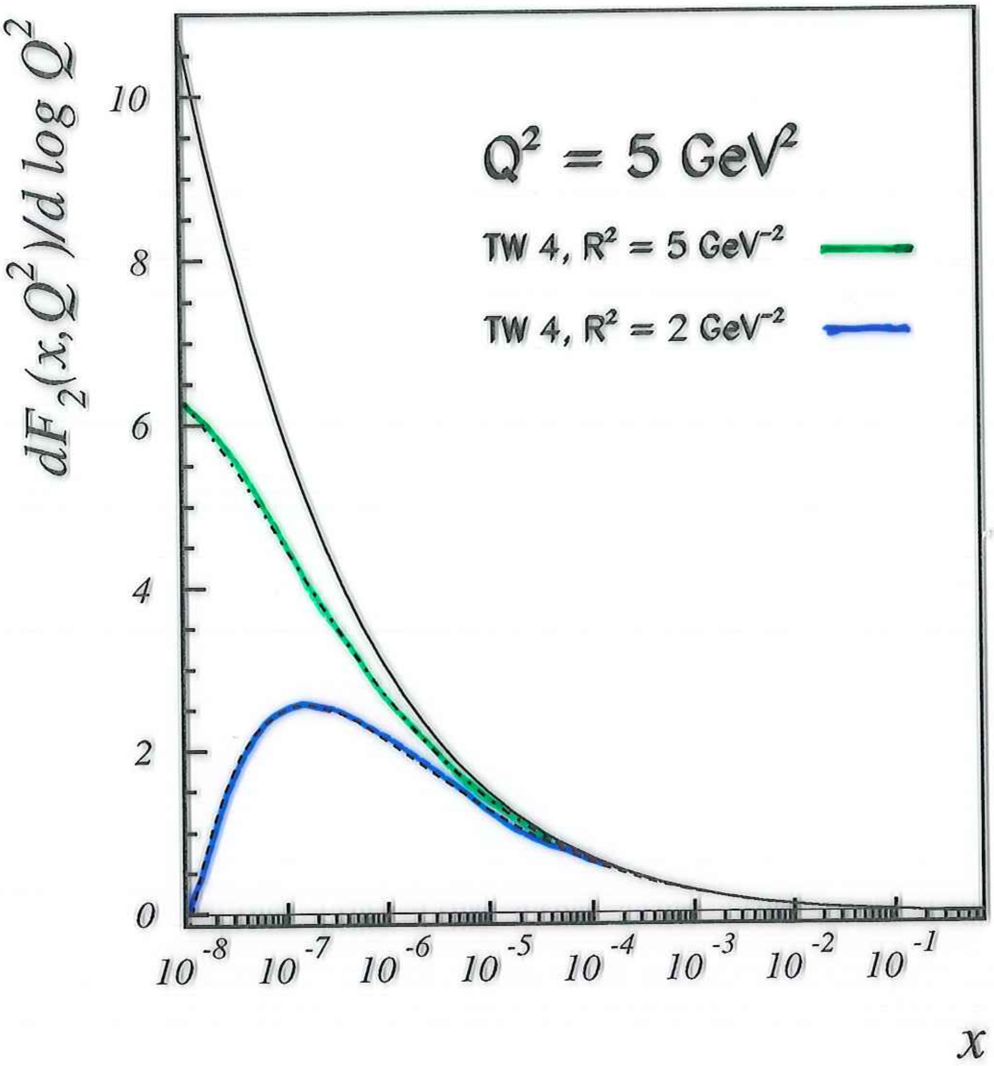


Figure 1a: The slope $dF_2(x,Q^2)/d\log Q^2$ at $Q^2 = 5\,\mathrm{GeV}^2$. Full line: leading order twist−2 contributions (parametrization Ref. [21]). Dash-dotted line: Eq. (??) with twist−4 mass scale $R^2 = 5\,\mathrm{GeV}^2$, and dashed line: $R^2 = 5\,\mathrm{GeV}^2$.

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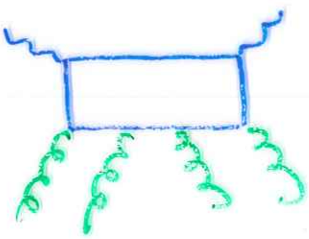
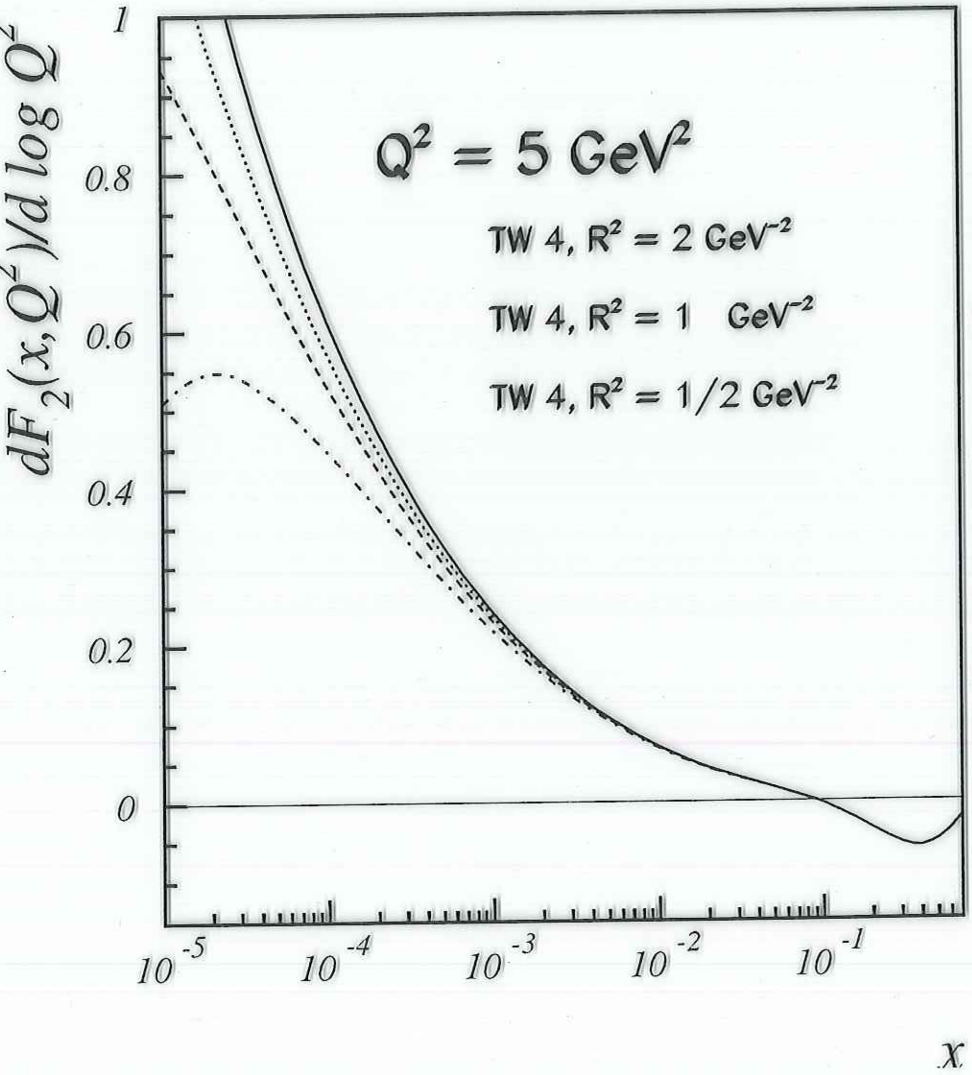
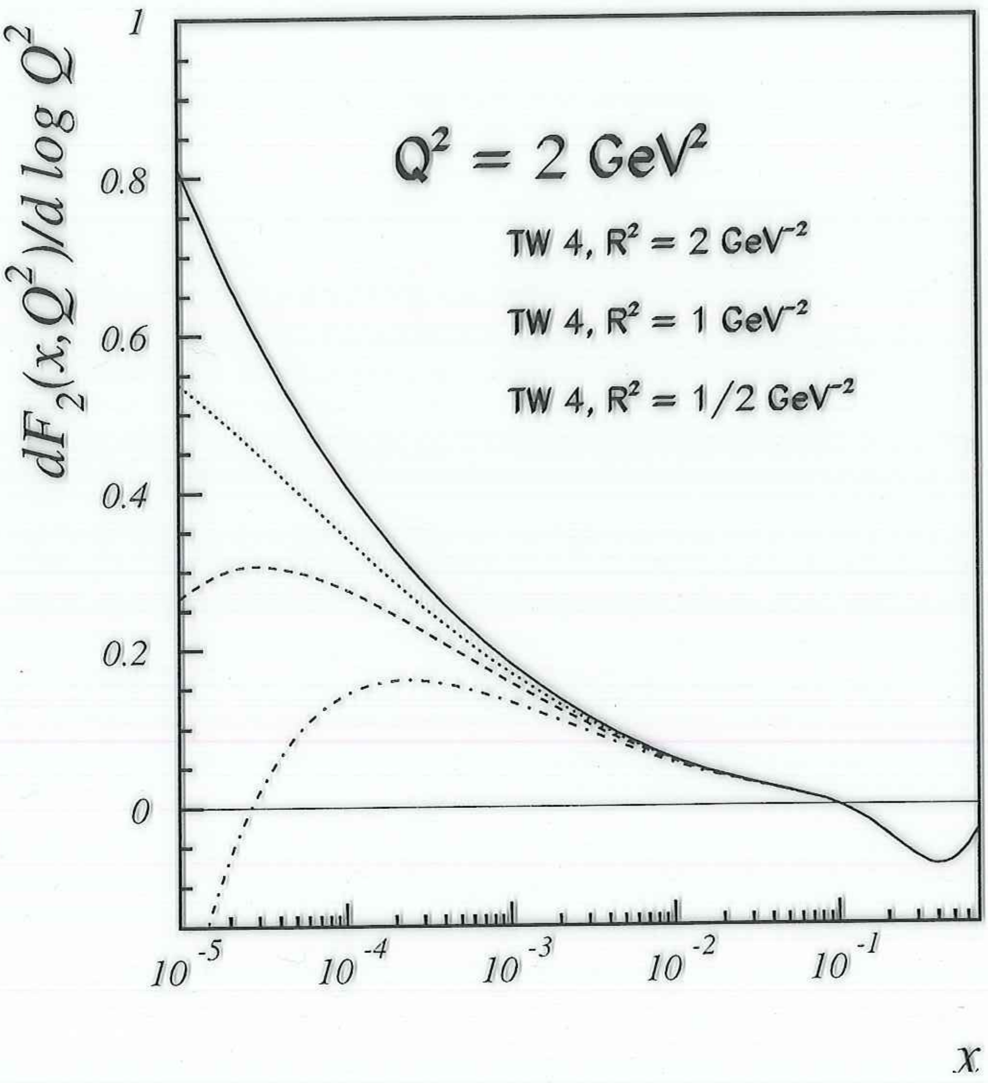


+ etc.

COEFFICIENT FUNCTION

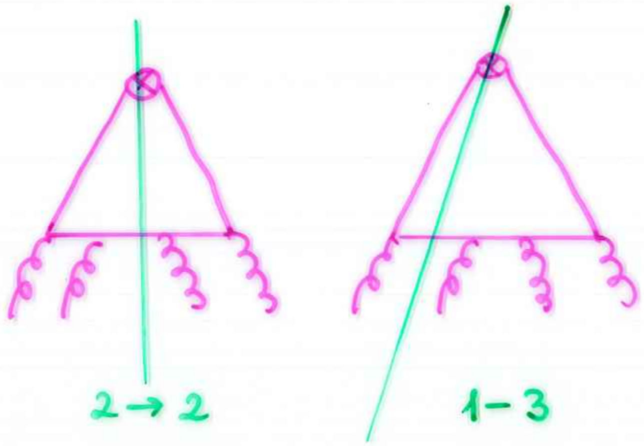

$dF_2(x,Q^2)/d\log Q^2$
$Q^2 = 5$ GeV^2
TW 4, $R^2 = 2$ GeV^{-2}
TW 4, $R^2 = 1$ GeV^{-2}
TW 4, $R^2 = 1/2$ GeV^{-2}
1
0.8
0.6
0.4
0.2
0
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
x



$dF_2(x,Q^2)/d\log Q^2$
$Q^2 = 2\ \mathrm{GeV}^2$
TW 4, $R^2 = 2\ \mathrm{GeV}^{-2}$
TW 4, $R^2 = 1\ \mathrm{GeV}^{-2}$
TW 4, $R^2 = 1/2\ \mathrm{GeV}^{-2}$
1
0.8
0.6
0.4
0.2
0
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
x

3. A MINUS SIGN

ASIDE THE $2 \to 2$ TRANSITION THERE ARE $1-3$ TRANSITIONS



etc. + VIRTUAL TERMS.

ONE MAY SHOW THAT THE FUNCTION P_{1-3} IS RELATED TO P_{2-2} BY

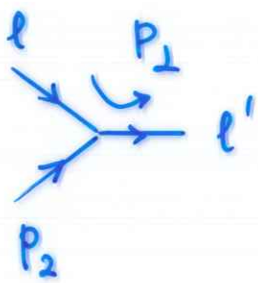
$$P_{1-3}(x_1, x_2, x_1', x_2') = - P_{2-2}(x_1, x_2, x_1', x_2')$$

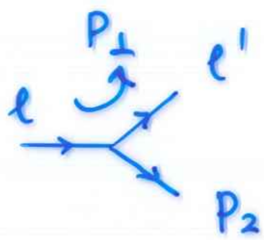
THE SIGN RESULTS FROM AN ENERGY DENOMINATOR.

IN $P^{GG \to q\bar{q}}$: NO VIRTUAL CORRECTIONS

BUT INTERFERENCE TERM.

$2 \to 2$ $\qquad\qquad$ $1 \to 3$





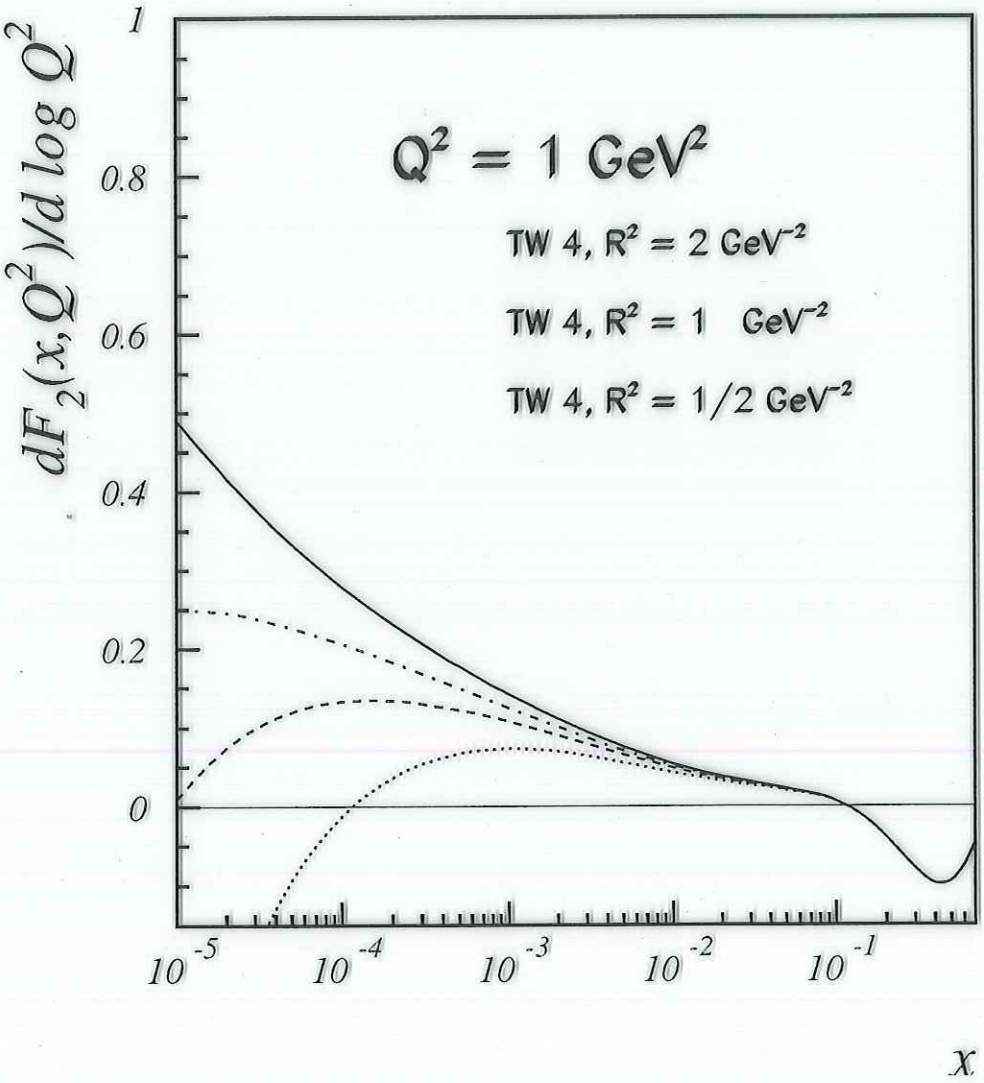
$x_\ell < x_{\ell'}$ $\qquad\qquad$ $x_\ell > x_{\ell'}$ $\qquad\qquad$ TOPT

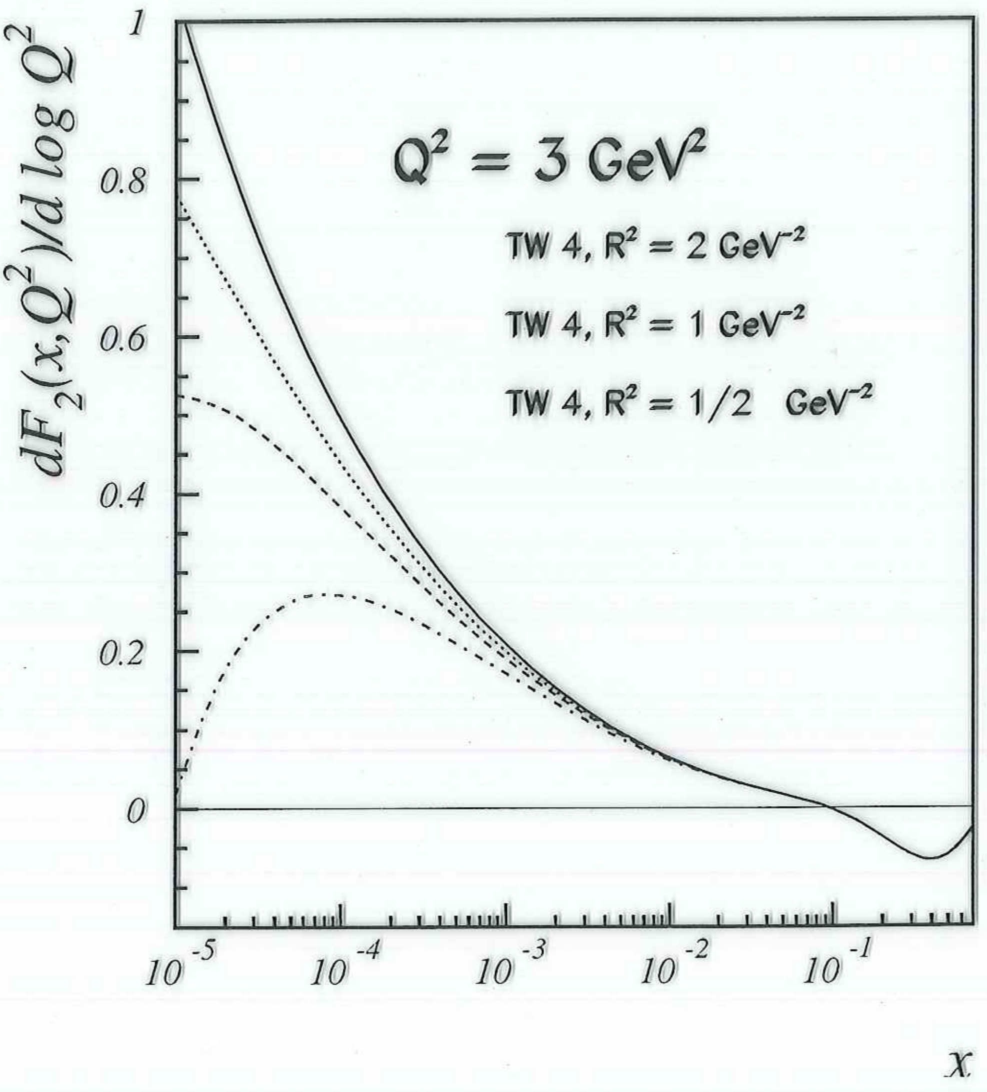
$P \to \infty$

$$\frac{1}{\frac{p_\perp^2}{2P}} \; \frac{1}{\frac{1}{x_\ell} - \frac{1}{x_{\ell'}}}$$

CHANGES SIGN.

4. FIRST NUMERICAL RESULTS





$Q^2 = 3\ GeV^2$
TW 4, $R^2 = 2\ GeV^{-2}$
TW 4, $R^2 = 1\ GeV^{-2}$
TW 4, $R^2 = 1/2\ GeV^{-2}$
$dF_2(x,Q^2)/d\log Q^2$
1
0.8
0.6
0.4
0.2
0
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
x

5. CONCLUSIONS

1) THE TWIST 4 RECOMBINATION CORRECTIONS WERE CALCULATED TO $\partial F_2 / \partial \log Q^2$ PROVIDED $R^2 < \infty$.

2) ANTISCREENING AND SCREENING CORRECTIONS OCCUR. THE SLOPE IS LOWERED @ SMALL x.

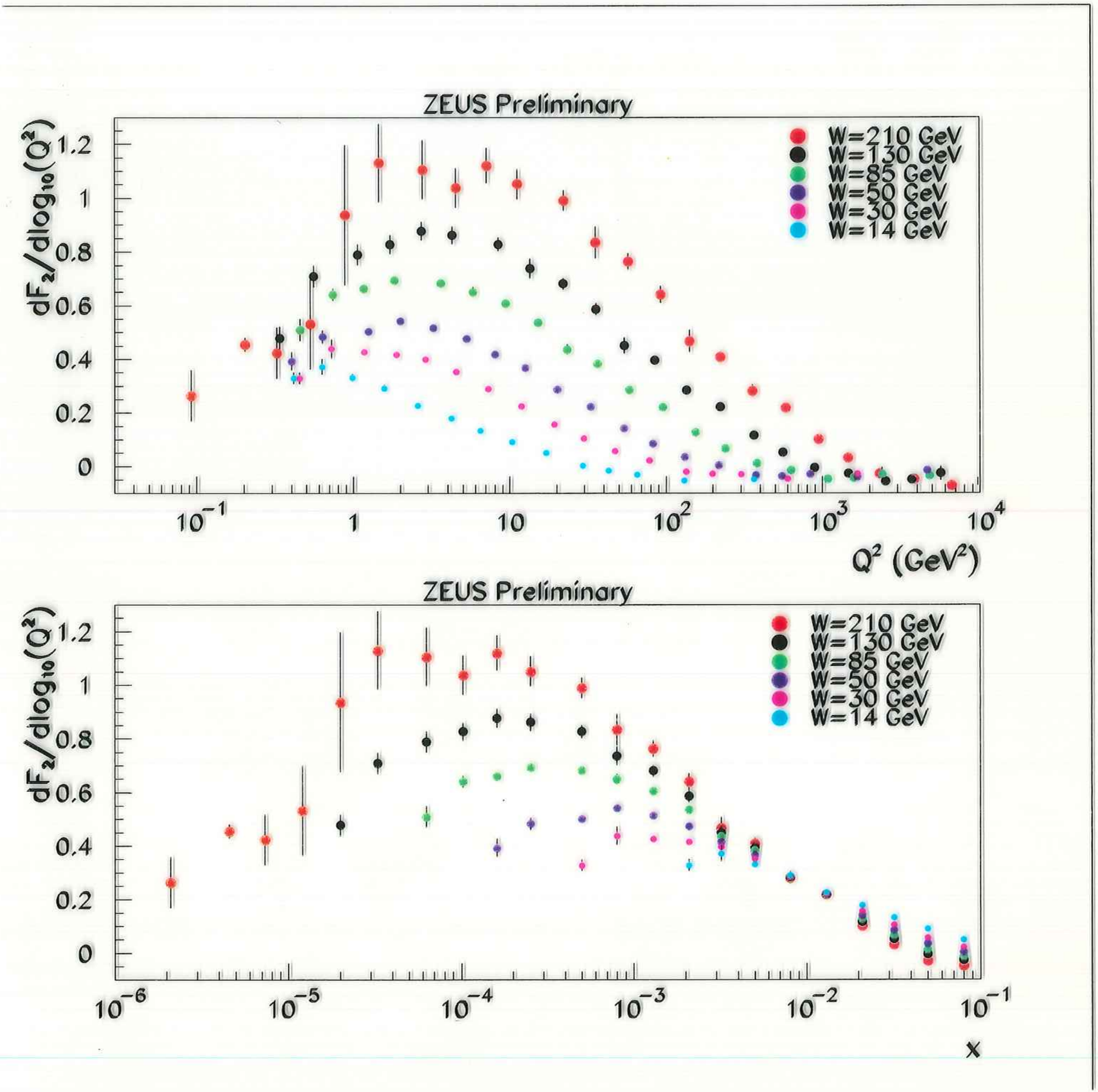
3) ENERGY-MOMENTUM IS PRESERVED.

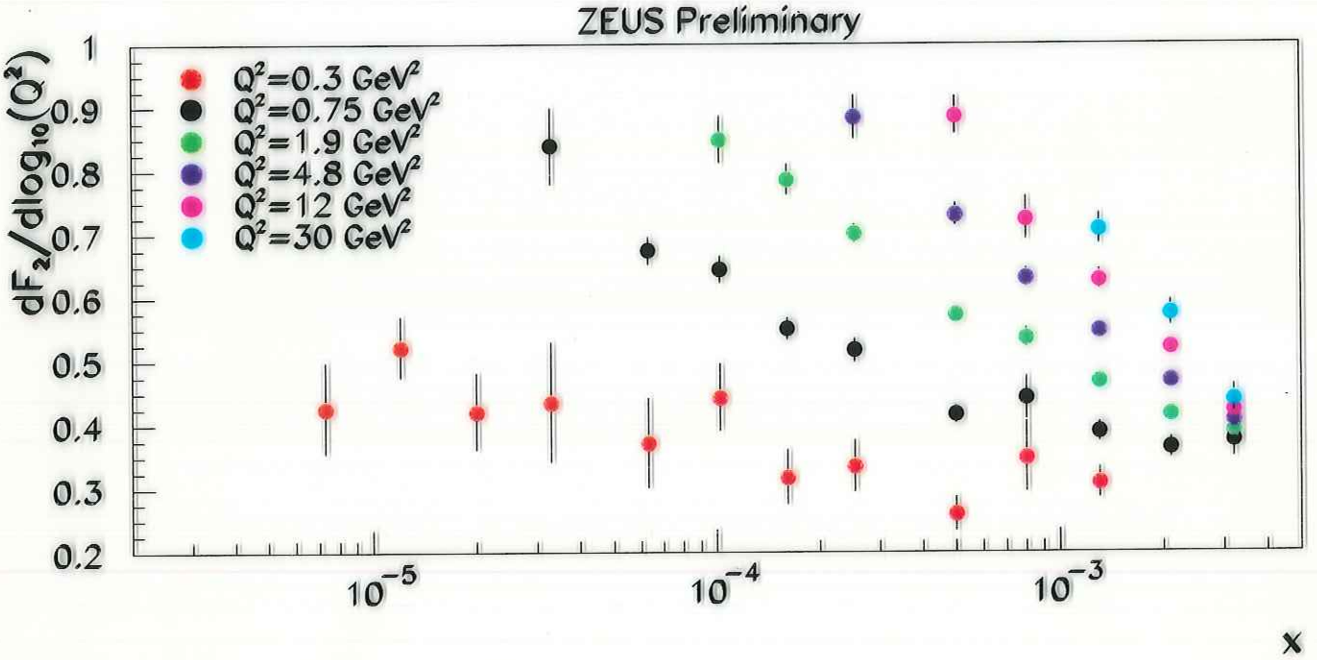
4) A NEW FUNCTION G_2 OCCURS TO BE MEASURED BY EXPERIMENT.

5) THE OBSERVED TURN-OVERS IN $\partial F_2 / \partial \log Q^2$ @ FIXED W (ZEUS) ARE WIDELY TWIST 2. THIS IS NOT A HIGHER TWIST SIGNATURE.

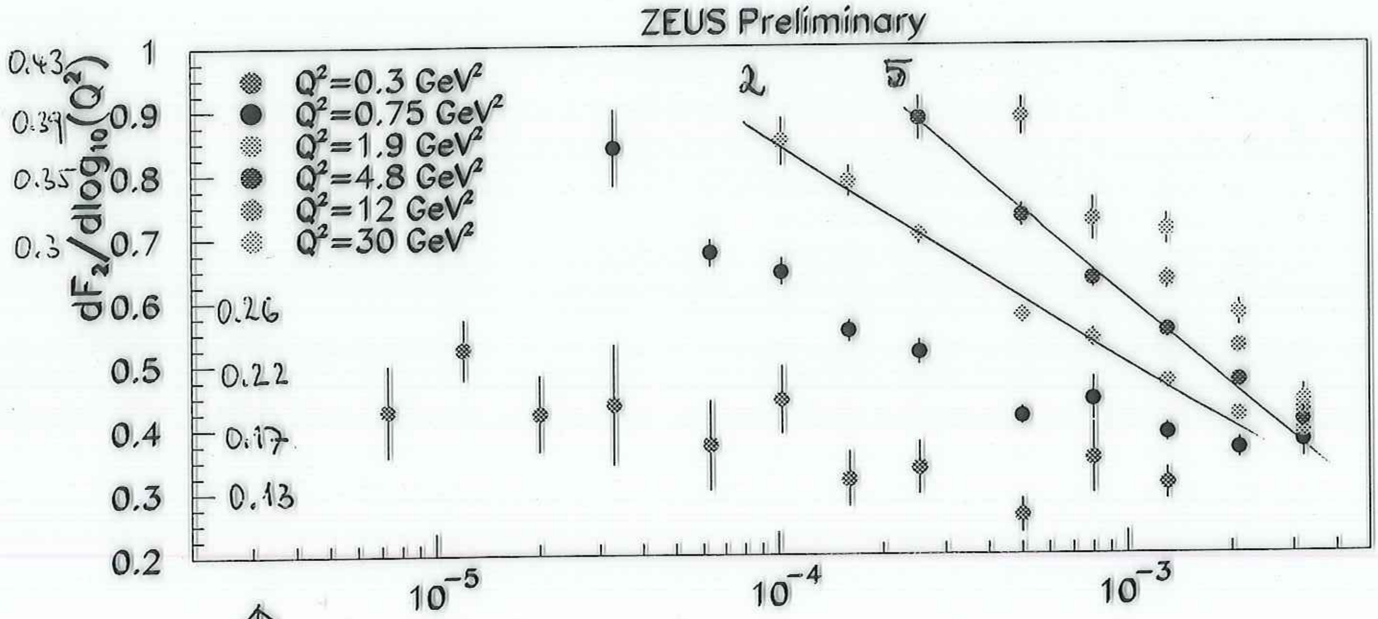
6) RECOMBINATION CORRECTIONS NEED A DEDICATED SEARCH IN EXPERIMENT.

CURRENT DATA:

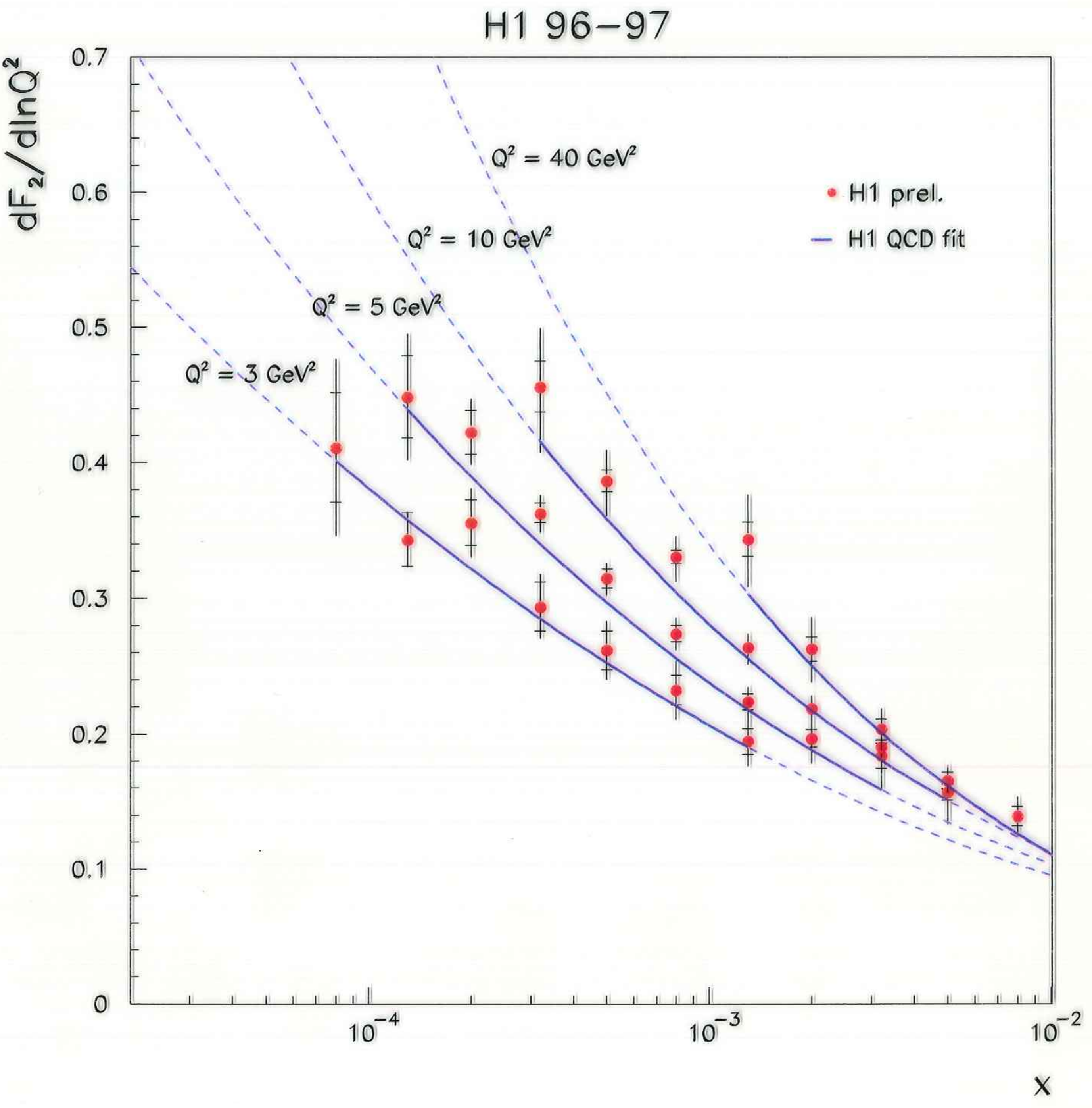




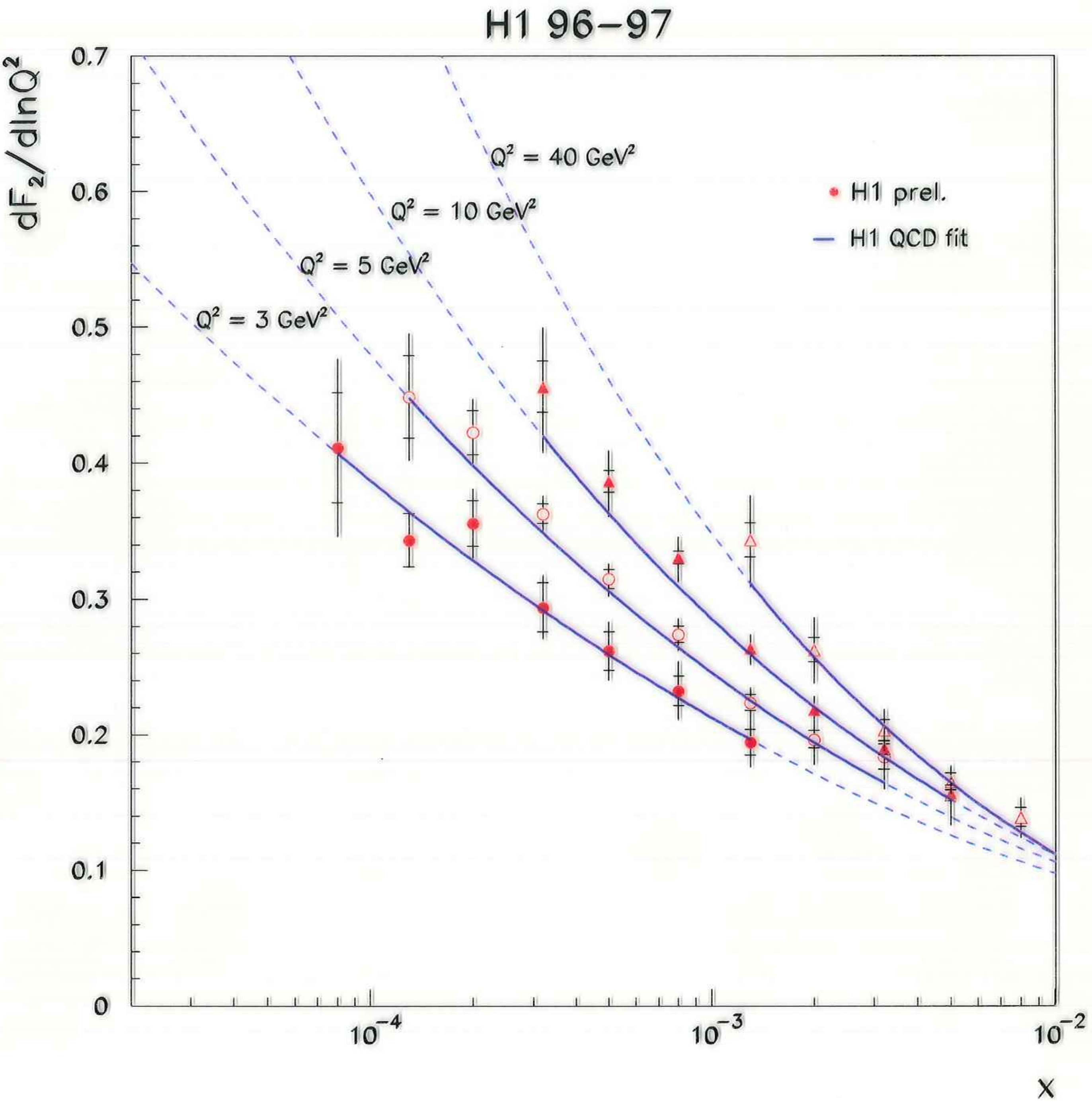
ZEUS Preliminary
$dF_2/dlog_{10}(Q^2)$
Q^2=0.3 GeV^2
Q^2=0.75 GeV^2
Q^2=1.9 GeV^2
Q^2=4.8 GeV^2
Q^2=12 GeV^2
Q^2=30 GeV^2
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
10^{-5}
10^{-4}
10^{-3}
x



↑ $\partial F_2 / \partial \log(Q^2)$ AS FOR NORMAL USE.



H1 96−97
$dF_2/d\ln Q^2$
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$Q^2 = 40$ GeV^2
$Q^2 = 10$ GeV^2
$Q^2 = 5$ GeV^2
$Q^2 = 3$ GeV^2
H1 prel.
H1 QCD fit
10^{-4}
10^{-3}
10^{-2}
x



H1 96-97
$dF_2/d\ln Q^2$
Q^2 = 40 GeV^2
Q^2 = 10 GeV^2
Q^2 = 5 GeV^2
Q^2 = 3 GeV^2
H1 prel.
H1 QCD fit
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10^{-4}
10^{-3}
10^{-2}
x