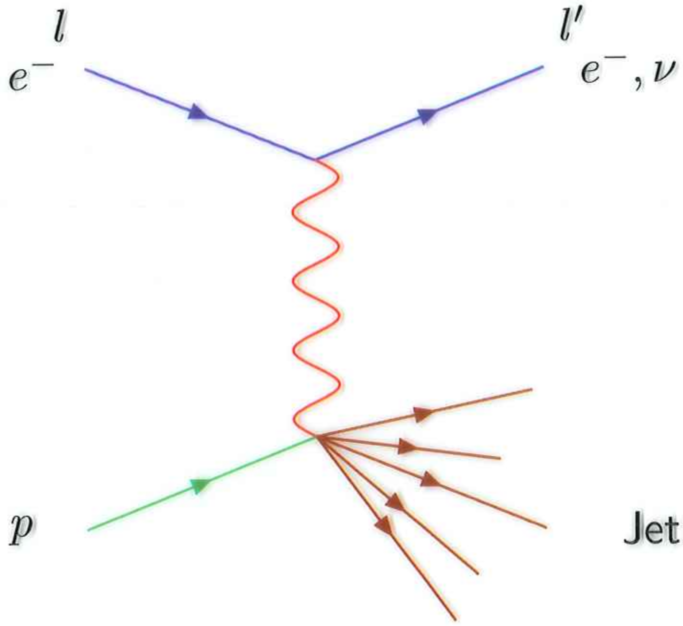
On the Way to QCD Precision Test with Deep Inelastic Scattering

Johannes Blümlein

DESY

OBERWÖLZ 2006

Deeply Inelastic Scattering



space-like process :

$$
\begin{aligned}
q^2 &= (l - l')^2 = -Q^2 < 0 \\
W^2 &= (p + q)^2 \geq M_p^2
\end{aligned}
$$

$$x = \frac{Q^2}{2p.q}, \qquad y = \frac{p.q}{p.l}$$

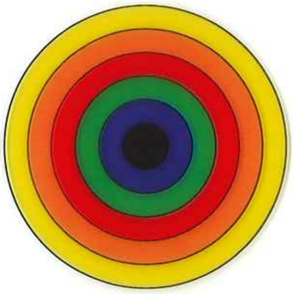
$$0 \leq x, y \leq 1$$

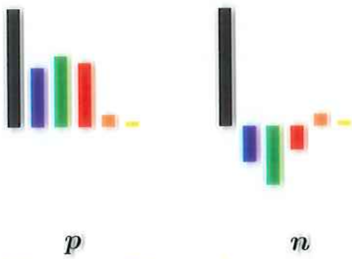
The Resolution of the Nucleon Microscope

$$\Delta x \sim \frac{1}{|Q|} = \frac{1}{\sqrt{-q^2}}$$

Examples :

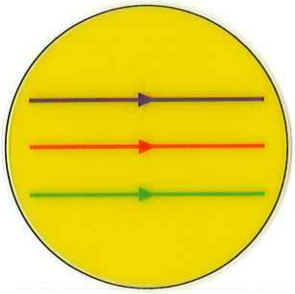
$Q^2 \sim 0.5 \cdot M_p^2$





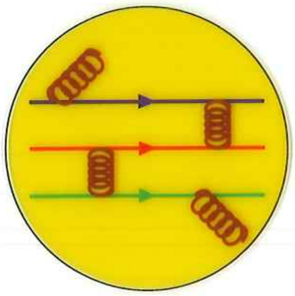
Charge Distribution

$Q^2 \sim 3 \cdot M_p^2$



Scaling

$Q^2 \sim 10 \ldots 500 \cdot M_p^2$



Violation of Scaling

If there are New Compositeness Scales, one may find them in the Future.

$$Q^2 > 10^4\ \mathrm{GeV}^2, \qquad 1\ \mathrm{GeV}^2 \sim M_p^2$$

When is a Parton ?

S. DRELL: **Infinite Momentum Frame:** P - large

$$\tau_{\rm int} \ll \tau_{\rm life}$$

$$\tau_{\rm int} \sim \frac{1}{q_0} = \frac{4Px}{Q^2(1-x)}$$

$$\tau_{\rm life} \sim \frac{1}{\sum_i E_i - E} = \frac{2P}{\sum_i (k_{\perp i}^2 + M_i^2)/x_i - M^2} \simeq \frac{2Px(1-x)}{k_\perp^2}$$

$$\frac{\tau_{\rm int}}{\tau_{\rm life}} = \frac{2k_\perp^2}{Q^2(1-x)^2}$$

Stay away from $x \rightarrow 0$, since xP becomes too small.

Stay away from $x \rightarrow 1$.

$$Q^2 \gg k_\perp^2.$$

MAIN RESEARCH OBJECTIVES :

- ☞ Precise Measurement of $\alpha_s(M_Z^2)$
- ☞ Reveal polarized and unpolarized parton densities at highest precision
- ☞ Precision tests of QCD
- ☞ Find novel sub-structures

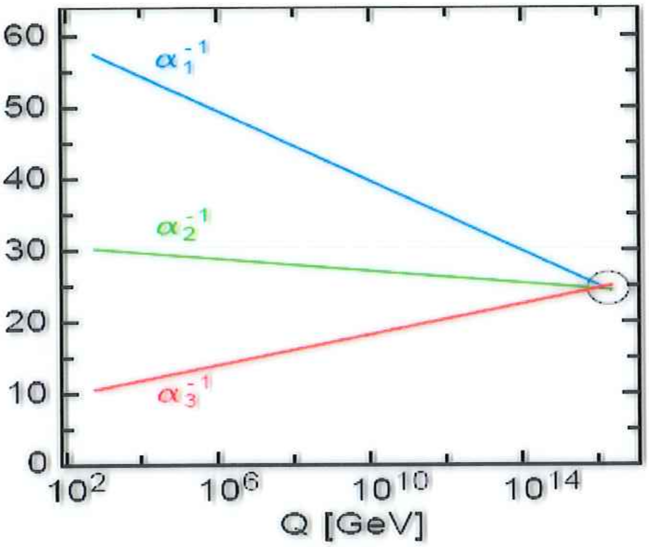
$\Longrightarrow$ Perturbative QCD :

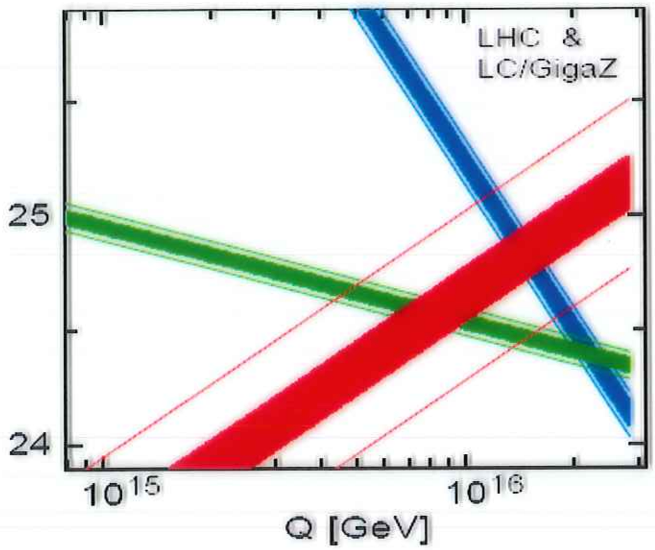
NNLO calculations using new technologies

$\Longrightarrow$ Lattice QCD :

Calculation of certain non-perturbative quantities a priori

UNIFICATION OF FORCES AND α_s





P. Zerwas, 2004

$$\frac{\delta\alpha(0)}{\alpha(0)} \sim 3 \cdot 10^{-11} \qquad \frac{\delta\alpha_w}{\alpha_w} \sim 7 \cdot 10^{-4} \qquad \frac{\delta\alpha_s(M_Z^2)}{\alpha_s(M_Z^2)} \sim 2 \cdot 10^{-2}$$

2. Basic Techniques

$$\frac{d\sigma^{\rm DIS}}{dxdy} \propto \sum_{s'} \overline{|M|^2} = \frac{1}{Q^4} \quad L_{\mu\nu} \quad W^{\mu\nu}, \qquad \text{pure } \gamma \text{ exchange}.$$

$L_{\mu\nu}$	–	calculable
$W^{\mu\nu}$	–	not calculable

Parameterize: according to the symmetries *P, T, C, etc.*

$$W^{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1(x,Q^2) + \frac{1}{M_p^2} \widehat{P}_\mu \widehat{P}_\nu W_2(x,Q^2) + \ldots .$$

$$\widehat{P}_\mu = p_\mu - \frac{q.p}{q^2} q_\mu .$$

THE PARTON MODEL :

R.P. Feynman, 1969; J.D. Bjorken, E.A. Paschos, 1969

ANSATZ:

$W_i(x,Q^2)$ is obtained as an integral over the momentum distributions of LOCAL SUB-COMPONENTS, THE PARTONS.

$$W_2(x,Q^2) = \sum_i \int_0^1 dx_i f(x_i) x_i e_i^2 \delta\left(\frac{q.p_i}{M^2} - \frac{Q^2}{2M}\right)$$

$\Longrightarrow$ STRONG CORRELATION BETWEEN $p.q$ AND Q^2

$\Longrightarrow$ "MICRO CANONICAL ENSEMBLE"

$f_i(x)$ - DISTRIBUTION FUNCTION

$$q.p_i = x_i p.q, \quad 2p.q = Q^2/x, \quad M\nu = p.q$$

$$\nu W_2(x,Q^2) = \sum_i e_i^2 x f_i(x) \equiv F_2(x) \ .$$

Bjorken Limit :

$$Q^2 \rightarrow \infty, \qquad \nu \rightarrow \infty$$

$$x = \text{const.}$$

Scaling :

$$MW_1(\nu,Q^2) \rightarrow F_1(x)$$
$$\nu W_2(\nu,Q^2) \rightarrow F_2(x)$$

The Light Cone Expansion :

More general approach, allowing for higher twist.

Brandt, Preparata, Zimmermann, Frishman, Christ et al.

$$W_{\mu\nu}(p.q) = \int d^4x e^{iqx} \langle p \| [j_\mu(x), j_\nu(0)] \| p \rangle$$

$$\begin{aligned} T\left[j_\mu(x), j_\nu(0)\right] &= \frac{x^2 g_{\mu\nu} - 2x_\mu x_\nu}{\pi^4 (x^2 - i\varepsilon)^4} + O_{\mu\nu} \\ &\quad -i\frac{x^\lambda \sigma_{\mu\lambda\nu\rho} O_V^\rho(x,0)}{2\pi^2(x^2 - i\varepsilon)} - i\frac{x^\lambda \varepsilon_{\mu\lambda\nu\rho} O_{V5}^\rho(x,0)}{2\pi^2(x^2 - i\varepsilon)} \end{aligned}$$

$$\begin{aligned} O_V^\mu(x,y) &= :\overline{\psi(x)}\gamma^\mu\psi(y) - \overline{\psi(y)}\gamma^\mu\psi(x): \\ O_{V5}^\mu(x,y) &= :\overline{\psi(x)}\gamma^\mu\gamma_5\psi(y) - \overline{\psi(y)}\gamma^\mu\gamma_5\psi(x): \\ O^{\mu\nu}(x,y) &= :\overline{\psi(x)}\gamma^\mu\psi(x)\overline{\psi}(y)\gamma^\nu\psi(x): \\ \psi(x) &= \psi(0) + x^\mu \left[\partial_\mu \psi(x)\right]_{x=0} + \frac{1}{2!} x^\mu x^\nu \left[\partial_\mu \partial_\nu \psi(x)\right]_{x=0} \\ &\quad + \ldots \\ O_{V,V5}^\mu(x,0) &= \sum_{n=0}^{\infty} \frac{1}{n!} x^{\mu_1} \ldots x^{\mu_n} O_{V,V5,\mu_1,\ldots,\mu_n}^\mu(0) \end{aligned}$$

$\Longrightarrow$ Calculate anomalous dimensions for Operators.

$\Longrightarrow$ Only safe way to Higher Twists

Twist 2: LCE $\simeq$ Parton Model

The Light Cone Expansion :

Non-forward scattering:

$$\gamma^* + p_1 \to \gamma'^* + p_2$$

$$T_{\mu\nu}(p_+, p_-, q) = i \int d^4x \, e^{iqx} \, \langle p_2, S_2 \left| T(J_\mu(x/2) J_\nu(-x/2)) \right| p_1, S_1 \rangle \; .$$

$$\begin{aligned} p_+ &= p_2 + p_1, & p_- = p_2 - p_1 = q_1 - q_2, \\ q &= \frac{1}{2}(q_1 + q_2), & p_1 + q_1 = p_2 + q_2 \; , \end{aligned}$$

Generalized Bjorken Limit:

$$\nu = q p_+ \longrightarrow \infty, \qquad Q^2 = -q^2 \longrightarrow \infty \; ,$$

Scaling Variables:

$$\xi = \frac{Q^2}{q p_+}, \qquad \eta = \frac{q p_-}{q p_+} = \frac{q_1^2 - q_2^2}{2\nu}$$

$$\Longrightarrow \qquad RT\left(J_\mu(x/2) J_\nu(-x/2) S\right)$$

$$J_\mu(x) = \overline{\psi}(x) \gamma_\mu \lambda^{\rm em} \psi(x) \; ,$$

$$
\begin{aligned}
&RT(J_\mu(x/2)J_\nu(-x/2)S) \\
&= i : \overline{\psi}(x/2)\{\hat{S}_{\mu\nu\rho\sigma} - i\varepsilon_{\mu\nu\rho\sigma}\gamma_5\}\gamma^\sigma(i\partial_x^\rho D^c(x))c^a\lambda_f^a\psi(-x/2) : \\
&- [(x/2,\ \mu) \leftrightarrow (-x/2,\ \nu)] \\
&+ \text{higher order terms}
\end{aligned}
$$

$$
\begin{aligned}
&\hat{S}_{\mu\nu\rho\sigma} \equiv -S_{\mu\rho\nu\sigma} = g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\rho\nu} \\
&\varepsilon_{\mu\nu\rho\sigma} \quad \text{Levi} - -\text{Civita symbol}
\end{aligned}
$$

$$
\begin{aligned}
&RT\left(J_\mu(x/2)J_\nu(-x/2)S\right) = \\
&\frac{1}{2\pi^2}\frac{c^a x^\rho}{(x^2 - i\varepsilon)^2}\int_{-\infty}^{+\infty} d\kappa_1 \int_{-\infty}^{+\infty} d\kappa_2 \\
&\times \left\{\frac{i}{2}\left[\overline{\psi}(\kappa_1 x)\lambda_f^a\gamma^\sigma\psi(\kappa_2 x) - \overline{\psi}(\kappa_2 x)\lambda_f^a\gamma^\sigma\psi(\kappa_1 x)\right](-\hat{S}_{\mu\nu\rho\sigma})\Delta_-(\kappa_1,\kappa_2)\right. \\
&+\frac{i}{2}\left[\overline{\psi}(\kappa_1 x)\gamma_5\lambda_f^a\gamma^\sigma\psi(\kappa_2 x) + \overline{\psi}(\kappa_2 x)\gamma_5\lambda_f^a\gamma^\sigma\psi(\kappa_1 x)\right] i\varepsilon_{\mu\nu\rho\sigma}\Delta_+(\kappa_1,\kappa_2) \\
&+ \ \ldots\ ,
\end{aligned}
$$

Wilson coefficient :

$$
\Delta_\pm(\kappa_1,\kappa_2) = \left[\delta(\kappa_1 - \tfrac{1}{2})\delta(\kappa_2 + \tfrac{1}{2}) \pm \delta(\kappa_2 - \tfrac{1}{2})\delta(\kappa_1 + \tfrac{1}{2})\right] .
$$

Light-cone vector :

$$
\tilde{x} = x + \frac{\zeta}{\zeta^2}\left(\sqrt{(x\zeta)^2 - x^2\,\zeta^2} - (x\zeta)\right), \qquad \tilde{x}^2 = 0
$$

$$RT\left(J_\mu(x/2)J_\nu(-x/2)S\right) \approx \int_{-1}^{+1} d^2\underline{\kappa}\, C_\Gamma(x^2,\underline{\kappa};\mu^2) RT\left(O^\Gamma(\kappa_1\tilde{x},\kappa_2\tilde{x})S\right) +$$

$$O^\Gamma(\kappa_1\tilde{x},\kappa_2\tilde{x}) = \int \frac{dp_1}{(2\pi)^4}\frac{dp_2}{(2\pi)^4} e^{i\kappa_1\tilde{x}p_1 + i\kappa_2\tilde{x}p_2} :\overline{\psi}(p_1)\Gamma\psi(p_2):$$

$$\begin{aligned} O^\Gamma(\kappa_1\tilde{x},\kappa_2\tilde{x}) &= \sum_{n_1 n_2} \frac{\kappa_1^{n_1}}{n_1!}\frac{\kappa_2^{n_2}}{n_2!} O^\Gamma_{n_1 n_2}(\tilde{x}), \\ O^\Gamma_{n_1 n_2}(\tilde{x}) &= \frac{\partial^{n_1}}{\partial\kappa_1^{n_1}}\frac{\partial^{n_2}}{\partial\kappa_2^{n_2}} O^\Gamma(\kappa_1\tilde{x},\kappa_2\tilde{x})\Big|_{\kappa_1=\kappa_2=0} \end{aligned}$$

$$(\tilde{x}\vec{\partial})^n \equiv \tilde{x}^{\mu_1}...\tilde{x}^{\mu_n}\vec{\partial}_{\mu_1}...\vec{\partial}_{\mu_n} \quad \text{(axial gauge).}$$

$$O^\Gamma_{n_1 n_2}(\tilde{x}) = \overline{\psi}(0)(\overleftarrow{\partial}\,\tilde{x})^{n_1}\Gamma(\tilde{x}\,\overrightarrow{\partial})^{n_2}\psi(0)\ .$$

$$\begin{aligned} & RT(J_\mu(x/2)J_\nu(-x/2)S) \approx \frac{1}{2}\int D\kappa \\ & \times\Big[C_a(x^2,\kappa_+,\kappa_-,\mu^2)\Big(g_{\mu\nu}O^a(\kappa_+\tilde{x},\kappa_-\tilde{x},\mu^2) - \tilde{x}_\mu O^a_\nu(\kappa_+\tilde{x},\kappa_-\tilde{x},\mu^2) \\ & \qquad -\tilde{x}_\nu O^a_\mu(\kappa_+\tilde{x},\kappa_-\tilde{x},\mu^2)\Big) \\ & + i\, C_{a,5}(x^2,\kappa_+,\kappa_-,\mu^2)\varepsilon_{\mu\nu}{}^{\rho\sigma}\tilde{x}_\rho O^a_{5,\sigma}(\kappa_+\tilde{x},\kappa_-\tilde{x},\mu^2)\Big] + ... \end{aligned}$$

Measure:

$$\begin{aligned} D\kappa &= d\kappa_1 d\kappa_2 \theta(1-\kappa_1)\theta(1+\kappa_1)\theta(1-\kappa_2)\theta(1+\kappa_2) \\ &= 2d\kappa_+ d\kappa_- \theta(1+\kappa_+ +\kappa_-)\theta(1+\kappa_+ -\kappa_-) \\ &\quad \theta(1-\kappa_+ +\kappa_-)\theta(1-\kappa_+ -\kappa_-) \end{aligned}$$

$$C_a(x^2,\kappa_+,\kappa_-) = \frac{1}{2\pi^2}\frac{c_a}{(x^2-i\varepsilon)^2}\delta(\kappa_+)\left[\delta(\kappa_- - \tfrac{1}{2}) - \delta(\kappa_- + \tfrac{1}{2})\right],$$
$$C_{a,5}(x^2,\kappa_+,\kappa_-) = \frac{1}{2\pi^2}\frac{c_a}{(x^2-i\varepsilon)^2}\delta(\kappa_+)\left[\delta(\kappa_- - \tfrac{1}{2}) + \delta(\kappa_- + \tfrac{1}{2})\right]$$

More general approach, allowing for higher twist.

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O^{\mu\nu}(x,y) &= :\overline{\psi(x)}\gamma^\mu\psi(x)\overline{\psi}(y)\gamma^\nu\psi(x): \\
\psi(x) &= \psi(0) + x^\mu\left[\partial_\mu\psi(x)\right]_{x=0} + \frac{1}{2!}x^\mu x^\nu\left[\partial_\mu\partial_\nu\psi(x)\right]_{x=0} \\
&\quad + \dots \\
O_{V,V5}^\mu(x,0) &= \sum_{n=0}^{\infty}\frac{1}{n!}x^{\mu_1}\dots x^{\mu_n} O_{V,V5,\mu_1,\dots,\mu_n}^\mu(0)
\end{aligned}$$

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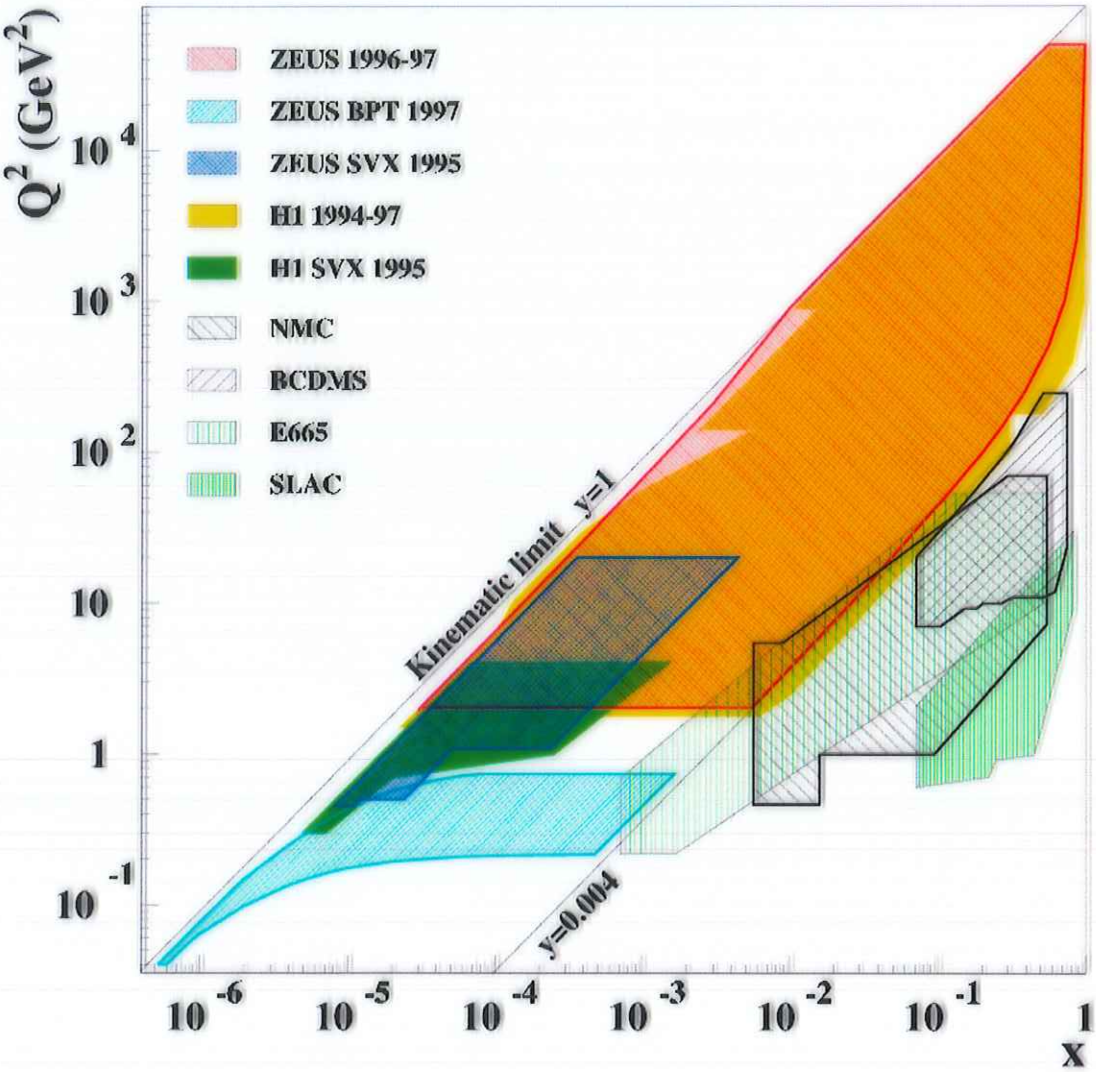
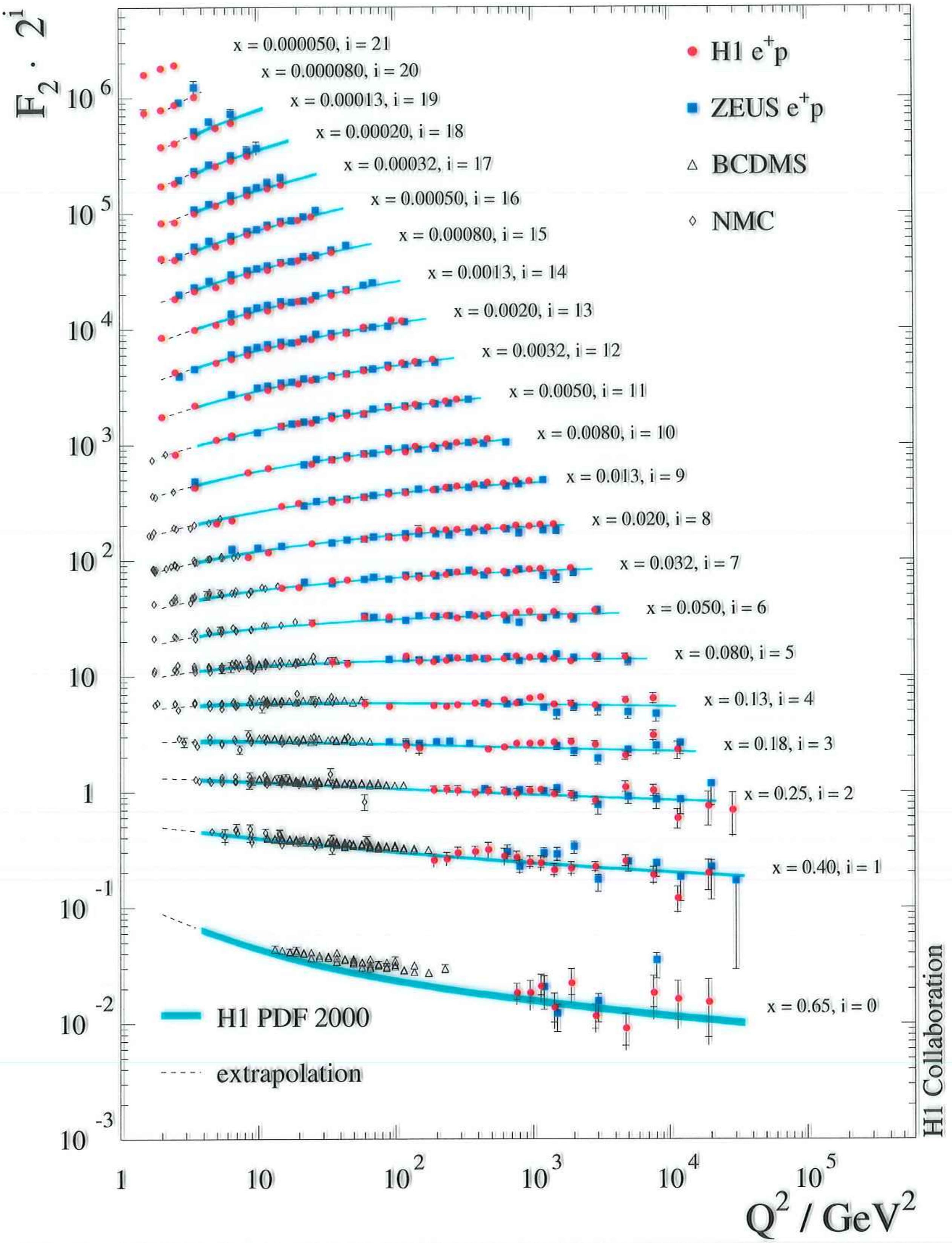
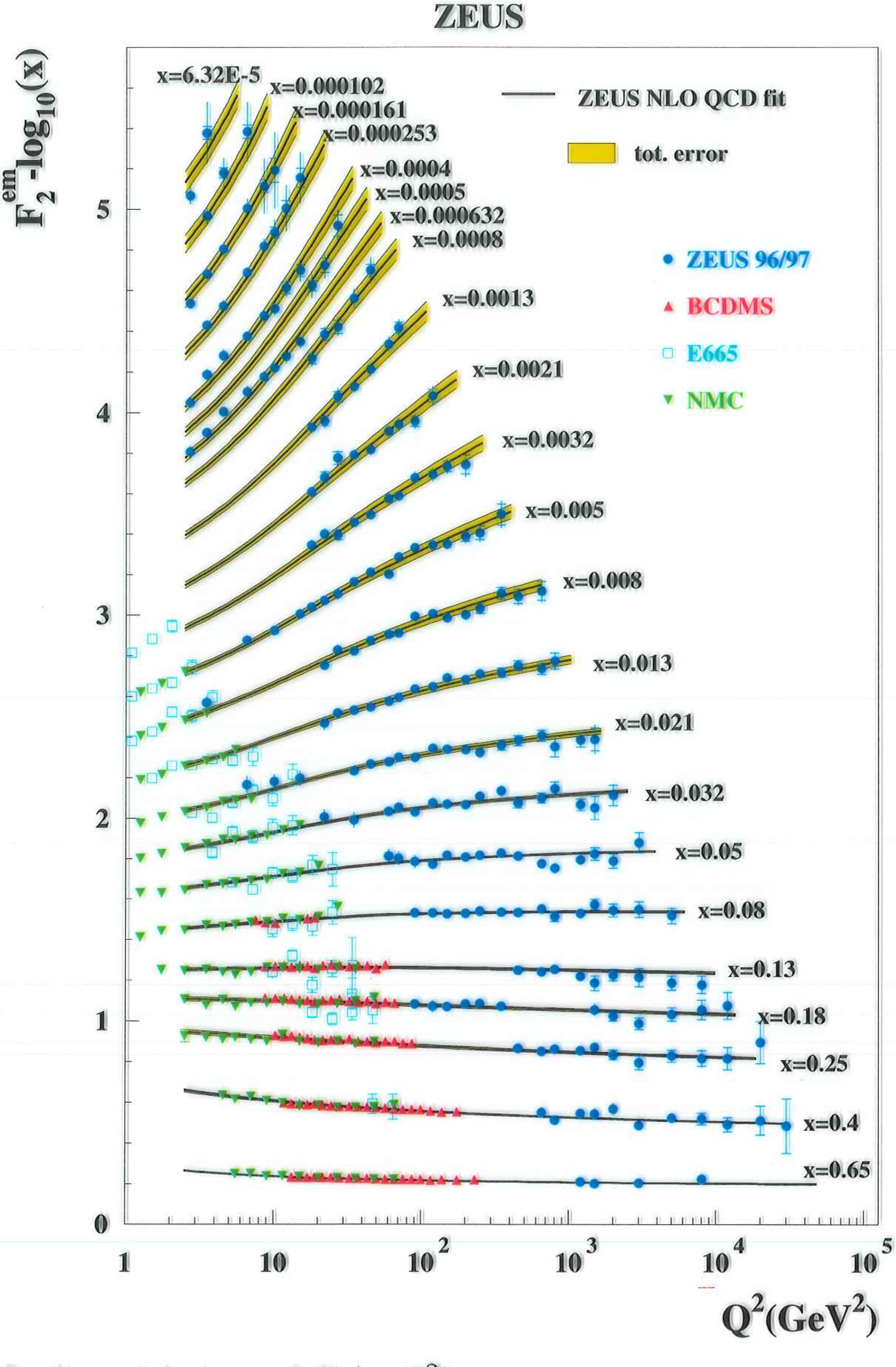
Twist 2: LCE $\simeq$ PARTON MODEL

Kinematic Domain

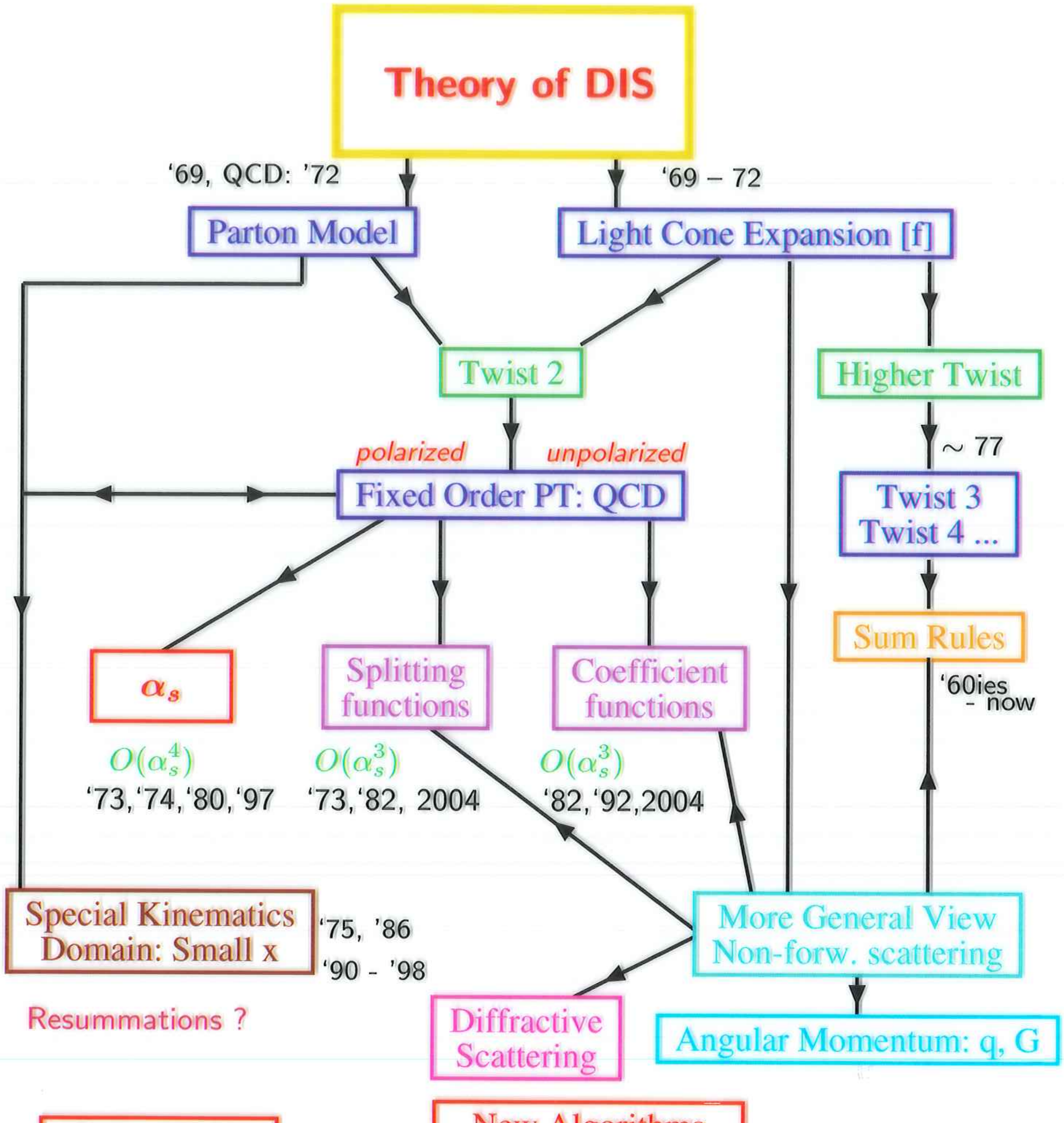


H1, ZEUS + fixed target data





Scaling violations of $F_2(x, Q^2)$.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Novel Mathematics

3. QCD Perturbation Theory to $O(\alpha_s^3)$, $\Lambda_{\rm QCD}$ and the PDF's

How can we measure $\alpha_s(Q^2)$ from the scaling violations of Structure Functions?

$$\begin{aligned}
F_j(x,Q^2) &= \hat{f}_i(x,\mu^2) \otimes \sigma_j^i\left(\alpha_s, \frac{Q^2}{\mu^2}, x\right) \\
& \quad \uparrow \text{bare pdf} \qquad \uparrow \text{sub}-\text{system cross}-\text{sect.} \\
&= \underbrace{\hat{f}_i(x,\mu^2) \otimes \Gamma_k^i\left(\alpha_s(R^2), \frac{M^2}{\mu^2}, \frac{M^2}{R^2}\right)}_{\text{finite pdf}\equiv \mathrm{f_k}} \\
& \quad \otimes \underbrace{C_j^k\left(\alpha_s(R^2), \frac{Q^2}{\mu^2}, \frac{M^2}{R^2}, x\right)}_{\text{finite Wilson coefficient}}
\end{aligned}$$

Move to Mellin space :

$$F_j(N) = \int_0^1 dx x^{N-1} F_j(x)$$

Diagonalization of the convolutions $\otimes$ into ordinary products.

Renormalization Group Equations :

$$\left[M\frac{\partial}{\partial M}+\beta(g)\frac{\partial}{\partial g}-2\gamma_{\psi}(g)\right]F_i(N) = 0$$

$$\left[M\frac{\partial}{\partial M}+\beta(g)\frac{\partial}{\partial g}+\gamma_{\kappa}^{N}(g)-2\gamma_{\psi}(g)\right]f_k(N) = 0$$

$$\left[M\frac{\partial}{\partial M}+\beta(g)\frac{\partial}{\partial g}-\gamma_{\kappa}^{N}(g)\right]C_j^k(N) = 0$$

Callan–Symnanzik equations for mass factorization

$\equiv$ Altarelli–Parisi evolution equations

x-space :

$$\frac{d}{d\log(\mu^2)}\begin{pmatrix} q^+(x,Q^2) \\ G(x,Q^2)\end{pmatrix} = \frac{\alpha_s}{2\pi}\boldsymbol{P}(x,\alpha_s)\otimes\begin{pmatrix} q^+(x,Q^2) \\ G(x,Q^2)\end{pmatrix}$$

$$\boldsymbol{P}(x,\alpha_s) = \boldsymbol{P}^{(0)}(x)+\frac{\alpha_s}{2\pi}\boldsymbol{P}^{(1)}(x)+\left(\frac{\alpha_s}{2\pi}\right)^2\boldsymbol{P}^{(2)}(x)+\dots$$

Evolution equs.: 3 non-singlet, 1 singlet

Separation of non-singlet and singlet quark contributions is **essential.**

SEPTEMBER 11, 2001
VICTIMS

This site is dedicated to the victims of September 11, 2001 tragedy.

THE SUN POSTER

This poster made using photomosaic technique and contains the pictures of the victims! more...

BE A SPONSOR OF THE POSTER FOR VICTIMS' FAMILIES

Victim of FLIGHT 77
William Caswell, 54, Silver Spring, Md.

september11victims.com

Physicist, U.S. Navy
Confirmed dead, American Flight 77 - Pentagon, airline passenger

Visitors Comments:

Janet Edghill 09/09/2002 3:22:50 PM
Commenter Email and IP address is in file

I had the pleasure of working with Bill for three years in a Navy Program Office. I remember him for his easy-going style and goofy grin. I wish his family well, and remember how proud he was of his daughter.

Benjamin Koester 09/17/2002 9:00:51 AM
Commenter Email and IP address is in file

www.september11victims.com
September 11,2002

Dear Family of William Caswell,
My name is Benjamin Koester. I'm an eighth grader at Newport Middle School. I'm privileged to be writing to you because of what your son did, by giving his own life for hundreds or even thousands, by trying to take over the plane and forcing the terrorist to bring the plane into a field.
When I heard the news, I was in my Math class. I was stunned when I heard what happened. I never thought anyone would do that to America. However I think our country is closer together after what happened. Our country is united as a family now.
I know your family is very hurt, upset, and emotional. However your loved one died as a hero, saving others liver by giving his. So be proud of what your loved one has done. I thank you, for what he has done. Thank You.

Fellow American,
Benjamin Koester

3.1. Running Coupling Constant

$$\frac{\partial a_s(\mu^2)}{\partial \log \mu^2} = -\beta_0 a_s^2 - \beta_1 a_s^3 - \beta_2 a_s^4 - \beta_3 a_s^5 + O(a_s^6)$$

$$a_s \equiv \frac{g_{\rm ren}^2}{(4\pi)^2} = \frac{\alpha_s}{2\pi}$$

The values of the β_k :

$$\beta_0 = 11 - \frac{2}{3} N_f$$ GROSS, POLITZER, WILCZEK, T'HOOFT, 1973

DISCOVERY OF ASYMPTOTIC FREEDOM :

NOBEL LAUREATES 2004

$$\beta_1 = 102 - \frac{38}{3} N_f$$ CASWELL(†11.9.01), JONES, 1974

$$\beta_2 = \frac{2857}{2} - \frac{5033}{18} N_f + \frac{325}{54} N_f^2$$

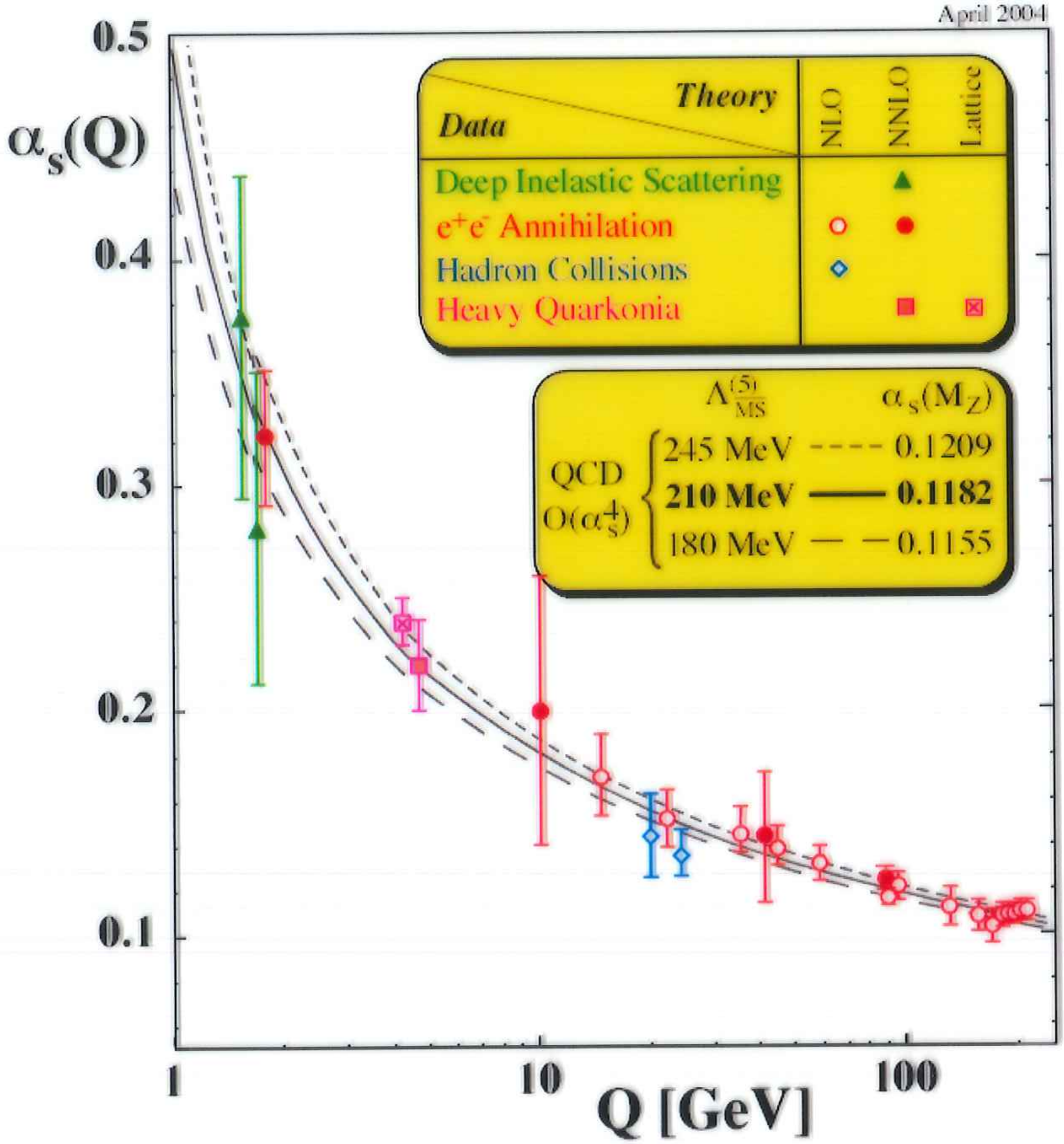
TARASOV, VLADIMIROV, ZHARKOV, 1981

LARIN, VERMASEREN, 1992

$$\beta_3 = \left(\frac{149753}{6} + 3564\zeta_3\right) - \left(\frac{1078361}{162} + \frac{6508}{27}\zeta_3\right) N_f + \left(\frac{50065}{162} + \frac{6472}{81}\zeta_3\right) N_f^2 + \frac{1093}{729} N_f^3$$

VAN RITBERGEN, VERMASEREN, LARIN, 1997

THE SOLUTION OF THE RGE LEADS TO A FALLING COUPLING CONSTANT AS SCALES INCREASE.



S. Bethke, LL2004.

3.2. Splitting Functions

$O(\alpha_s)$ unpolarized:

$$
\begin{aligned}
P_{\rm NS}^{(0)}(z) \equiv P_{qq}^{(0)}(z) &= C_F \left[\frac{1+z^2}{1-z}\right]_+ \\
P_{qg}^{(0)}(z) &= T_f \left[(1-z)^2 + z^2\right] \\
P_{gq}^{(0)}(z) &= C_F \frac{1+(1-z)^2}{z} \\
P_{gg}^{(0)}(z) &= 2C_A \left[\frac{1-z}{z} + \frac{z}{(1-z)_+} + z(1-z)\right] + \frac{1}{2}\beta_0 \delta(1-z)
\end{aligned}
$$

QED : P_{qq} Fermi, 1924 P_{gq} Williams, 1933; Weizsäcker, 1934

Gross, Wilczek; Georgi, Politzer, 1973;

further: Lipatov, 1975; Altarelli, Parisi, 1977; Kim, Schilcher, 1977; Dokshitser, 1977

$O(\alpha_s)$ polarized:

$$
\begin{aligned}
\Delta P_{qq}^{(0)}(z) &= P_{qq}^{(0)}(z) \\
\Delta P_{qg}^{(0)}(z) &= T_f \left[(1-z)^2 - z^2\right] \\
\Delta P_{gq}^{(0)}(z) &= C_F \frac{1-(1-z)^2}{z} \\
\Delta P_{gg}^{(0)}(z) &= 2C_A \left[\left(\frac{1}{1-z}\right)_+ + 1 - 2z\right] + \frac{1}{2}\beta_0 \delta(1-z)
\end{aligned}
$$

Ito, 1975; K. Sasaki, 1975; Ahmed & Ross 1975,1976;

correct: Altarelli, Parisi, 1977.

no terms $\propto 1/z$.

2 LOOP :

UNPOLARIZED: 1977 – 1992 !

FLORATOS, D. ROSS, SACHRAIDA, 1977-79; CURCI, FURMANSKI, PERTONZIO, 1980; FURMANSKI, PETRONZIO, 1980; GONZALEZ-ARROYO, LOPEZ, YNDURAIN, 1979, 1980; FLORATOS, KOUNNAS, LACAZE, 1981ABC; VAN NEERVEN, HAMBERG, 1992;

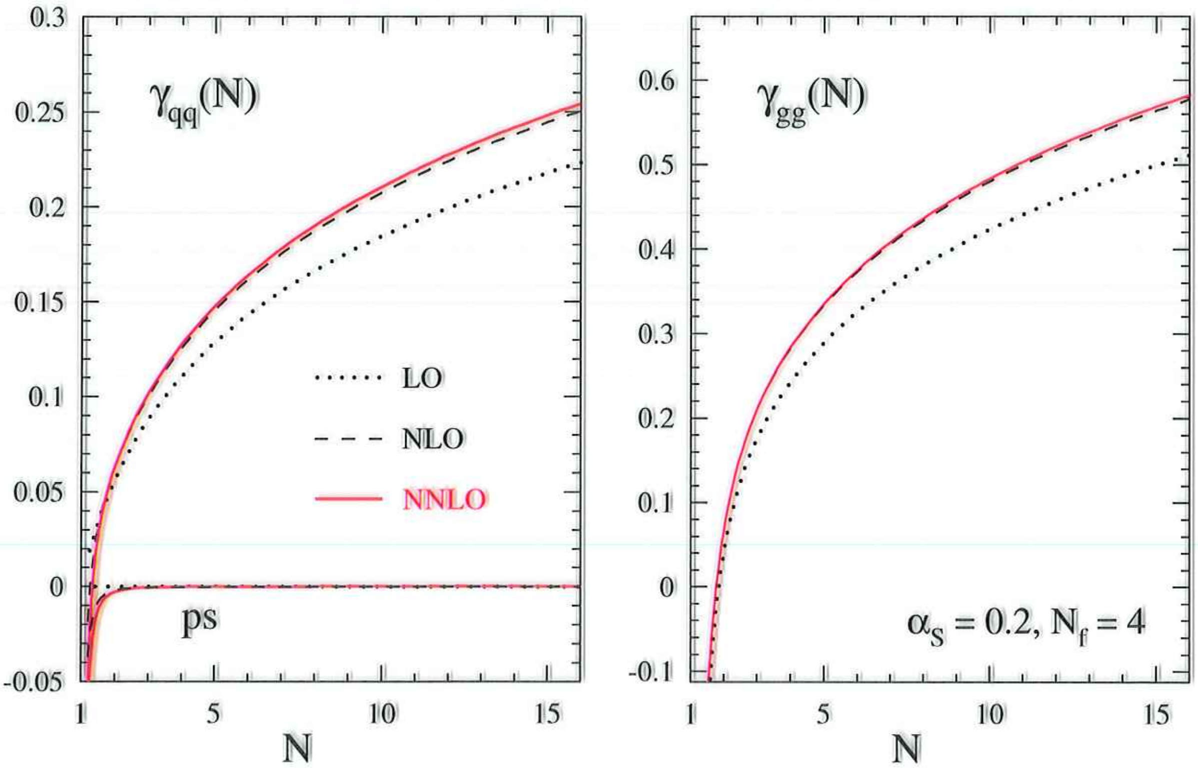
POLARIZED:

ZIJLSTRA, VAN NEERVEN, 1994; MERTIG, VAN NEERVEN, 1995; VOGELSANG 1995.

3 LOOP :

UNPOLARIZED:

MOMENTS : LARIN, NOGUEIRA, VAN RITBERGEN, VERMASEREN, 1994, 1997; RETEY, VERMASEREN, 2001; J.B., VERMASEREN, 2004.
COMPLETE : MOCH, VERMASEREN, VOGT, 2004.



3.3. Coefficient Functions

$O(\alpha_s)$ unpolarized:

$$C_{F_2^q}^{(1)}(z) = C_F \left\{ \frac{1+z^2}{1-z} \left[\ln\left(\frac{1-z}{z}\right) - \frac{3}{4} \right] + \frac{1}{4}(9+5z) \right\}_+$$

$$C_{F_2^g}^{(1)}(z) = 2 N_f T_f \left\{ \left[z^2 + (1-z)^2 \right] \ln\left(\frac{1-z}{z}\right) - 1 + 8z(1-z) \right\}$$

$$C_{F_1^q}^{(1)}(z) = C_{F_2^q}^{(1)}(z) - C_F \cdot 2z$$

$$C_{F_1^g}^{(1)}(z) = C_{F_2^g}^{(1)}(z) - 8 N_f T_f z(1-z)$$

$$C_{F_3^q}^{(1)}(z) = C_{F_2^q}^{(1)}(z) - C_F(1+z)$$

Furmanski, Petronzio, 1982: correct form.

$O(\alpha_s)$ polarized:

$$C_{g_1^q}^{(1)}(z) = C_{F_1^q}^{(1)}(z)$$

$$C_{g_1^g}^{(1)}(z) = 4 N_f T_f \left\{ [2z-1] \ln\left(\frac{1-z}{z}\right) + 3 - 4z \right\}$$

Altarelli, Ellis, Martinelli, 1979; Humpert, van Neerven, 1981; Bodwin Qui, 1990.

2 Loop :

polarized, unpolarized:

Zijlstra, van Neerven 1992–1994;

Moments: Moch, Vermaseren, 1999

unpolarized, heavy flavor:

Laenen, Riemersma, Smith, van Neerven, 1993, 1994

Mellin Space: Alekhin, J.B., 2004

3 Loop :

unpolarized:

Moments : Larin, Nogueira, van Ritbergen, Vermaseren, 1994, 1997; Retey, Vermaseren, 2001; J.B., Vermaseren, 2004.

Complete : Moch, Vermaseren, Vogt, ~~in preparation~~. 2005.

4. New Mathematics in Perturbation Theory

Consider hard scattering processes in massless field theories:

QCD, QED, $m_i \to 0$

Factorization Theorem Leading Twist:

The cross section σ factorizes as

$$\sigma = \sum_k \sigma_{k,W} \otimes f_k$$

σ_W perturbative Wilson Coefficient

f non-perturbative Parton Density

$\otimes$ Mellin convolution

$$\begin{aligned} [A \otimes B](x) &= \int_0^1 dx_1 \int_0^1 dx_2 \delta(x - x_1 x_2) A(x_1) B(x_2) \\ \mathbf{M}[A \otimes B](N) &= \mathbf{M}[A](N) \cdot \mathbf{M}[B](N) \end{aligned}$$

with the Mellin transform :

$$\mathbf{M}[f(x)](N) = \int_0^1 dx x^{N-1} f(x), \quad Re[N] > c$$

Observation :

Feynman Amplitudes seem to obey the **Mellin Symmetry**

i.e. to significantly simplify in **Mellin Space** van Neerven, Zijlstra 1992

$$c_{2,-}^{(2)}(x) = C_F\,(C_F - C_A/2)\,\times$$

$$\Bigg\{\frac{1+x^2}{1-x}\Bigg[\Big[4\ln^2(x) - 16\ln(x)\ln(1+x) - 16\mathrm{Li}_2(-x) - 8\zeta_2\Big]\ln(1-x)$$

$$+\Big[-2\ln^2(x) + 20\ln(x)\ln(1+x) - 8\ln^2(1+x) + 8\mathrm{Li}_2(1-x) + 16\mathrm{Li}_2(-x) - 8\Big]\ln(x)$$

$$-16\ln(1+x)\mathrm{Li}_2(-x) - 8\zeta_2\ln(1+x) - 16\left[\mathrm{Li}_3\left(-\frac{1-x}{1+x}\right) - \mathrm{Li}_3\left(\frac{1-x}{1+x}\right)\right]$$

$$-16\mathrm{Li}_2(1-x) + 8S_{1,2}(1-x) + 8\mathrm{Li}_3(-x) - 16S_{1,2}(-x) + 8\zeta_3\Bigg]$$

$$+(4+20x)\Big[\ln^2(x)\ln(1+x) - 2\ln(x)\ln^2(1+x) - 2\zeta_2\ln(1+x) - 4\ln(1+x)\mathrm{Li}_2(-x)$$

$$+2\mathrm{Li}_3(-x) - 4S_{1,2}(-x) + 2\zeta_3\Big] + \left(32 + 32x + 48x^2 - \frac{72}{5}x^3 + \frac{8}{5x^2}\right)$$

$$\times\left[\mathrm{Li}_2(-x) + ln(x)\ln(1+x)\right] + 8(1+x)\left[\mathrm{Li}_s(1-x) + \ln(x)\ln(1-x)\right] + 16(1-x)\ln(1-x)$$

$$+\left(-4 - 16x - 24x^2 + \frac{36}{5}x^3\right)\ln^2(x) + \frac{1}{5}\left(-26 - 106x + 72x^2 - \frac{8}{x}\right)\ln(x)$$

$$+\left(-4 + 20x + 48x^2 - \frac{72}{5}x^3\right)\zeta_2 + \frac{1}{5}\left(-162 + 82x + 72x^2 + \frac{8}{x}\right)\Bigg\}$$

.... several other pages for $c_2^{(+)}(x), c_2^G(x), c_L^{(q,G)}(x)$

$\Longrightarrow$ 77 Functions @ 2 Loops

$\Longrightarrow$ partly rather complicated arguments

$\Longrightarrow$ relations are not directly visible ...

The 77 functions do roughly correspond in number to the number of all possible harmonic sums up to weight w=4: 80.

3 LOOP COEFFICIENT FCTS: UP TO 728 OBJECTS!

GOAL: SIMPLICITY

W. of Occam

MULTIPLE HARMONIC SUMS TO LEVEL 6 :

THE SIMPLEST EXAMPLE :

$$P_{qq}(x) = \left(\frac{1+x^2}{1-x}\right)_+ = \frac{2}{(1-x)_+} + \ldots$$

$$\int_0^1 dx \frac{x^{N-1}}{(1-x)_+} = -\sum_{k=0}^{N-2} \int_0^1 dx x^k = -\sum_{k=1}^{N-1} \frac{1}{k} = -S_1(N-1)$$

Alternating sums :

$$S_{-1}(N-1) = (-1)^{N-1} \mathbf{M}\left[\frac{1}{1+x}\right](N) - \ln(2) = \int_0^1 dx \frac{x^{N-1}}{(1-x)_+} = \sum_{k=1}^{N-1} \frac{(-1)^k}{k}$$

(Finite for $N \to \infty$.)

General case :

$$S_{a_1,\ldots,a_l}(N) = \sum_{k_1=1}^{N} \frac{(\mathrm{sign}(a_1))^{k_1}}{k_1^{|a_1|}} \sum_{k_2=1}^{k_1} \frac{(\mathrm{sign}(a_2))^{k_2}}{k_2^{|a_2|}} \ldots$$

Vermaseren, 1997

All Mellin transforms occurring in massless Field Theories for 1-Parameter Quantities can be represented by Harmonic Sums

(at least to 3–loop order).

Theory of Words

Can we count the Basis in simpler way ? $\Longrightarrow$ YES.

Free Algebras and Elements of the Theory of Codes

$\Longrightarrow$ **Particle Physics**

Only the multiplication relation and the Index structure matters

$\mathfrak{A} = \{a, b, c, d, \ldots\}$ **Alphabet**

$a < b < c < d < \ldots$ **ordered**

$\mathfrak{A}^*(\mathfrak{A})$ **Set of all words W**

$W = a_1 \cdot a_2 \cdot a_{27} \ldots a_{532} \equiv$ **concatenation product (nc)**

$W = p \cdot x \cdot s$ **p = prefix; s = suffix**

Definition:

A Lyndon word is smaller than any of its suffixes.

Theorem:[Radford, 1979]

The shuffle algebra $K\langle\mathfrak{A}\rangle$ is freely generated by the Lyndon words.
I.e. the number of Lyndon words yields the number of basic elements.

Examples :

$\{a, a, \ldots, a, b\} = aaa \ldots ab$ 1 Lyndon word for these sets

$n \quad a's: \quad n_{basic}/n_{all} = 1/n \qquad n \equiv$ depth of the sums

Symmetries lead to a smaller fraction.

Is there a general Counting Relation ?

E. Witt, 1937

$$l_n(n_1,\ldots,n_q) = \frac{1}{n}\sum_{d|n_i}\mu(d)\frac{(n/d)!}{(n_1/d)!\ldots(n_q/d)!}, \qquad \sum_i n_i = n$$

$\mu(k)$ Möbius function

2nd Witt formula.

The Length of the Basis is a function mainly of the Depth.

Observation: Sums with index -1 do not occur.

$$N_{\neg -1}(w) = \frac{1}{2}\left[\left(1+\sqrt{2}\right)^w + \left(1-\sqrt{2}\right)^w\right]$$

$$N_{\neg -1}^{\rm basic}(w) = \frac{2}{w}\sum_{d|w}\mu\left(\frac{w}{d}\right)N_{\neg -1}(d)$$

J.B., 2004; Further Reduction: Structural Relations.

Weight	Sums	a-basic	Sums $\neg -1$	a-basic	str. Rel.	Fraction
1	2	2	1	0	0	0.0
2	6	3	3	0	0	0.0
3	18	8	7	2	2	0.1111
4	54	18	17	5	3	0.0555
5	162	48	41	14	8	0.0494
6	486	116	99	28	?	$<$0.0576
	728	195	168	49	$<$41	$<$0.0563

The Basic Functions :

The final set of functions:

Trivial functions:

$$S_{\pm k}(N) \quad \longrightarrow \quad \psi^{(k-1)}(N+1)$$

For $w = 1, 2$ no non–trivial functions contribute to the anomalous dimensions and Wilson coefficients.

Non–trivial functions:

$N = 3$: Two–Loop anomalous dimensions

$$\mathbf{M}\left[\frac{\mathrm{Li}_2(x)}{1+x}\right](N)$$

Yndurain et al., 1980

$N = 4$: Two–Loop Wilson Coefficients

$$\mathbf{M}\left[\frac{\ln(1+x)}{1+x}\right](N), \quad \mathbf{M}\left[\frac{\mathrm{Li}_2(x)}{1-x}\right](N), \quad \mathbf{M}\left[\frac{\mathrm{S}_{1,2}(x)}{1\pm x}\right](N)$$

Structure Fct.: J.B., S. Moch, 2003,

Drell-Yan, Higgs-Prod., Fragmentation: J.B., V. Ravindran, 2004.

$N = 5$: Three–Loop Anomalous Dimensions

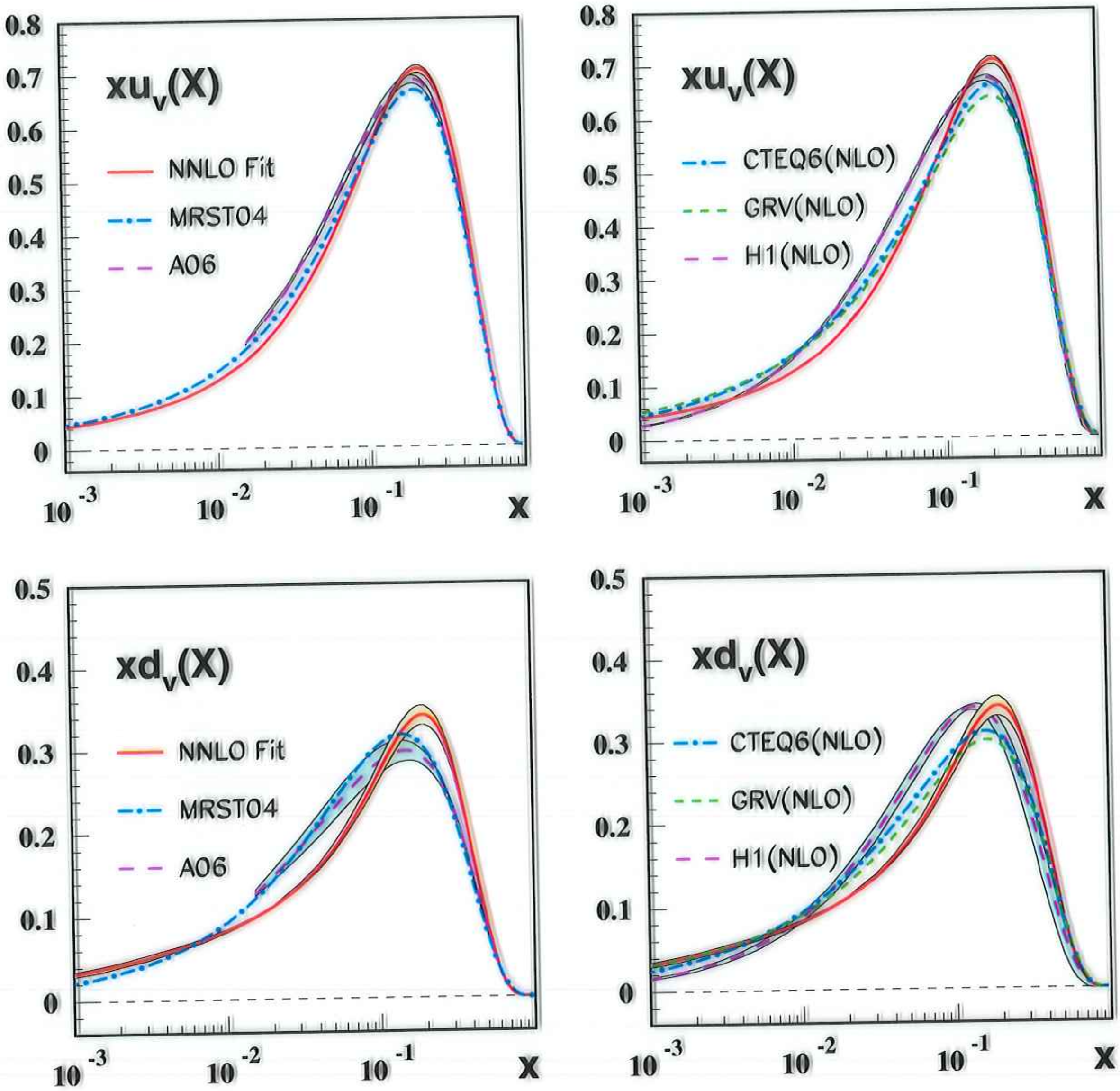
$$\mathbf{M}\left[\frac{\mathrm{Li}_4(x)}{1\pm x}\right](N), \quad \mathbf{M}\left[\frac{S_{1,3}(x)}{1+x}\right](N), \quad \mathbf{M}\left[\frac{S_{2,2}(x)}{1\pm x}\right](N),$$

$$\mathbf{M}\left[\frac{S_{2,2}(-x)-\mathrm{Li}_2^2(-x)/2}{1\pm x}\right](N), \quad \mathbf{M}\left[\frac{\mathrm{Li}_2^2(x)}{1+x}\right](N)$$

J.B., S. Moch, 2004.

Essentially 14 Functions seem to rule the single scale processes of massless QCD.

5. QCD NS-Analysis to 3 Loops



$$W^2 > 12.5\,\mathrm{GeV}^2, Q^2 > 4\,\mathrm{GeV}^2$$

N^3LO :

$$\alpha_s(M_Z^2) = 0.1141^{+0.0020}_{-0.0022}$$

J.B., H. Böttcher, A. Guffanti, 2006 (JULY) hep-ph/0607200

9 Tables

Experiment	x	Q^2, GeV^2	F_2^p	F_2^p *cuts*	F_2^p *HT*	Norm
BCDMS (100)	0.35 – 0.75	11.75 – 75.00	51	21	10	1.005
BCDMS (120)	0.35 – 0.75	13.25 – 75.00	59	32	4	0.998
BCDMS (200)	0.35 – 0.75	32.50 – 137.50	50	28	0	0.998
BCDMS (280)	0.35 – 0.75	43.00 – 230.00	49	26	0	0.998
NMC (comb)	0.35 – 0.50	7.00 – 65.00	15	14	6	1.000
SLAC (comb)	0.30 – 0.62	7.30 – 21.39	57	57	259	1.013
H1 (hQ2)	0.40 – 0.65	200 – 30000	26	26	0	1.020
ZEUS (hQ2)	0.40 – 0.65	650 – 30000	15	15	0	1.007
proton			322	227	279	
Experiment	x	Q^2, GeV^2	F_2^d	F_2^d *cuts*	F_2^d *HT*	Norm
BCDMS (120)	0.35 – 0.75	13.25 – 99.00	59	32	4	1.001
BCDMS (200)	0.35 – 0.75	32.50 – 137.50	50	28	0	0.998
BCDMS (280)	0.35 – 0.75	43.00 – 230.00	49	26	0	1.003
NMC (comb)	0.35 – 0.50	7.00 – 65.00	15	14	6	1.000
SLAC (comb)	0.30 – 0.62	10.00 – 21.40	59	59	268	0.990
deuteron			232	159	278	
Experiment	x	Q^2, GeV^2	F_2^{NS}	F_2^{NS} (*cuts*)	F_2^{NS} *HT*	Norm
BCDMS (120)	0.070 – 0.275	8.75 – 43.00	36	30	0	0.983
BCDMS (200)	0.070 – 0.275	17.00 – 75.00	29	28	0	0.999
BCDMS (280)	0.100 – 0.275	32.50 – 115.50	27	26	0	0.997
NMC (comb)	0.013 – 0.275	4.50 – 65.00	88	53	0	1.000
SLAC (comb)	0.153 – 0.293	4.18 – 5.50	28	28	1	0.994
non – singlet			208	165	1	
total			762	551	558	

Table 1: Number of data points for the non–singlet QCD analysis with their x and Q^2 ranges. In the first column are given (in parentheses) the beam momentum in GeV of the the respective data set (number), a flag whether the data come from a combined analysis of all beam momenta (comb) or whether the data are taken at high momentum transfer (hQ2). The fourth column (F_2) contains the number of data points according to the cuts: $Q^2 > 4$ GeV^2, $W^2 > 12.5$ GeV^2, $x > 0.3$ for F_2^p and F_2^d and $x < 0.3$ for F_2^{NS}. The reduction of the number of data points by the additional cuts on the BCDMS data ($y > 0.3$) and on the NMC data ($Q^2 > 8$ GeV^2) are given in the 5th column (F_2 *cuts*). The 6th column (F_2 *HT*) contains the number of data points in the range 4 GeV^2 $< W^2 <$ 12.5 GeV^2 used to fit the higher twist coefficients $C_{HT}(x)$ for the proton and deuteron data. In the last column the normalization shifts (see text) are listed.

Fully Correlated Error Calculation

- The fully correlated 1σ error for the parton density f_q as given by Gaussian error propagation is

$$\sigma(f_q(x)^2 = \sum_{i,j=1}^{n_p} \left(\frac{\partial f_q}{\partial p_i} \frac{\partial f_q}{\partial p_j} \right) \text{cov}(p_i, p_j) \; , \tag{1}$$

  where the $\partial f_q / \partial p_i$ are the derivatives of f_q w.r.t. the parameters p_i and the $\text{cov}(p_i, p_j)$ are the elements of the covariance matrix as determined in the fit.

- The derivatives $\partial f_q / \partial p_i$ at the input scale Q_0^2 can be calculated analytically. Their values at Q^2 are given by evolution.

- The derivatives evolved in $\mathrm{MELLIN\text{-}N}$ space are transformed back to x–space and can then be used according to the error propagation formula above.

$\Longrightarrow$ As an example the derivative of $f(x, a, b)$ w.r.t. parameter a in $\mathrm{MELLIN\text{–}N}$ space reads:

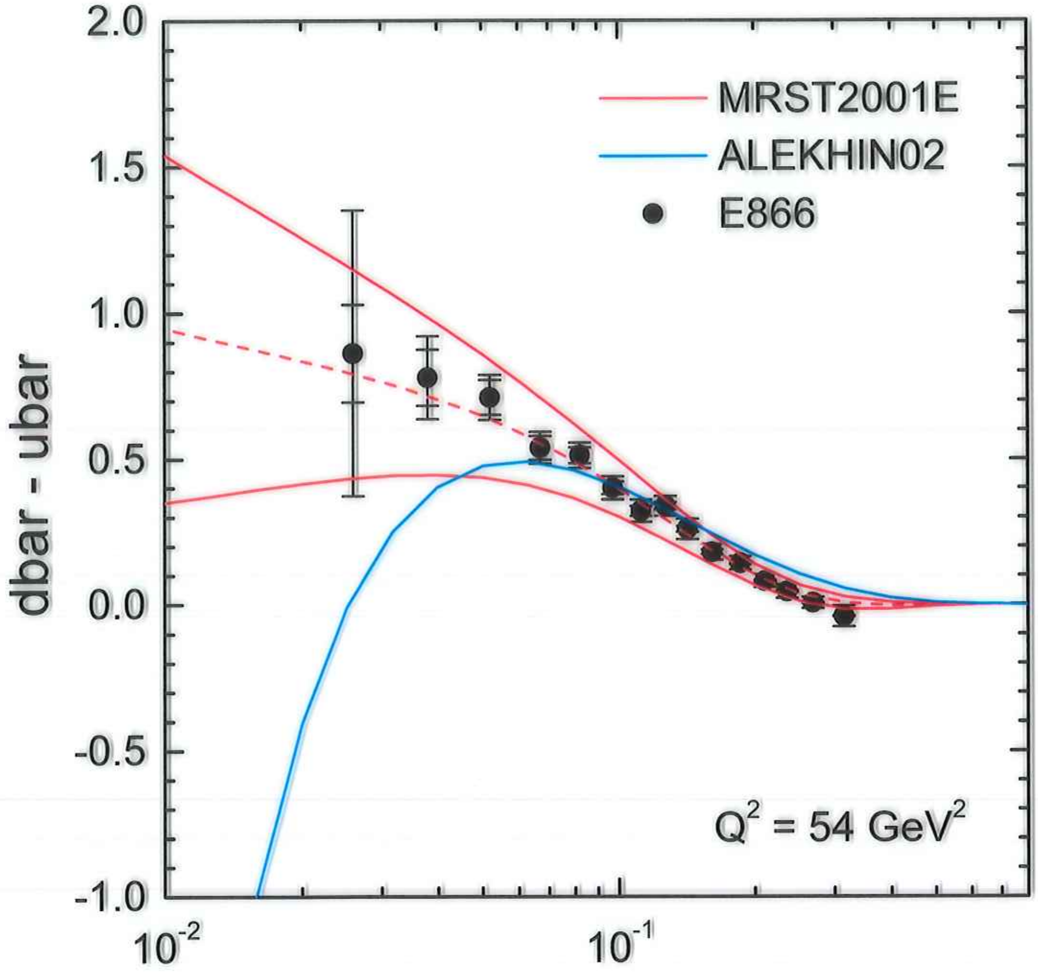
		NLO	NNLO	N^3LO
u_v	a	0.274 ± 0.027	0.291 ± 0.008	0.298 ± 0.008
	b	3.909 ± 0.040	4.013 ± 0.037	4.032 ± 0.037
	ρ	6.003	6.227	6.042
	γ	35.089	35.629	35.492
d_v	a	0.461 ± 0.030	0.488 ± 0.033	0.500 ± 0.034
	b	5.683 ± 0.228	5.878 ± 0.239	5.921 ± 0.243
	ρ	−3.699	−3.639	−3.618
	γ	16.491	16.445	16.414
$\Lambda_{\rm QCD}^{\rm N_f=4}$, MeV		265 ± 27	226 ± 25	234 ± 26
χ^2/ndf		$484/546 = 0.89$	$472/546 = 0.86$	$461/546 = 0.84$

Table 2: Parameter values of the NLO, NNLO and N^3LO non–singlet QCD fit at $Q_0^2 = 4$ GeV^2. The values without error have been fixed after a first minimization since the data do not constrain these parameters well enough (see text).

NNLO	$\Lambda_{\rm QCD}^{\rm N_f=4}$	a_{u_v}	b_{u_v}	a_{d_v}	b_{d_v}
$\Lambda_{QCD}^{(4)}$	**6.45E-4**				
a_{u_v}	9.03E-5	**5.75E-5**			
b_{u_v}	-3.37E-4	1.55E-4	**1.40E-3**		
a_{d_v}	1.92E-4	-8.97E-6	-4.69E-4	**1.07E-3**	
b_{d_v}	9.19E-4	5.82E-5	-3.30E-3	7.21E-3	**5.72E-2**

Table 3: The covariance matrix of the NNLO non–singlet QCD fit at $Q_0^2 = 4$ GeV^2.

$\overline{d} - \overline{u}$

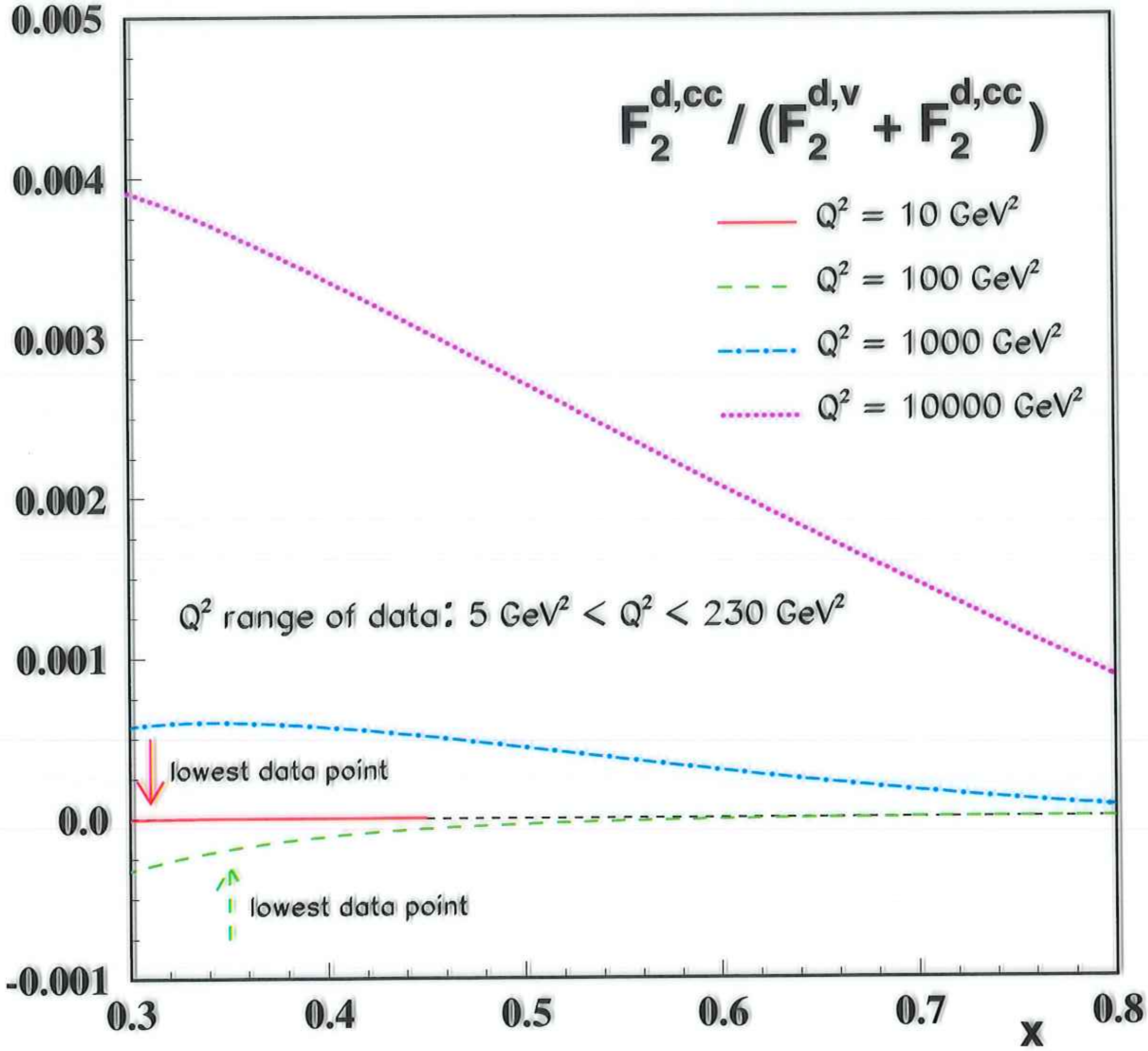


x$(\overline{d}(x) - \overline{u}(x)) = 1.195x^{1.24}(1-x)^{9.10}(1 + 14.05x - 45.52x^2)$

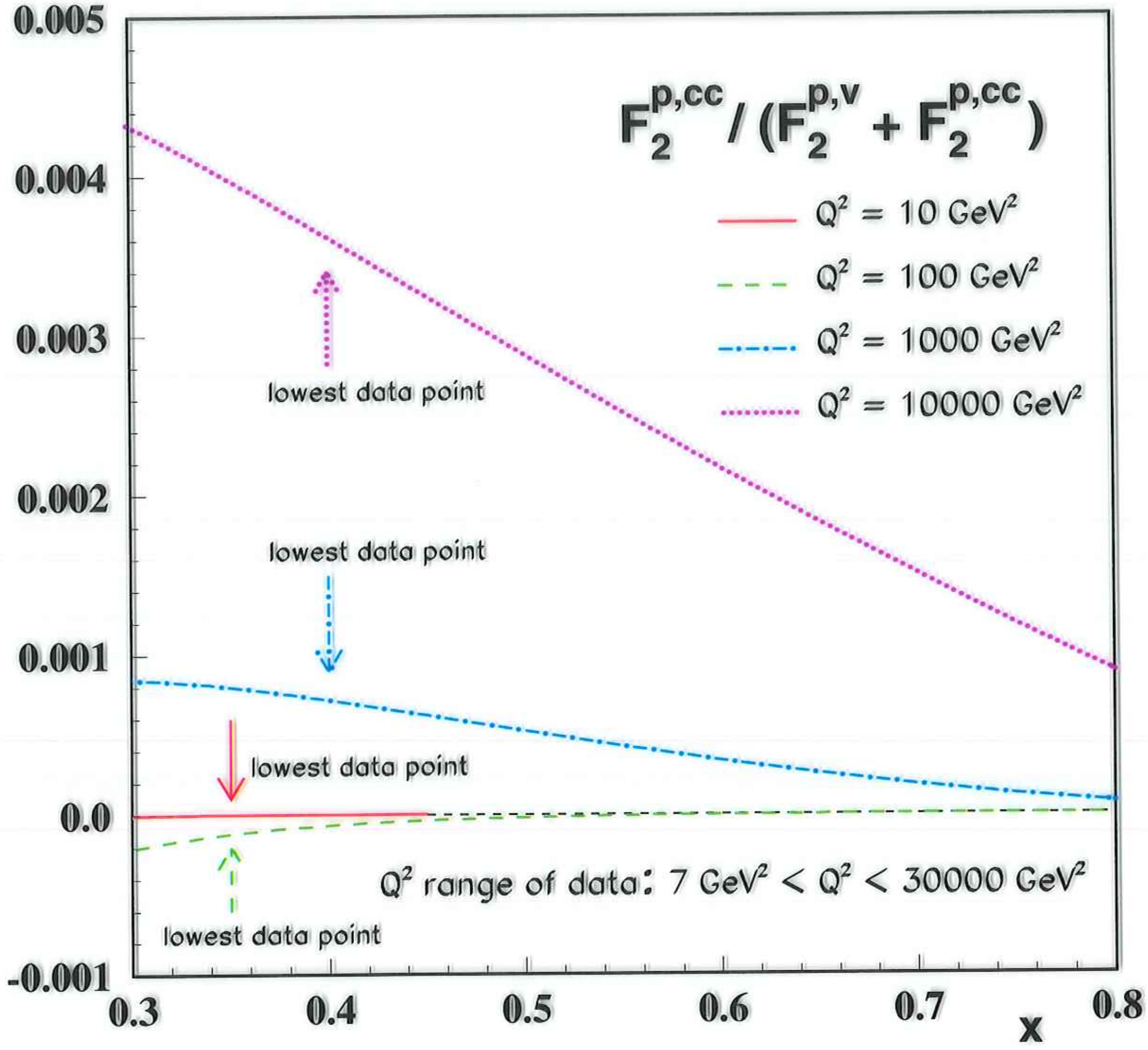
$Q^2 = 1$ GeV^2

MRST

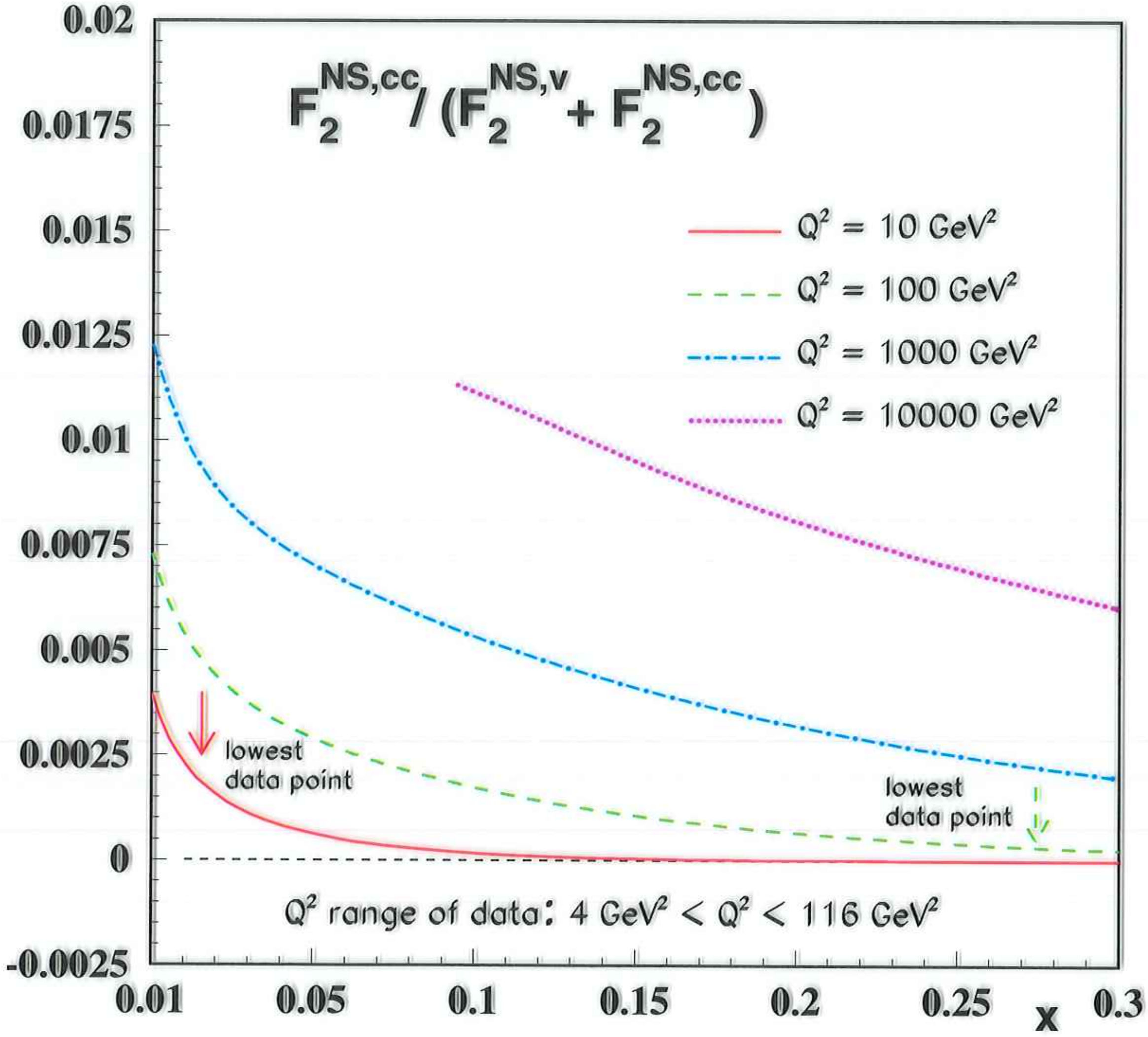
Heavy Flavor NS-contributions



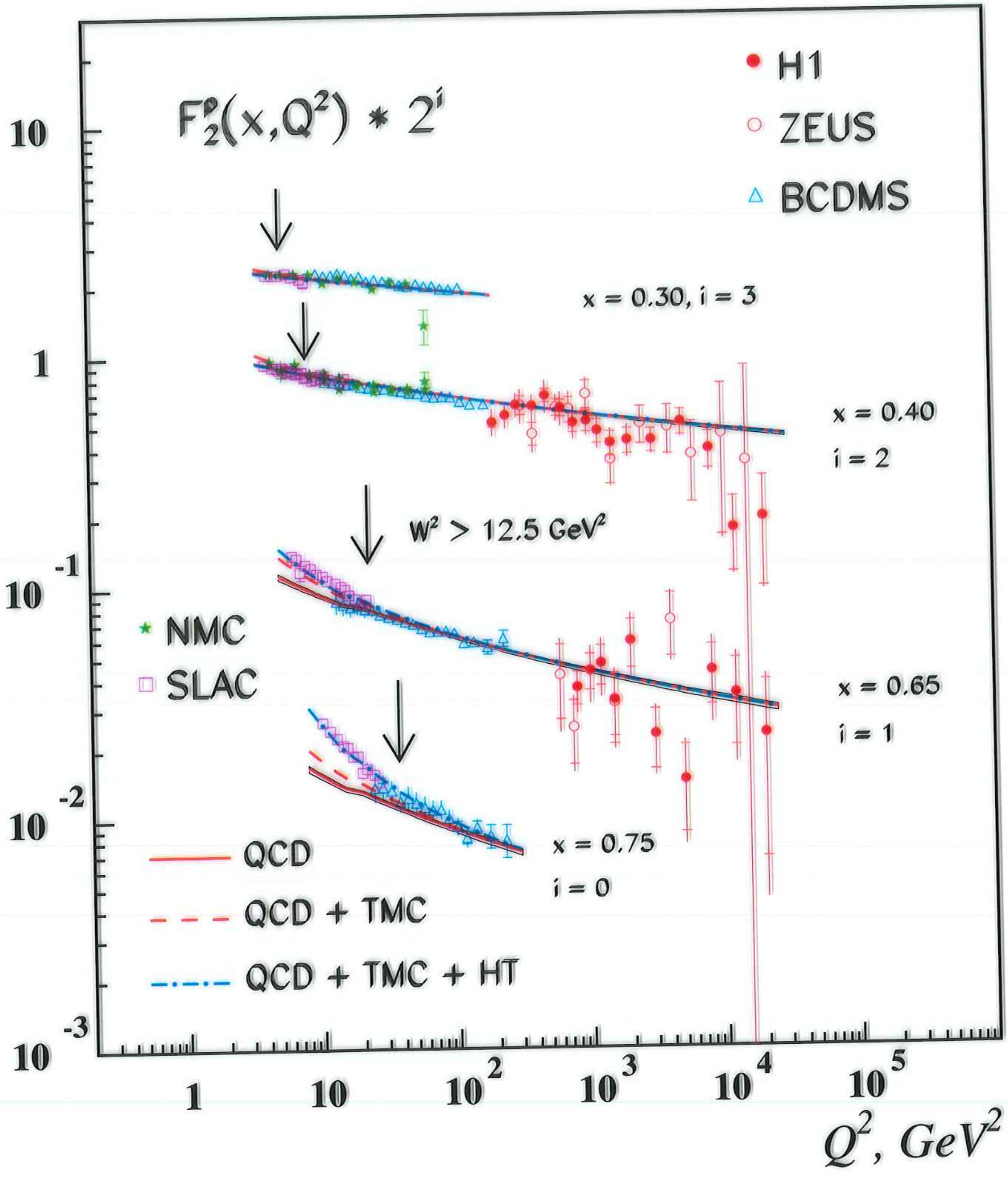
Heavy Flavor NS-contributions



Heavy Flavor NS-contributions

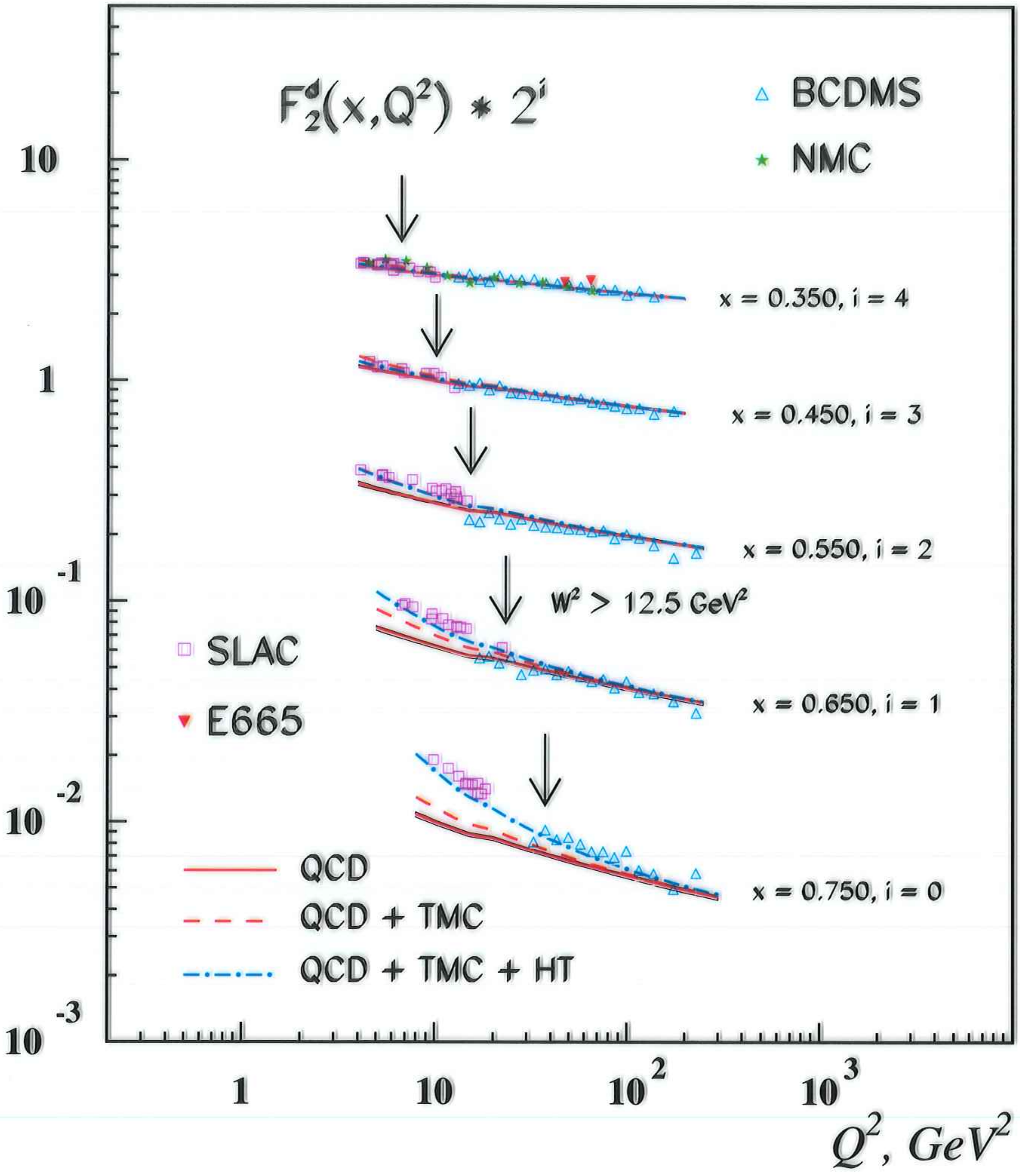


Non-Singlet 3-Loop QCD Analysis



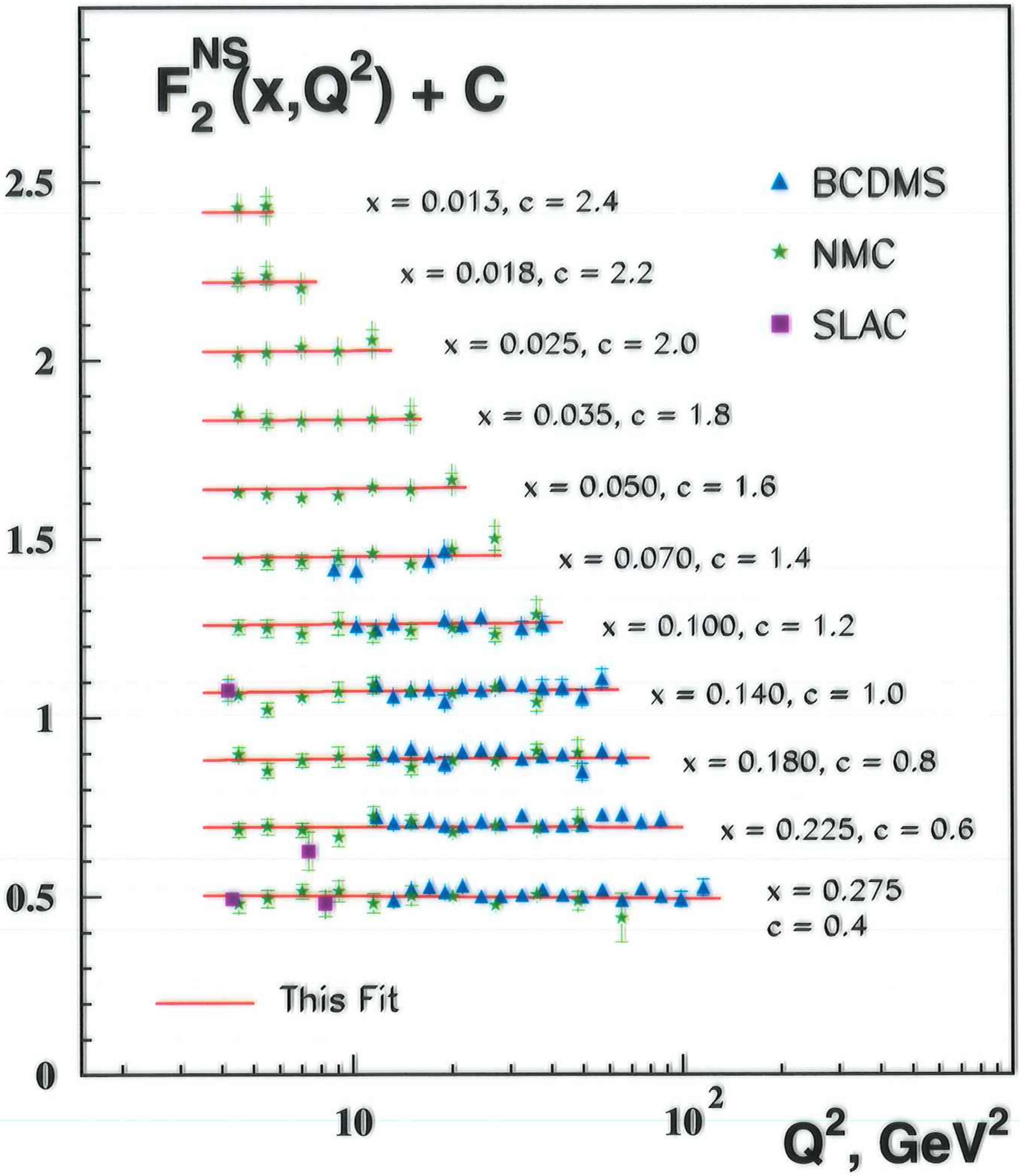
$x > 0.3$

Non-Singlet 3-Loop QCD Analysis



$x > 0.3$

NON-SINGLET 3-LOOP QCD ANALYSIS



$x < 0.3$

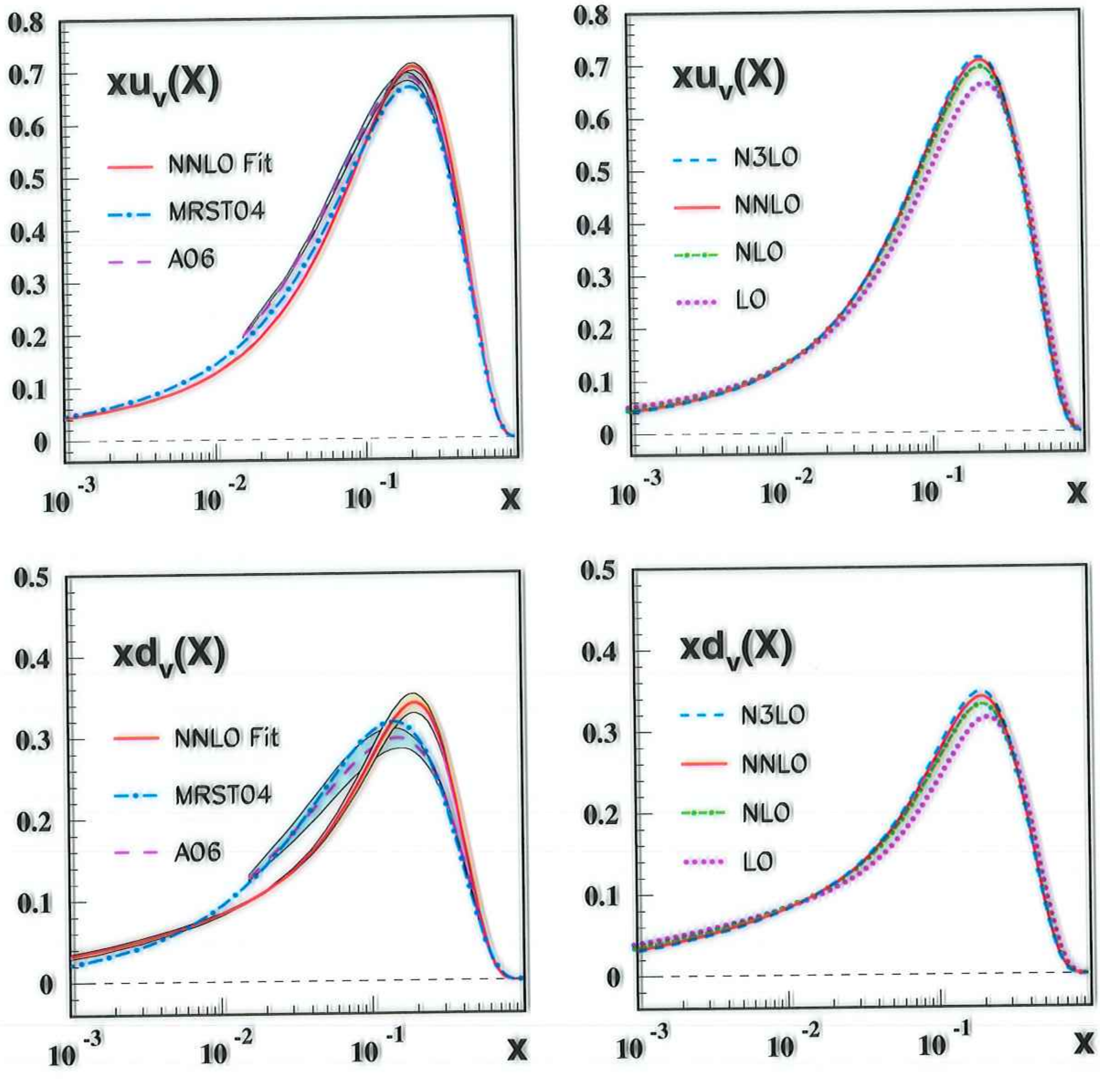


Figure 10: Left panels: The parton densities xu_v and xd_v at the input scale $Q_0^2 = 4.0$ GeV^2 (solid line) compared to results obtained from NNLO analyses by MRST (dashed–dotted line) [49] and A06 (dashed line) [13]. The shaded areas represent the fully correlated 1σ statistical error bands. Right panels: Comparison of the same parton densities at different orders in QCD as resulting from the present analysis.

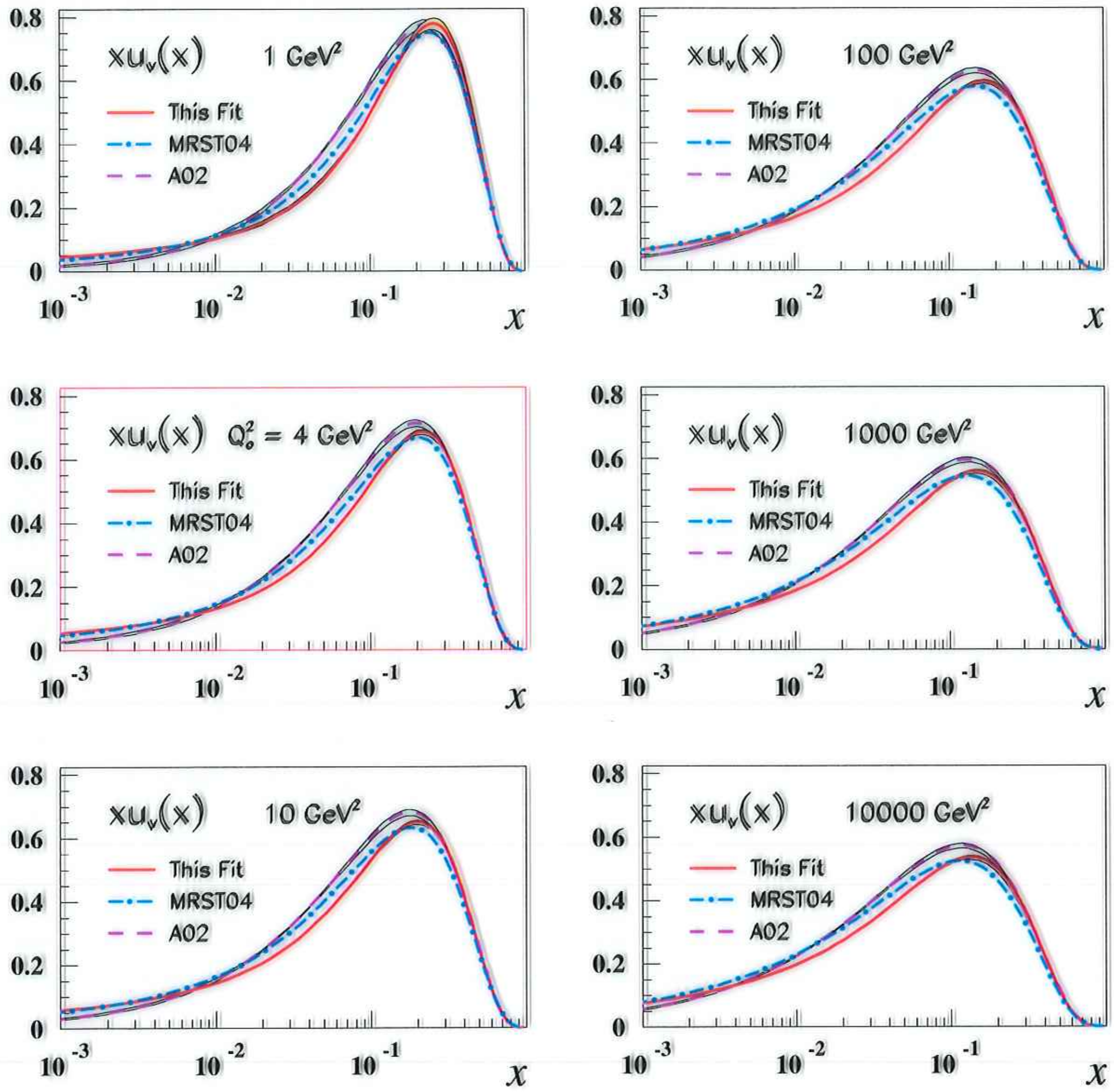


Figure 11: The parton density xu_v at NNLO evolved up to $Q^2 = 10,000$ GeV^2 (solid lines) compared to results obtained by MRST (dashed–dotted line) [49] and A02 (dashed line) [12]. The shaded areas represent the fully correlated 1σ statistical error bands.

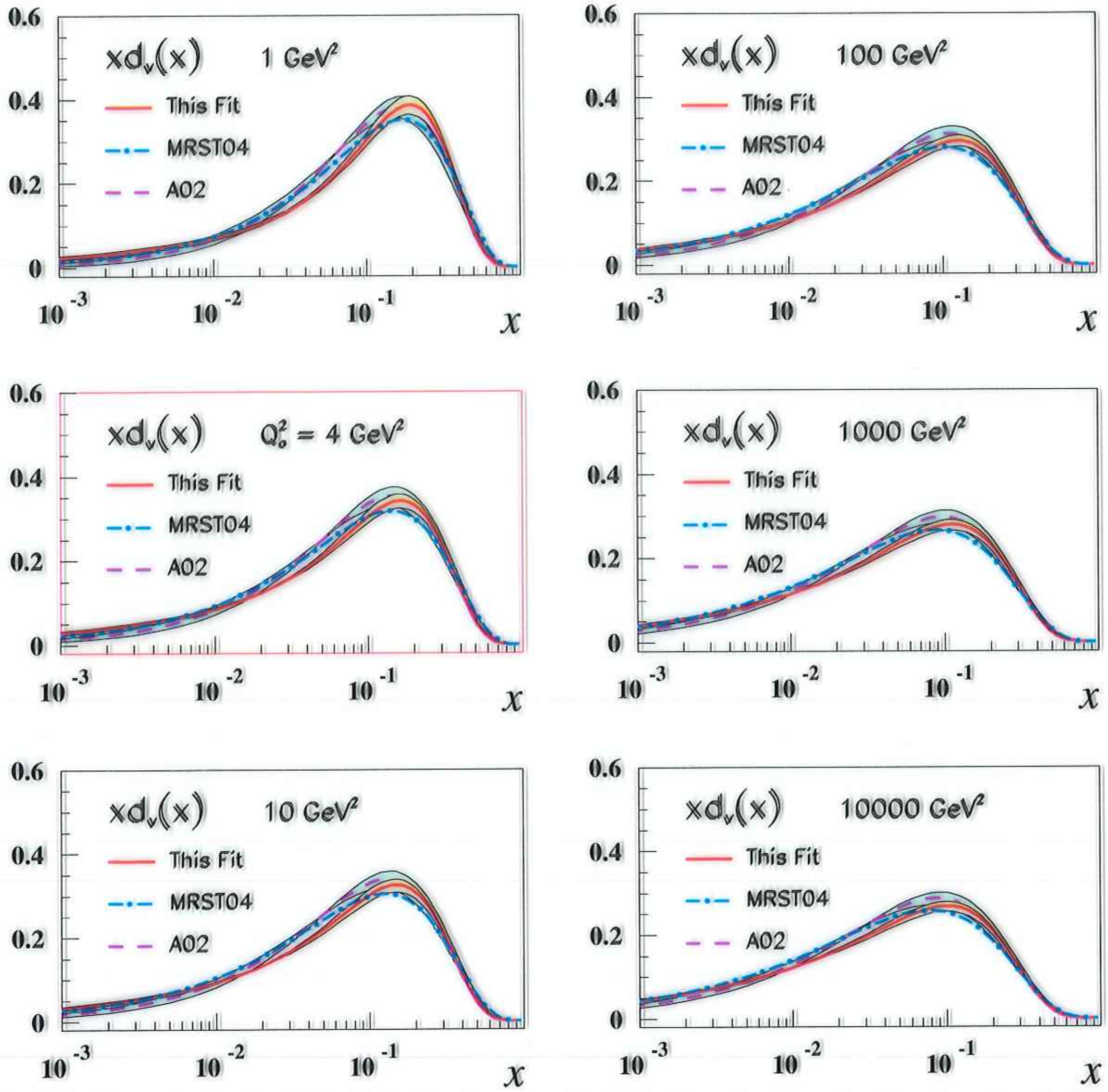


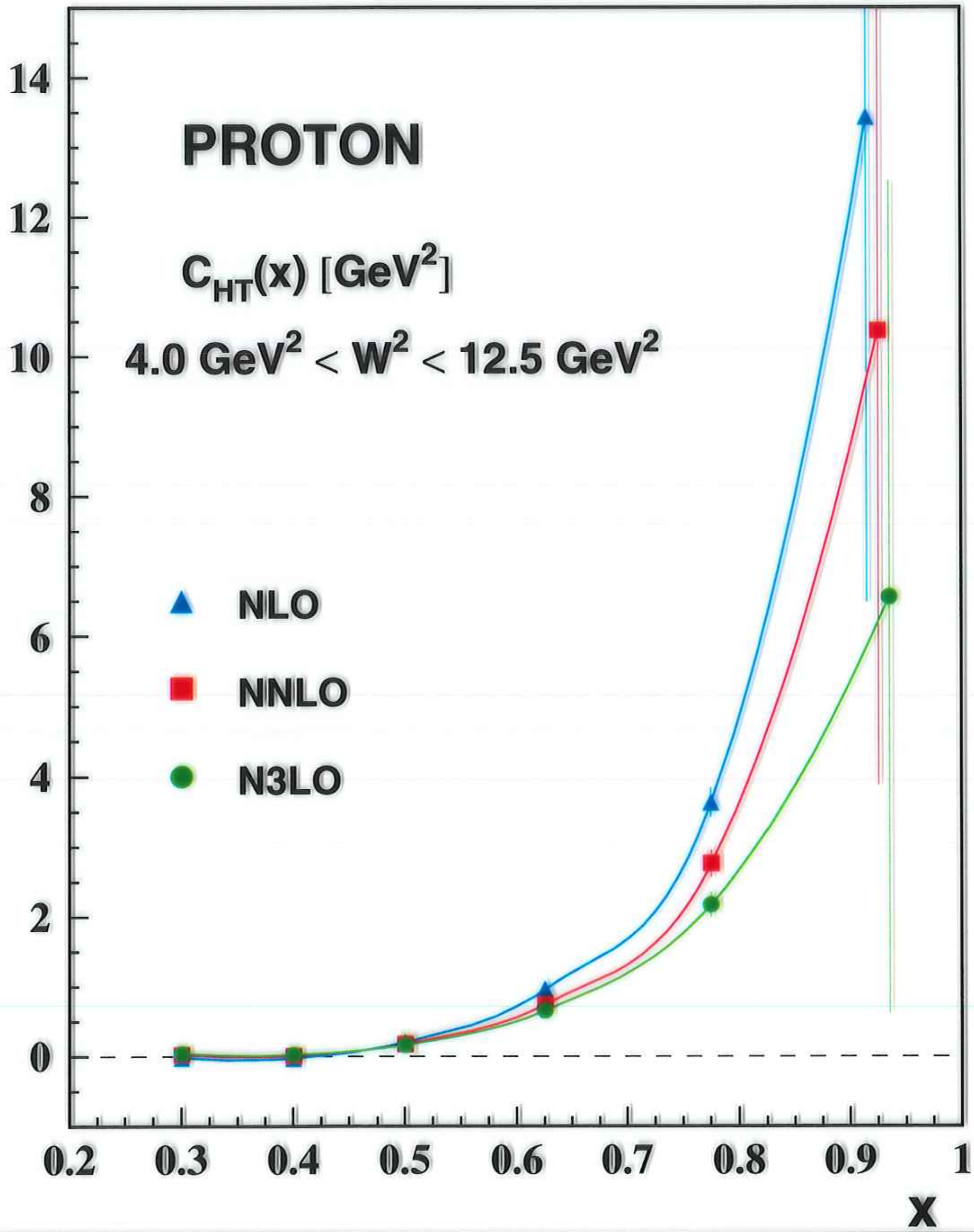
Figure 12: The parton density xd_v at NNLO evolved up to $Q^2 = 10,000$ GeV^2 (solid lines) compared to results obtained by MRST (dashed–dotted line) [49] and A02 (dashed line) [12]. The shaded areas represent the fully correlated 1σ statistical error bands.

f	n	N^3LO	NNLO	MRST04	A02	A06
u_v	2	0.3006 ± 0.0031	0.2986 ± 0.0029	0.285	0.304	0.2947
	3	0.0877 ± 0.0012	0.0871 ± 0.0011	0.082	0.087	0.0843
	4	0.0335 ± 0.0006	0.0333 ± 0.0005	0.032	0.033	0.0319
d_v	2	0.1252 ± 0.0027	0.1239 ± 0.0026	0.115	0.120	0.1129
	3	0.0318 ± 0.0009	0.0315 ± 0.0008	0.028	0.028	0.0275
	4	0.0106 ± 0.0004	0.0105 ± 0.0004	0.009	0.010	0.0092
$u_v - d_v$	2	0.1754 ± 0.0041	0.1747 ± 0.0039	0.171	0.184	0.182
	3	0.0559 ± 0.0015	0.0556 ± 0.0014	0.055	0.059	0.057
	4	0.0229 ± 0.0007	0.0228 ± 0.0007	0.022	0.024	0.023

Table 4: Comparison of low order moments at $Q_0^2 = 4$ GeV^2 from our non–singlet N^{3}LO and NNLO QCD analyses with the NNLO analyses MRST04 [49], A02 [12] and A06 [13] derived from global analyses, resp. combined singlet/non-singlet fits.

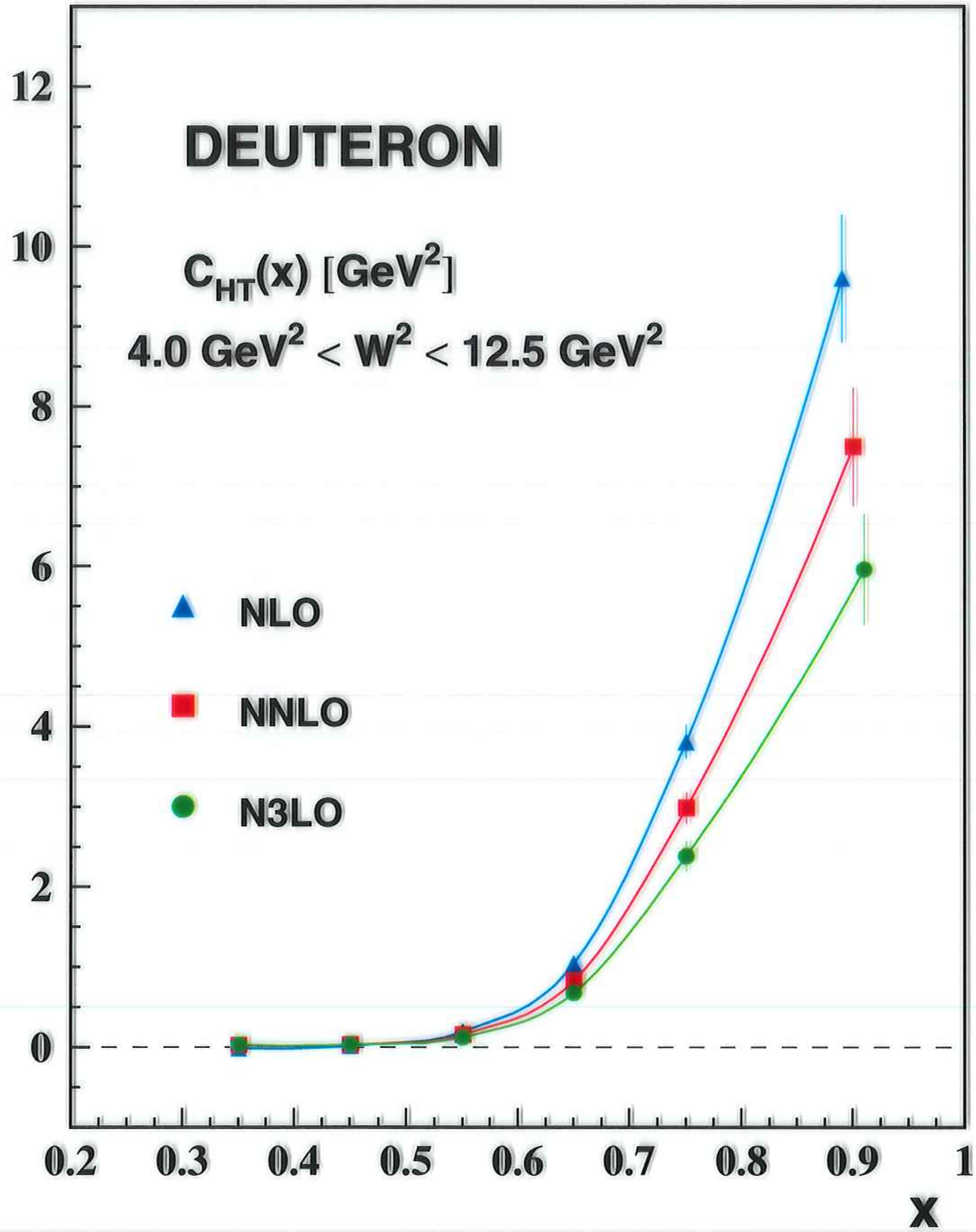
Higher Twist Contributions:

$4 < W^2 < 12.5\,\mathrm{GeV}^2, Q^2 > 4\,\mathrm{GeV}^2$



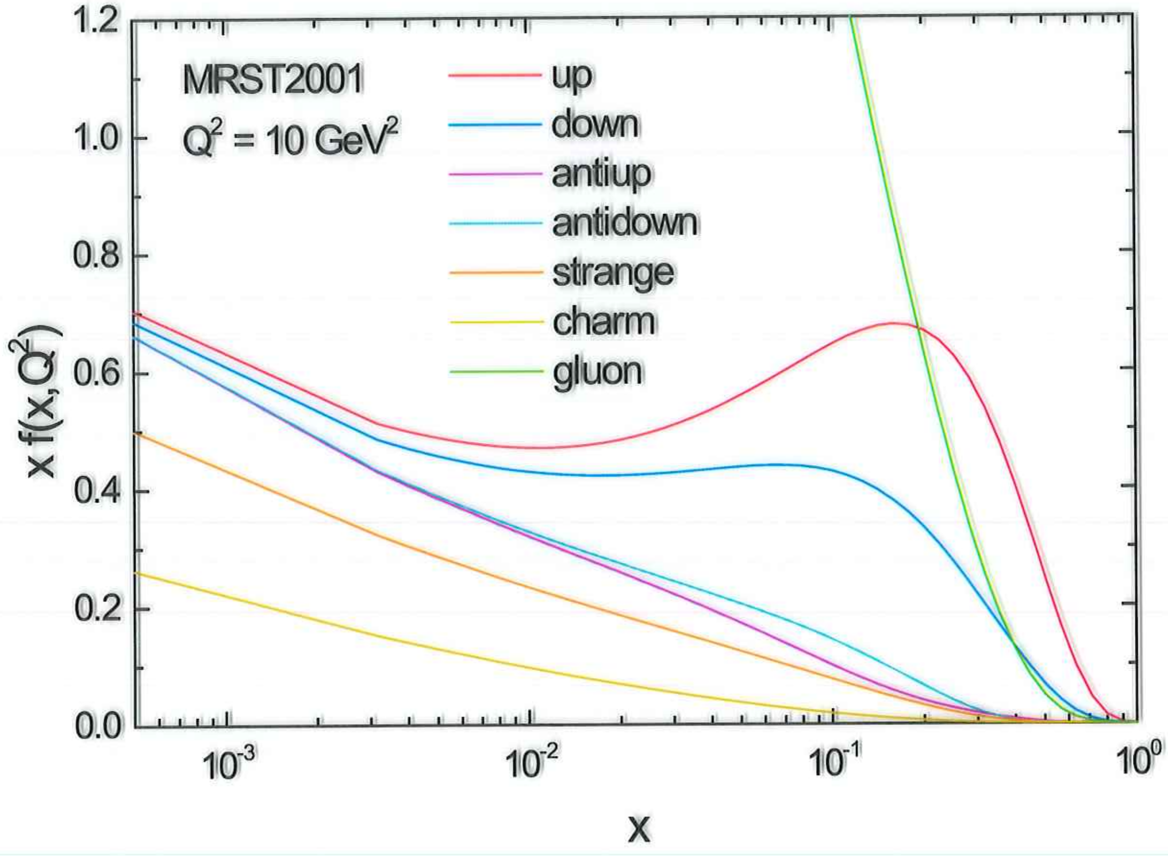
Higher Twist Contributions:

$$4 < W^2 < 12.5\,\mathrm{GeV}^2, Q^2 > 4\,\mathrm{GeV}^2$$



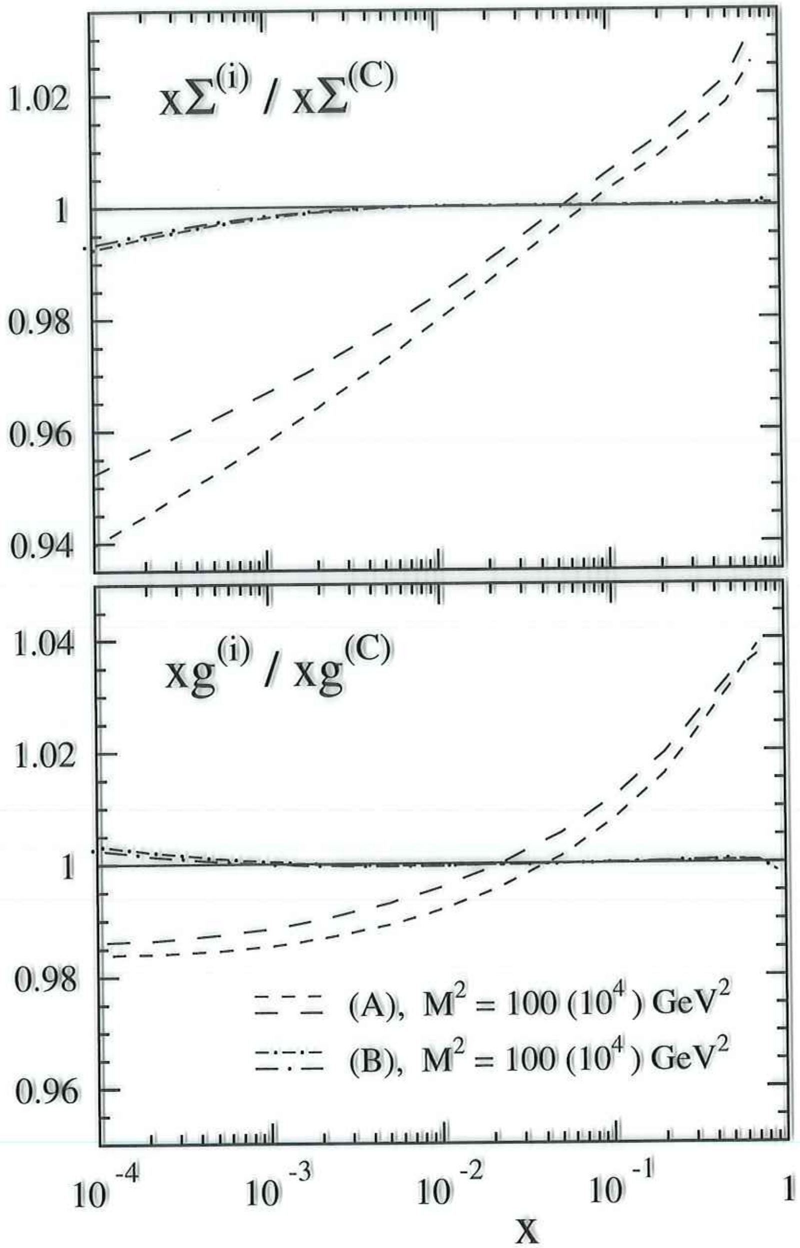
6. The Singlet Sector

Parton Densities: Relative Size



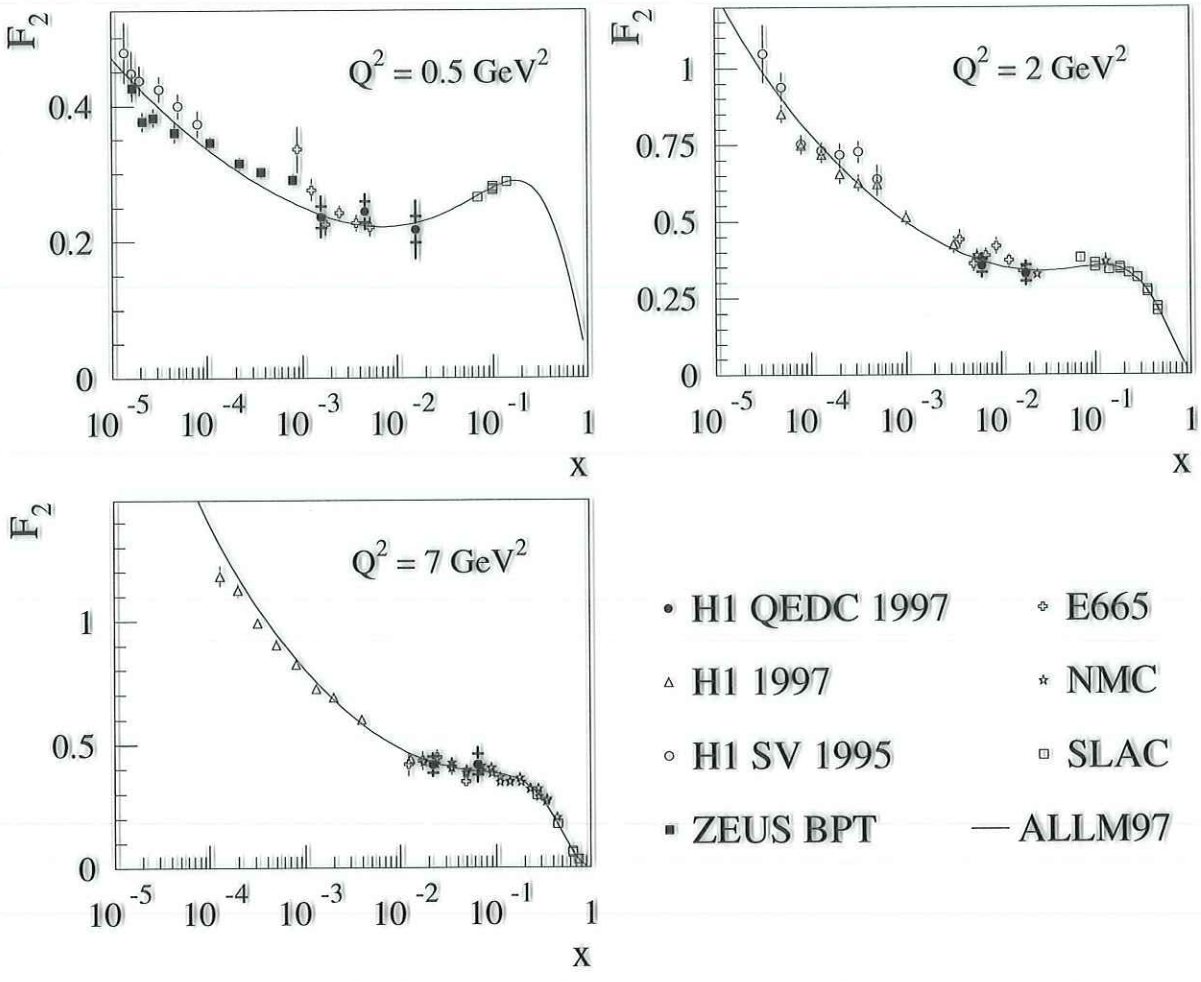
PILE–UP EFFECTS:

Iterative vs Exact Solution of Evolution Equations

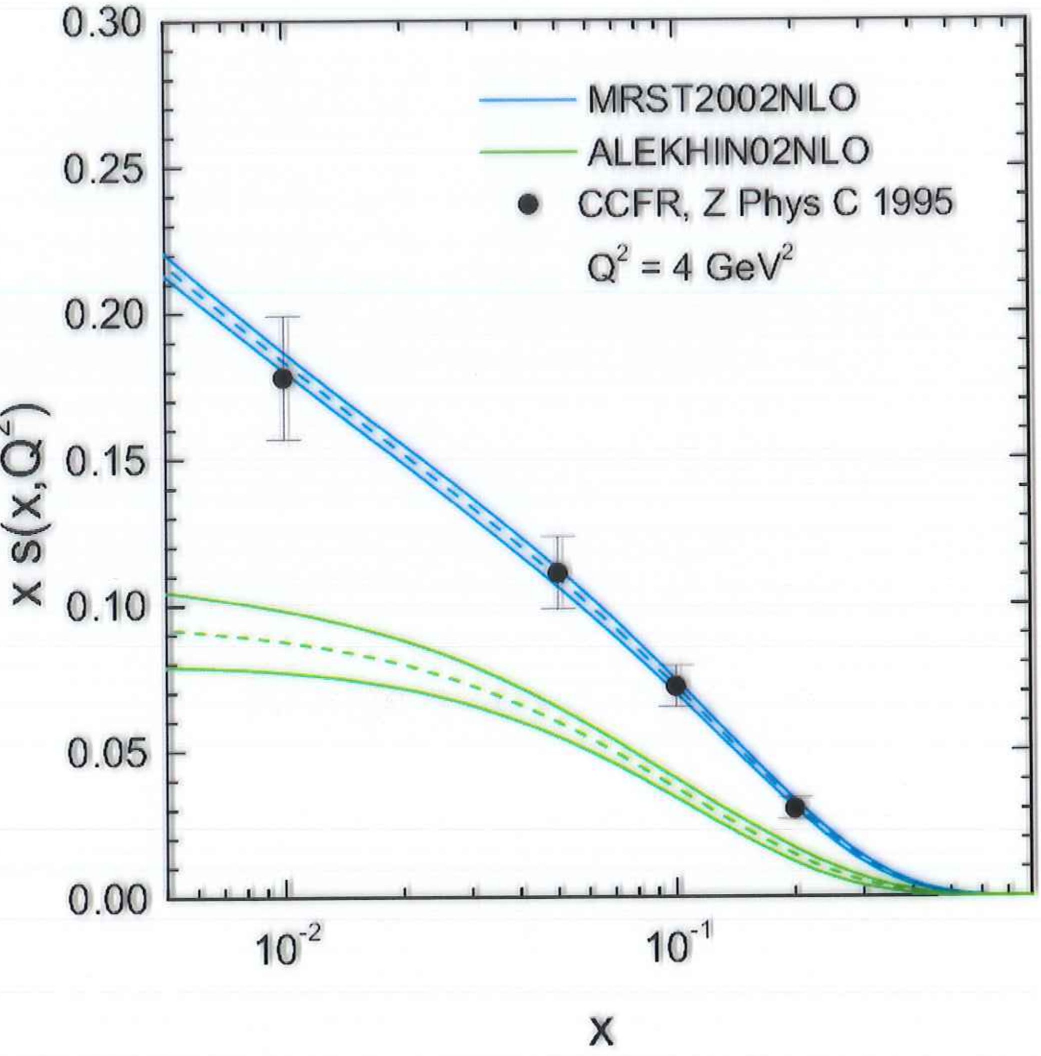


Blümlein, Riemersma, van Neerven, Vogt, 1996

x rise of $F_2(x, Q^2)$ at low Q^2 :



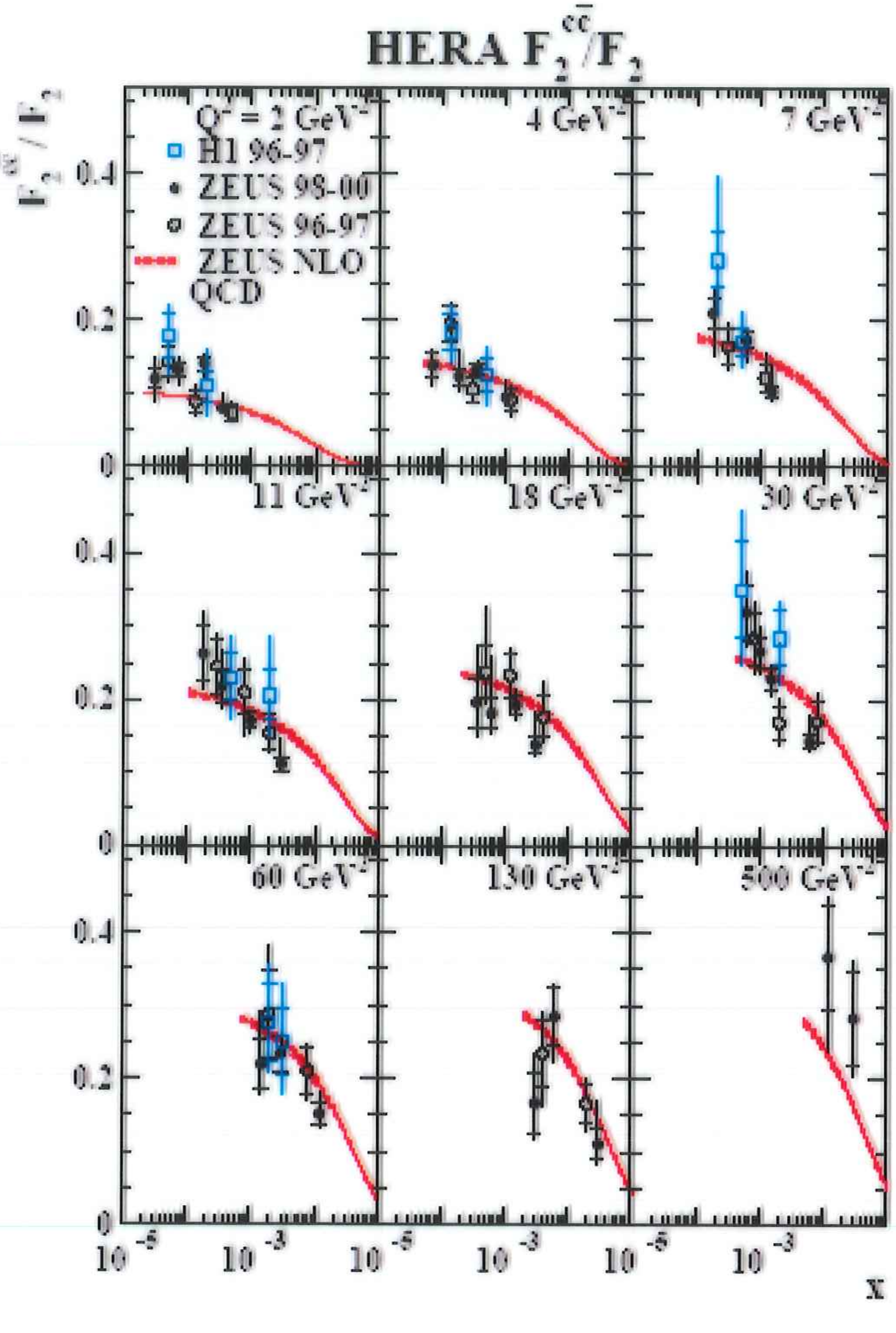
Strange quark distribution



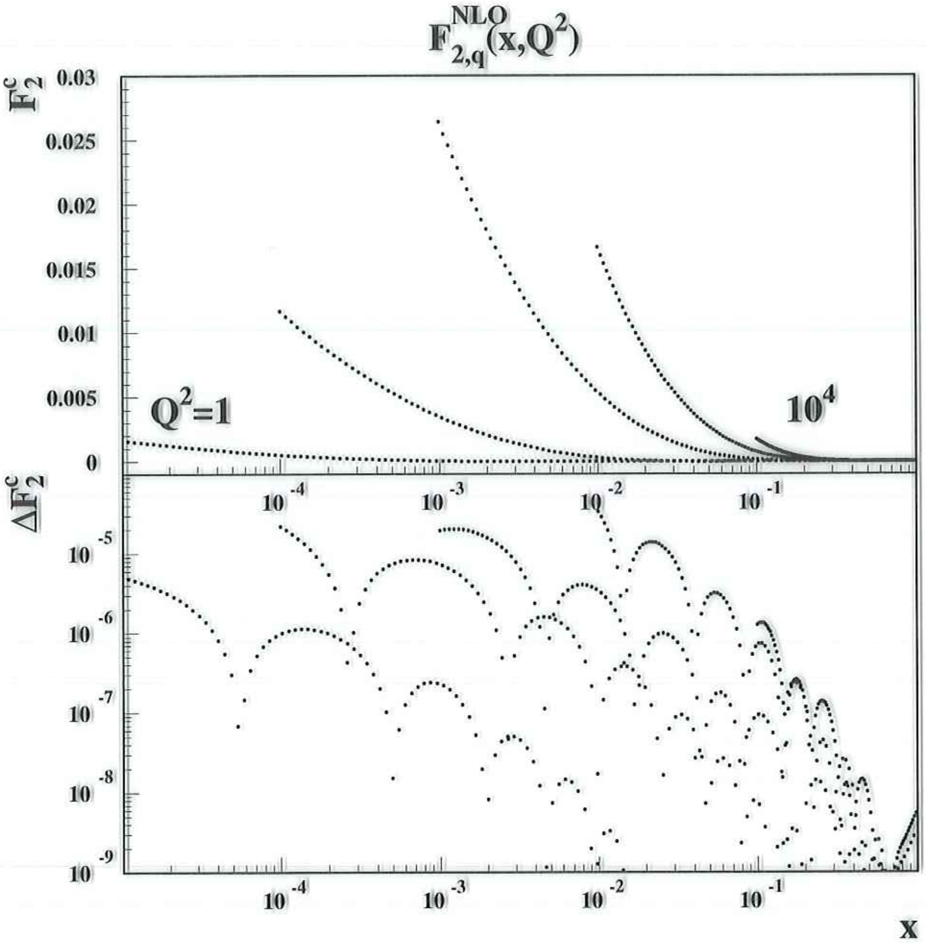
- CCFR : iron target, EMC effect. How large ?

CAN HERMES MEASURE $s(x, Q^2)$?

$c\bar{c}$ Structure Function F_2



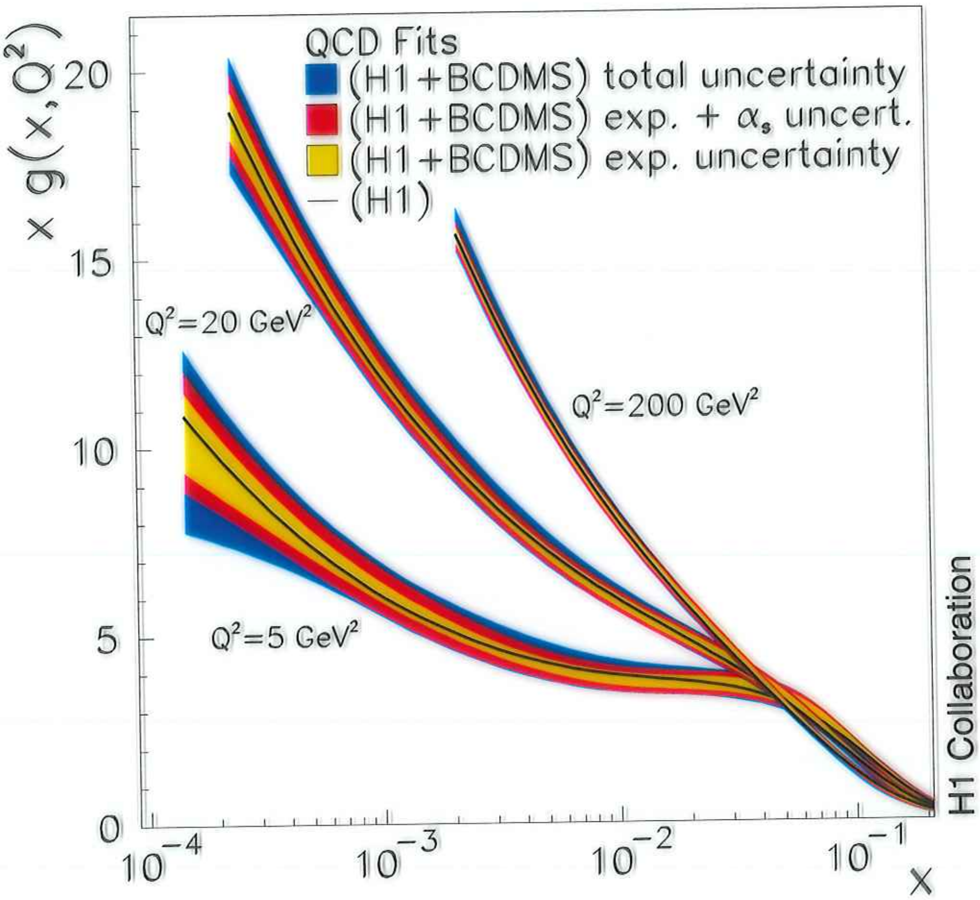
Mellin-space representation :

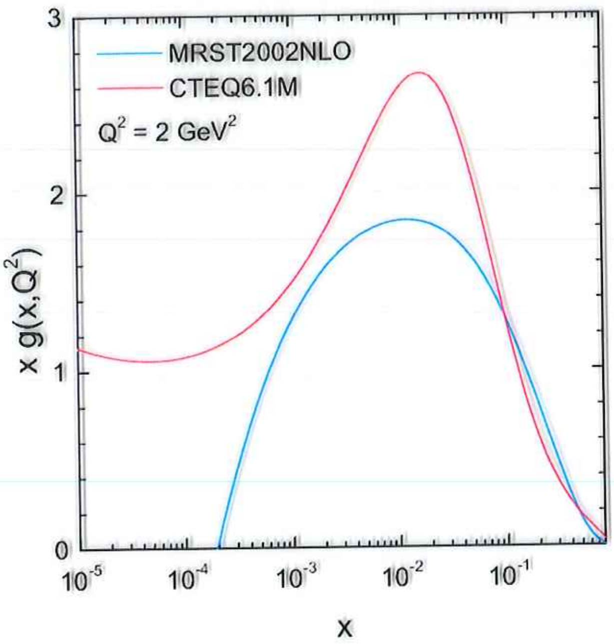


S. Alekhin and J.B., 2004

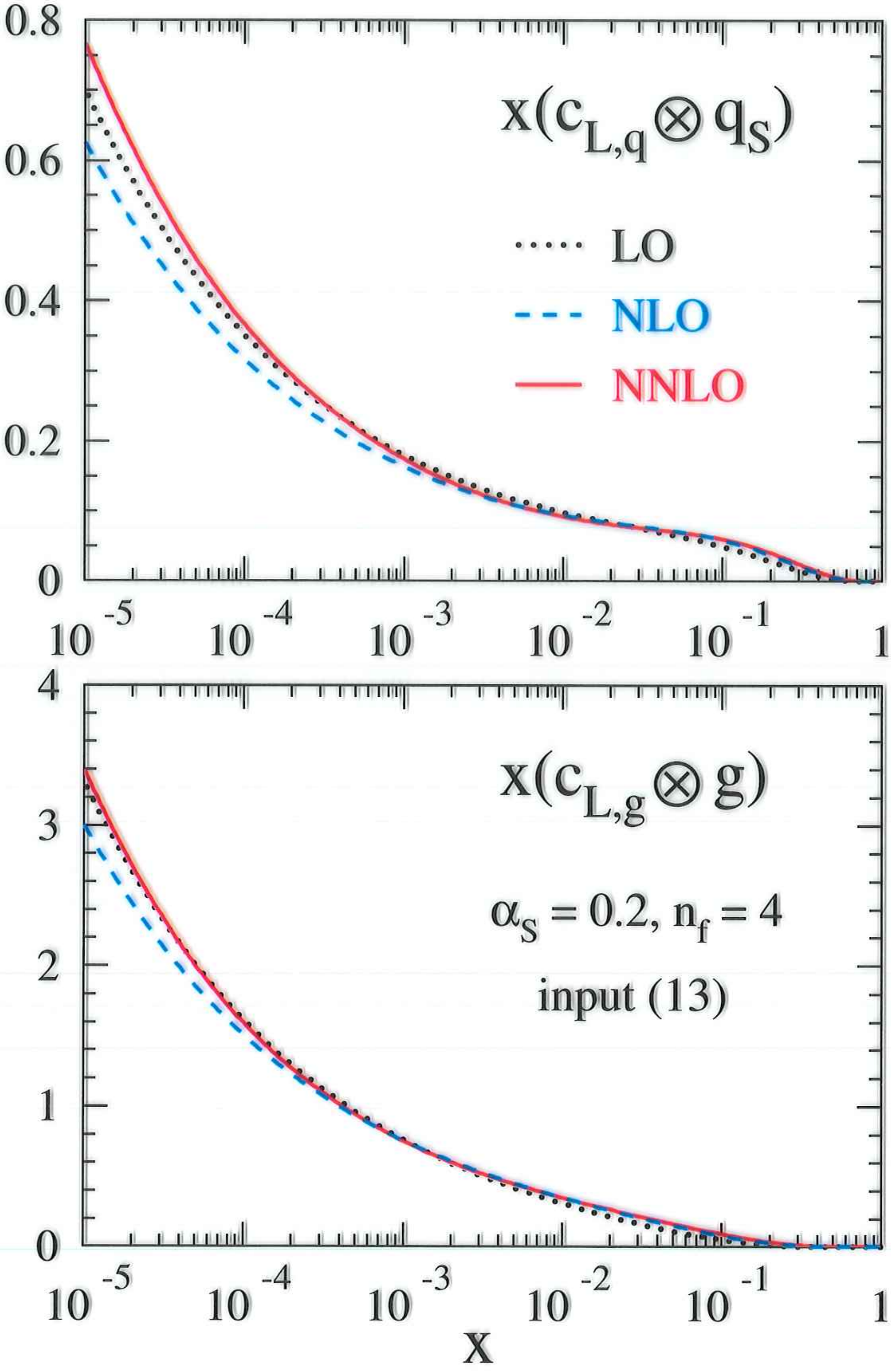
- necessary for scheme-invariant evolution.
- fast and accurate access to heavy flavor Wilson coefficients.

Gluon Density



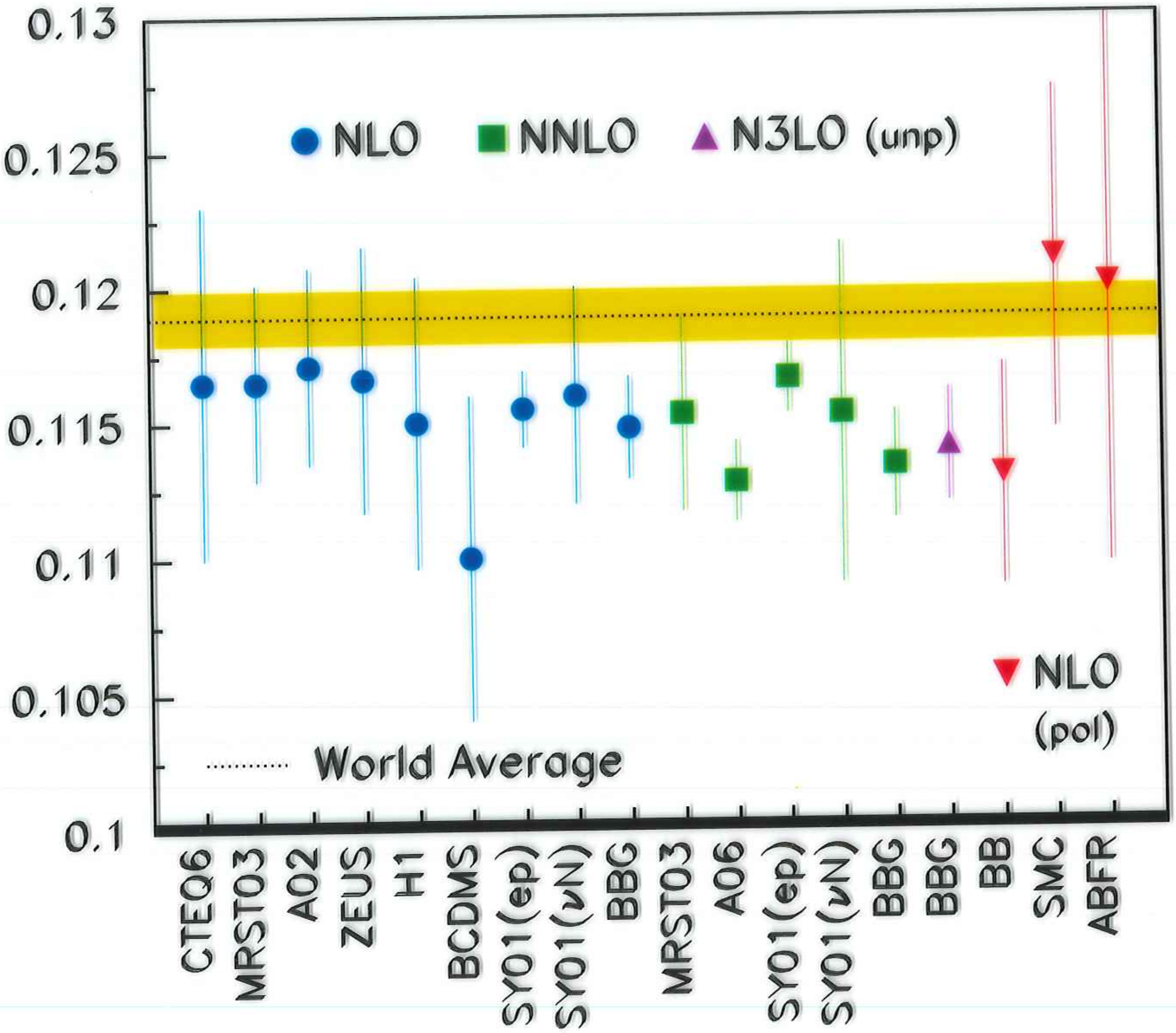


$F_L(x, Q^2)$



Moch, Vermasern, Vogt, hep-ph/0411112

7. DIS: $\alpha_s(M_Z^2)$



	$\Lambda_{\rm QCD}^{\rm N_f=4}$, MeV	$\alpha_s(M_Z^2)$		
NLO	265 ± 27	0.1148	$^{+0.0019}_{-0.0019}$	(expt)
NNLO	226 ± 25	0.1134	$^{+0.0019}_{-0.0021}$	(expt)
→ $\rm N^3LO$	234 ± 26	0.1141	$^{+0.0020}_{-0.0022}$	(expt)

Table 5: $\Lambda_{\rm QCD}^{\rm N_f=4}$ and $\alpha_s(M_Z^2)$ at NLO, NNLO and N^{3}LO.

	$\alpha_s(M_Z^2)$	expt	theory	model	Ref.
NLO					
CTEQ6	0.1165	± 0.0065			[19]
MRST03	0.1165	± 0.0020	± 0.0030		[11]
A02	0.1171	± 0.0015	± 0.0033		[12]
ZEUS	0.1166	± 0.0049		± 0.0018	[56]
H1	0.1150	± 0.0017	± 0.0050	$^{+0.0009}_{-0.0005}$	[5]
BCDMS	0.110	± 0.006			[2]
GRS	0.112				[18]
BBG	0.1148	$\pm$ 0.0019			
BB (pol)	0.113	± 0.004	$^{+0.009}_{-0.006}$		[58]
NNLO					
MRST03	0.1153	± 0.0020	± 0.0030		[11]
A02	0.1143	± 0.0014	± 0.0009		[12]
SY01(ep)	0.1166	± 0.0013			[14]
SY01(νN)	0.1153	± 0.0063			[14]
GRS	0.111				[18]
A06	0.1128	$\pm$ 0.0015			[13]
BBG	0.1134	$^{+0.0019}_{-0.0021}$			
N^{3}LO					
BBG	0.1141	$^{+0.0020}_{-0.0022}$			

Table 6: Comparison of $\alpha_s(M_Z^2)$ values from NLO, NNLO, and N^{3}LO QCD analyses.

LATTICE RESULTS $N_f = 2$

α - COLLAB.:

$\Lambda = 245 \pm 16 \pm 16$ MeV

QCDSF:

$\Lambda = 249 \pm^{13}_{8} \pm 17$ MeV

8. Future Avenues

HERA:

- Collect high luminosity for $F_2(x, Q^2)$, $F_2^{c\bar{c}}(x, Q^2)$, $g_2^{c\bar{c}}(x, Q^2)$, and measure $h_1(x, Q^2)$.
- Measure : $F_L(x, Q^2)$. This is a key-question for HERA.

RHIC & LHC:

- Improve constraints on gluon and sea–quarks: polarized and unpolarized.

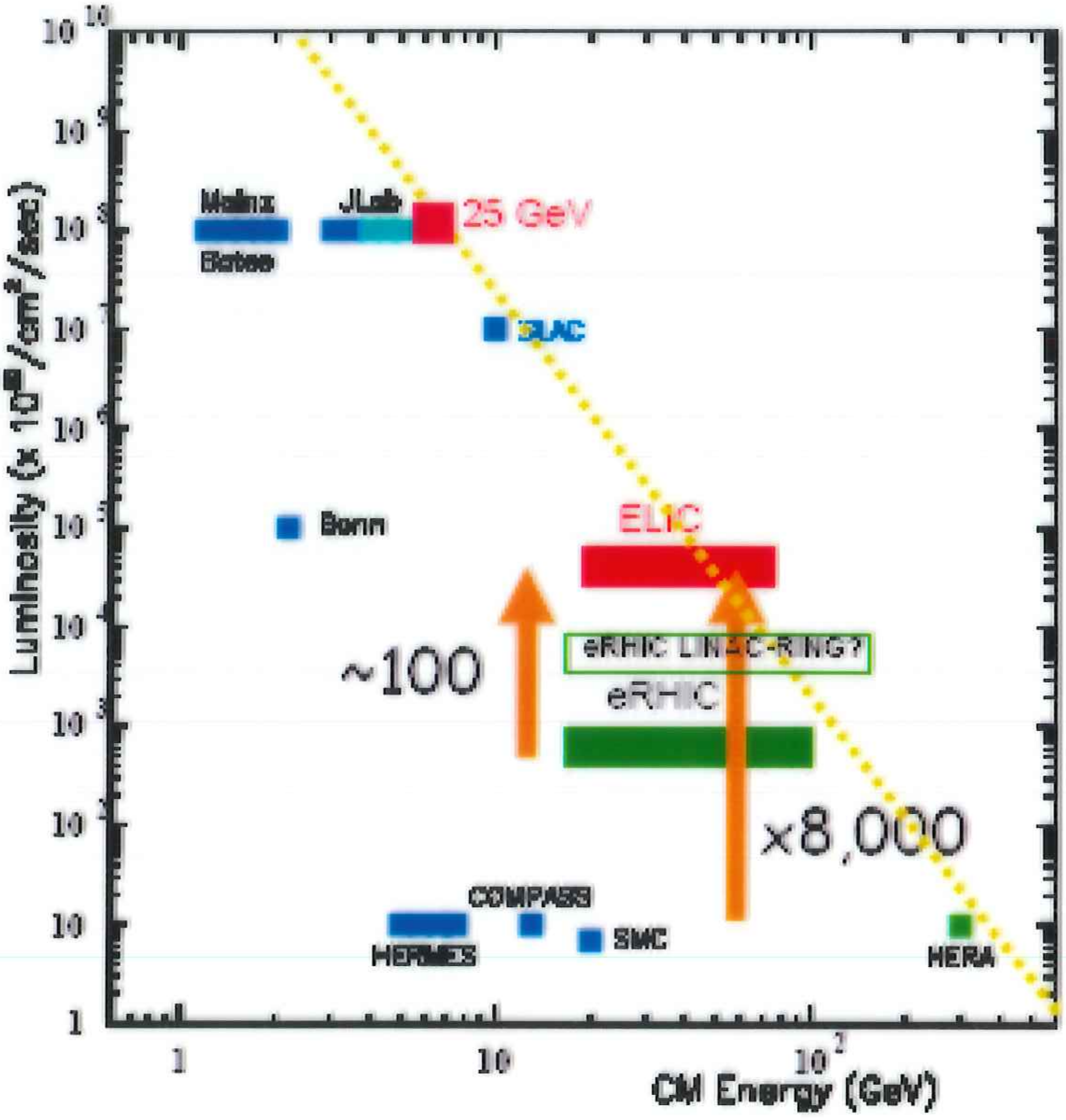
JLAB:

- High precision measurements in the large x domain at unpolarized and polarized targets; supplements HERA's high precision measurements at small x.

ELIC:

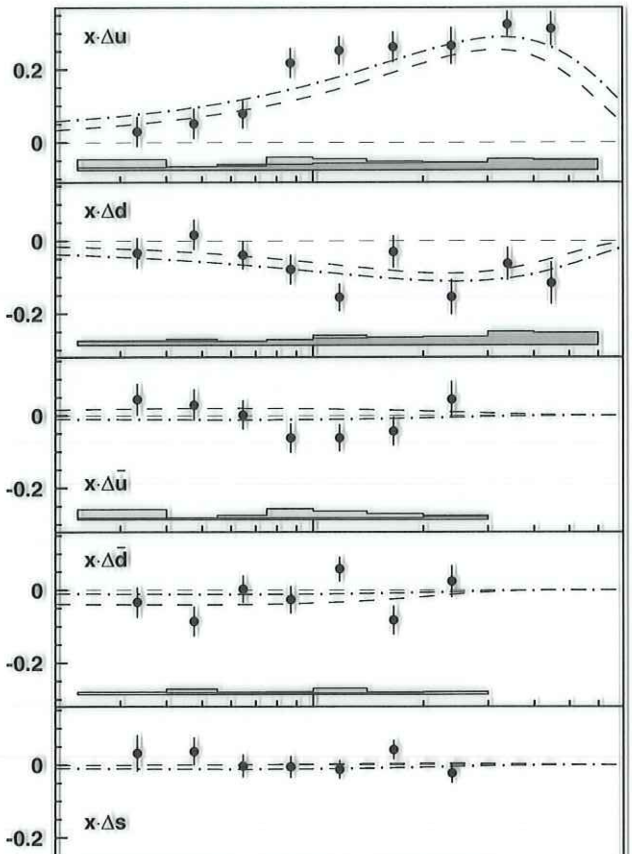
- High precision measurements in the medium x domain; both unpolarized and polarized

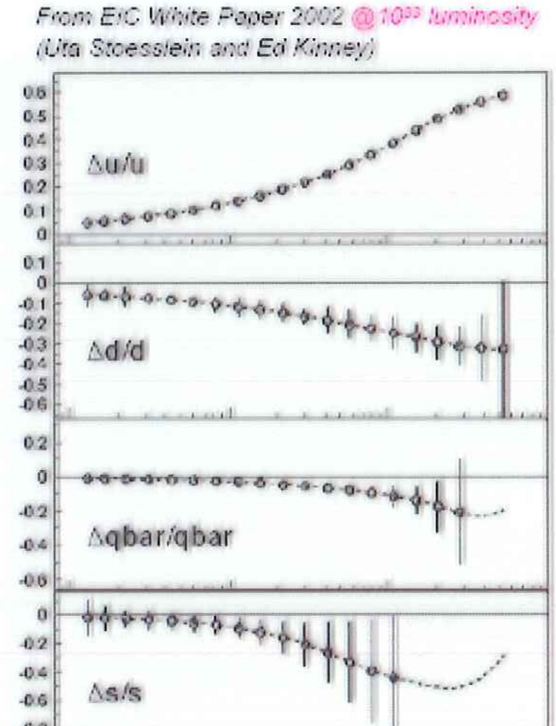
THE QUEST FOR LARGE LUMINOSITY !



.... allows very precise measurements

Example : Flavor Separation of polarized PDF's





HERMES EIC

- What is the correct value of $\alpha_s(M_z^2)$? $\overline{\text{MS}}$-analysis vs. scheme-invariant evolution helps. Compare non–singlet and singlet analysis; careful treatment of heavy flavor.[Theory & Experiment]

- Flavor Structure of Sea-Quarks: More studies needed.[All Experiments]

- Revisit polarized data upon arrival of the 3-loop anomalous dimensions; NLO heavy flavor contributions needed.[Theory]

- QCD at Twist 3: $g_2(x, Q^2)$, semi–exclusive Reactions [High Precision polarized experiments, JLAB, EIC]

- Comparison with Lattice Results: α_s, Moments of Parton Distributions, Angular Momentum.

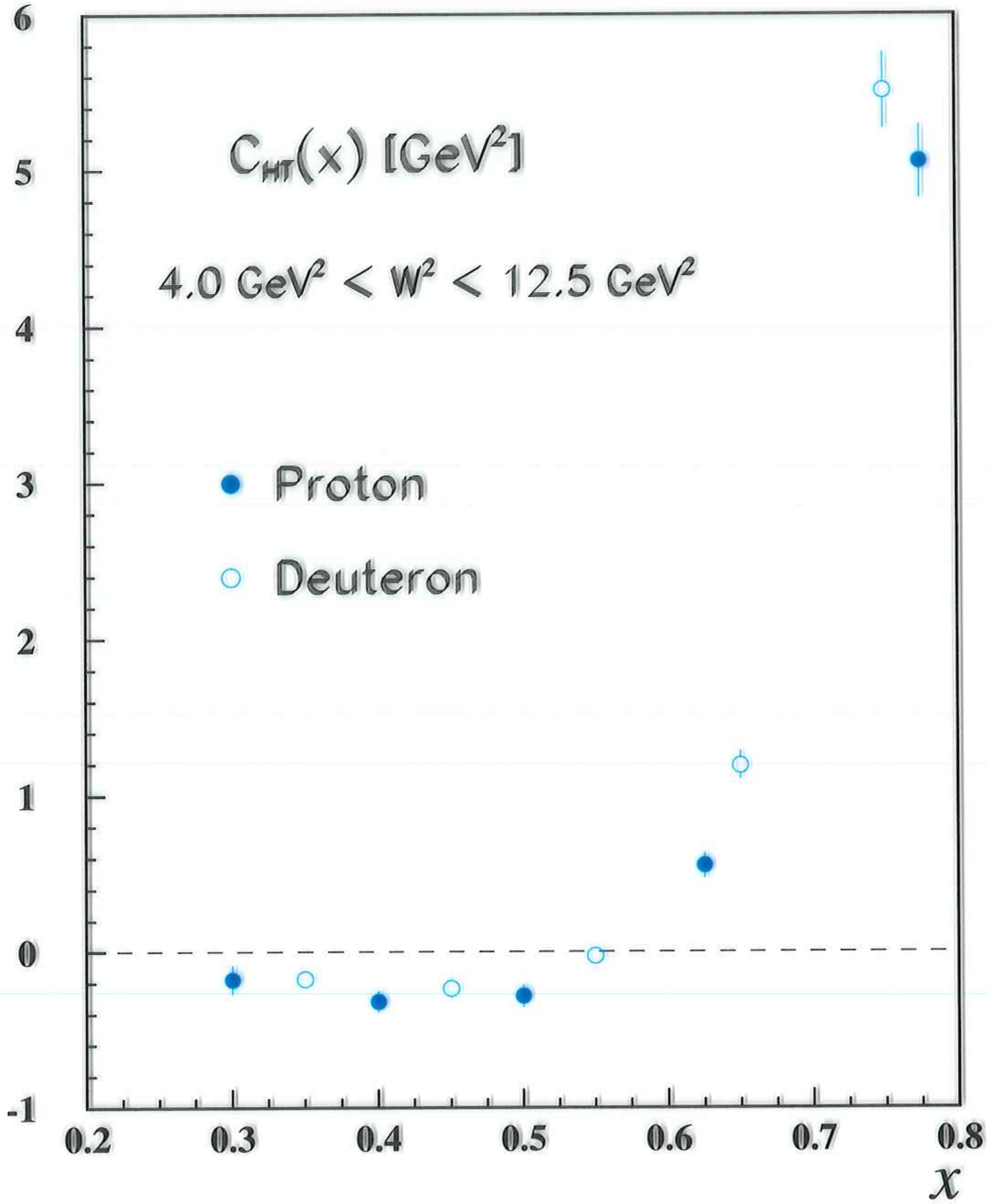
- Calculation of more hard scattering reactions at the 3–loop level: ILC, LHC

- Further perfection of the mathematical tools:
  $\Longrightarrow$ Algorithmic simplification of Perturbation theory in higher orders.

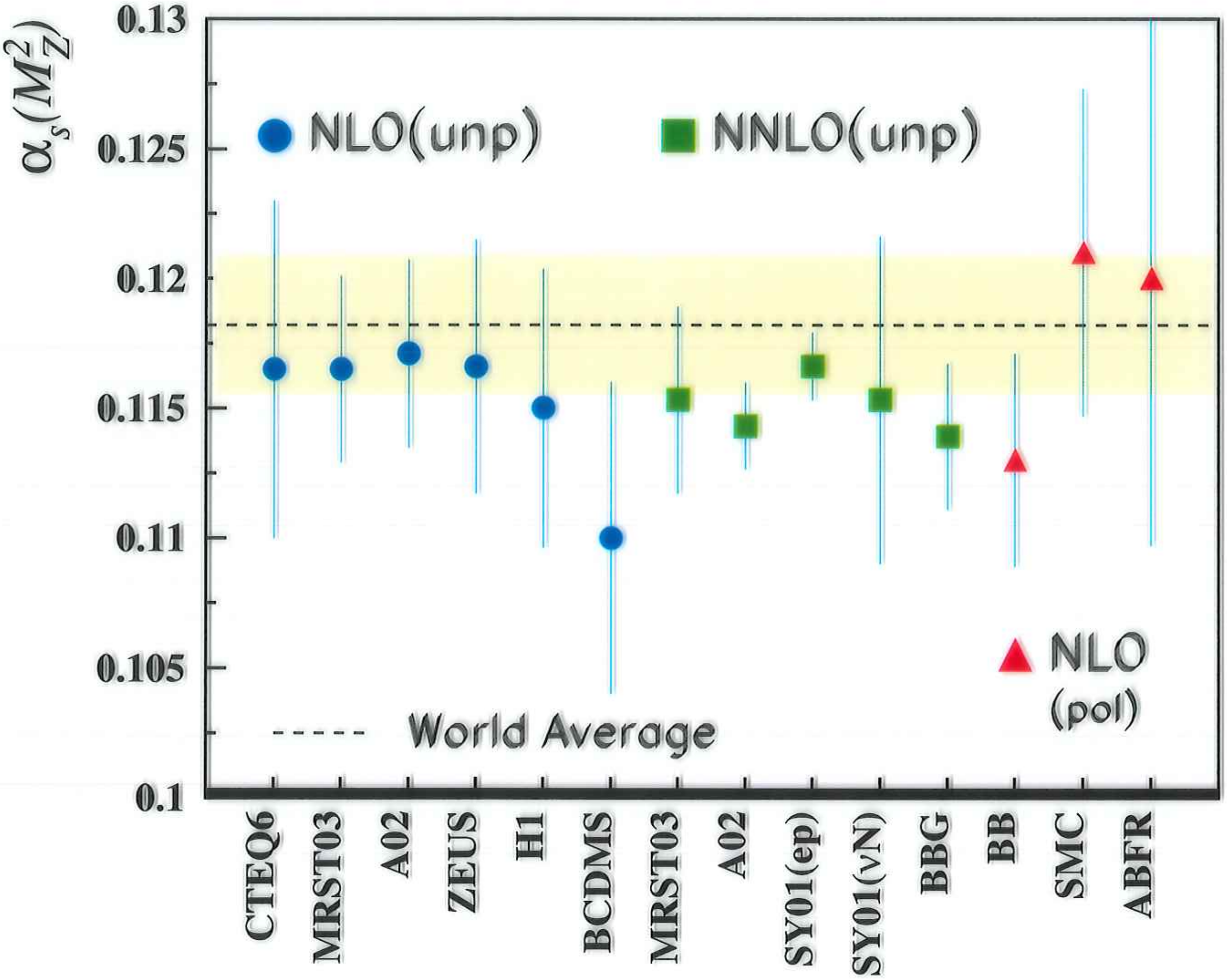
- Even higher order corrections needed ?

Higher Twist Contributions:

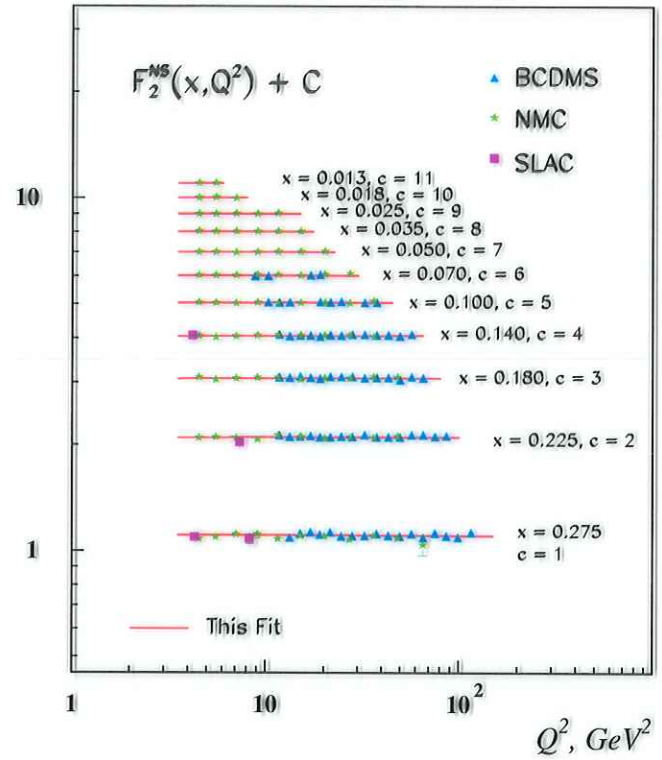
$4 < W^2 < 12.5\,\text{GeV}^2, Q^2 > 4\,\text{GeV}^2$



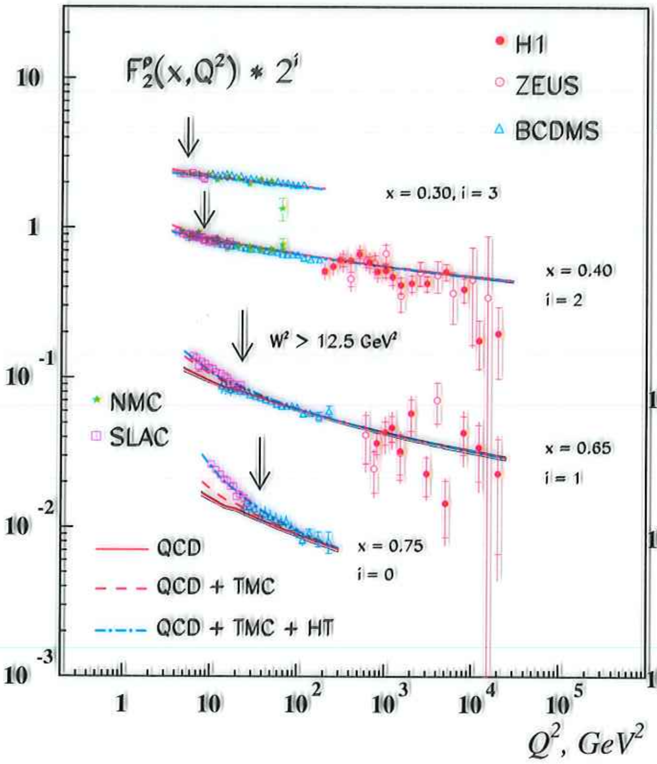
DIS: $\alpha_s(M_Z^2)$



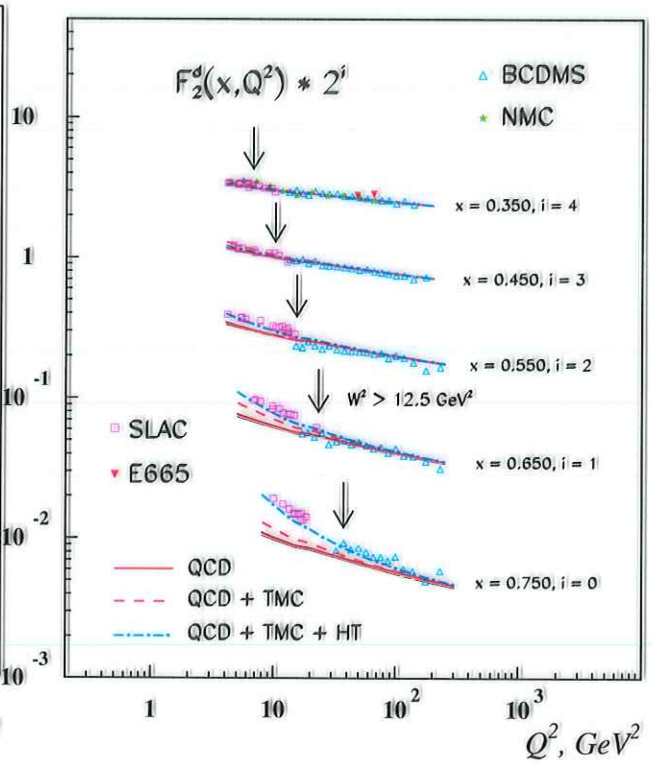
Non-Singlet 3-Loop QCD Analysis



$x > 0.3$



proton



deuteron