QCD Analysis of Polarized Deep Inelastic Scattering Data and New Polarized Parton Distributions

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OUTLINE:

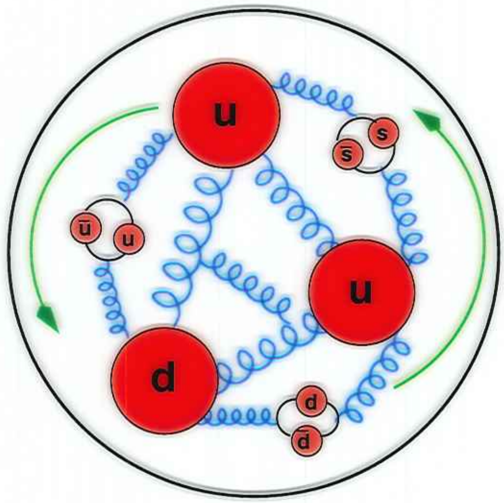
- Motivation
- QCD Analysis Formalism
- World Data
- Error Calculation
- Parton Distributions with Errors
- Λ_{QCD} and $\alpha_s(M_Z^2)$
- Factorization Scheme Invariant Evolution
- Moments - Comparison QCD with Lattice
- Conclusion

Ref.: [hep-ph/0203155], Nucl. Phys. **B** in print

NUFACT 02, LONDON , JULY 2002

Introduction: The Spin of the Proton

CONTRIBUTIONS TO THE SPIN OF THE PROTON:



$$S_z = \frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_G$$

WITH

$$\Delta\Sigma = (\Delta u + \Delta\bar{u}) + (\Delta d + \Delta\bar{d}) + (\Delta s + \Delta\bar{s})$$

$\Longrightarrow$ POLARIZED DEEP INELASTIC LEPTON NUCLEON SCATTERING (INCL. AND SEMI-INCL.)

$\Delta\Sigma$ IS FOUND TO BE SMALL IN INCLUSIVE DIS EXPERIMENTS (EMC, SMC, SLAC, HERMES)

$$\frac{1}{2}\Delta\Sigma \approx 0.1 \dots 0.2$$

THE MISSING CONTRIBUTIONS TO S_z:

- ΔG : FROM OPEN CHARM, HIGH p_T PAIRS, ... PRODUCTION, ALSO FROM QCD NLO ANALYSES OF INCL. DATA
- L_q, L_G : FROM MEASUREMENTS OF DEEP INELASTIC NON–FORWARD SCATTERING (HOPEFULLY)
  DVCS (GPDs): $J_q = \Delta\Sigma + L_q$
  $\longrightarrow$ BRIDGE TO L_q !

Table 2 : A comparison of different structure function relations (twist 2) derived in the literature with the results obtained in the local OPE. The signs in the last column mark agreement or disagreement.

	sum rule	ref.	$m_q = 0$
1	$g_4 = 2xg_5$	[10,11,16,23]	+
2	$12x[(g_1+g_2)^{ep} - (g_1+g_2)^{en}] = g_3^{\nu n} - g_3^{\nu p}$	[9,11,12]	−
3	$12x[g_2^{ep} - g_2^{en}] = (g_3 - 2g_4)^{\nu n} - (g_3 - 2g_4)^{\nu p}$		−
4	$12x(g_1^{ep} - g_1^{en}) = g_4^{\nu n} - g_4^{\nu p}$	[12]	+
5	$3\int_0^1 dx(g_1^{ep} - g_1^{en}) - \int_0^1 dx(g_1^{\nu p} + g_1^{\nu n}) = -\frac{1}{6}g_A^8$		−
6	$6\int_0^1 dx(g_2^{ep} - g_2^{en}) - \int_0^1 dx(g_1^{\nu p} - g_1^{\nu n}) = -g_A^*$		+
7	$12\int_0^1 dx(g_2^{ep} - g_2^{en}) - \int_0^1 \frac{dx}{x}(g_4^{\nu p} - g_4^{\nu n}) = -2g_A$		+
8	$12x[g_1^{ep} - g_1^{en}] = g_3^{\nu n} - g_3^{\nu p}$		−
9	$\int_0^1 dx[(g_1+g_2)^{\bar{\nu}p} - (g_1+g_2)^{\nu p}] = g_A^*$	[17]	+
10	$\int_0^1 \frac{dx}{x}[g_3^{\bar{\nu}p} + g_3^{\nu p}] = -g_A^{8*}$		+
11	$\int_0^1 dx g_2^{\gamma} = 0$	[24]	
12	$\int_0^1 dxx(g_1 + 2g_2)^{\nu p - \bar{\nu}p} = 0$	[13]	+
13	$\int_0^1 dx(g_3 - 2xg_5)^{\nu p + \bar{\nu}p} = 0$		+
14	$\int_0^1 dx(g_4 - g_3)^{\nu p + \bar{\nu}p} = 0$		+
15	$\int_0^1 dx(g_5^{\nu p} - g_5^{\nu n}) = g_A$	[9]	+

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17	$\int_0^1 dx \frac{(g_3^{\nu p} - g_3^{\nu n})}{x} = 2 g_A$		−
18	$g_4 - g_3 = 2x g_5$	[15]	−
19	$\int_0^1 dx x^n (\frac{n-7}{n+1} g_4 + 2 g_3) = 0$	[16]	−
20	$g_3 = 2x g_5$	[6]	−
21	$g_3 = g_4$		−
22	$g_2^{\gamma} = g_2^{\gamma Z} = 0$		−
23	$g_1^{W^\pm} = -2 g_2^{W^\pm}$		−
24	$\int_0^1 dx (g_3 - g_4)^{(\nu+\bar{\nu}),\gamma,Z} = 0$	[23]	
25	$\int_0^1 dx g_2^{\nu+\bar{\nu}} = 0$		
26	$\int_0^1 dx x\, [g_1 + 2 g_2]^{W^- - W^+} = 0$		+
27	$\int_0^1 dx (g_3 - 2x g_5)^{(\nu+\bar{\nu}),\gamma,Z} = 0$		+
28	$\int_0^1 dx \frac{g_3^{\nu p} - g_3^{\nu n}}{x} = 4 g_A$	this paper	+
29	$24x[(g_1 + g_2)^{ep} - (g_1 + g_2)^{en}] = g_3^{\nu n} - g_3^{\nu p}$		+
30	$24x[g_2^{ep} - g_2^{en}] = (g_3 - 2g_4)^{\nu n} - (g_3 - 2g_4)^{\nu p}$		+
31	$\int_0^1 dx (g_1^{ep} + g_1^{en}) - \frac{2}{9} \int_0^1 dx (g_1^{\nu p} + g_1^{\nu n}) = \frac{1}{18} g_A^8$		+

Motivation

WHAT IS THE NUCLEON'S SPIN MADE OFF ?

EMC (1987):

$$\sum_{i=1}^{3} [\Delta q_i + \Delta \overline{q}_i] << \frac{1}{2} \qquad \text{Today} : 0.14 \quad Q^2 = 4 \text{ GeV}^2$$

Violation of the Ellis-Jaffe sum rule :

$$\int_0^1 dx g_1^{ep(n)}(x) = \frac{g_A}{12}\left[\pm 1 + \frac{5}{3}\frac{3(F/D) - 1}{(F/D) + 1}\right]$$

The E-J sumrule is non-fundamental, since :

$$\int_0^1 dx g_1(x) = \frac{1}{6} I_3^N (F + D) + \frac{1}{36}(3F - D) + \frac{2}{9}\Gamma_0^{5,N}$$

(e.g. Van Neerven, Ravindran, 2001)

Non-conservation of the axial current, due to the ABJ-anomaly, leads to a scale dependence of $\Gamma_0^{5,N}$, which cannot be associated to a hadron quantum number.

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5	$3\int_0^1 dx(g_1^{ep} - g_1^{en}) - \int_0^1 dx(g_1^{\nu p} + g_1^{\nu n}) = -\frac{1}{6}g_A^8$		−
6	$6\int_0^1 dx(g_2^{ep} - g_2^{en}) - \int_0^1 dx(g_1^{\nu p} - g_1^{\nu n}) = -g_A^*$		+
7	$12\int_0^1 dx(g_2^{ep} - g_2^{en}) - \int_0^1 \frac{dx}{x}(g_4^{\nu p} - g_4^{\nu n}) = -2g_A$		+
8	$12x[g_1^{ep} - g_1^{en}] = g_3^{\nu n} - g_3^{\nu p}$		−
9	$\int_0^1 dx[(g_1+g_2)^{\bar{\nu}p} - (g_1+g_2)^{\nu p}] = g_A^*$	[17]	+
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31	$\int_0^1 dx (g_1^{ep} + g_1^{en}) - \frac{2}{9} \int_0^1 dx (g_1^{\nu p} + g_1^{\nu n}) = \frac{1}{18} g_A^8$		$+$

System : $g_1(x,Q^2), \partial g_1/\partial t(x,Q^2)$

Leading Order :

$$\begin{aligned}
K_{22}^{N(0)} &= 0 \\
K_{2d}^{N(0)} &= -4 \\
K_{d2}^{N(0)} &= \frac{1}{4}\left(\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)}\right) \\
K_{dd}^{N(0)} &= \gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)}
\end{aligned}$$

Next-to-Leading Order : [W. Furmanski and R. Petronzio, Z. Phys. **C 11** (1982) 293.]

$$\begin{aligned}
K_{22}^{N(1)} &= K_{2d}^{N(1)} = 0 \\
K_{d2}^{N(1)} &= \frac{1}{4}\left[\gamma_{gg}^{N(0)}\gamma_{qq}^{N(1)} + \gamma_{gg}^{N(1)}\gamma_{qq}^{N(0)} - \gamma_{qg}^{N(1)}\gamma_{gq}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(1)}\right] \\
&\quad -\frac{\beta_1}{2\beta_0}\left(\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} - \gamma_{gq}^{N(0)}\gamma_{qg}^{N(0)}\right) \\
&\quad +\frac{\beta_0}{2}C_{2,q}^{N(1)}\left(\gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)} - 2\beta_0\right) \\
&\quad -\frac{\beta_0}{2}\frac{C_{2,g}^{N(1)}}{\gamma_{qg}^{N(0)}}\left[\gamma_{qq}^{N(0)\,2} - \gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} + 2\gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)} - 2\beta_0\gamma_{qq}^{N(0)}\right] \\
&\quad -\frac{\beta_0}{2}\left(\gamma_{qq}^{N(1)} - \frac{\gamma_{qq}^{N(0)}\gamma_{qg}^{N(1)}}{\gamma_{qg}^{N(0)}}\right) \\
K_{dd}^{N(1)} &= \gamma_{qq}^{N(1)} + \gamma_{gg}^{N(1)} - \frac{\beta_1}{\beta_0}\left(\gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)}\right) + 4\beta_0 C_{2,q}^{N(1)} - 2\beta_1 \\
&\quad -\frac{2\beta_0}{\gamma_{qg}^{N(0)}}\left[C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} - 2\beta_0\right) - \gamma_{qg}^{N(1)}\right]
\end{aligned}$$

'Prediction' of Moments

		QCD Scenario 1	
	n	value at $Q^2 = 4\,GeV^2$	value out of measured range
Δu	-1	0.851 ± 0.075	0.152|4E−4
	0	0.160 ± 0.014	8E−4|3E−4
	1	0.055 ± 0.006	1E−5|3E−4
	2	0.024 ± 0.003	0|3E−4
Δd	-1	-0.415 ± 0.124	-0.144|-7E−5
	0	-0.050 ± 0.022	-7E−4|-6E−5
	1	-0.015 ± 0.009	-1E−5|-5E−5
	2	-0.006 ± 0.005	0|-5E−5
$\Delta \bar{q}$	-1	-0.074 ± 0.017	-0.04|0
	0	-0.003 ± 0.001	-2E−4|0
	1	−4E−4 ± 1E−4	0|0
	2	−8E−5 ± 2E−5	0|0
ΔG	-1	1.026 ± 0.549	0.04|1E−5
	0	0.184 ± 0.103	5E−4|1E−5
	1	0.050 ± 0.028	1E−5|1E−5
	2	0.017 ± 0.010	0|1E−5

7+1 parameter NLO fit: $\Lambda_{QCD}^{(4)} \Rightarrow \alpha_s(M_Z^2)$

$\Lambda_{QCD}^{(4)}$ [Gev]	Scenario 1		Scenario 2	
	value	error	value	error
FS/RS=1.0/1.0	0.235	± 0.053	0.240	± 0.060
FS/RS=0.5/1.0	0.188	− 0.047	0.195	− 0.045
FS/RS=2.0/1.0	0.296	+ 0.061	0.298	+ 0.058
FS/RS=1.0/0.5	0.349	+ 0.114	0.363	+ 0.123
FS/RS=1.0/2.0	0.174	− 0.061	0.174	− 0.066

- Sc. 1:

$$\alpha_s(M_Z^2) = 0.113 \begin{array}{c}+0.004\\-0.004\end{array} \begin{array}{c}+0.004\\-0.004\end{array} \begin{array}{c}+0.008\\-0.005\end{array}$$

(fit) (fac) (ren)

- Sc. 2:

$$\alpha_s(M_Z^2) = 0.114 \begin{array}{c}+0.004\\-0.005\end{array} \begin{array}{c}+0.004\\-0.004\end{array} \begin{array}{c}+0.008\\-0.006\end{array}$$

- SMC: $0.121 \pm 0.002(stat) \pm 0.006(syst + theor)$

E154: $0.108 - 0.116 \quad (bad\, for \geq 0.120)$

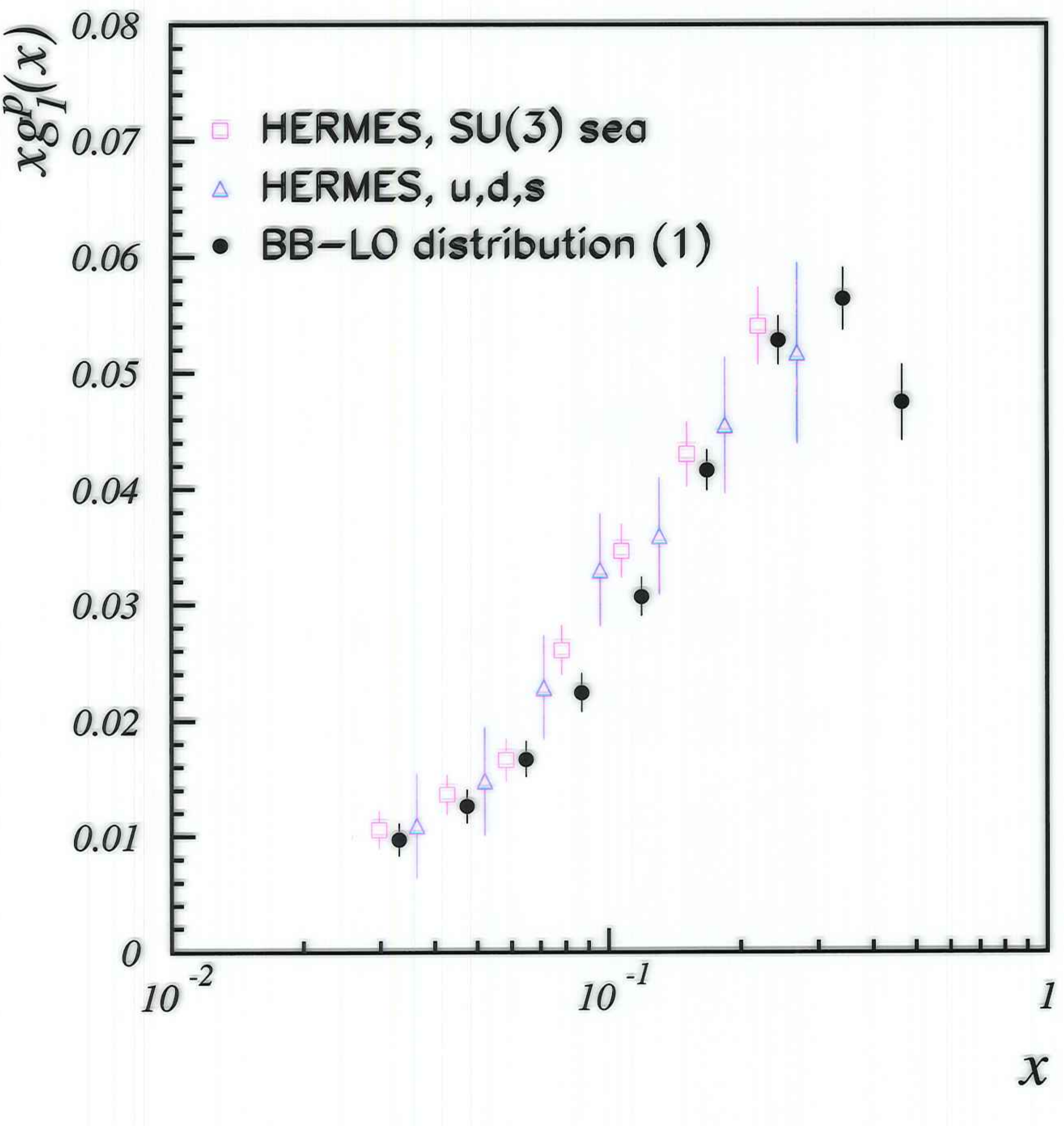
ABFR: $0.120 \begin{array}{c}+0.004\\-0.005\end{array} (exp) \begin{array}{c}+0.009\\-0.006\end{array} (theor)$

⇒ world average (PDG): 0.118 ± 0.002

⇒ H1 + BCDMS data: [Eur.Phys.J. **C21**(2001)33]

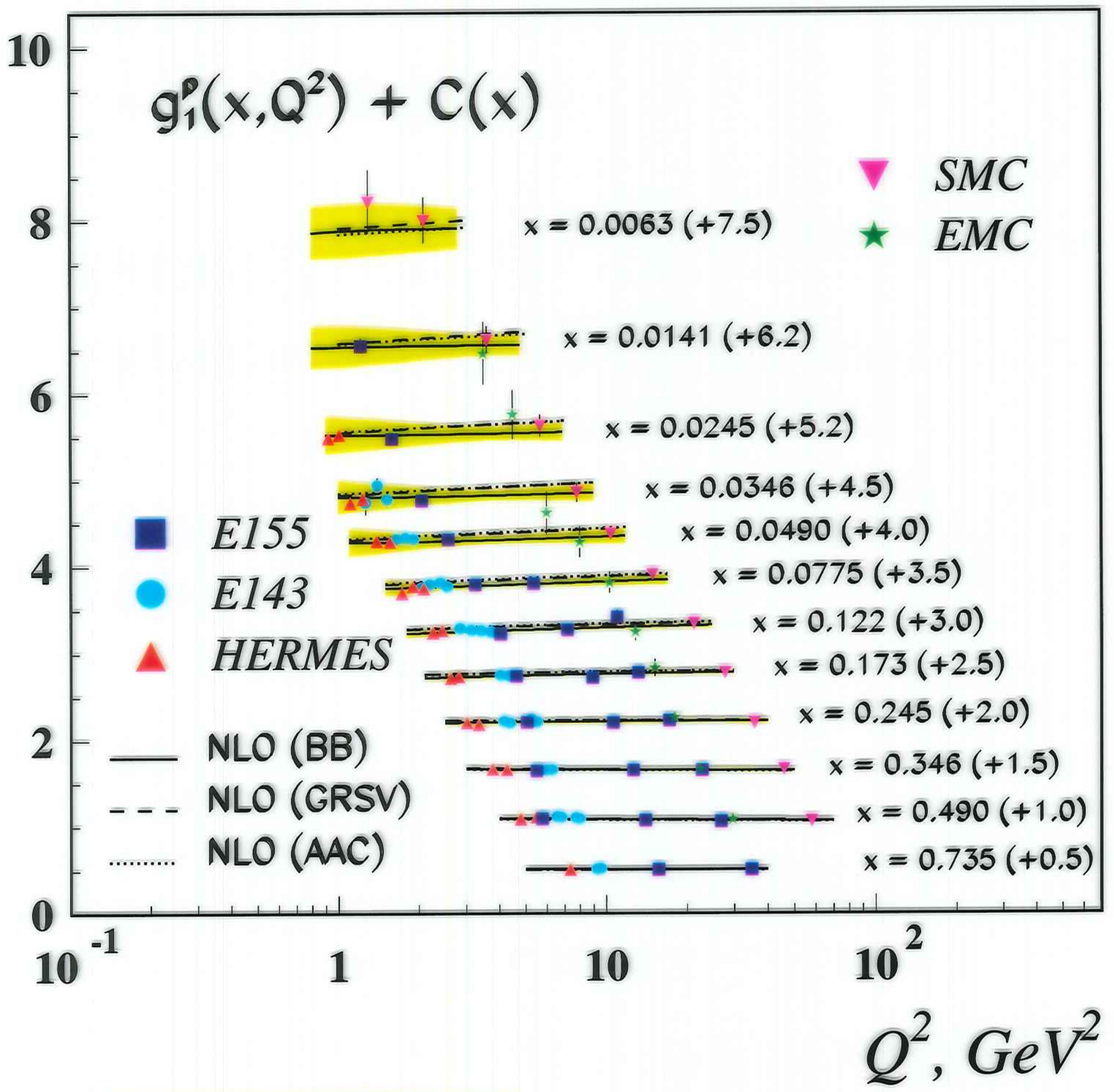
$0.1150 \pm 0.0017(exp) \begin{array}{c}+0.0009\\-0.0005\end{array} (model) \pm 0.0050(thy)$

Comparison with Δq from Semi-Incl. Data



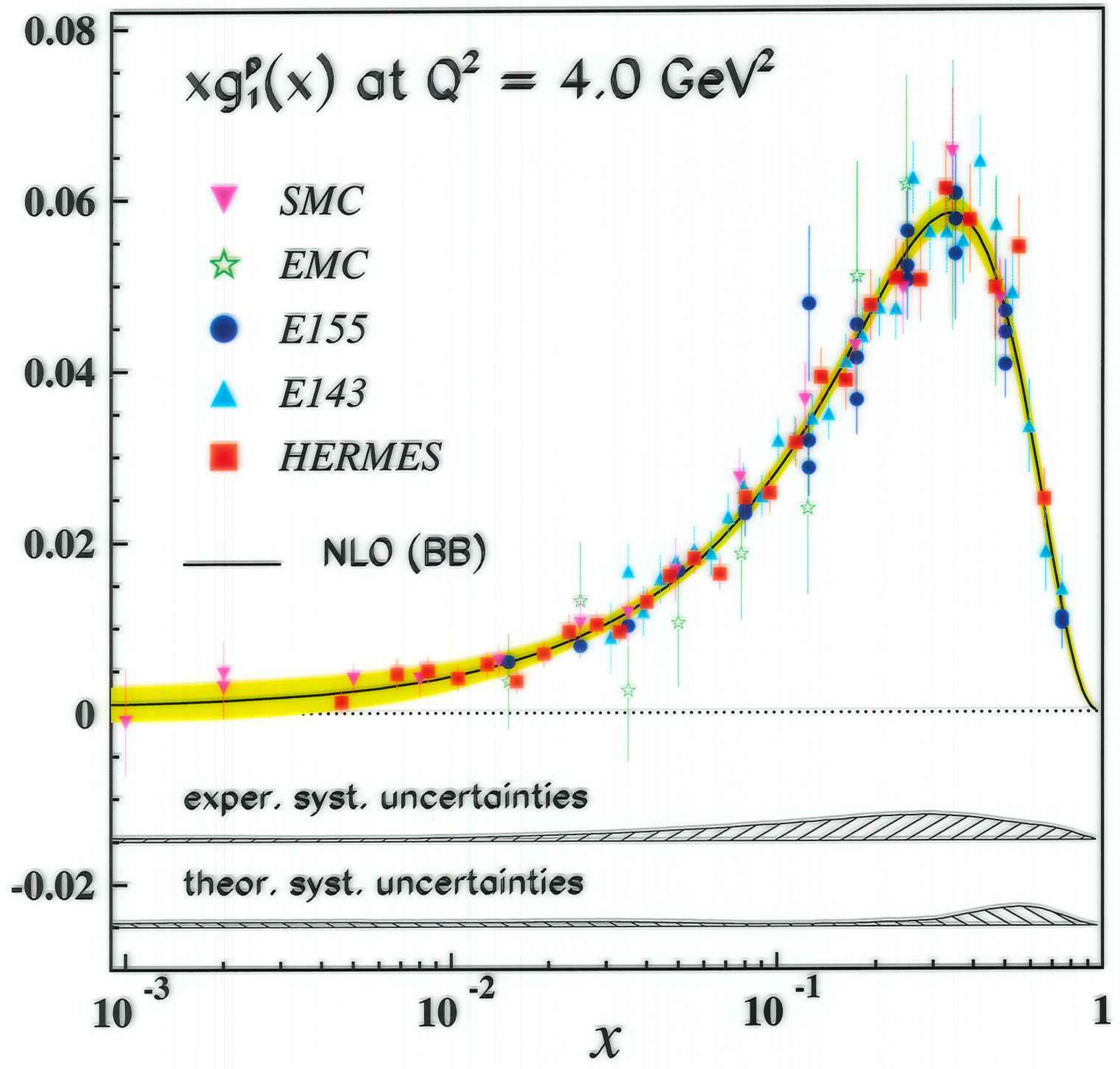
⟹ z–range in the Semi–Incl. Analysis: $0.2 < z < 0.7$

$g_1^p(x)$ versus Q^2



⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

$xg1p(x)$ with error bands



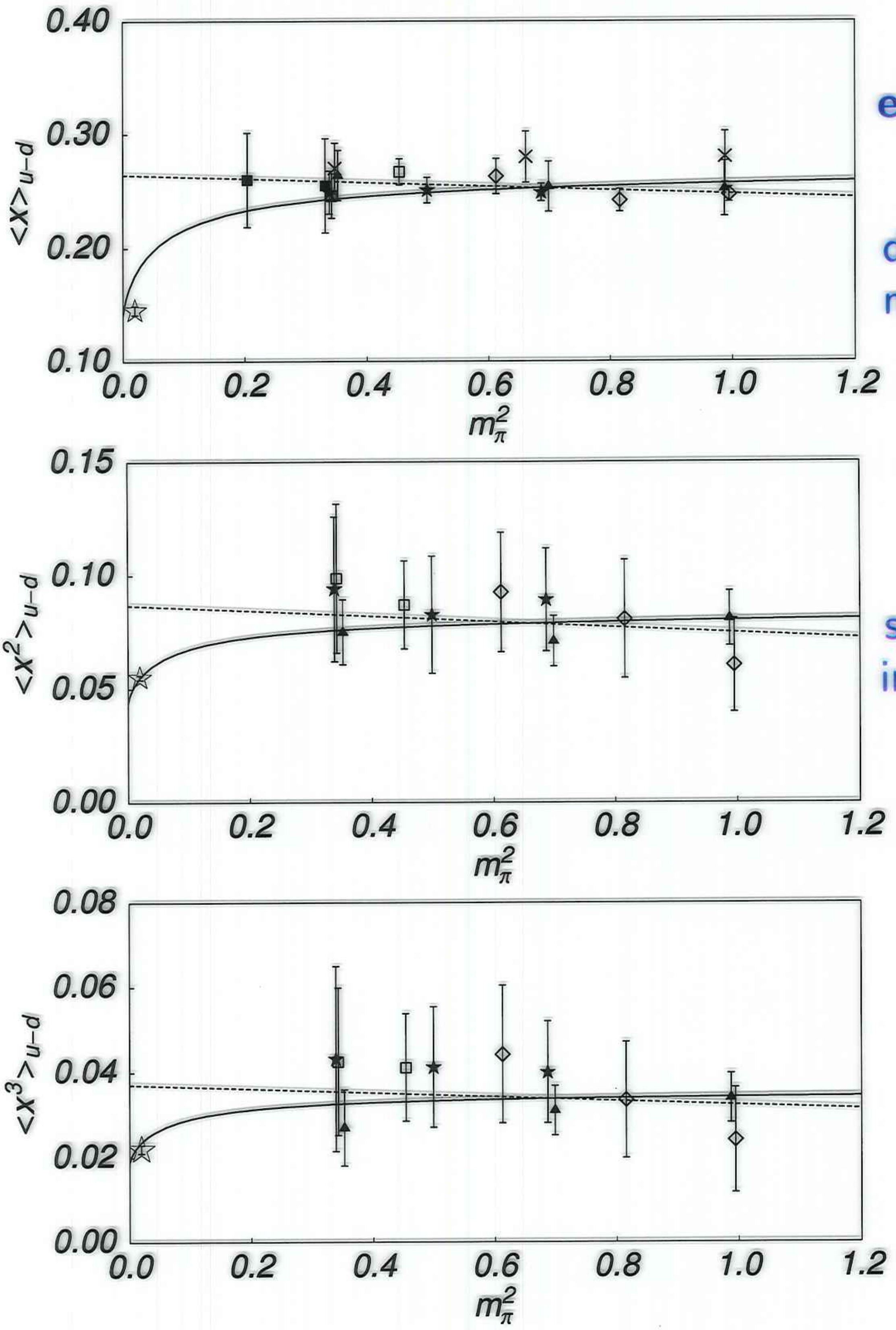
⇒ Yellow error band: Fully correlated 1σ statistical error band at the input scale $Q_0^2 = 4.0\,GeV^2$.

Comparison of Moments (2)

	BB Scenario 1	AAC	GRSV	ABFR
Δu_v	0.926 ± 0.071	0.921	0.928	$\eta_u =$
Δd_v	-0.341 ± 0.123	-0.341	-0.342	0.692
Δu	0.851 ± 0.075	0.859	0.840	$\eta_d =$
Δd	-0.415 ± 0.124	-0.404	-0.430	-0.418
$\Delta \overline{q}$	-0.074 ± 0.017	-0.063	-0.088	
ΔG	1.026 ± 0.549	0.683	0.808	1.262

Comparison of the first moments of the polarized parton densities in NLO in the $\overline{\mathrm{MS}}$ scheme at $Q^2 = 4$ GeV^2 for different sets of recent parton parameterizations. For the ABFR-analysis the values $\eta_{u,d}$ are the first moments of $\Delta u + \Delta \overline{u}$ and $\Delta d + \Delta \overline{d}$, respectively, and $\Delta s + \Delta \overline{s} = -0.081$.

Lattice: The lowest moments of $u - d$



extrapolations:

dashed:
naive linear

solid:
improved chiral

Ref.: M. Detmold, W. Melnitchouk, A.W. Thomas; hep-lat/0206001.

Fac. Scheme Invariant Combinations

- Instead of PROCESS-INDEPENDENT SCHEME-DEPENDENT Evolution Equations for PARTONS one may think of PROCESS-DEPENDENT SCHEME-INDEPENDENT Evolution Equations for OBSERVABLES, F_A, F_B.

  ⇒ The input densities are measured! Control over the input directly.

  ⇒ No ΔG–Ansatz necessary.

  ⇒ A one parameter fit only – Λ_{QCD}.

Evolution Equations : [J. Blümlein, V. Ravindran, and W. L. van Neerven, Nucl. Phys **B586** (2000) 349.]

$$\frac{\partial}{\partial t}\begin{pmatrix} F_A^N \\ F_B^N \end{pmatrix} = -\frac{1}{4}\begin{pmatrix} K_{AA}^N & K_{AB}^N \\ K_{BA}^N & K_{BB}^N \end{pmatrix}\begin{pmatrix} F_A^N \\ F_B^N \end{pmatrix}$$

evolution variable :

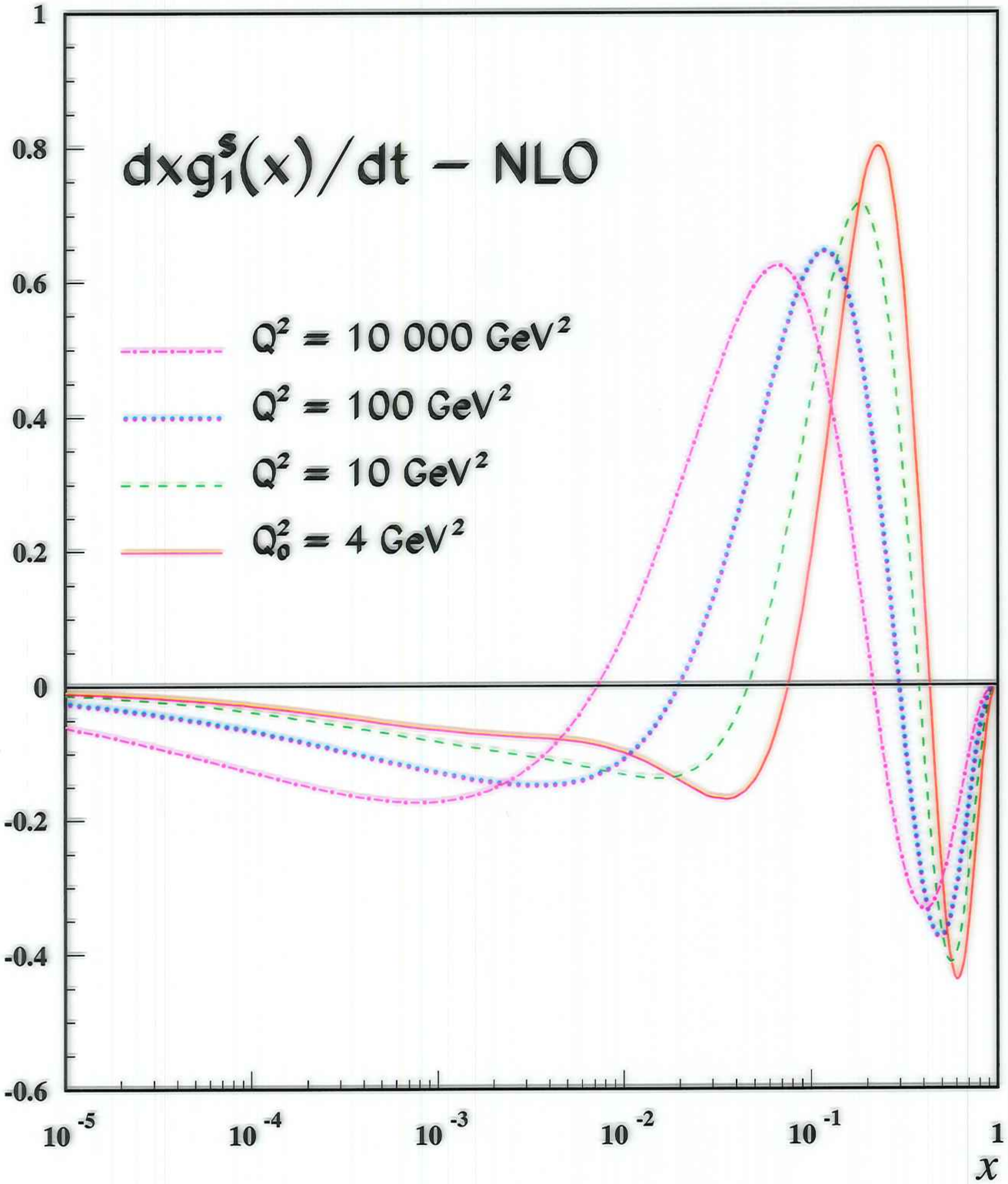
$$t = -\frac{2}{\beta_0}\log\left(\frac{a_s(Q^2)}{a_s(Q_0^2)}\right)$$

⟹ The evolution kernels K_{IJ}^N are also Physical Quantities! The Factorization Scheme Independence holds order by order.

The Renormalization Scale Dependence disappears only with more higher orders.

⟹ A possible choice: $F_A = g_1$ and $F_B = \partial g_1/\partial t$.

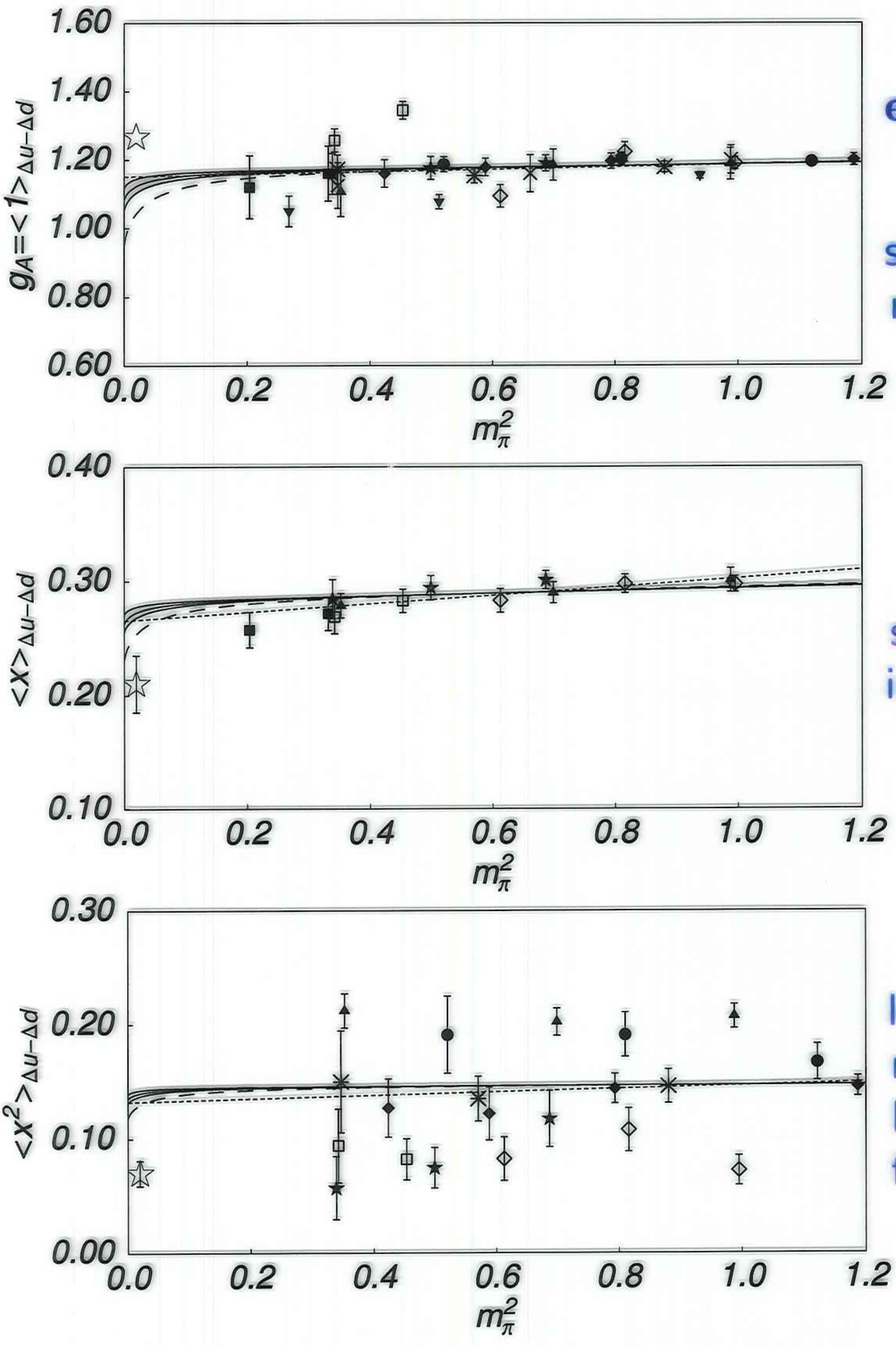
$\partial xg_1^S/\partial t(x,Q^2)$ and shift of $\Lambda_{QCD}^{(4)}$



$Sc1:\ \Lambda_{QCD}^{(4)}: 0.235 \rightarrow 0.223\ ,\ \alpha_s(M_Z^2): 0.113 \rightarrow 0.112$

$Sc2:\ \Lambda_{QCD}^{(4)}: 0.240 \rightarrow 0.228\ ,\ \alpha_s(M_Z^2): 0.114 \rightarrow 0.113$

Lattice: The lowest moments of $\Delta u - \Delta d$



extrapolations:

short–dashed:
naive linear

solid:
improved chiral

long-dashed:
no Δ and
LNA coeff.
from χPT

Ref.: M. Detmold, W. Melnitchouk, A.W. Thomas; hep-lat/0206001.

Conclusions

- A LO and NLO QCD Analysis of the current World-Data of Polarized Structure Functions was performed.

- New parametrizations of the parton densities including their fully correlated 1σ Error Bands were derived. They are available via a fast **FORTRAN** program for the range:

$$1\ GeV^2 < Q^2 < 10^6\ GeV^2 \text{ and } 10^{-9} < x < 1.$$

- The following values for $\alpha_s(M_Z^2)$ were obtained:

  - Scenario 1:

$$\alpha_s(M_Z^2) = 0.113 \begin{array}{c} +0.004 \\ -0.004 \end{array} \text{(fit)} \begin{array}{c} +0.004 \\ -0.004 \end{array} \text{(fac)} \begin{array}{c} +0.008 \\ -0.005 \end{array} \text{(ren)}\ ,$$

  - Scenario 2:

$$\alpha_s(M_Z^2) = 0.114 \begin{array}{c} +0.004 \\ -0.005 \end{array} \text{(fit)} \begin{array}{c} +0.004 \\ -0.004 \end{array} \text{(fac)} \begin{array}{c} +0.008 \\ -0.006 \end{array} \text{(ren)}\ ,$$

compatible with results from other QCD analyses and with the world average.

Conclusions (cont'd)

- First steps in a factor. scheme invariant QCD evolution based on the structure function $g_1(x, Q^2)$ and $\partial g_1(x, Q^2)/\partial \log Q^2$ were performed yielding similar results for $\alpha_s(M_Z^2)$.

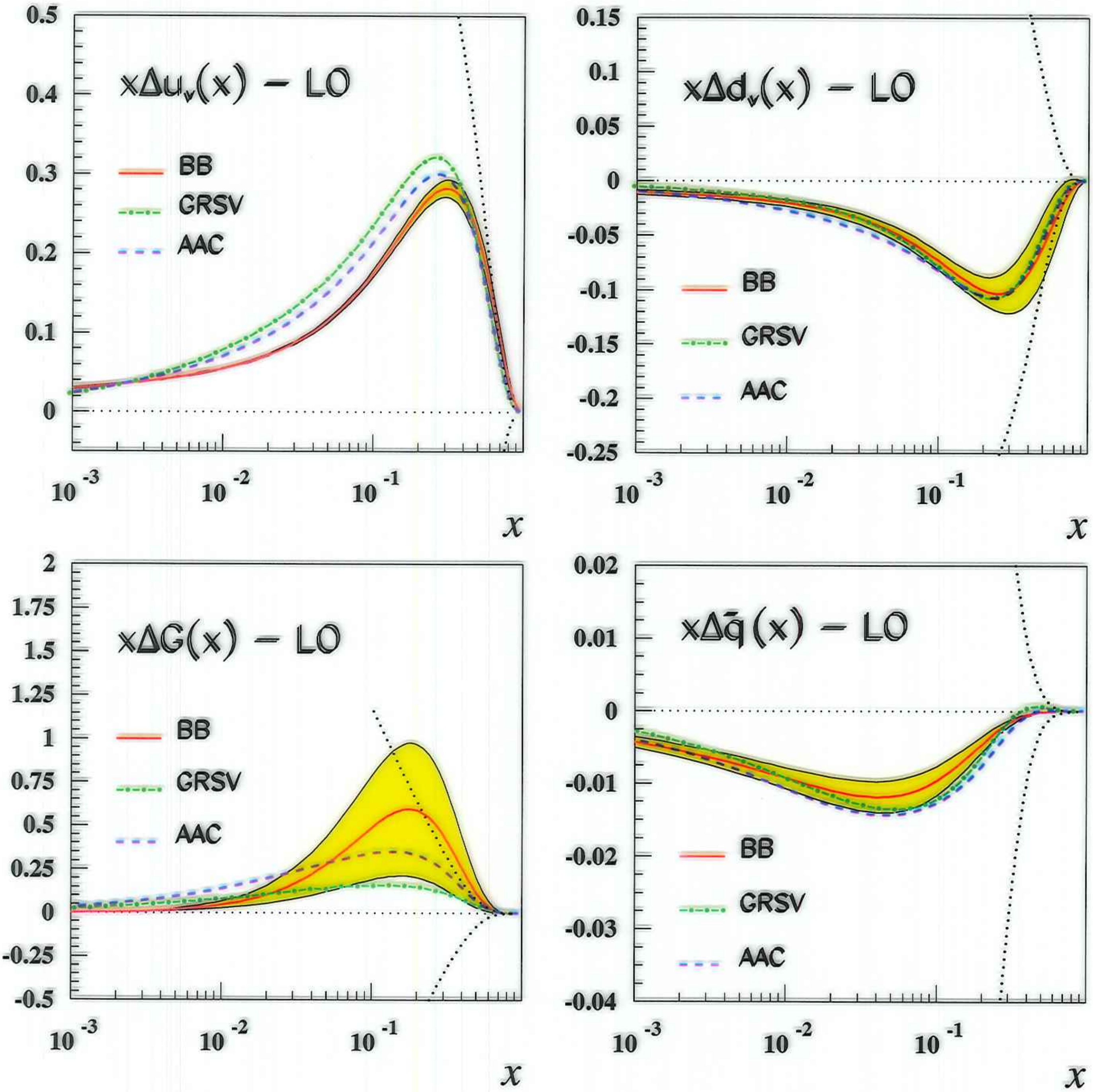
  Such an analysis is a very promising way to proceed in the future, since it allows to extract Λ_{QCD} fixing all the input distributions by direct measurement.

- Comparing the lowest moments with values from Lattice simulations the errors improved during recent years and the values became closer. The chiral extrapolation $m_\pi^2 \to 0$ seems to be flat. However, more work has yet to be done in the future on systematic effects and even more precise experimental data are welcome to improve precision.

- The evanescent spin puzzle lead to both a much deeper experimental and theoretical understanding of the nucleon at short distances, and, hopefully will in the future.

Pol. Parton Densities at $Q_0^2 = 4.0\,GeV^2$

- 7+1 Parameter Fit based on the Asymmetry Data:



⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation at the input scale Q_0^2.

⇒ Dark dotted line: Unpolarized Parton Distribution ('Positivity Limit') taken from GRV.

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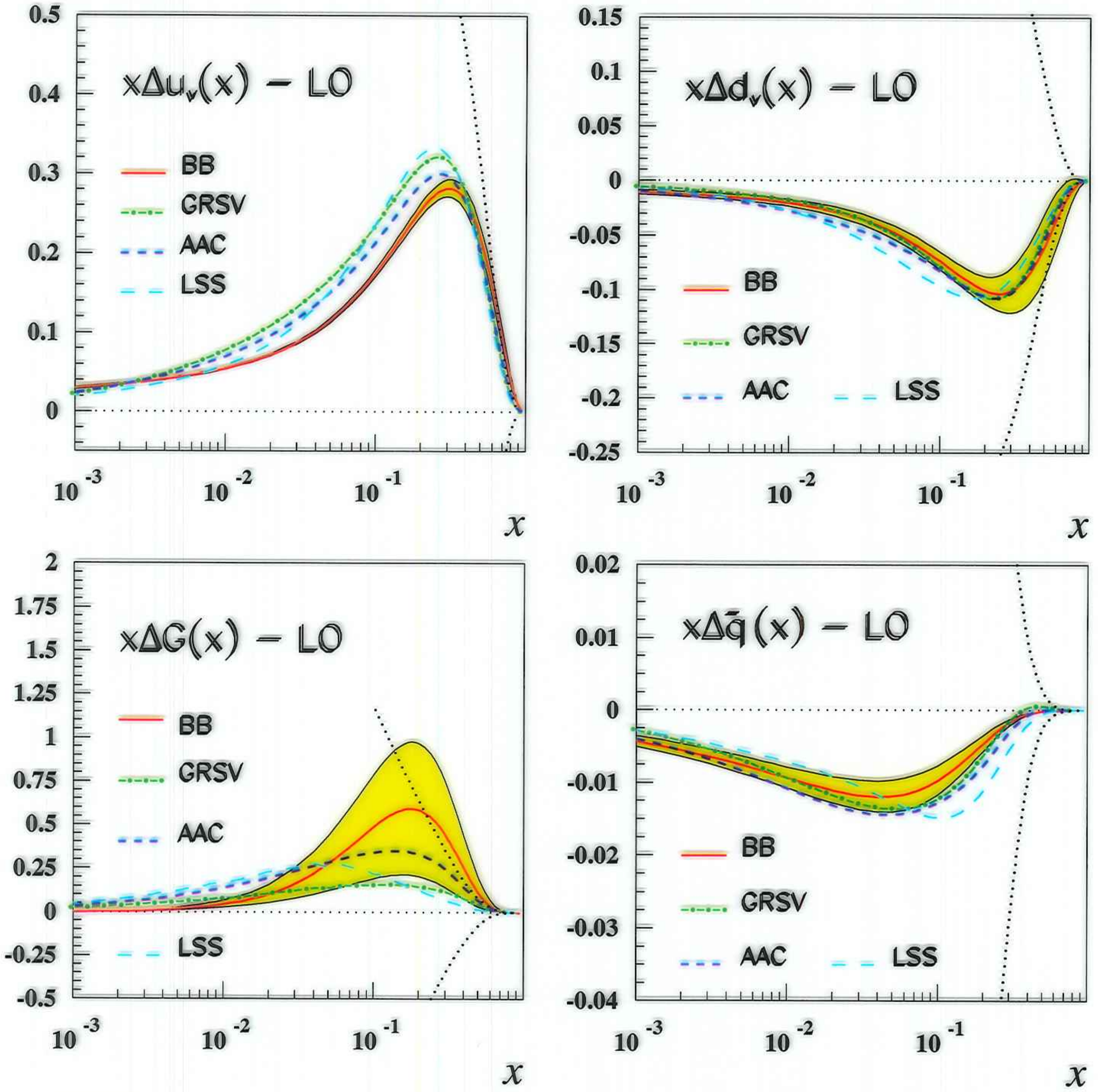

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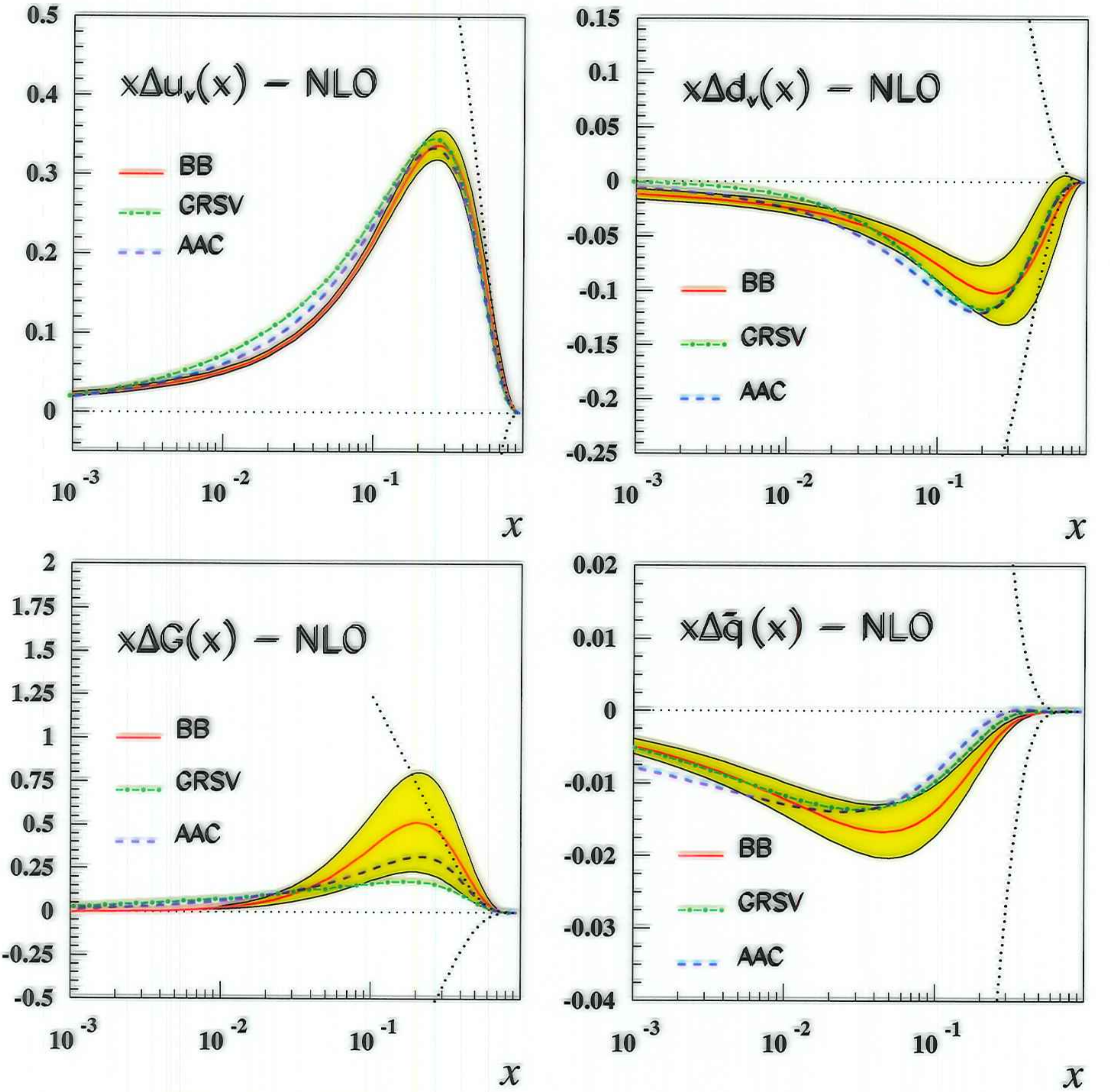
Pol. Parton Densities at $Q_0^2 = 4.0\,GeV^2$

- 7+1 Parameter Fit based on the Asymmetry Data:

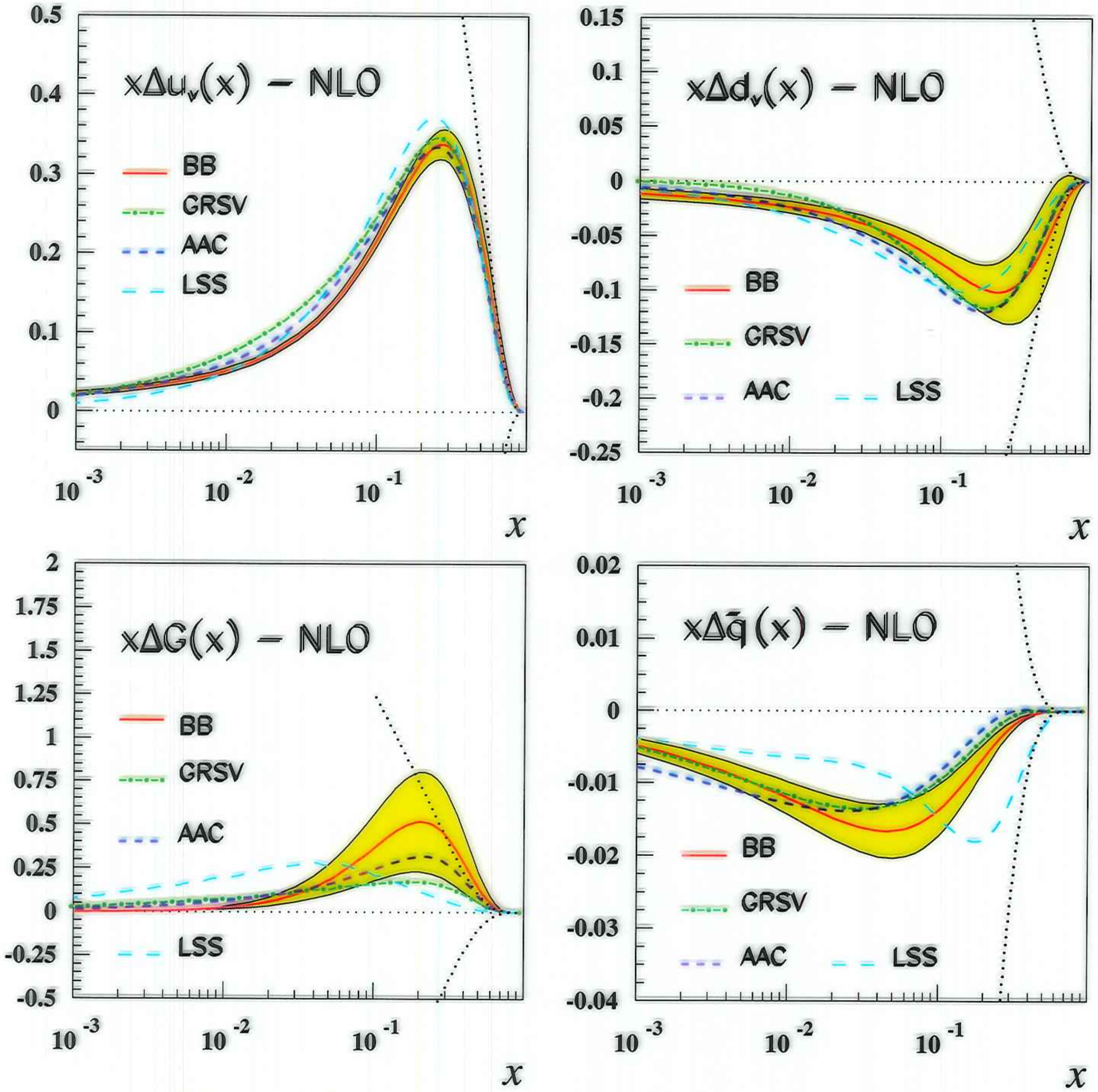


⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation at the input scale Q_0^2.

⇒ Dark dotted line: Unpolarized Parton Distribution ('Positivity Limit') taken from GRV.

Pol. Parton Densities at $Q_0^2 = 4.0\,GeV^2$

- 7+1 Parameter Fit based on the Asymmetry Data:

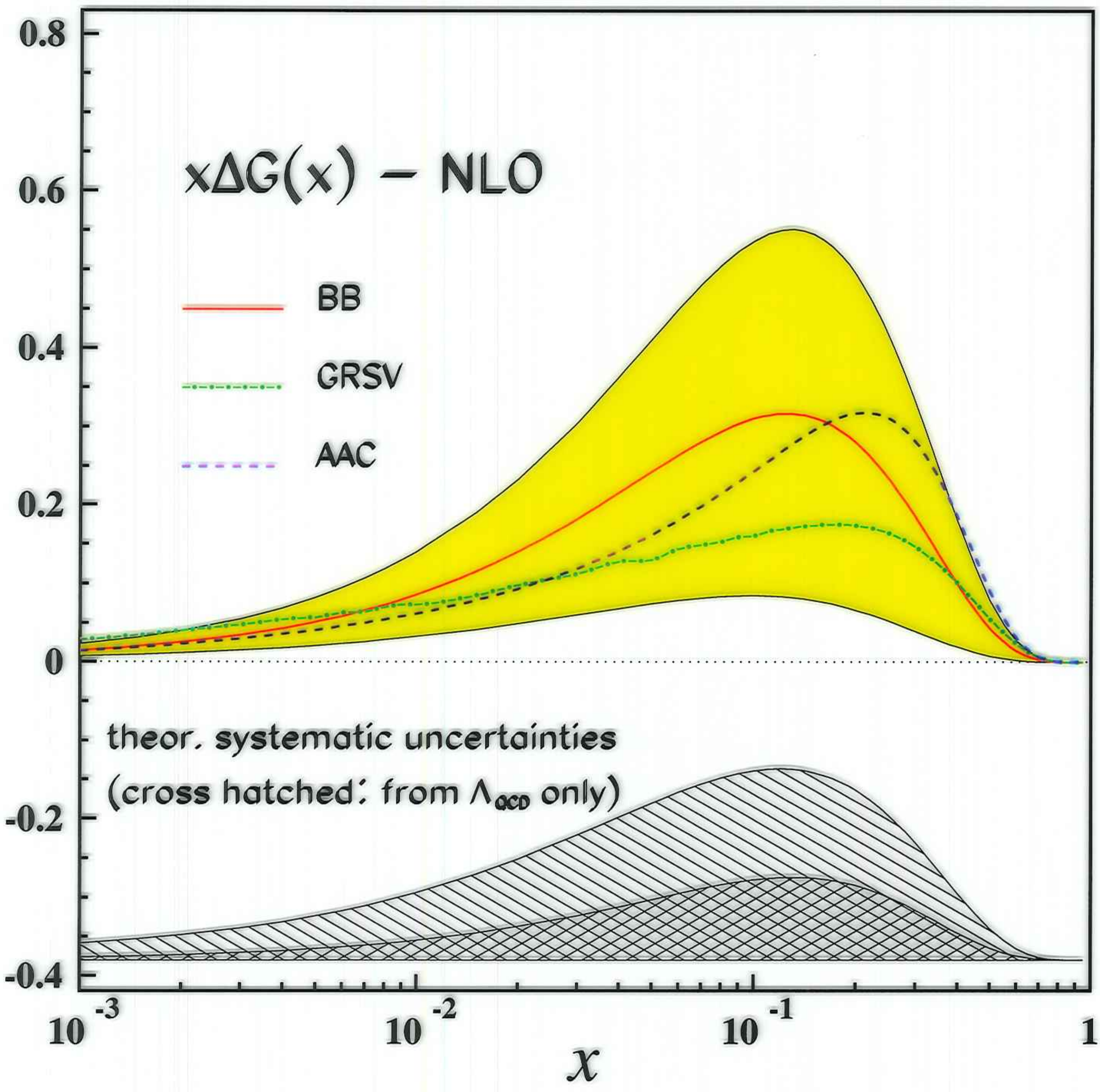


⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation at the input scale Q_0^2.

⇒ Dark dotted line: Unpolarized Parton Distribution ('Positivity Limit') taken from GRV.

Pol. Parton Densities at $Q_0^2 = 4.0\,GeV^2$
Scenario 2

- Gluon density of the present analysis.



⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation at the input scale Q_0^2.