Teupitz,
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Leptoquarks at e^+e^- Colliders

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1. **Classification of Leptoquark States**
2. **e^+e^- Annihilation**

   2.1. **QED Corrections & Beamstrahlung**

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3. **$e\gamma$ Scattering**
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1. Classification of Leptoquark States

- B and L conserving
- family–diagonal
- $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant couplings

BUCHMÜLLER, RÜCKL, WYLER 1987

leptoquark (Φ)	spin	F	colour	T_3	Q_{em}	$\lambda_L(lq)$	$\lambda_R(lq)$	$\lambda_L(\nu q)$
S_1	0	-2	$\bar{3}$	0	1/3	g_{1L}	g_{1R}	$-g_{1L}$
$\tilde{S}_1$	0	-2	$\bar{3}$	0	4/3	0	$\tilde{g}_{1R}$	0
$\vec{S}_3$	0	-2	$\bar{3}$	$+1$	4/3	$-\sqrt{2}g_{3L}$	0	0
				0	1/3	$-g_{3L}$	0	$-g_{3L}$
				-1	$-2/3$	0	0	$\sqrt{2}g_{3L}$
R_2	0	0	3	1/2	5/3	h_{2L}	h_{2R}	0
				$-1/2$	2/3	0	$-h_{2R}$	h_{2L}
$\tilde{R}_2$	0	0	3	1/2	2/3	$\tilde{h}_{2L}$	0	0
				$-1/2$	$-1/3$	0	0	$\tilde{h}_{2L}$
$V_{2\mu}$	1	-2	$\bar{3}$	1/2	4/3	g_{2L}	g_{2R}	0
				$-1/2$	1/3	0	g_{2R}	g_{2L}
$\tilde{V}_{2\mu}$	1	-2	$\bar{3}$	1/2	1/3	$\tilde{g}_{2L}$	0	0
				$-1/2$	$-2/3$	0	0	$\tilde{g}_{2L}$
$U_{1\mu}$	1	0	3	0	2/3	h_{1L}	h_{1R}	h_{1L}
$\tilde{U}_{1\mu}$	1	0	3	0	5/3	0	$\tilde{h}_{1R}$	0
$\vec{U}_{3\mu}$	1	0	3	$+1$	5/3	$\sqrt{2}h_{3L}$	0	0
				0	2/3	$-h_{3L}$	0	h_{3L}
				-1	$-1/3$	0	0	$\sqrt{2}h_{3L}$

9 + 9 STATES

$$\mathcal{L} = \mathcal{L}^f_{|F|=2} + \mathcal{L}^f_{F=0} + \mathcal{L}^{\gamma,Z,g}$$

$$\mathcal{L}^{\gamma,Z,g} = \sum_{scalars} \left[(D^\mu \Phi)^\dagger (D_\mu \Phi) - M^2 \Phi^\dagger \Phi \right] + \sum_{vectors} \left[-\frac{1}{2} G^\dagger_{\mu\nu} G^{\mu\nu} + M^2 \Phi^{\mu\dagger} \Phi_\mu \right]$$

$$D_\mu = \partial_\mu - ieQ^\gamma A_\mu - ieQ^Z Z_\mu - ig_s \frac{\lambda_a}{2} \mathcal{A}^a_\mu$$

$$\begin{aligned} \mathcal{L}^f_{F=0} &= (h_{2L} \bar{u}_R l_L + h_{2R} \bar{q}_L i\tau_2 e_R) R_2 + \tilde{h}_{2L} \bar{d}_R l_L \tilde{R}_2 \\ &+ (h_{1L} \bar{q}_L \gamma^\mu l_L + h_{1R} \bar{d}_R \gamma^\mu e_R) U_{1\mu} \\ &+ \tilde{h}_{1R} \bar{u}_R \gamma^\mu e_R \tilde{U}_{1\mu} + h_{3L} \bar{q}_L \vec{\tau} \gamma^\mu l_L \vec{U}_{3\mu} + h.c. \end{aligned}$$

$$\begin{aligned} \mathcal{L}^f_{|F|=2} &= (g_{1L} \bar{q}^c_L i\tau_2 l_L + g_{1R} \bar{u}^c_R e_R) S_1 \\ &+ \tilde{g}_{1R} \bar{d}^c_R e_R \tilde{S}_1 + g_{3L} \bar{q}^c_L i\tau_2 \vec{\tau} l_L \vec{S}_3 \\ &+ (g_{2L} \bar{d}^c_R \gamma^\mu l_L + g_{2R} \bar{q}^c_L \gamma^\mu e_R) V_{2\mu} \\ &+ \tilde{g}_{2L} \bar{u}^c_R \gamma^\mu l_L \tilde{V}_{2\mu} + h.c., \end{aligned}$$

Decay Pattern for Pair Production

JB, RŮCKL 93

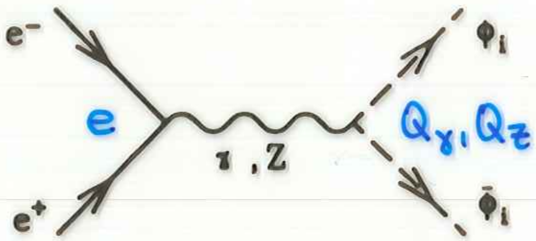
states	$l^+l^- + 2jets$	$l\nu + 2jets$	$\nu\overline{\nu} + 2jets$
S_1 U_1	$\frac{4}{9}$ 1 $\frac{1}{4}$	$\frac{4}{9}$ 0 $\frac{1}{2}$	$\frac{1}{9}$ 0 $\frac{1}{4}$
$R_2^{2/3}$ $V_2^{1/3}$	$\frac{1}{4}$ 1 0	$\frac{1}{2}$ 0 0	$\frac{1}{4}$ 0 1
$S_3^{1/3}$ $U_3^{2/3}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
$\tilde{S}_1$ $S_3^{4/3}$ $R_2^{5/3}$ $\tilde{R}_2^{2/3}$ $V_2^{4/3}$ $\tilde{V}_2^{1/3}$ $\tilde{U}_1$ $U_3^{5/3}$	1	0	0
$S_3^{-2/3}$ $\tilde{R}_2^{-1/3}$ $\tilde{V}_2^{-2/3}$ $U_3^{-1/3}$	0	0	1

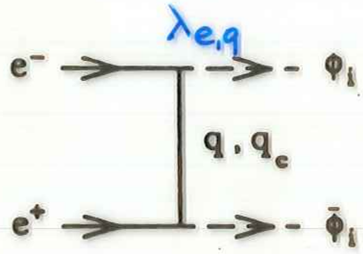
Table 3: Branching ratios for final states arrising from the decays of leptoquarks associated with the first ($l = e$) and second ($l = \mu$) family. The sequence of branching fractions given in the second and third row refers to the assumptions $\lambda_L = \lambda_R, \lambda_L = 0$, and $\lambda_R = 0$, respectively.

2. e^+e^- Annihilation

1) PAIR PRODUCTION:

BORN:





JB, RÜCKL

S: HEWETT, RIZZO
SCHAILE, ZERWAS

s   s - t   t

$$\sigma_{scalar}(s) = \frac{\pi\alpha^2\beta^3}{2s}\sum_{a=L,R}\left\{|\kappa_a(s)|^2 + \left(\frac{\lambda_a}{e}\right)^2 Re[\kappa_a(s)]F_1(\beta) + \left(\frac{\lambda_a}{e}\right)^4 F_2(\beta)\right\}$$

$$\sigma_{vector}(s) = \frac{\pi\alpha^2\beta}{2M_\Phi^2}\sum_{a=L,R}\left\{|\kappa_a(s)|^2\tilde{F}_1(\beta) + \left(\frac{\lambda_a}{e}\right)^2 Re[\kappa_a(s)]\tilde{F}_2(\beta) + \left(\frac{\lambda_a}{e}\right)^4 \tilde{F}_3(\beta)\right\}$$

(M.COUPL.) (γ, Z)

SCALAR:

$$F_1(\beta) = \frac{3}{2}\left(\frac{1+\beta^2}{\beta^2} - \frac{(1-\beta^2)^2}{2\beta^3}\ln\frac{1+\beta}{1-\beta}\right)$$

$$F_2(\beta) = 3\left(-\frac{1}{\beta^2} + \frac{1+\beta^2}{2\beta^3}\ln\frac{1+\beta}{1-\beta}\right)$$

VECTOR:

$$\tilde{F}_1(\beta) = \beta^2\left(\frac{7-3\beta^2}{4}\right)$$

$$\tilde{F}_2(\beta) = \frac{15}{4} + 2\beta^2 - \frac{3}{4}\beta^4 - \frac{3}{8\beta}(1-\beta^2)^2(5-\beta^2)\ln\frac{1+\beta}{1-\beta}$$

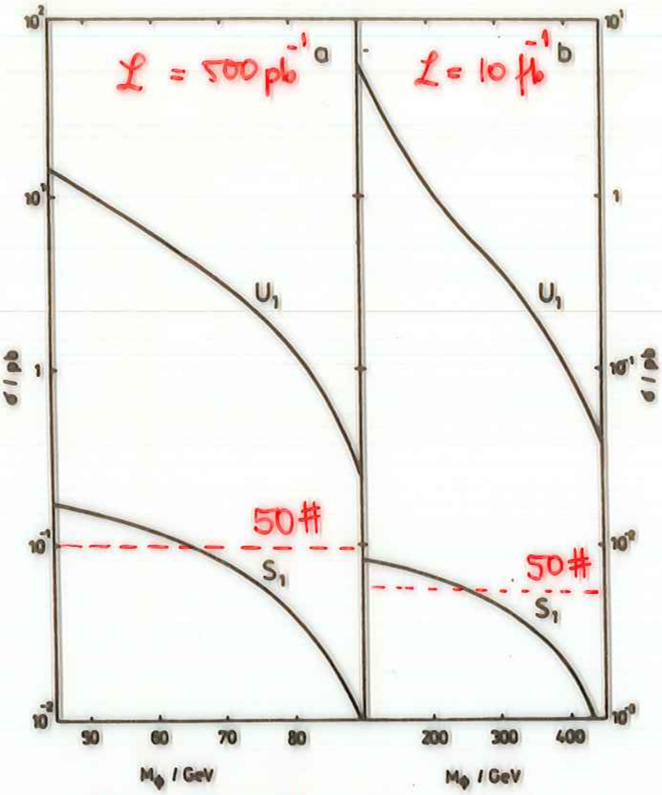
$|t|^2 \infty$

$$\tilde{F}_3(\beta) = 3(1+\beta^2) + \frac{\beta^2}{4}\frac{s}{M_\Phi^2} + \frac{3}{2\beta}(1-\beta^4)\ln\frac{1+\beta}{1-\beta}$$

YANG-MILLS VECTOR COUPLING: EBOLI et al. 93

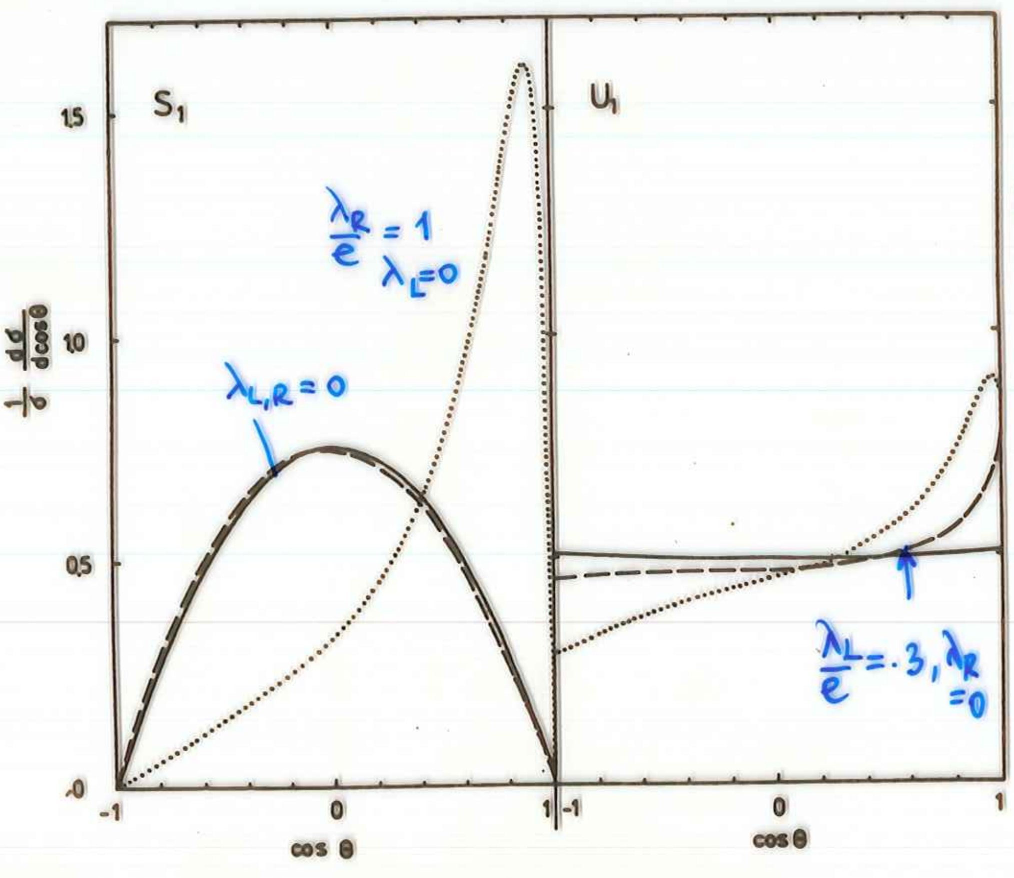
LARGE (λ/e) DEMANDED $\leftrightarrow$ HERA!

JB, R. Rückl.



$\sqrt{s}$ = 190 GeV
LEP200

$\sqrt{s}$ = 1 TeV
Linac



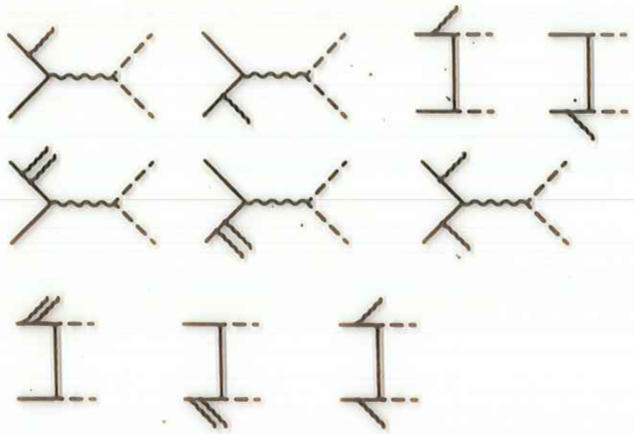
$\sqrt{s}$ = 1 TeV

M.C. FOR VECTORS

2.1. QED Corrections & Beamstrahlung

(LLA)

$\propto \left(\frac{\alpha}{2\pi}\log\frac{s}{m_e^2}\right)^n$



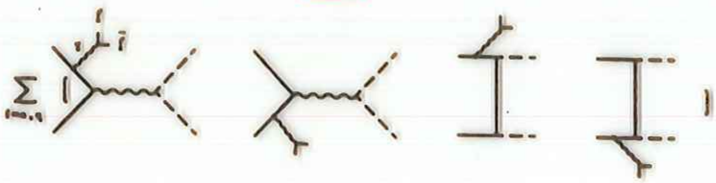
BREMSSTRAHLUNG

a)

JB, 93.

e^+e^- PAIRS

b)



ALL $f\bar{f}$ PAIRS

c)

$$P_{ee}^{(0)}(z) = \delta(1-z)\left[\frac{3}{2} + 2\ln\Delta\right] + \theta(1-\Delta-z)\frac{1+z^2}{1-z}$$

a$_1$) $O(\alpha)$

$$\begin{aligned}\frac{1}{2}\left[P_{ee}^{(0)} \otimes P_{ee}^{(0)}\right](z) &= \delta(1-z)\left[2\ln^2\Delta + 3\ln\Delta + \frac{9}{8} - 2\zeta(2)\right] \\ &+ \theta(1-\Delta-z)\left\{\frac{1+z^2}{1-z}\left[2\ln(1-z) - \ln z + \frac{3}{2}\right]\right. \\ &+ \left.\frac{1}{2}(1+z)\ln z - (1-z)\right\}\end{aligned}$$

a$_2$) $O(\alpha^2)$

$$\frac{1}{2}\left[P_{e\gamma}^{(0)} \otimes P_{\gamma e}^{(0)}\right](z) \equiv P_{e\gamma e}^{(1)}(z) = (1+z)\ln z + \frac{1}{2}(1-z) + \frac{2}{3}\frac{1}{z}(1-z^3)$$

b) $O(\alpha^2)$

$$P_{f\bar{f}}^{(1)}(z) = N_c(f)e_f^2\frac{1}{3}P_{ee}^{(0)}(z)\theta\left(1 - z - \frac{4m_f}{\sqrt{s}}\right)$$

c) $O(\alpha^2)$

$$P_{soft}^{(3)}(z) = b(1-z)^{b-1}(1+\delta_1+\delta_2) - \frac{b(1+\delta_1) + b^2\ln(1-z)}{1-z}$$

$b = (2\alpha/\pi)(L_m - 1)$, $\delta_1 = (3\alpha/2\pi)L_m$ and $\delta_2 = (\alpha/\pi)^2[9/8 - 2\zeta(2)]L_m^2$

expon.

Bremsstrahlung:
$\mathcal{O}(\alpha)$, $\mathcal{O}(\alpha^2)$, soft exponentiation

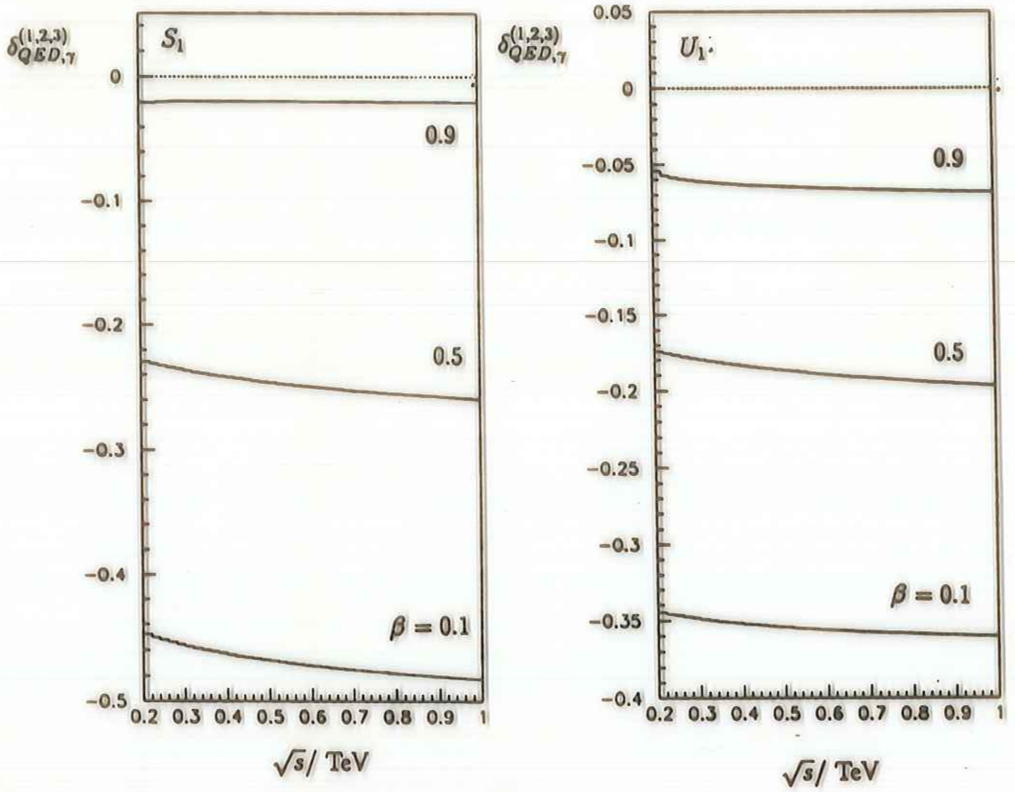
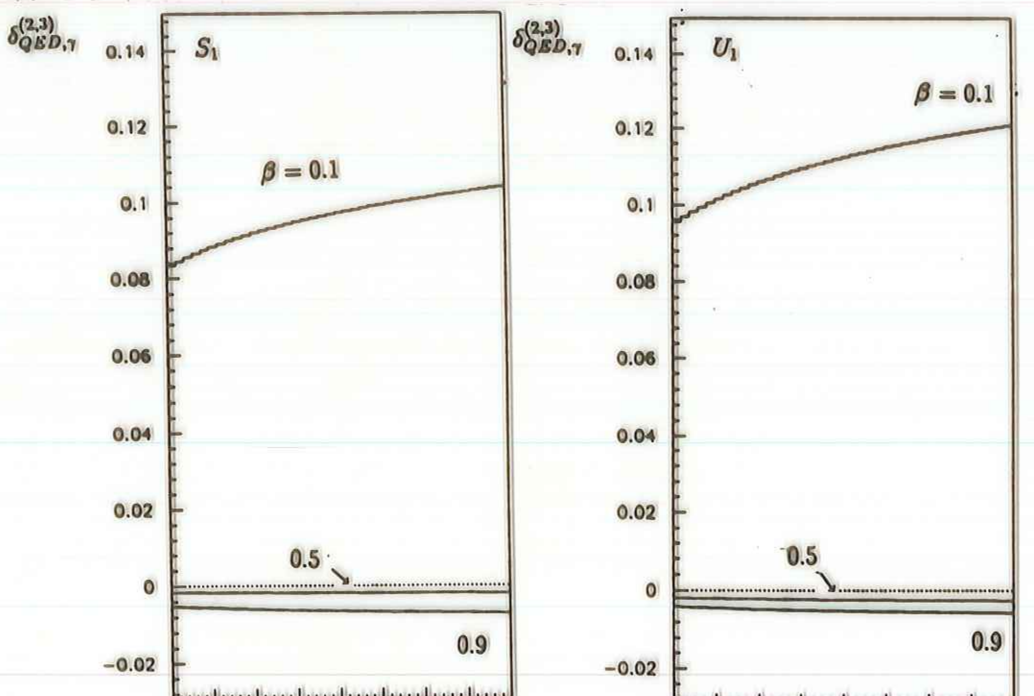


Figure 3: QED initial state Bremsstrahlung contributions (figure 2a) up to $\mathcal{O}(\alpha^2)$ + soft exponentiation for the scalar (S_1) and vector (U_1) leptoquark pair production cross section. Here we assumed $\lambda_L = 0$ and $\lambda_R = e$ for the leptoquark-fermion couplings.

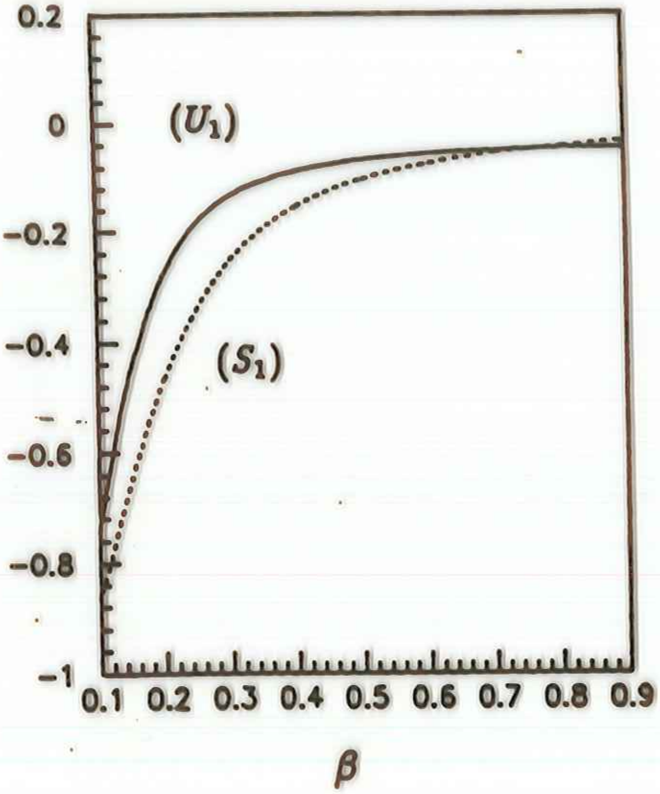


⊕ 10..12 %.

Figure 4: $\mathcal{O}(\alpha^2)$ + soft exponentiation terms of figure 3 only.

Beamstrahlung

$$\Delta\sigma_{BS}(s) = \int_0^1 dz \underbrace{\frac{1}{\mathcal{L}}\frac{d\mathcal{L}}{dz}}_{\text{LUMI. DISTR.}}(z)\sigma^{(0)}(zs)\theta\left(z - \frac{4M^2}{s}\right) - \sigma^{(0)}(s).$$



$\frac{1}{\mathcal{L}}\frac{d\mathcal{L}}{dz}$:

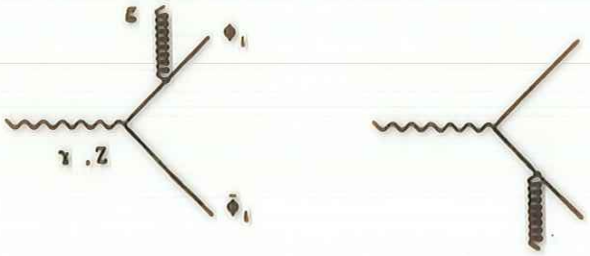
ORTEU et al. 93

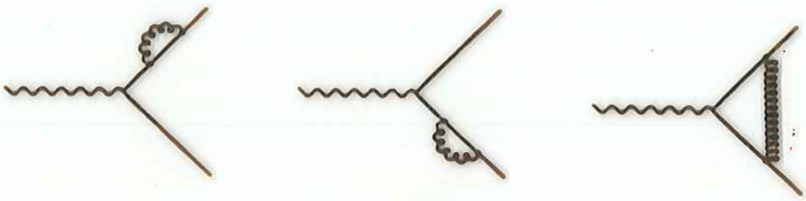
Figure 6: Beamstrahlung correction for a narrow band machine for the pair production cross section of scalar (S_1) and vector (U_1) leptoquarks as a function of β for $\sqrt{s} = 300\,\text{GeV}$.

2.2. QCD Corrections: Scalars

$\lambda_L, \lambda_R \ll 1$

FSR:





JB. '93

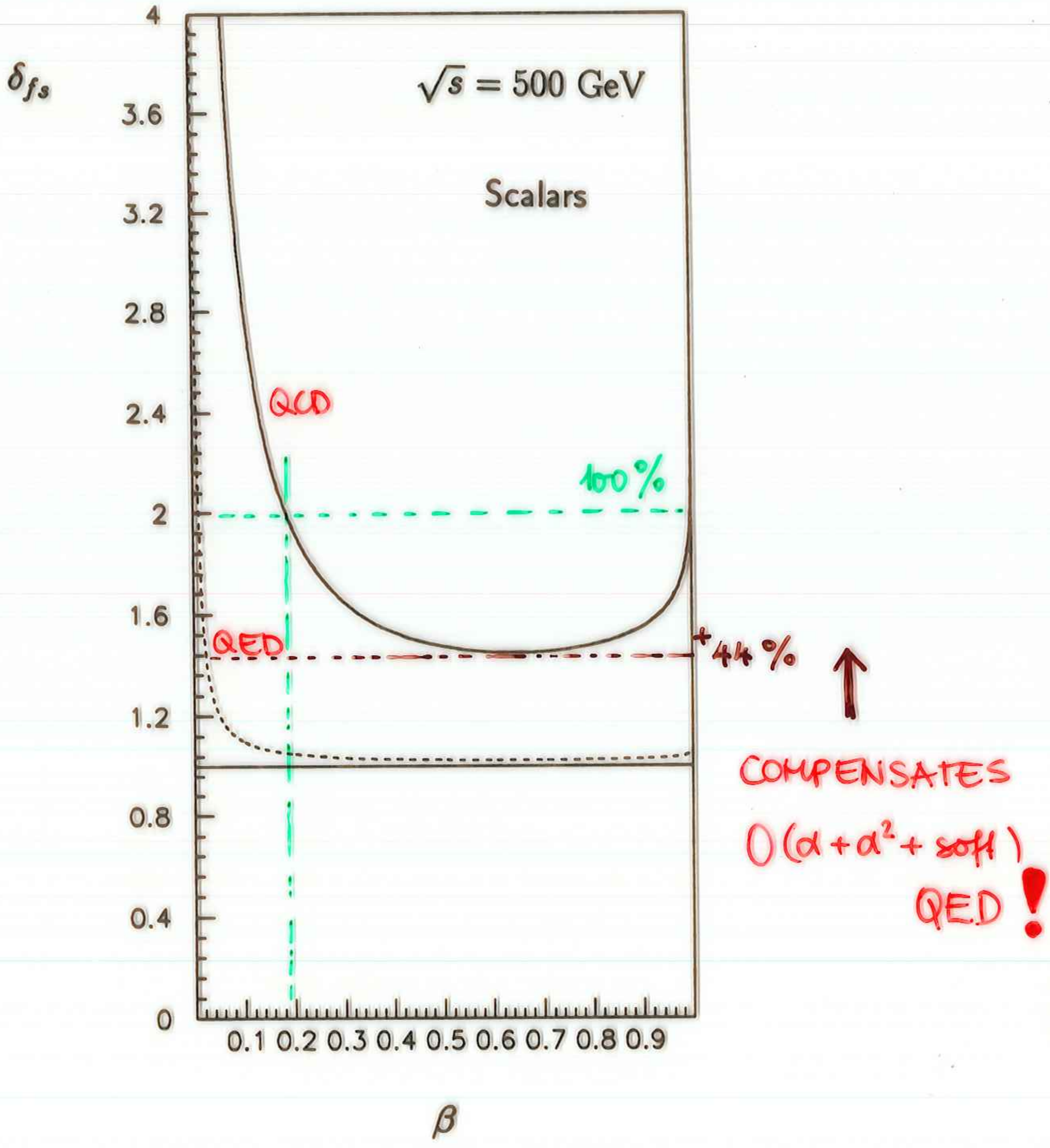
Scalars

$$
\begin{aligned}
\mathcal{F}_s(\beta) &= \frac{1+\beta^2}{\beta}\left[4Li_2\left(\frac{1-\beta}{1+\beta}\right) + 2Li_2\left(-\frac{1-\beta}{1+\beta}\right) - 3\ln\frac{2}{1+\beta}\ln\frac{1+\beta}{1-\beta} - 2\ln\beta\ln\frac{1+\beta}{1-\beta}\right] \\
&- 3\ln\left(\frac{4}{1-\beta^2}\right) - 4\ln\beta + \frac{1}{\beta^3}\left[\frac{5(1+\beta^2)^2}{4} - 2\right]\ln\frac{1+\beta}{1-\beta} + \frac{3}{2}\frac{1+\beta^2}{\beta^2}
\end{aligned}
$$

(SCHWINGER) 70ies

$$\sigma^{(1,QCD)}_{scalar}(s) = \sigma^{(0)}_{scalar}(s)\left\{1 + \frac{4\alpha_s}{3\pi}\mathcal{F}_s(\beta)\right\}$$ QCD

$$\sigma^{(1,QED,fs)}_{scalar}(s) = \sigma^{(0)}_{scalar}(s)\left\{1 + \frac{\alpha}{\pi}\mathcal{F}_s(\beta)\right\}$$ QED

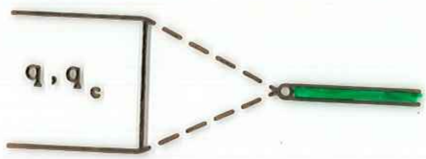


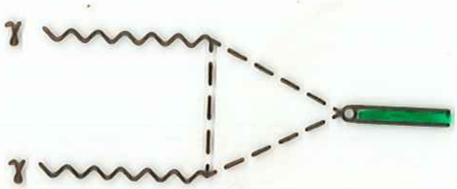
δ_{fs}
$\sqrt{s} = 500$ GeV
Scalars
QCD
100%
QED
+44%
COMPENSATES
O(α + α² + soft)
QED !
β

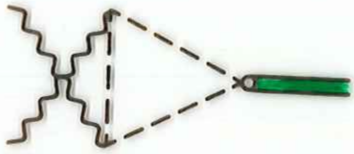
Formation of Bound States ?

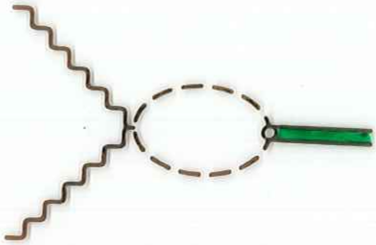
'Leptoquarkonia'

$\underline{\beta \ll 1}$







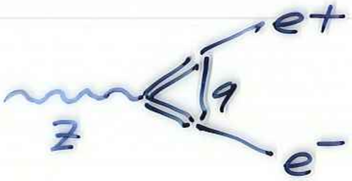


CONTINUUM CROSS SECT.:

$$\boxed{\begin{aligned} \sigma(e^+e^- \to \Phi_s\overline{\Phi}_s) &\propto \beta^3 \\ \sigma(e^+e^- \to \Phi_v\overline{\Phi}_v) &\propto \beta^3 \\ \sigma(\gamma\gamma \to \Phi_s\overline{\Phi}_s) &\propto \beta \\ \sigma(\gamma\gamma \to \Phi_v\overline{\Phi}_v) &\propto \beta \end{aligned}} \tag{14}$$

$\longleftrightarrow$ BEAM SPREAD.

LEPTOQUARK LOOP EFFECTS ?



, ...

3. $e\gamma$ Scattering

• Single Leptoquark Production in e^+e^- Annihilation:

$\sigma \propto \left(\frac{\lambda}{e}\right)^2$!

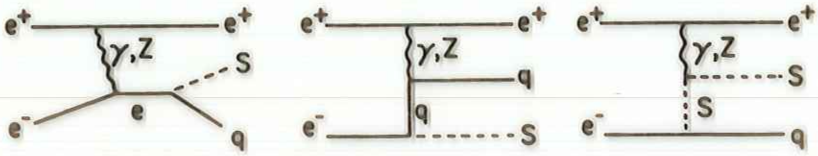


Fig. 1. Diagrams giving the dominant contribution to $e^+e^- \to Se^+q$

HEWETT PAKVASA '89
WWA

BELANGER et al. '93

DONCHESKI, GODFREY '93

S

FULL DIRECT:

$\mathcal{L} = 10\ fb^{-1}$

50#
BELANGER et al.

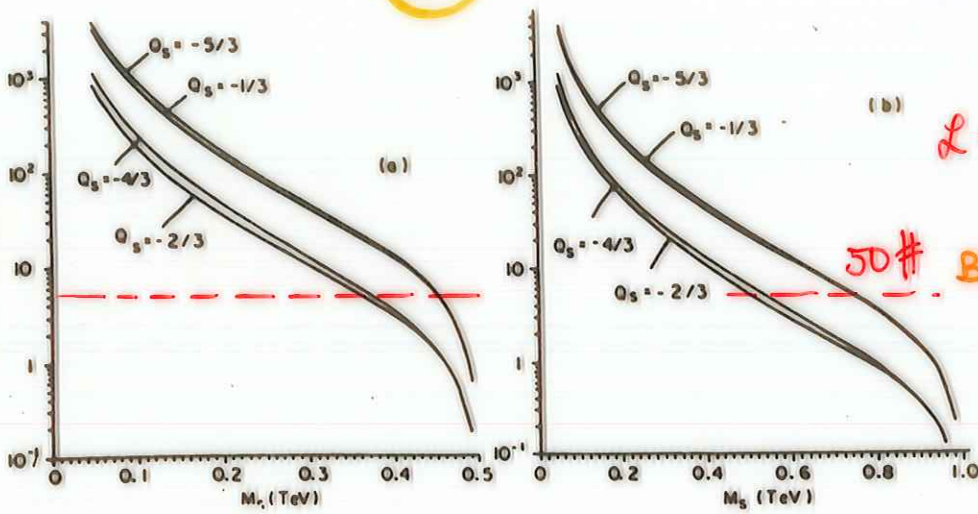


Fig. 2. Cross-sections for single leptoquark production in e^+e^- at (a) $\sqrt{s}$ = 500 GeV (b) $\sqrt{s}$ = 1 TeV for the four different leptoquark charges and $k = 1$ $\equiv \lambda/e$

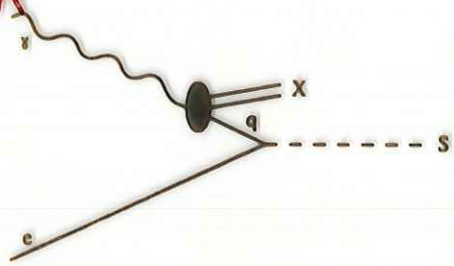
DON CHESKI/ GODFREY

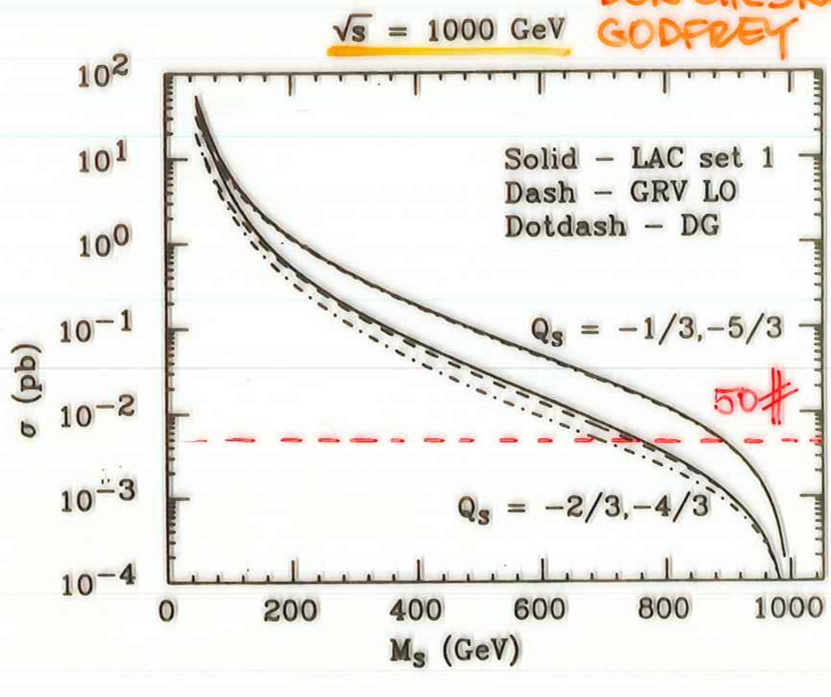
RESOLVED ONLY:

(COMBINATION NEEDED!)

J.B., SCHULER

WWA



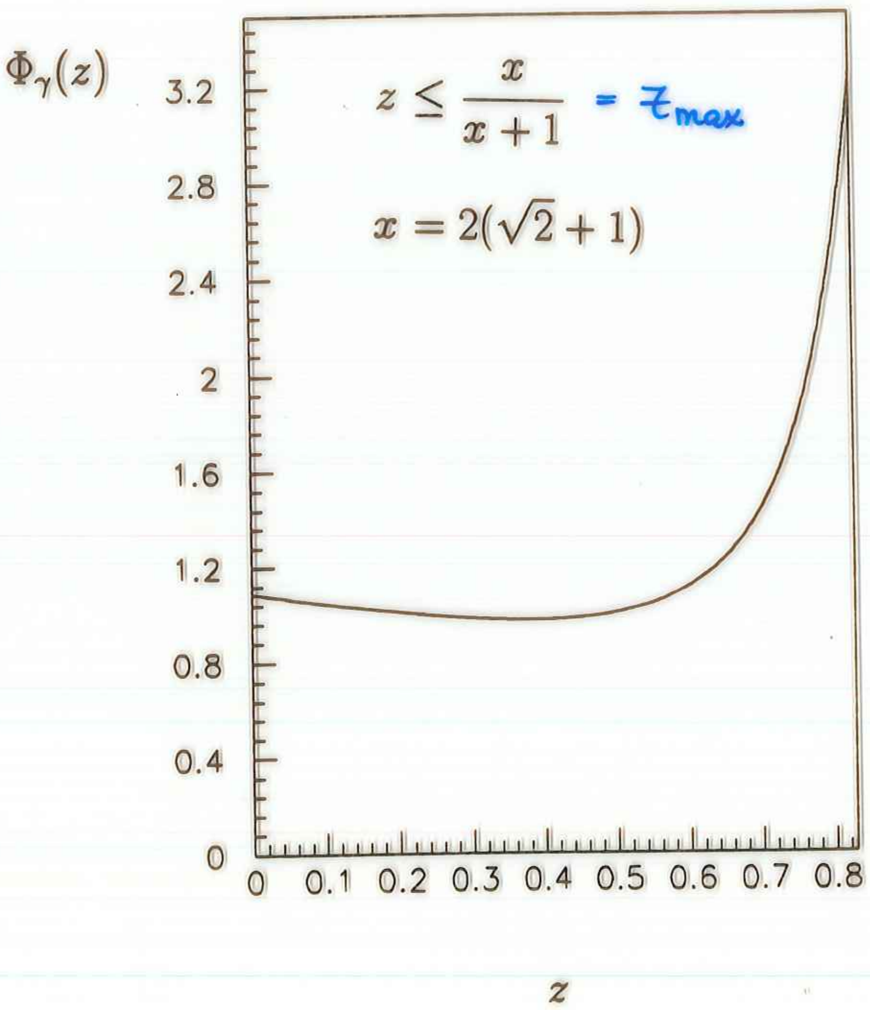


L- COMPTON SPECTRUM :

$$\Phi_\gamma(z) = \frac{1}{N(x)}\left[1 - z + \frac{1}{1-z} - \frac{4z}{x(1-z)} + \frac{4z^2}{x^2(1-z)^2}\right]$$

$$N(x) = \frac{16 + 32x + 18x^2 + x^3}{2x(1+x)^2} + \frac{x^2 - 4x - 8}{x^2}\ln(1+x)$$

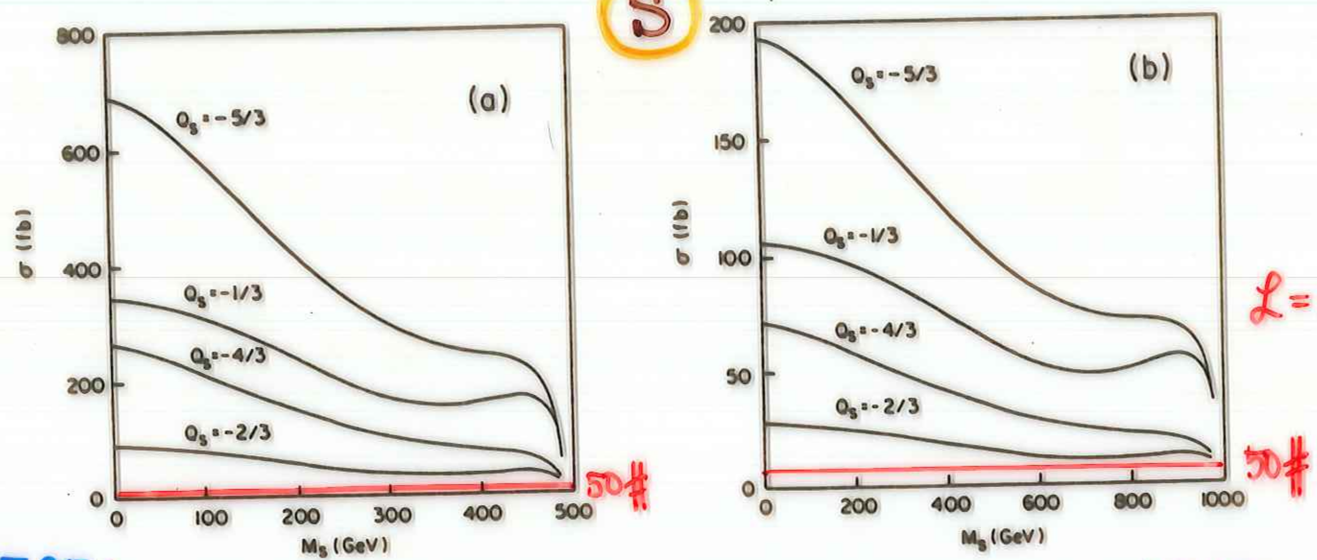
GINZBURG et al. 1983, 84.



⊗ COMPTON BACKSCATTERING SPECTRUM.

- Single Leptoquark Production in $e\gamma$ Scattering:

S



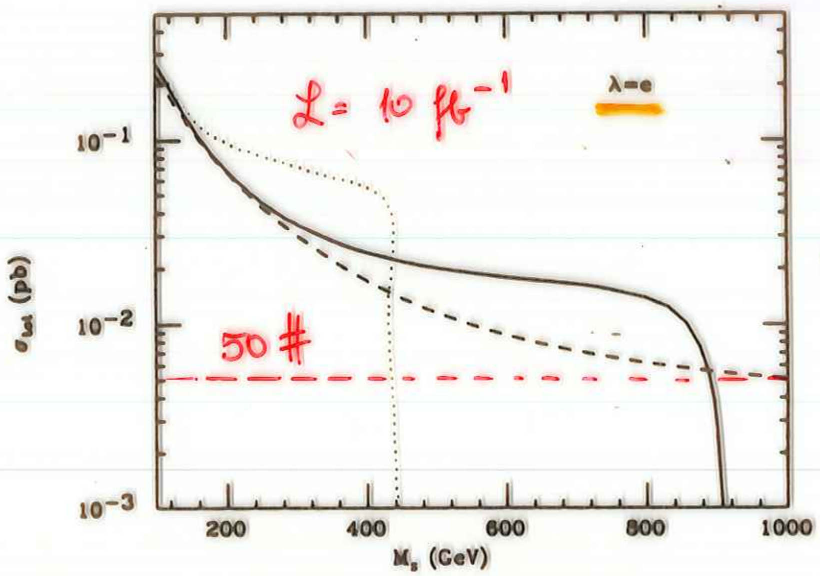
DIRECT:

LONDON NADEAU '93

FIG. 2. Cross section for single leptoquark production in $e\gamma$ collisions at (a) $\sqrt{s} = 500$ GeV, (b) $\sqrt{s} = 1$ TeV, for the four possible LQ charges, $Q_S = -\frac{1}{3}, -\frac{2}{3}, -\frac{4}{3}, -\frac{5}{3}$. The results are given for $k = 1$.

EBOLI et al. '93

DIRECT & RESOLVED



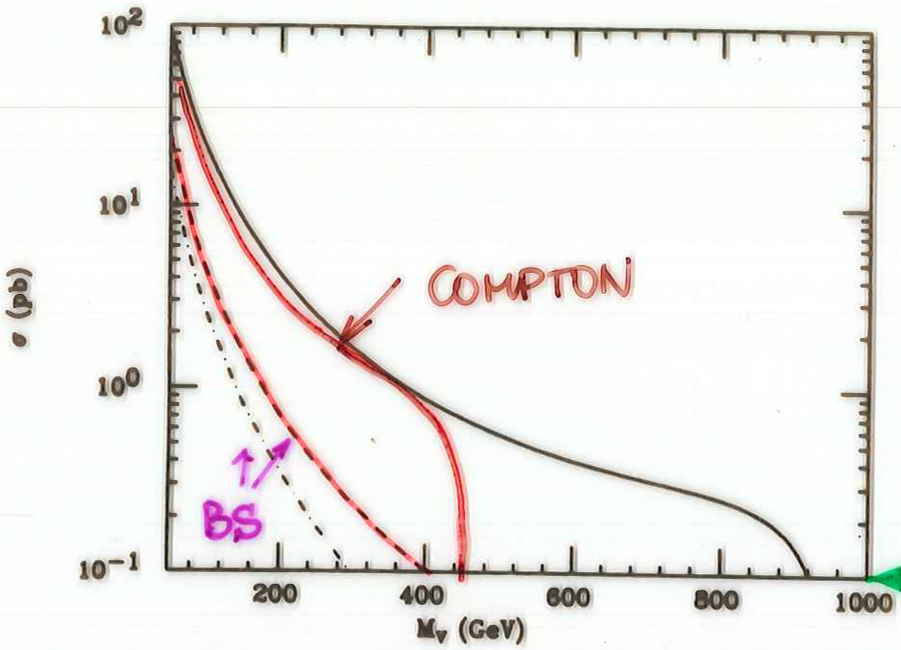
(WUDKA : LQ's $S^{1/3}$)

Fig. 5. Total cross section for the process $e^+e^- \to e + \gamma(g_\gamma) \to S + \text{jet}$ as a function of M_S with $\lambda = e$ and $\sqrt{s} = 500$ GeV (dotted line), $\sqrt{s} = 1000$ GeV (solid line), and $\sqrt{s} = 2000$ GeV (dashed line). We added the contributions from the subprocesses (17) and (18).

V

(YANG-MILLS TYPE COUPLING ONLY!)

DIRECT:



CIEZA-MONTALVO, EBOLI '93

$\mathcal{L} = 10\,fb^{-1}$

10^3 #

FIG. 5. Total cross section for the process $e^-\gamma \to V^- q$ as a function of M_V: (a) laser backscattering at $\sqrt{s} = 500$ GeV (dotted line); (b) laser backscattering at $\sqrt{s} = 1000$ GeV (solid line); (c) bremsstrahlung at $\sqrt{s} = 500$ GeV (dot-dashed line); (d) bremsstrahlung at $\sqrt{s} = 1000$ GeV (dashed line).

4. $\gamma\gamma$ Scattering

JB, EB

$U_{em}(1)$ invariant Lagrangian:

$$\mathcal{L} = \mathcal{L}_s + \mathcal{L}_v \tag{1}$$

with

$$\mathcal{L}_s = \sum_{scalars} \left[(D^\mu \Phi)^\dagger (D_\mu \Phi) - M_s^2 \Phi^\dagger \Phi \right] \tag{2}$$

and

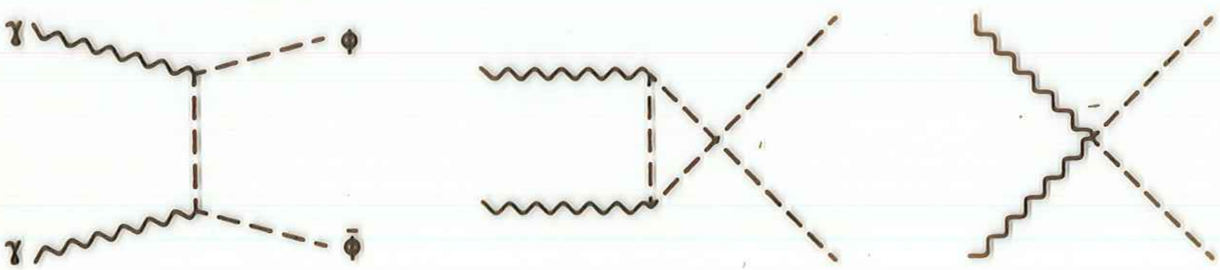
$$\mathcal{L}_v = \sum_{vectors} \left\{ -\frac{1}{2} G^\dagger_{\mu\nu} G^{\mu\nu} + M_v^2 \Phi^\dagger_\mu \Phi^\mu - ie \left[(1-\kappa_A) \Phi^\dagger_\mu \Phi_\nu F^{\mu\nu} + \frac{\lambda_A}{M_v^2} G^\dagger_{\sigma\mu} G^\mu_\nu F^{\nu\sigma} \right] \right. \tag{3}$$

Field strength tensors:

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, \\ G_{\mu\nu} &= D_\mu \Phi_\nu - D_\nu \Phi_\mu \end{aligned} \tag{4}$$

Covariant derivative:

$$D_\mu = \partial_\mu - ieQ^\gamma A_\mu \tag{5}$$



Anomalous couplings: κ_A and λ_A

$\Updownarrow$

anomalous magnetic moment μ_Φ and electric quadrupole moment q_Φ of leptoquarks:

$$\boxed{\begin{aligned} \mu_{\Phi,A} &= \frac{eQ_\gamma}{2M_\Phi}(2 - \kappa_A + \lambda_A) \\ q_{\Phi,A} &= -\frac{eQ_\gamma}{M_\Phi^2}(1 - \kappa_A - \lambda_A) \end{aligned}} \tag{6}$$

Scalar Leptoquarks

$$\frac{d\hat{\sigma}_s}{d\cos\theta} = \frac{\pi\alpha^2}{\hat{s}} Q_\Phi^4 N_c \beta \left\{ 1 - \frac{2(1-\beta^2)}{1-\beta^2\cos^2\theta} + \frac{2(1-\beta^2)^2}{(1-\beta^2\cos^2\theta)^2} \right\} \tag{7}$$

$$\hat{\sigma}_s(\hat{s},\beta) = \frac{\pi\alpha^2}{\hat{s}} Q_\Phi^4 N_c \left\{ 2(2-\beta^2)\beta - (1-\beta^4)\log\left|\frac{1+\beta}{1-\beta}\right| \right\} \tag{8}$$

with $\beta = \sqrt{1 - 4M_\Phi^2/\hat{s}}$.

(SHAPE : UK 40'ies)

Vector Leptoquarks

$$\frac{d\hat{\sigma}_v}{d\cos\theta} = \frac{\pi\alpha^2}{\hat{s}} Q_\Phi^4 N_c \sum_{j=0}^{14} \chi_j(\kappa_A,\lambda_A) \frac{F_j(\hat{s},\beta,\cos\theta)}{(1-\beta^2\cos^2\theta)^2} \tag{9}$$

with

($\leftrightarrow$ W^+W^-, CHOI, SCHREMPP).

$$\begin{array}{rrrrrrr}
\sum_{j=0}^{14}\chi_j(\kappa_A,\lambda_A)F_j & = & F_0 & + & \kappa_A F_1 & + & \kappa_A^2 F_2 \\
 & + & \kappa_A^3 F_3 & + & \kappa_A^4 F_4 & + & \lambda_A F_5 \\
 & + & \lambda_A^2 F_6 & + & \lambda_A^3 F_7 & + & \lambda_A^4 F_8 \\
 & + & \kappa_A\lambda_A F_9 & + & \kappa_A\lambda_A^2 F_{10} & + & \kappa_A\lambda_A^3 F_{11} \\
 & + & \kappa_A^2\lambda_A F_{12} & + & \kappa_A^3\lambda_A F_{13} & + & \kappa_A^2\lambda_A^2 F_{14}
\end{array}$$

$$\hat{\sigma}_v = \frac{2\pi\alpha^2}{M_v^2} Q_\Phi^4 N_c \sum_{j=0}^{14} \chi_j(\kappa_A,\lambda_A) \widetilde{F}_j(\hat{s},\beta) \tag{10}$$

where

$$\widetilde{F}_j = \frac{M_v^2}{\hat{s}} \int_0^\beta d\xi \frac{F_j(\xi = \beta\cos\theta)}{(1-\xi^2)^2} \tag{11}$$

(SPECIAL κ, λ CHOICES : LONG LIST OF REFS: W^+W^-, YM, MC. e.g. EBOLI et al.)

(ANALYTIC RESULT)

$$
\begin{aligned}
\tilde{F}_0 &= \beta\left(\frac{11}{2}-\frac{9}{4}\beta^2+\frac{3}{4}\beta^4\right)-\frac{3}{8}\left(1-\beta^2-\beta^4+\beta^6\right)\ln\left|\frac{1+\beta}{1-\beta}\right| \qquad \text{YM}\\
\tilde{F}_1 &= -8\beta-\frac{3}{2}\left(1-\beta^2\right)\log\left|\frac{1+\beta}{1-\beta}\right| \qquad \propto \kappa_A\\
\tilde{F}_2 &= 3\beta+\frac{1}{4}\beta\frac{\hat{s}}{M_\Phi^2}+\left(\frac{7}{2}-2\beta^2\right)\log\left|\frac{1+\beta}{1-\beta}\right|\\
\tilde{F}_3 &= -\frac{1}{4}\beta\frac{\hat{s}}{M_\Phi^2}+\left(-2+\frac{3}{4}\beta^2\right)\log\left|\frac{1+\beta}{1-\beta}\right|\\
\tilde{F}_4 &= -\frac{1}{96}\beta+\frac{5}{48}\beta\frac{\hat{s}}{M_\Phi^2}+\frac{4-\beta^2}{16}\log\left|\frac{1+\beta}{1-\beta}\right|\\
\tilde{F}_5 &= -\left(1-\beta^2\right)\log\left|\frac{1+\beta}{1-\beta}\right| \qquad \propto \lambda_A\\
\tilde{F}_6 &= -\frac{1}{6}\beta+\frac{17}{12}\beta\frac{\hat{s}}{M_\Phi^2}+\left(-3-\frac{\beta^2}{2}+\frac{1}{2}\frac{\hat{s}}{M_\Phi^2}\right)\log\left|\frac{1+\beta}{1-\beta}\right|\\
\tilde{F}_7 &= -\beta+\frac{11}{6}\beta\frac{\hat{s}}{M_\Phi^2}-\frac{1}{3}\beta\frac{\hat{s}^2}{M_\Phi^4}-\frac{3+\beta^2}{4}\log\left|\frac{1+\beta}{1-\beta}\right|\\
\tilde{F}_8 &= -\frac{1}{96}\beta+\frac{59}{80}\beta\frac{\hat{s}}{M_\Phi^2}-\frac{113}{320}\beta\frac{\hat{s}^2}{M_\Phi^4}+\frac{43}{960}\beta\frac{\hat{s}^3}{M_\Phi^6}+\left(-\frac{1}{2}-\frac{1}{16}\beta^2+\frac{1}{8}\frac{\hat{s}}{M_\Phi^2}\right)\log\left|\frac{1+\beta}{1-\beta}\right|\\
\tilde{F}_9 &= 2\beta+\left(2+\beta^2\right)\log\left|\frac{1+\beta}{1-\beta}\right| \qquad \propto \kappa_A\cdot\lambda_A\\
\tilde{F}_{10} &= 2\beta-\frac{7}{3}\beta\frac{\hat{s}}{M_\Phi^2}+\left(3+\frac{5}{4}\beta^2-\frac{1}{2}\frac{\hat{s}}{M_\Phi^2}\right)\log\left|\frac{1+\beta}{1-\beta}\right|\\
\tilde{F}_{11} &= \frac{1}{24}\beta-\frac{59}{48}\beta\frac{\hat{s}}{M_\Phi^2}+\frac{5}{32}\beta\frac{\hat{s}^2}{M_\Phi^4}+\frac{5+\beta^2}{4}\log\left|\frac{1+\beta}{1-\beta}\right|\\
\tilde{F}_{12} &= -\beta-\frac{1}{2}\beta\frac{\hat{s}}{M_\Phi^2}+\left(-\frac{1}{4}-\frac{7}{4}\beta^2\right)\log\left|\frac{1+\beta}{1-\beta}\right|\\
\tilde{F}_{13} &= \frac{1}{24}\beta+\frac{1}{3}\beta\frac{\hat{s}}{M_\Phi^2}-\frac{1}{4}\left(1-\beta^2\right)\log\left|\frac{1+\beta}{1-\beta}\right|\\
\tilde{F}_{14} &= -\frac{1}{16}\beta+\frac{11}{96}\beta\frac{\hat{s}}{M_\Phi^2}+\frac{17}{192}\beta\frac{\hat{s}^2}{M_\Phi^4}+\left(\frac{1}{8}\frac{\hat{s}}{M_\Phi^2}-\frac{3}{4}-\frac{3}{8}\beta^2\right)\log\left|\frac{1+\beta}{1-\beta}\right| \qquad (12)
\end{aligned}
$$

Tree–level unitarity:

$$
\begin{aligned}
\lambda_A &= 0\\
\kappa_A^2\left[\left(\kappa_A-\frac{6}{5}\right)^2+\frac{24}{25}\right] &= 0. \qquad (13)
\end{aligned}
$$

Since κ_A, λ_A real $\rightarrow \kappa_A \equiv \lambda_A \equiv 0$.

HOWEVER: SEARCH AT THRESHOLD $4M_\phi^2 \simeq \hat{s}$!

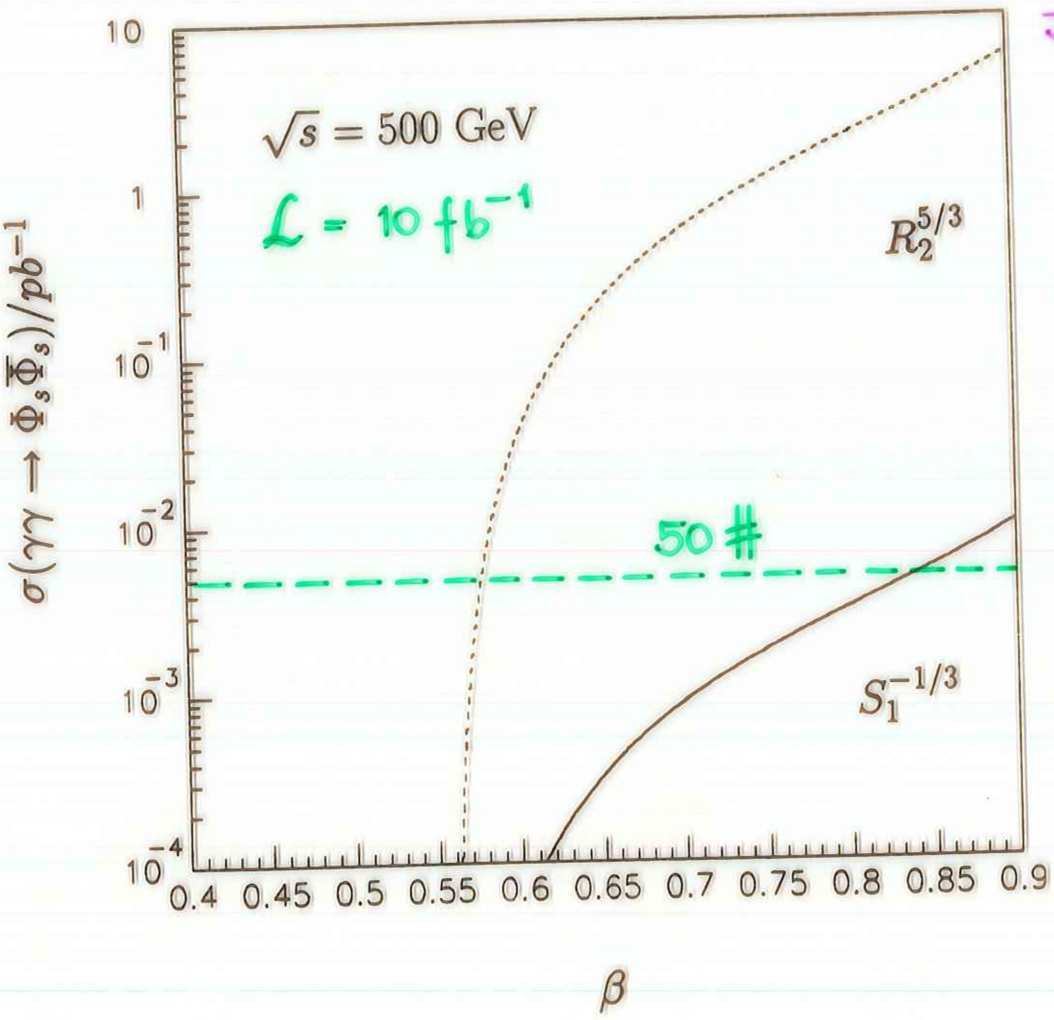
$\hat{s} \not\gg M_\phi^2$.

$$\sigma_{S,V} = \int_0^{z_{max}} dz_1 \int_0^{z_{max}} dz_2 \; \phi_\gamma(z_1)\, \phi_\gamma(z_2)\, \hat{\sigma}_{S,V}(\hat{s})\, \Theta(\hat{s} - 4m_\phi^2)$$

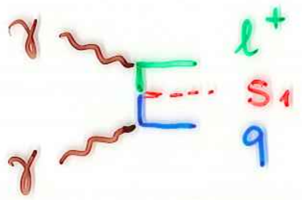
$$\hat{s} = S z_1 z_2$$

SCALARS :

J.B. 1993

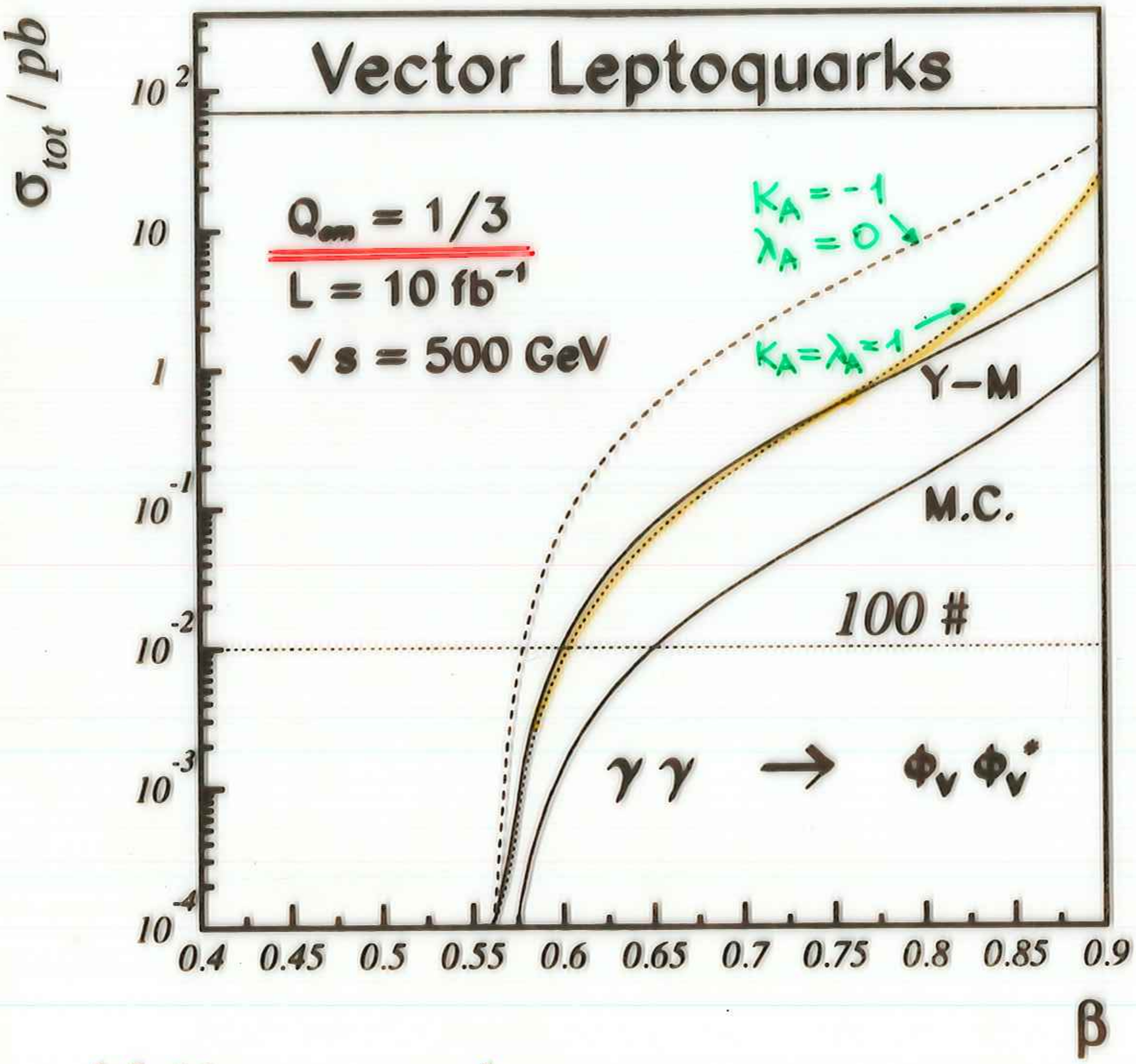


∴ SINGLE PRODUCTION : BELANGER et al. 93



$\propto \left(\frac{\lambda}{e}\right)^2$

J.B., E.B. 1994



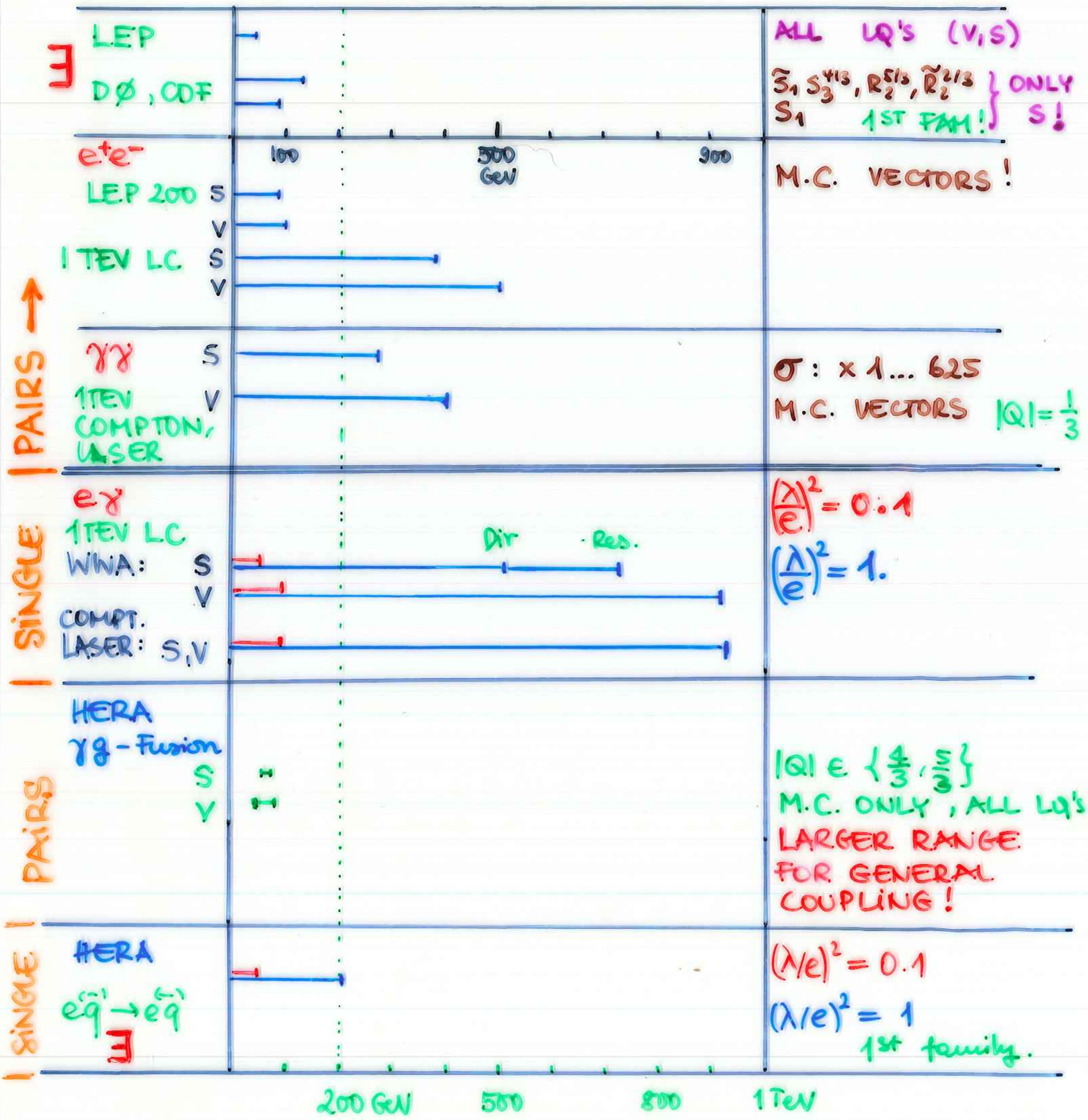
Y–M : $\kappa_A \equiv \lambda_A \equiv 0$

M.C. : $\kappa_A = 1$, $\lambda_A = 0$

$\sigma(\phi_V) \propto Q^4_{\phi_V}$

1: .... : 625

5. Present Bounds & Search Potential at e^+e^- Colliders



Sensitivity to Quantum Numbers :

Process	LQ	Quantum Numbers
e^+e^- Annihilation	S,V	$Q^{\gamma}_{\Phi}, Q^{Z}_{\Phi}, \lambda_{L,R}$
$\gamma\gamma$ Collider	S	Q^{γ}_{Φ}
	V	$Q^{\gamma}_{\Phi}, \kappa_A, \lambda_A$
$e\gamma$ Collider	S,V	$\lambda_{L,R}, Q^{\gamma}_{\Phi}$
ep Collider	single LQ	$\lambda_{L,R}$
	pairs : S	Q^{γ}_{Φ}
	pairs : V	$Q^{\gamma}_{\Phi}, \kappa_A, \kappa_G, \lambda_A, \lambda_G$
$p\overline{p}$ Collider	V	κ_G, λ_G