Leading Order Radiative Corrections to Deep Inelastic *ep* Scattering to $\mathcal{O}(\alpha^2)$ for Different Kinematical Variables

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DESY – Zeuthen, FRG

1. The Different Variables
2. The Corrections up to $\mathcal{O}(\alpha^2 L^2)$
3. Numerical Results
4. Conclusions

1. The Different Variables

Goal:

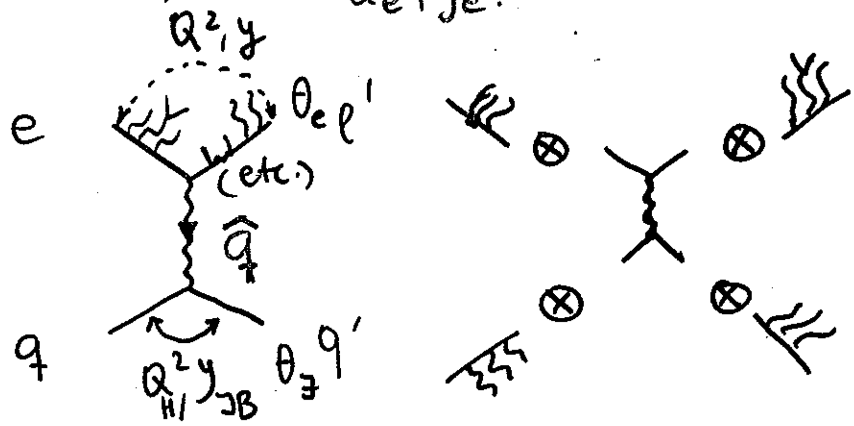
Measurement of a Born Cross section: $2 \rightarrow 2$ Reaction

↪ Integrating over the DOF of the radiated Photon(s).

↪ Different Correction Functions for Different Variables are obtained !

CALCULATE : K-FACTORS : $\delta_{NC,CC}(x,y) = \frac{\sigma^{(0)} + \sigma^{corr}}{\sigma^{(0)}}$.

- Double Angle Method θ_e, θ_J } (ZEUS)
- θ_e & y_J
- Jet Measurement: NC Q^2_J, y_J
- Jet Measurement: CC -"-
- Mixed Variables (Q^2_e, y_J) } H1
- (Lepton Measurement) Q^2_e, y_e.



$\hat{V}$ SUBSYSTEM VARIABLE

V 'TREE LEVEL' VARIABLE

	$\hat{s}$	$\hat{Q}^2$	$\hat{y}$	$\mathcal{J}(x,y,z)$
lepton measurement	zs	Q^2z	$(z+y-1)/z$	$y/(z+y-1)$
jet measurement	zs	$Q^2(1-y)/(1-y/z)$	y/z	$(1-y)/(z-y)$
mixed variables	zs	Q^2z	y/z	1
double angle method	zs	Q^2z^2	y	z
y_{JB} and θ_e	zs	$Q^2z(z-y)/(1-y)$	y/z	$(z-y)/(1-y)$

Table 1: The shifted variables for different types of cross section measurement

- z_0 :

LEPTON MEASUREMENT
JET MEASUREMENT
$\hat{x}(z_0) = 1$

MIXED VARIABLES
θ_e, y_J
$z_0 = y$

DOUBLE ANGEL :
$z_0 \equiv 0.$

} $\hat{\partial}(\hat{x} \to 0, \hat{Q}^2 \to 0)$! for $z \to z_0$!

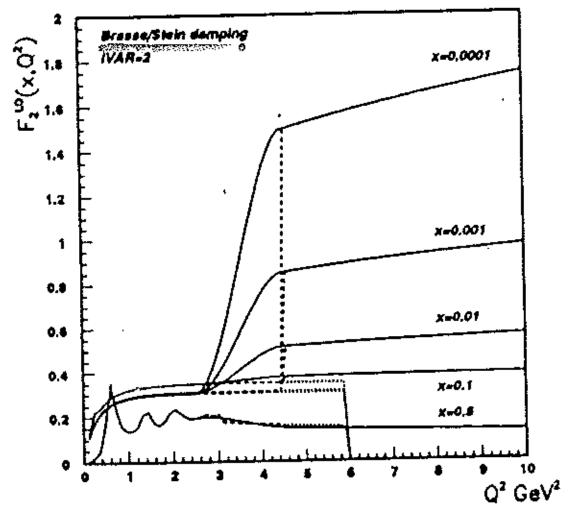
BUT:

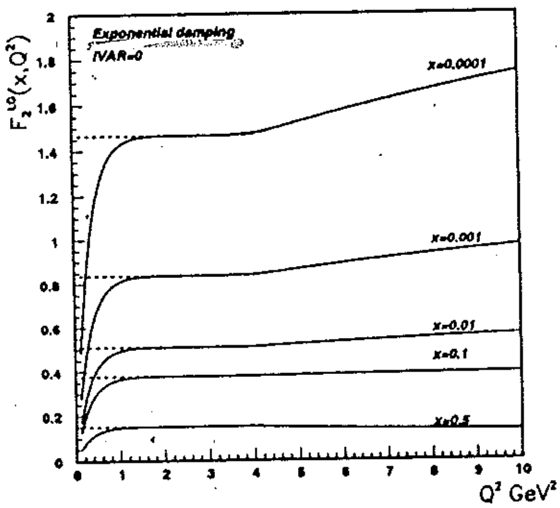
$$2E_e = E'_e(1-\cos\theta_e) + E_J(1-\cos\theta_J) \geq A \quad ! \qquad (3)$$

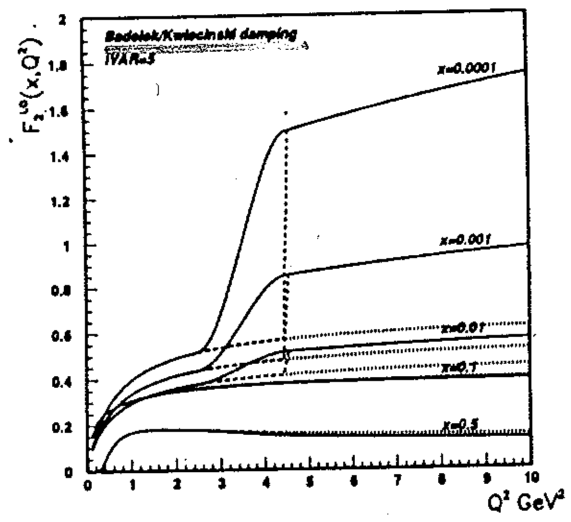
FORTUNATELY: $z_0 = \dfrac{A}{2E_e}$.

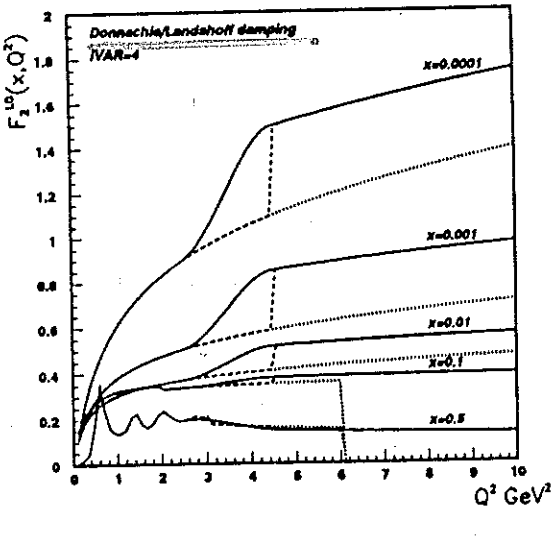
$\Longrightarrow$ ZEUS: THIS HELPS <u>ONLY</u> IN THE CASE OF THE DOUBLE ANGEL METHOD !

F_2 AT LOW Q^2


Brasse/Stein damping
IVAR=2
x=0.0001
x=0.001
x=0.01
x=0.1
x=0.5
$F_2^{LO}(x,Q^2)$
Q^2 GeV^2


Exponential damping
IVAR=0
x=0.0001
x=0.001
x=0.01
x=0.1
x=0.5
$F_2^{LO}(x,Q^2)$
Q^2 GeV^2


Badelek/Kwiecinski damping
IVAR=5
x=0.0001
x=0.001
x=0.01
x=0.1
x=0.5
$F_2^{LO}(x,Q^2)$
Q^2 GeV^2


Donnachie/Landshoff damping
IVAR=4
x=0.0001
x=0.001
x=0.01
x=0.1
x=0.5
$F_2^{LO}(x,Q^2)$
Q^2 GeV^2

2. The Corrections up to $\mathcal{O}(\alpha^2)$

Contributions:

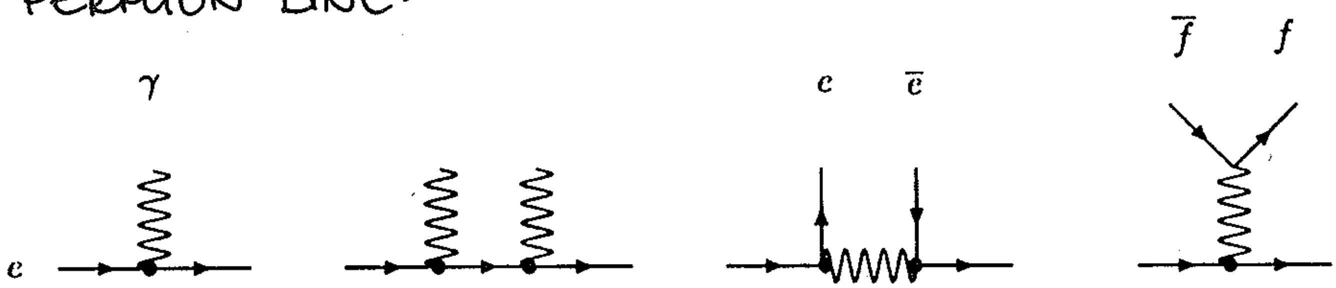
1. Bremsstrahlung: Diagrams a,b
2. Electron Pair Production: Diagram c
3. Fermion Pair Production: Diagram d , $f = e, \mu, \tau, u, d, s, c, b$

The Radiator-Method is applied.

Meaning of the bullet: Collinear Bremsstrahlung contribution **including** soft & virtual corrections.

An individual consideration of initial and final state bremsstrahlung is possible.

- APPLY THE RENORMALIZATION GROUP EQUATION → TRIVIAL COEFFICIENT FUNCTIONS IN $O(\alpha^N L^N)$
- FACTORIZATION FOR EACH CHARGED ('MASSLESS') FERMION LINE.



(a) NS | (b) NS | (c) S | (d) RUNNING α_{QED}

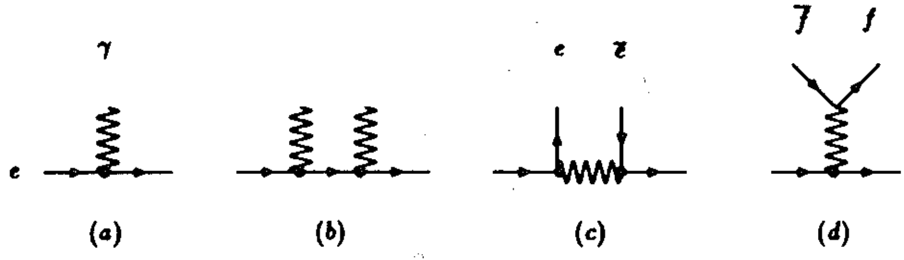


Figure 1: Diagrams contributing to the radiative corrections up to $\mathcal{O}(\alpha^2 L^2)$.

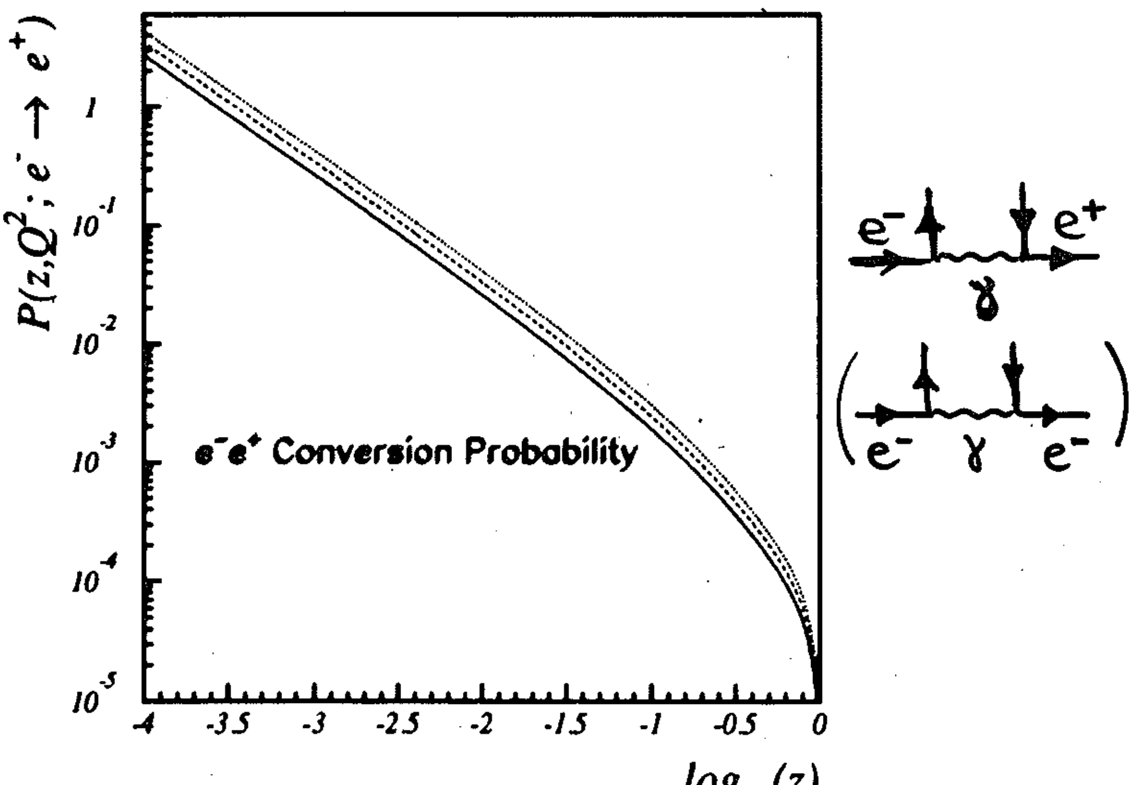


Figure 2: $e^- \to e^+$ transition probability for different values of Q^2. Full line: $Q^2 = 10\,\text{GeV}^2$, dashed line: $Q^2 = 100\,\text{GeV}^2$, and dotted line: $Q^2 = 1000\,\text{GeV}^2$.

BORN O(α)

$$\frac{d^2\sigma^{(2)}}{dxdy} = \frac{d^2\sigma^{(0)}}{dxdy} + \frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right)\int_0^1 P_{ee}^{(1)}(z)\left\{\theta(z-z_0)\mathcal{J}(x,y,z)\frac{d^2\sigma^{(0)}}{dxdy}\Big|_{x=\hat{x},y=\hat{y},s=\hat{s}} - \frac{d^2\sigma^{(0)}}{dxdy}\right\}$$

$$+ \frac{1}{2}\left[\frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right)\right]^2\int_0^1 P_{ee}^{(2,1)}(z)\left\{\theta(z-z_0)\mathcal{J}(x,y,z)\frac{d^2\sigma^{(0)}}{dxdy}\Big|_{x=\hat{x},y=\hat{y},s=\hat{s}} - \frac{d^2\sigma^{(0)}}{dxdy}\right\}$$

$$+ \left(\frac{\alpha}{2\pi}\right)^2\int_{z_0}^1\left\{\ln^2\left(\frac{Q^2}{m_e^2}\right)P_{ee}^{(2,2)}(z) + \sum_{f=l,q}\ln^2\left(\frac{Q^2}{m_f^2}\right)P_{ee,f}^{(2,3)}(z)\right\}\mathcal{J}(x,y,z)\frac{d^2\sigma^{(0)}}{dxdy}\Big|_{x=\hat{x},y=\hat{y},s=\hat{s}}$$

O(α²)

$$\mathcal{J}(x,y,z) = \begin{vmatrix} \partial\hat{x}/\partial x & \partial\hat{y}/\partial x \\ \partial\hat{x}/\partial y & \partial\hat{y}/\partial y \end{vmatrix}. \qquad (2)$$

z < 1

O(α)

$$P_{ee}^{(1)}(z) = \frac{1+z^2}{1-z} \qquad (4)$$

O(α²L²)

$$P_{ee}^{(2,1)}(z) = \frac{1}{2}\left[P_{ee}^{(1)}\otimes P_{ee}^{(1)}\right](z)$$
$$= \frac{1+z^2}{1-z}\left[2\ln(1-z) - \ln z + \frac{3}{2}\right] + \frac{1}{2}(1+z)\ln z - (1-z) \qquad (5)$$

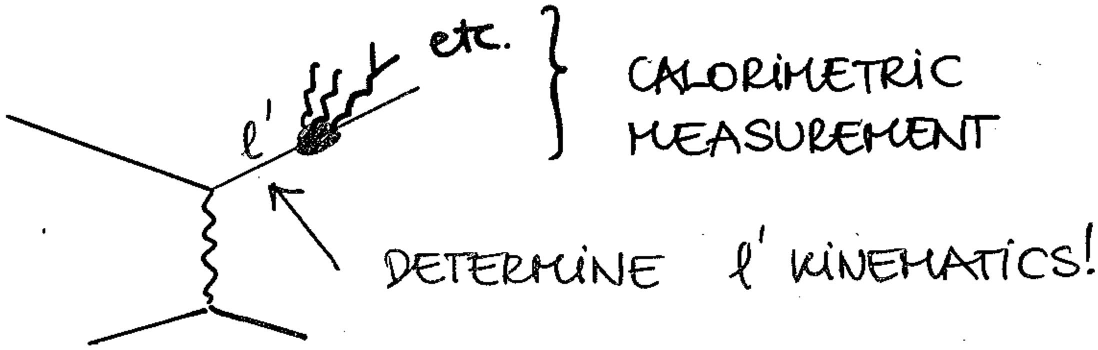
$$P_{ee}^{(2,2)}(z) = \frac{1}{2}\left[P_{e\gamma}^{(1)}\otimes P_{\gamma e}^{(1)}\right](z)$$
$$\equiv (1+z)\ln z + \frac{1}{2}(1-z) + \frac{2}{3}\frac{1}{z}(1-z^3) \qquad (6)$$

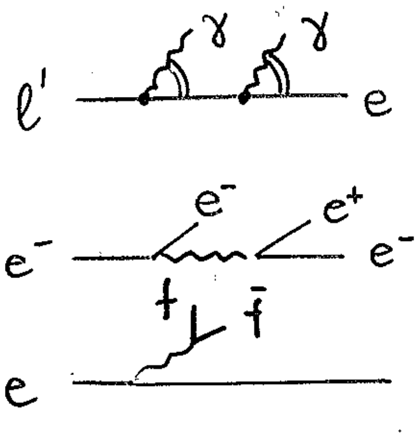
$$P_{ee,f}^{(2,3)}(z) = N_c(f)e_f^2\frac{1}{3}P_{ee}^{(1)}(z)\theta\left(1 - z - \frac{2m_f}{E_e}\right) \qquad (7)$$

$$\times\left[1 - \exp(-A^2\hat{Q}^2)\right] \quad \text{with} \quad A^2 = 3.37\,\mathrm{GeV}^{-2}. \qquad (12)$$

REMARK :

FSR : $\mathcal{O}(\alpha^2)$ FROM LEPTONS.

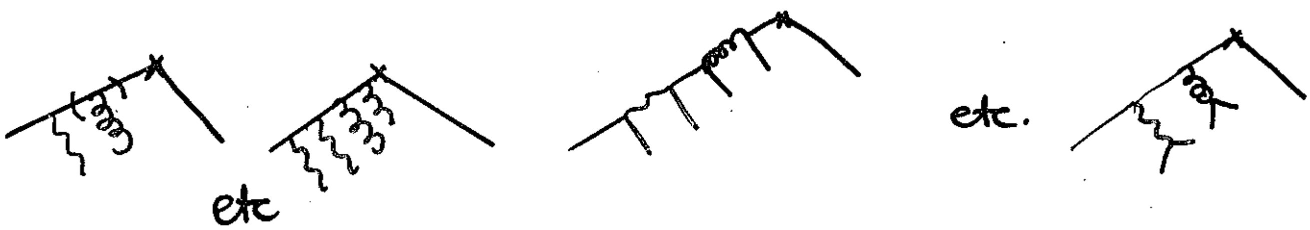




$$E_{\ell'} = E_e + E_{\gamma_i} + E_{e^+e^-} + \sum_i E_{f_i\bar{f}_i}$$

COLLECT ALL RADIATED ENERGY IN THE ANGULAR VICINITY OF e'!

RADIATION OF THE QUARK LINES:



⟹ ABSORB INTO SCALING VIOL. OF THE QUARK DISTR. (S & NS) (& RUNNING α_{QED} EFFECTS ARE TAKEN INTO ACC.).

SOFT EXPONENTIATION :

SOLVE : LO - GRIBOV LIPATOV eq. (NS) FOR $z \to 1$

$$D_{NS}(z,Q^2) = \zeta(1-z)^{\zeta-1}\frac{\exp\left[\frac{1}{2}\zeta\left(\frac{3}{2}-2\gamma_E\right)\right]}{\Gamma(1+\zeta)} \tag{8}$$

with

$$\zeta = -3\ln\left[1-(\alpha/3\pi)\ln(Q^2/m_e^2)\right] \tag{9}$$

(RUNNING α_{QED} !)

↓ THESE TERMS WERE TAKEN INTO ACC. ALREADY !

$$P_{ee}^{>2,\,soft}(z,Q^2) = D_{NS}(z,Q^2) - \frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right)\frac{2}{1-z}\left\{1+\frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right)\left[\frac{11}{6}+2\ln(1-z)\right]\right\} \tag{10}$$

and[6]

$$\frac{d^2\sigma^{(>2,soft)}}{dxdy} = \int_0^1 dz\,P_{ee}^{(>2)}(z)\left\{\theta(z-z_0)\mathcal{J}(x,y,z)\frac{d^2\sigma^{(0)}}{dxdy}\Big|_{x=\hat{x},y=\hat{y},s=\hat{s}} - \frac{d^2\sigma^{(0)}}{dxdy}\right\} \tag{11}$$

⟶ NOTE: NO 'UNIQUE' EXPONENTIATION EXISTS !

3. Numerical Results

- UPDATE: $O(\alpha)$ K-FACTORS

  ⟶ GOOD NUMERICAL AGREEMENT BETWEEN COMPLETE $O(\alpha)$ & $O(\alpha L)$.

- $O(\alpha^2 L^2)$ IN ALL VARIABLES

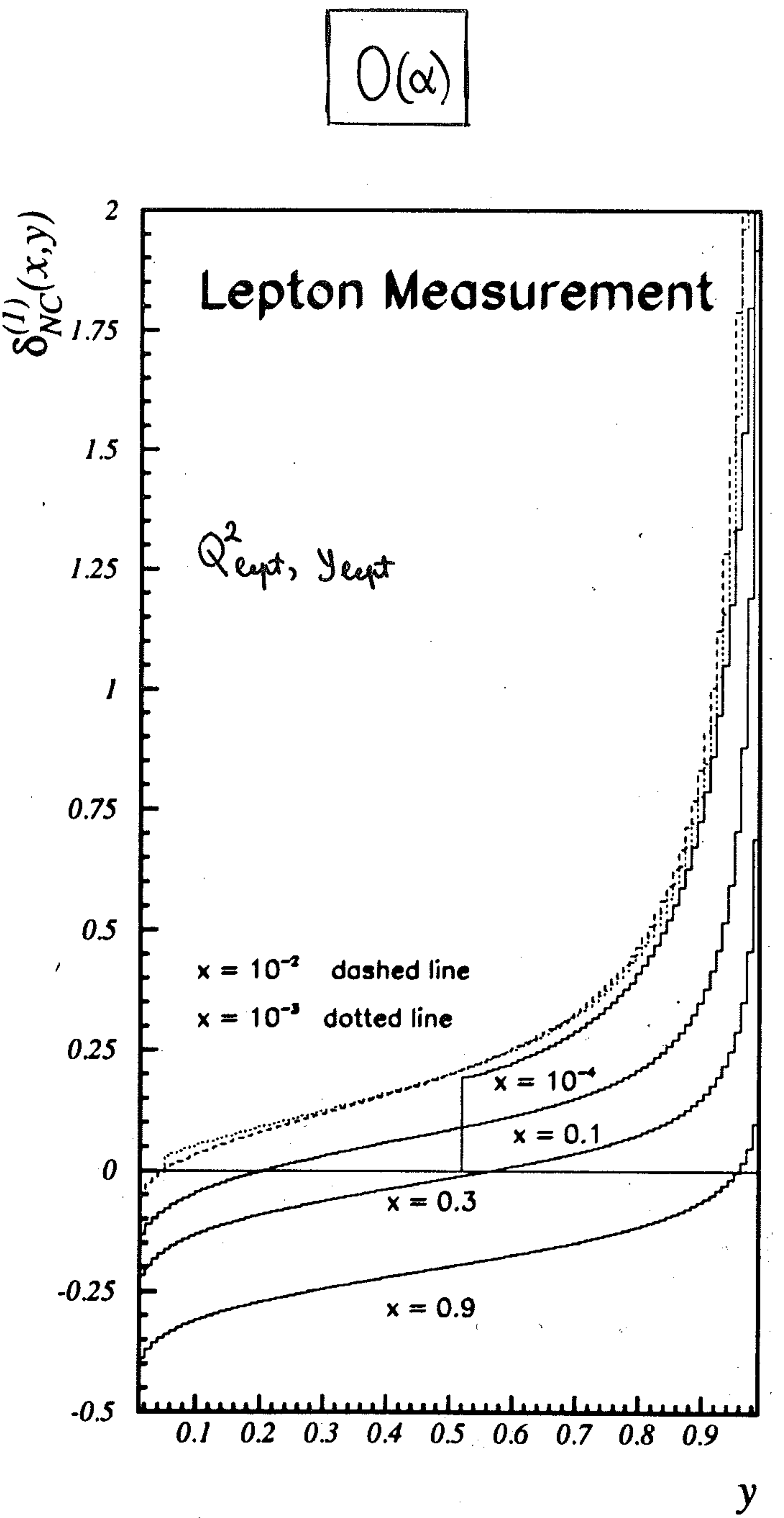
- CONSIDER ISR ONLY (LEPTONS)

  ⟶ FSR INTEGRATED BY THE CALORIMETRIC MEASUREMENT

  ⟶ QUARK LINE RAD ⟶ $O(\frac{\alpha}{\alpha_s})$ MOD. OF SCAL. VIOL.

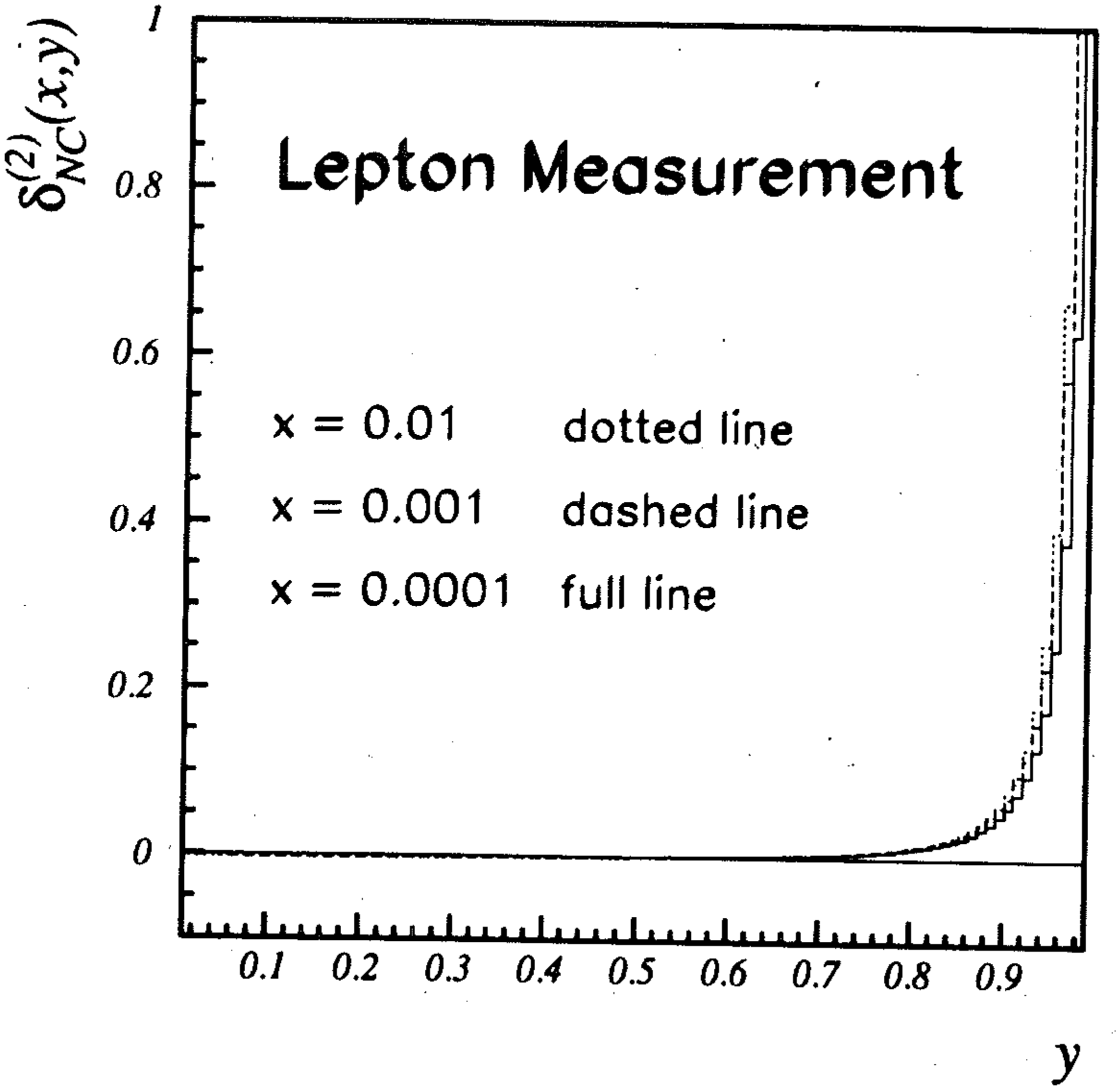
  ⟶ COMPTON PEAK: EXCLUS. TREATMENT !

- PDF'S: MRS D^-, SIM. RES: MRSH, CTEQ2.

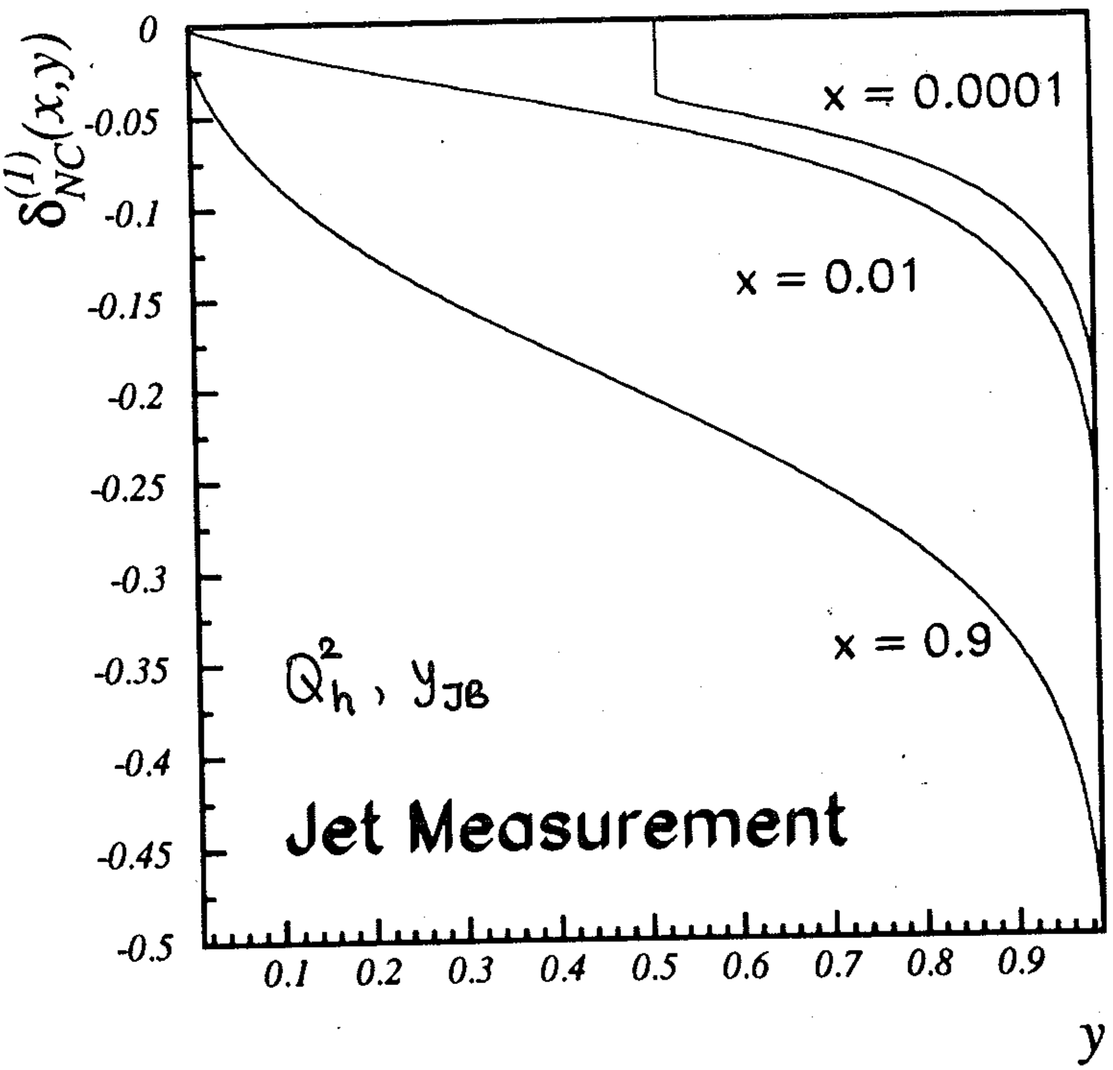


O(α)
Lepton Measurement
$\delta_{NC}^{(1)}(x,y)$
Q^2_{lept}, y_{lept}
x = 10⁻² dashed line
x = 10⁻³ dotted line
x = 10⁻⁴
x = 0.1
x = 0.3
x = 0.9
y
2
1.75
1.5
1.25
1
0.75
0.5
0.25
0
-0.25
-0.5
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

$O(\alpha^2 L^2)$



HIGH y RANGE:
IMPORTANT FOR THE
MEASUREMENT OF $F_L(x,Q^2)$
$\propto$ $G(x,Q^2)$!



$\delta_{NC}^{(1)}(x,y)$
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
-0.4
-0.45
-0.5
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
y
x = 0.0001
x = 0.01
x = 0.9
Q_h^2, y_{JB}
Jet Measurement

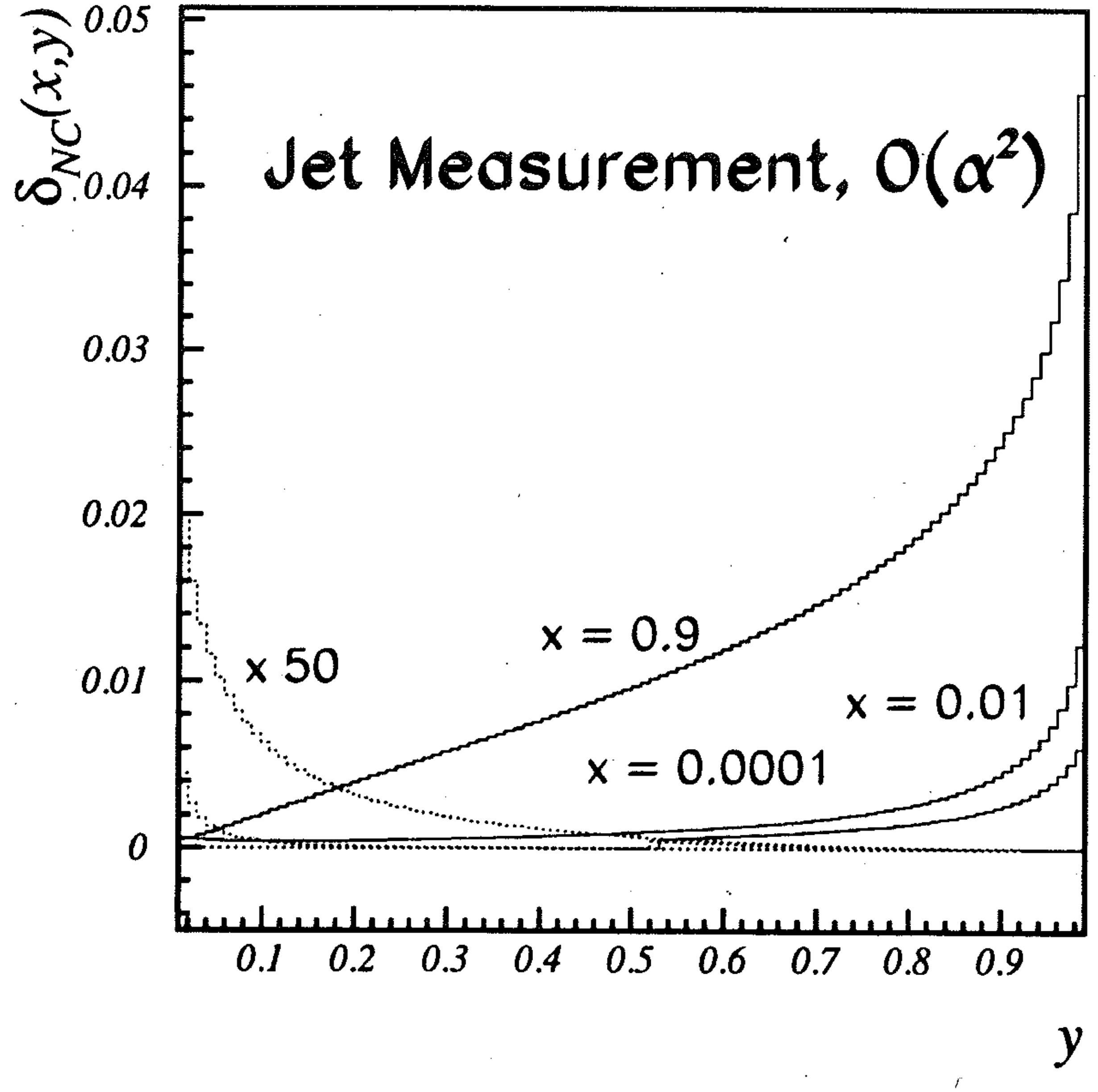
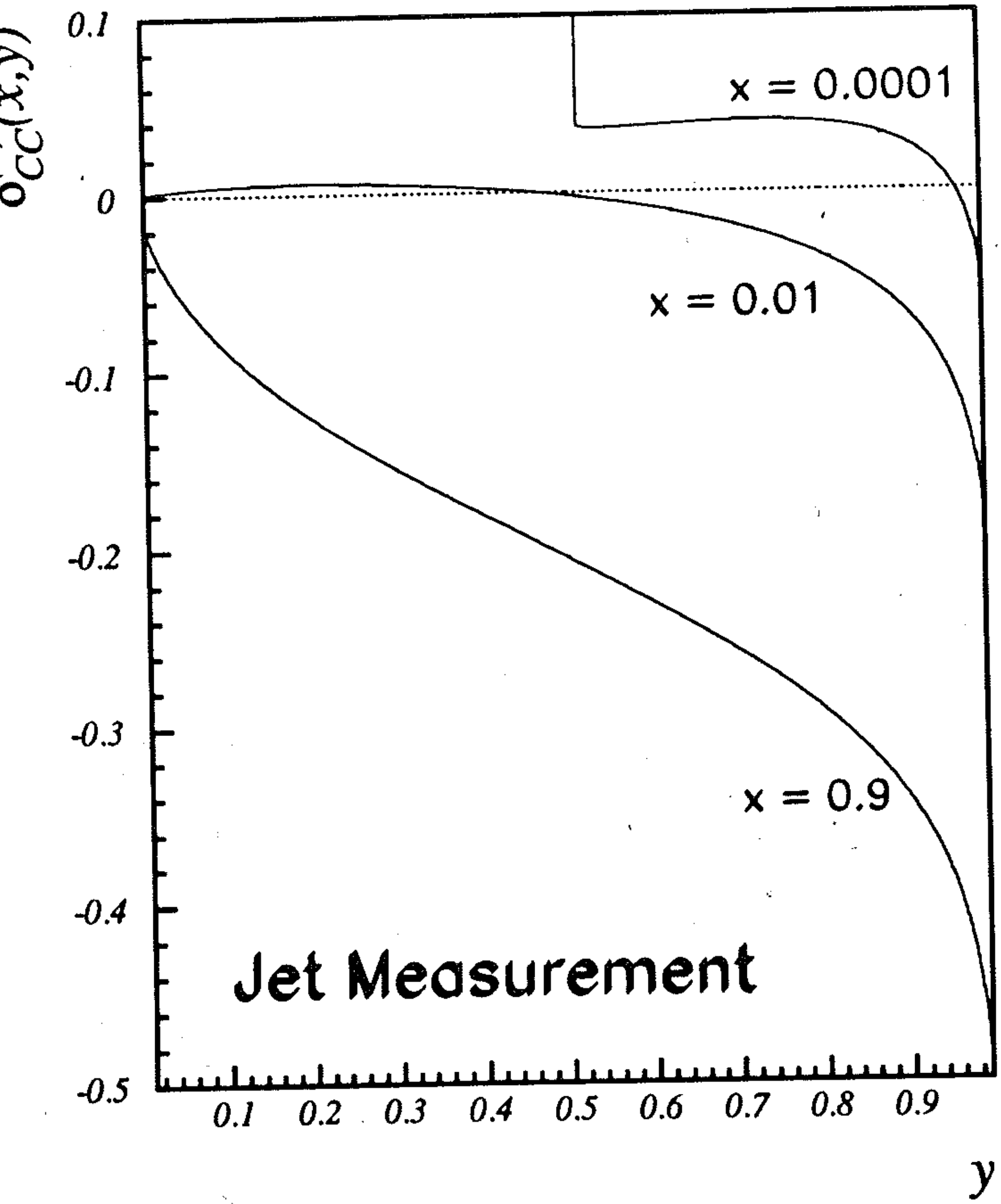


Figure 3: Leptonic initial state radiative corrections $\delta_{NC}(x,y) = (d\sigma^{(2+>2,soft)}/dxdy)/(d\sigma^0/dxdy)$ in LLA for e^-p deep inelastic scattering in the case of jet measurement for $\sqrt{s} = 314\,\text{GeV}$, $\mathcal{A} = 0$, and $Q^2 \geq 5\,\text{GeV}^2$. Full lines: $\mathcal{O}(\alpha^2)$ corrections; dotted lines: contributions due to $e^- \to e^+$ conversion eq. (13), $\delta_{NC}^{e^- \to e^+}(x,y) = (d\sigma^{(2,e^- \to e^+)}/dxdy)/(d\sigma^0/dxdy)$ scaled by $\times 50$; upper line: $x = 0.01$, middle line: $x = 0.0001$, lower line $x = 0.9$.



$\delta_{CC}^{(1)}(x,y)$
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
x = 0.0001
x = 0.01
x = 0.9
Jet Measurement
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
y

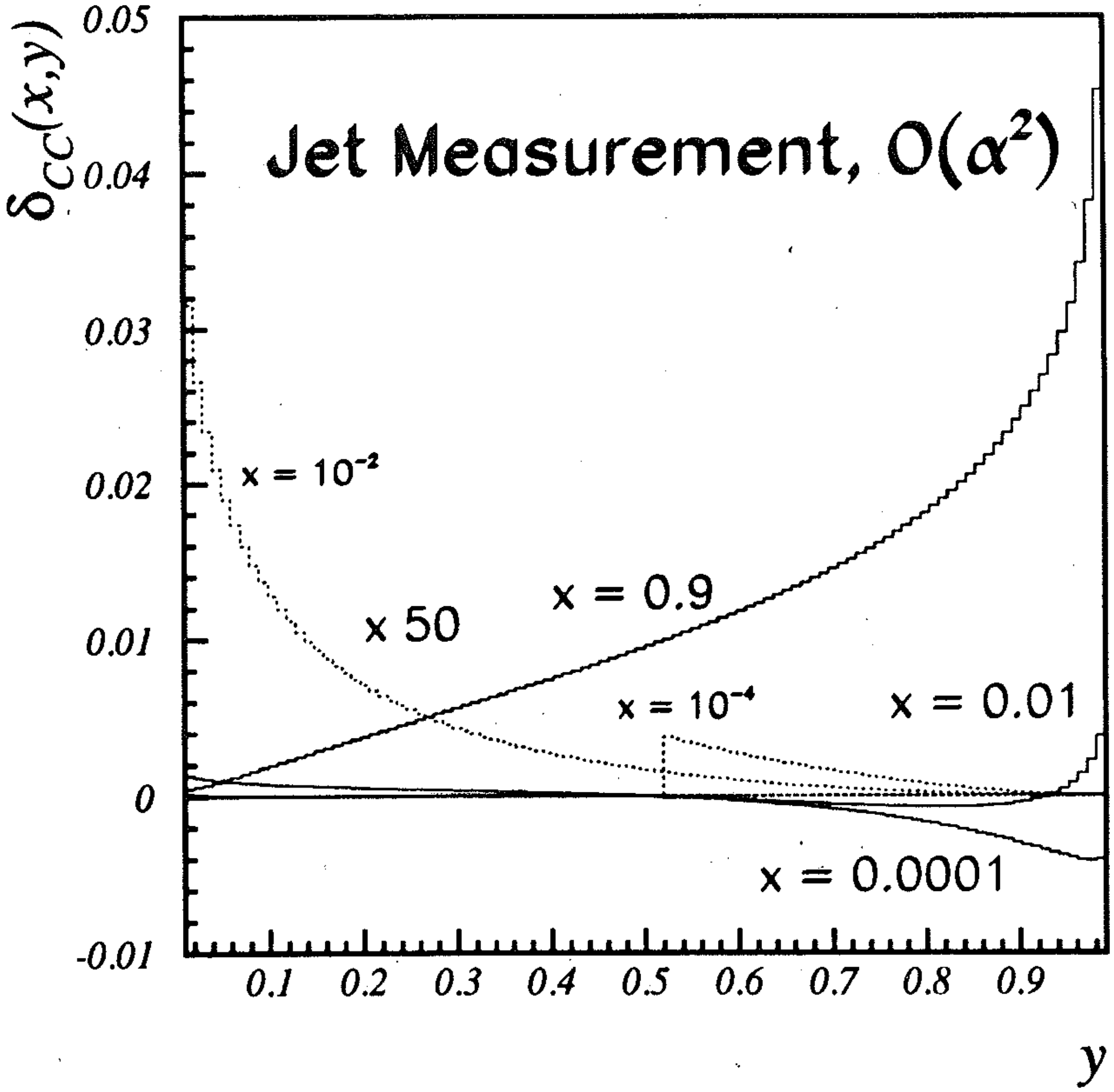
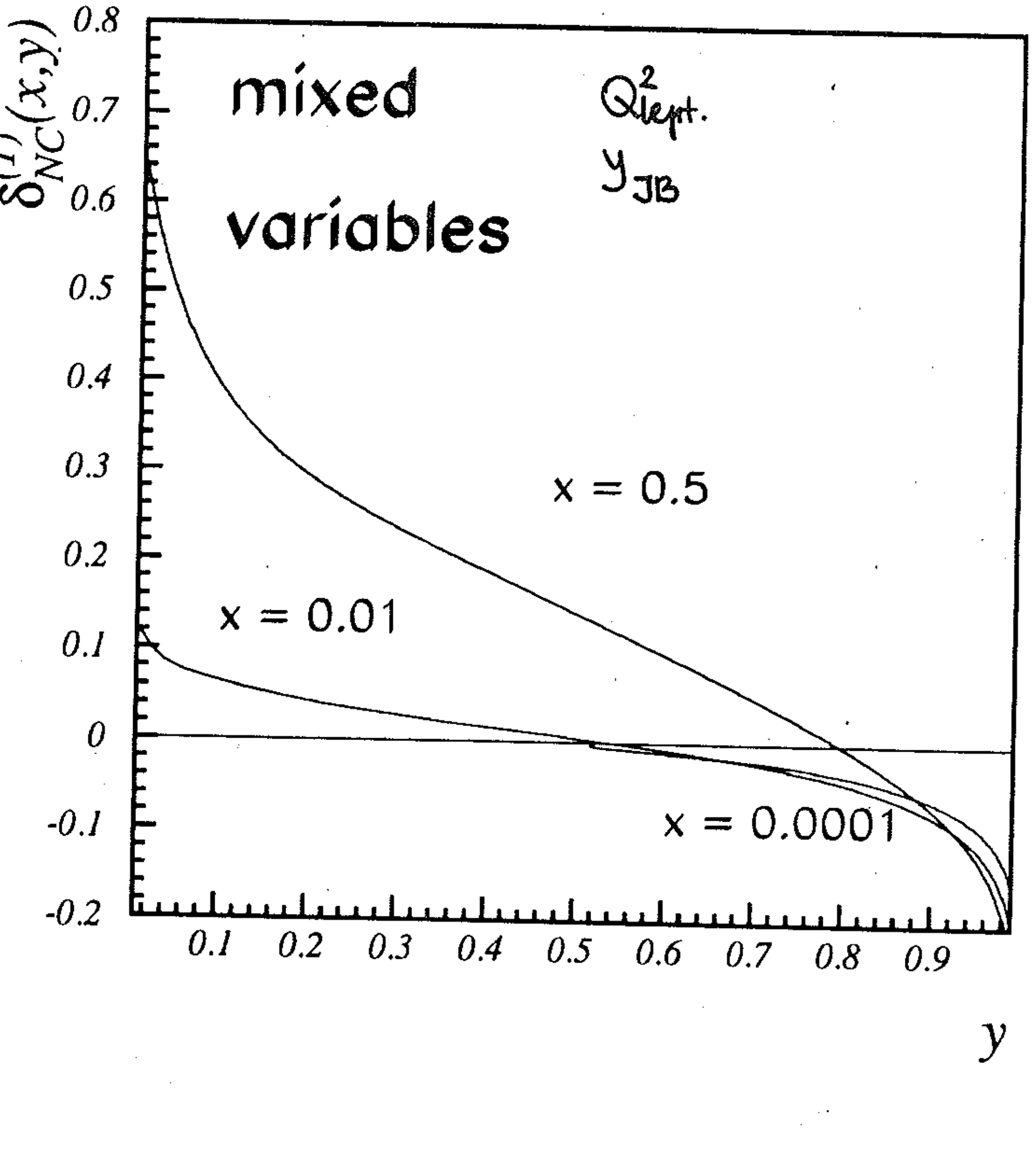


Figure 4: $\delta_{CC}(x,y) = (d\sigma_{CC}^{(2+>2,soft)}/dxdy)/(d\sigma_{CC}^{0}/dxdy)$ for deep inelastic e^-p scattering in the case of jet measurement. Dotted lines: $\delta_{CC}^{e^-\to e^+}(x,y)$. The other parameters are the same as in figure 3.



$\delta_{NC}^{(1)}(x,y)$
mixed
variables
$Q^2_{lept.}$
y_{JB}
x = 0.5
x = 0.01
x = 0.0001
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
y

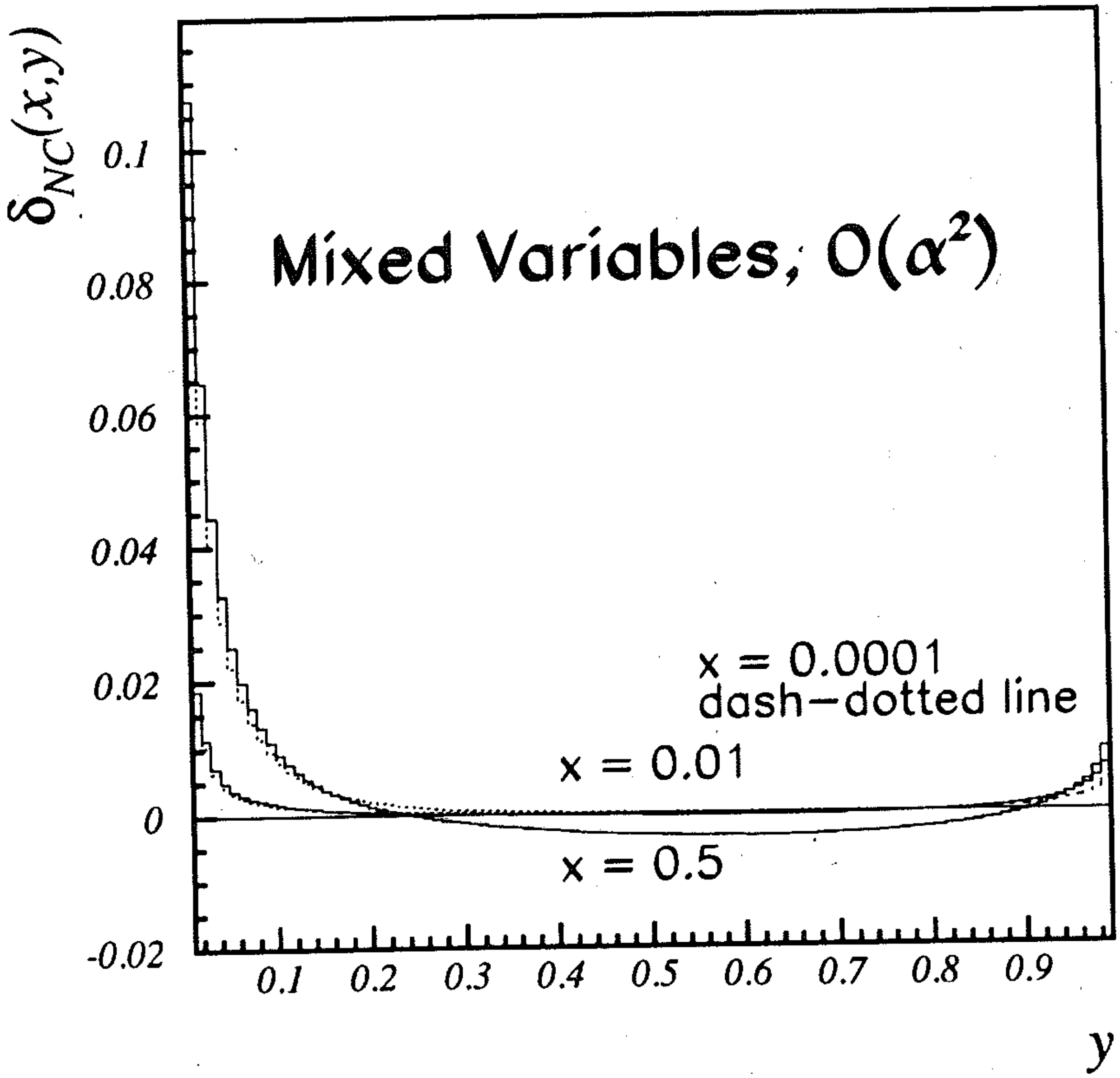


Figure 5: $\delta_{NC}(x,y)$ for the case of mixed variables. Dotted lines: $\delta_{NC}^{e^- \to e^+}(x,y)$; upper line: $x = 0.5$, lower line $x = 0.01$. The other parameters are the same as in figure 3.

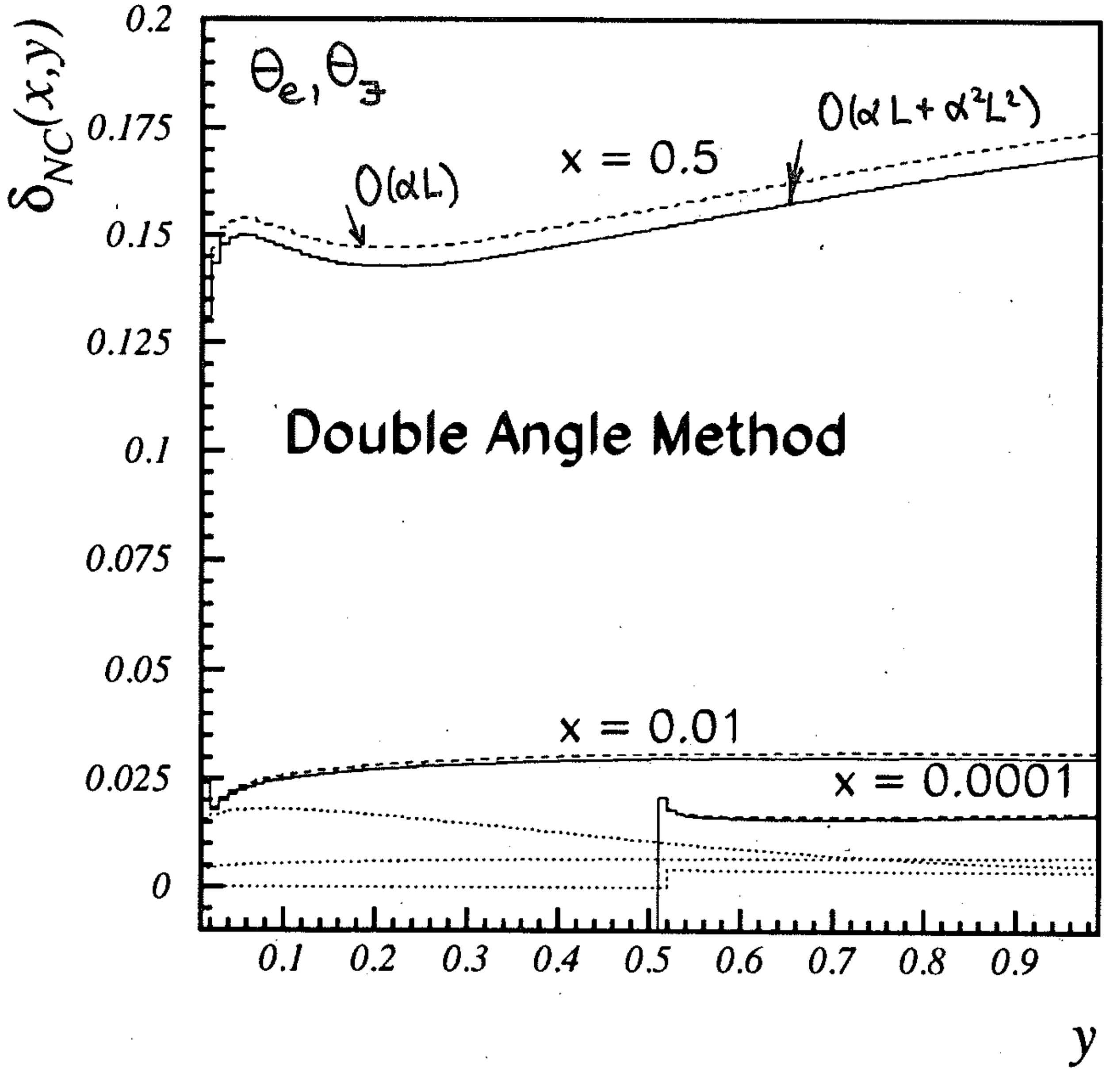


Figure 6: $\delta_{NC}(x,y)$ for the case of the double angle method for $A = 35\,\mathrm{GeV}$. Full lines: $\delta_{NC}^{(1+2+>2,soft)}(x,y)$, dashed lines: $\delta_{NC}^{(1)}(x,y)$. Dotted lines: $\delta_{NC}^{e^-\to e^+}(x,y)$ scaled by $\times 100$; upper line: $x = 0.5$, middle line: $x = 0.01$, lower line: $x = 0.0001$. The other parameters are the same as in figure 3.

A DANGEROUS CASE: θ_e & y_J

RESCALING: ISR

$$\hat{Q}^2 = Q^2 z \frac{z-y}{1-y}$$

$$\hat{x} = x \frac{z(z-y)}{1-y}$$

$$\boxed{z_0 = y}$$

ZEUS: $\boxed{z_0 = \max\left\{\frac{35\ \mathrm{GeV}}{2E_e}, y\right\}}$

$\delta_{NC}(x,y)$ JUMPS! AT $y \gtrsim \frac{\mathcal{A}}{2E_e}$, $\mathcal{A} = 35$ GeV.

$\frac{\sigma(Q^2 \& x \to 0)}{\sigma(Q^2, x)}$! NO CONTROL ON INPUT AT ALL !

→ UNFORTUNATE CHOICE OF VARIABLES.

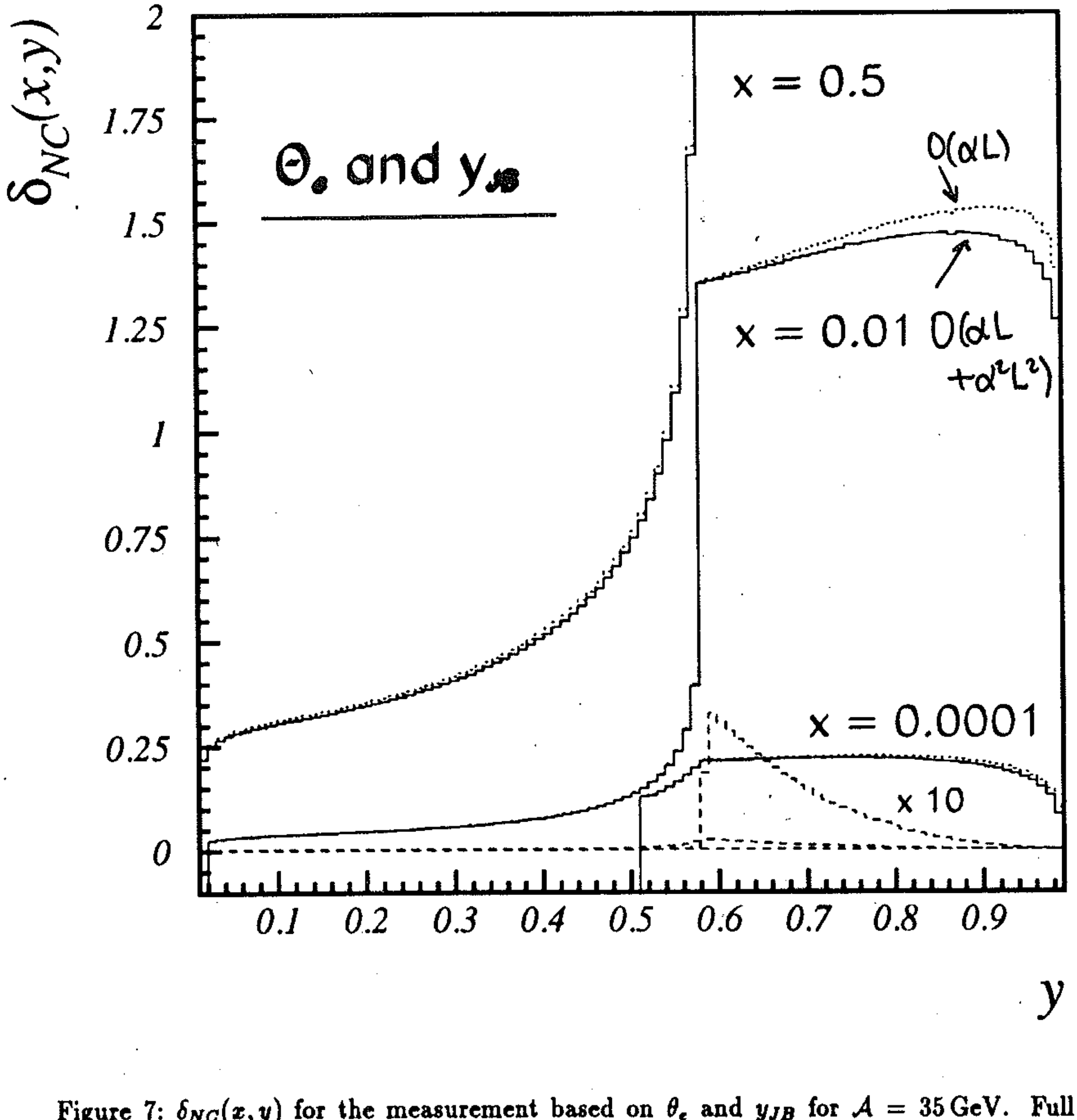
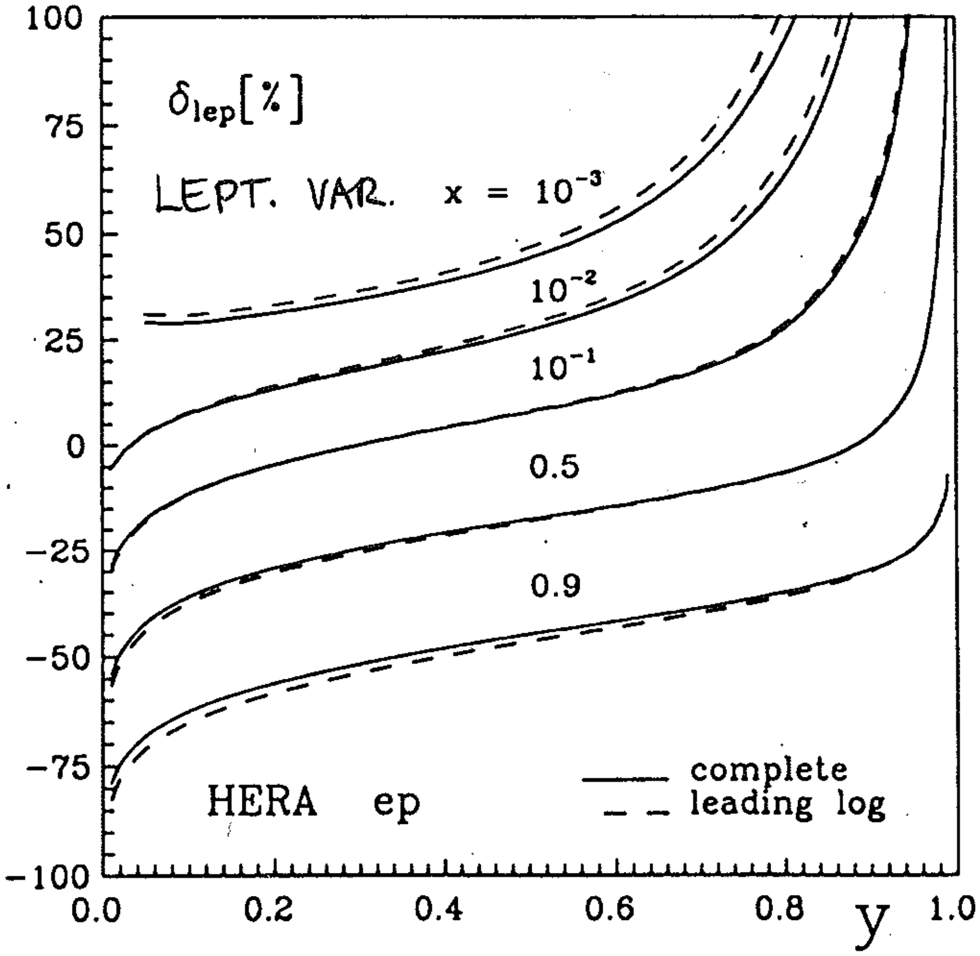
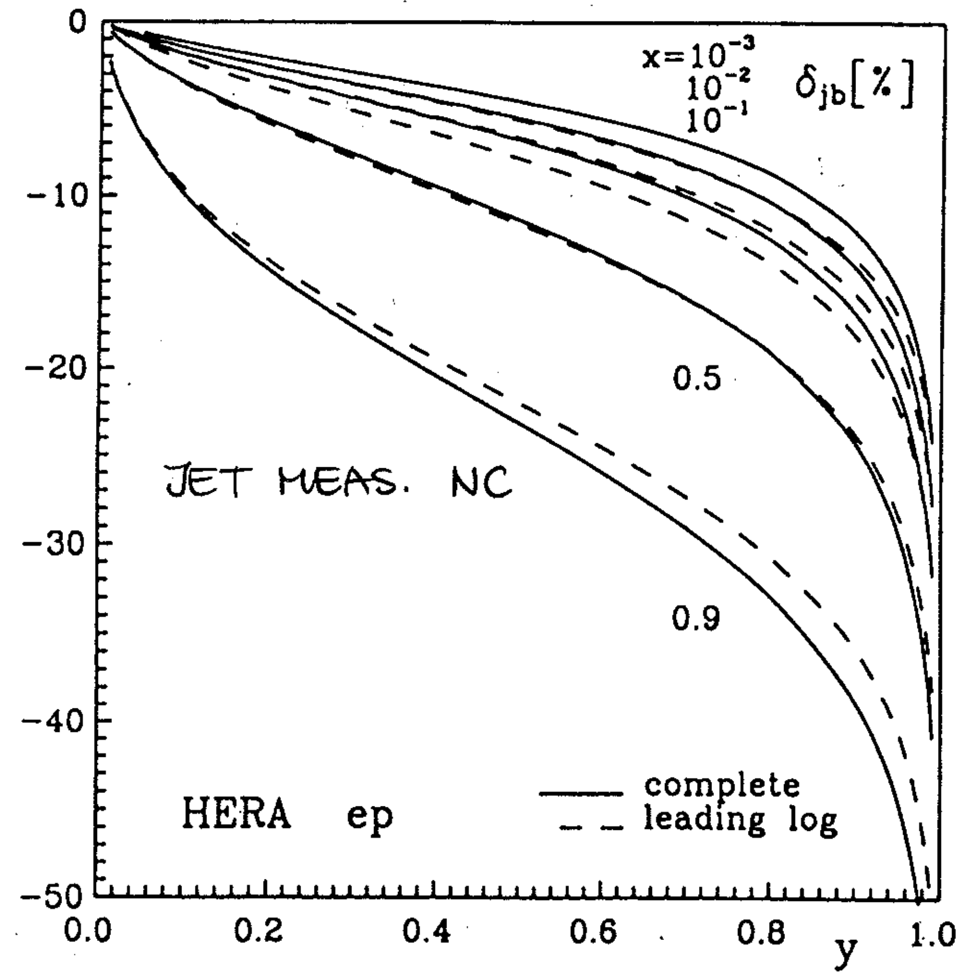
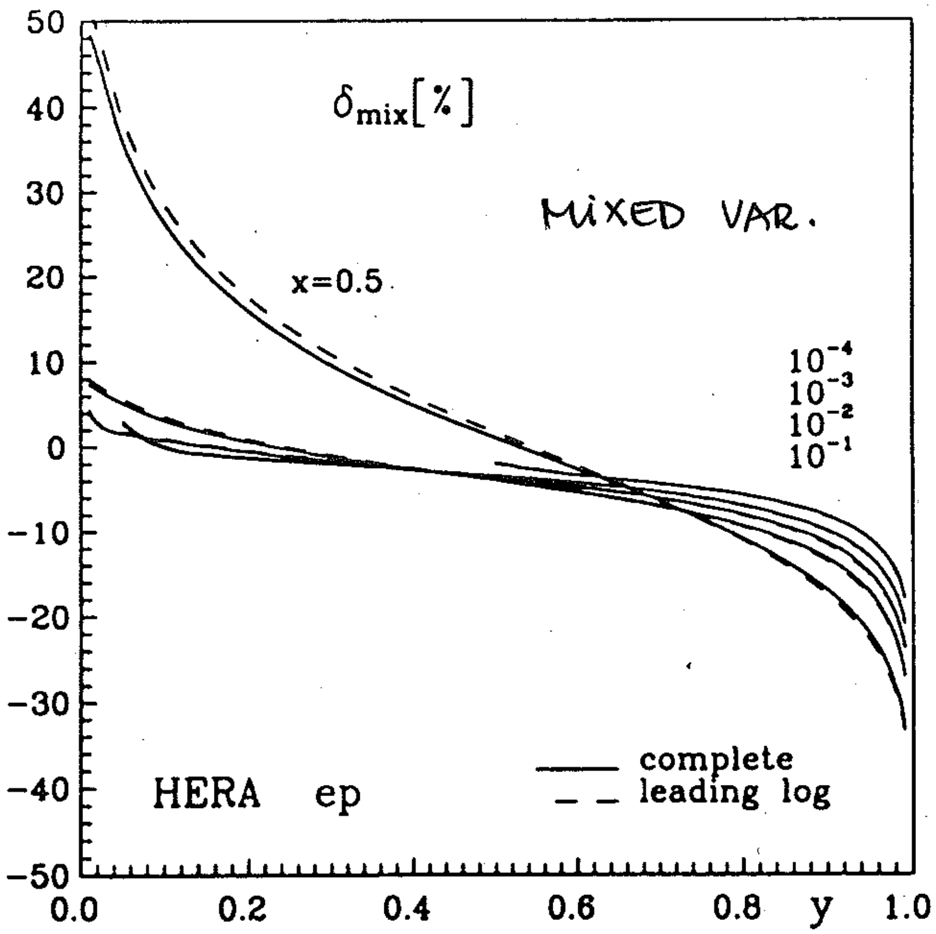


Figure 7: $\delta_{NC}(x,y)$ for the measurement based on θ_e and y_{JB} for $\mathcal{A} = 35\,\mathrm{GeV}$. Full lines: $\delta_{NC}^{(1+2+>2,soft)}(x,y)$, dotted lines: $\delta_{NC}^{(1)}(x,y)$. Dashed lines: $\delta_{NC}^{e^-\to e^+}(x,y)$; upper line: $x = 0.5$, middle line: $x = 10^{-2}$, lower line: $x = 10^{-4}$. The other parameters are the same as in figure 3.

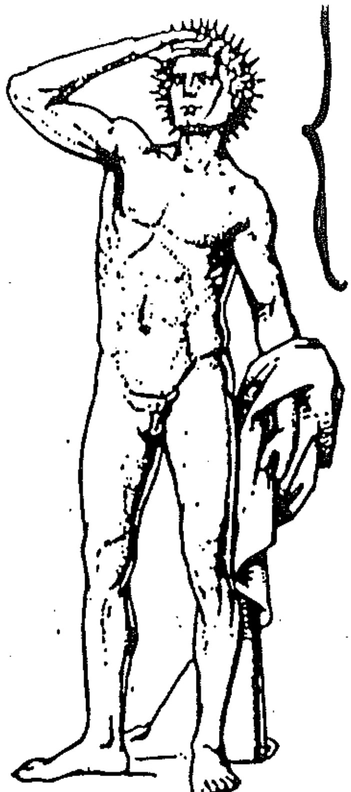
Comparison with a Full $\mathcal{O}(\alpha)$ Calculation
TERAD, D.Y. Bardin et al.







$\delta_{mix}[\%]$
MIXED VAR.
x=0.5
10^{-4}
10^{-3}
10^{-2}
10^{-1}
HERA ep
complete
leading log
y
50
40
30
20
10
0
−10
−20
−30
−40
−50
0.0
0.2
0.4
0.6
0.8
1.0



HADRON
ELECTRON
LEAD-
ING
ORDER
CORRECTIONS

⊕

TERAD 91
BARDIN
RIEMANN
AKHUDOV
CHRISTOVA
KALINOVSKAYA

HELIOS
JB.

⊕ UPGRADES QED
QCD
new variables ...

ARBUZOV
BARDIN
BLÜMLEIN
KALINOVSKAYA
RIEMANN

HECTOR

HADRON
ELECTRON
CODE
TO CALCULATE 1st &, HIGHER
ORDER
RADIATIVE CORRECTIONS

POLYDAMAS ADVICES HECTOR TO MAKE THE ASSAULT ONTO THE CAMP OF THE GREEKS ON FOOT.
engraving by J. FLAXMAN 1780ies.

4. Conclusions

1. THE $O(\alpha L)$ AND $O(\alpha^2 L^2)$ RADIATIVE CORRECTIONS HAVE BEEN CALCULATED FOR:
   - LEPTONIC VARIABLES
   - JET MEASUREMENT: NC & CC
   - MIXED VARIABLES
   - DOUBLE ANGLE METHOD
   - VARIABLES BASED ON θ_e, y_{JB}.

2. THE DOMINANCE & STABILITY OF RC'S IN $O(\alpha)$ IS ESTABLISHED, EXCEPT OF THE HIGH y RANGE FOR LEPT. VARIABLES & THE (θ_e, y_{JB}) CASE.

3. THE DOUBLE ANGLE METHOD IS THE IDEAL WAY TO MEASURE $d^2\sigma/dxdy^{BORN}$ WITH RESPECT TO RC'S, DUE TO THEIR FLAT BEHAVIOUR & SMALLNESS.

4. THE METHOD BASED ON θ_e & y_{JB} IS PROBLEMATIC DUE TO A JUMP AT THE CUT THRESHOLD y_{crit}. THE REASON FOR THIS IS THE MAPPING $d^2\hat{\sigma} \longrightarrow d^2\hat{\sigma}(x=0, Q^2=0)$.

5. THE INCLUSION OF THE $O(\alpha^2 L^2)$ IS REQU. TO REACH ACCURACY AT THE % LEVEL.

6. THE USE OF RGE METHODS CONSIDERABLY SIMPLIFIES THE CALCULATION OF DOMINANT TERMS & PROVIDES A FASTER WAY TO RECOGNIZE INSTABLE MAPPINGS UNDER RC IF COMPARED TO FULL CALCULATIONS.