Theoretical Status of Nucleon Structure Functions

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DESY

1. Introduction
2. Scaling Violations up to 3 Loops
3. Small x Resummation
4. Recombination Corrections
5. Polarised Structure Functions
6. Some Sum Rules
7. Scaling Violations in Diffractive Scattering
8. Structures behind Feynman Diagrams
9. Conclusions

1. Introduction

GOALS:

PDF'S : UNPOLARIZED $\longrightarrow$ ~1% $u_v \ldots d_v$

some % G, S

POLARIZED $\longrightarrow$ Δu_v, Δd_v

ΔG

as precise as possible

$\longrightarrow$ unfold: INDIV. SEA QUARK DENSITIES

$\longrightarrow$ MEASURE δq

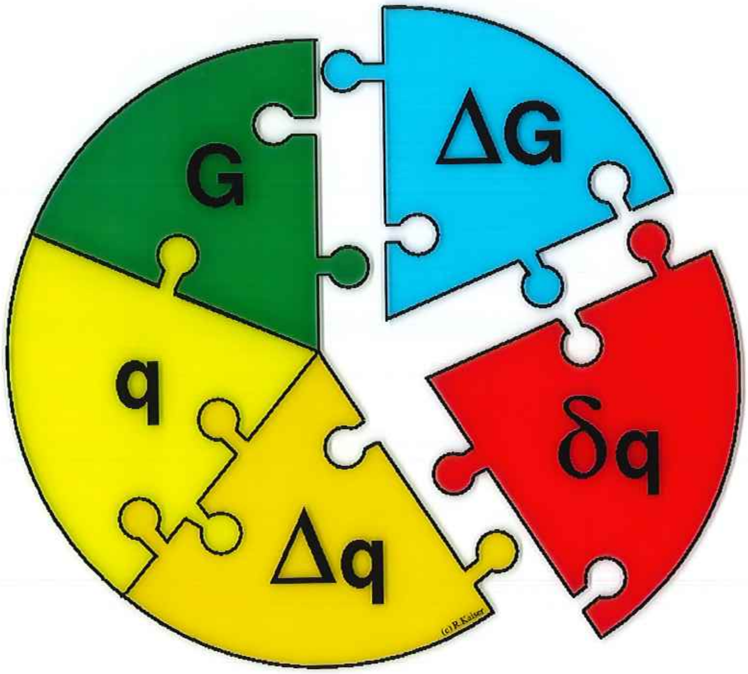
$\longrightarrow$ ... TWIST 3, SEE TWIST 4 ?? (PERHAPS)

$$\Rightarrow \Delta \alpha_s(M_Z) = \pm 1\%$$

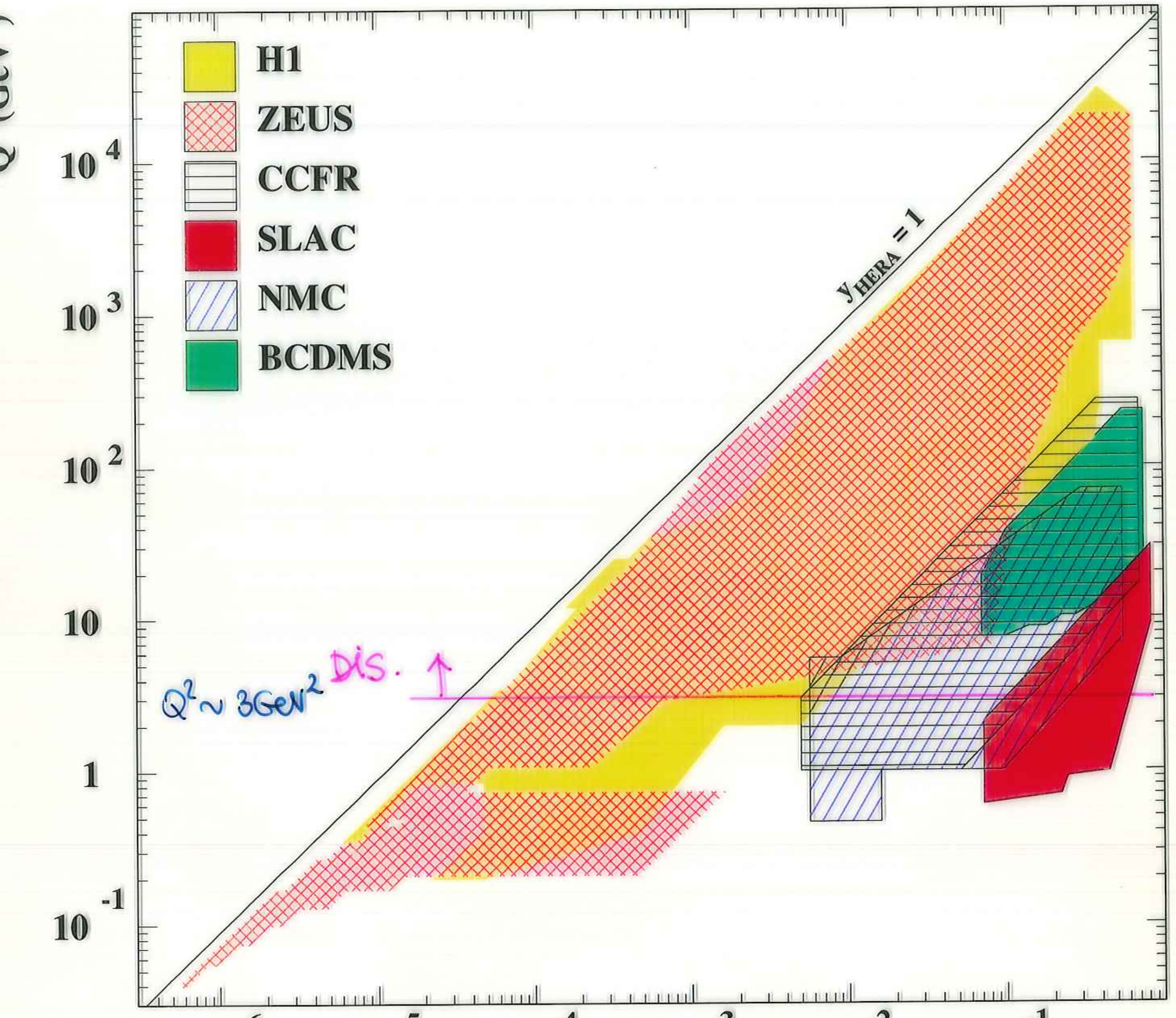
Motivation

WHY DO WE STUDY POLARIZED DEEP INELASTIC SCATTERING ?

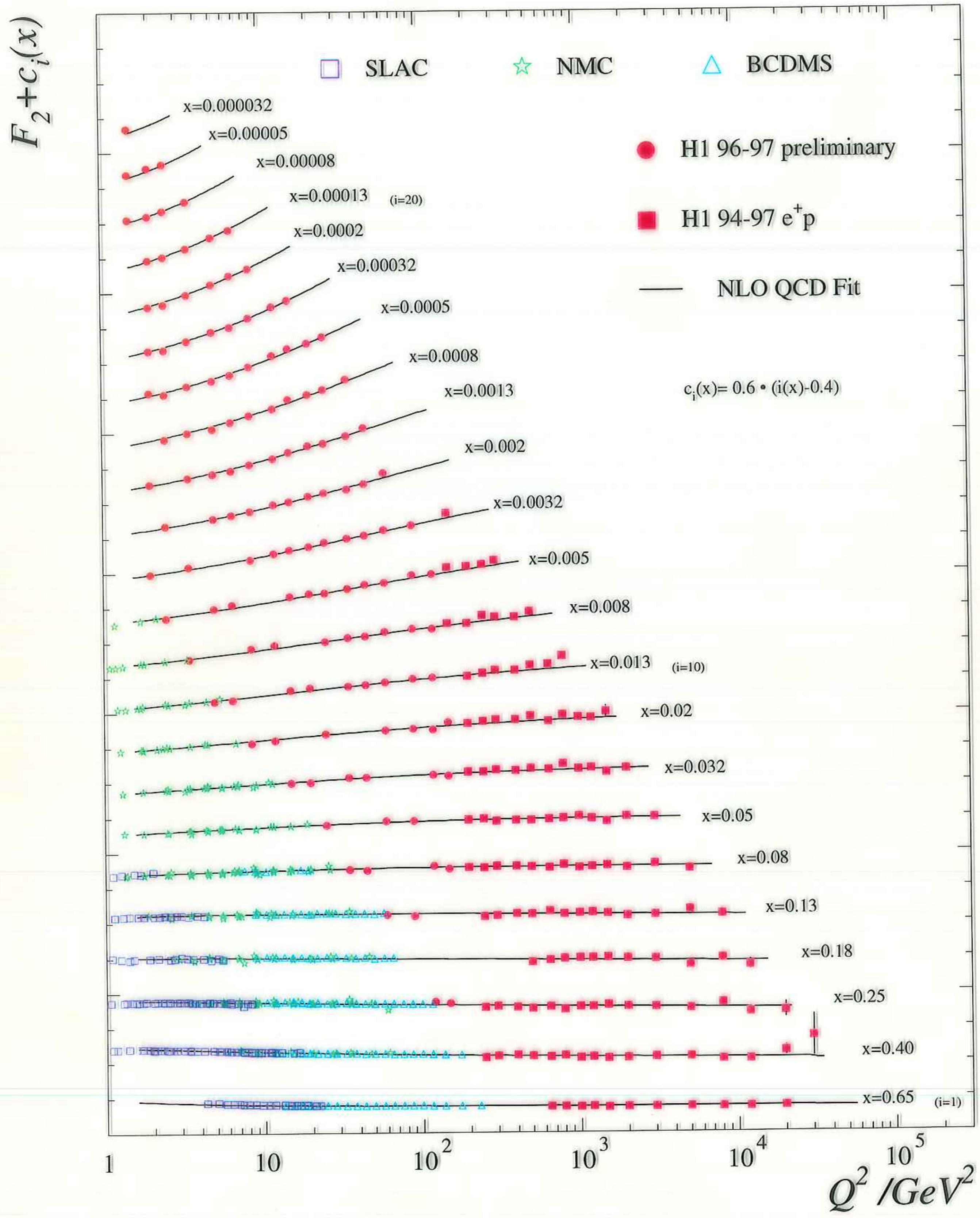
- Study of short distance structure of nucleon spin
- Understand and finally solve the spin puzzle



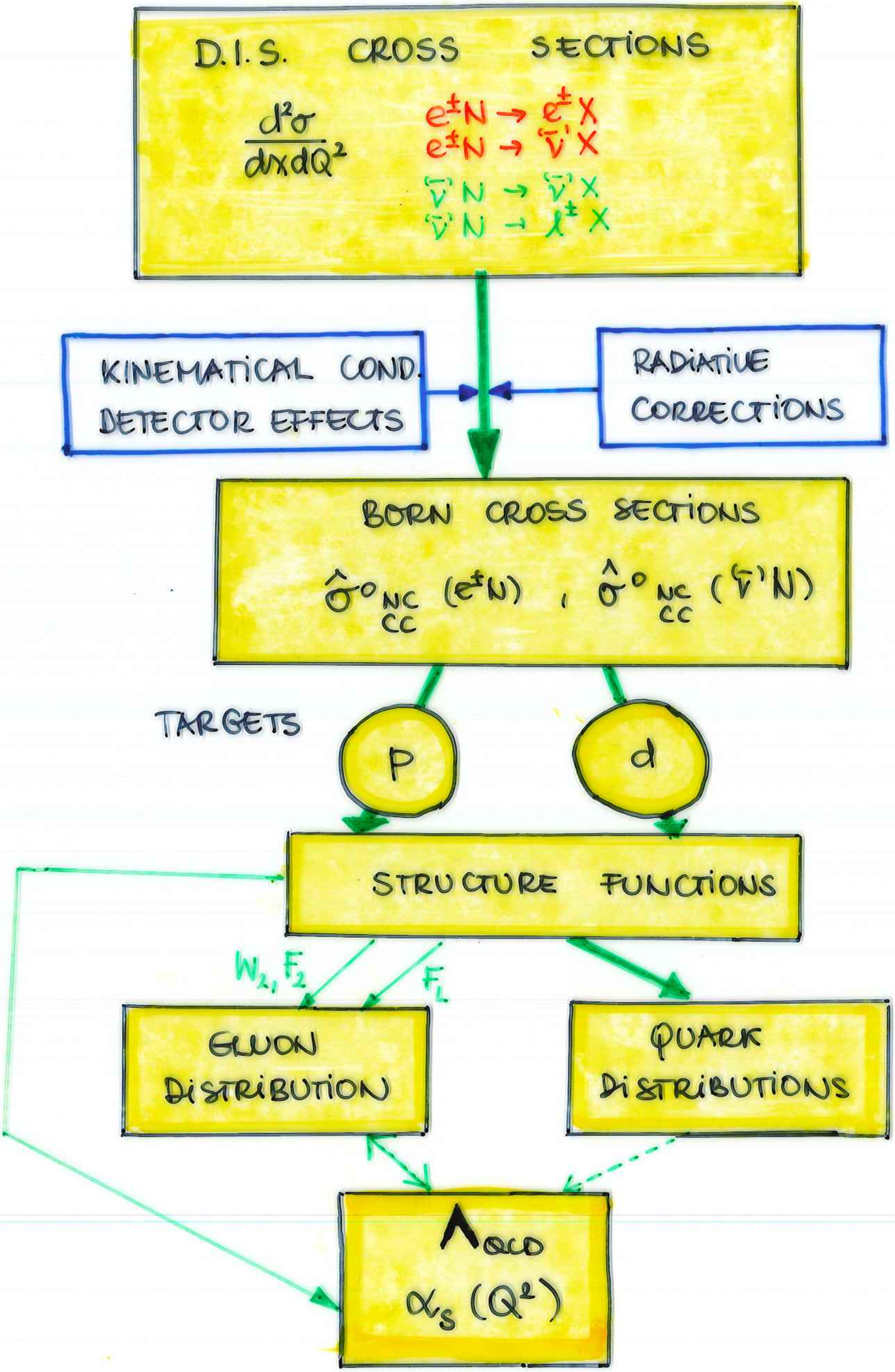
- Test of perturbative QCD in spin sector: Λ_{QCD}
- Test of fundamental and less fundamental sum rules



Q^2 (GeV^2)
H1
ZEUS
CCFR
SLAC
NMC
BCDMS
$y_{HERA} = 1$
10^4
10^3
10^2
10
1
10^{-1}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
x
$Q^2 \sim 3GeV^2$
DIS.

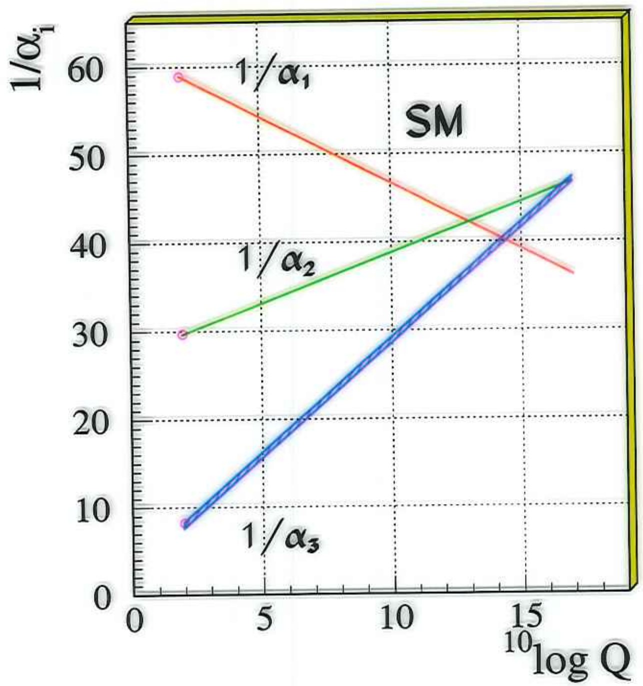


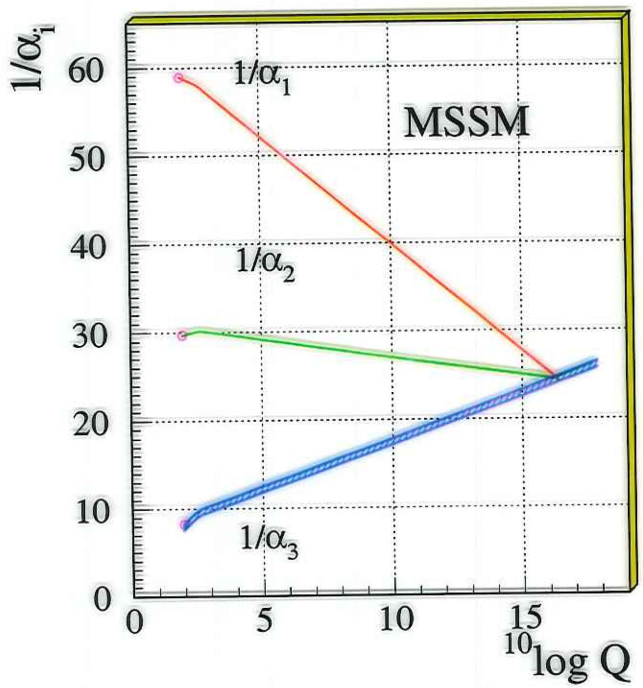
$F_2+c_i(x)$
SLAC
NMC
BCDMS
H1 96-97 preliminary
H1 94-97 e+p
NLO QCD Fit
$c_i(x)= 0.6 \cdot (i(x)-0.4)$
x=0.000032
x=0.00005
x=0.00008
x=0.00013
(i=20)
x=0.0002
x=0.00032
x=0.0005
x=0.0008
x=0.0013
x=0.002
x=0.0032
x=0.005
x=0.008
x=0.013
(i=10)
x=0.02
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.40
x=0.65
(i=1)
1
10
10^2
10^3
10^4
10^5
$Q^2\ /GeV^2$



D.I.S. CROSS SECTIONS
$\frac{d^2\sigma}{dx\,dQ^2}$
$e^{\pm}N \to e^{\pm}X$
$e^{\pm}N \to \overset{(-)}{\nu} X$
$\overset{(-)}{\nu} N \to \overset{(-)}{\nu} X$
$\overset{(-)}{\nu} N \to l^{\pm} X$
KINEMATICAL COND.
DETECTOR EFFECTS
RADIATIVE CORRECTIONS
BORN CROSS SECTIONS
$\hat{\sigma}^{0}_{NC,CC}(e^{\pm}N)$, $\hat{\sigma}^{0}_{NC,CC}(\overset{(-)}{\nu}N)$
TARGETS
p
d
STRUCTURE FUNCTIONS
W_2, F_2
F_L
GLUON DISTRIBUTION
QUARK DISTRIBUTIONS
Λ_{QCD}
$\alpha_s(Q^2)$

Unification of the Coupling Constants in the SM and the minimal MSSM





de Boer '02

CURRENTLY : $\Delta \alpha_s (M_Z^2)_{TH} = \pm 5\%$

WANTED : 1%

⟶ QCD @ 3 LOOPS

2. Scaling Violations up to 3 Loops

- WHAT DO WE KNOW ?
- APPLICATIONS TO DATA
- CONSEQUENCES FOR α_s etc.
- COMPARISON WITH LATTICE RESULTS

3. QCD PERTURBATION THEORY TO $O(\alpha_s^3)$

HOW CAN WE MEASURE $\alpha_s(Q^2)$ FROM THE SCALING VIOLATIONS OF STRUCTURE FUNCTIONS?

$$F_j(x, Q^2) = \hat{f}_j(x) \otimes \sigma_i^j(\alpha_s, Q^2/\mu^2, x)$$

↑ BARE PARTONS ↑ SUBSYSTEM CROSS CT.

$$= \hat{f}_i(x) \otimes \Gamma_k^i\left(\alpha_s, \frac{M^2}{\mu^2}, \frac{M^2}{R^2}\right) \otimes C_j^k\left(\alpha_s(R^2), \frac{Q^2}{M^2}, \frac{M^2}{R^2}, x\right)$$

↑ OME ↑ WILSON COEFF.

finite.

$$F_j(N) = \int_0^1 dx\, x^{N-1} F_j(x) \qquad \text{(DIAG.)}$$

2 RGE's:

- RENORMALIZATION SC. R (UV)
- FACTORIZATION SC. M (COLLINEAR)

MAIN OBJECTIVES :

- PRECISE MEASUREMENT OF $\alpha_s(M_Z)$
- REVEAL POLARIZED & UNPOLARIZED PARTON DENSITIES WITH HIGH PRECISION
- FIND NOVEL STRUCTURES
- PROBE QCD

→ PERTURBATIVE QCD : HO
NEW TECHNOLOG.

→ LATTICE STUDIES

SOURCES OF SCALING VIOLATION:

- HIGHER LOOP ORERS α_s^n
- HIGHER TWIST OPERATORS $(\Lambda_i^2/Q^2)^N$
- QUARK MASS TERMS (c,b) $(m_q^2/Q^2)^N$
- TARGET MASS CORRECTIONS $(M^2/Q^2)^N$

⟶ LEADING TWIST IS TO BE ISOLATED

CALCULATE, CONSTRAIN & ELIMINATE THE OTHER TERMS.

$\Longrightarrow$ $\alpha_s(Q^2)$

$q^2 < 0$

AT HIGH PRECISION.

4.1. The Running Coupling Constant

RGE FOR THE STRONG COUPLING CONSTANT:

$$a_s = g_s^2 / 16\pi^2$$

$$\frac{\partial a_s}{\partial \log \mu^2} = -\beta_0 a_s^2 - \beta_1 a_s^3 - \beta_2 a_s^4 - \beta_3 a_s^5 + O(a_s^6)$$

COLOR FACTORS: (QCD: $N_c = 3$, $N_A = 8$)

$\beta_0, \beta_1, \beta_2$ } $C_A = N_c$

$$C_F = \frac{N_c^2 - 1}{2N_c}, \quad T_F = \frac{1}{2}$$

β_3

firstly at 4-loops.

$$\frac{d_A^{abcd} d_A^{abcd}}{N_A} = \frac{N_c^2(N_c^2 + 36)}{24}$$

$$\frac{d_A^{abcd} d_F^{abcd}}{N_A} = \frac{N_c(N_c^2 + 6)}{48}$$

$$\frac{d_F^{abcd} d_F^{abcd}}{N_A} = \frac{N_c^4 - 6N_c^2 + 18}{96 N_c^2}$$

$$\beta_0 = 11 - \frac{2}{3} N_f$$

> 0

GROSS, WILCZEK 1973

POLITZER 1973

T'HOOFT

DISCOVERY OF ASYMPTOTIC FREEDOM

$$\beta_1 = 102 - \frac{38}{3} N_f$$

CASWELL 1974

JONES 1974

FIG

$$\beta_2 = \frac{2857}{2} - \frac{5033}{18} N_f + \frac{325}{54} N_f^2$$

TARASOV, VLADIMIROV, ZHARKOV 1980

LARIN, VERMASEREN 1993

$$\beta_3 = \left(\frac{149753}{6} + 3564\,\zeta_3\right) - \left(\frac{1078361}{162} + \frac{6508}{27}\,\zeta_3\right) N_f + \left(\frac{50065}{162} + \frac{6472}{81}\,\zeta_3\right) N_f^2 + \frac{1093}{729} N_f^3$$

VAN RITBERGEN, VERMASERN, LARIN 1997.

4.2. The Splitting Functions and Evolution Equations

$O(\alpha_s)$: LO UNPOLARIZED

$$P^{(0)}_{NS}(z) \equiv P^{(0)}_{qq}(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$$

$$P^{(0)}_{qg}(z) = T_f\left[(1-z)^2 + z^2\right]$$

$$P^{(0)}_{gq}(z) = C_F\,\frac{1+(1-z)^2}{z}$$

$$P^{(0)}_{gg}(z) = 2C_G\left[\frac{1-z}{z} + \frac{z}{(1-z)_+} + z(1-z) + \frac{1}{2}\beta_0\delta(1-z)\right]$$

GROSS, WILCZEK 1973

GEORGI, POLITZER 1973

LIPATOV 1975

DOKSHITSER 1977

ALTARELLI, PARISI 1977

KIM, SCHILCHER 1977, 78

$$\int_0^1 dz\, z^{N-1} P^{(0)}_{ab}(z) = -\frac{1}{4}\gamma^{(0)}_{ab}(N)$$

CONNECTION TO ANOMALOUS DIMENSIONS.

$O(\alpha_s)$: LO POLARIZED

$$\hat{P}^{(0)}_{NS,qq}(z) = \hat{P}^{(0)}_{qq,S}(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$$

$$\hat{P}^{(0)}_{qg}(z) = T_f\left[4z - 2\right]$$

$$\hat{P}^{(0)}_{gq}(z) = C_F\left[2 - z\right]$$

$$\hat{P}^{(0)}_{gg}(z) = C_A\left[\left(\frac{2}{1-z}\right)_+ + 2 - 4z + \frac{1}{2}\beta_0\,\delta(1-z)\right]$$

ITO 1975

K. SASAKI 1975

AHMED, G. ROSS 1975, 76

ALTARELLI, PARISI 1977

NO TERMS $\propto \frac{1}{z}$.

$O(\alpha_s^2)$: NLO UNPOLARIZED

FLORATOS, D ROSS, SACHRAIDA 1977-79

CURCI, FURMANSKI, PETRONZIO 1980

FURMANSKI, PETRONZIO 1980

GONZALEZ-ARROYO, LOPEZ, YNDURAIN 1979, 80

FLORATOS, KOUNNAS, LACAZE 1981abc

VAN NEERVEN, HAMBERG 1992

∴ NS :

$$P_{qq}^{(1)}(z) = \underline{C_F^2}\, P_F(z) + \underline{\tfrac{1}{2} C_F C_A}\, P_G(z) + \underline{C_F N_f T_f}\, P_{N_f}(z)$$

$$P_{q\bar{q}}^{(1)}(z) = \underline{\left[C_F^2 - \tfrac{1}{2} C_F C_A\right]}\, P_A(z)$$

$$P_F(z) = -2\left(\frac{1+z^2}{1-z}\right)\ln z \ln(1-z) - \left(\frac{3}{1-z} + 2z\right)\ln z - \frac{1}{2}(1+z)\ln^2 z$$

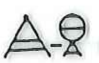
$$P_G(z) = \left(\frac{1+z^2}{1-z}\right)\left[\ln^2 z + \frac{11}{3}\ln z + \frac{67}{9} - \frac{1}{3}\pi^2\right] + 2(1+z)\ln z + \frac{40}{3}(1-z)$$

$$P_{N_f}(z) = -\frac{2}{3}\left[\frac{1+z^2}{1-z}\left(\ln z + \frac{5}{3}\right) + 2(1-z)\right]$$

$$P_A(z) = 2\,\frac{1+z^2}{1-z}\int_{z/(1+z)}^{1/(1+z)} \frac{du}{u}\ln\left(\frac{1-u}{u}\right) + 2(1+z)\ln z + 4(1-z)$$

TABLE 1

Detailed contribution of various diagrams to $\Gamma_{qq}(x, \alpha, 1/\epsilon)$

		C_F^2						$\frac{1}{2}C_F C_G$					$\frac{1}{2}N_F C_F$
	$\Gamma_{qq}(x, \alpha, 1/\epsilon)$						SUM					SUM	
$1/\epsilon$ A	$\frac{1+x^2}{1-x}$	$7-\frac{2}{3}\pi^2$	$-7+\frac{2}{3}\pi^2$	0	0	0	0	$7-\frac{2}{3}\pi^2$	0	$-11+\pi^2$	$\frac{103}{9}-\frac{2}{3}\pi^2$	$\frac{67}{9}-\frac{1}{3}\pi^2$	$-\frac{10}{9}$
	$\frac{1+x^2}{1-x}\ln^2 x$	-2	1	-1	2	0	0	-1	1	1	0	1	0
	$(1+x)\ln x$	0	$-\frac{7}{2}$	2	-1	0	$-\frac{5}{2}$	$\frac{7}{2}$	-2	$\frac{1}{2}$	0	2	0
	$1-x$	3	-11	0	3	0	-5	11	0	-1	$\frac{10}{3}$	$\frac{40}{3}$	$-\frac{4}{3}$
B	$(1+x)\ln^2 x$	0	0	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	0	0	0	0	0
	$\frac{1+x^2}{1-x}\ln^2(1-x)$	0	0	0	0	0	0	0	0	2	-2	0	0
	$\frac{1+x^2}{1-x}\ln x\ln(1-x)$	-4	2	0	0	0	-2	-2	0	6	-4	0	0
	$\frac{1+x^2}{1-x}\ln x$	0	$-\frac{3}{2}$	0	0	0	$-\frac{3}{2}$	$\frac{3}{2}$	0	$-\frac{3}{2}$	$\frac{11}{3}$	$\frac{11}{3}$	$-\frac{2}{3}$
	$\frac{1+x^2}{1-x}\ln(1-x)$	3	-3	4	-4	0	0	3	-4	5	-4	0	0
	$(1-x)\ln x$	-4	2	0	3	0	1	-2	0	2	0	0	0
	$(1-x)\ln(1-x)$	0	0	0	0	0	0	0	0	4	-4	0	0
	$I_1\frac{1+x^2}{1-x}$	4	-4	0	0	0	0	4	0	-8	4	0	0
	$I_0\frac{1+x^2}{1-x}$	0	0	4	-4	0	0	0	-4	8	-4	0	0
	$I_0\frac{1+x^2}{1-x}(\ln x+\ln(1-x))$	-4	4	0	0	0	0	-4	0	8	-4	0	0
	$I_0(1-x)$	-4	4	0	0	0	0	-4	0	8	-4	0	0

Appropriate colour factors are shown in the first line. Terms of type A satisfy the Gribov-Lipatov relation while those of type B break it.

SINGLET : γ_8^{ij}

$$
\begin{aligned}
\gamma_8^{\psi\psi} = {} & a_s C_F \frac{9883}{1260} \\
& + a_s^2 \Big[C_F C_A \frac{25870049}{762048} + C_F^2 (-\frac{27040578211}{4000752000}) + n_f C_F (-\frac{36241943}{4762800}) \Big] \\
& + a_s^2 fl_{02} n_f C_F (-\frac{40333}{8164800}) \\
& + a_s^3 \Big[C_F C_A^2 (\frac{8101059985033}{41150592000} + \frac{2510407}{132300}\zeta_3) + C_F^2 C_A (-\frac{3662576699059}{112021056000} - \frac{2510407}{44100}\zeta_3) \\
& \quad + C_F^3 (-\frac{109308710097437993}{6351593875200000} + \frac{2510407}{66150}\zeta_3) + n_f C_F C_A (-\frac{1578915745223}{72013536000} - \frac{19766}{315}\zeta_3) \\
& \quad + n_f C_F^2 (-\frac{91675209372043}{1680315840000} + \frac{19766}{315}\zeta_3) + n_f^2 C_F (-\frac{38920977797}{18003384000}) \Big] \\
& + a_s^3 fl_{02} \Big[n_f C_F C_A (-\frac{343248329803}{2592487296000} - \frac{1369}{1890}\zeta_3) + n_f C_F^2 (\frac{39929737384469}{90737055360000} + \frac{1369}{1890}\zeta_3) \\
& \quad + n_f^2 C_F (-\frac{13131081443}{108020304000}) \Big] \\
= {} & a_s 10.4582010582 \\
& + a_s^2 \Big(123.7764525165 - 10.1458366227 n_f \Big) \\
& + a_s^2 fl_{02} \Big(-0.0065864851 n_f \Big) \\
& + a_s^3 \Big(2164.0918358230 - 352.3116595904 n_f - 2.8824934836 n_f^2 \Big) \\
& + a_s^3 fl_{02} \Big(-1.6821565188 n_f - 0.1620816452 n_f^2 \Big)
\end{aligned}
$$

$$
\begin{aligned}
\gamma_8^{\psi G} = {} & a_s n_f (-\frac{37}{180}) \\
& + a_s^2 \Big[n_f C_F (-\frac{51090517}{16329600}) + n_f C_A (\frac{100911011}{40824000}) \Big] \\
& + a_s^3 \Big[n_f C_F C_A (\frac{4896295442015177}{129624364800000} - \frac{515201}{18900}\zeta_3) + n_f C_F^2 (-\frac{4374484944665803}{226842638400000} + \frac{749}{108}\zeta_3) \\
& \quad + n_f C_A^2 (-\frac{24648658224523}{1157360400000} + \frac{64021}{3150}\zeta_3) + n_f^2 C_F (\frac{7903297846481}{12962436480000}) + n_f^2 C_A (\frac{10379424541}{22044960000}) \Big] \\
= {} & a_s \Big(-0.2055555556 n_f \Big) \\
& + a_s^2 \Big(3.2439572229 n_f \Big) \\
& + a_s^3 \Big(28.7612614990 n_f + 2.2254331118 n_f^2 \Big)
\end{aligned}
$$

$$
\begin{aligned}
\gamma_8^{G\psi} = {} & a_s C_F (-\frac{37}{126}) \\
& + a_s^2 \Big[C_F C_A (-\frac{58805263}{28576800}) + C_F^2 (\frac{331619149}{400075200}) + n_f C_F (-\frac{12613}{238140}) \Big] \\
& + a_s^3 \Big[C_F C_A^2 (-\frac{840976971727}{129624364800} - \frac{58649}{6615}\zeta_3) + C_F^2 C_A (-\frac{16504689458671}{907370553600} + \frac{58649}{2205}\zeta_3) \\
& \quad + C_F^3 (\frac{12876352060509647}{635159387520000} - \frac{117298}{6615}\zeta_3) + n_f C_F C_A (-\frac{3105820553}{6751269000} + \frac{296}{63}\zeta_3) \\
& \quad + n_f C_F^2 (\frac{8498139408671}{9073705536000} - \frac{296}{63}\zeta_3) + n_f^2 C_F (\frac{339184373}{600112800}) \Big] \\
= {} & a_s \Big(-0.3915343915 \Big) \\
& + a_s^2 \Big(-6.7576035061 - 0.0706195235 n_f \Big) \\
& + a_s^3 \Big(-134.7055041700 + 12.3754453990 n_f + 0.7536013741 n_f^2 \Big)
\end{aligned}
$$

$$
\begin{aligned}
\gamma_8^{GG} = {} & a_s \Big[C_A (\frac{319}{45}) + n_f (\frac{2}{3}) \Big] \\
& + a_s^2 \Big[C_A^2 (\frac{2223694}{91125}) + n_f C_F (\frac{685883}{326592}) + n_f C_A (-\frac{623687}{68040}) \Big] \\
& + a_s^3 \Big[+ C_A^3 (\frac{1381390082227}{10333575000}) + n_f C_F C_A (-\frac{220111823810087}{2592487296000} + \frac{81941}{945}\zeta_3) \\
& \quad + n_f C_F^2 (-\frac{14058417959723}{22684263840000} - \frac{37}{54}\zeta_3) + n_f C_A^2 (\frac{2080130771161}{102876480000} - \frac{162587}{1890}\zeta_3) \\
& \quad + n_f^2 C_F (-\frac{1747563703}{1687817250}) + n_f^2 C_A (-\frac{4209708 49}{128595600}) \Big] \\
= {} & a_s \Big(21.2666666667 + 0.6666666667 n_f \Big) \\
& + a_s^2 \Big(219.6240987654 - 24.6992643216 n_f \Big) \\
& + a_s^3 \Big(3609.3541896322 - 673.9430658122 n_f - 11.2013383657 n_f^2 \Big)
\end{aligned}
$$

$O(\alpha_s^3)$: THE FIRST MOMENTS

LARIN, VERMASEREN, VAN RITBERGEN 1994

— " — + NOGUEIRA 1997

	Tree	1-loop	2-loops	3-loops	Lorentz projections
qγqγ	1	3	27	413	2
qφqφ		1	24	697	1
gγgγ		2	20	366	2
hγhγ			2	53	2
gφgφ	1	11	241	7219	1
hφhφ		1	36	1266	1
TOTAL	3	23	399	10846	

DIAGRAMS

Table 1. Number of diagrams and Lorentz tensor structures in the classes qγqγ, qφqφ, gγgγ, hγhγ, gφgφ and hφhφ. Notation: q = quark, g = gluon, h = ghost, γ = photon, φ = scalar particle that couples only to gluons.

NET 1 YEAR OF CPU TIME (96)

2000: RETEY, VERMASEREN: 6 MOMENTS NOW.

NON - SINGLET : γ_{10}

$$
\begin{aligned}
\gamma_{10}^{\rm ns} = {} & a_s C_F \left(\frac{12055}{1386}\right) \\
& + a_s^2 \left[C_F C_A \left(\frac{19524247733}{523908000}\right) + C_F^2 \left(-\frac{9579051036701}{1331250228000}\right) + n_f C_F \left(-\frac{2451995507}{288149400}\right)\right] \\
& + a_s^3 \left[C_F C_A^2 \left(\frac{94091568579766453}{435681892800000} + \frac{151796299}{8004150}\zeta_3\right)\right. \\
& + C_F^2 C_A \left(-\frac{16389982059548833}{465937579800000} - \frac{151796299}{2668050}\zeta_3\right) \\
& + C_F^3 \left(-\frac{22077113008087364405687}{1278663181493544000000} + \frac{151796299}{4002075}\zeta_3\right) \\
& + n_f C_F C_A \left(-\frac{9007773127403}{389001690000} - \frac{48220}{693}\zeta_3\right) + n_f C_F^2 \left(-\frac{75522073210471127}{1230075210672000} + \frac{48220}{693}\zeta_3\right) \\
& \left. + n_f^2 C_F \left(-\frac{27995901056887}{1119812520520000}\right)\right]
\end{aligned}
$$

IMPROVEMENTS NEEDED:

- CALCULUS OF FINITE (IFINITE) MULTIPLE HARMONIC SUMS
- EULER-ZAGIER VALUES

BROADHURST, KREIMER
VERMASEREN, VAN RITBERGEN
BLÜMLEIN

$O(\alpha_s^2)$: NLO POLARIZED

ZIJLSTRA, VAN NEERVEN 1994 $\hat{P}_{qq}^{(1)}$, $\hat{P}_{qG}^{(1)}$

MERTIG, VAN NEERVEN 1995

VOGELSANG 1995

$O(\alpha_s^3)$: NNLO

COMPLETE RESULTS ARE NOT YET AVAILABLE BOTH FOR THE POLARIZED AND UNPOLARIZED CASE.

→ NEEDED TO CONTROL $\Delta\alpha_s^{THY} \to \pm 0.002$ @ $Q = M_Z$.

(SCHEME DEPENDENCE)

→ HERA : $\delta\alpha_s^{stat+sys} = \pm 0.002$ CAN BE ACHIEVED

4.3. Coefficient Functions

PROCESS-DEPENDENT QUANTITIES.

$O(\alpha_s)$: UNPOLARIZED

$$C^{(1)}_{F_2 q}(z) = C_F \left[\frac{1+z^2}{1-z}\left[\log\left(\frac{1-z}{z}\right) - \frac{3}{4}\right] + \frac{1}{4}(9+5z)\right]_+$$

$$C^{(1)}_{F_2 g}(z) = 2N_f T_f \left\{ \left[z^2 + (1-z)^2\right] \log\left(\frac{1-z}{z}\right) - 1 + 8z(1-z) \right\}$$

$$C^{(1)}_{F_1 q}(z) = C^{(1)}_{F_2 q} - C_F \cdot 2z$$

$$C^{(1)}_{F_1 g}(z) = C^{(1)}_{F_2 g} - 8N_f T_f z(1-z)$$

$$C^{(1)}_{F_3 q}(z) = C^{(1)}_{F_2}(z) - C_F(1+z).$$

FURMANSKI, PETRONZIO 1982 (AND VARIOUS AUTHORS BEFORE (ERRORS, SOMETIMES))

BARDEEN, BURAS, MUTA, DUKE ✓

$O(\alpha_s)$: POLARIZED

$$C^{(1)}_{g_1 q} = C^{(1)}_{F_1 q}$$

$$C^{(1)}_{g_1 g} = 4N_f T_f \left\{ (2z-1)\log\left(\frac{1-z}{z}\right) + 3 - 4z \right\}.$$

ALTARELLI, ELLIS, MARTINELLI 1979
HUMPERT, VAN NEERVEN 1981; BODWIN, QIU 1990

$O(\alpha_s^2)$: ZIJLSTRA, VAN NEERVEN 1992 (UNPOL.) ; 1994 POLARIZED.

The coefficient functions $C_i(x,Q^2)$ read

$$\begin{aligned} C_{\rm NS}(z,Q^2) &= a_s c_{L,q}^{(1)}(z) + a_s^2 c_{L,q}^{(2),NS}(z) \\ C_{\rm S}(z,Q^2) &= a_s^2 c_{L,q}^{(2),PS}(z) \\ C_{\rm g}(z,Q^2) &= a_s c_{L,g}^{(1)}(z) + a_s^2 c_{L,g}^{(2)}(z)\ , \end{aligned} \tag{25}$$

where $a_s = \alpha_s(Q^2)/(4\pi)$. The leading order coefficient functions are given by [38]

C_L :

$$c_{L,q}^{(1)}(z) = 4C_F z \tag{26}$$

$$c_{L,g}^{(1)}(z) = 8N_f z(1-z)\ . \tag{27}$$

In the $\overline{\rm MS}$ scheme the NLO coefficient functions read [21, 22] [6]

$$\begin{aligned} c_{L,q}^{(2),NS}(z) &= 4C_F(C_A-2C_F)z\Bigg\{4\frac{6-3z+47z^2-9z^3}{15z^2}\ln z \\ &-4{\rm Li}_2(-z)[\ln z - 2\ln(1+z)] - 8\zeta(3) - 2\ln^2 z[\ln(1+z)+\ln(1-z)] \\ &+4\ln z\ln^2(1+z) - 4\ln z{\rm Li}_2(z) + \frac{2}{5}(5-3z^2)\ln^2 z \\ &-4\frac{2+10z^2+5z^3-3z^5}{5z^3}[{\rm Li}_2(-z)+\ln z\ln(1+z)] \\ &+4\zeta(2)\left[\ln(1+z)+\ln(1-z)-\frac{5-3z^2}{5}\right] + 8{\rm S}_{1,2}(-z) + 4{\rm Li}_3(z) \\ &+4{\rm Li}_3(-z) - \frac{23}{3}\ln(1-z) - \frac{144+294z-1729z^2+216z^3}{90z^2}\Bigg\} \\ &+8C_F^2 z\Bigg\{{\rm Li}_2(z)+\ln^2 z - 2\ln z\ln(1-z) + \ln^2(1-z) - 3\zeta(2) \\ &-\frac{3-22z}{3z}\ln z + \frac{6-25z}{6z}\ln(1-z) - \frac{78-355z}{36z}\Bigg\} \\ &-\frac{8}{3}C_F N_f z\left\{2\ln z - \ln(1-z) - \frac{6-25z}{6z}\right\}, \end{aligned} \tag{28}$$

$$\begin{aligned} c_{L,q}^{(2),PS}(z) &= \frac{16}{9z}C_F N_f\Big\{3(1-2z-2z^2)(1-z)\ln(1-z) + 9z^2[{\rm Li}_2(z)+\ln^2 z - \zeta(2)] \\ &\quad +9z(1-z-2z^2)\ln z - 9z^2(1-z) - (1-z)^3\Big\}, \end{aligned} \tag{29}$$

$$\begin{aligned} c_{L,g}^{(2)}(z) &= C_F N_f\Bigg\{16z[{\rm Li}_2(1-z)+\ln z\ln(1-z)] \\ &+\left(-\frac{32}{3}z+\frac{64}{5}z^3+\frac{32}{15z^2}\right)[{\rm Li}_2(-z)+\ln z\ln(1+z)] + (8+24z-32z^2)\ln(1-z) \\ &-\left(\frac{32}{3}z+\frac{32}{5}z^3\right)\ln^2 z + \frac{1}{15}\left(-104-624z+288z^2-\frac{32}{z}\right)\ln z \\ &+\left(-\frac{32}{3}z+\frac{64}{5}z^3\right)\zeta(2) - \frac{128}{15} - \frac{304}{5}z + \frac{336}{5}z^2 + \frac{32}{15z}\Bigg\} \\ &+C_A N_f\Big\{-64{\rm Li}_2(1-z) + (32z+32z^2)[{\rm Li}_2(-z)+\ln z\ln(1+z)] \\ &+(16z-16z^2)\ln^2(1-z) + (-96z+32z^2)\ln z\ln(1-z) \\ &+\left(-16-144z+\frac{464}{3}z^2+\frac{16}{3z}\right)\ln(1-z) + 48z\ln^2 z + (16+128z-208z^2)\ln z \end{aligned}$$

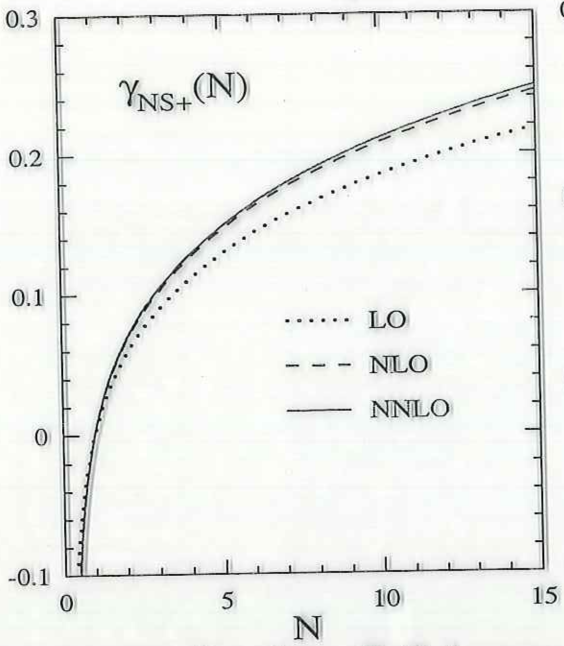
[6] Previous calculations [39, 40] turned out to be partly incorrect, whereas agreement was shown between Refs. [21, 22] and [41]. Refs. [40] were later corrected in Ref. [42].

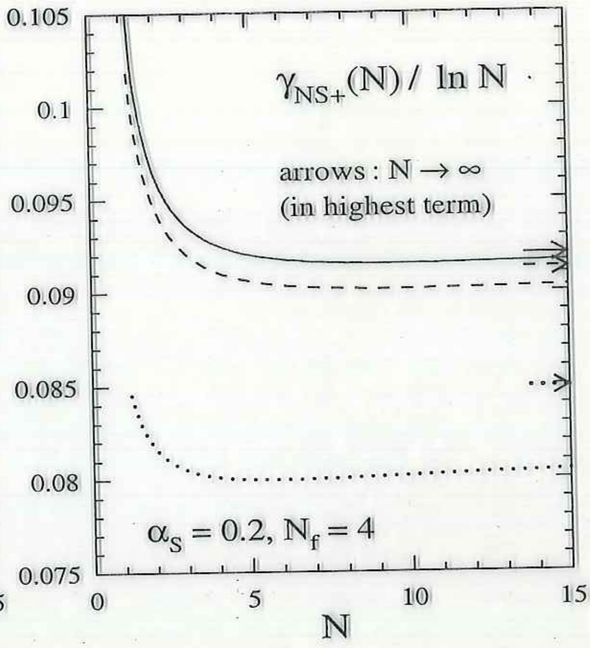
$$
\begin{aligned}
C^{G}_{L,8} = {}& a_s n_f \left(\tfrac{4}{45}\right) + a_s^2 n_f C_F \left(-\tfrac{51097}{51030}\right) + a_s^2 n_f C_A \left(\tfrac{7712869}{2551500}\right) \\
& + a_s^3 \mathrm{fl}^g_{11} n_f^2 \frac{d^{abc}d^{abc}}{N_A} \left(\tfrac{3665714041}{285768000} + \tfrac{77209}{1575}\zeta_3 - \tfrac{208}{3}\zeta_5\right) \\
& + a_s^3 n_f C_F C_A \left(-\tfrac{520855237960033}{7129340064000} - \tfrac{6119609}{519750}\zeta_3 + \tfrac{128}{3}\zeta_5\right) \\
& + a_s^3 n_f C_F^2 \left(\tfrac{2384408424295187}{71293400640000} + \tfrac{7723411}{779625}\zeta_3 - \tfrac{128}{3}\zeta_5\right) \\
& + a_s^3 n_f C_A^2 \left(\tfrac{27404278602137}{289340100000} + \tfrac{20438}{23625}\zeta_3 - \tfrac{32}{3}\zeta_5\right) \\
& + a_s^3 n_f^2 C_F \left(\tfrac{124374980290567}{35646700320000} - \tfrac{608}{1485}\zeta_3\right) + a_s^3 n_f^2 C_A \left(-\tfrac{11324757281}{1377810000} - \tfrac{8}{45}\zeta_3\right) \\
= {}& a_s \left(0.0888888889 n_f\right) + a_s^2 \left(7.7335451042 n_f\right) \\
& + a_s^3 \left(-0.2322211886 n_f^2 \mathrm{fl}^g_{11} + 592.3307972098 n_f - 21.3033368117 n_f^2\right).
\end{aligned}
$$

$$
\begin{aligned}
C^{\mathrm{ns}}_{L,10} = {}& a_s C_F \left(\tfrac{4}{11}\right) + a_s^2 n_f C_F \left(-\tfrac{163679}{114345}\right) + a_s^2 C_F C_A \left(\tfrac{89670761}{8731800} - \tfrac{48}{11}\zeta_3\right) \\
& + a_s^2 C_F^2 \left(-\tfrac{1999510607}{528273900} + \tfrac{96}{11}\zeta_3\right) \\
& + a_s^3 \mathrm{fl}_{11} n_f \frac{d^{abc}d^{abc}}{n} \left(-\tfrac{5073093424963}{528099264000} - \tfrac{1820773}{363825}\zeta_3 + \tfrac{160}{11}\zeta_5\right) \\
& + a_s^3 n_f C_F C_A \left(-\tfrac{1761835769882273 23}{1699159381920000} + \tfrac{55485434}{1216215}\zeta_3\right) \\
& + a_s^3 n_f C_F^2 \left(\tfrac{9048874326307637}{190368782604000} - \tfrac{1174256}{15015}\zeta_3\right) + a_s^3 n_f^2 C_F \left(\tfrac{63272639}{11320155}\right) \\
& + a_s^3 C_F C_A^2 \left(\tfrac{2366034921481985137}{6796637527680000} - \tfrac{95022195887}{187297110}\zeta_3 + \tfrac{3760}{11}\zeta_5\right) \\
& + a_s^3 C_F^2 C_A \left(-\tfrac{323139848004267269}{3354750574560000} + \tfrac{22904191}{17325}\zeta_3 - \tfrac{14240}{11}\zeta_5\right) \\
& + a_s^3 C_F^3 \left(-\tfrac{88756238669864596 7383}{316621359226972 8000} - \tfrac{357031607224}{468242775}\zeta_3 + \tfrac{13440}{11}\zeta_5\right) \\
= {}& a_s \left(0.4848484848\right) + a_s^2 \left(32.0176594698 - 1.9085982480 n_f\right) \\
& + a_s^3 \left(-2.3976416945 n_f \mathrm{fl}_{11} + 2081.2132221274\right.
\end{aligned}
$$

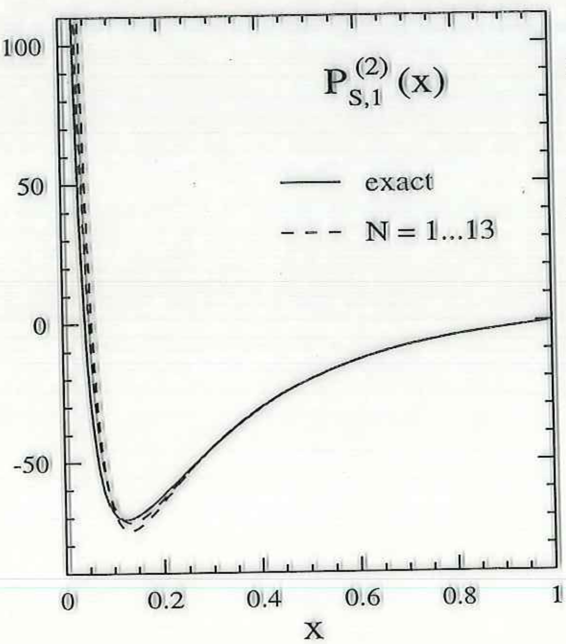
3 Loop Anomalous Dimensions

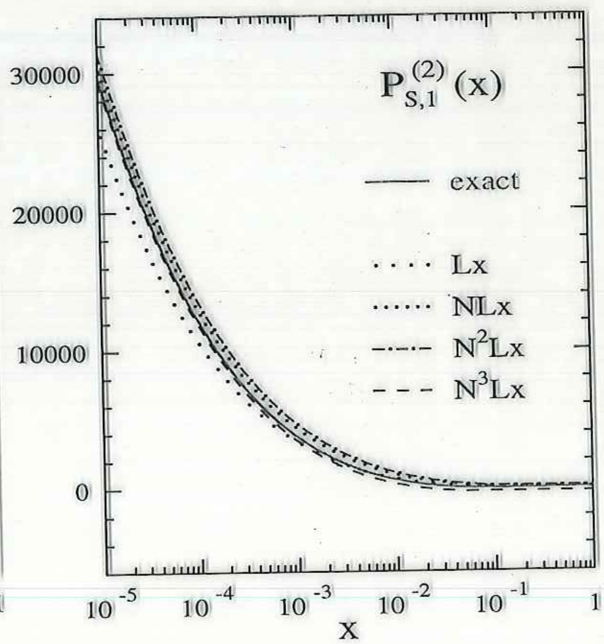
S. Moch, J. Vermaseren, A. Vogt: non-singlet: hep-ph/0403192 :





A new contribution @ 3 loops :

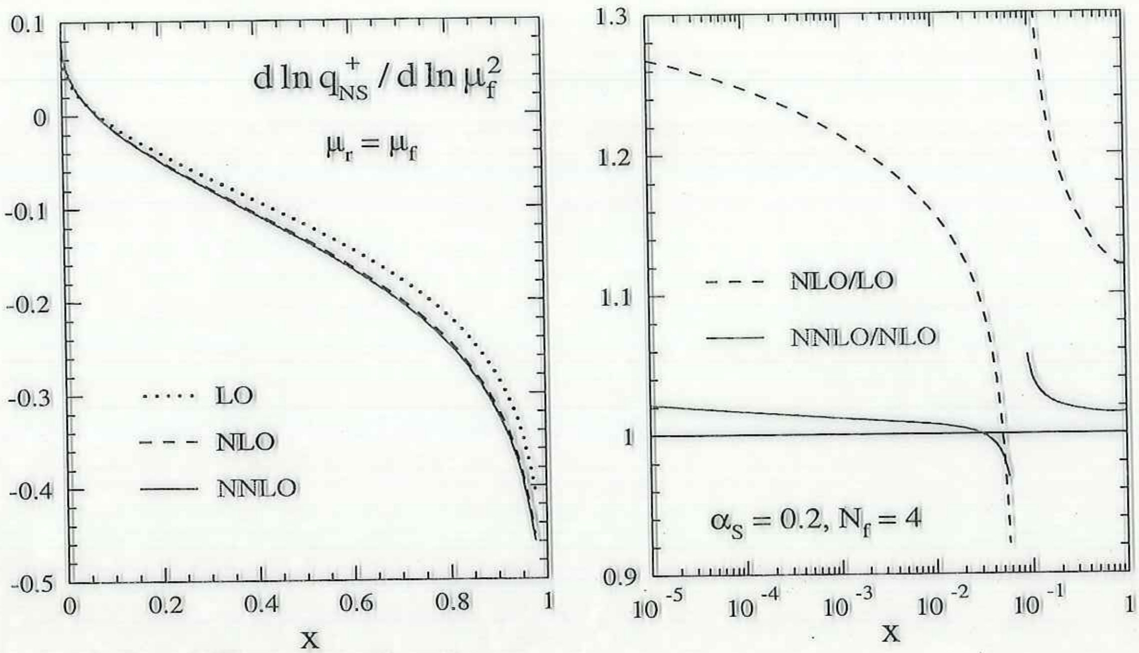




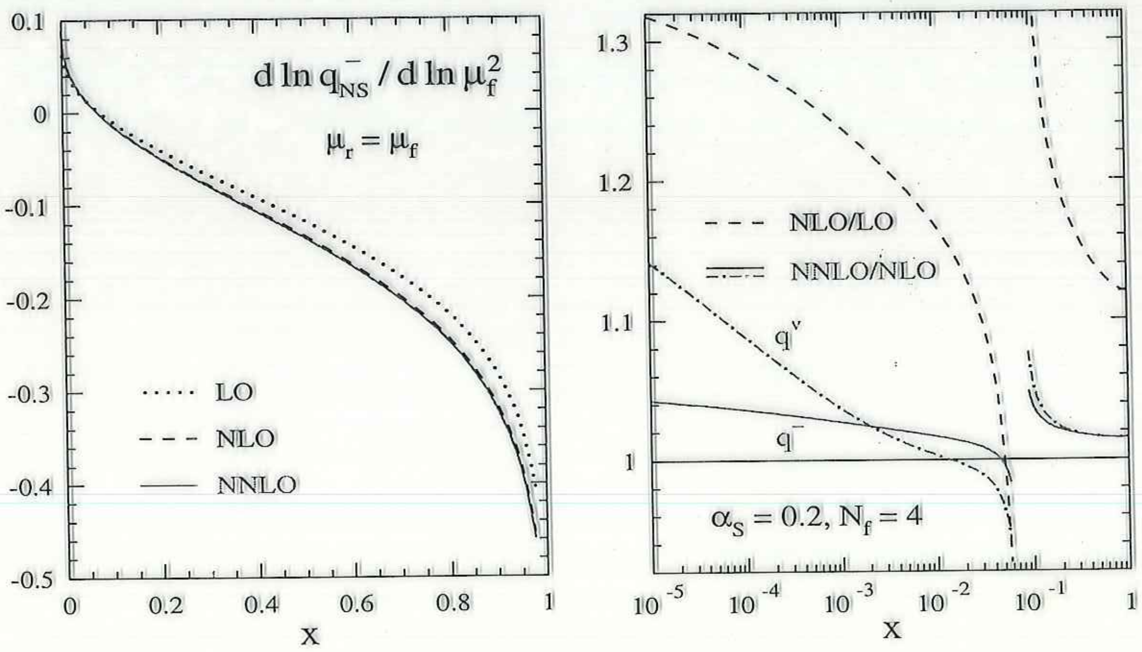
3 Loop Anomalous Dimensions

S. Moch, J. Vermaseren, A. Vogt: non-singlet: hep-ph/0403192 :

Slope of the NS^+ distribution



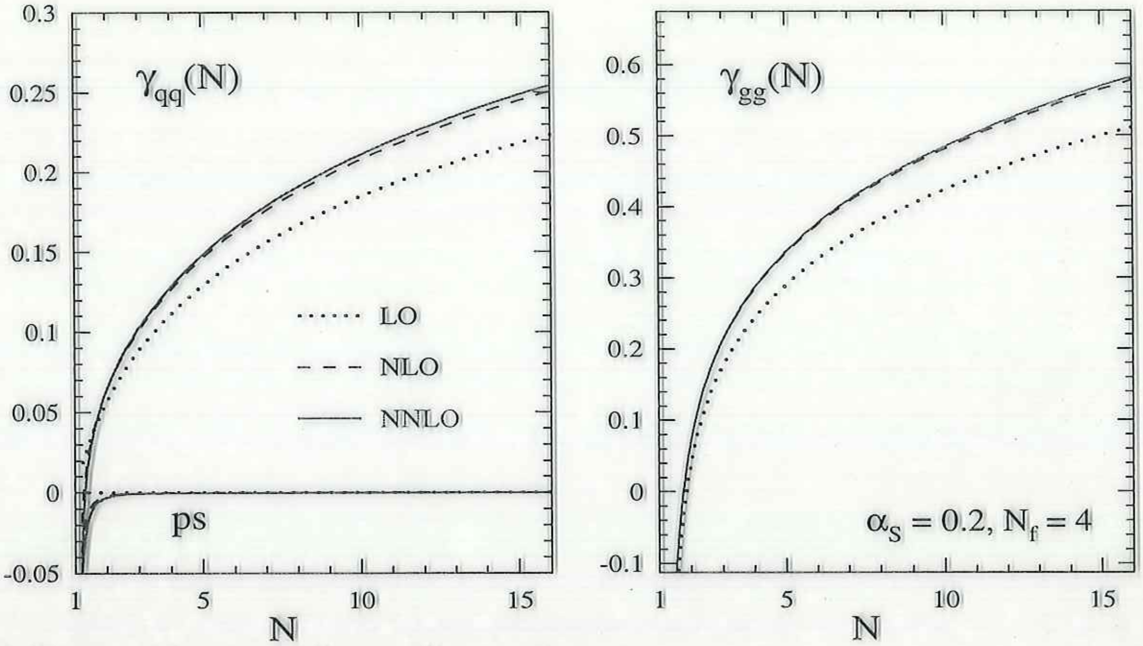
Slope of the NS^- distribution



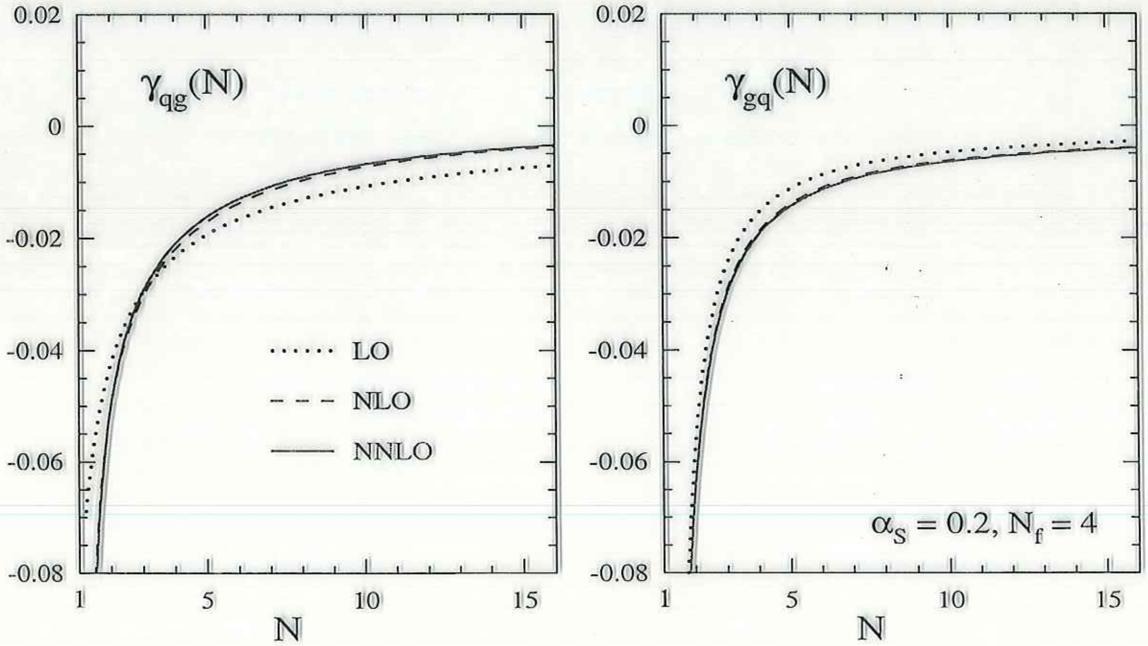
3 Loop Anomalous Dimensions

S. Moch, J. Vermaseren, A. Vogt: singlet: hep-ph/0403192 :

Diagonal anomalous dimensions



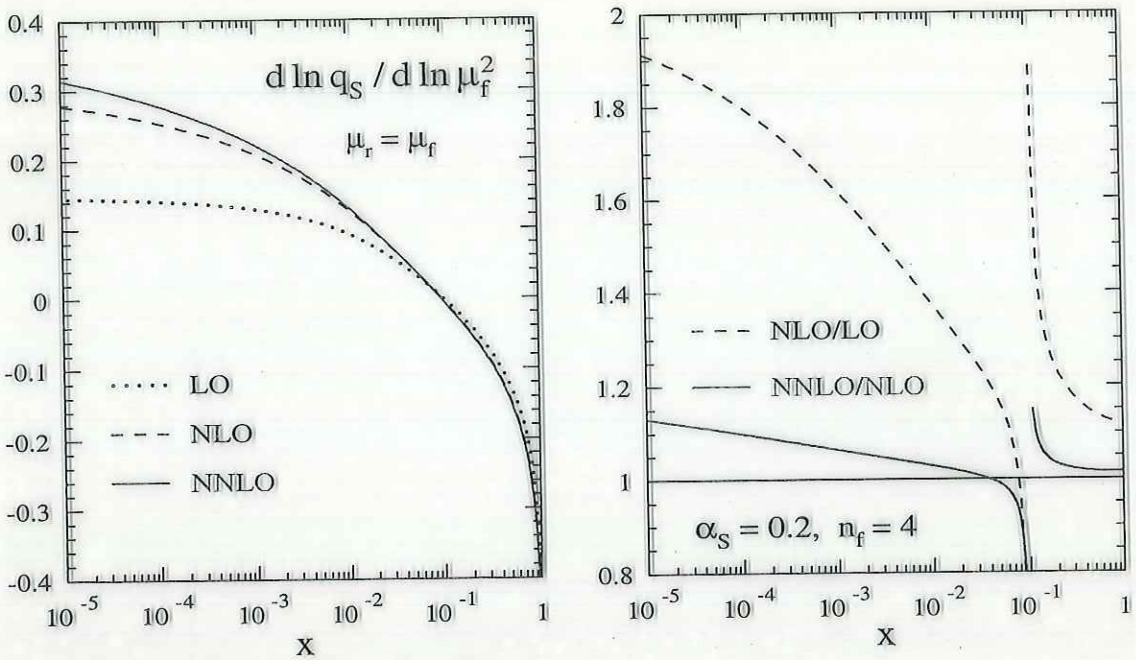
Off-diagonal anomalous dimension



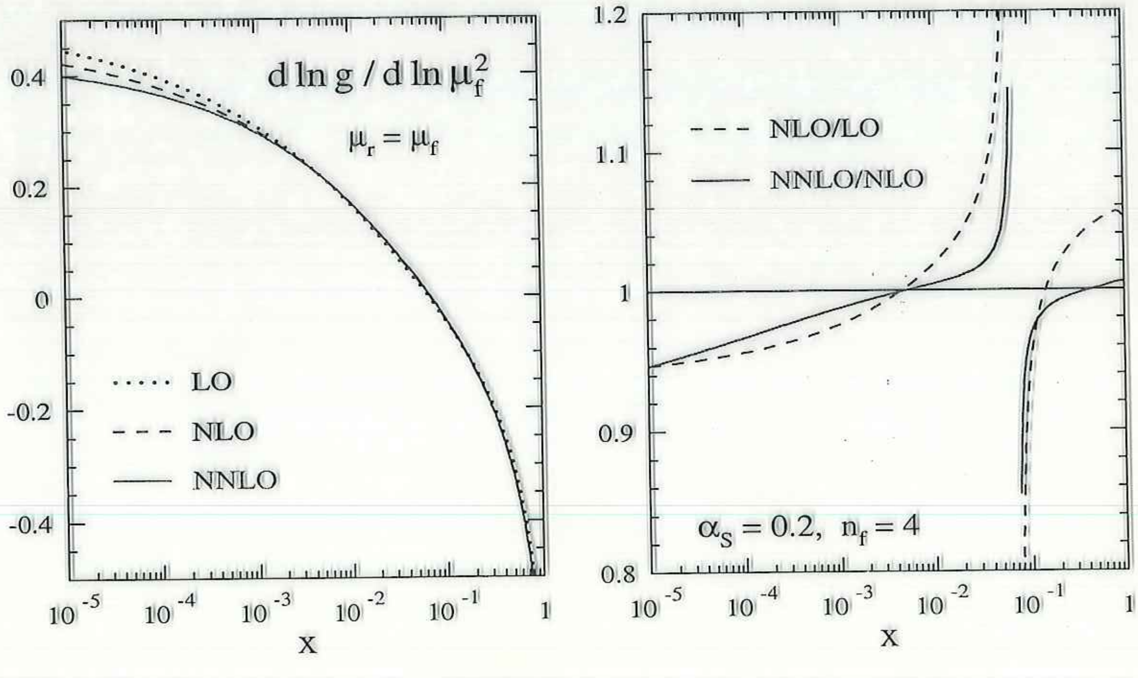
3 Loop Anomalous Dimensions

S. Moch, J. Vermaseren, A. Vogt: singlet: hep-ph/0403192 :

Slope of the singlet distribution



Clope of the gluon distribution



CHECK: CALCULATE 16TH MOMENT

Results :

$$\gamma_{16}^{(0)} = \frac{64419601}{6126120}\,\mathrm{CF}$$

$$\begin{aligned}\gamma_{16}^{(1)} = &-\frac{1176525373840303}{112588038763200}\,\mathrm{CF\,NF} + \frac{2154615916612989}{484994628518400}\,\mathrm{CF\,CA}\\ &-\frac{36890244529287813828 77}{4598185573520098 56000}\,\mathrm{CF}^2\end{aligned}$$

$$\begin{aligned}&\gamma_{16}^{(2)} =\\ &\left(\frac{59290512768143}{1563722760600}\,\zeta_3 - \frac{58552930270652300886778705063429867}{3451337970612452534317096673280000}\right)\mathrm{CF}^3\\ &+\left(-\frac{150184218240603886594365 59}{579371382263532418560000} - \frac{64419601}{765765}\,\zeta_3\right)\mathrm{CF\,CA\,NF}\\ &+\left(\frac{16704237280839842078788254 67}{6488959481351563087872000} + \frac{59290512768143}{3127445521200}\,\zeta_3\right)\mathrm{CF\,CA}^2\\ &-\frac{5559466349834573157251}{2069183508084044352000}\,\mathrm{CF\,NF}^2\\ &+\left(-\frac{1229794646000775781127856064477}{30335885575318557435801600000} - \frac{59290512768143}{1042481840400}\,\zeta_3\right)\mathrm{CF}^2\mathrm{CA}\\ &+\left(-\frac{7154359967798515534255135 5451}{938967886855098206346240000} + \frac{64419601}{765765}\,\zeta_3\right)\mathrm{CF}^2\mathrm{NF}\end{aligned}$$

Agreement with : Moch, Vermaseren, Vogt, hep-ph/0403192.

C_L^{NS}

```
                          /                     2
              4 cf a      |  29393927457809 cf      39366889 cf nf   48 z3 cf ca
    coef  := ------  +   |- ------------------ - -------------- - -----------
               17         \   44659922042736           39054015          17

                                                 2\          /
          55969347000169 cf ca     96 z3 cf  |  2         |
        + -------------------- + ---------| a   +    |
            8209544493150             17       /          \

       /39360 z5    196256899828170631 z3    750828182127677149812644729011091 9\
       |-------- - --------------------- - ----------------------------------|
       \   17         133698296031300       136478982354388524292425984000 00 /

         3     /29604550101013356532203920715 9677    40160 z5    2253147763389895 z3\
       cf   + |--------------------------------- - -------- + -------------------|
               \ 936620467137960460830374400000       17         1188429298056     /

                  2    /35291373463211704531604 63    44651224 z3\          2
       ca cf   + |-------------------------- - -----------| nf cf   +
                        \1367960208122229321600 00        765765     /

       /  1634895686765221 z3    146079249942710013949328037 1    10240 z5\    2
       |- ------------------- + ---------------------------- + --------| ca   cf
       \    2673965920626       825604219725533696448000 0        17      /

                                   2
          895967716232 cf nf
        + ------------------
            209134250325

                                                                                  \
                /   4495805144658565385501573689      43594330672 z3\              |  3
        +  |- ---------------------------- + --------------| ca nf cf| a
                \   5779229538078735875136 0000        1249937325    /             /

> quit;
bytes used=3658008, alloc=2227816, time=0.09
```

C_2^{MS}

```
                4047739719 cf a    //44426674163044428879366970127    24439538 z3\   2
coef  :=  ---------------- +  ||------------------------------ + -----------| cf
                190590400      \\ 3219318469217479564615680000      255255    /

        /1791830840849829422278308 7   113298677 z3\
      + |-------------------------- - ------------| cf ca
        \ 594227058731828121600000       1021020   /

        14356837276190747211117 7 cf nf\  2    //59290512768143 z4   27643576 z5
      - -------------------------------| a  + ||----------------- - -----------
            2758911344112059136000     /      \\  3127445521200        21879

        30368133975995097250846772938425059765591616 89
      + ------------------------------------------------
          80344580160407759334216478634033479680000000

        149434192694045086538740 3 z3\    3   /59290512768143 z4
      + -----------------------------| cf  + |------------------
           5956740402060127680000     /       \  6254891042400

        26286537788347572655880093551503319033 3   47187263 z5
      + --------------------------------------- + -----------
          56646805852503848671021043712000000        51051

        153550504691714823 13 z3\       2
      - -----------------------| cf ca
           49914030518352 00     /

        /72273849359996703123187898849 99   64419601 z3\      2    /
      + |--------------------------------- + -----------| cf nf  + |
        \ 76056398835262954714045440000     20675655   /           \

      77500266271187687528450917608900514652427 41   28494820041389214915 31 z3
      --------------------------------------------- - -------------------------
        1652500620329242273431025887166464000000        67411671216729840 00

        983963 z5    59290512768143 z4\    2          /
      + ---------- - -----------------| cf   ca +  |
          21879        2084963680800  /             \

       552298563960959 z3    40732072413484931961522220799335575 29    64419601 z4
      - ------------------ - ---------------------------------------- + -----------
          4021001384400        352977746994455372827884887040000 0       1531530

      \    2
      | cf   nf +
      /

      /598788865585667 z3    64419601 z4    58281163492154299564717935869853654 7\
      |------------------ - ----------- - --------------------------------------|
      \  1850495446800        1531530       40462004180359891907872174080000 0   /

                  \   3
      cf ca nf| a
                  /

> quit;
bytes used=9005724, alloc=4717728, time=0.08
```

LAST DIAGRAM: 56 days CPU!

JUNE 14th, 6:30 a.m. after ~ 600 CPU hours!

USE, 3 HIGH PERFORMANCE PC'S 2.6-3GHz
(1 64b OPTERON)
1: 4.2 Tbyte RAID SYSTEM

NON-SINGLET ANALYSIS:

Fit Results

WITH: H. BÖTTCHER.

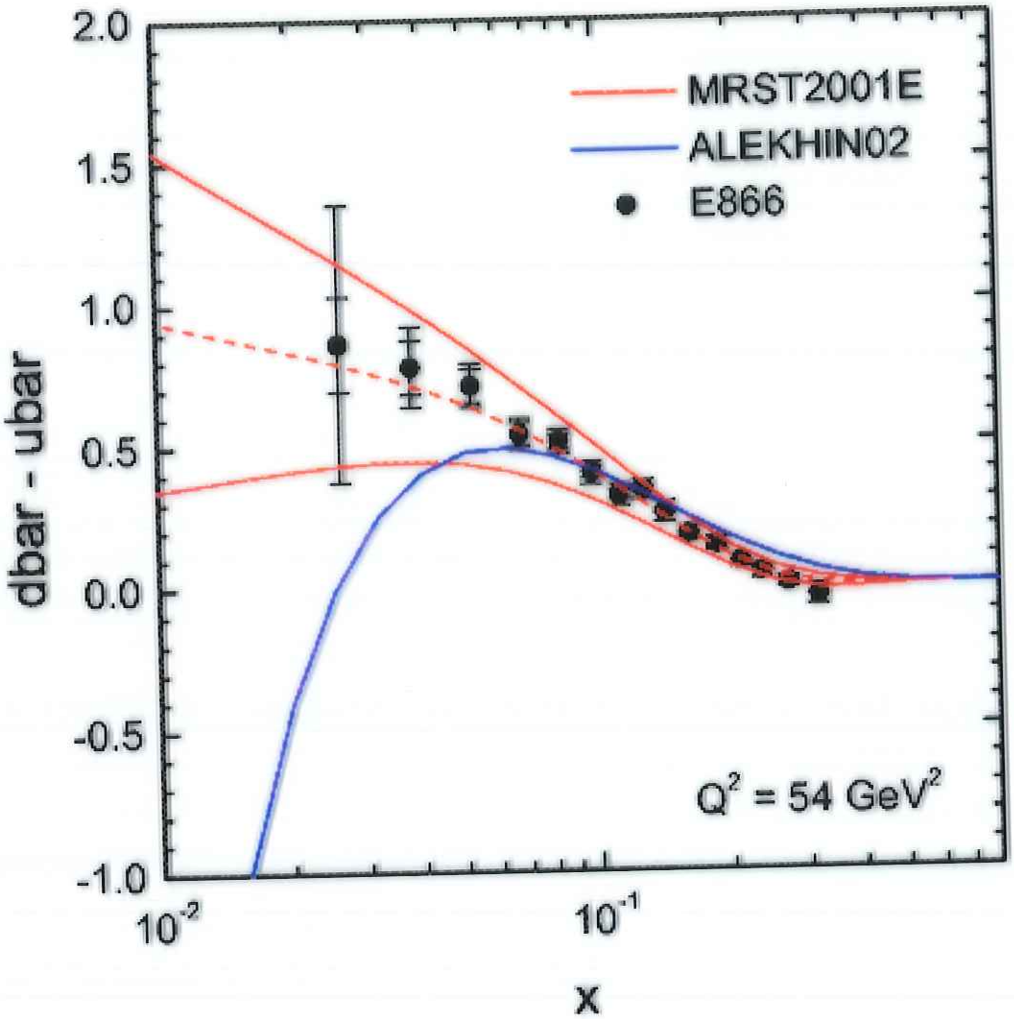
- Parameter values at the input scale $Q_0^2 = 4.0\,GeV^2$

$$xq_i(x, Q_0^2) = A_i x^{a_i}(1-x)^{b_i}(1+\rho_i x^{\frac{1}{2}} + \gamma_i x)$$

u_v	a	0.285 ± 0.007
	b	4.011 ± 0.045
	ρ	-1.108
	γ	26.283
d_v	a	0.339 ± 0.041
	b	5.160 ± 0.292
	ρ	0.895
	γ	18.179
$\Lambda_{QCD}^{(4)}$		$219 \pm 31\ MeV$
χ^2/ndf		= 688/757 = 0.91

- Covariance Matrix at the input scale $Q_0^2 = 4.0\,GeV^2$

NLO					
	$\Lambda_{QCD}^{(4)}$	a_{u_v}	b_{u_v}	a_{d_v}	b_{d_v}
$\Lambda_{QCD}^{(4)}$	**9.86E-4**				
a_{u_v}	5.21E-5	**5.11E-5**			
b_{u_v}	-7.08E-4	2.07E-4	**2.06E-3**		
a_{d_v}	6.06E-5	-1.45E-4	-1.07E-3	**1.72E-3**	
b_{d_v}	-2.61E-4	-8.01E-4	-6.94E-3	1.12E-2	**8.50E-02**



- evidence for
  - $\bar{d} > \bar{u}$
  - $x(\bar{d} - \bar{u}) \to 0$

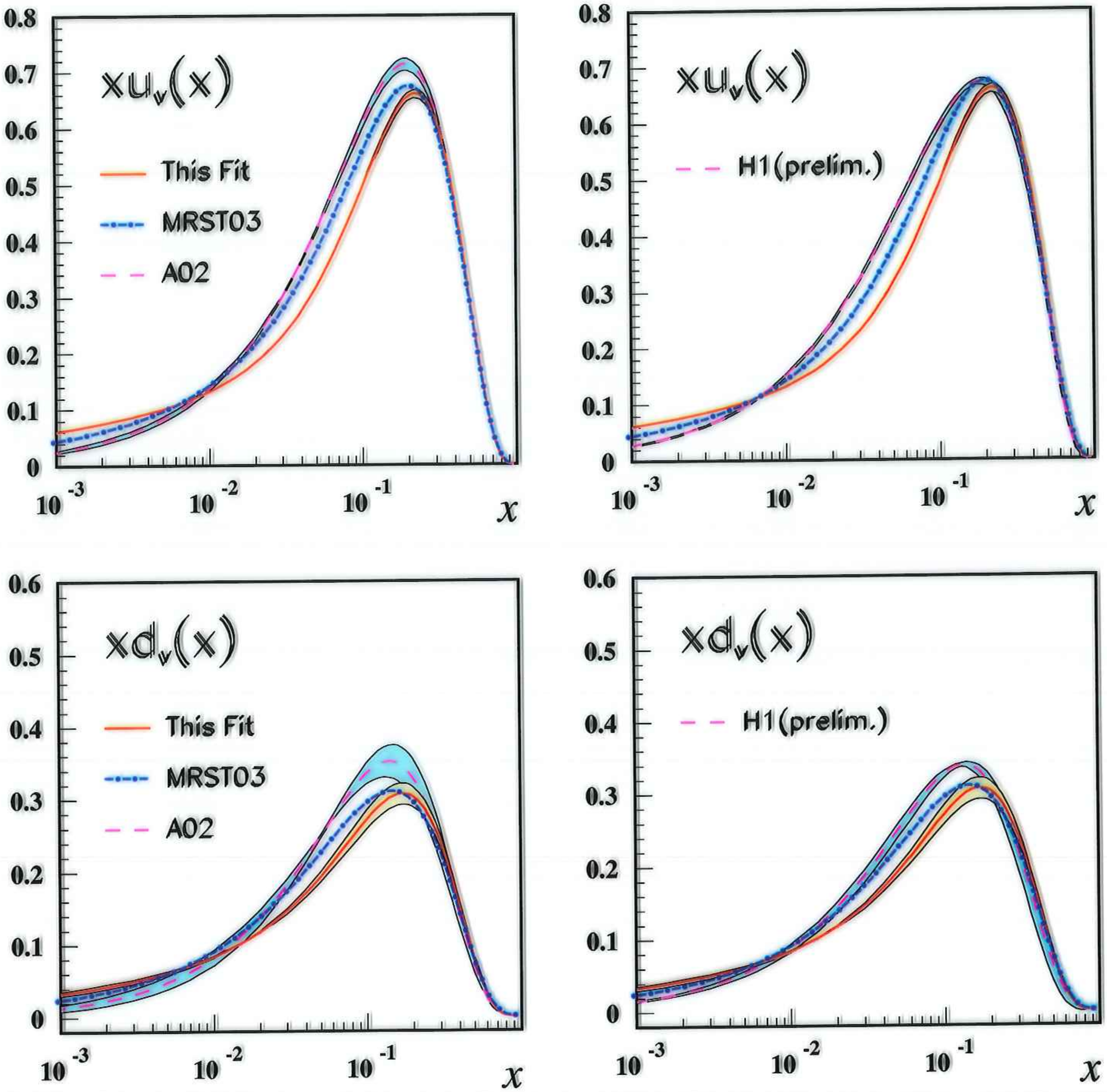
  as $x \to 0$?

- HERA3 (ep and ed DIS at small x) could provide an interesting measurement!

J. STIRLING

Valence Parton Densities at $Q_0^2 = 4.0\,GeV^2$

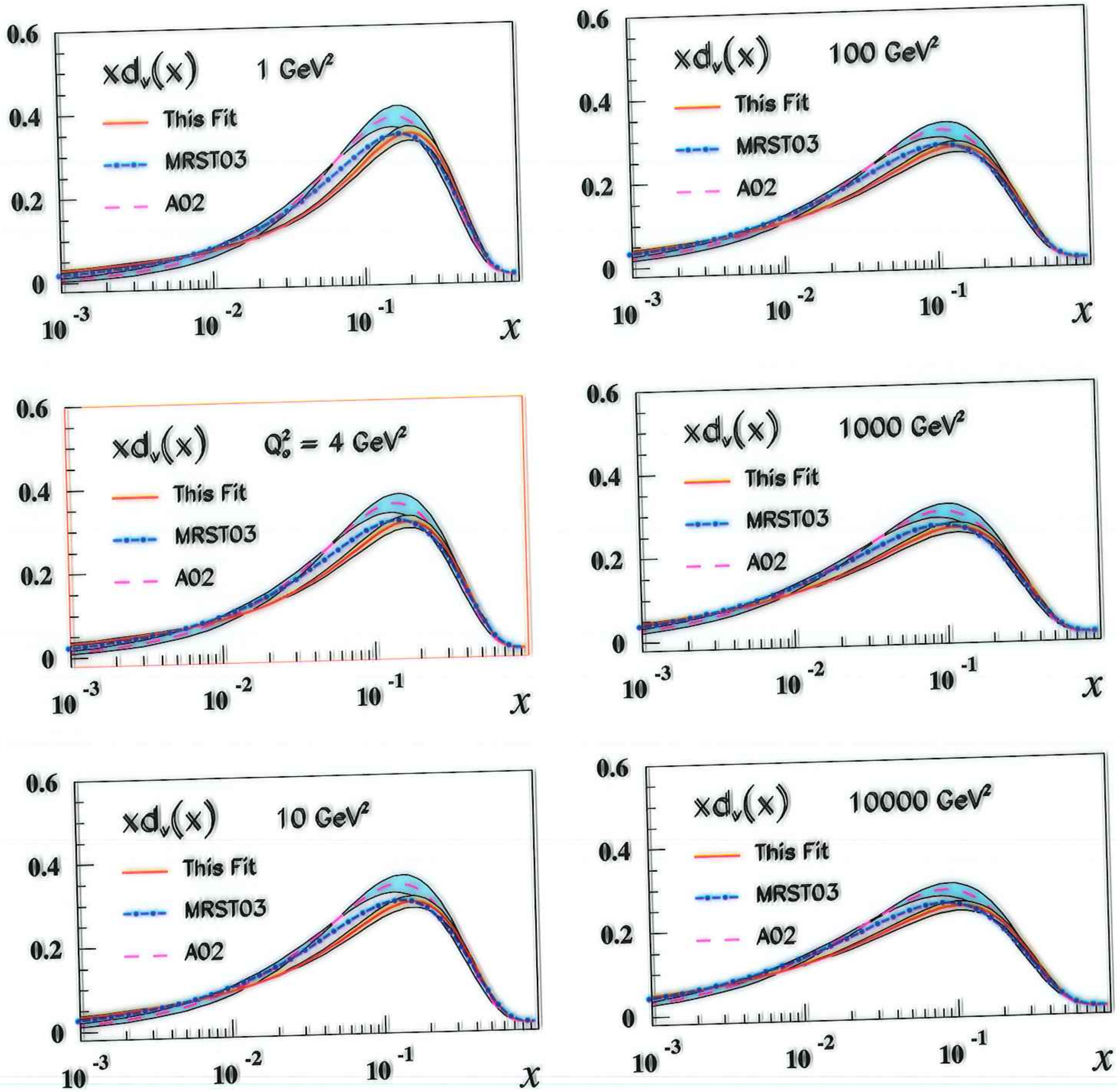
- 4+1 parameter non-singlet fit to F_2 data:



⇒ Yellow error band: Fully correlated 1σ error as given by Gaussian error propagation.

Evolution of the Parton Density $xd_v(x)$

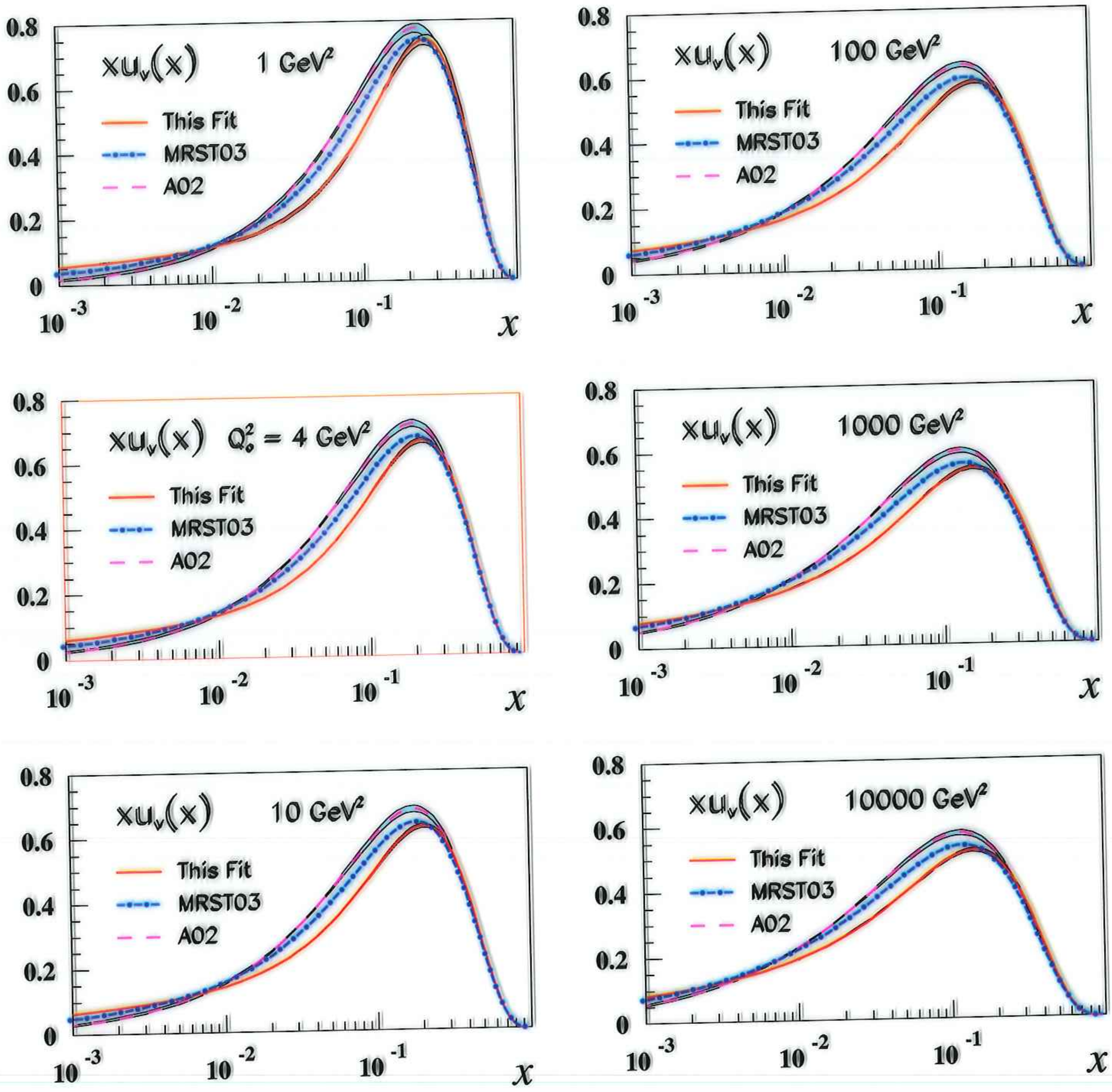
- 4+1 parameter non-singlet fit to F_2 data:



⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

Evolution of the Parton Density $xu_v(x)$

- 4+1 parameter non-singlet fit to F_2 data:



⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

	$\Lambda^{(4)}_{QCD}$, MeV	$\alpha_s(M_Z^2)$
This Fit	219 ± 31	0.113 $^{+0.003}_{-0.002}$ (expt)

⇒ Comparison with global QCD analyses (significant sea and gluon contributions and correlations):

	$\alpha_s(M_Z^2)$	expt	theory	model	Ref.
NLO					
CTEQ6	0.1165	±0.0065			[1]
MRST03	0.1165	±0.0020	±0.0030		[2]
A02	0.1171	±0.0015	±0.0033		[3]
ZEUS	0.1166	±0.0049		±0.0018	[4]
H1	0.1150	±0.0017	±0.0050	$^{+0.0009}_{-0.0005}$	[5]
BCDMS	0.111	±0.006			[6]
BB (pol)	0.113	±0.004	$^{+0.009}_{-0.005}$		[7]
NNLO					
SY01(ep)	0.1166	±0.0013			[8]
SY01(νN)	0.1153	±0.0063			[8]
MRST03	0.1153	±0.0020	±0.0030		[2]
A02	0.1143	±0.0014	±0.0009		[3]

[1]: CTEQ Collab.: J.Pumplin et al., JHEP 0207:012 (2002). [2]: MRST Collab.: A.D.Martin et al., hep-ph/0307262. [3]: S.Alekhin, hep-ph/0211096. [4]: ZEUS Collab.: S.Chekanov et al., Phys.Rev.**D67** (2003) 012007. [5]: H1 Collab.: C.Adloff et al., Eur.Phys. **C21** (2001) 33. [6]: BCDMS Collab.: A.C.Benvenuti et al., Phys.Lett. **223** (1989) 490. [7] J. Blümlein and H. Böttcher, Nucl. Phys. **B636** (2002) 225. [8] J. Santiago and F.J. Yndurain, Nucl. Phys. **B611** (2001) 447.

Comparison of Moments at $Q^2 = 4.0\,GeV^2$

f	n	This Fit	MRST03	A02
u_v	2	0.284 ± 0.003	0.289	0.304
	3	0.085 ± 0.002	0.084	0.087
	4	0.033 ± 0.001	0.032	0.033
d_v	2	0.114 ± 0.004	0.113	0.120
	3	0.029 ± 0.001	0.028	0.028
	4	0.010 ± 0.001	0.010	0.010
$u_v - d_v$	2	0.170 ± 0.005	0.176	0.184
	3	0.056 ± 0.002	0.056	0.059
	4	0.023 ± 0.001	0.023	0.024

		QCD	Lattice
f	n	This Fit	QCDSF
$u_v - d_v$	2	0.170 ± 0.005	$0.191 \pm 0.012^{*)}$

$$\Longrightarrow \Gamma_f(Q^2) = \int_0^1 x^{n-1} f(x, Q^2)\,dx$$

Lattice simulation: Scale $\mu^2 = 1/a^2 \sim 4\,GeV^2$.

$*$) G.Schierholz, private communication.

Scheme Invariant Combinations

Evolution Equations of Structure or Fragmentation Functions do normally exhibit FACTORIZATION AND RENORMALIZATION SCHEME DEPENDENCES. INSTEAD OF PROCESS-INDEPENDENT SCHEME-DEPENDENT EVOLUTION EQUATIONS FOR PARTONS ONE MAY THINK OF PROCESS-DEPENDENT SCHEME-INDEPENDENT EVOLUTION EQUATIONS FOR **Observables.**

Evolution Equations :

$$\frac{\partial}{\partial t}\left(\begin{array}{c} F_A^N \\ F_B^N \end{array}\right) = -\frac{1}{4}\left(\begin{array}{cc} K_{AA}^N & K_{AB}^N \\ K_{BA}^N & K_{BB}^N \end{array}\right)\left(\begin{array}{c} F_A^N \\ F_B^N \end{array}\right),$$

evolution variable

$$t = -\frac{2}{\beta_0}\ln\left(\frac{a_s(Q^2)}{a_s(Q_0^2)}\right),$$

physical evolution kernels

$$K_{IJ}^N = \left[-4\frac{\partial C_{I,m}^N(t)}{\partial t}\left(C^N\right)_{m,J}^{-1}(t) - \frac{\beta_0 a_s(Q^2)}{\beta(a_s(Q^2))}C_{I,m}^N(t)\gamma_{mn}^N(t)\left(C^N\right)_{n,J}^{-1}(t)\right]$$

with

$$K_{IJ}^N = \sum_{n=0}^{\infty} a_s^n(Q^2)\left(K^N\right)_{IJ}^{(n)}$$

Possible choices for F_A and F_B are F_2 and $\partial F_2/\partial t$ or F_2 and F_L. For these sets of physical observables we will examine the crossing-behaviour from S to T-Channel.

The dependence on the **renormalization scheme** is only removed if the perturbation series is summed to all orders.

System : $F_2(x,Q^2), \partial F_2/\partial t(x,Q^2)$

Leading Order :

$$
\begin{aligned}
K_{22}^{N(0)} &= 0\\
K_{2d}^{N(0)} &= -4\\
K_{d2}^{N(0)} &= \frac{1}{4}\left(\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)}\right)\\
K_{dd}^{N(0)} &= \gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)}
\end{aligned}
$$

Next-to-Leading Order :

[Furmanski, Petronzio 1982]

$$
\begin{aligned}
K_{22}^{N(1)} &= K_{2d}^{N(1)} = 0\\
K_{d2}^{N(1)} &= \frac{1}{4}\left[\gamma_{gg}^{N(0)}\gamma_{qq}^{N(1)} + \gamma_{gg}^{N(1)}\gamma_{qq}^{N(0)} - \gamma_{qg}^{N(1)}\gamma_{gq}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(1)}\right]\\
&\quad - \frac{\beta_1}{2\beta_0}\left(\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} - \gamma_{gq}^{N(0)}\gamma_{qg}^{N(0)}\right)\\
&\quad + \frac{\beta_0}{2}C_{2,q}^{N(1)}\left(\gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)} - 2\beta_0\right)\\
&\quad - \frac{\beta_0}{2}\frac{C_{2,g}^{N(1)}}{\gamma_{qg}^{N(0)}}\left[(\gamma_{qq}^{N(0)})^2 - \gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} + 2\gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)} - 2\beta_0\gamma_{qq}^{N(0)}\right]\\
&\quad - \frac{\beta_0}{2}\left(\gamma_{qq}^{N(1)} - \frac{\gamma_{qq}^{N(0)}\gamma_{qg}^{N(1)}}{\gamma_{qg}^{N(0)}}\right)
\end{aligned}
\tag{1}
$$

$$K_{dd}^{N(1)} = \gamma_{qq}^{N(1)} + \gamma_{gg}^{N(1)} - \frac{\beta_1}{\beta_0}\left(\gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)}\right) + 4\beta_0 C_{2,q}^{N(1)} - 2\beta_1$$
$$- \frac{2\beta_0}{\gamma_{qg}^{N(0)}}\left[C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} - 2\beta_0\right) - \gamma_{qg}^{N(1)}\right]$$

System : $F_2(x,Q^2), F_L(x,Q^2)$

$(\widetilde{F}_L^N \equiv F_L^N/(a_s(Q^2)C_{L,g}^{N(1)}))$

Leading Order :

[Catani 1997]

$$K_{22}^{N(0)} = \gamma_{qq}^{N(0)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\gamma_{qg}^{N(0)}$$
$$K_{2L}^{N(0)} = \gamma_{qg}^{N(0)}$$
$$K_{L2}^{N(0)} = \gamma_{gq}^{N(0)} - \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\right)^2 \gamma_{qg}^{N(0)}$$
$$K_{LL}^{N(0)} = \gamma_{gg}^{N(0)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\gamma_{qg}^{N(0)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right)$$

Next-to-Leading Order :

[BRvN 2000]

$$K_{22}^{N(1)} = \gamma_{qq}^{N(1)} - \frac{\beta_1}{\beta_0}\gamma_{qq}^{N(0)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\left(\gamma_{qg}^{N(1)} - \frac{\beta_1}{\beta_0}\gamma_{qg}^{N(0)}\right)$$
$$+ \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}C_{2,g}^{N(1)}\gamma_{qq}^{N(0)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}C_{2,g}^{N(1)}\gamma_{gg}^{N(0)}$$

$$
\begin{aligned}
& - \left[\frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}} + \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \right)^2 C_{2,g}^{N(1)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}} \right] \gamma_{qg}^{N(0)} \\
& + C_{2,g}^{N(1)} \gamma_{gq}^{N(0)} + 2\beta_0 \left(C_{2,q}^{N(1)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} C_{2,g}^{N(1)} \right) \\
K_{2L}^{N(1)} = \; & \gamma_{qg}^{N(1)} - \frac{\beta_1}{\beta_0} \gamma_{qg}^{N(0)} - C_{2,g}^{N(1)} (\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}) + 2\beta_0 C_{2,g}^{N(1)} \\
& + \left(C_{2,q}^{N(1)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} C_{2,g}^{N(1)} - \frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}} \right) \gamma_{qg}^{N(0)} \\
K_{L2}^{N(1)} = \; & \gamma_{gq}^{N(1)} - \frac{\beta_1}{\beta_0} \gamma_{gq}^{N(0)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \left(\gamma_{qq}^{N(1)} - \frac{\beta_1}{\beta_0} \gamma_{qq}^{N(0)} \right) \\
& - \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \right)^2 \left(\gamma_{qg}^{N(1)} - \frac{\beta_1}{\beta_0} \gamma_{qg}^{N(0)} \right) \\
& - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \left(\gamma_{gg}^{N(1)} - \frac{\beta_1}{\beta_0} \gamma_{gg}^{N(0)} \right) \\
& + \left[\frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} C_{2,q}^{N(1)} + \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \right)^2 C_{2,g}^{N(1)} \right] \gamma_{qq}^{N(0)} \\
& - \left[\left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \right)^3 C_{2,g}^{N(1)} + 2 \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}} - \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \right)^2 \frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}} \right. \\
& \left. - \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \right)^2 C_{2,q}^{N(1)} \right] \gamma_{qg}^{N(0)} \\
& + \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} C_{2,g}^{N(1)} - C_{2,q}^{N(1)} + \frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}} \right) \gamma_{gq}^{N(0)}
\end{aligned}
$$

$$
\begin{aligned}
& -\left[\frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}} + \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\right)^2 C_{2,g}^{N(1)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} C_{2,q}^{N(1)}\right] \gamma_{qg}^{N(0)} \\
& +2\beta_0 \left(\frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}}\right)
\end{aligned}
$$

$$
\begin{aligned}
K_{LL}^{N(1)} = \; & \gamma_{gg}^{N(1)} - \frac{\beta_1}{\beta_0} \gamma_{gg}^{N(0)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \left(\gamma_{qg}^{N(1)} - \frac{\beta_1}{\beta_0} \gamma_{qg}^{N(0)}\right) \\
& - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} C_{2,g}^{N(1)} \gamma_{qq}^{N(0)} + \left[\frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}} \right. \\
& \left. + \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\right)^2 C_{2,g}^{N(1)}\right] \gamma_{qg}^{N(0)} \\
& - C_{2,g}^{N(1)} \gamma_{gq}^{N(0)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} C_{2,g}^{N(1)} \gamma_{gg}^{N(0)} + 2\beta_0 \frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}}
\end{aligned}
$$

4. Recombination Corrections

ESSENTIAL AT LOWER Q^2.

TWIST 4... : COEFFICIENT FUNCTIONS
& ANOMALOUS DIMENSIONS

- FIRST STEPS TO A COMPLETE CALCULATION
  R. L. JAFFE, SOLDATE, ...
  K. ELLIS, FURMANSKI, PETRONZIO 1982, 83
  J. BARTELS et al. 1999
  JB, ZHU, RAVINDRAN, RUAN PL B 504 (2001) 235.
  (QCD diagr.)

- GRIBOV jun., LEVIN, RYSKIN 1981
  A. MUELLER, QIU 1986
  GLAUBER BASED

→ HOW COME SATURATION EFFECTS
IN SINGLE NUCLEONS ABOUT ?

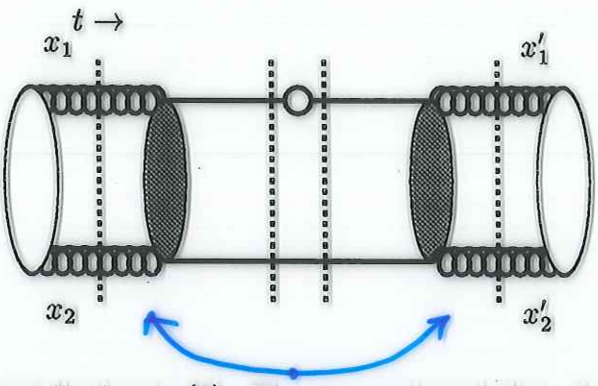


Figure 1: Direct diagrams contributing to (5). The grey oval symbolizes the set of diagrams in Figure 3. Orthogonal dashed lines stand for the time ordering. The separated white ovals symbolize the two parts of the non-perturbative 4–gluon density. x_1, x_2, x_1' and x_2' are the longitudinal momentum fractions, with $x_1 + x_2 - x_1' - x_2' = 0$. The circle stands for the forward subprocess $\gamma^* + q \rightarrow \gamma^* + q$ through which the virtual photon couples to the amplitude.

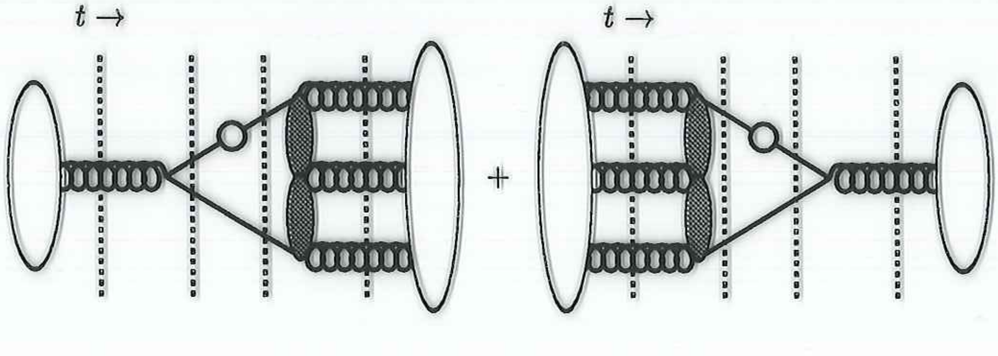


Figure 2: Interference diagrams associated to the process in Figure 1.

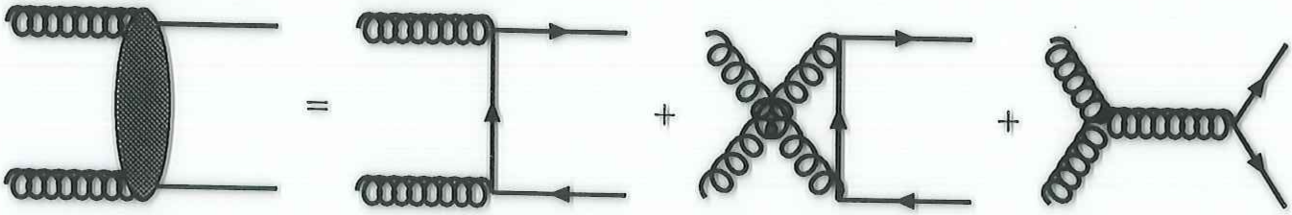


Figure 3: Diagrams symbolized by grey ovals in Figure 1 and 2.

IN THE SYMMETRIC CASE: $x_1 = x_2$ DGR. 3 VANISHES.

THE 4 G → 2Q COEFFICIENT FUNCTION:

$$\propto \frac{dp_\perp^2}{(p_\perp^2+m^2)^2} :$$

→ TOPT.

$$C_{G\to q}^{4\to 2,\, 2\to 2}(x_1; x) = \frac{1}{96}\,\frac{(2x_1 - x)^2}{x_1^5}\left[14x^2 - 3x\,x_1 + 18x_1^2\right]$$

ONE MAY SHOW THAT:

$$C_{G,q}^{4\to 2,\, 2\to 2} = -\,C_{G,q}^{4,2;\, 1\to 3} = -\,C_{G,q}^{4,2;\, 3\to 1}$$

AT ALL x !

4. Numerical Results

PREDICTION FOR THE SLOPE OF F_2.

$$\frac{\partial F_2(x,Q^2)}{\partial \log Q^2} = \frac{\partial F_2^{\tau=2}}{\partial \log Q^2}$$

ANTISCREENING $\longrightarrow$

$$+\left(\frac{\alpha_s}{2\pi}\right)^2 \frac{1}{Q^2} \int_{x/2}^{1/2} dx_1 \left(\frac{x}{x_1}\right) C^{4\to 2,2\to 2}(x_1,x) \cdot G_2(x_1)$$

SCREENING $\longrightarrow$

$$-2\left(\frac{\alpha_s}{2\pi}\right)^2 \frac{1}{Q^2} \int_{x}^{1/2} dx_1 \left(\frac{x}{x_1}\right) C^{4\to 2,2\to 2}(x_1,x) \cdot G_2(x_1)$$

$$\frac{\partial F_2^{\tau=2}}{\partial \log Q^2} = \left(\frac{\alpha_s}{2\pi}\right) \times \left\{ \sum_q e_q^2 \left[P_{qq} \otimes (q+\bar{q})\right](x) + \left[\sum_q e_q^2\right]\left[P_{qG} \otimes G_1\right](x) \right\}$$

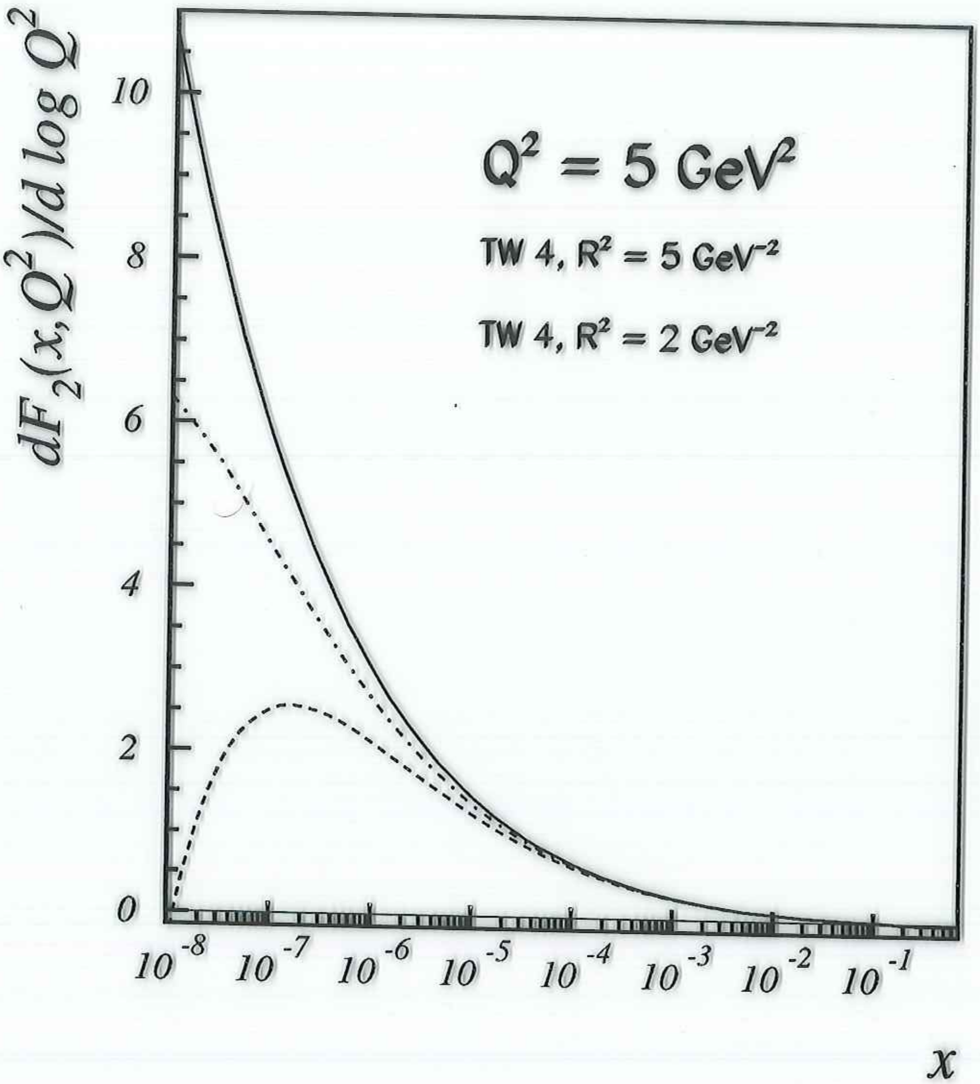


Figure 4a: The slope $dF_2(x, Q^2)/d\log Q^2$ at $Q^2 = 5\,\text{GeV}^2$. Full line: leading order twist–2 contributions (parameterization Ref. [24]). Dash-dotted line: Eq. (7) with twist–4 mass scale $R^2 = 5\,\text{GeV}^{-2}$, and dashed line: $R^2 = 2\,\text{GeV}^{-2}$.

"LEADING" POLE EXPANSION FAILS.

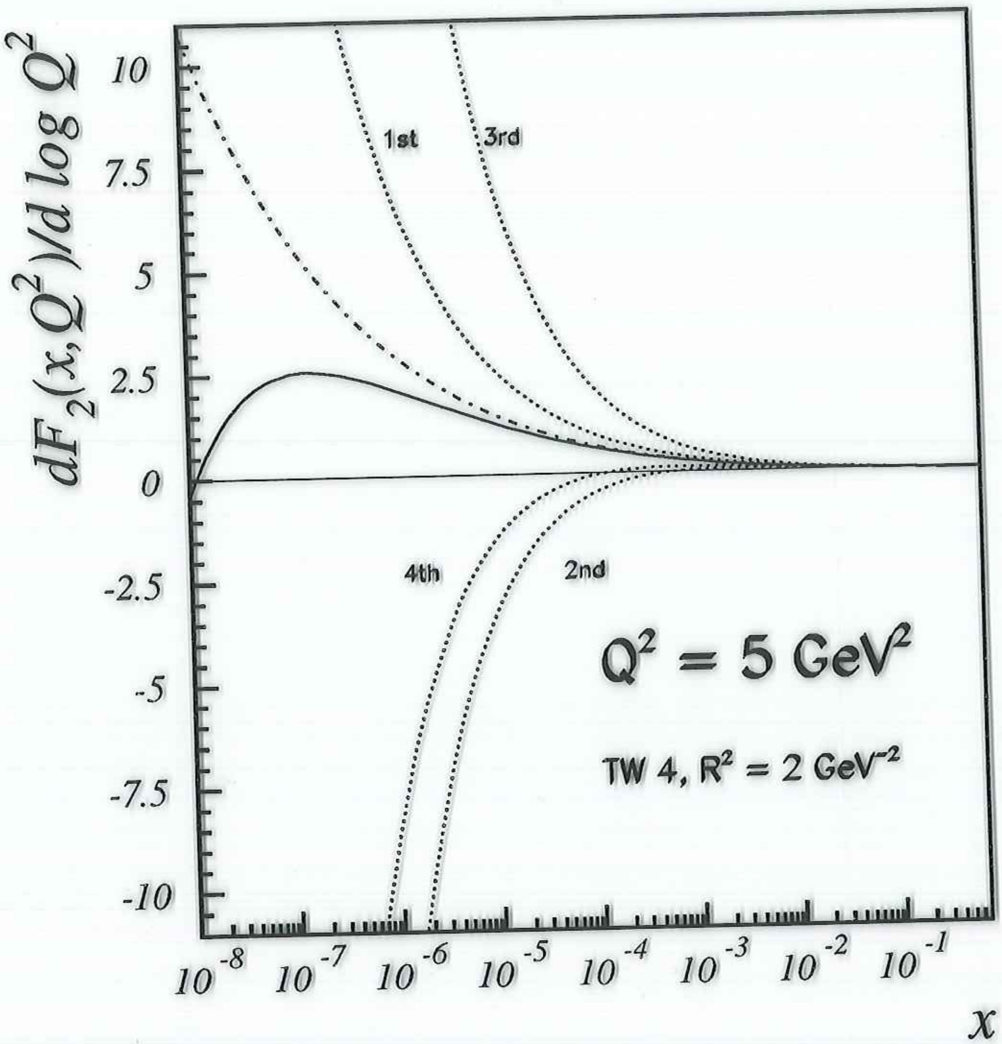


Figure 5: Comparison of the slope $dF_2(x,Q^2)/d\log Q^2$ at $Q^2 = 5\,\mathrm{GeV}^2$ and twist–4 mass scale $R^2 = 2\,\mathrm{GeV}^{-2}$, Eq. (7) (full line) with the corresponding results obtained approximating the coefficient function Eq. (5) by the sequence of contributing powers. 1st: z^0, 2nd: z etc. (dotted lines). Dash-dotted line: twist–2 contribution.

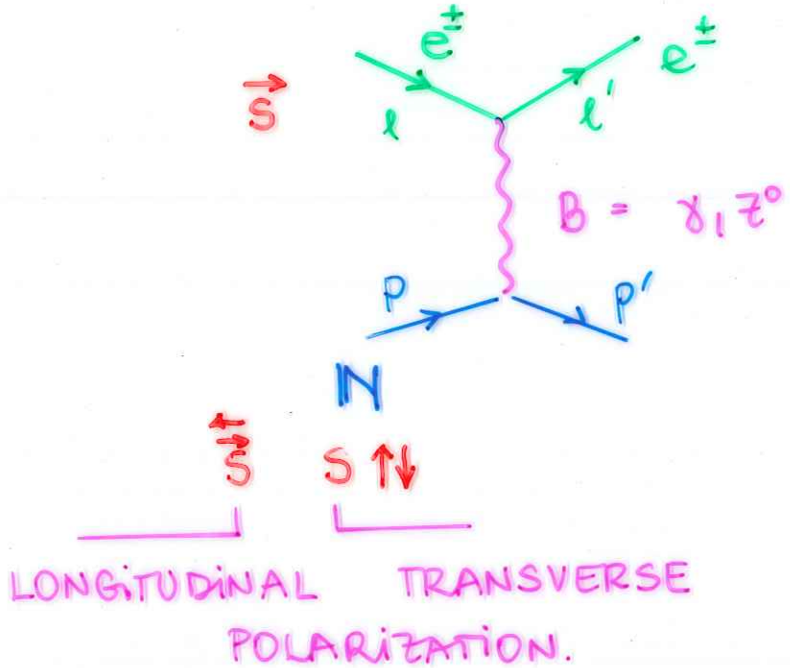
$$C_{G\to q}\left(z = \frac{x}{x_1}\right) = \frac{1}{96}\frac{1}{x_1}(2-z)^2\left[14z^2 - 3z + 18\right]$$

$$= \frac{1}{x_1}\frac{1}{96}\sum_{k=0}^{4} a_k z^k.$$

Recall: $P_{gg}^{(0)} \to \frac{6}{z}$

$P_{gq}^{(0)} \to C_F \cdot \frac{2}{z}$

5. Polarized DIS and the Next Twist



$$\frac{d^2\sigma^L}{dx\,dy} \underset{|\gamma^*|^2}{\sim} \left[-2y\left(2-y-\frac{2xyM^2}{S}\right)x\,g_1(x,Q^2) + \frac{8yx^2M^2}{S}\,g_2(x,Q^2)\right]$$

$$\frac{d^2\sigma^T}{dx\,dy} \underset{|\gamma^*|^2}{\sim} \sqrt{\frac{M^2}{S}}\sqrt{xy\left(1-y-\frac{xyM^2}{S}\right)}\left[-2y\,x\,g_1(x,Q^2) - 4x\,g_2(x,Q^2)\right]$$

(MORE TERMS FOR ELECTROWEAK CURRENT)

UNFOLD : g_1 & g_2 EXPERIMENTALLY.

WHAT IS THEIR STRUCTURE ACCORDING TO THE

LIGHT CONE EXPANSION ?

LOWEST ORDER QCD:

- TWO OPERATOR EXPECTATION VALUES

$$a_n \quad , \quad d_n \qquad n = \text{SPIN}$$

↑ TWIST 2 — ↑ TWIST 3 + HIGHER TWISTS $\sim 1/Q^{2k}$

$|\gamma^*|^2$ 2 SF'S , $\Delta q + \Delta \bar{q}$ → 1 RELATION

$|\gamma + Z^0|^2$ 5 SF'S , $\Delta q \pm \Delta \bar{q}$ → 3 RELATIONS

MORE PRECISELY: ONE REL. PER LOWEST TWIST

$|\gamma^*|^2$:

$$g_2^{II}(x,Q^2) = -g_1^{II}(x,Q^2) + \int_x^1 \frac{dy}{y} g_1^{II}(y,Q^2)$$

WANDZURA, WILCZEK 1977

$$g_1^{III}(x,Q^2) = \frac{4M^2x^2}{Q^2}\left[g_2^{III}(x,Q^2) - 2\int_x^1 \frac{dy}{y} g_2^{III}(y,Q^2)\right]$$

JB, TKABLADZE 1998

(ALL MASSES NEED TO BE RESUMMED.)

$|\gamma + Z^0|^2$:

DICUS 1972 } TWIST 2 2 REL.
JB, KOCHELEV 1996 }
JB, TKABLADZE 1998 TWIST 3 OTHER 2 REL.

EXPERIMENTAL STUDY POSSIBLE AT SLAC SOON.

$\Lambda_{QCD}^{(4)} \Longleftrightarrow \alpha_s(M_Z^2)$
8 parameter fit

$\Lambda_{QCD}^{(4)}$	$A1$		$g1$	
	VALUE	ERROR	VALUE	ERROR
FS/RS=1.0/1.0	0.235	± 0.060	0.242	± 0.067
FS/RS=0.5/1.0	0.185	– 0.050	0.193	– 0.049
FS/RS=2.0/1.0	0.293	+ 0.058	0.318	+ 0.076
FS/RS=1.0/0.5	0.330	+ 0.095	0.349	+ 0.107
FS/RS=1.0/2.0	0.175	– 0.060	0.187	– 0.055
SYST. ERROR $\Longrightarrow$		+ 0.121 – 0.077		+ 0.130 – 0.084

- $A1$: $\alpha_s(M_Z^2) = 0.113 \begin{smallmatrix} +0.004 \\ -0.005 \end{smallmatrix} (exp) \begin{smallmatrix} +0.004 \\ -0.004 \end{smallmatrix} (fac) \begin{smallmatrix} +0.007 \\ -0.005 \end{smallmatrix} (ren)$

- $g1$: $\alpha_s(M_Z^2) = 0.114 \begin{smallmatrix} +0.005 \\ -0.006 \end{smallmatrix} (exp) \begin{smallmatrix} +0.005 \\ -0.004 \end{smallmatrix} (fac) \begin{smallmatrix} +0.008 \\ -0.005 \end{smallmatrix} (ren)$

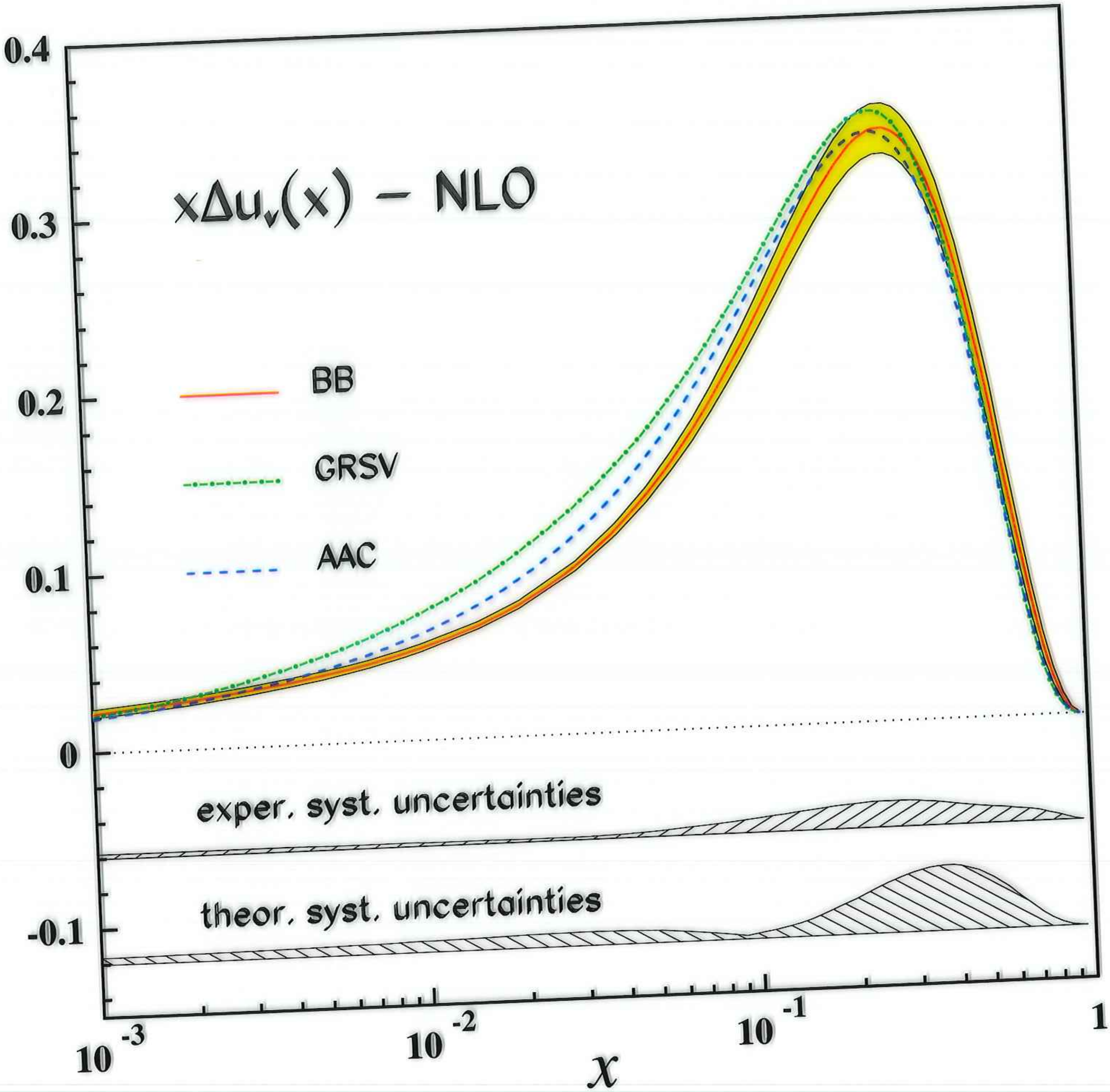
- SMC: $0.121 \pm 0.002(stat) \pm 0.006(syst + theor)$

  E154: $0.108 - 0.116 (> 0.120\ bad)$

  ABFR: $0.120 \begin{smallmatrix} +0.004 \\ -0.005 \end{smallmatrix} (exp) \begin{smallmatrix} +0.009 \\ -0.006 \end{smallmatrix} (theor)$

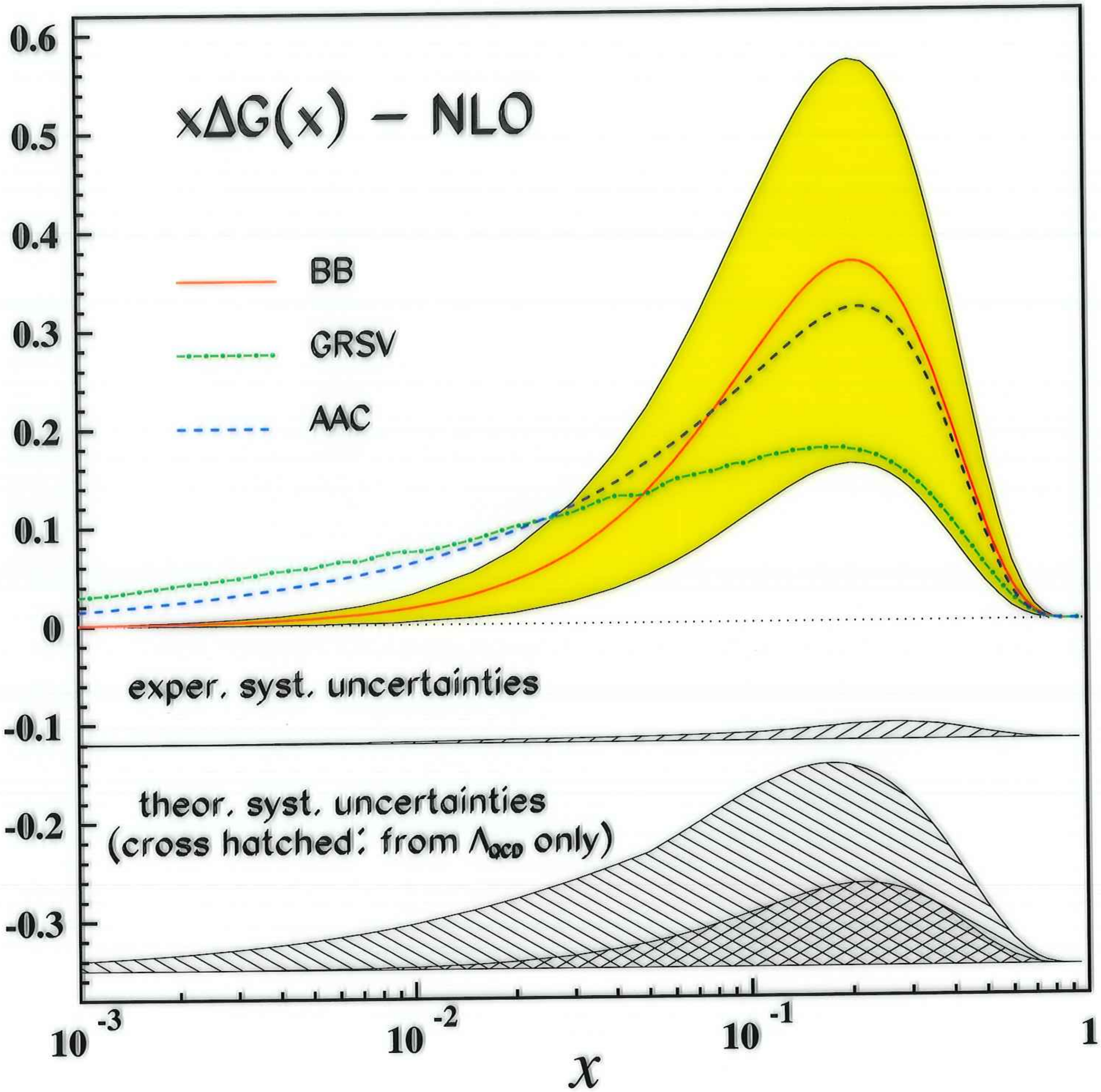
  WORLD AVERAGE: 0.118 ± 0.002

$x\Delta u_v(x)$ with error bands



$\Rightarrow$ Yellow error band: Fully correlated 1σ statistical error band at the input scale $Q_0^2 = 4.0\,GeV^2$.

$x\Delta G(x)$ with error bands



⇒ Yellow error band: Fully correlated 1σ statistical error band at the input scale $Q_0^2 = 4.0\,GeV^2$.

Comparison of Moments

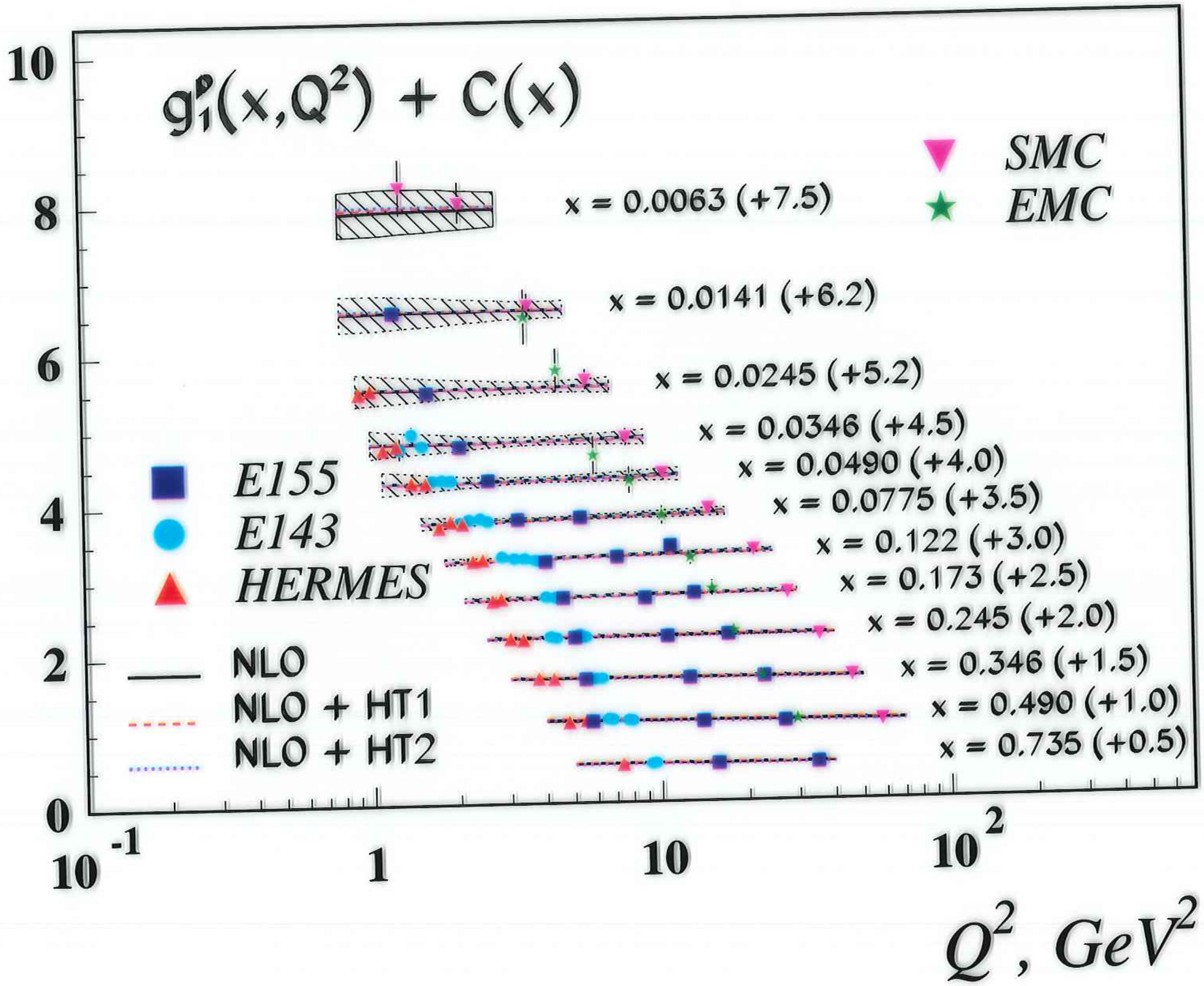
Δf	n	QCD Scenario 1 moment at $Q^2 = 4\,GeV^2$	lattice results QCDSF	lattice results LHPC/ SESAM
Δu_v	-1	0.926 ± 0.071	0.889(29)	0.860(69)
	0	0.163 ± 0.014	0.198(8)	0.242(22)
	1	0.055 ± 0.006	0.041(9)	0.116(42)
Δd_v	-1	-0.341 ± 0.123	-0.236(27)	-0.171(43)
	0	-0.047 ± 0.021	-0.048(3)	-0.029(13)
	1	-0.015 ± 0.009	-0.028(2)	0.001(25)
Δu–Δd	-1	1.267 ± 0.142	1.14(3)	1.031(81)
	0	0.210 ± 0.025	0.246(9)	0.271(25)
	1	0.070 ± 0.011	0.069(9)	0.115(49)

$\Longrightarrow \Gamma_{\Delta f}(Q^2) = \int_0^1 x^{n+1} \Delta f(x, Q^2) dx$

Lattice simulation: Scale $\mu^2 = 1/a^2 \sim 4\,GeV^2$. For the $n = 0, 1$ values of the QCDSF Coll. no continuum extrapolation was performed.

[**Refs:** M.Göckeler et al., QCDSF Coll., Phys.Rev. **D53** (1996) 2317; Phys.Lett. **B414** (1997) 340; hep-ph/9711245; Phys.Rev. **D63** (2001) 074506; S.Capitani et al., Nucl.Phys.(Proc. Suppl.) **B79** (1999) 548; S.Güsken et al., SESAM Coll., hep-lat/9901009; D.Dolgov et al., LHPC and SESAM Coll., hep-lat/0201021.]

$g_1^p(x)$ + Higher Twist - Scenario 1

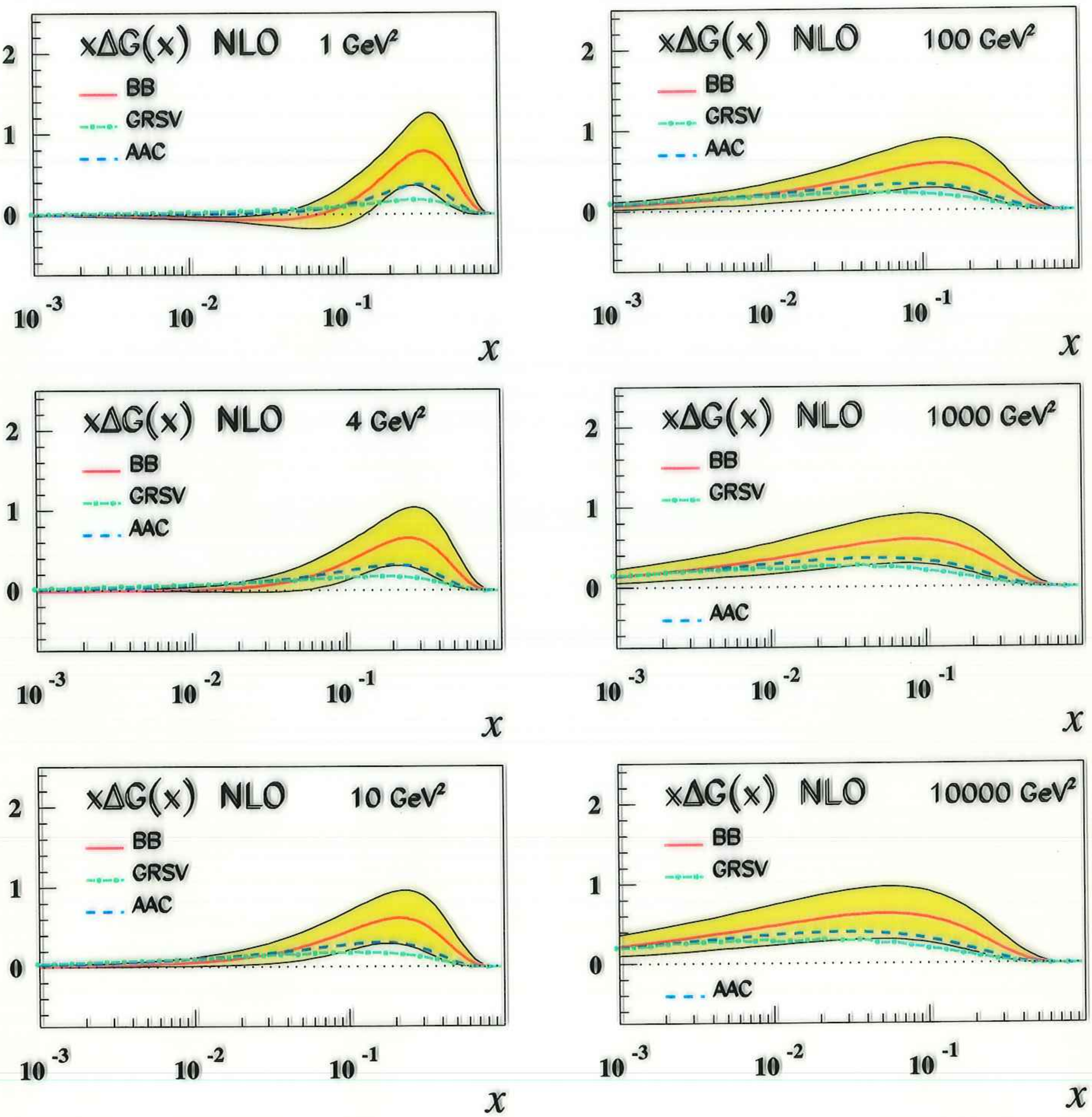


⇒ Hatched error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

- Higher Twist Contribution: $g_1(x, Q^2)[1 + HT(x, Q^2)]$
  - HT1: $1/Q^2(x^a(1-x)^b)$
  - HT2: $1/Q^2(a + bx + cx^2)$

Evolution of Polarized Parton Densities

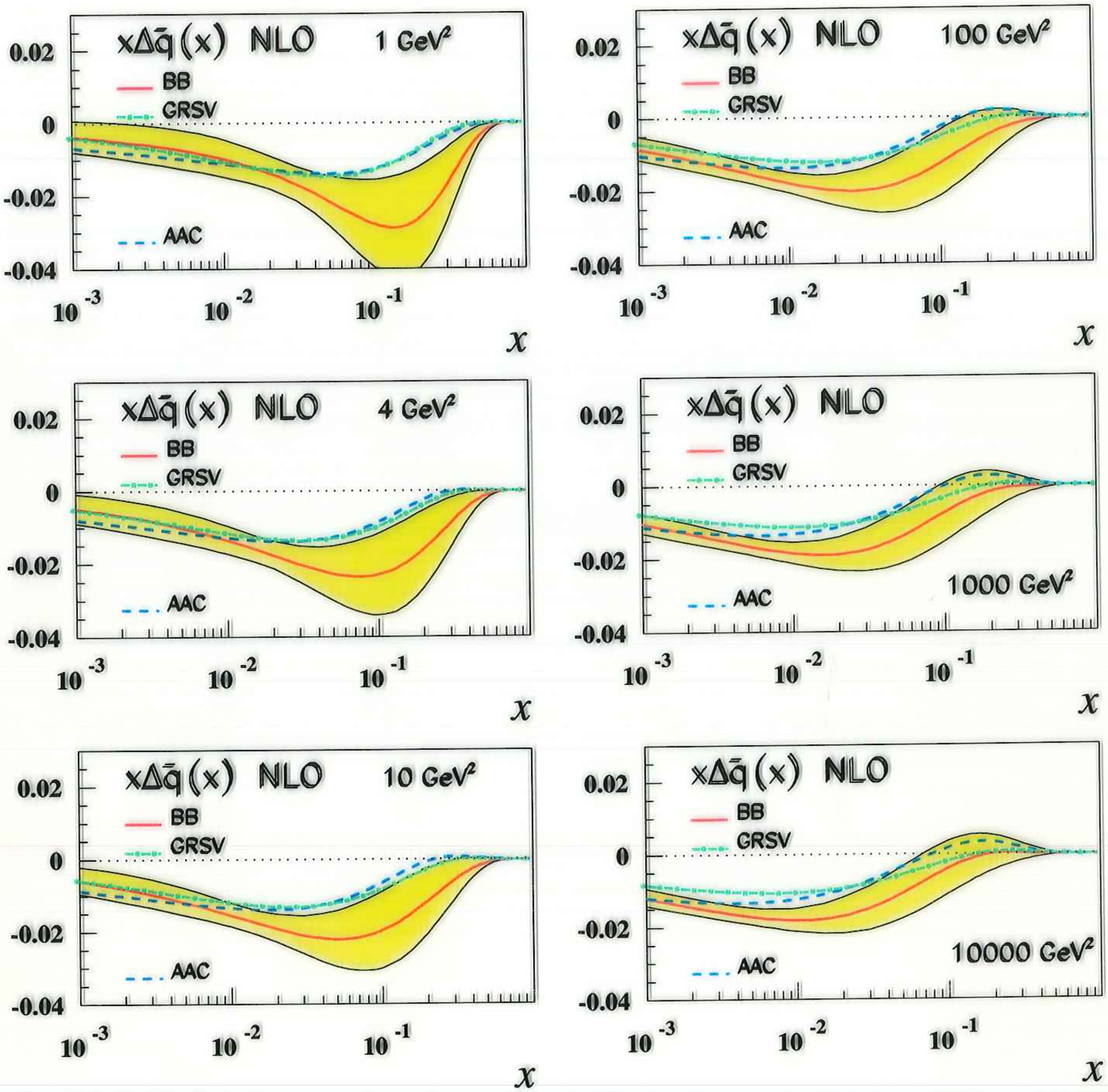
- 8 Parameter Fit based on $A1(g1/F1)$ Data:



⇒ Yellow Error Band evolved to the Q^2 indicated.

Evolution of Polarized Parton Densities

- 8 Parameter Fit based on $A1(g1/F1)$ Data:



⇒ Yellow Error Band evolved to the Q^2 indicated.

TARGET MASS CORRS.

JB, A. TKABLADZE '99

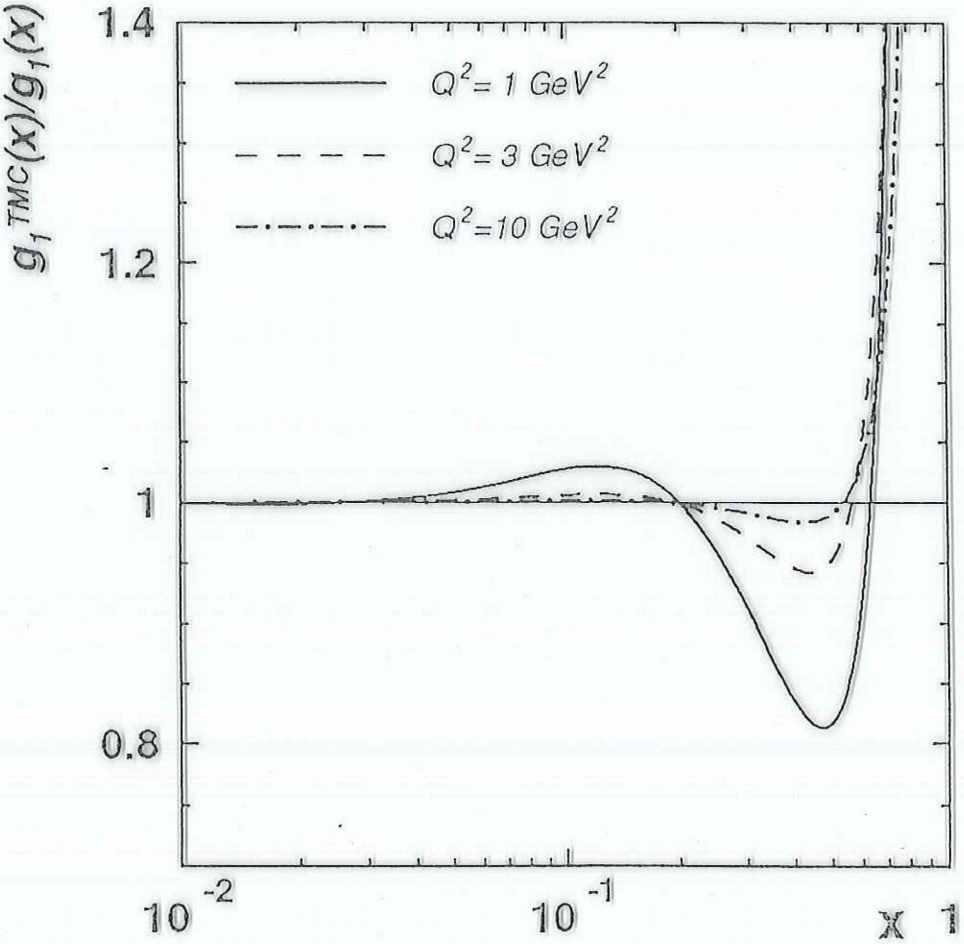


Figure 1. The ratio $g_1^{TMC}(x)/g_1(x)$ versus x.

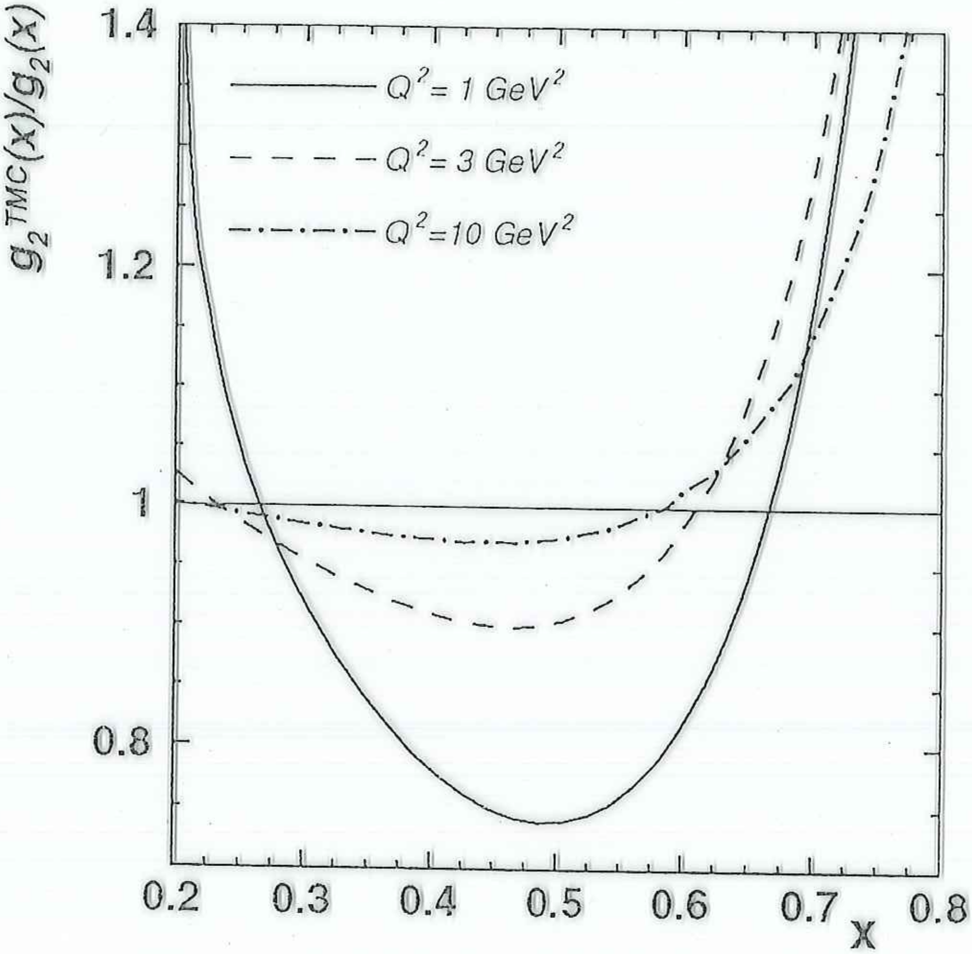


Figure 2. The ratio $g_2^{TMC}(x)/g_2(x)$ versus x.

6. Some Sum Rules

- KEY ROLE OF CERTAIN INTEGRALS OVER STRUCTURE FUNCTIONS

$\longrightarrow$ MEASURE α_s

1) GROSS-LLEWELLYN SMITH SR:

$$\int_0^1 dx\, F_3^{\bar{\nu}p+\nu p}(x) = 6\left[1 - \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2\left[-\frac{55}{12} + \frac{1}{3}N_f\right]\right.$$
$$+ \left(\frac{\alpha_s}{\pi}\right)^3\left[-\frac{13841}{216} - \frac{44}{9}\zeta_3 + \frac{55}{2}\zeta_5\right.$$
$$\left.\left. + \left(\frac{10029}{1296} + \frac{91}{54}\zeta_3 - \frac{5}{3}\zeta_5\right)N_f - \frac{115}{648}N_f^2\right]\right]$$

2) POL. BJORKEN SR:

$$\int_0^1 dx\, g_1^{ep-en}(x) = \frac{1}{3}\left|\frac{g_V}{g_A}\right|\left[\ldots + \left(\frac{\alpha_s}{\pi}\right)^3\left[\ldots \left(\frac{10339}{1296} + \frac{61}{54}\zeta_3 \ldots\right)N_f \ldots\right]\right]$$

VERMASEREN, LARIN 1991

3) UNPOL. BJORKEN SR:

$$\int_0^1 dx\, F_1^{\bar{\nu}p-\nu p}(x) = 1 - \frac{2}{3}\left(\frac{\alpha_s}{\pi}\right) + \left(\frac{\alpha_s}{\pi}\right)^2\left(-\frac{23}{6} + \frac{8}{27}N_f\right)$$
$$+ \left(\frac{\alpha_s}{\pi}\right)^3\left[-\frac{4075}{108} + \frac{622}{27}\zeta_3 - \frac{680}{27}\zeta_5\right.$$
$$\left. + \left(\frac{3565}{648} - \frac{59}{27}\zeta_3 + \frac{10}{3}\zeta_5\right)N_f - \frac{155}{372}N_f^2\right].$$

TWIST 2:

POLARIZED.

WANDZURA-WILCZEK '77

$$g_2(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \frac{dy}{y} g_1(y,Q^2)$$

STANDS: TM
HEAVY FLAVOR } DIS
DIFFRACTION
NON-FORWARD SCATTERING.

$$g_4 = 2x g_5 + \Delta_D$$

$$g_3^{(4)} = 2x \int_x^1 \frac{dy}{y^2} g_4(y)$$

JB, KOCHELEV '96

TWIST 3:

$$g_1(x,Q^2) = \frac{4M^2x^2}{Q^2}\left[g_2(x,Q^2) - 2\int_x^1 \frac{dy}{y} g_2(y,Q^2)\right]$$

$$\frac{4M^2x^2}{Q^2} g_3(x,Q^2) = g_4(x,Q^2)\left(1 + \frac{4M^2x^2}{Q^2}\right) + 3\int_x^1 \frac{dy}{y} g_4(y,Q^2)$$

$$2x g_5(x,Q^2) = -\int_x^1 \frac{dy}{y} g_4(y,Q^2)$$

JB, TKABLADZE '98

7. Scaling Violations in Diffractive Scattering

ABRAMOWICZ, DAINTON '96.

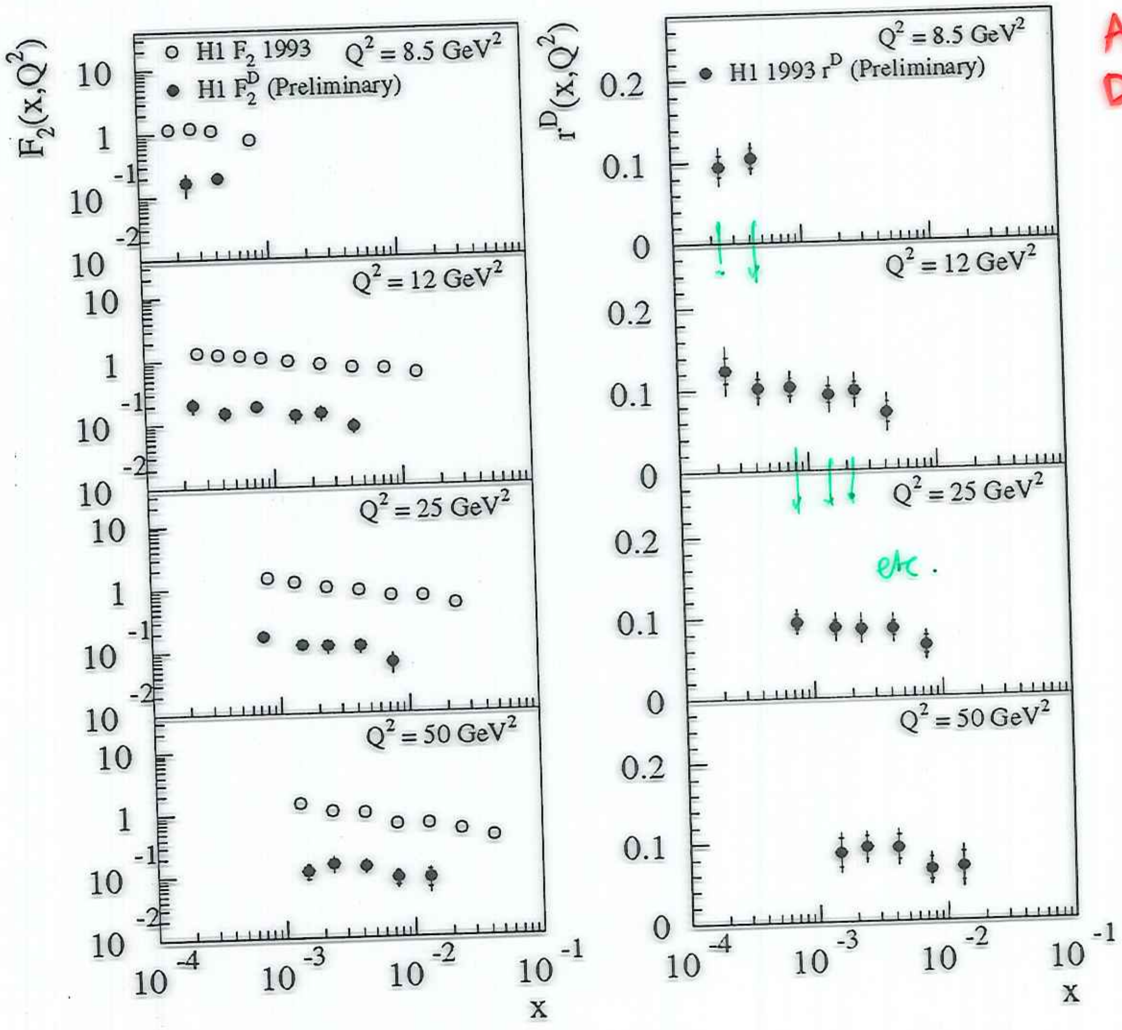
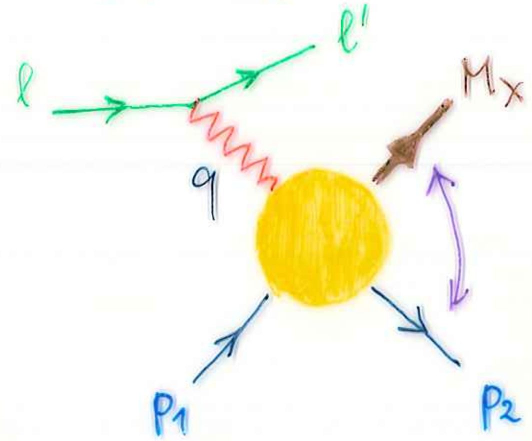


Figure 3. The structure function $F_2^D(x, Q^2)$ and the ratio $r^D = F_2^D(x, Q^2)/F_2(x, Q^2)$ for $x_{\mathbb{P}} < 0.05$. The result is for deep-inelastic diffraction in which the proton does not dissociate. Approximately one third of deep-inelastic diffractive interactions are consistent with proton dissociation.

2. Lorentz Structure

$$d^5\sigma_{\mathrm{DIFFR}} = \frac{1}{2(S-M^2)}\,\frac{1}{4}\,dPS^{(3)}\sum_{\mathrm{spins}}\frac{e^4}{Q^4}\,L_{\mu\nu}W^{\mu\nu}$$



$$\Delta\eta \sim \log\frac{1}{x_{\mathbb{P}}}$$

$$x = \frac{Q^2}{W^2+Q^2-M^2} \qquad t = +(p_1-p_2)^2, \qquad M_X^2 = (q+p_1-p_2)^2$$

$$W^2 = (q+p_1)^2$$

$$x_{\mathbb{P}} = -\frac{2\eta}{1-\eta} \geq x$$

$$\eta = \frac{q.(p_2-p_1)}{q.(p_2+p_1)} \in \left[-1, \frac{-x}{2-x}\right]$$

$$W_{\mu\nu} = \left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)W_1 + \hat{P}_{1\mu}\hat{P}_{1\nu}\frac{W_3}{M^2} + \hat{P}_{2\mu}\hat{P}_{2\nu}\frac{W_4}{M^2}$$

$$+\left[\hat{P}_{1\mu}\hat{P}_{2\nu}+\hat{P}_{1\nu}\hat{P}_{2\mu}\right]\frac{W_5}{M^2} + W^{POL}$$

$$\hat{P}_{i\mu} = p_{i\mu} - q_\mu\frac{p_i.q}{q^2}.$$

4. Evolution Equations

HOW DO DIFFRACTIVE PARTON DENSITIES EVOLVE ?

START WITH GENERAL NON-FORWARD FORMALISM, TO TRACE η-DEPENDENCE.

$$\frac{d}{d\log\mu^2} O^A_{(S)}(\kappa_+\tilde{x}, \kappa_-\tilde{x};\mu^2) = \int Dk' \, \gamma^{AB}(\kappa_+, \kappa_-; \kappa_+', \kappa_-';\mu^2) * O_{B(S)}(\kappa_+'\tilde{x}, \kappa_-'\tilde{x})$$

THE OME $\langle p_1, -p_2 | O | p_1, -p_2\rangle$ ARE INTRODUCED.

$$f^A(\vartheta,\eta) = \int \frac{d\kappa_-\tilde{x}p_-}{2\pi} e^{i\kappa_-\tilde{x}p_-\vartheta} \langle p_1, -p_2 | O^A | p_1, -p_2\rangle \cdot (\tilde{x}p_-)^{1-d_A}$$

$d_A = 1 : q$
$d_A = 2 : G$

$\longrightarrow$ NO κ_+ DEPENDENCE FOR THIS PROJECTION.

$$\gamma^{AB}(\kappa_+, \kappa_-, \kappa_+', \kappa_-') \longrightarrow \gamma^{AB}(0, \kappa_-, \kappa_+', \kappa_-').$$

ALL-ORDER RESCALING PROPERTY:

$$\gamma^{AB}(k_+, k_-, k'_+, k'_-) = \sigma^{d_{AB}} \gamma^{AB}(\sigma k_+, \sigma k_-, \sigma k'_+, \sigma k'_-)$$

$$d_{AB} = 2 + d_A - d_B .$$

↷

$$\int Dk' \, k_-^{d_B - d_A} \gamma^{AB}\left(0, 1, \frac{k'_+}{k_-}, \frac{k'_-}{k_-}, \mu^2\right)$$

$$= \int D\alpha \, k_-^{d_B - d_A} \hat{K}^{AB}(\alpha_1, \alpha_2, \mu^2)$$

FURTHER CONVERSION:

$$\mu^2 \frac{d}{d\mu^2} f^A(\vartheta; \eta; \mu^2) = \int_0^1 du \int_{\vartheta}^{-\mathrm{sign}(\vartheta)/\eta} d\vartheta' \, \tilde{O}^{AB}(u\vartheta' - \vartheta) \, \hat{\hat{K}}^{AB}(u, \mu^2) \, f_B(\vartheta', \eta, \mu^2)$$

$$\tilde{O}^{AB}(u\vartheta' - \vartheta) = \begin{cases} \delta(u\vartheta' - \vartheta) & A = B \\ \partial_u \delta(u\vartheta' - \vartheta) & A = q, \; B = G \\ \theta(u\vartheta' - \vartheta)/\vartheta & A = G, \; B = q \end{cases}$$

↷ EVOLUTION EQUATION

EVOLUTION EQUATION IN

$$\vartheta = z_- + \frac{1}{\eta} z_+$$

$$\mu^2 \frac{\partial}{\partial \mu^2} f^A(\vartheta, \eta, \mu^2) = \int_{\vartheta}^{-\mathrm{sign}(\vartheta)/\eta} \frac{d\vartheta'}{\vartheta'} P^{AB}\left(\frac{\vartheta}{\vartheta'}, \mu^2\right) f_B(\vartheta', \eta, \mu^2)$$

ACTION OF THE ABSORPTION CONDITION:

$$\delta(\vartheta - 2\beta)$$

$$\mu^2 \frac{\partial}{\partial \mu^2} f_A^D(\beta, x_{I\!P}, \mu^2) = \int_{\beta}^{1} \frac{d\beta'}{\beta'} P^{AB}\left(\frac{\beta}{\beta'}, \mu^2\right) f_B(\beta', x_{I\!P}, \mu^2)$$

$x_{I\!P}$ IS A BARE PARAMETER (LIKE η), AND DOES NOT EVOLVE.

THE EVOLUTION AFFECTS β.

5. Conclusions

- DIFFRACTIVE ep SCATTERING DEPENDS ON 4 UNPOLARIZED & 8 POLARIZED SF'S.
- FOR $t, M^2 \to 0$ 2 UNPOL. + 2 POL. SF'S CONTRIBUTE.
- THE ASSOCIATE OME'S MAY BE FORMED APPLYING MUELLER'S OPTICAL THEOREM
- THE STUDY OF THE COMPTON AMPLITUDE FOR THE NEW 2-PARTICLE HADR. INITIAL STATE YIELDS

$$2x F_1(\beta, x_{\mathbb{P}}, \mu^2) = F_2(\beta, x_{\mathbb{P}}, \mu^2)$$

$$G_2(\beta, x_{\mathbb{P}}, \mu^2) = -G_1(\beta, x_{\mathbb{P}}, \mu^2) + \int_\beta^1 \frac{d\beta'}{\beta'} G_1(\beta', x_{\mathbb{P}}, \mu^2)$$

MODIFIED CALLAN-GROSS AND WANDZURA-WILCZEK RELATIONS. IN $O(\alpha_s^0)$.

- AT TWIST 2 THE USUAL EVOLUTION EQS. ARE OBTAINED ; $x_{\mathbb{P}}\ (\eta)$ BEHAVES AS A PLAIN PARAMETER.
- THE METHOD APPLIES TO ALL TWISTS. THE EVOLUTION IS ALWAYS FORWARD
- LGT: WELL SUITED PRESCRIPTION FOR LATTICE MEASUREMENTS.

8. Structures behind Feynman Diagrams

ARE HIGHER ORDER RESULTS

SIMPLE AFTER ALL ?

van Neerven, Zijlstra 1992

$$c_{2,-}^{(2)}(x) = C_F\,(C_F - C_A/2) \times$$
$$\Bigg\{\frac{1+x^2}{1-x}\Bigg[\Big[4\ln^2(x) - 16\ln(x)\ln(1+x) - 16\mathrm{Li}_2(-x) - 8\zeta_2\Big]\ln(1-x)$$
$$+\Big[-2\ln^2(x) + 20\ln(x)\ln(1+x) - 8\ln^2(1+x) + 8\mathrm{Li}_2(1-x) + 16\mathrm{Li}_2(-x) - 8\Big]\ln(x)$$
$$-16\ln(1+x)\mathrm{Li}_2(-x) - 8\zeta_2\ln(1+x) - 16\left[\mathrm{Li}_3\left(-\frac{1-x}{1+x}\right) - \mathrm{Li}_3\left(\frac{1-x}{1+x}\right)\right]$$
$$-16\mathrm{Li}_2(1-x) + 8S_{1,2}(1-x) + 8\mathrm{Li}_3(-x) - 16S_{1,2}(-x) + 8\zeta_3\Bigg]$$
$$+(4+20x)\Big[\ln^2(x)\ln(1+x) - 2\ln(x)\ln^2(1+x) - 2\zeta_2\ln(1+x) - 4\ln(1+x)\mathrm{Li}_2(-x)$$
$$+2\mathrm{Li}_3(-x) - 4S_{1,2}(-x) + 2\zeta_3\Big] + \left(32 + 32x + 48x^2 - \frac{72}{5}x^3 + \frac{8}{5x^2}\right)$$
$$\times\,[\mathrm{Li}_2(-x) + ln(x)\ln(1+x)] + 8(1+x)\,[\mathrm{Li}_s(1-x) + \ln(x)\ln(1-x)] + 16(1-x)\ln(1-x)$$
$$+\left(-4 - 16x - 24x^2 + \frac{36}{5}x^3\right)\ln^2(x) + \frac{1}{5}\left(-26 - 106x + 72x^2 - \frac{8}{x}\right)\ln(x)$$
$$+\left(-4 + 20x + 48x^2 - \frac{72}{5}x^3\right)\zeta_2 + \frac{1}{5}\left(-162 + 82x + 72x^2 + \frac{8}{x}\right)\Bigg\}$$

.... several other pages for $c_2^{(+)}(x), c_2^G(x), c_L^{(q,G)}(x)$

$\Longrightarrow$ 77 Functions @ 2 Loops

- $\Longrightarrow$ partly rather complicated arguments
- $\Longrightarrow$ relations are not directly visible ...

The 77 functions do roughly correspond in number to the number of all possible harmonic sums up to weight w=4: 80.

3. Multiple Harmonic Sums to Level 6

The simplest example :

$$P_{qq}(x) = \left(\frac{1+x^2}{1-x}\right)_+ = \frac{2}{(1-x)_+} + \dots$$

$$\int_0^1 dx \frac{x^{N-1}}{(1-x)_+} = -\sum_{k=0}^{N-2} \int_0^1 dx x^k = -\sum_{k=1}^{N-1} \frac{1}{k} = -S_1(N-1)$$

Alternating sums :

$$S_{-1}(N-1) = (-1)^{N-1}\mathbf{M}\left[\frac{1}{1+x}\right](N) - \ln(2) = \int_0^1 dx \frac{x^{N-1}}{(1+x)_+} = \sum_{k=1}^{N-1} \frac{(-1)^k}{k}$$

(Finite for $N \rightarrow \infty$.)

General case :

$$S_{a_1,\dots,a_l}(N) = \sum_{k_1=1}^{N} \frac{(\mathrm{sign}(a_1))^{k_1}}{k_1^{|a_1|}} \sum_{k_2=1}^{N} \frac{(\mathrm{sign}(a_2))^{k_2}}{k_2^{|a_2|}} \dots$$

Vermaseren, 1997

All Mellin transforms occurring in massless Field Theories for 1-Parameter Quantities can be represented by Harmonic Sums

(at least to 3–loop order).

Algebraic Relations

First relation:

L. Euler, 1775

$$S_{m,n} + S_{n,m} = S_m \cdot S_n + S_{m+n}, \quad m, n > 0$$

Generalized to alternating sums by

$$\begin{aligned} S_{m,n} + S_{n,m} &= S_m \cdot S_n + S_{m \wedge n}, \\ m \wedge n &= [|m| + |n|] \, \mathrm{sign}(m)\mathrm{sign}(n) \end{aligned}$$

Ternary relations: Sita Ramachandra Rao, 1984,

4-ary relation: J.B., Kurth, 1998.

These & other relations hold widely independent of their **Value** and **Type**.

Determined by :
- Index Structure
- Multiplication Relation

The Formalism applies as well to the Harmonic Polylogarithms. Remiddi, Vermaseren, 1999.

Application to QED: T. Riemann et al., 2004

Linear Representations of Mellin Transform by Harmonic Sums:

$$\mathbf{M}[F_w(x)](N) = S^w_{k_1,\ldots,k_m}(N) + P\left(S^{\tau'}_{k_1,\ldots,k_r}, \sigma^{\tau''}_{k_1,\ldots,k_p}\right)$$

$w = \sum_{i=1}^m |k_i|$ Weight

$\tau', \tau'' < w$ P is a polynomial.

w	#	Σ	
1	2	2	
2	6	8	
3	18	26	2 Loop anom. Dimensions
4	54	80	2 Loop Wilson Coefficients
5	162	242	3 Loop anom. Dimensions
6	486	728	3 Loop Wilson Coefficients
	$2 \cdot 3^{w-1}$	$3^w - 1$	

Theory of Words

Can we count the Basis in simpler way ? $\Longrightarrow$ YES.

Free Algebras and Elements of the Theory of Codes

$\Longrightarrow$ **Particle Physics**

Only the multiplication relation and the Index structure matters

$\mathfrak{A} = \{a, b, c, d, \ldots\}$ **Alphabet**

$a < b < c < d < \ldots$ **ordered**

$\mathfrak{A}^*(\mathfrak{A})$ **Set of all words W**

$W = a_1 \cdot a_2 \cdot a_{27} \ldots a_{532} \equiv$ **concatenation product (nc)**

$W = p \cdot x \cdot s$ **p = prefix; s = suffix**

Definition:

A Lyndon word is smaller than any of its suffixes.

Theorem:[Radford, 1979]

The shuffle algebra $K\langle\mathfrak{A}\rangle$ is freely generated by the Lyndon words.
I.e. the number of Lyndon words yields the number of basic elements.

Examples :

$\{a, a, \ldots, a, b\} = aaa \ldots ab$ 1 Lyndon word for these sets

n-1 $a's$: $n_{basic}/n_{all} = 1/n$ $n \equiv$ depth of the sums

$\{a,a,a,b,b,b\}$ $aaabbb, aababb, aabbab$ 3 Lyndon words

$n_{basic}/n_{all} = 3/20 < 1/6.$ Symmetries lead to a smaller fraction.

Is there a general Counting Relation ?

E. Witt, 1937

$$l_n(n_1,\ldots,n_q) = \frac{1}{n}\sum_{d\|n_i}\mu(d)\frac{(n/d)!}{(n_1/d)!\ldots(n_q/d)!}, \qquad \sum_i n_i = n$$

$\mu(k)$ Möbius function

2nd Witt formula.

The Length of the Basis is a function mainly of the Depth.

$$l_6(\{a,a,a,b,b,b\}) = \frac{1}{6}\left[\mu(1)\frac{6!}{3!3!} + \mu(3)\frac{2!}{1!1!}\right] = 3$$

$$n_6(\{a,a,a,b,b,b\}) = \frac{6!}{2!3!} = 20$$

Weight	# Sums	Cum. # Sums	# Basic Sums	Cum. # Basic Sums	Cum. Fraction
1	2	2	0	0	0.0
2	6	8	1	1	0.1250
3	18	26	6	7	0.2692
4	54	80	16	23	0.2875
5	162	242	46	69	0.2851
6	486	728	114	183	0.2513

⇑ 2nd Witt formula

4. A Quadratic Law ?

The anomalous dimensions and Wilson coefficients for $m_i = 0$ can be expressed in terms of multiple harmonic sums to 3–loop order.

What are the irreducible functions behind this representation ?

We will not count Euler's Γ-function neither all derivations of the functions occurring.

The final set of functions:

Trivial functions:

$$S_{\pm k}(N) \quad \longrightarrow \quad \psi^{(k-1)}(N+1)$$

For $w = 1, 2$ no non–trivial functions contribute to the anomalous dimensions and Wilson coefficients.

Non–trivial functions:

$N = 3$: Two–Loop anomalous dimensions

$$\mathbf{M}\left[\frac{\mathrm{Li}_2(x)}{1+x}\right](N)$$

Yndurain et al., 1980

$N = 4$: Two–Loop Wilson Coefficients

$$\mathbf{M}\left[\frac{\mathrm{Li}_2(x)}{1-x}\right](N), \quad \mathbf{M}\left[\frac{\mathrm{Li}_3(x)}{1+x}\right](N), \quad \mathbf{M}\left[\frac{\mathrm{S}_{1,2}(x)}{1 \pm x}\right](N)$$

J.B., S. Moch, 2003,

also: J.B., V. Ravindran, 2004.

$N = 5$: Three–Loop Anomalous Dimensions

$$\mathbf{M}\left[\frac{\ln(1+x)}{1+x}\right](N), \quad \mathbf{M}\left[\frac{\mathrm{Li}_4(x)}{1\pm x}\right](N), \quad \mathbf{M}\left[\frac{S_{1,3}(x)}{1+x}\right](N),$$
$$\mathbf{M}\left[\frac{S_{2,2}(x)}{1\pm x}\right](N), \quad \mathbf{M}\left[\frac{S_{2,2}(-x)-\mathrm{Li}_2^2(-x)/2}{1\pm x}\right](N),$$
$$\mathbf{M}\left[\frac{\mathrm{Li}_2^2(x)}{1+x}\right](N)$$

J.B., S. Moch, 2004.

The number of **Non-trivial Basic Functions** seems to grow as :

$$\mathbf{N_w} = \theta(\mathbf{w}-\mathbf{2})\cdot[\mathbf{w}-\mathbf{2}]^{\mathbf{2}}$$

Essentially 14 Functions seem to rule the single scale processes of massless QCD.

This is a rather small number if compared to the number of possible harmonic sums $3^w - 1$.

9. Conclusions

1. QCD FITS TO DIS DATA ARE POSSIBE NOW AT 3 LOOP ORDERS : UNPOL. DATA.
   → POL. ANOM DIMS. TO COME.

2. $\Delta\alpha_s^{THEO} \lesssim 1\%$ IS WITHIN REACH.
   THEORY ERRORS ON PDF'S : POL & UNPOL. WILL DIMINUISH SOON. ALSO: SCHEME INV. EVOL.

3. SMALL x EFFECTS : VERY SMALL IN THE HERA REGION
   - NOT LEADING IN THE SENSE OF RESUMM. DUE TO EQUAL ORDER SUBLEADING TERMS. + ...

4. MATHEMATICAL STRUCTURE OF HO CORRS: DRASTIC REDUCTION IN COMPLEXITY
   14 FUNCTIONS & $\Gamma(x)$ + THEIR DERIVATIVES RULE $O(\alpha_s^3)$! FOR STRUCTURE FUNCTIONS

5. MORE DATA WANTED:
   $g_2, \delta q$; EW POL. SF'S → HE ν FACTORIES.
   EVERYWHERE : LARGE x : @ $W^2 > 4\ GeV^2$