Universal QED Corrections to Polarized Electron Scattering in Higher Orders

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DESY

1. **Motivation**
2. **Solution of Evolution Equation to $O([\alpha L(Q^2)]^5)$**
   - 2.1. Non-Singlet Case
   - 2.2. Singlet Case
3. **Resummation of the $O((\alpha \ln^2(z))^k)$ Terms**
   - 3.1. Non-Singlet Case
   - 3.2. Singlet Case
4. **Conclusions**

Work in common with H. Kawamura DESY 02-016.

1. Motivation

- QED CORRECTIONS TO HE-PROCESSES ARE MUTUAL.

  $s \gg m_i^2 \quad (m_e^2)$ LARGE LOG's

- UNIVERSAL CONTRIBUTIONS

A) $\longrightarrow$ SINGLE LEG RADIATION

EVOLUTION EQ. $\rightarrow$ MASS SINGULARITIES

$$O[(\alpha L)^k]$$

B) RESUMMATION OF UNIVERSAL TERMS IN THE CONFORMAL LIMIT : $(m \rightarrow 0)$

$$a = \frac{\alpha}{4\pi} = \text{CONST.} \qquad \text{"}\beta \rightarrow 0\text{"}$$

POL. SCATTERING: $O[(\alpha \ln^2 z)^k]$

THESE ARE UNIVERSAL CLASSES, WHICH HAVE TO BE RESUMMED ALWAYS.

UNLIKE IN QCD (DIS) STRAIGHT FORWARD NUMERICAL SOLUTIONS ARE NOT POSSIBLE; SINCE

$$D^{e^\pm}(a_0, z) = \delta(1-z)$$

RATHER THAN

$$D(a_0, z)_{QCD} \propto (1-z)^a \quad , \quad a \gtrsim 3$$

$\longrightarrow$ PERTURBATIVE, ANALYTIC SOLUTION OF EVOLUTION EQS.

$$D^{e^\pm}(a, z) = D^{e^\pm}_{NS}(a, z) + D^{e^\pm}_{\Sigma}(a, z)$$

$$D^{e^-}_{NS}(a, z) = - D^{e^+}_{NS}(a, z) = \frac{1}{2}\left[D^{e^-}(a, z) - D^{e^+}(a, z)\right]$$

$$D^{e^\pm}_{\Sigma}(a, z) = \frac{1}{2}\left[D^{e^-}(a, z) + D^{e^+}(a, z)\right]$$

2. Solution of Evolution Equation to $O([\alpha L]^5)$

QED : AP EQUATIONS

$$\frac{\partial}{\partial \log Q^2} D_{NS}^{e^\pm}(a, z) = a P_{NS}^{(0)}(z) \otimes D_{NS}^{e^\pm}(a, z)$$

$$\frac{\partial}{\partial \log Q^2} \begin{pmatrix} D_\Sigma^e(a,z) \\ D_\Sigma^\gamma(a,z) \end{pmatrix} = a \mathbb{P}^{(0)}(z) \otimes \begin{pmatrix} D_\Sigma^e(a,z) \\ D_\Sigma^\gamma(a,z) \end{pmatrix}$$

$$\frac{da}{d\log Q^2} = -\beta_0 a^2 + \ldots.$$

$$\beta_0 \underset{e}{=} -\frac{4}{3}$$

SOLUTION THROUGH EVOLUTION OPERATORS.

INITIAL CONDITIONS :

$$D_{NS,\Sigma}^e(a_0, z) = \delta(1-z)$$

$$D_\Sigma^\gamma(a_0, z) = 0 \qquad \text{(electron case.)}$$

2.1. NON-SINGLET SOLUTION

$$D_{NS}^{e}(a,z) = \delta(1-z) + \sum_{k=1}^{\infty} \frac{P_0^{(k)}(z)}{k!}\left(-\frac{1}{\beta_0}\log\frac{a}{a_0}\right)^k$$

$$P_0^{(k)}(z) = \bigotimes_{k=1}^{k} P_0(z) \quad , \quad P_0 = \left(\frac{1+z^2}{1-z}\right)_+$$

LIMIT OF NO RUNNING:

$$-\frac{1}{\beta_0}\log\left(\frac{a}{a_0}\right) = \frac{1}{\beta_0}\log\left(1-\beta_0 a_0 L\right) \approx a_0 L$$

$$L = \log\left(\frac{s}{m_e^2}\right)$$

$\longrightarrow$ CALCULATE $P_0^{(k)}(z)$ FOR A SERIES OF ORDERS , ... $k=5$

NS
- $k=3$ SKRZYPEK 92
- $k=3$ JEZABEK 92
- $k=5$ PRZYBYCIEN 93
- $k=5$ ARBUZOV 99 (E).

$k=2$ SHUMEIKO et al. /98 (E)

SOME TECHNICAL ASPECTS:

- HARMONIC SUMS JB, KURTH '98

→ NEW ONES + MELLIN TRANSFORMS.

$$\mathbf{M}\left[\frac{\mathrm{S}_{2,2}(1-x)}{1-x}\right](N) = 2\zeta(5)-\zeta(2)\zeta(3)-\frac{1}{4}\zeta(4)S_1(N-1)-S_{1,1,3}(N-1) \\ +\zeta(3)S_{1,1}(N-1)$$

$$\begin{aligned}\frac{S_2^2(N)}{N} &= 2\,\mathbf{M}\left[2\mathrm{S}_{2,2}(x)+\ln(x)[\mathrm{Li}_3(x)-2\mathrm{S}_{1,2}(x)-2\mathrm{S}_{1,2}(1-x)]-3\mathrm{Li}_4(x)\right](N) \\ &+\mathbf{M}\left[4\ln(1-x)\mathrm{Li}_3(x)+\mathrm{Li}_2^2(x)-2\ln(x)\ln(1-x)\mathrm{Li}_2(x)\right](N) \\ &-\mathbf{M}\left[\ln^2(x)[\mathrm{Li}_2(1-x)-\zeta(2)]+5\mathrm{S}_{1,3}(1-x)\right](N) \\ &+2\,\mathbf{M}\left[2\zeta(3)[\ln(x)-\ln(1-x)]+\zeta(2)^2\right](N)\end{aligned}$$

$$\begin{aligned}\frac{S_2(N)S_1^2(N)}{N} &= \mathbf{M}\left[\frac{1}{3}\ln(x)\ln^3(1-x)-2\zeta(4)\right](N) \\ &+\mathbf{M}\left[\zeta(2)\ln^2(1-x)-2\ln(1-x)\mathrm{S}_{1,2}(1-x)+2\mathrm{S}_{1,3}(x)\right](N) \\ &-\mathbf{M}\left[2\mathrm{S}_{2,2}(1-x)-\mathrm{Li}_2^2(1-x)\right](N) \\ &-\Big\{2\,\mathbf{M}\left[2\mathrm{S}_{2,2}(x)+\ln(x)[\mathrm{Li}_3(x)-2\mathrm{S}_{1,2}(x)-2\mathrm{S}_{1,2}(1-x)]-3\mathrm{Li}_4(x)\right](N) \\ &+\mathbf{M}\left[4\ln(1-x)\mathrm{Li}_3(x)+\mathrm{Li}_2^2(x)-2\ln(x)\ln(1-x)\mathrm{Li}_2(x)\right](N) \\ &-\mathbf{M}\left[\ln^2(x)[\mathrm{Li}_2(1-x)-\zeta(2)]+5\mathrm{S}_{1,3}(1-x)\right](N) \\ &+2\,\mathbf{M}\left[2\zeta(3)[\ln(x)-\ln(1-x)]+\zeta(2)^2\right](N)\Big\}\end{aligned}$$

JB, HK 02: SYSTEMATIC TABULATION OF ALL CONVOLUTIONS.

e.g.

$$\frac{1}{(1-x)_+} \otimes \ln(x)\ln^2(1-x) =$$

$$\begin{aligned}
&- 2S_{2,2}(x) - 2S_{2,2}(1-x) - 2S_{1,3}(x) + 12 S_{1,3}(1-x) \\
&+ 12\,\mathrm{Li}_4(x) - \mathrm{Li}_2^2(1-x) \\
&+ 2\ln(x)\ln(1-x)\left[2\,\mathrm{Li}_2(1-x) - \zeta_2\right] \\
&+ 6\ln^2(x)\left[\mathrm{Li}_2(1-x) - \zeta_2\right] + \frac{2}{3}\ln(x)\ln^3(1-x) \\
&+ 2\ln^3(x)\ln(1-x) + 6\ln(1-x)\,S_{1,2}(1-x) \\
&+ 4\ln(x)\left[3S_{1,2}(1-x) - \mathrm{Li}_3(1-x) - 2\zeta_3\right] \\
&- \frac{19}{2}\zeta_4
\end{aligned}$$

$\longrightarrow$ $P_{NS}^{(5)}(z)$ FILLS ONE PAGE AFTER COMPACTIFICATION.

$\longrightarrow$ BE MORE COMPACT.

COMPACT NON-SINGLET SOLUTION:

$$D_{\rm NS}(x,\beta) = \left[\underbrace{\frac{\exp[\beta/2(3/4-\gamma_E)]}{\Gamma(1+\beta/2)}\frac{\beta}{2}(1-x)^{\beta/2-1}}_{\text{SOFT EXP.}} \underbrace{\frac{\mathrm{I}_1\left((-\beta\ln(x))^{1/2}\right)}{[-\beta\ln(x)]^{1/2}}}_{\text{SMALL z EXP.}} \underbrace{\sum_{n=0}^{\infty}\left(\frac{\beta}{2}\right)^n \Psi_n(x)}_{\text{MEDIUM z.}} \right]_+$$

$$\beta = \frac{2}{\pi}\int_{m_e^2}^{s}\frac{ds'}{s'}\alpha(s'). \sim \frac{2\alpha}{\pi}L + \text{running.}$$

with

$$\begin{aligned}
\Psi_0(x) &= 1+x^2\\
\Psi_1(x) &= -\frac{1}{2}\left[(1-x)^2+x^2\ln(x)\right]\\
\Psi_2(x) &= \frac{1}{4}(1-x)\left[1-x-x\ln(x)+(1+x)\mathrm{Li}_2(1-x)\right]\\
\Psi_3(x) &= -\frac{1}{48}\Big\{6(1-x^2)[2\mathrm{Li}_3(1-x)+\mathrm{Li}_2(1-x)]+5(x-1)^2\\
&\quad +(1+7x^2)[\ln(x)\mathrm{Li}_2(1-x)+2\mathrm{S}_{1,2}(1-x)]-\left(\frac{1}{2}+6x-\frac{13}{2}x^2\right)\ln(x)+\frac{1}{12}x^2\ln^3(x)\Big\}\\
\Psi_4(x) &= \frac{1}{96}\Big\{(1-x^2)\Big[24\mathrm{Li}_4(1-x)+12\mathrm{Li}_3(1-x)-\frac{5}{2}\mathrm{S}_{1,3}(1-x)\\
&\qquad -12\mathrm{S}_{2,2}(1-x)-\frac{3}{2}\ln(x)\mathrm{S}_{1,2}(1-x)-\frac{1}{4}\ln^2(x)\mathrm{Li}_2(1-x)+7\mathrm{Li}_2(1-x)\Big]\\
&\quad +4(1+x^2)\mathrm{Li}_2^2(1-x)+(1-8x+7x^2)\Big[\ln(x)\mathrm{Li}_2(1-x)+2\mathrm{S}_{1,2}(1-x)\Big]\\
&\quad +2(1+7x^2)\ln(x)\mathrm{Li}_3(1-x)-\left(\frac{3}{4}+5x-\frac{23}{4}x^2\right)\ln(x)\\
&\quad -\frac{1}{12}x(1-x)\ln^3(x)-\frac{1}{48}x^2\ln^4(x)+(1-x)^2\left[\frac{7}{2}+\frac{1}{8}\ln^2(x)\right]\Big\}
\end{aligned}$$

2.1. SINGLET SOLUTION

STRUCTURE:

$$\mathbb{E}_S(a,z) = \mathbb{1}\,\delta(1-z) + \sum_{k=1}^{\infty} \frac{1}{k!}\, \mathbb{P}_0^{(k)}(z) \left(-\frac{1}{\beta_0} \log\left(\frac{a}{a_0}\right)\right)^k$$

$$a = \frac{\alpha}{4\pi}$$

$$\mathbb{P}_0^{(k)}(z) = \bigotimes_{l=1}^{k} \cdot\, \mathbb{P}_0(z)$$

k-fold matrix multiplication & Mellin convolution

INITIAL CONDITION FOR $e^{\pm}$-RADIATION:

$$\begin{pmatrix} \delta(1-z) \\ 0 \end{pmatrix}$$

$$\mathbb{D}_S(a,z) = \mathbb{E}_S(a,z) \otimes \begin{pmatrix} \delta(1-z) \\ 0 \end{pmatrix}$$

$$D_S^e(a,z) = (1,0) \cdot \mathbb{D}_S(a,z)$$

BOTH e & γ TERMS HAVE TO BE CONSIDERED BEFORE CARRYING OUT THE LAST CONTRACTION.

RUNNING MAY BE MADE EXPLICIT:

$$\begin{aligned}
\boldsymbol{E}_0(a,x) \;=\;& \mathbf{1}\,\delta(1-x) + \boldsymbol{P}_0(x) a_0 L + \left(\frac{1}{2}\,\boldsymbol{P}_0^{(1)}(x) + \frac{2}{3}\,\boldsymbol{P}_0(x)\right)(a_0 L)^2 \\
&+ \left(\frac{1}{6}\,\boldsymbol{P}_0^{(2)}(x) + \frac{2}{3}\,\boldsymbol{P}_0^{(1)}(x) + \frac{16}{27}\,\boldsymbol{P}_0(x)\right)(a_0 L)^3 \\
&+ \left(\frac{1}{24}\,\boldsymbol{P}_0^{(3)}(x) + \frac{1}{3}\,\boldsymbol{P}_0^{(2)}(x) + \frac{22}{27}\,\boldsymbol{P}_0^{(1)}(x) + \frac{16}{27}\,\boldsymbol{P}_0(x)\right)(a_0 L)^4 \\
&+ \left(\frac{1}{120}\,\boldsymbol{P}_0^{(4)}(x) + \frac{1}{9}\,\boldsymbol{P}_0^{(3)}(x) + \frac{14}{27}\,\boldsymbol{P}_0^{(2)}(x) + \frac{80}{81}\,\boldsymbol{P}_0^{(1)}(x) + \frac{256}{405}\,\boldsymbol{P}_0(x)\right)(a_0 L)^5\,,
\end{aligned}$$

$$-\beta_0 \log\left(\frac{a}{a_0}\right) \xrightarrow[\text{expanded}]{} \frac{1}{\beta_0} \log\left(1-\beta_0 a_0 L\right)$$

RESULT UP TO $O(a_0 L)$

ORDER PARAMETER.

2.2. SINGLET CASE

$$\mathbb{P}_{(0)} = \begin{pmatrix} 2\left(\frac{1+x^2}{1-x}\right)_+ & -4(1-2x) \\ 2(1-x) & -\frac{4}{3}\delta(1-x) \end{pmatrix}$$

$$\mathbb{P}_{(1)} = \begin{pmatrix} P_{NS(1)}(x) + P_{S(11)} & P_{S12} \\ P_{S21} & P_{S22} \end{pmatrix}$$

$$P_{S11} = 8\left[5(1-x) + 2(1+x)\ln(x)\right]$$

$$P_{S12} = 4\left[2(2x-1)\left[2\ln(1-x) - \ln(x)\right] + \frac{13-8x}{3}\right]$$

$$P_{S21} = 4\left[(2-x)\left[2\ln(1-x) + \ln(x)\right] - \frac{8-13x}{6}\right]$$

$$P_{S22} = 8\left[5(1-x) + 2(1+x)\ln(x) + \frac{2}{9}\delta(1-x)\right]$$

$$\vdots$$

$$\begin{aligned}
P^{(4)}_{1,1}(x) = \; & P^{(4)}_{\mathrm{NS}} + 32\Bigg[\left(\frac{73678}{27} + 704\zeta(2) + 640\zeta(3)\right)(1-x) \\
& -\left(\frac{15212}{9} + 960\zeta(2)\right)(1-x)\ln(1-x) \\
& +\left(\frac{36568}{27} - \frac{9068}{27}x + 416\zeta(2) - 544\zeta(2)x + 256(1+x)\zeta(3)\right)\ln(x) \\
& -704(1-x)\ln^2(1-x) - \left(\frac{8000+20672x}{9} + 384(1+x)\zeta(2)\right)\ln(x)\ln(1-x) \\
& +\left(\frac{2696+4472x}{9} + 96(1+x)\right)\ln^2(x) + 320(1-x)\ln^3(1-x) \\
& -(416-544x)\ln(x)\ln^2(1-x) - (152-88x)\ln^2(x)\ln(1-x) \\
& +\frac{40}{9}(8-7x)\ln^3(x) + 128(1+x)\ln(x)\ln^3(1-x) - 96(1+x)\ln^2(x)\ln^2(1-x) \\
& +\frac{4}{3}(1+x)\ln^4(x) - \left(\frac{14336}{9} + 384\zeta(2)\right)(1+x)\mathrm{Li}_2(1-x) \\
& +128(1+x)\ln(1-x)\mathrm{Li}_2(1-x) - 720(1-x)\ln(x)\mathrm{Li}_2(1-x) \\
& +384(1+x)\ln^2(1-x)\mathrm{Li}_2(1-x) - 96(1+x)\ln^2(x)\mathrm{Li}_2(1-x) \\
& -128(1+x)\mathrm{Li}_3(1-x) - 768(1+x)\ln(1-x)\mathrm{Li}_3(1-x) + 768(1+x)\mathrm{Li}_4(1-x) \\
& -(816-944x)\mathrm{S}_{1,2}(1-x) + 384(1+x)\ln(1-x)\mathrm{S}_{1,2}(1-x) \\
& -256(1+x)\ln(x)\mathrm{S}_{1,2}(1-x) - 384(1+x)\mathrm{S}_{22}(1-x) - 192(1+x)\mathrm{S}_{13}(1-x)\Bigg]
\end{aligned}$$

$$
\begin{aligned}
P_{1,2}^{(4)}(x) \;=\; 32\Bigg[& -\frac{1912189}{648} + \frac{958405}{324}x - \frac{7852-7064x}{9}\zeta(2) + \frac{640-128x}{3}\zeta(3) - 48(1-2x)\zeta(4) \\
&+ \left(-\frac{57280}{27} + \frac{58364}{27}x - 64(5-x)\zeta(2) - 256(1-2x)\zeta(3)\right)\ln(1-x) \\
&+ \left(-\frac{37633}{27} - \frac{100213}{27}x - 368\zeta(2) - 656\zeta(2)x + 128(1-2x)\zeta(3)\right)\ln(x) \\
&+ \left(\frac{7852-7064x}{9} + 192(1-2x)\zeta(2)\right)\ln^2(1-x) \\
&- \left(\frac{9220-4616x}{9} + 192(1-2x)\zeta(2)\right)\ln(x)\ln(1-x) \\
&+ \left(-\frac{4703}{18} + \frac{4442}{9}x - 24(1-2x)\zeta(2)\right)\ln^2(x) + \frac{64}{3}(5-x)\ln^3(1-x) \\
&+(368+656x)\ln(x)\ln^2(1-x) - \frac{616+1000x}{3}\ln^2(x)\ln(1-x) \\
&- \left(20 + \frac{89}{3}x\right)\ln^3(x) - 32(1-2x)\ln^4(1-x) + 64(1-2x)\ln(x)\ln^3(1-x) \\
&+24(1-2x)\ln^2(x)\ln^2(1-x) - \frac{40}{3}(1-2x)\ln^3(x)\ln(1-x) - \frac{5}{12}(1-2x)\ln^4(x) \\
&-(152+272x)\mathrm{Li}_2(1-x) + (1056+1248x)\ln(1-x)\mathrm{Li}_2(1-x) \\
&-\frac{32}{3}(4+x)\ln(x)\mathrm{Li}_2(1-x) + 288(1-2x)\ln(x)\ln(1-x)\mathrm{Li}_2(1-x) \\
&-16(1-2x)\ln^2(x)\mathrm{Li}_2(1-x) - (1056+1248x)\mathrm{Li}_3(1-x) \\
&-288(1-2x)\ln(x)\mathrm{Li}_3(1-x) + 352(1-2x)\ln(1-x)\mathrm{S}_{1,2}(1-x) \\
&+80(1-2x)\ln(x)\mathrm{S}_{1,2}(1-x) - 32(1-2x)\mathrm{Li}_2^2(1-x) + 48(9+13x)\mathrm{S}_{1,2}(1-x) \\
&-224(1-2x)\mathrm{S}_{22}(1-x) + 224(1-2x)\mathrm{S}_{13}(1-x)\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
P_{2,1}^{(4)}(x) \;=\; 32\Bigg[& \frac{958405}{648} - \frac{1912189}{1296}x + \frac{3532-3926x}{9}\zeta(2) - \frac{64}{3}(1-5x)\zeta(3) + 24(2-x)\zeta(4) \\
&+ \left(\frac{41849+94913x}{54} + 296\zeta(2) + 344\zeta(2)x - 64(2-x)\zeta(3)\right)\ln(x) \\
&+ \left(\frac{29182-28640x}{27} + 32(1-5x)\zeta(2) + 128(2-x)\zeta(3)\right)\ln(1-x) \\
&+ \left(-\frac{3532-3926x}{9} - 96(2-x)\zeta(2)\right)\ln^2(1-x) \\
&+ \left(\frac{4756-3242x}{9} + 96(2-x)\zeta(2)\right)\ln(x)\ln(1-x) \\
&+ \left(\frac{1609}{9} - \frac{6071}{36}x + 12(2-x)\zeta(2)\right)\ln^2(x) - \frac{32}{3}(1-5x)\ln^3(1-x) \\
&-(296+344x)\ln(x)\ln^2(1-x) + \frac{436+484x}{3}\ln^2(x)\ln(1-x) + \left(\frac{35}{2} + \frac{14}{3}x\right)\ln^3(x) \\
&+16(2-x)\ln^4(1-x) - 32(2-x)\ln(x)\ln^3(1-x) - 12(2-x)\ln^2(x)\ln^2(1-x) \\
&+\frac{20}{3}(2-x)\ln^3(x)\ln(1-x) + \frac{5}{24}(2-x)\ln^4(x) + (136+76x)\mathrm{Li}_2(1-x) \\
&-(624+528x)\ln(1-x)\mathrm{Li}_2(1-x) - \frac{16}{3}(1+4x)\ln(x)\mathrm{Li}_2(1-x) \\
&-144(2-x)\ln(x)\ln(1-x)\mathrm{Li}_2(1-x) + 8(2-x)\ln^2(x)\mathrm{Li}_2(1-x) \\
&+48(13+11x)\mathrm{Li}_3(1-x) + 144(2-x)\ln(x)\mathrm{Li}_3(1-x) + 16(2-x)\mathrm{Li}_2^2(1-x) \\
&-312(1+x)\mathrm{S}_{1,2}(1-x) - 176(2-x)\ln(1-x)\mathrm{S}_{1,2}(1-x) \\
&-40(2-x)\ln(x)\mathrm{S}_{1,2}(1-x) + 112(2-x)\mathrm{S}_{22}(1-x) - 112(2-x)\mathrm{S}_{13}(1-x)\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
P_{2,2}^{(4)}(x) \;=\; 32\Bigg[& \left(\frac{256273}{108} + \frac{628}{3}\zeta(2) + 160\zeta(3)\right)(1-x) - (1417 + 240\zeta(2))(1-x)\ln(1-x) \\
& + \left(\frac{32002 - 6257x}{27} + \frac{352 - 368x}{3}\zeta(2) + 64(1+x)\zeta(3)\right)\ln(x) \\
& - \frac{628}{3}(1-x)\ln^2(1-x) - \left(\frac{2150 + 3406x}{3} + 96(1+x)\zeta(2)\right)\ln(x)\ln(1-x) \\
& + \left(\frac{763 + 626x}{3} + 24(1+x)\zeta(2)\right)\ln^2(x) + 80(1-x)\ln^3(1-x) \\
& - \frac{352 - 368x}{3}\ln(x)\ln^2(1-x) - \frac{394 - 386x}{3}\ln^2(x)\ln(1-x) \\
& + \frac{50}{9}(5 - 4x)\ln^3(x) + 32(1+x)\ln(x)\ln^3(1-x) \\
& - 24(1+x)\ln^2(x)\ln^2(1-x) - \frac{20}{3}(1+x)\ln^3(x)\ln(1-x) + \frac{7}{6}(1+x)\ln^4(x) \\
& - (926 + 96\zeta(2))(1+x)\mathrm{Li}_2(1-x) + \frac{16}{3}(1+x)\ln(1-x)\mathrm{Li}_2(1-x) \\
& - 380(1-x)\ln(x)\mathrm{Li}_2(1-x) + 96(1+x)\ln^2(1-x)\mathrm{Li}_2(1-x) \\
& - 44(1+x)\ln^2(x)\mathrm{Li}_2(1-x) + \frac{16}{3}(1+x)\mathrm{Li}_3(1-x) \\
& - 192(1+x)\ln(1-x)\mathrm{Li}_3(1-x) + 192(1+x)\mathrm{Li}_4(1-x) \\
& - \frac{1252 - 1268x}{3}\mathrm{S}_{1,2}(1-x) + 96(1+x)\ln(1-x)\mathrm{S}_{1,2}(1-x) \\
& - 104(1+x)\ln(x)\mathrm{S}_{1,2}(1-x) - 96(1+x)\mathrm{S}_{22}(1-x) - 88(1+x)\mathrm{S}_{13}(1-x) \\
& - \frac{32}{243}\delta(1-x)\Bigg]
\end{aligned}
$$

3. Resummation of the $O((\alpha \ln^2(z))^k)$ Terms

SINGLET TERMS :

$$\mathbb{P}(x,a)_{x\to 0} = \sum_{l=0}^{\infty} \mathbb{P}^{(l)}_{x\to 0} \; a^{l+1} \ln^{2l}(x)$$

$$= \frac{1}{8\pi^2} \mathbb{M}^{-1}\left[\mathbb{F}_0(N,a)\right](x)$$

QCD:
BARTELS, ERMOLAEV, RYSKIN '96
JB, VOGT '96

$$\mathbb{F}_0(N,a) = 16\pi^2 \frac{a}{N} \mathbb{M}_0 - \frac{8a}{N^2} \mathbb{F}_8(N,a) \mathbb{G}_0 + \frac{1}{8\pi^2}\frac{1}{N} \mathbb{F}_0^2(N,a)$$

$$\mathbb{F}_8(N,a) \underset{\text{QED}}{=} 4\pi^2 \left(1 - \sqrt{1 - \frac{8a}{N^2}}\right) \mathbb{M}_8$$

$$\mathbb{M}_0 = \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix} \quad \mathbb{M}_8 = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \quad \mathbb{G}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$P_0 = 2 \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix}$$

$$P_1 = 2 \begin{pmatrix} -4 & -2 \\ 2 & -4 \end{pmatrix}$$

$$P_2 = \frac{2}{3} \begin{pmatrix} -13 & 10 \\ -10 & 4 \end{pmatrix}$$

$$\vdots$$

SYMMETRY RELATION:

$$\mathbf{P}^{(l)}_{f\gamma} = -\mathbf{P}^{(l)}_{\gamma f}.$$

The first 15 coefficients are listed in Table 1.

l	$P^{(l)}_{ff}$	$P^{(l)}_{fg}$	$P^{(l)}_{gg}$
0	0.2000000000E+01	−0.4000000000E+01	0.0000000000E+00
1	−0.1400000000E+01	−0.4000000000E+01	−0.8000000000E+01
2	−0.8666666667E+01	0.6666666667E+01	−0.2666666667E+01
3	0.1444444444E+01	0.2444444444E+01	0.2044444444E+01
4	0.7888888889E+00	−0.2825396825E+00	0.4634920635E+00
5	0.5537918871E−02	−0.1008112875E+00	−0.3626102293E−01
6	−0.1216503661E−01	−0.5536849981E−03	−0.9307893752E−02
7	−0.5063924905E−03	0.8396483000E−03	−0.3947845218E−04
8	0.4517396184E−04	0.2736857499E−04	0.4555056936E−04
9	0.2611470148E−05	−9.1960462641E−05	0.1192943787E−05
10	−0.4136771391E−07	−0.9306140011E−07	−0.6994805938E−07
11	−0.4259181787E−08	0.1228221436E−08	−0.2774664782E−08
12	−0.1716660622E−10	0.1073980741E−09	0.3096908050E−10
13	0.2808782930E−11	0.3728781778E−12	0.2327857737E−11
14	0.3713452748E−13	−0.5274589147E−13	0.7034915420E−14

Table 1: The coefficients of the matrices $\mathbf{P}^{(l)}_{x\to 0}$

NON-SINGLET CASE:

$$\mathbb{M}[P_{NS,z\to 0}](N,a) = \frac{N}{2}\left\{1-\sqrt{1+\frac{8a}{N^2}\left[1-2\sqrt{1-\frac{8a}{N^2}}\right]}\right\}$$

JB, VOGT '96

$$P_{NS,z\to 0}(z,a) = \sum_{k=0}^{\infty} c_k a^{k+1} \ln^{2k}(z).$$

k	c_k
0	2.0000E+0
1	−6.0000E+0
2	−3.3333E+0
3	−0.4222E+0
4	+1.5873E−3
5	+2.8571E−3
6	+1.4000E−4
7	−3.8468E−7
8	−2.0649E−7
9	−6.1484E−9

SUBLEADING TERMS:

$$-12\frac{a^2}{N^3}\left[1-\frac{2}{9}N\right] \ldots$$

FROM:

BURGERS, BERENDS, VAN NEERVEN '89

>...≫

CONTRIBUTION TO RC CORRECTION:

- FULL HO-SOLUTION MECHANISM OF EVOLUTION EQ. REQUIRED

$\propto \alpha^n \log^{2n}(z) \rightarrow$ n th ORDER TERM.

$$D_{NS, z\to 0}(z,Q^2) = \sum_{k=1}^{\infty} c_k \int_{m_e^2}^{Q^2} \frac{dq^2}{q^2} a^{k+1}(q^2) \ln^{2k}(z)$$

$\rightarrow$ $\boxed{\text{NO ANALYTIC SOLUTION POSSIBLE IN SINGLET CASE}}$

SPLITTING MATRICES OF DIFFERENT ORDERS IN a DO NOT COMMUTE.

JB, VOGT '98 : ORDER-BY-ORDER U-MATRIX SOLUTION IN N-SPACE.

POSSIBLE, SINCE: $M[\ln^2(z)] \sim \frac{1}{N^3}$ etc.

SUFFICIENT DAMPING FOR $Re[N] \rightarrow -\infty$.

$$D_S(a,z) = U(a,z) \otimes E_0(a,a_0,z) \otimes U^{-1}(a_0,z)$$

$E_0(a,a_0,z)$ — RESUMMATION OF $O((aL)^k)$

$$R_k(z) = \frac{1}{\beta_0} P_k(x) \quad , \; k \geq 0$$

$$\hat{R}_1(z) = R_1(z)$$

$$\hat{R}_k(z) = R_k(z) + \sum_{l=1}^{k-1} R_{k-l}(z) \otimes U_k(z) \; , \; k > 1$$

$$[U_1, R_0] = R_1 + U_1$$

$\vdots$

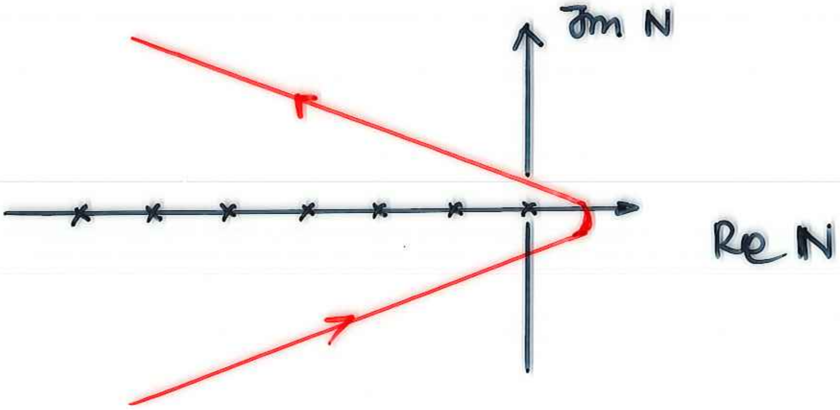
$$[U_k, R_0] = R_k + \sum_{i=1}^{k-1} R_{k-i} U_i + k U_k$$

$\longrightarrow$ SOLVE FOR U_k. MELLIN SPACE

$$U(a,z) = \mathbb{1} \cdot \delta(1-z) + \sum_{k=1}^{\infty} a^k U_k(z)$$

z-SPACE.

SIMILARLY FOR $U^{-1}(a_0,z)$.



MELLIN INVERSION.

4. Conclusions

1) THE $O((\alpha L)^5)$ CORRECTION TO POLARIZED (∥) EMISSION OFF MASSLESS FERMIONS & PHOTONS WAS CALCULATED IN ANALYTIC FORM USING HARMONIC SUMS.
2) BOTH THE FLAVOR NON-SINGLET & SINGLET CONTRIBUTIONS WERE DERIVED.

3) A SECOND CLASS OF POTENTIALLY LARGE UNIVERSAL CORRECTIONS CONCERNS THE TERMS $\propto (\alpha \ln^2 z)^k$.
THEY RESULT FROM NON-LINEAR (MATRIX) EQUATIONS.

4) THE IMPACT OF THESE CONTRIBUTIONS HAS TO BE DERIVED NUMERICALLY IN THE SINGLET CASE DUE TO $[P_k, P_l] \neq 0,\ k \neq l$.
THIS IS POSSIBLE USING A MELLIN TECHNIQUE, SINCE $\mathbf{M}[\ln^2 z](N) \propto \frac{1}{N^3}$ IS SUFFICIENTLY DAMPED FOR $\mathrm{Re}\,N \to -\infty$.