

Non-Singlet QCD Analysis of the Structure Function F_2 in 3-Loops

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OUTLINE:

- Motivation
- QCD Formalism
- World Data
- Error Calculation
- Parton Distributions
- Λ_{QCD} and $\alpha_s(M_Z^2)$
- Moments - Comparison: QCD and Lattice
- Conclusion



IN COLLABORATION WITH
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First results: [hep-ph/0407089](https://arxiv.org/abs/hep-ph/0407089)

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Motivation

- Having the recently completed **NNLO anomalous dimensions** available (*) a consistent 3-loop analysis of the unpolarized deep inelastic Structure Functions can be carried out. \implies **Precision Tests of QCD**
- The present Analysis concentrates on the **Non-Singlet evolution** only to firstly obtain an accurate as possible picture for the **valence quark distributions**.
- The aim of the analysis is to determine $\alpha_s(Q^2)$ and the valence quark distributions with **Correlated Errors** to 3-loop accuracy.
- An NNLO QCD analysis of the structure function $F_2(x, Q^2)$ may allow to reduce the theoretical error in determining $\alpha_s(Q^2)$ to at least the level of the experimental error.
- Comparison of QCD analysis results with results from recent lattice simulations concerning **low order moments** has shown **astonishing agreement in the polarized case** [Ref.: J.Blümlein and H. Böttcher, Nucl. Phys. **B636** (2002) 225].
- With the **steadily improving Lattice Simulations** such a Comparison might become feasible soon also for unpolarized valence quark densities.

(*) **Ref.:** S.Moch, J.A.M.Vermaseren and A.Vogt, hep-ph/0403192.

Motivation

- We aim at parameterizations of the parton densities and their fully correlated 1σ error bands which are directly applicable to determine 'experimental' errors for other observables in hard scattering processes.
- Having the recently completed **NNLO anomalous dimensions** available (*) a consistent 3-loop analysis of the unpolarized deep inelastic Structure Functions can be carried out.

⇒ Precision Tests of QCD

- The present Analysis concentrates on the **Non-Singlet** evolution only to firstly obtain an accurate as possible picture for the **valence quark distributions**.
- The aim of the analysis is to determine $\alpha_s(Q^2)$ and the valence quark distributions with **Correlated Errors** to 3-loop accuracy.
- An NNLO QCD analysis of the structure function $F_2(x, Q^2)$ may allow to reduce the theoretical error in determining $\alpha_s(Q^2)$ to at least the level of the experimental error.

(*) **Ref.:** S.Moch, J.A.M.Vermaseren and A.Vogt, hep-ph/0403192.

Motivation, cont'd

- Comparison of QCD analysis results with results from recent lattice simulations concerning both QCD parameters and low order moments has shown astonishing agreement in the polarized case:

[Ref.: J.Blümlein and H. Böttcher, Nucl. Phys. **B636** (2002) 225]

Δf	n	QCD	Lattice	
		moment at $Q^2 = 4 \text{ GeV}^2$	QCDSF	LHPC/ SESAM
Δu_v	1	0.926 fixed	0.889(29)	0.860(69)
	2	0.163 ± 0.014	$0.198(8)$	$0.242(22)$
	3	0.055 ± 0.006	0.041(9)	0.116(42)
Δd_v	1	-0.341 fixed	-0.236(27)	-0.171(43)
	2	-0.047 ± 0.021	$-0.048(3)$	$-0.029(13)$
	3	-0.015 ± 0.009	-0.028(2)	0.001(25)

- With the steadily improving Lattice Simulations a Comparison of the results of the present analysis and the measurement of the lowest Moments for unpolarized valence quark densities may become feasible soon.

QCD Analysis Formalism

- In Mellin-N space the non-singlet parts of the structure function $F_2(N, Q^2)$ are written as

$$F_2^{\pm, V}(N, Q^2) = \left[1 + C_1(N)a + C_2(N)a^2 \right] f^{\pm, V}(N, Q^2),$$

where $f^{\pm, V}(N, Q^2)$ stand for the non-singlet (NS) quark combinations.

- The quark combinations to be considered are

$$\Delta^{\pm} = (u \pm \bar{u}) - (d \pm \bar{d})$$

$$V = (u - \bar{u}) + (d - \bar{d})$$

and the non-singlet parts of F_2 are then given by :

$$F_2^{NS} \propto \frac{1}{3}\Delta^+ \propto \frac{1}{3}(u_v - d_v - 2(\bar{d} - \bar{u}))$$

$$F_2^{p, V} \propto \frac{5}{18}V + \frac{1}{6}\Delta^- \propto \frac{4}{9}u_v + \frac{1}{9}d_v$$

$$F_2^{d, V} \propto \frac{5}{18}V \propto \frac{1}{2}\left(\frac{5}{9}u_v + \frac{5}{9}d_v\right).$$

QCD Analysis Formalism cont'd

- Solving the evolution equation for $F_2^{\pm, V}$ up to 3-Loops gives

$$\begin{aligned}
 F_2^{\pm, V}(Q^2) &= F_2^{\pm, V}(Q_0^2) \left(\frac{a}{a_0} \right)^{-P_0/\beta_0} \\
 &\quad \left\{ 1 - \frac{1}{\beta_0}(a - a_0) \left[P_1^{\pm} - \frac{\beta_1}{\beta_0} P_0 - C_1 \beta_0 \right] \right. \\
 &\quad - \frac{1}{2\beta_0} (a^2 - a_0^2) \left[P_2^{\pm, V} - \frac{\beta_1}{\beta_0} P_1^{\pm} \right. \\
 &\quad \left. \left. + \left(\frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{\beta_0} \right) P_0 - 2C_2 \beta_0 - C_1 \beta_1 + C_1^2 \beta_0 \right] \right. \\
 &\quad \left. + \frac{1}{2\beta_0^2} (a - a_0)^2 \left(P_1^{\pm} - \frac{\beta_1}{\beta_0} P_0 - C_1 \beta_0 \right)^2 \right\},
 \end{aligned}$$

where C_i are the Wilson coefficients (*), P_i the splitting functions, and $a = \alpha_s/4\pi$ with $a_0 = a(Q_0^2)$.

(*) C_2 : W.L. van Neerven and A.Vogt, Nucl.Phys. **B568**(2000)263.

- The evolution of the parton densities u_v , d_v , and $\bar{d} - \bar{u}$ goes without the Wilson coefficient terms.

Parameterization of the Non-Singlet Part

- Choice of the parameterization of the parton densities at the input scale of $Q_0^2 = 4 \text{ GeV}^2$:

$$xu_v(x, Q_0^2) = A_{u_v} x^{a_u} (1-x)^{b_u} (1 - 1.108x^{\frac{1}{2}} + 26.283x)$$

$$xd_v(x, Q_0^2) = A_{d_v} x^{a_d} (1-x)^{b_d} (1 + 0.895x^{\frac{1}{2}} + 18.179x)$$

and as adopted from MRST:

⇒ figure

[Ref.: Eur.Phys.J. C23(2002)73]

$$x(\bar{d} - \bar{u})(x, Q_0^2) = 1.195x^{1.24}(1-x)^{9.10} (1 + 14.05x - 45.52x^2)$$

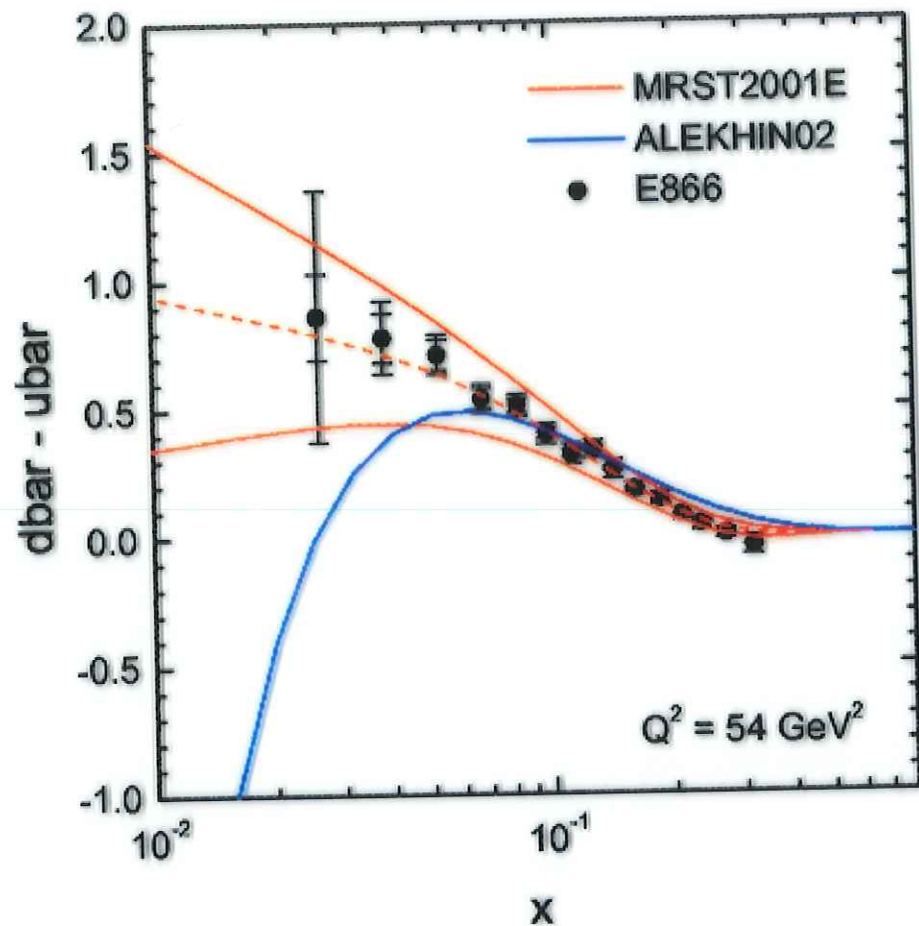
- The normalization constants A_{u_v} and A_{d_v} are fixed by the conservation of the number of valence quarks:

$$\int_0^1 u_v(x) dx = 2, \quad \int_0^1 d_v(x) dx = 1.$$

⇒ Finally 4 parameters are to be determined in the fit:

$$u_v: a_u, b_u, \quad d_v: a_d, b_d$$

and in addition Λ_{QCD} .



- evidence for
 - $\bar{d} > \bar{u}$
 - $x(\bar{d} - \bar{u}) \rightarrow 0$
- as $x \rightarrow 0$?

Ref.: J. Stirling, 'Phenomenology of Parton Distributions',
 Talk given at KITP Conference: Collider Physics,
 Santa Barbara, USA, January - April, 2004.

The F_2 Data Handling

- **R correction:**

$$\text{BCDMS}(R_{QCD}) \implies \text{BCDMS}(R_{1998})$$

[Ref: E143 Collaboration, K.Abe et al., Phys.Lett.**B452**(1999)194.]

- **Deuteron Data correction:**

Fermi motion and offshell correction

[Ref: W.Melnitchouk and A.W.Thomas, Phys.Lett.**B377**(1996)11.]

- **Kinematic cuts:**

$$0.3 < x < 1.0 \text{ for } F_2^p \text{ and } F_2^d$$

$$0.0 < x < 0.3 \text{ for } F_2^{ns} = 2(F_2^p - F_2^d)$$

$$4.0 < Q^2 < 30000 \text{ GeV}^2, W^2 > 12.5 \text{ GeV}^2$$

- **Normalization uncertainties:**

We allow for a **Relative Normalization Shift** between the different data sets within the normalization uncertainties quoted by the experiments or assumed accordingly (**fitted** and **then fixed**).

The World Data on F_2

<i>Experiment</i>	x	Q^2, GeV^2	F_2	<i>Norm</i>
BCDMS (100)	0.35 – 0.75	11.75 – 75.00	51	1.016
BCDMS (120)	0.35 – 0.75	13.25 – 75.00	59	1.009
BCDMS (200)	0.35 – 0.75	32.50 – 137.50	50	1.012
BCDMS (280)	0.35 – 0.75	43.00 – 230.00	49	1.014
NMC (comb)	0.35 – 0.50	7.00 – 65.00	15	1.003
SLAC (comb)	0.30 – 0.62	7.30 – 21.39	57	1.017
H1 (hQ2)	0.40 – 0.65	200 – 30000	26	1.018
ZEUS (hQ2)	0.40 – 0.65	650 – 30000	15	1.001
<i>proton</i>			322	
BCDMS (120)	0.35 – 0.75	13.25 – 99.00	59	0.987
BCDMS (200)	0.35 – 0.75	32.50 – 137.50	50	0.985
BCDMS (280)	0.35 – 0.75	43.00 – 230.00	49	0.987
NMC (comb)	0.35 – 0.50	7.00 – 65.00	15	0.980
SLAC (comb)	0.30 – 0.62	10.00 – 21.40	59	0.980
<i>deuteron</i>			232	
BCDMS (120)	0.070 – 0.275	8.75 – 43.00	36	1.000
BCDMS (200)	0.070 – 0.275	17.00 – 75.00	29	1.000
BCDMS (280)	0.100 – 0.275	32.50 – 115.50	27	1.000
NMC (comb)	0.013 – 0.275	4.50 – 65.00	88	1.000
SLAC (comb)	0.153 – 0.293	4.18 – 5.50	28	1.000
<i>non – singlet</i>			208	
<i>total</i>			762	

- CUTS:** $0.3 < x < 1.0$ for F_2^p and F_2^d
 $0.0 < x < 0.3$ for $F_2^{ns} = 2(F_2^p - F_2^d)$
 $4.0 < Q^2 < 30000 GeV^2$, $W^2 > 12.5 GeV^2$

Error Treatment

⇒ **Problem:** Systematic errors are known to be partly correlated.

- **Experimental Errors in this analysis:**

For all data sets we used the simplest procedure by adding the **statistical and total systematic errors in quadrature**. Only fits with a **Positive Definite Covariance Matrix** are accepted to be able to

⇒ calculate the **Fully Correlated 1σ Error Bands** by Gaussian error propagation.

- **χ² Expression:**

$$\chi^2 = \sum_{i=1}^{n^{exp}} \left[\frac{(N_i - 1)^2}{(\Delta N_i)^2} + \sum_{j=1}^{n^{data}} \frac{(N_i F_{2,j}^{data} - F_{2,j}^{theor})^2}{(\Delta F_{2,j}^{data})^2} \right]$$

Fully Correlated Error Calculation

- The fully correlated 1σ error for the parton density f_q as given by Gaussian error propagation is

$$\sigma(f_q(x))^2 = \sum_{i,j=1}^{n_p} \left(\frac{\partial f_q}{\partial p_i} \frac{\partial f_q}{\partial p_j} \right) \text{cov}(p_i, p_j),$$

where the $\partial f_q / \partial p_i$ are the derivatives of f_q w.r.t. the parameters p_i and the $\text{cov}(p_i, p_j)$ are the elements of the covariance matrix as determined in the fit.

- The derivatives $\partial f_q / \partial p_i$ at the input scale Q_0^2 can be calculated **analytically**. Their values at Q^2 are given by **evolution**.
 - The derivatives evolved in **MELLIN-N space** are transformed back to **x -space** and can then be used according to the error propagation formula above.
- ⇒ As an example the derivative of $f(x, a, b)$ w.r.t. parameter a in MELLIN-N space reads:

Derivatives in MELLIN-N space

The general form of the derivative of the MELLIN moment w.r.t. parameter a for complex values of N is

$$\frac{\partial \mathbf{M}[f(x, a, b)](N)}{\partial a} = A \frac{\partial \bar{\mathbf{M}}}{\partial a} + \bar{\mathbf{M}} \frac{\partial A}{\partial a},$$

with $\bar{\mathbf{M}} = \mathbf{M}/A$ and A the normalization constant.

$$\begin{aligned} \frac{\partial \bar{\mathbf{M}}}{\partial a} &= \{ [\Psi(a - 1 + N) - \Psi(a + N + b)] \\ &+ \gamma \frac{a - 1 + N}{a + N + b} (\Psi(a + N) - \Psi(a + N + b + 1)) \} \\ &B(a - 1 + N, b + 1) \\ &+ \rho [\Psi(a - \frac{1}{2} + N) - \Psi(a + \frac{1}{2} + N + b)] \\ &B(a - \frac{1}{2} + N, b + 1) \end{aligned}$$

$$\frac{\partial A}{\partial a} = -AZ_a/X_a = -CZ_a/X_a^2$$

$$\begin{aligned} Z_a &= [\Psi(a) - \Psi(a + b + 1)]B(a, b + 1) + \rho [\Psi(a + 1/2) \\ &- \Psi(a + \frac{1}{2} + b + 1)]B(a + \frac{1}{2}, b + 1) + \gamma \\ &\times [\Psi(a + 1) - \Psi(a + 1 + b + 1)]B(a + 1, b + 1) \end{aligned}$$

$$X_a = B(a, b + 1) + \rho B(a + \frac{1}{2}, b + 1) + \gamma B(a + 1, b + 1)$$

with C the number of valence quarks, i.e. **2** for \mathbf{u}_v and **1** for \mathbf{d}_v .

Fit Results

- Parameter values at the input scale $Q_0^2 = 4.0 \text{ GeV}^2$

$$xq_i(x, Q_0^2) = A_i x^{a_i} (1-x)^{b_i} (1 + \rho_i x^{\frac{1}{2}} + \gamma_i x)$$

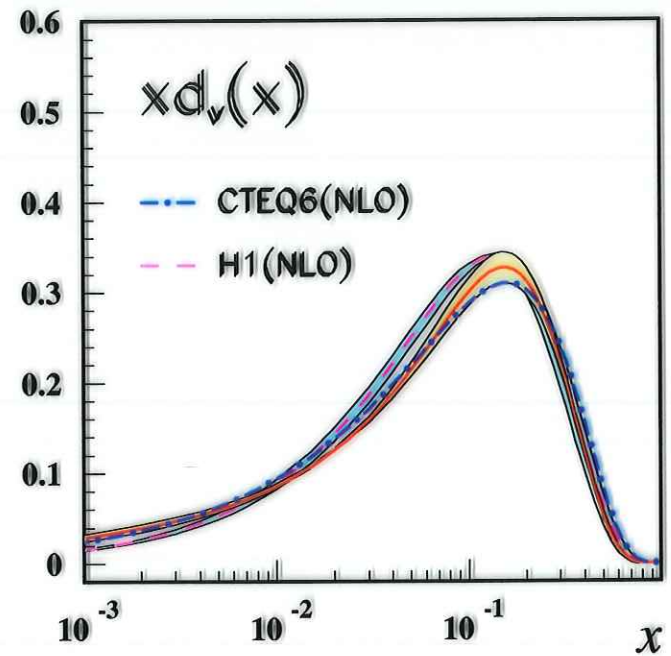
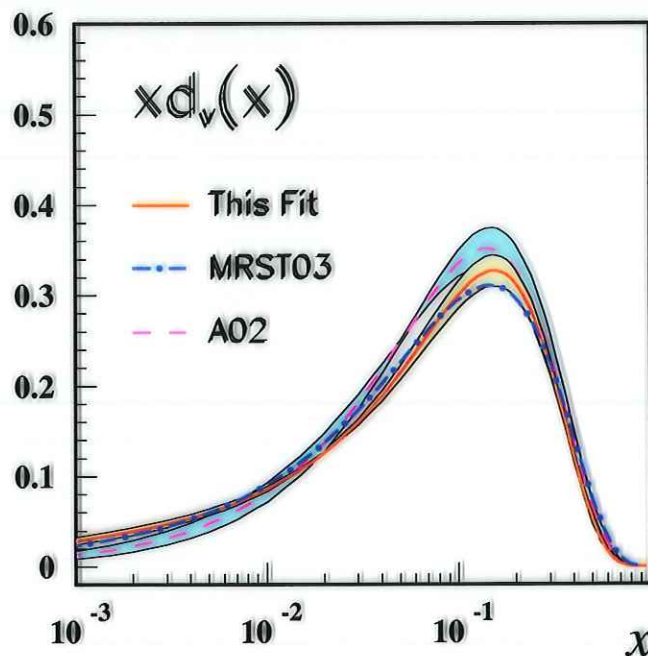
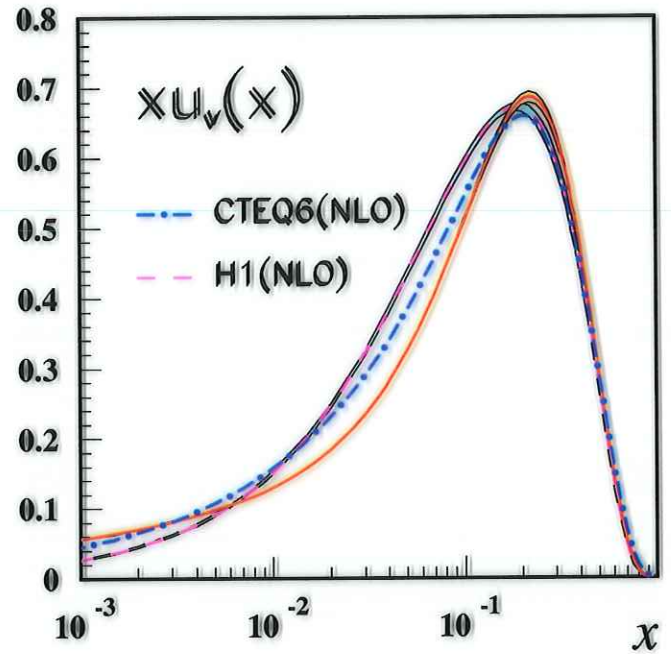
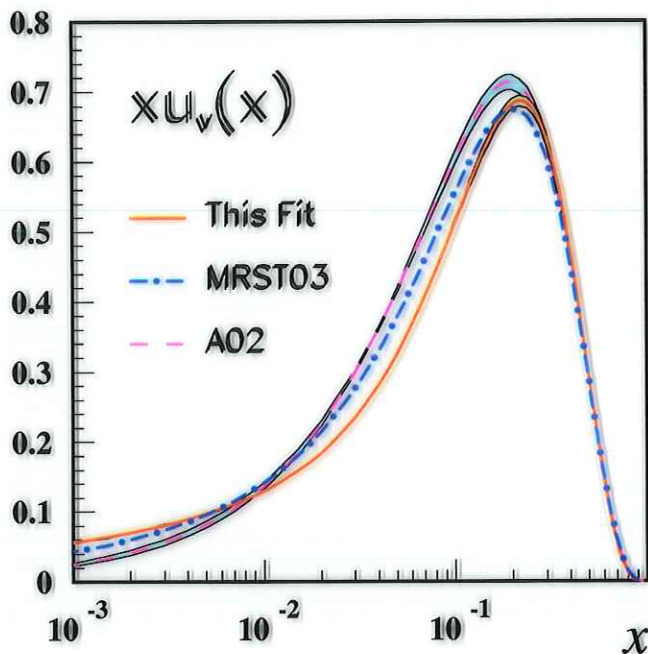
u_v	a	0.314 ± 0.007
	b	4.199 ± 0.032
	ρ	-1.108
	γ	26.283
d_v	a	0.413 ± 0.047
	b	6.196 ± 0.332
	ρ	0.895
	γ	18.179
$\Lambda_{QCD}^{(4)}$		$227 \pm 30 \text{ MeV}$
$\chi^2/ndf = 652/757 = 0.86$		

- Covariance Matrix at the input scale $Q_0^2 = 4.0 \text{ GeV}^2$

	$\Lambda_{QCD}^{(4)}$	a_{u_v}	b_{u_v}	a_{d_v}	b_{d_v}
$\Lambda_{QCD}^{(4)}$	9.27E-4				
a_{u_v}	1.09E-4	5.52E-5			
b_{u_v}	-1.13E-4	1.63E-4	9.94E-4		
a_{d_v}	2.23E-4	-1.40E-5	-4.85E-4	2.18E-3	
b_{d_v}	1.25E-3	2.53E-4	-2.83E-3	1.43E-2	1.11E-1

Valence Parton Densities at $Q_0^2 = 4.0 \text{ GeV}^2$

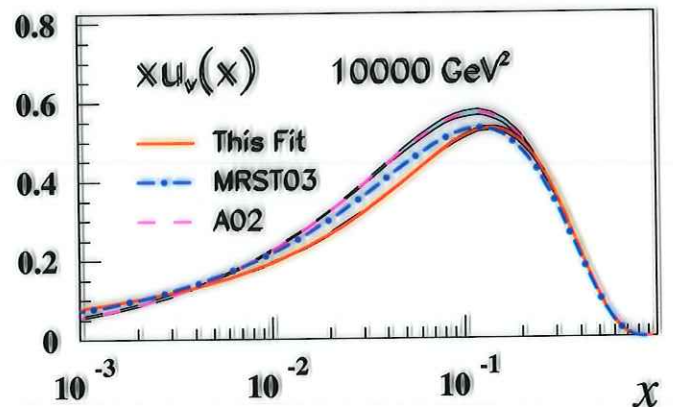
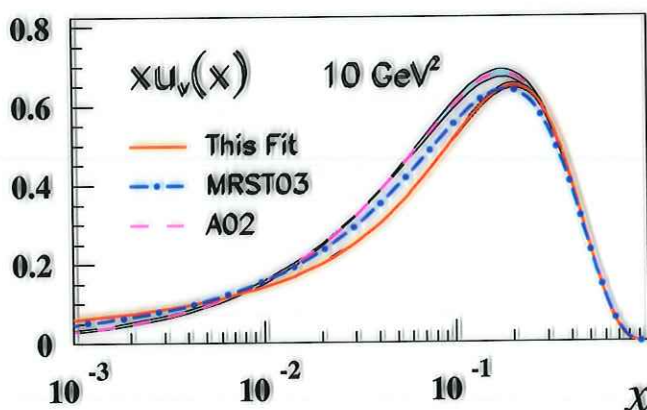
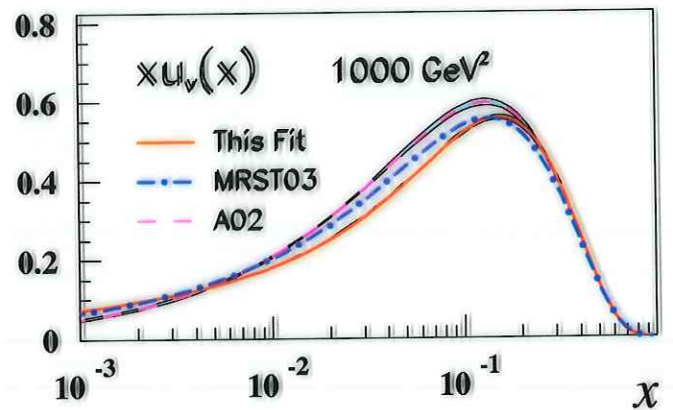
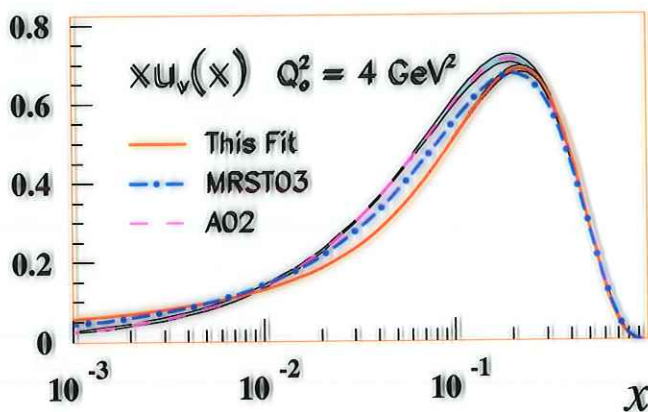
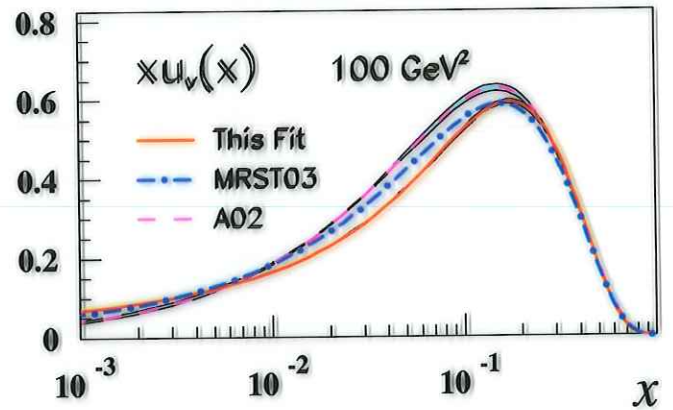
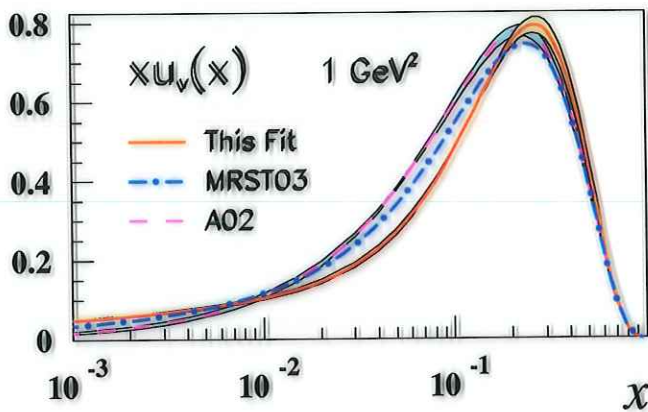
- 4+1 parameter non-singlet fit to F_2 data:



⇒ **Yellow error band:** Fully correlated 1σ error as given by Gaussian error propagation.

Evolution of the Parton Density $xu_v(x)$

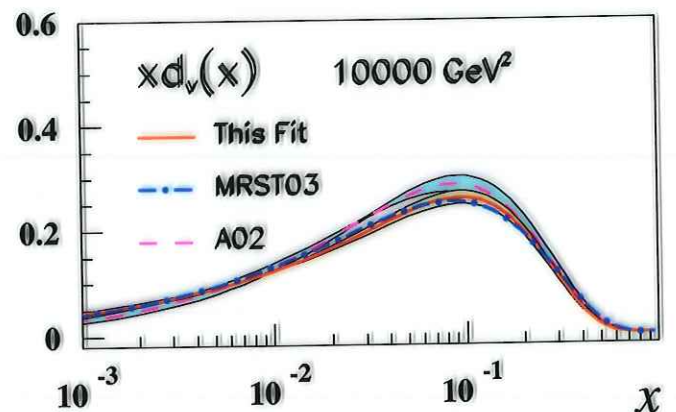
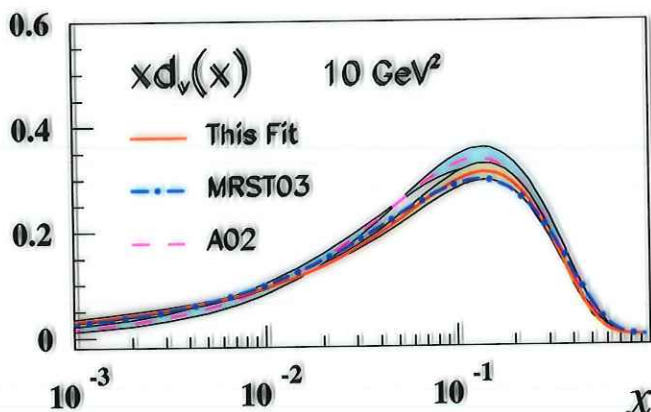
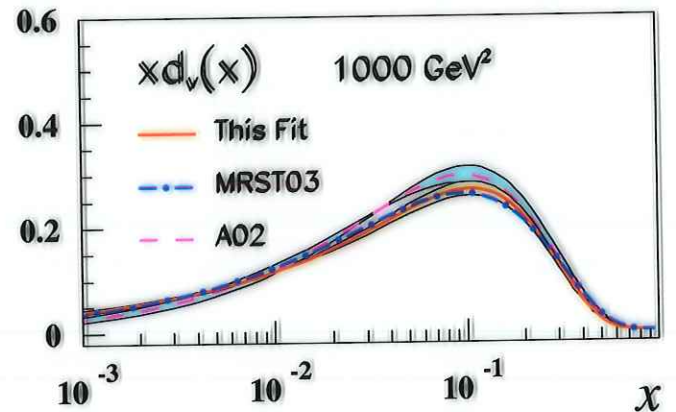
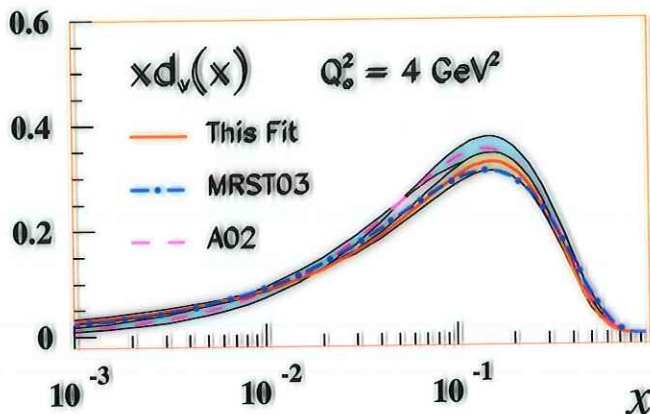
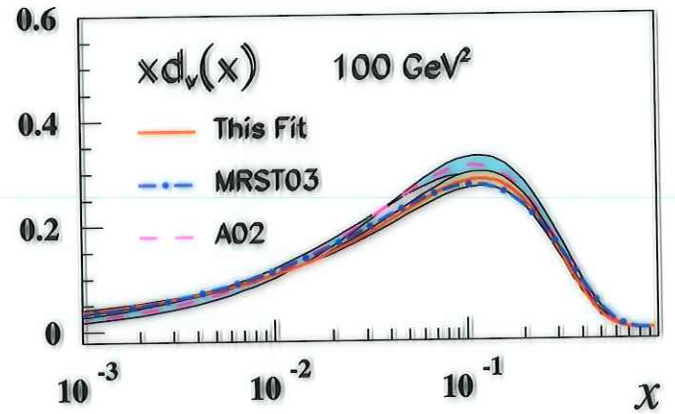
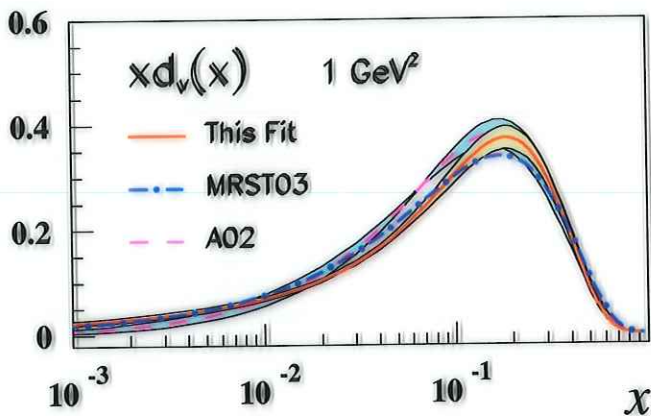
- 4+1 parameter non-singlet fit to F_2 data:



⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

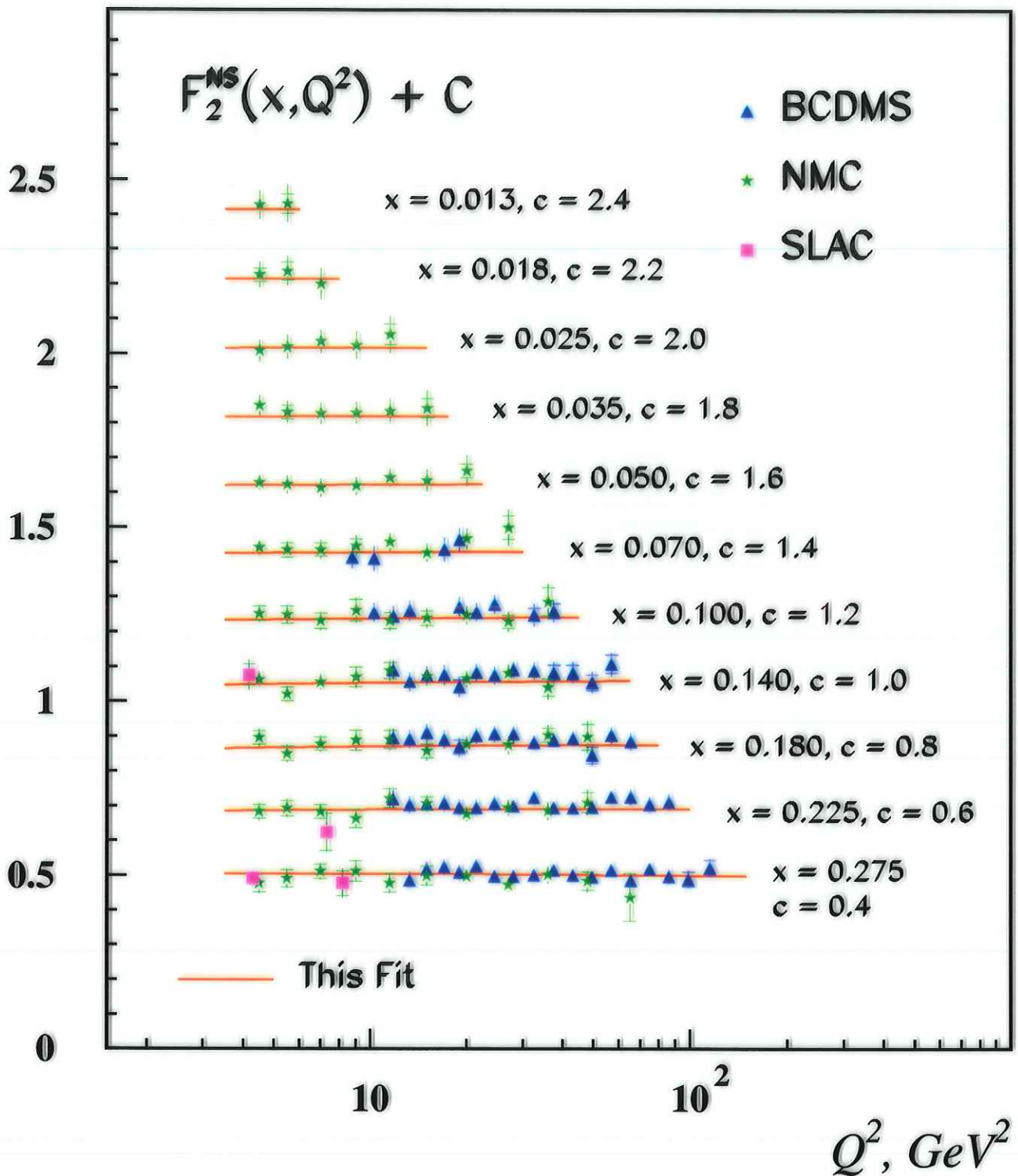
Evolution of the Parton Density $x d_v(x)$

- 4+1 parameter non-singlet fit to F_2 data:



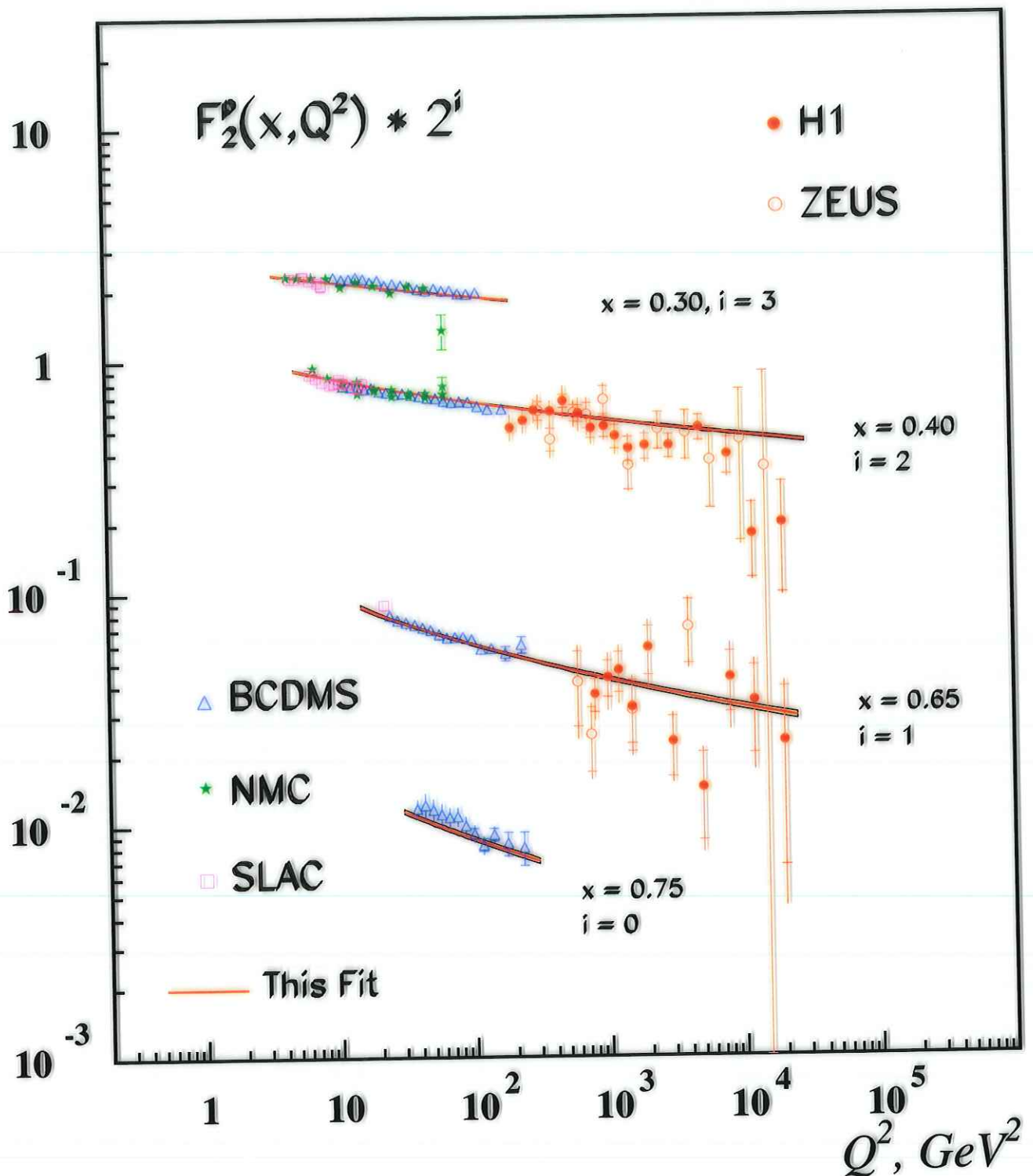
⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

$F_2^{NS}(x)$ versus Q^2



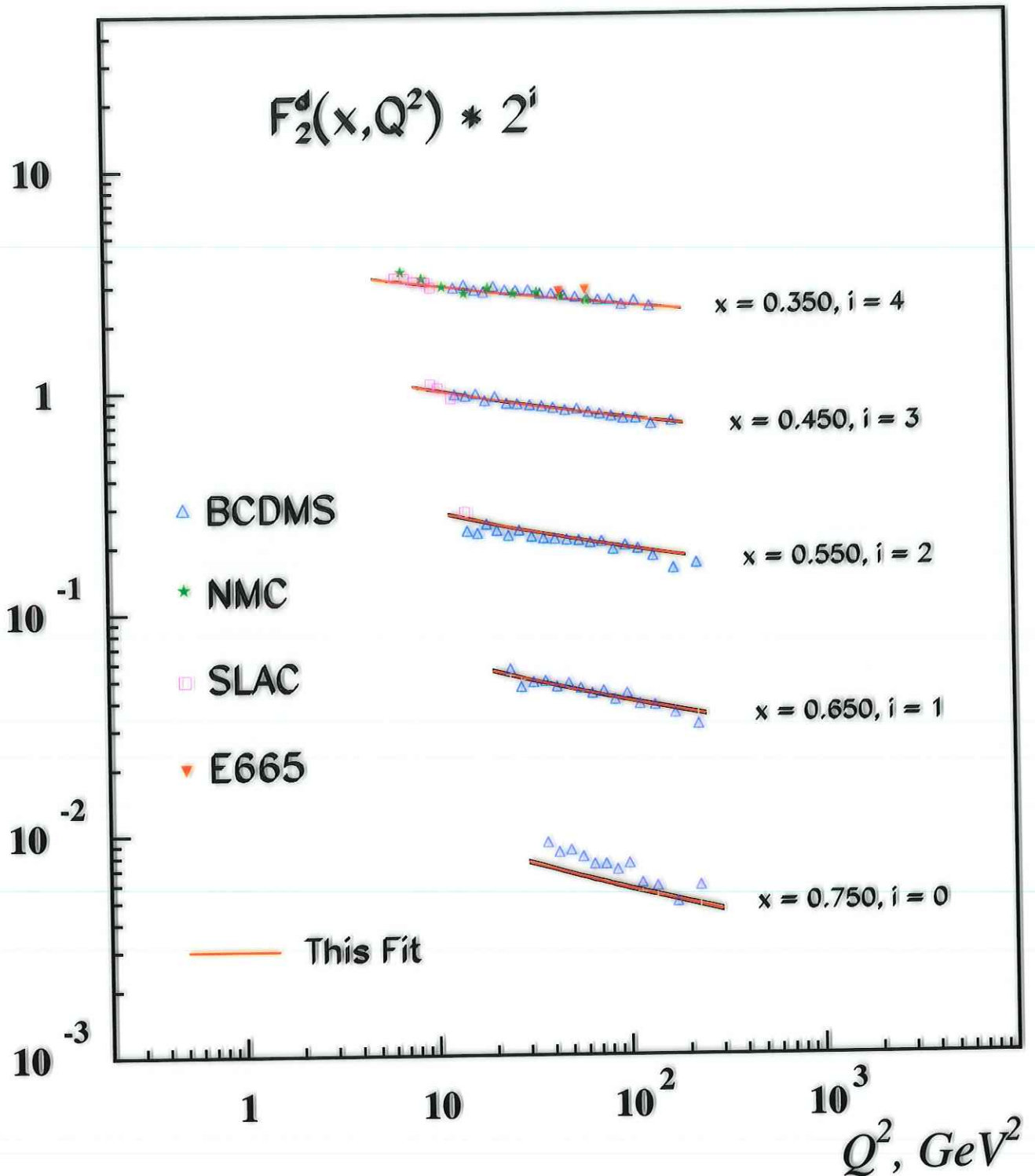
⇒ **Yellow error band:** Fully correlated 1σ Gaussian error propagation through the evolution equation.

$F_2^p(x)$ versus Q^2



⇒ **Yellow error band:** Fully correlated 1σ Gaussian error propagation through the evolution equation.

$F_2^d(x)$ versus Q^2



⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

TMC and HT

- **Target Mass Correction (TMC):**

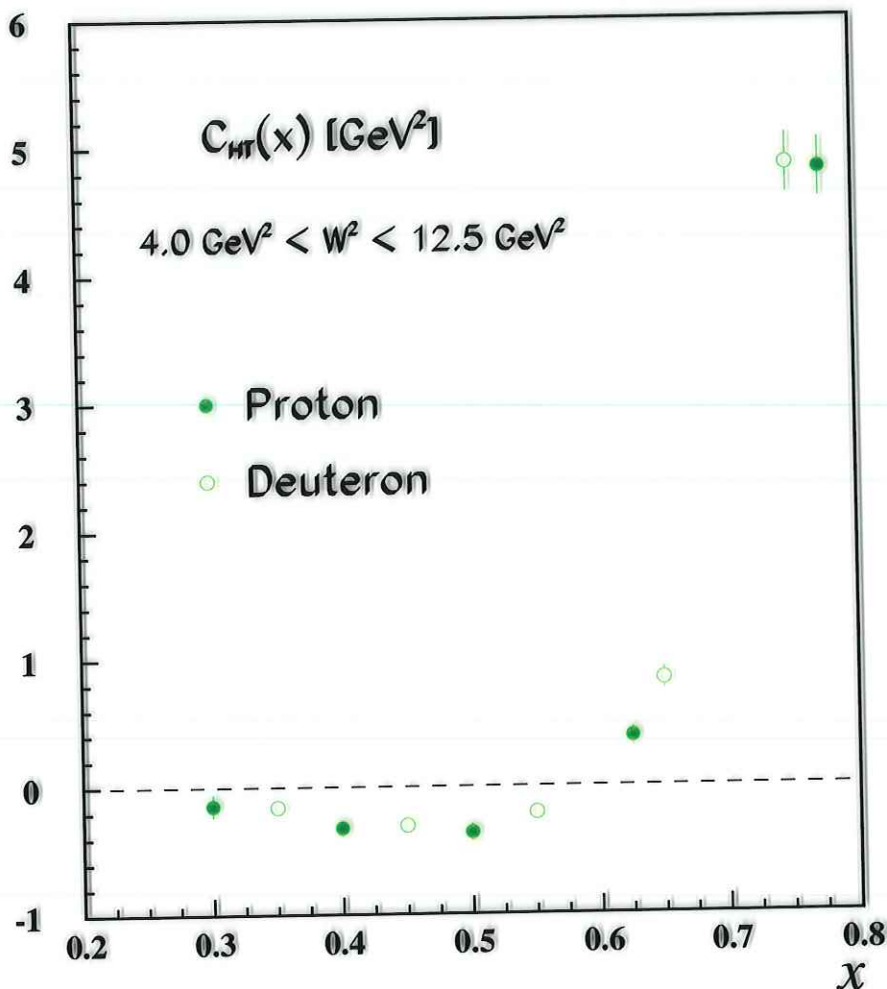
$$F_2^{TMC}(x, Q^2) = F_2^{QCD}(x, Q^2) \times TMC$$

[Ref: H.Georgi and H.D.Politzer, Phys.Rev.**D14**(1976)1829.]

- **Higher Twist (HT):**

$$F_2^{HT}(x_i, Q^2) = F_2^{QCD}(x_i, Q^2) \times C_{HT}(x_i)/Q^2$$

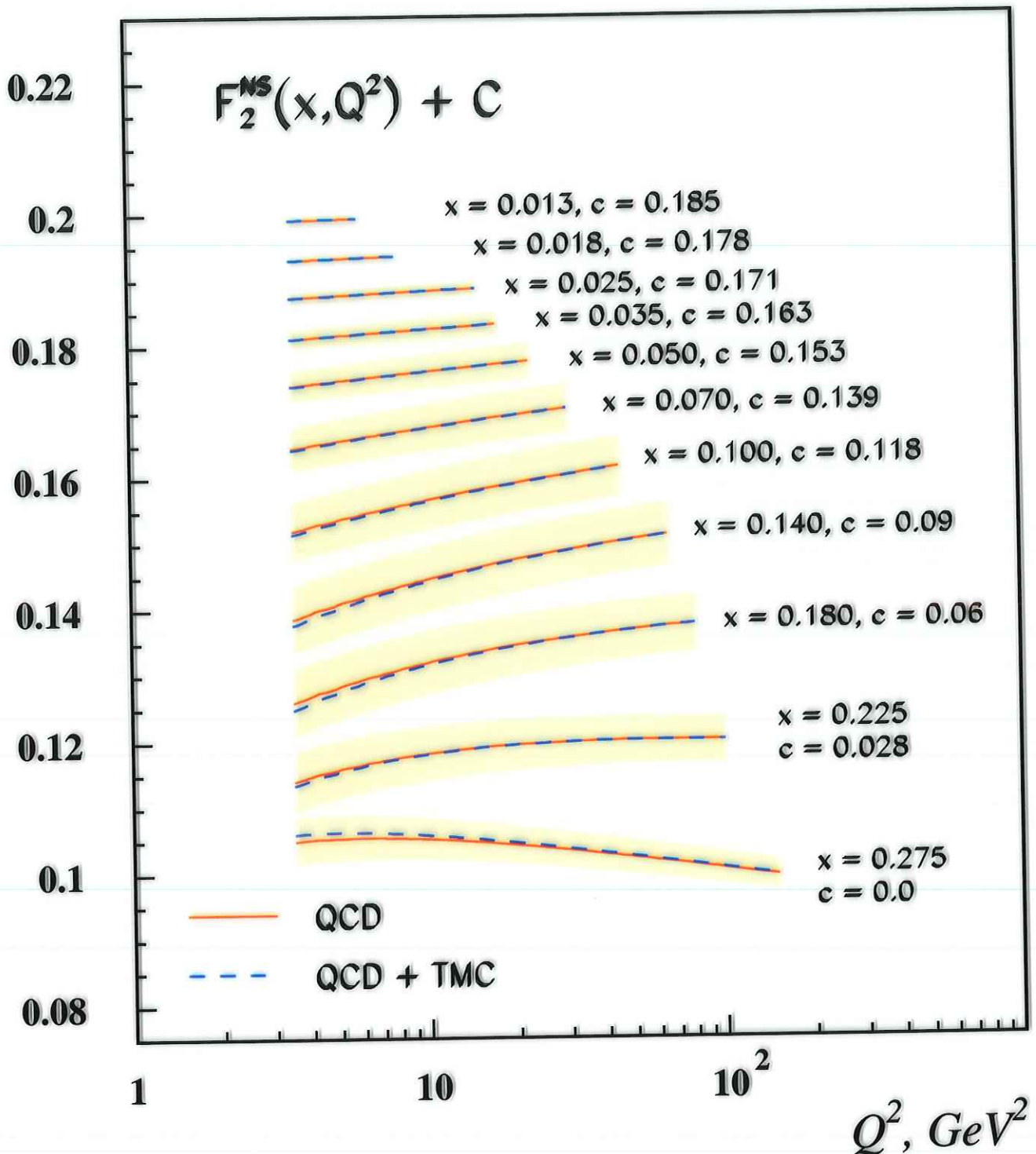
[Ref: M.Virchaux and A.Milsztajn, Phys.Lett.**B274**(1992)221.]



Fit in intervals
of x .
QCD parameters
fixed.

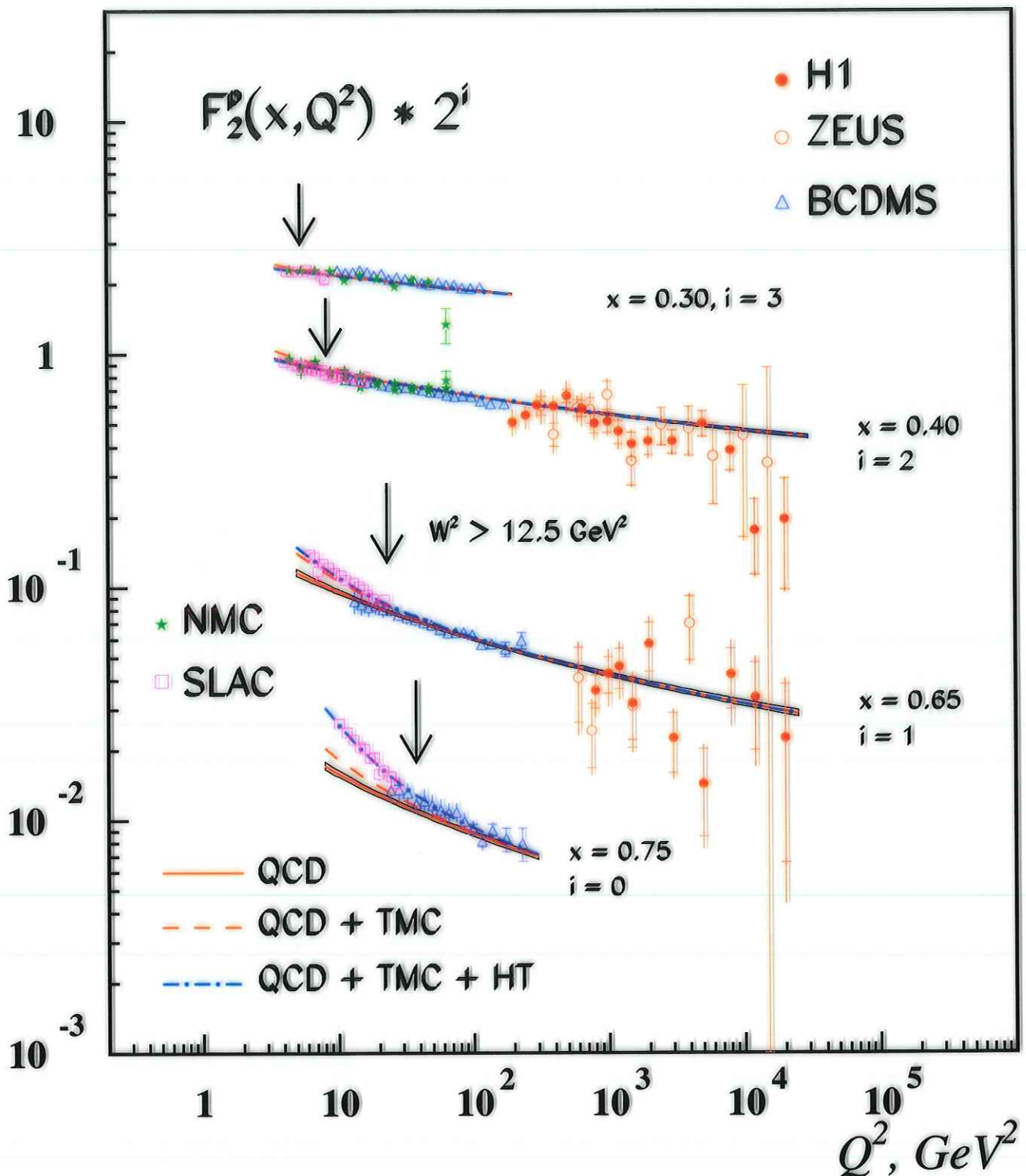
$$F_2(x, Q^2) = F_2^{TMC}(x, Q^2) + F_2^{HT}(x, Q^2)$$

$F_2^{NS}(x)$ versus Q^2



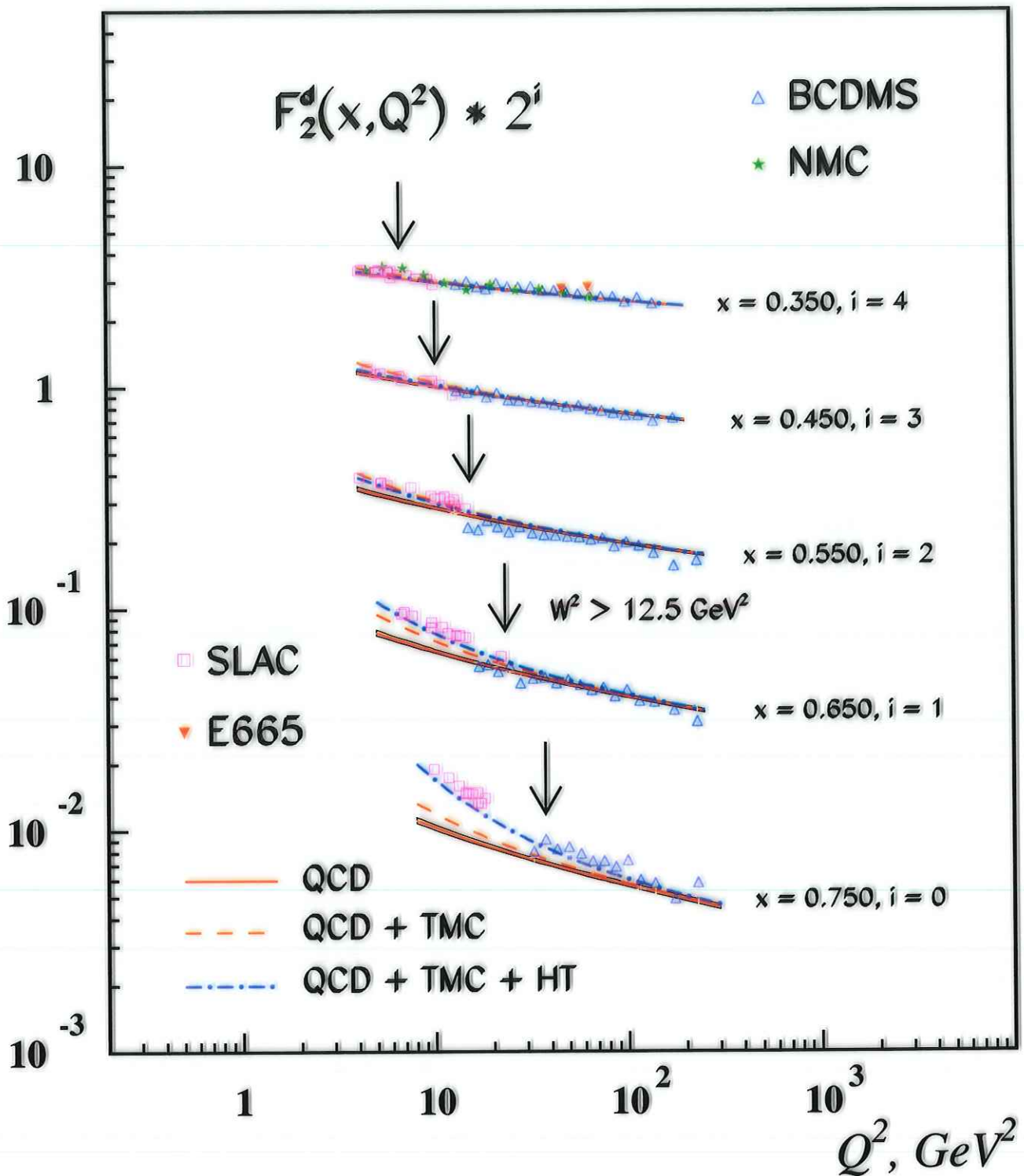
⇒ **Yellow error band:** Fully correlated 1σ Gaussian error propagation through the evolution equation.

$F_2^p(x)$ versus Q^2



⇒ **Yellow error band:** Fully correlated 1σ Gaussian error propagation through the evolution equation.

$F_2^d(x)$ versus Q^2



⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

$$\Lambda_{QCD}^{(4)} \Rightarrow \alpha_s(M_Z^2)$$

	$\Lambda_{QCD}^{(4)}$, MeV	$\alpha_s(M_Z^2)$
This Fit	227 ± 30	0.1135 $\begin{matrix} +0.0023 \\ -0.0026 \end{matrix}$ (expt)

$\Rightarrow \Lambda_{QCD}^{(4)}$ stable against a variation of the Q^2 -cut on the data (4, 7, 10 GeV^2).

\Rightarrow latest world average: 0.1182 ± 0.0027

Ref.: S.Bethke, LL2004, Zinnowitz, April 25-30, 2004

Comparison of $\alpha_s(M_Z^2)$

\Rightarrow This Fit: $\alpha_s(M_Z^2) = 0.1135^{+0.0023}_{-0.0026}$ (expt)

- Comparison with other QCD analyses (significant sea and gluon contributions and correlations):

	$\alpha_s(M_Z^2)$	expt	theory	model	Ref.
NLO					
CTEQ6	0.1165	± 0.0065			[1]
MRST03	0.1165	± 0.0020	± 0.0030		[2]
A02	0.1171	± 0.0015	± 0.0033		[3]
ZEUS	0.1166	± 0.0049		± 0.0018	[4]
H1	0.1150	± 0.0017	± 0.0050	$+0.0009$ -0.0005	[5]
BCDMS	0.110	± 0.006			[6]
BB (pol)	0.113	± 0.004	$+0.009$ -0.006		[7]
NNLO					
MRST03	0.1153	± 0.0020	± 0.0030		[2]
A02	0.1143	± 0.0014	± 0.0009		[3]
SY01(ep)	0.1166	± 0.0013			[8]
SY01(ν N)	0.1153	± 0.0063			[8]

[1]: CTEQ Coll.: J.Pumplin et al., JHEP 0207:012 (2002). [2]: MRST Coll.: A.D.Martin et al., hep-ph/0307262. [3]: S.Alekhin, hep-ph/0211096. [4]: ZEUS Coll.: S.Chekanov et al., Phys.Rev. **D67** (2003) 012007. [5]: H1 Coll.: C.Adloff et al., Eur.Phys. **C21** (2001) 33. [6]: BCDMS Coll.: A.C.Benvenuti et al., Phys.Lett. **237** (1990) 592. [7]: J.Blümlein and H.Böttcher, Nucl.Phys. **B636** (2002) 225. [8]: J.Santiago and F.J.Yndurain, Nucl.Phys. **B611** (2001) 447.

Comparison of Moments at $Q^2 = 4.0 \text{ GeV}^2$

f	n	This Fit	MRST03	A02
u_v	2	0.289 ± 0.003	0.289	0.304
	3	0.085 ± 0.001	0.084	0.087
	4	0.0324 ± 0.0004	0.032	0.033
d_v	2	0.109 ± 0.004	0.113	0.120
	3	0.025 ± 0.001	0.028	0.028
	4	0.0076 ± 0.0004	0.010	0.010
$u_v - d_v$	2	0.180 ± 0.005	0.176	0.184
	3	0.060 ± 0.001	0.056	0.059
	4	0.0248 ± 0.0006	0.023	0.024

f	n	QCD	Lattice
		This Fit	QCDSF
$u_v - d_v$	2	0.180 ± 0.005	$0.191 \pm 0.012^{*)}$

$$\Rightarrow \Gamma_f(Q^2) = \int_0^1 x^{n-1} f(x, Q^2) dx$$

Lattice simulation: Scale $\mu^2 = 1/a^2 \sim 4 \text{ GeV}^2$.

*) G.Schierholz, private communication.

Summary

- A **NON-SINGLET** QCD ANALYSIS OF THE STRUCTURE FUNCTION $F_2(x, Q^2)$ BASED ON THE NON-SINGLET WORLD DATA HAS BEEN PERFORMED TO 3-LOOPS.

- THE VALUE FOR $\alpha_s(M_Z^2)$ WAS DETERMINED TO BE:

$$\alpha_s(M_Z^2)|_{\text{NS}} = 0.1135 \begin{array}{l} +0.0023 \\ -0.0026 \end{array} \text{ (EXPT)},$$

COMPATIBLE WITH RESULTS FROM OTHER QCD ANALYSES AND YET WITH THE WORLD AVERAGE.

- **CORRELATED ERRORS** ON ALL FIT-PARAMETERS WERE DETERMINED AND THEIR PROPAGATION THROUGH THE EVOLUTION EQUATIONS WAS PERFORMED ANALYTICALLY.
- ANALYSES BY OTHER GROUPS ARE MAINLY BASED ON COMBINED SINGLET AND NON-SINGLET FITS AND ARE THEREFORE SUBJECT TO CORRELATIONS BETWEEN THE **SEA** AND **GLUON** PARAMETERS.
- WE CALCULATED THE LOW MOMENTS OF THE DISTRIBUTIONS $u_v, d_v, u_v - d_v$ WITH CORRELATED ERRORS WHICH MAY BE COMPARED TO UPCOMING **LATTICE MEASUREMENTS**.