HH 25/9/95

ACCESSING $g_1(x, Q^2)$ IN POLARIZED ep COLLISIONS AT HERA

J. BLÜMLEIN, DESY

1) BEHAVIOUR OF POL. PDF'S (SMALL X & WITH Q^2)

2) NLO P_{ij}'s

3) IRE & SMALL x BEHAVIOUR

4) g_1^{ep} AT HERA ($\vec{p}\,\vec{e}$).

G. LADINSKY

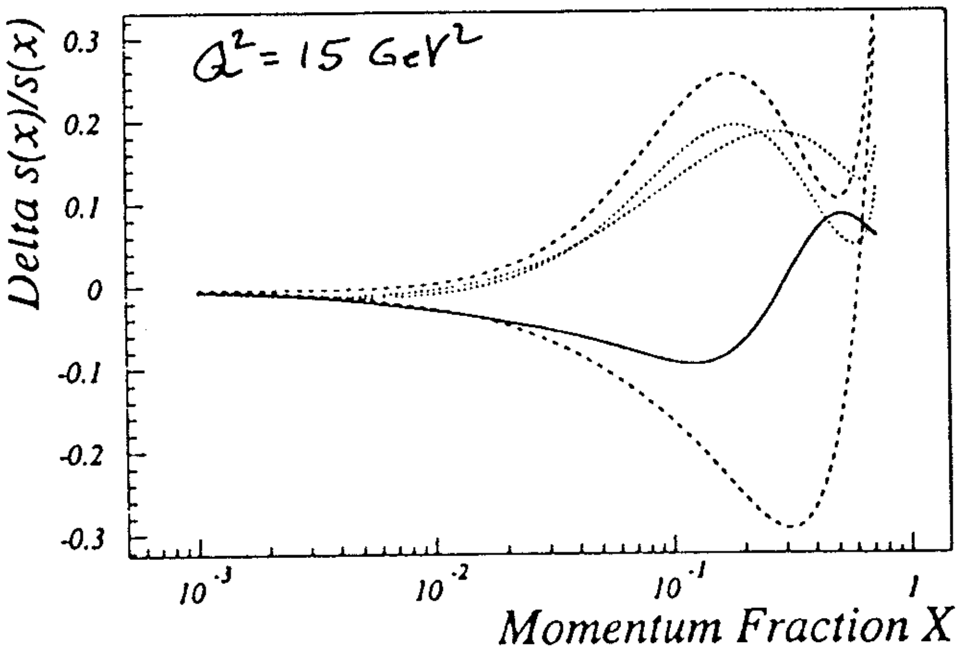
PARTIAL LISTING OF
POLARIZED PARTON DISTRIBUTION FUNCTIONS

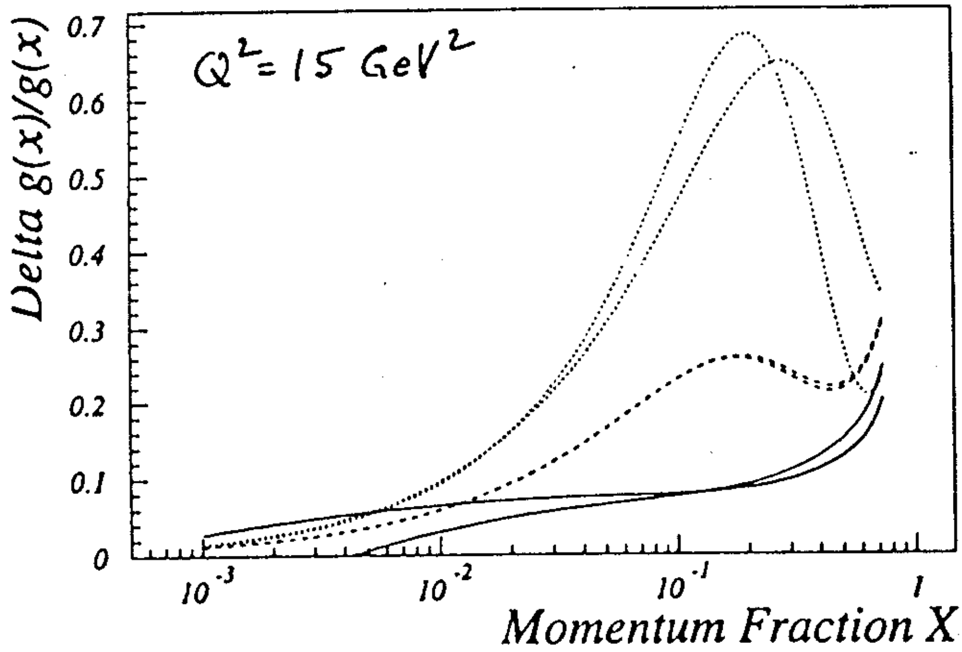
ΔPDFs	
Authors	Reference
Gluck, Reya and Vogelsang	preprint, DO-TH 95/11
Bourrely and Soffer	Nucl. Phys. B445 (1995) 341
de Florian, et al.	Phys. Rev. D51 (1995) 37
Gehrmann and Stirling	Z. Phys. C65 (1995) 461
Brodsky, et al.	Nucl. Phys. B441 (1995) 197
Nadolsky	Z. Phys. C63 (1994) 601
Chiappetta, et al.	Z. Phys. C59 (1993) 629
de Florian, et al.	Phys. Lett. B319 (1993) 285
H.-Y. Cheng and C.F. Wai	Phys. Rev. D46 (1992) 125
Chiappetta and Nardulli	Z. Phys. C51 (1991) 435
Gluck, Reya and Vogelsang	Nucl. Phys. B329 (1990) 347
Gupta, et al.	Z. Phys. C46 (1990) 111
H.-Y. Cheng and S.-N. Lai	Phys. Rev. D41 (1990) 91

+ NLO

* LIBRARY OF LO PDF'S AVAILABLE.

G. LADINSKY


Q^2 = 15 GeV^2
Delta s(x)/s(x)
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
10^-3
10^-2
10^-1
1
Momentum Fraction X


Q^2 = 15 GeV^2
Delta g(x)/g(x)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10^-3
10^-2
10^-1
1
Momentum Fraction X

JB

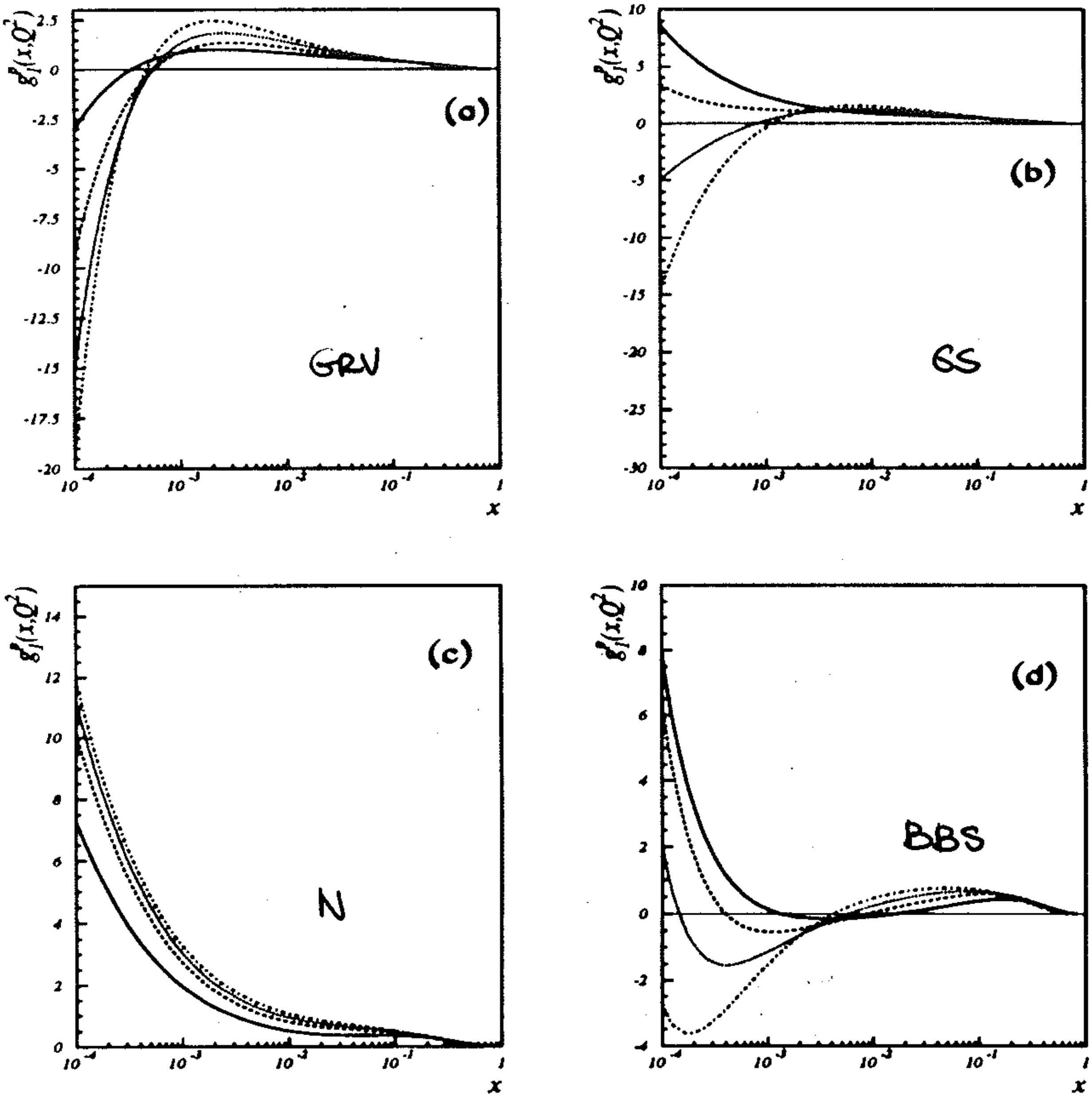


Figure 1: The structure function $g_1^p(x, Q^2)$ in the range $x > 10^{-4}$. Full line: $Q^2 = 10\,\mathrm{GeV}^2$, dashed line: $Q^2 = 10^2\,\mathrm{GeV}^2$, dotted line: $Q^2 = 10^3\,\mathrm{GeV}^2$, dash–dotted line: $Q^2 = 10^4\,\mathrm{GeV}^2$. The parametrizations are: (a) ref. [5], (b) ref. [6], (c) ref. [7], (d) ref. [8].

JB

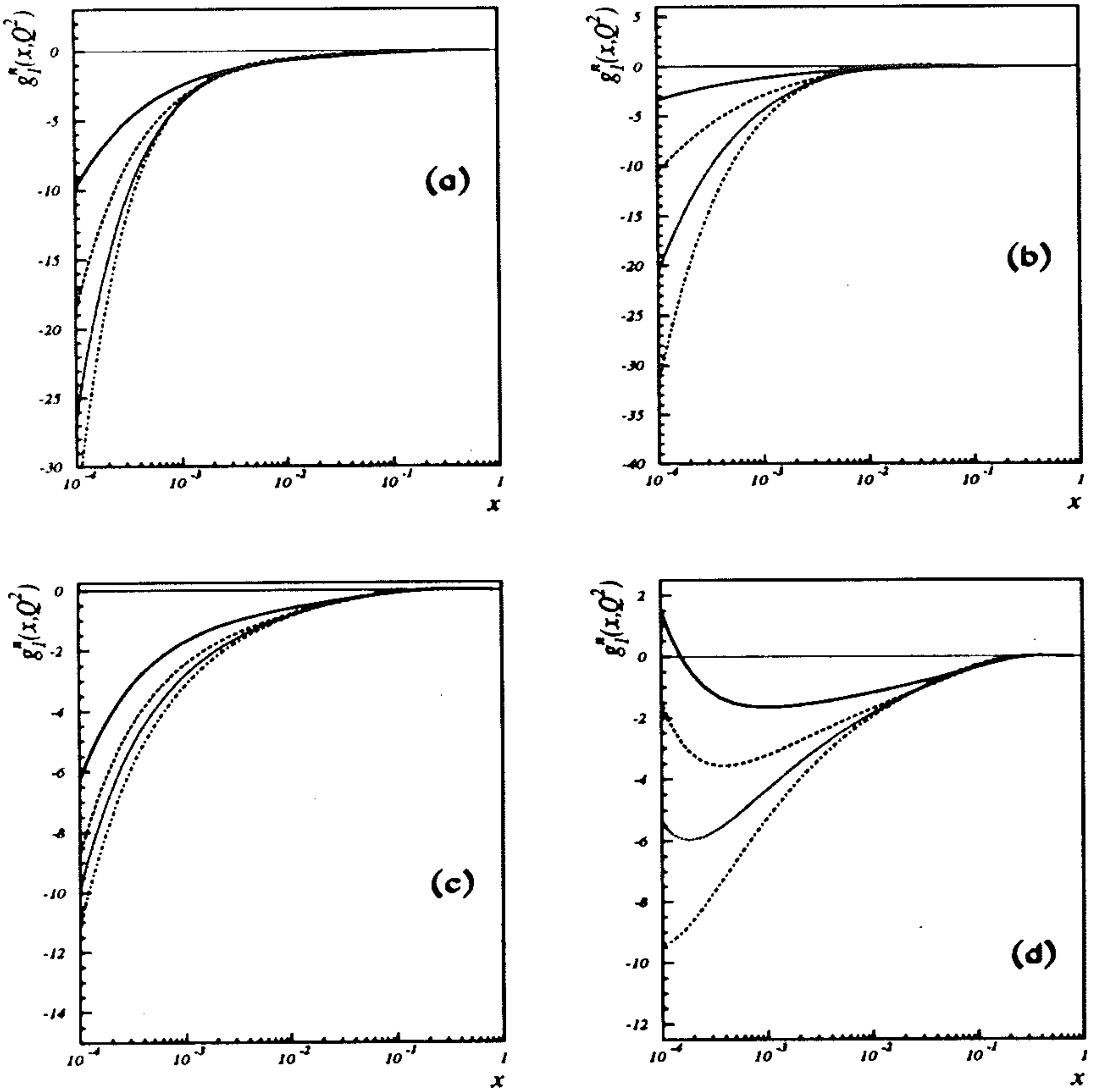
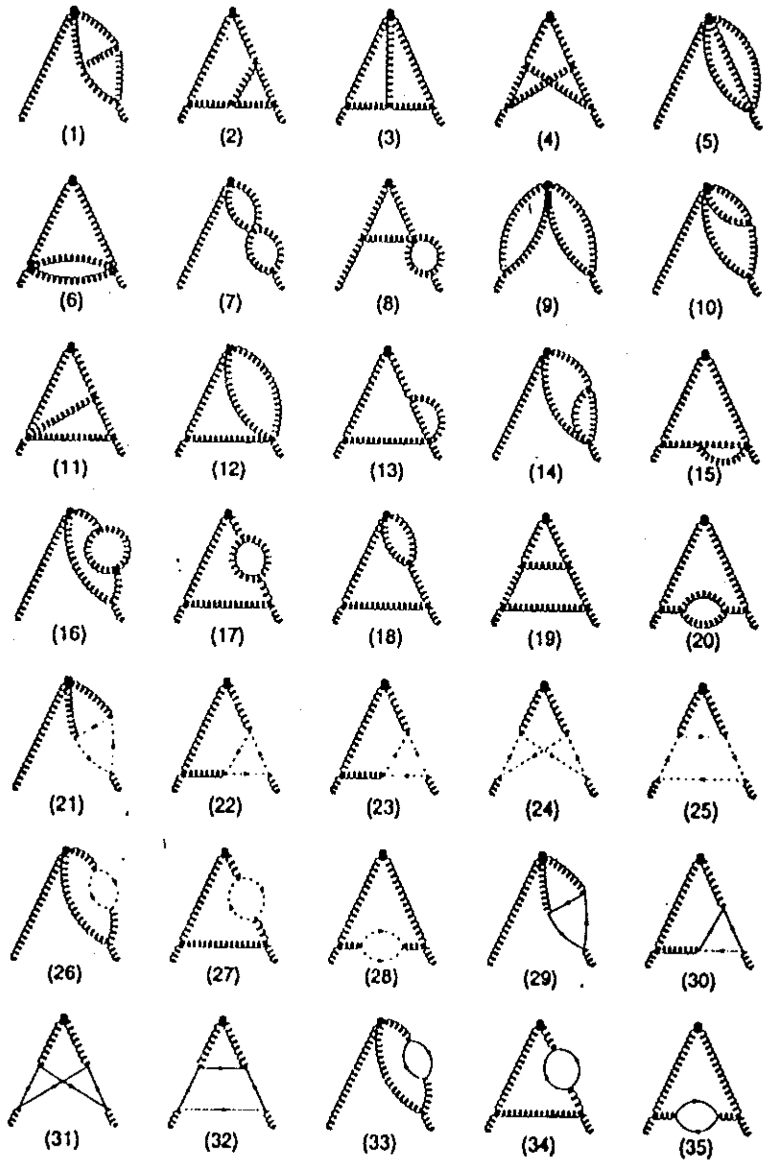


Figure 2: The structure function $g_1^n(x, Q^2)$ in the range $x > 10^{-4}$. Full line: $Q^2 = 10\,\mathrm{GeV}^2$, dashed line: $Q^2 = 10^2\,\mathrm{GeV}^2$, dotted line: $Q^2 = 10^3\,\mathrm{GeV}^2$, dash–dotted line: $Q^2 = 10^4\,\mathrm{GeV}^2$. The parametrizations are: (a) ref. [5], (b) ref. [6], (c) ref. [7], (d) ref. [8].

MERTIG, VAN NEERVEN: NLO ANOM. FOR g_1 EVOL.

∴ P_{gg} DIAGRAMS:



$$P^{(1)}_{PS,qq} = C_F T_f \left[-16(1+x)\ln^2 x - 16(1-3x)\ln x + 16(1-x)\right].$$

• LEAD. SING. STRUCT. : D. GROSS' CONJ. '74
NLO CONFIRMED.

$$\begin{aligned}
P^{(1)}_{S,qg} = {} & 4\,C_A T_f\,[-8(1+2x)\mathrm{Li}_2(-x) - 8\zeta(2) - 8(1+2x)\ln x\ln(1+x) \\
& + 4(1-2x)\ln^2(1-x) - 4(1+2x)\ln^2 x \\
& -16\,(1-x)\ln(1-x) + 4\,(1+8x)\ln x - 44\,x + 48] \\
& +4\,C_F T_f\,\left[8\,(1-2x)\zeta(2) - 4\,(1-2x)\ln^2(1-x)\right. \\
& + 8\,(1-2x)\ln x\ln(1-x) - 2\,(1-2x)\ln^2 x \\
& \left.+16\,(1-x)\ln(1-x) - 2(1-16x)\ln x + 4 + 6x\right],
\end{aligned} \tag{3.66}$$

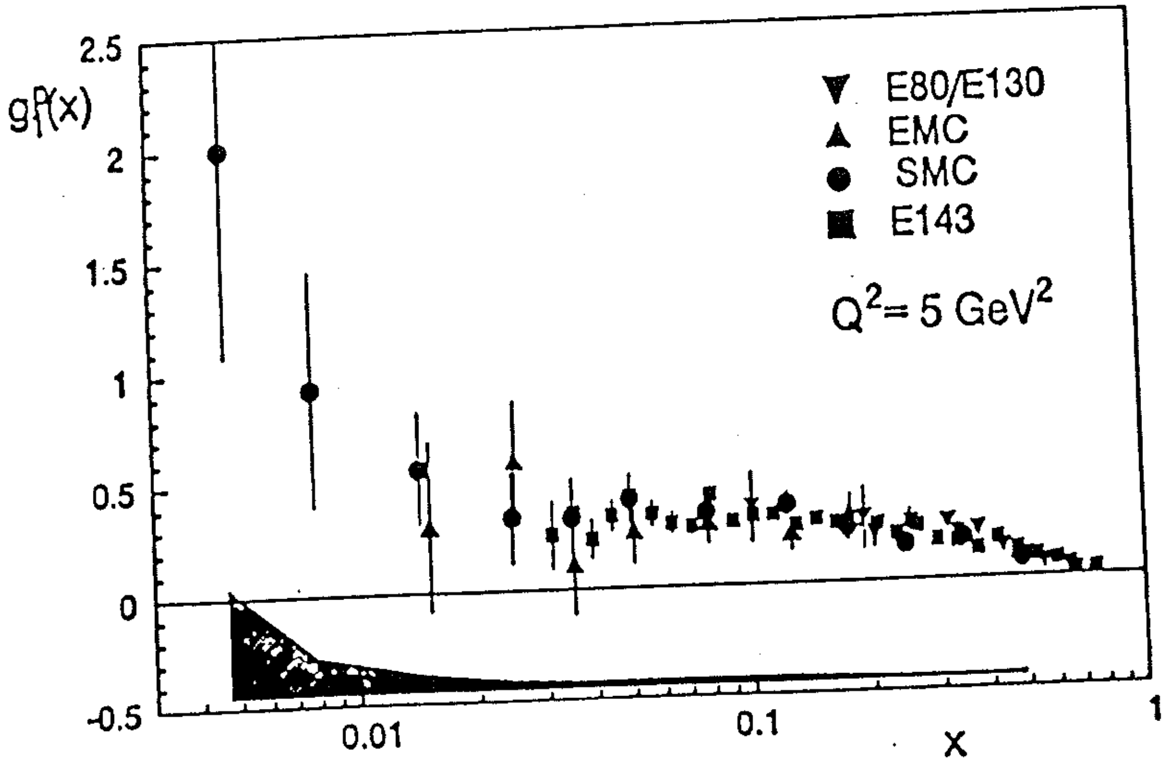
$$\begin{aligned}
P^{(1)}_{S,gq} = {} & C_A C_F\,\left[16\,(2+x)\mathrm{Li}_2(-x) + 16\,x\,\zeta(2) + 8\,(2-x)\ln^2(1-x)\right. \\
& + 16\,(2+x)\ln x\ln(1+x) + 8\,(2+x)\ln^2 x \\
& + 16\,(x-2)\ln x\ln(1-x) + \left(\frac{80}{3} + \frac{8}{3}x\right)\ln(1-x) \\
& \left.+8\,(4-13\,x)\ln x + \frac{328}{9} + \frac{280}{9}x\right] \\
& +C_F^2\,\left[8\,(x-2)\ln^2(1-x) - 4\,(x-2)\ln^2 x - 164 + 128\,x\right. \\
& \left.-8\,(x+2)\ln(1-x) - 4\,(20+7\,x)\ln x\right] \\
& +C_F T_f\,\left[-\frac{32}{9}(4+x) + \frac{32}{3}(x-2)\ln(1-x)\right],
\end{aligned} \tag{3.67}$$

$$\begin{aligned}
P^{(1)}_{S,gg} = {} & C_A^2\,\left[\left(64x + \frac{32}{1+x} + 32\right)\mathrm{Li}_2(-x) + \left(64x - 16\left(\frac{1}{1-x}\right)_+ + \frac{16}{1+x}\right)\zeta(2)\right. \\
& + \left(\frac{8}{1-x} - \frac{8}{1+x} + 32\right)\ln^2 x + \left(64x + \frac{32}{1+x} + 32\right)\ln x\ln(1+x) \\
& + \left(64x - \frac{32}{1-x} - 32\right)\ln x\ln(1-x) + \left(\frac{232}{3} - \frac{536}{3}x\right)\ln x \\
& \left.+ \frac{536}{9}\left(\frac{1}{1-x}\right)_+ - \frac{388}{9}x - \frac{148}{9} + \delta(1-x)(24\zeta(3) + \frac{64}{3})\right] \\
& +C_A T_f\,\left[-\frac{160}{9}\left(\frac{1}{1-x}\right)_+ - \frac{32}{3}(1+x)\ln x - \frac{448}{9} + \frac{608}{9}x\right. \\
& \left.-\frac{32}{3}\delta(1-x)\right] \\
& +C_F T_f\,\left[-16(1+x)\ln^2 x + 16(x-5)\ln x - 80(1-x)\right. \\
& \left.-8\delta(1-x)\right].
\end{aligned} \tag{3.68}$$

R. VOSS

From A_1 to g_1

- Remember: $g_1 \propto A_1 F_2 / 2x[1+R]$
- Assumptions:
  - $R(x,Q^2)$: use SLAC parametrisation
  - $F_2(x,Q^2)$: use NMC parametrisation
  - Assume A_1 to be Q^2-independent
- QCD predicts Q^2-dependence of A_1 – formalism exists but depends on polarised gluon distribution
- Present status of $g_1{}^p$:



- Observe a rise at small x???

RESUMMATION OF TERMS $\propto \alpha_s \ln^2 x$:

RYSKIN, ERMOLAEV, MANAYENKOV
BARTELS, RYSKIN, ERMOLAEV

- equs. derived by KIRSCHNER, LIPATOV 1983 (more applications later).

IRE

- confirm $\propto \alpha_s \frac{1}{2\pi} C_F$ term : $\lim_{x \to 0} \frac{\alpha_s}{2\pi} C_F \frac{1+x^2}{1-x}$

- relate this to HO terms in $\alpha_s \ln^2 x$, large correction. $\sim 2 \ldots 10$.

At which x dominance of these terms ?

RESUMMATION OF $O(\alpha_s \ln^2 x)$ TERMS:

NS$_\pm$ EVOLUTION J.B., DESY 95-175

- NS : MOST SINGULAR TERMS AS $x \to 0$ (TWIST 2)
- S : " " " " " FOR $\underline{g_1}$.

<u>EVOLUTION EQU. FOR STRUCTURE FUNCTIONS</u>

$$F^{\pm}_{NS,i}(x, Q^2) = C^{(i)\pm}_{NS}(x, \alpha_s) \otimes f^{\pm}_{NS}(x, \alpha_s) \qquad (\overline{MS})$$

$$\frac{\partial F^{\pm}_{NS}}{\partial \log Q^2} = \left[\frac{\partial C^{\pm}_{NS}}{\partial \log Q^2} \otimes C^{\pm^{-1}}_{NS} + P^{\pm}_{NS}(x, \alpha_s) \right] \otimes F^{\pm}_{NS}$$

<u>SPLITTING FCT.:</u> $q^2 < 0$:
$x < 1$

$$P^{\pm}_{NS} = P_{qq} \pm P_{q\bar{q}}$$

$$P_{qq} = \frac{\alpha_s}{2\pi} C_F \frac{1+x^2}{1-x} + \left(\frac{\alpha_s}{2\pi}\right)^2 \left[C_F^2 P_F + \frac{1}{2} C_F C_G P_G + C_F N_f T_R P_{N_f}(x) \right] + O(\alpha_s^3)$$

$$P_{q\bar{q}} = \left(\frac{\alpha_s}{2\pi}\right)^2 \left(C_F^2 - \frac{1}{2} C_F C_G\right) P_A + O(\alpha_s^3)$$

$$P_F = -\frac{1}{2} \log^2 x + \dots \qquad P_A = \log^2 x$$

$$P_G = \log^2 x + \dots$$

$$P_{N_f} = + \dots + \underline{0} \log^2 x \qquad + O(\alpha_s^3)$$

$$\lim_{x \to 0} P_{\pm}(x) =:$$

$$P_+ \to \frac{\alpha_s}{2\pi} C_F + \frac{1}{2} \left(\frac{\alpha_s}{2\pi}\right)^2 C_F^2 \ln^2 x$$

$$P_- \to \frac{\alpha_s}{2\pi} C_F + \left(\frac{\alpha_s}{2\pi}\right)^2 \left(-\frac{3}{2} C_F^2 + C_F C_G\right) \ln^2 x.$$

COEFF. FKT.:

$-Q^2 \equiv q^2 < 0$:

$$C_{NS}^{+} = \delta(1-x) + \frac{\alpha_s}{2\pi} C_F \left[\frac{1+x^2}{1-x}\left(\ln\frac{1-x}{x} - \frac{3}{4}\right) + \frac{1}{4}(9+5x)\right]_+ + \ldots$$

$$C_{NS}^{-} = C_{NS}^{+} - C_F \frac{\alpha_s}{2\pi}(1+x) + \ldots$$

$$\frac{\partial C_{NS}^{\pm}}{\partial \log Q^2} \otimes C_{NS}^{\pm^{-1}} = - \frac{\alpha_s^2}{(2\pi)^2} \frac{\beta_0}{2} C_F \left[\frac{1+x^2}{1-x}\left(\ln\frac{1-x}{x} - \frac{3}{4}\right) + \frac{1}{4}(9+5x) - \delta_-(1+x)\right]$$

$$\lim_{x\to 0} \frac{\partial C_{NS}^{\pm}}{\partial \log} \otimes C_{NS}^{\pm^{-1}} = - \frac{\alpha_s^2}{(2\pi)^2}\frac{\beta_0}{2} C_F \cdot \underline{0} \cdot \ln^2 x + O(\alpha_s^3).$$

ASYMPTOTIC PART OF THE EVOLUTION EQ:

$$\boxed{\begin{aligned} \frac{\partial F^{\pm}_{NS\,|x\to 0}}{\partial \log Q^2} &= P^{\pm}_{NS,\,x\to 0}(x,\alpha_s) \otimes F^{\pm}_{NS\,|x\to 0} \\ P_{+,x\to 0} &= \frac{\alpha_s}{2\pi} C_F + \frac{1}{2}\left(\frac{\alpha_s}{2\pi}\right)^2 C_F^2 \ln^2 x \\ P_{-,x\to 0} &= \frac{\alpha_s}{2\pi} C_F + \left(\frac{\alpha_s}{2\pi}\right)^2 \left[-\frac{3}{2} C_F^2 + C_F C_G\right] \ln^2 x \end{aligned}}$$

IRE : LIPATOV 1982
* KIRSCHNER, LIPATOV 1983

CF ALSO:
RYSKIN et al. '95
BARTELS et al. '95

LO resummation for $F_{NS}^{\pm}$ (not for $f_{NS}^{\pm}$!)

Mellin transform:

$$M[P_{+,x\to 0}](\omega) = \frac{\omega}{2}\left\{1-\sqrt{1-\frac{2\alpha_s C_F}{\pi\omega^2}}\right\} \qquad (*)$$

$$M[P_{-,x\to 0}](\omega) = \frac{\omega}{2}\left\{1-\sqrt{1-\frac{2\alpha_s C_F}{\pi\omega^2}\left[1-\frac{2N_c\alpha_s}{\pi\omega}\frac{d}{d\omega}\phi\right]}\right\}$$

$$\phi := \ln\left(e^{\frac{z^2}{4}} D_{-\frac{1}{2N_c^2}}(z)\right)$$

$$z = \omega/\omega_V\,, \quad \omega_V = \sqrt{\bar{\alpha}_s/2}\,, \quad \bar{\alpha}_s = \frac{N_c\alpha_s}{\pi}.$$

EXPAND & USE: $M\left[\ln^k \frac{1}{x}\right](\omega) = \frac{k!}{\omega^{k+1}}.$

$$P_{+,x\to 0} = \frac{\alpha_s C_F}{2\pi} + \frac{1}{2}\left(\frac{\alpha_s C_F}{2\pi}\right)^2 \ln^2 x + \ldots$$

$$P_{-,x\to 0} = \underbrace{\frac{\alpha_s C_F}{2\pi}}_{\uparrow\ \text{KNOWN}} + \underbrace{\left(\frac{\alpha_s}{2\pi}\right)^2 C_F\left[\frac{1}{N_c}+\frac{1}{2}C_F\right]}_{\uparrow\ \text{NEW}} \ln^2 x + \ldots$$

SINCE:

$$C_G - \frac{3}{2}C_F \equiv \frac{1}{N_c} + \frac{1}{2}C_F$$

IN SU(N),

(*) AGREES TO ALL (NTLO) KNOWN ORDERS WITH THE RESULT ON F_{NS} EVOLUTION FOR $Q^2 > 0$.

NOTE THAT A DIFFERENT RESULT $O(\alpha_s^2)$ IS OBTAINED FOR THE TIMELIKE REGION $q^2>0$:

(VIOL. OF GRIBOV - LIPATOV rel.)

- NO CONTRIBUTION DUE TO $\frac{\partial C_{NS}^{\pm\,T}}{\partial \log Q^2} \otimes C_{NS}^{T\pm\,-1}$

- $$P^T_{\pm, x\to 0} = P^S_{\pm, x\to 0} - 2\left(\frac{\alpha_s}{2\pi}\right)^2 C_F^2 \ln^2 x .$$

THE RESUMMATION DUE TO K&L DOES NOT APPLY FOR $q^2>0$.

JB

$g_1(x, Q^2)$ AT HERA

$\lambda_e \equiv \lambda_p = 0.8$; $\int \mathcal{L}\, dt = 60\ pb^{-1}$ (30 per pol.)

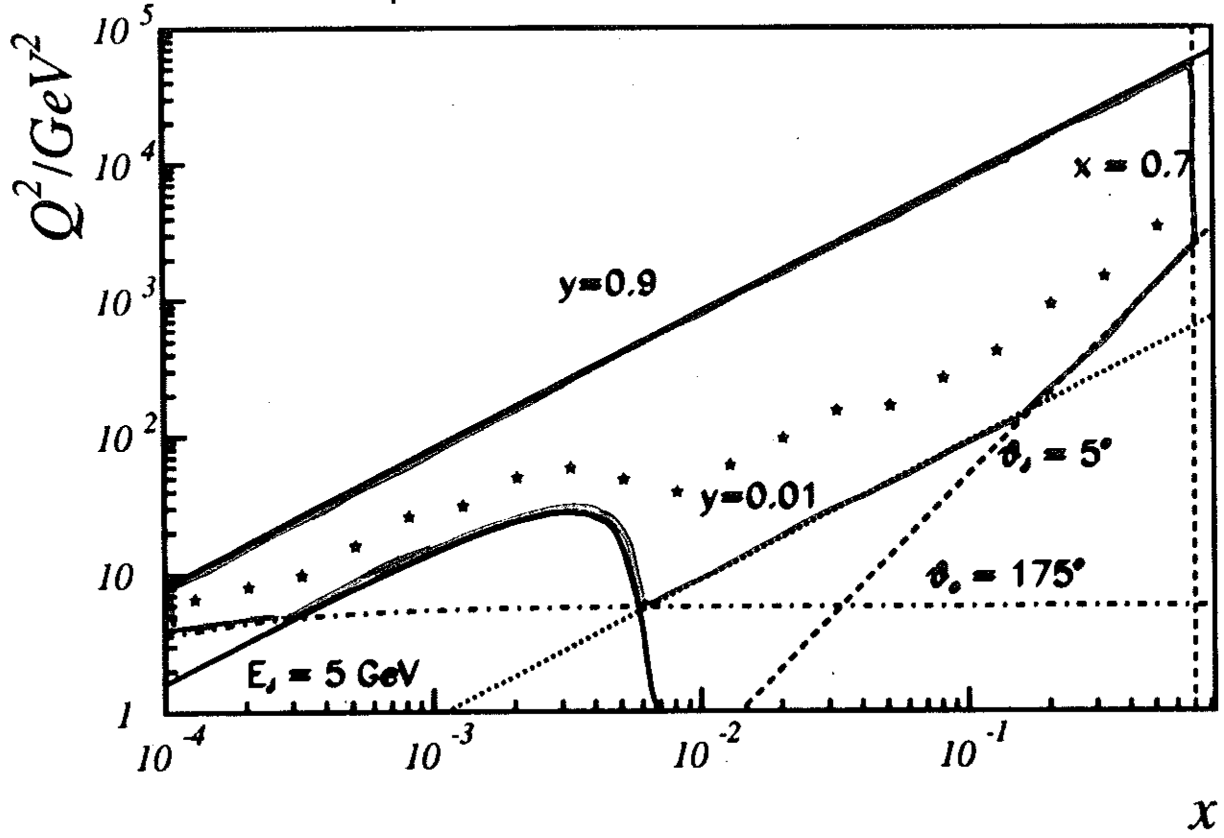


Figure 3: The accessible kinematical range for neutral current deep inelastic scattering at HERA; $E_p = 820\,\mathrm{GeV}$, $E_e = 27.6\,\mathrm{GeV}$. The stars indicate the values of $\langle Q^2 \rangle$ at a given value of x for neutral current deep inelastic scattering.

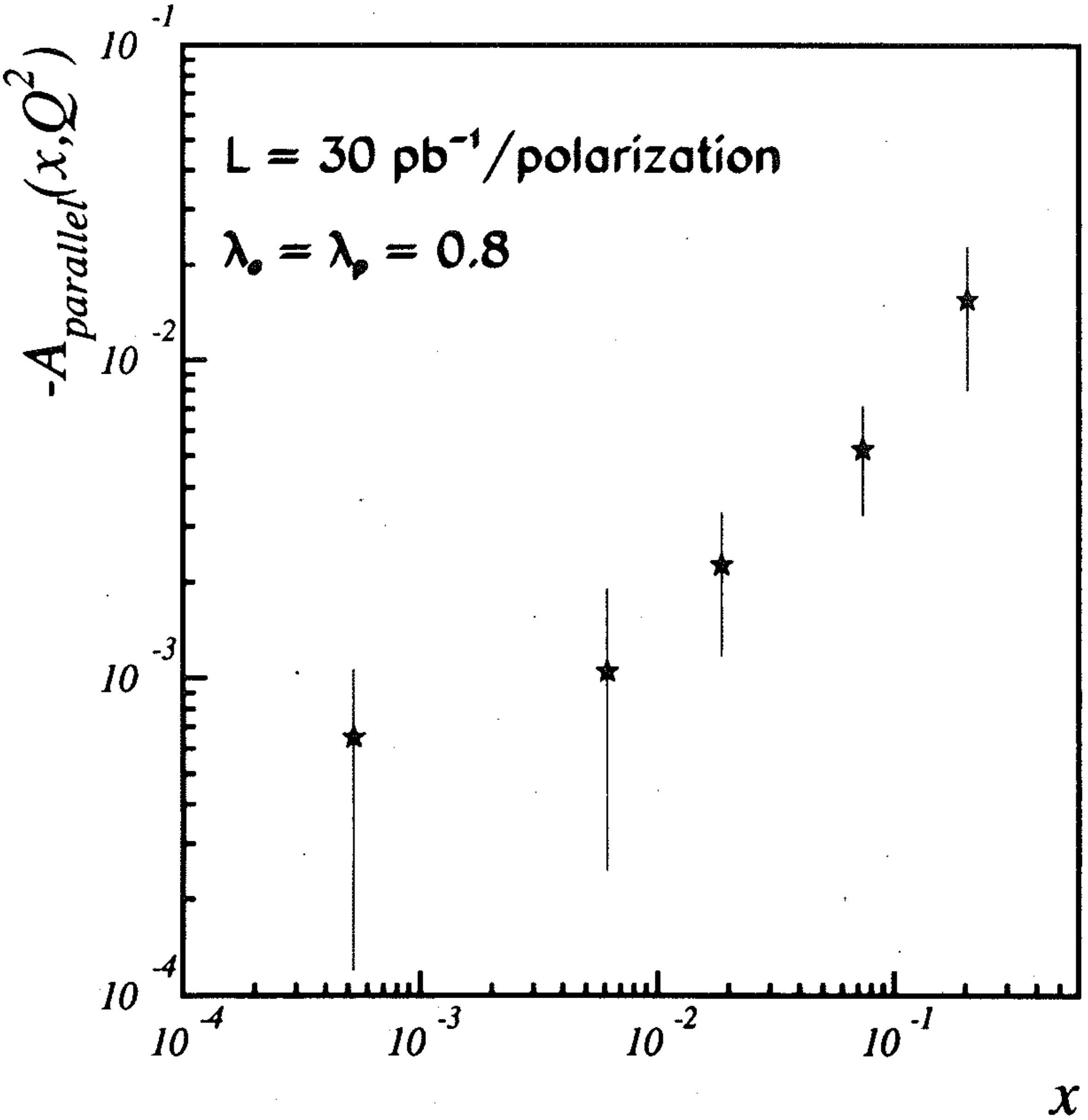


Figure 4: Statistical precision of a measurement of $-A_{\|}(x,\langle Q^2\rangle)$ in the kinematical domain of HERA. The data points represent averages over the accessible Q^2 range and were calculated using the parametrizations [6, 9].

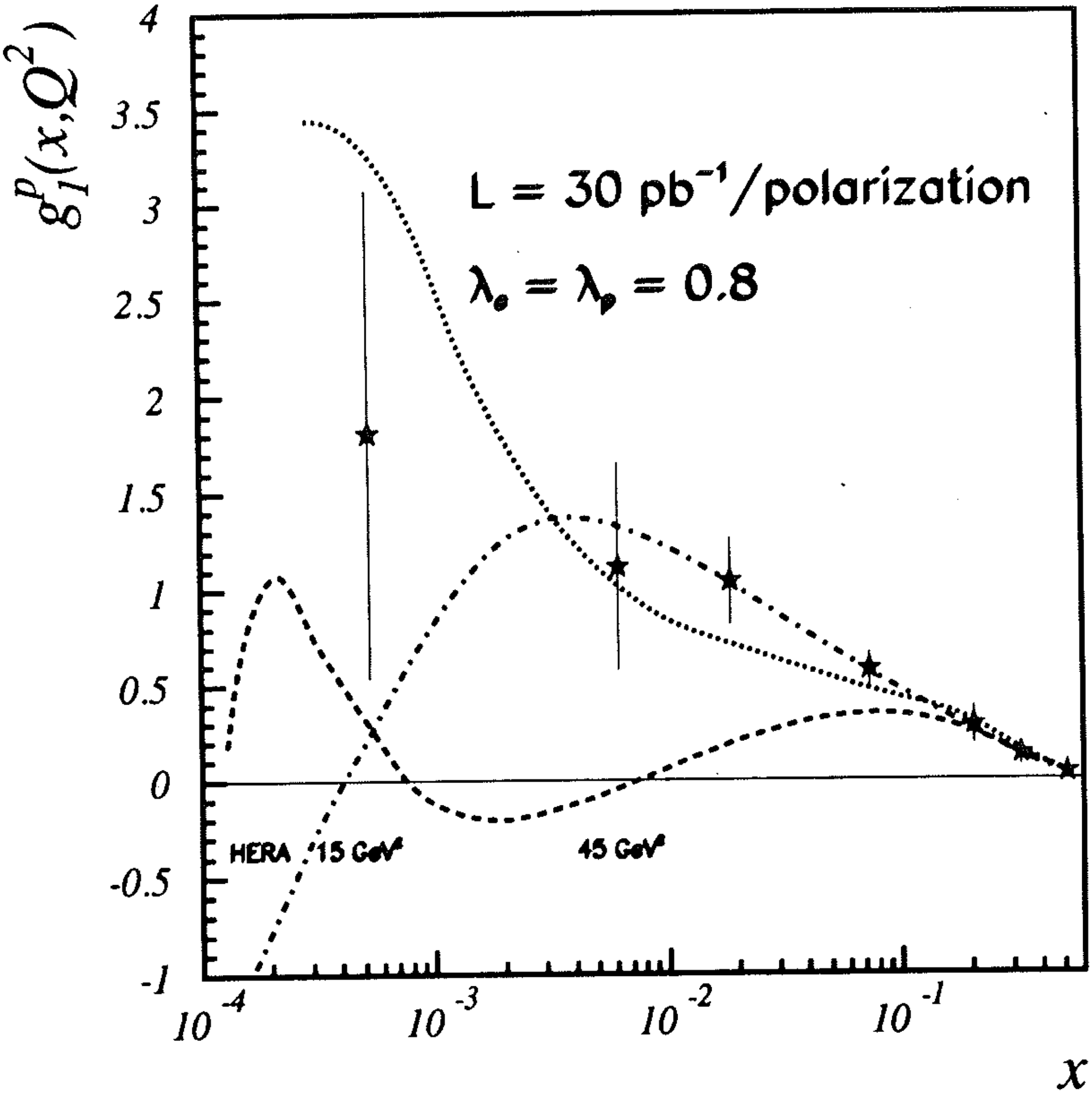


Figure 5: Statistical precision of a measurement of $g_1^p(x,Q^2)$ in the kinematical domain of HERA. The data points represent averages over the accessible Q^2 range and were calculated using the parametrization [6]. The dashed, dotted line, and dash–dotted line correspond to the values of $g_1^p(x,\langle Q^2\rangle)$ for the parametrizations [8], [7], and [5], respectively.

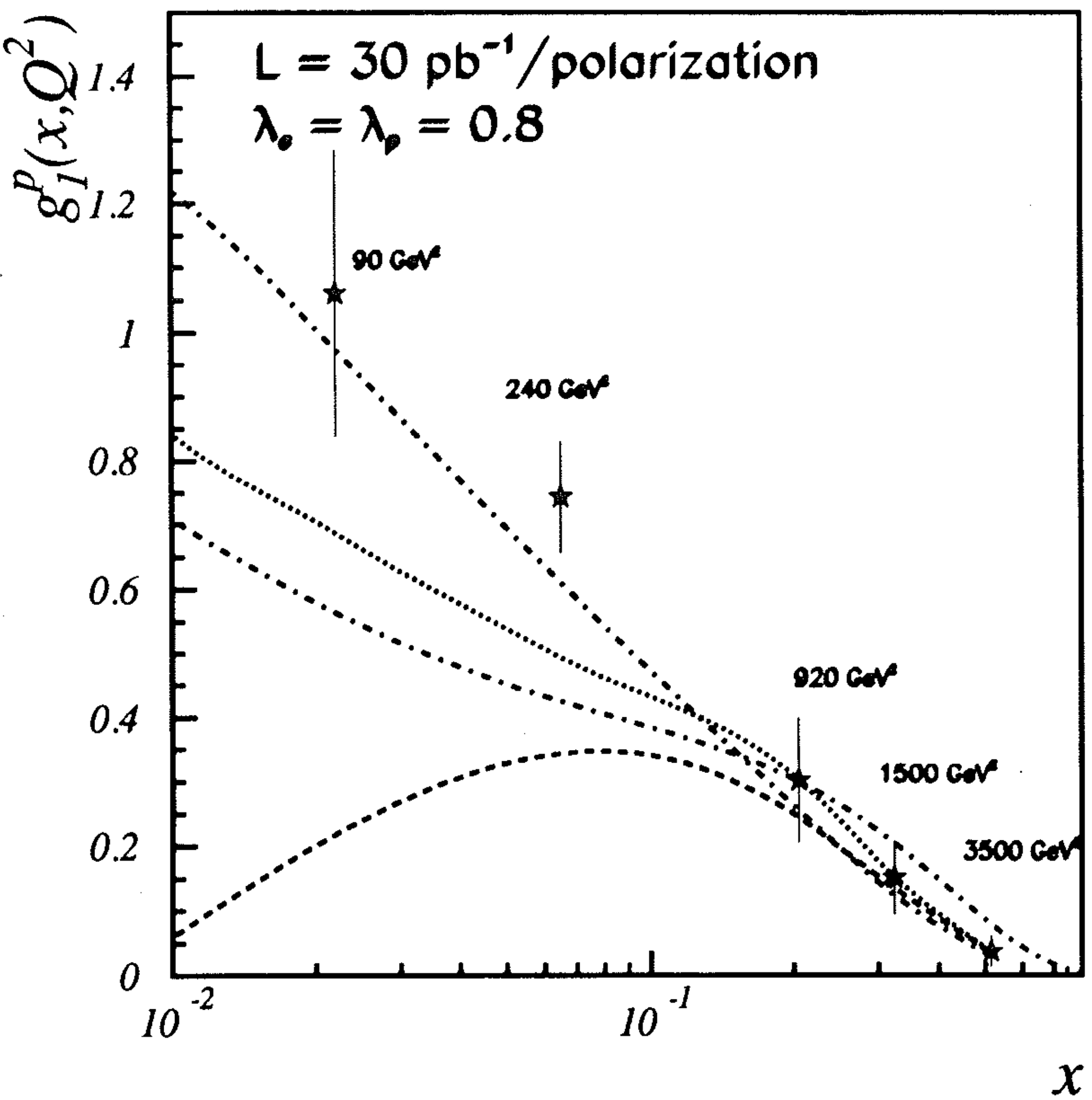


Figure 6: Statistical precision of a measurement of $g_1^p(x, Q^2)$ in the kinematical domain of HERA at larger values of x. The data points represent averages over the accessible Q^2 range and were calculated using the parametrization [6]. The dashed, dotted, and upper dash–dotted line correspond to the values of $g_1^p(x, \langle Q^2 \rangle)$ for the parametrizations [8], [7], and [5], respectively. The lower dash–dotted line shows $g_1^p(x, Q_0^2)$ for $Q_0^2 = 4\,\mathrm{GeV}^2$ for parametrization [5].