Hamburg
September 1994

# Small $x$ Contributions to Structure Functions using $k_\perp$ Factorization

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# 1 Introduction

- **In the small $x$ range a *novel* behaviour of nucleon structure functions is expected.**
  Among the possible dynamical effects are: Screening and effects due to non strong $k_\perp$ ordering.

- **This requires to consider rather:**

$$\int^{\mu^2} dK^2 \frac{\partial x G(x, K^2)}{\partial K^2} \qquad \text{CF. S. DRELL} \quad (1)$$

**than**

$$x G(x, \mu^2) \tag{2}$$

- To obtain results which are **consistent with perturbative QCD** a consistent factorization relation has to be derived.

- Furthermore, approximations of the $x$ behaviour of the $k_\perp$ dependend coefficient functions or (additional) low $Q^2$ 'cuts' bear the danger to obtain partly unphysical and misleading results (e.g. *negative* structure functions).

- The goal of the present paper is to find a consistent solution of the above problems and to extend earlier investigations (LEVIN AND RYSKIN 1991, ASKEW ET AL. 1993).

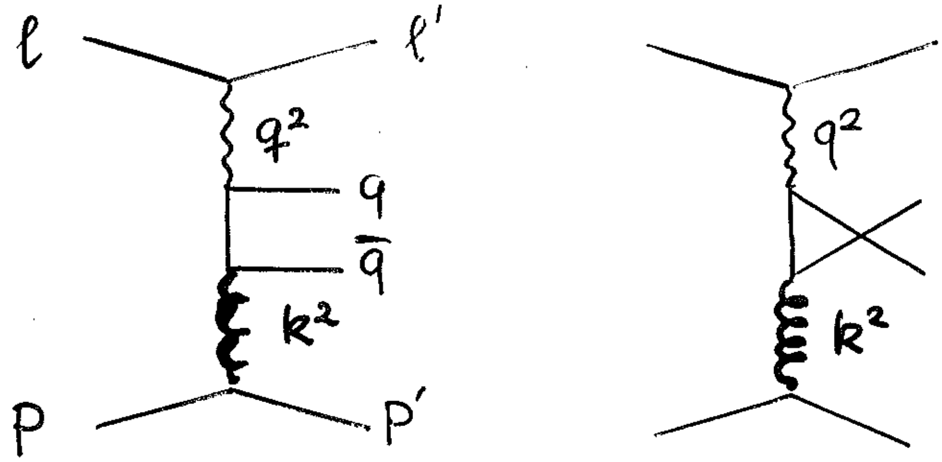
- We aim on a **general** formulation of the gluon contribution to the structure functions which offers the possibility to **unfold** the $k_\perp$ dependence of the gluon at small $x$. Theoretical predictions of its small $x$ behaviour can thus be confronted with the data with respect to the $k_\perp$ dependence directly.

- LARGE X : DO WE OBTAIN THE COLLINEAR PICTURE ? DGLAP ; $\hat{\sigma}(k_{\perp i}^2) \simeq \hat{\sigma}(0)$ ?
  $\longrightarrow$ NUM. ANALYSIS.

→ HOW TO DESCRIBE A $k^2$ DEPENDENT GLUON DISTRIBUTION ?

↔ RELATION TO HIGHER ORDER CORRECTIONS IN FIXED ORDER PERTURBATION THEORY.

## 2 Kinematics




$$q + k = p_1 + p_2 \tag{3}$$

The $2 \rightarrow 2$ process is described by the invariants:

$$\begin{aligned} \hat{s} &= (q+k)^2 = (p_1+p_2)^2 \\ \hat{t} &= (k-p_1)^2 = (q-p_2)^2 \\ \hat{u} &= (q-p_1)^2 = (k-p_2)^2 \end{aligned} \tag{4}$$

with

$$\hat{s} + \hat{t} + \hat{u} = -Q^2 - K^2 + m_1^2 + m_2^2 \tag{5}$$

and $Q^2 = -q^2$, $K^2 = -k^2$, $q' = q + xP$.

$$k_\mu = \xi q' + \eta P_\mu + k_{\perp\mu} \tag{6}$$

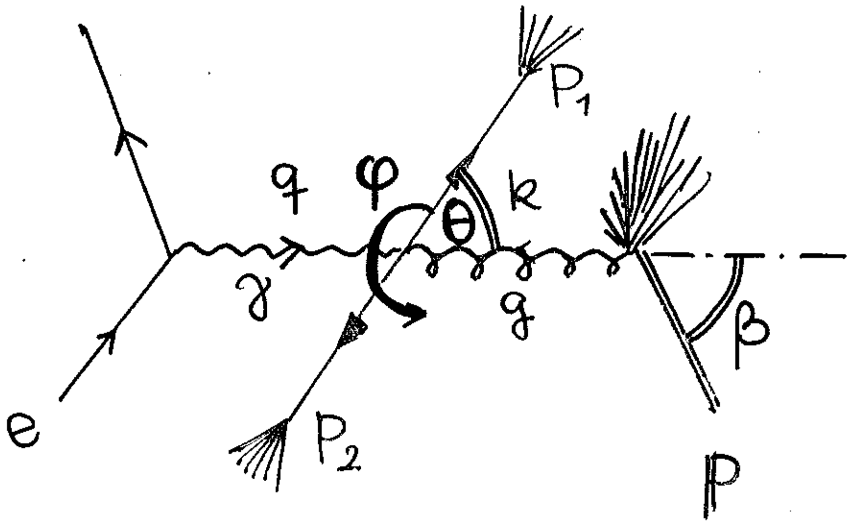
with $k_\perp.q' = k_\perp.P = 0$ and

$$x = \frac{Q^2}{2P.q} \tag{7}$$

$$\xi = \frac{2k.P}{2q'.P} \tag{8}$$

$$\eta = \frac{2k.q'}{2q'.P} \tag{9}$$

$\gamma g$ - CMS




(P carries $p_\perp$)

$\longrightarrow$ DERIVE THE x DEPENDENT COEFFICIENT FUNCTIONS.

EARLIER: CIAFALONI et al. $0^{th}$ MOMENT

$$k = z_3 P + z_4 q' + k_\perp^2$$

$$q = z_1 P + z_2 q' + q_\perp^2$$

ONE INTEGRATION TO BE DONE USING THE $0^{th}$ MOMENT, OTHERWISE NONTRIVIAL IN THIS REF. FRAME.

HIGH ENERGY LIMIT: $z_2 \gg z_4$

$z_3 \gg z_1$

and

$$K^2 = k^2 - \xi\eta\frac{Q^2}{x} \tag{10}$$

with $k_\perp^2 = -k^2$. between $P$ and $k$ is needed. We will assume that

$$\xi \ll \eta \tag{11}$$

which implies

$$\xi \approx 0 \qquad P.k \approx 0 \qquad k_\mu \approx \eta P_\mu + k_{\perp\mu} \qquad K^2 \approx k^2 \tag{12}$$

In the scaling limit, $\xi = 0$ and $k_\perp = 0$, the gluon momentum is

$$\underline{k_\mu = \eta P_\mu} \tag{13}$$

and the relations given below simplify accordingly. The general representation of the particle momenta is thus given by:

photon – gluon cms

$$\begin{aligned}
k &= (K_0, 0, 0, |\vec{k}|) \\
q &= (Q_0, 0, 0, -|\vec{k}|) \\
P &= E_p(1, \sin\beta, 0, \cos\beta) \\
p_1 &= (E_1, q_1\sin\theta\cos\varphi, q_1\sin\theta\sin\varphi, q_1\cos\theta) \\
p_2 &= (E_2, -q_1\sin\theta\cos\varphi, -q_1\sin\theta\sin\varphi, -q_1\cos\theta)
\end{aligned} \tag{14}$$

with

$$\begin{aligned}
K_0 &= \mathcal{E}(\hat{s}, -K^2, -Q^2) \\
Q_0 &= \mathcal{E}(\hat{s}, -Q^2, -K^2) \\
E_1 &= \mathcal{E}(\hat{s}, m_1^2, m_2^2) \\
E_2 &= \mathcal{E}(\hat{s}, m_2^2, m_1^2) \\
|\vec{k}| &= \mathcal{P}(\hat{s}, -Q^2, -K^2) \\
q_1 &= \mathcal{P}(\hat{s}, m_1^2, m_2^2) \\
\cos\theta &= \frac{2K_0E_1 + K^2 - m_1^2 + \hat{t}}{2|\vec{k}|q_1}
\end{aligned}$$

$$
\begin{aligned}
E_p &= \mathcal{E}(\hat{s}, 0, t_{qP'}) = \frac{Q^2}{2x\sqrt{\hat{s}}} \\
\cos\beta &= \frac{K_0}{|\vec{k}|} = \frac{1-\zeta/2}{\sqrt{1-x\zeta}} \qquad (15)
\end{aligned}
$$

where

$$
\begin{aligned}
\zeta &= \frac{4K^2 x}{Q^2} \qquad (16) \\
\mathcal{E}(a,b,c) &= \frac{a+b-c}{2\sqrt{a}} \\
\mathcal{P}(a,b,c) &= \sqrt{\frac{\lambda(a,b,c)}{4a}} \qquad (17)
\end{aligned}
$$

$$
\lambda(a,b,c) = (a-b-c)^2 - 4bc \qquad (18)
$$

$$
t_{qP'} = (q-P')^2 = \hat{s} - Q^2/x \qquad (19)
$$

$\varphi$ is defined by

$$
\cos\varphi = \left.\frac{(q \times P).(q \times p_1)}{|q \times P||q \times p_1|}\right|_{p_1=-p_2} \qquad (20)
$$

In the limit $m_{1,2} \to 0$ the 3–particle phase space is described by

$$
\begin{aligned}
dPS^{(3)} &= \frac{1}{128\pi^3}\frac{d\varphi_P}{2\pi}\frac{d\hat{s}d\hat{t}dK^2}{\lambda^{1/2}(\hat{s},-K^2,-Q^2)\lambda^{1/2}(s_\gamma,0,-Q^2)}\frac{d\varphi}{2\pi} \\
&\times\ \Theta\{-\mathcal{G}(s_\gamma,-K^2,\hat{s},0,-Q^2,0)\} \\
&\times\ \Theta\{-\mathcal{G}(\hat{s},\hat{t},0,-K^2,-Q^2,0)\}\Theta\{\hat{s}\} \qquad (21)
\end{aligned}
$$

with $\mathcal{G}(x,y,z,u,v,w)$ the Caley determinants.

$$
\int dPS^{(3)} = \frac{1}{128\pi^3}\int_{\eta_{min}}^{\eta_{max}} d\eta \int_{K^2_{min}(\eta)}^{K^2_{max}(\eta)} dK^2 \int_0^{2\pi}\frac{d\varphi_P}{2\pi}\frac{1}{2}\int_{-1}^{1} d\cos\theta \int_0^{2\pi}\frac{d\varphi}{2\pi} \qquad (22)
$$

The variables in (22) are bounded by:

$$\begin{aligned}
K^2_{min} &= 0 \\
K^2_{max} &= Q^2\frac{\eta - x}{x} \\
\eta_{min} &= x \\
\eta_{max} &= 1
\end{aligned} \tag{23}$$

# 3 Factorization

- The $k_\perp$ integration starts at $K^2 = 0$.
  However, a physical definition of $\partial xG(x, K^2)/\partial K^2$
  is ONLY possible for $K^2 \geq Q_0^2$ ! ↑VIRTUALITY.

- **The naive $k_\perp$ factorization fails.** (AKMS)

$$F_i(x, Q^2) = \int_0^1 \int_0^1 dx_1 dx_2 \delta(x - x_1 x_2) \int d^2\ \boldsymbol{k} f_i(x_1, K^2/Q^2) \frac{\partial x_2 G(x_2, K^2)}{\partial K^2} \tag{24}$$

(IF THE $k_\perp$ DISTRIBUTION OF THE GLUON IS INFRA-RED FINITE THERE IS NO PROBLEM!)

## One observes:

**for: $Q_0^2 \ll Q^2$**

**one has for $K^2 \leq Q_0^2$: $f_i^{qG}(\frac{K^2}{Q^2}, z) \Rightarrow f_i^{qG}(\frac{K^2}{Q^2} \to 0, z)$**

## The $k_\perp$ Factorization Relation is thus:

PERTURBATIVE RANGE!

$$\begin{aligned}
F_L^g(x, Q^2) &= \int_x^1 \frac{dz}{z} f_L^{g,0}(z) \frac{x}{z} G(z, Q_0^2) \\
&+ \int_x^1 \frac{dz}{z} \int_{Q_0^2}^{K_{max}^2} dK^2 f_L^g(z, \frac{K^2}{Q^2}) \frac{x}{z} \frac{\partial G(x/z, K^2)}{\partial K^2} \theta(K_{max}^2 - Q_0^2) \\
F_2^g(x, Q^2) &= F_2^{g,coll}(x, Q^2) \\
&+ \int_x^1 \frac{dz}{z} \int_{Q_0^2}^{K_{max}^2} dK^2 \left\{ f_2^g(z, \frac{K^2}{Q^2}) - f_2^g(z, \frac{\Lambda^2}{Q^2}) \theta(Q^2 - K^2) \right\} \\
&\times \frac{x}{z} \frac{\partial G(x/z, K^2)}{\partial K^2} \theta(K_{max}^2 - Q_0^2)
\end{aligned} \tag{25}$$

### The Scale of $\alpha_s$:

For the present leading order calculation we have choosen: $Q^2$.
Other characteristic scales would do as well.

# 4 The Structure Functions in the $k_\perp$ Factorization Scheme in $\mathcal{O}(\alpha_s)$

$$\frac{d^2\sigma}{dQ^2dy} = 2\pi\alpha^2 \frac{Ms}{(s-M^2)^2}\frac{1}{Q^4}L_{\mu\nu}W^{\mu\nu} \tag{26}$$

$$\begin{aligned} L_{\mu\nu} &= 2\left[l_\mu l'_\nu + l'_\mu l_\nu - g_{\mu\nu} l.l'\right] \\ W_{\mu\nu} &= \frac{1}{4\pi}\sum_n \langle P|J_\mu^{em\dagger}(0)|n\rangle\langle n|J_\nu^{em}(0)|P\rangle(2\pi)^4\delta^{(4)}(P+q-p_n) \end{aligned} \tag{27}$$

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)W_1(x,Q^2) + \frac{1}{M^2}\left[\left(P_\mu - \frac{P.q}{q^2}q_\mu\right)\left(P_\nu - \frac{P.q}{q^2}q_\nu\right)\right]W_2(x,Q^2) \tag{28}$$

$$\begin{aligned} F_2(x,Q^2) &= xT^1_{\mu\nu}W^{\mu\nu} = x\left(-g_{\mu\nu} + \frac{12x^2}{Q^2}P_\mu P_\nu\right)W^{\mu\nu} \\ F_L(x,Q^2) &= xT^2_{\mu\nu}W^{\mu\nu} = \frac{8x^3}{Q^2}P_\mu P_\nu W^{\mu\nu} \end{aligned} \tag{29}$$

$$-g^{\mu\nu}\widehat{W}_{\mu\nu} = 32\pi\alpha_s e_q^2\left\{\frac{(p_1.P)^2 + (p_2.P)^2}{\hat{t}\hat{u}} - \frac{Q^2}{K^2}\left[\frac{p_1.P}{\hat{t}} - \frac{p_2.P}{\hat{u}}\right]^2\right\} \tag{30}$$

$$P^\mu P^\nu \widehat{W}_{\mu\nu} = 64\pi\alpha_s e_q^2 \frac{1}{K^2}\left\{-2\frac{(p_1.P)^2(p_2.P)^2}{\hat{t}\hat{u}} + \frac{(p_1.P)^3(p_2.P)}{\hat{t}^2} + \frac{(p_1.P)(p_2.P)^3}{\hat{u}^2}\right\} \tag{31}$$

(AGREES WITH CCH)

$$\begin{aligned} f_L^g(z,\frac{K^2}{Q^2}) &= \frac{2}{\pi}\alpha_s(Q^2)\sum_{q=1}^{N_f/2}\left(e_{q_u}^2 + e_{q_d}^2\right)\left\{\frac{1}{64z}\left(\frac{Q^2}{K^2}\right)^2 G_{1L}(\omega,\beta)\right. \\ &+ \left. z\frac{1}{16}\frac{Q^2}{K^2}\left[G_{2L}(\omega,\beta) + G_{3L}(\omega,\beta)\log\left|\frac{1-\omega}{1+\omega}\right|\right]\right\} \end{aligned} \tag{32}$$

with

$$\omega = \sqrt{1 - 4K^2z/Q^2} \tag{33}$$

$$f_L^g(z,\frac{K^2}{Q^2}\to 0) = \frac{2}{\pi}\alpha_s(Q^2)\sum_{q=1}^{N_f/2}(e_{q_u}^2+e_{q_d}^2)\,\underline{\underline{z^2(1-z)}}$$

$$G_{1L}(\omega,\beta) = \sum_{k=0}^{4} \frac{1}{\omega^k} g_{1k}^{(L)}(\beta) \tag{34}$$

$$G_{2L}(\omega,\beta) = \sum_{k=2}^{4} \frac{1}{\omega^k} g_{2k}^{(L)}(\beta) \tag{35}$$

$$G_{3L}(\omega,\beta) = \sum_{k=1}^{5} \frac{1}{\omega^k} g_{3k}^{(L)}(\beta) \tag{36}$$

and

$$\boxed{\begin{aligned}
g_{10}^{(L)}(\beta) &= \frac{5}{2} + 3\cos^2\beta - \frac{3}{2}\cos^4\beta \\
g_{11}^{(L)}(\beta) &= 4\cos\beta - 12\cos^3\beta \\
g_{12}^{(L)}(\beta) &= 3 - 18\cos^2\beta + 15\cos^4\beta \\
g_{13}^{(L)}(\beta) &= -12\cos\beta + 20\cos^3\beta \\
g_{14}^{(L)}(\beta) &= -\frac{3}{2} + 15\cos^2\beta - \frac{35}{2}\cos^4\beta \\
g_{22}^{(L)}(\beta) &= 4 - 12\cos^2\beta \\
g_{23}^{(L)}(\beta) &= -24\cos\beta + 40\cos^3\beta \\
g_{24}^{(L)}(\beta) &= -3 + 30\cos^2\beta - 35\cos^4\beta \\
g_{31}^{(L)}(\beta) &= \frac{3}{4} + \frac{1}{2}\cos^2\beta + \frac{3}{4}\cos^4\beta \\
g_{32}^{(L)}(\beta) &= 2\cos\beta - 6\cos^3\beta \\
g_{33}^{(L)}(\beta) &= \frac{7}{2} - 15\cos^2\beta + \frac{15}{2}\cos^4\beta \\
g_{34}^{(L)}(\beta) &= -18\cos\beta + 30\cos^3\beta \\
g_{35}^{(L)}(\beta) &= -\frac{9}{4} + \frac{45}{2}\cos^2\beta - \frac{105}{4}\cos^4\beta
\end{aligned}} \tag{37}$$

$$\cos\beta = \frac{1 - \frac{2K^2 z}{Q^2}}{\sqrt{1 - \frac{4K^2 z^2}{Q^2}}} \qquad \omega = \sqrt{1 - \frac{4K^2 z}{Q^2}}$$

# 5 Numerical Results

- CONSIDER $F_L(x, Q^2)$ AS AN EXAMPLE.

- $\dfrac{\partial\, x\, G(x, K^2)}{\partial \log K^2}$ :

  DO'
  D$^{-}$'
  MRSA
  CTEQ2M
  GRV HO

  } SMOOTHED BY R. ROBERTS.
  NEW FIT!

- X DEPENDENCE
- $Q^2$ DEPENDENCE

  OF THE $K_\perp$ EFFECTS.

- $Q_0^2$ DEPENDENCE (?)

$\longrightarrow$ KIN. EFFECT TAKING INTO ACCOUNT THE $K_\perp^2$ LIMITS CORRECTLY.

## $K_\perp$ DEPENDENT GLUON DISTRIBUTION

HOW TO DESCRIBE IT ?

$K_\perp$ DEPENDENCE $\longleftarrow$ SHOUD DEPEND ON THE DYNAMICS ACCOUNTED FOR;

AP

BFKL

- UNFOLD $\mathcal{F}_g(x, K^2, Q^2, \mu^2)$ EXPERIMENTALLY FROM 2 JET CROSS SECTIONS, KNOWING $\hat{\sigma}(x, Q^2, K_\perp^2)$ !

- 'WHICH' GLUON DISTRIBUTIONS ARE COVERED ?

$\longrightarrow$ SINGLE GLUON DISTRIBUTIONS FACTORIZED BY:

$$\sum_\lambda \varepsilon_\mu^*(\lambda)\, \varepsilon_\nu(\lambda) = \frac{P_\mu P_\nu}{k^2} \, .$$

$\longrightarrow$ INCLUDES BOTH THE CASES OF THE LIPATOV AND THE AP DYNAMICS.

## A. GLAP EQUATIONS:

(moments)

$$\begin{pmatrix} \dfrac{\partial \Sigma_n}{\partial K^2} \\ \dfrac{\partial G_n}{\partial K^2} \end{pmatrix} = \frac{\alpha_S(K^2)}{2\pi\, K^2} \begin{pmatrix} P_{qq}^{(n)} & P_{qG}^{(n)} \\ P_{Gq}^{(n)} & P_{GG}^{(n)} \end{pmatrix} \begin{pmatrix} \Sigma_n \\ G_n \end{pmatrix}$$

GLUONS:

$$\frac{\partial G_n}{\partial K^2} \approx \underline{\frac{\alpha_s}{2\pi}\frac{2C_G}{K^2}} \left[\underline{\frac{1}{n-1}} + \frac{1}{2C_G}\left(P_{GG}^{(n)} - \frac{2C_G}{n-1}\right)\right] \underline{G_n(K^2)}$$

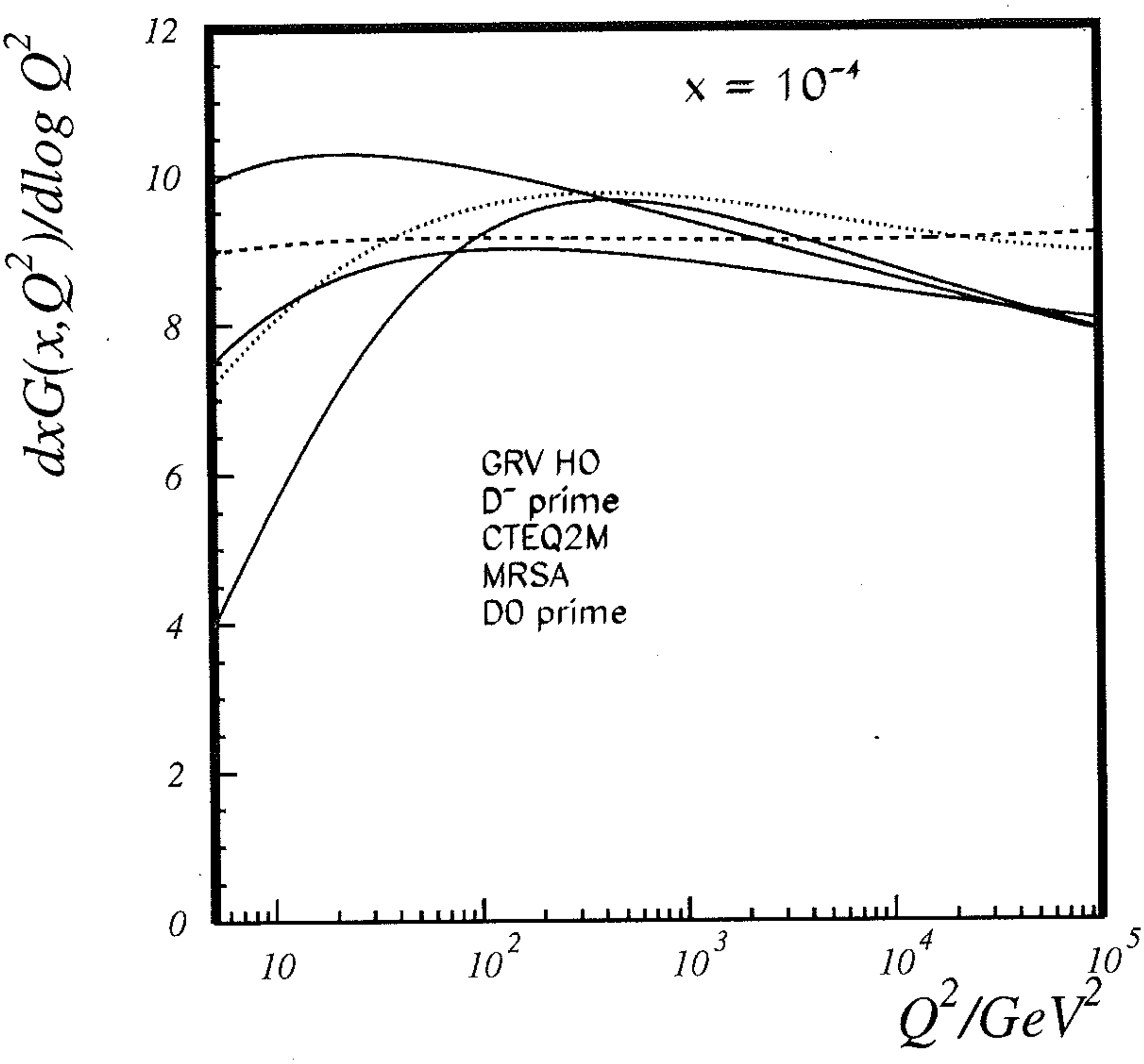
$$+ \underline{\frac{\alpha_s N_f}{2\pi}\frac{1}{k^2}}\left(\underline{\frac{2}{n-1}} + \cdots\right)\underline{\Sigma_n(K^2)}$$

$$\frac{\partial G_n}{\partial K^2} \approx \frac{\bar{\alpha}_s}{K^2}\frac{1}{n-1} G_n \qquad \text{(DOUBLE LOG)} \quad \bar{\alpha}_s = 3\alpha_s/\pi.$$

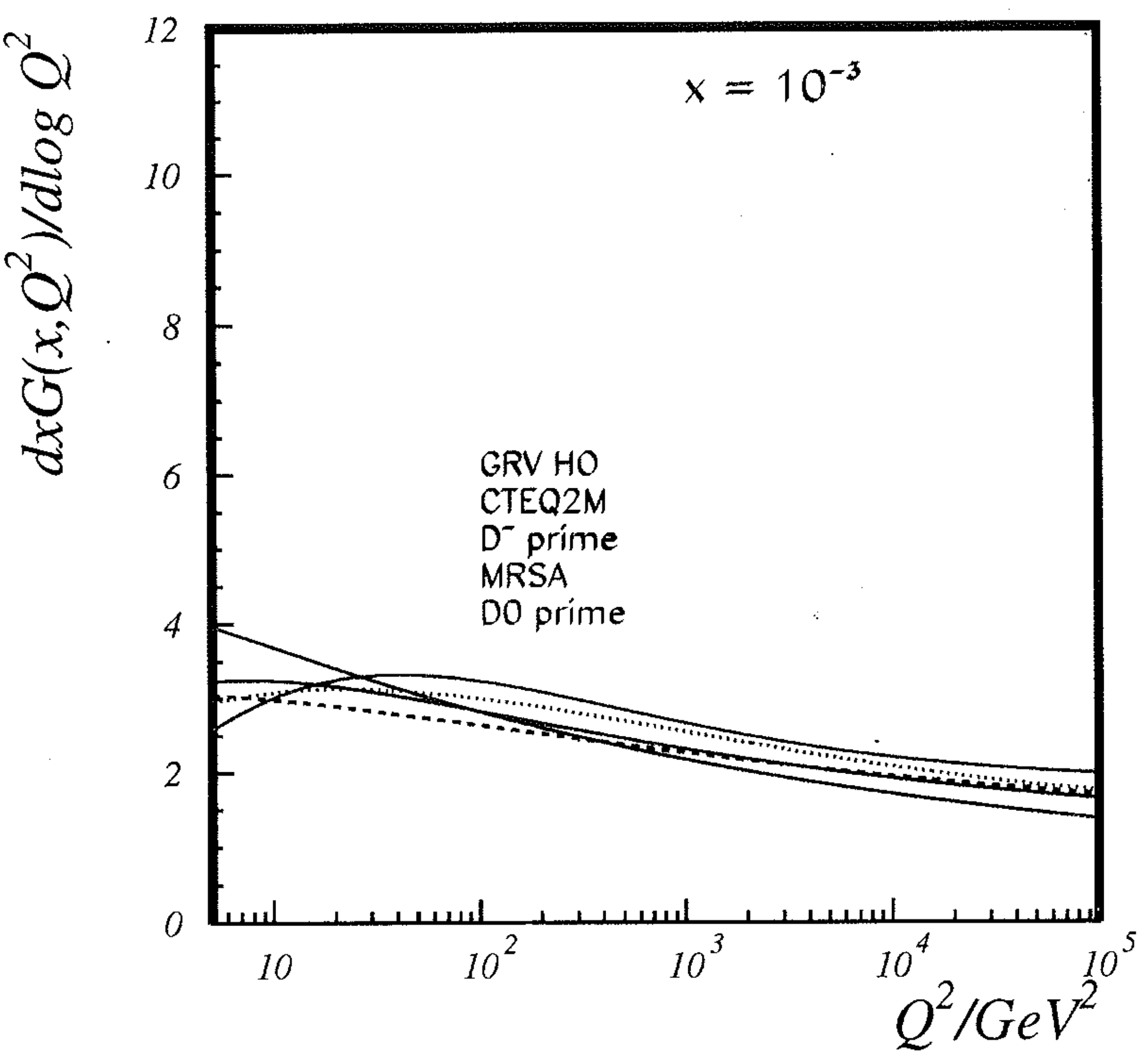
(ONE EXPECTS $\approx$ CONST. $\overset{\text{LOG.}}{\phantom{x}}$SLOPES FOR $G_n$ VS $Q^2$).

AT $x =$ const

Figs.



$dxG(x,Q^2)/dlog\ Q^2$
12
10
8
6
4
2
0
$x = 10^{-4}$
GRV HO
D⁻ prime
CTEQ2M
MRSA
D0 prime
10
$10^2$
$10^3$
$10^4$
$10^5$
$Q^2/GeV^2$



x = 10^-3
GRV HO
CTEQ2M
D^- prime
MRSA
D0 prime
dxG(x,Q^2)/dlog Q^2
Q^2/GeV^2
0
2
4
6
8
10
12
10
10^2
10^3
10^4
10^5

## GLAP DYNAMICS:

$$\mathcal{F}_g(x, K^2) = \frac{\partial \mathbb{G}(x, K^2)}{\partial K^2} \qquad \text{WITH}$$

$$\int_0^{K^2} dK'^2 \, \frac{\partial \mathbb{G}(x, K'^2)}{\partial K^2} \underset{\mathrm{Df}}{=} \mathbb{G}(x, K^2)$$

FOR $K^2$ IN THE PERTURBATIVE RANGE.

$\longrightarrow$ USE

$$F_L(x, Q^2) = f_L^{g,0}(x) \otimes \mathbb{G}(x, Q_0^2) + \int_x^1 \frac{dz}{z} \int_{Q_0^2}^{K^2_{max}} dK^2 \, f_L^g\left(z, \frac{K^2}{Q^2}\right) \frac{x}{z} \, \frac{\partial \mathbb{G}(x/z, K^2)}{\partial K^2} \, \Theta(K^2_{max} - Q_0^2).$$

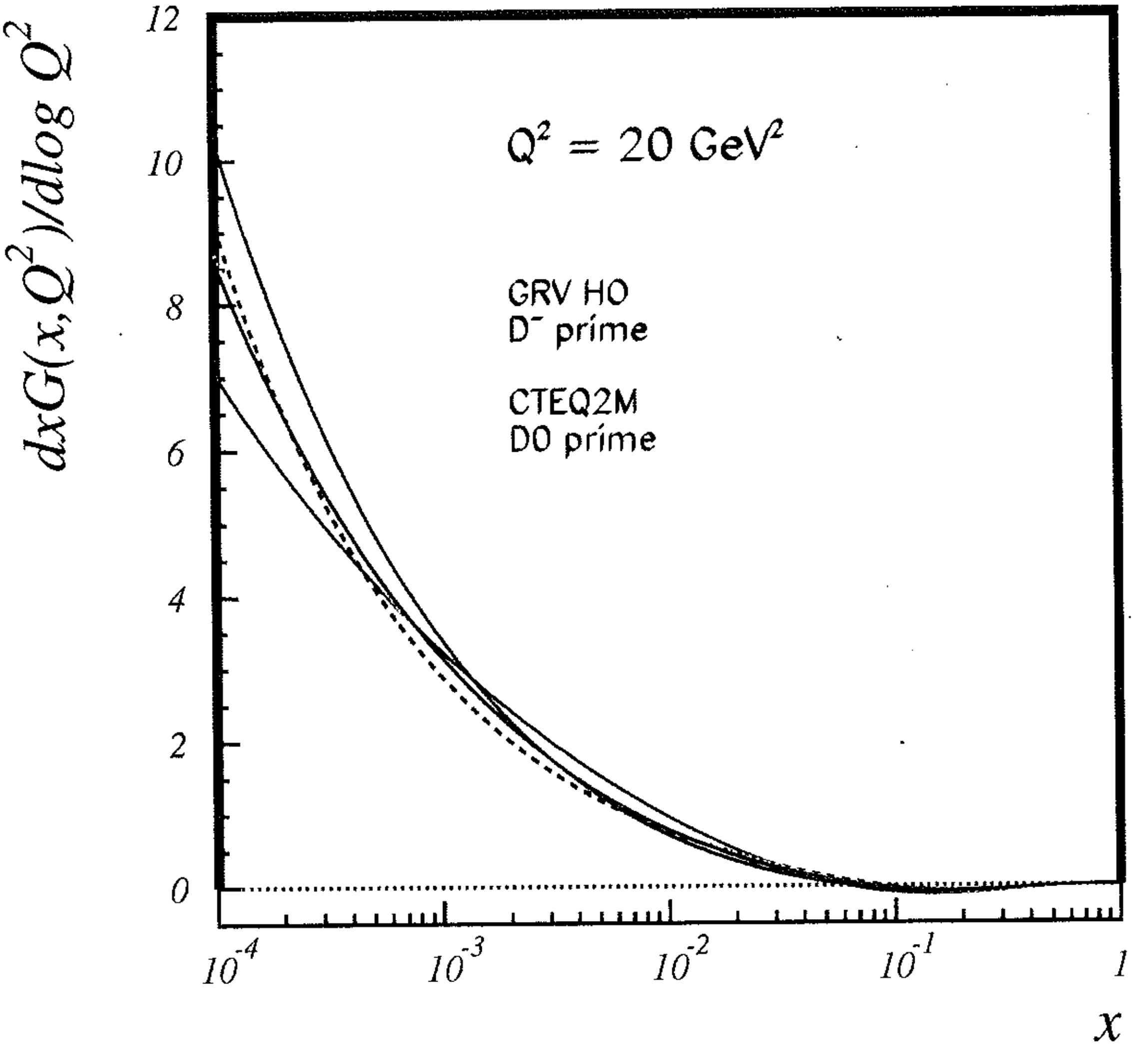
---

HOWEVER: GOING TO LARGE $x \gtrsim O(.1)$
$\partial \mathbb{G}(x, k^2) / \partial K^2$ BECOMES NEGATIVE
(FORTUNATELY IT IS ALSO SMALL!)

[ Fig. ]

$\longleftrightarrow$ $F_L$ STILL IS POSITIVE. (AKMS $\longrightarrow$ OTHER APPROACH, CUT AT $Q_0^2$ (WOULD) OBTAIN NEGATIVE VALUES).

$\Longrightarrow$ $\mathcal{F}_g$ HAS TO BE DIFFERENT FROM $\dfrac{\partial \mathbb{G}}{\partial K^2}$.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$10^{-4}$
$10^{-3}$
$10^{-2}$
$10^{-1}$
1
x

## B. BFKL EQUATION:

CONSIDER AGAIN:

$$\frac{\partial G_n^{(0)}}{\partial K^2} = \frac{\bar{\alpha}_s(\mu)}{K^2}\frac{1}{n-1}\, G_n(\mu^2) \equiv F_g^{(0)}(K^2,\mu)$$

FOR $\bar{\alpha}_s \ll 1$ THIS IS THE $1^{ST}$ EXPANSION TERM OF

$$\mathbb{F}_g(K^2,\mu) = \gamma(\bar{\alpha}_s)\,\frac{\bar{\alpha}_s}{k^2}\left(\frac{k^2}{\mu^2}\right)^{\gamma(\bar{\alpha}_s)} \tilde{G}(\mu^2)$$

(COLLINS, ELLIS)

$$\boxed{\begin{aligned} &n-1-\bar{\alpha}_s\, X(\gamma(\bar{\alpha}_s)) = 0 \\ &X(\beta) = 2\psi(1)-\psi(\beta)-\psi(1-\beta)\end{aligned}}$$

IMPLICIT EQUATION FOR THE BFLK 'ANOMALOUS' DIMENSION: MORE PRECISELY ONE SHOULD CALL THIS BFKL RESUMMATION OF THE $\left(\frac{\bar{\alpha}_s}{N}\right)^n$ TERMS IN THE MATRIX ELEMENT $\gamma_{GG}$ OF THE TWIST-2 ANOMALOUS DIMENSION.

IN THIS WAY INFORMATION OF THE BFKL EQUATION IS INCORPORATED INTO $\mathbb{F}_g(K^2,\mu)$.

REPRESENTATION:

$$\mathbb{F}_g(K^2,\mu) = \gamma(\bar{\alpha}_s)\,\frac{1}{k^2}\left(\frac{k^2}{\mu^2}\right)^{\gamma(\alpha_s)} \tilde{G}(\mu^2)$$

$$\tilde{G}_j(\mu^2) = \left[\ln\left(\frac{\mu}{\Lambda}\right)/\ln\left(\frac{\mu_0}{\Lambda}\right)\right]^{d_I(n)} \exp\left[\frac{1}{b(j-1)}\int_{\gamma_c^{N_0}}^{\gamma_c^{N}} d\gamma\,\gamma\,\frac{dX(\gamma)}{d\gamma}\right] G_j(\mu$$

$$d_I = d - \frac{1}{b(j-1)}$$

MORE GENERALLY ONE HAS TO ACCOUNT FOR BFKL & DGLAP EFFECTS IN A COMBINED EVOLUTION EQU.

- G. MARCHESINI, 'QCD AT 200 TEV', (PLENUM, NY, 1992), p. 183

$$\tilde{F}(x, Q_t, \bar{Q}) = \delta(1-x)\,\delta(Q_t)\,\Delta_s(\bar{Q}, Q_s)$$
$$+ \int \Delta_s(\bar{Q}, zq)\,\theta(\bar{Q} - zq)\,\frac{dz}{z}\frac{d^2q}{q^2}\,\theta\left(1 - z - \frac{Q_0}{q}\right)$$
$$\cdot \tilde{P}_g(z, q, Q_t)\,\tilde{F}\left(\frac{x}{z}, Q_t - (1-z)q, q\right).$$

$$\tilde{P}_g(z, q, Q_t) = \left[\frac{\bar{\alpha}_s}{1-z} + \frac{\bar{\alpha}_s}{z}\Delta_{ns}(z, q, Q_t)\right]$$

$$\Delta_{ns}(z, q, Q_t) = \exp\left[-\bar{\alpha}_s \int_z^1 \frac{dz}{z} \int_{(zq)^2}^{Q_t^2} \frac{dq^2}{q^2}\right]$$

$$\Delta_s(Q, zq) = \exp\left[-\int_{(zq)^2}^{Q^2} \frac{d\bar{q}^2}{\bar{q}^2} \int_0^{1-Q_0/\bar{q}} dz\,\frac{\bar{\alpha}_s}{1-z}\right]$$

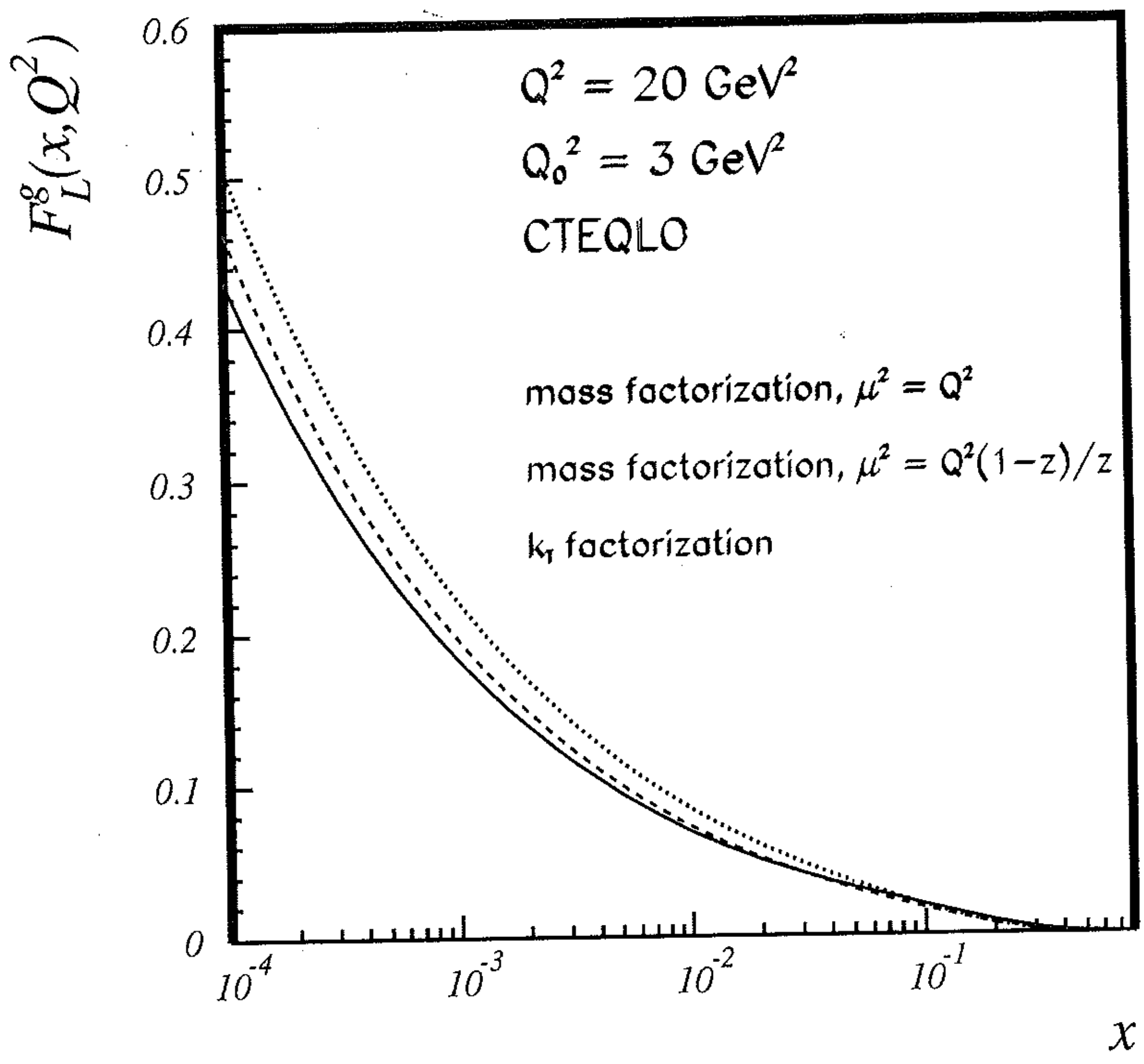
$\longrightarrow$ $\tilde{F}$ DESCRIBES AN EVOLUTION KERNEL;
(TO BE CONVOLUTED WITH AN INPUT DISTRIBUTION.)

(ZEE, WILCZEK, TREIMAN 1974.)

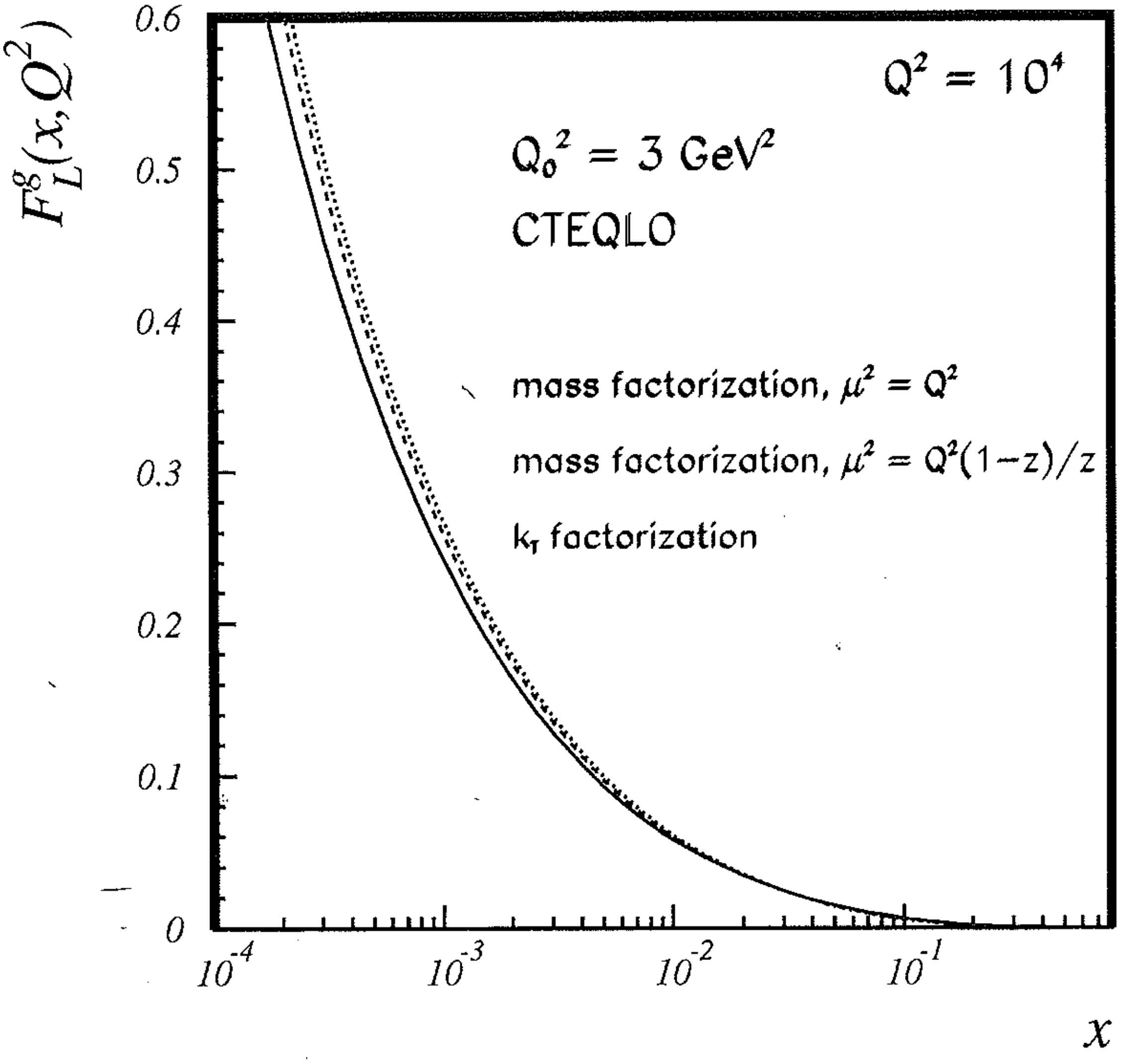
$$F_L^{g,0}(x,Q^2) = f_L^{g,0}(x,Q^2) \otimes G(x,Q^2)$$

$$F_L^{g,\tilde{0}}(x,Q^2) = f_L^{g,0}(x,Q^2) \otimes G_z(x,K^2_{max})$$

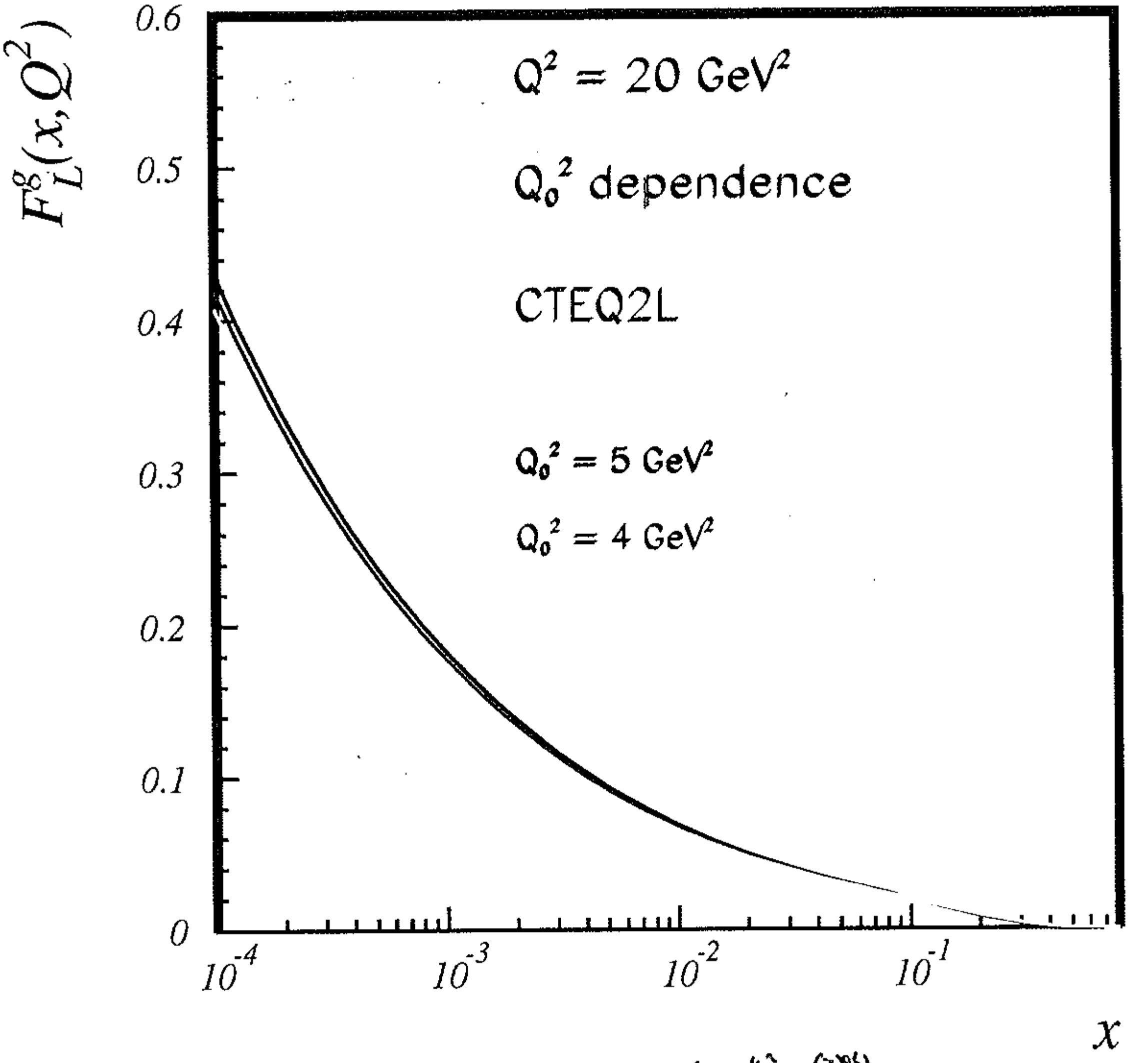
$$F_L^{g,k_\perp}(x,Q^2)_{|Q_0^2} = f_L^{g,0}(x) \otimes G(x,Q_0^2) + \int_x^1 \frac{dz}{z} \int_{Q_0^2}^{K^2_{max}} dK^2 \, f_L^g\left(z,\frac{K^2}{Q^2}\right) \frac{x}{z} \frac{\partial G(x/z,K^2)}{\partial K^2} \cdot \theta(K^2_{max}-Q^2).$$



$F_L^g(x,Q^2)$
0.6
0.5
0.4
0.3
0.2
0.1
0
$Q^2 = 20\ GeV^2$
$Q_0^2 = 3\ GeV^2$
CTEQLO
mass factorization, $\mu^2 = Q^2$
mass factorization, $\mu^2 = Q^2(1-z)/z$
$k_T$ factorization
$10^{-4}$
$10^{-3}$
$10^{-2}$
$10^{-1}$
$x$



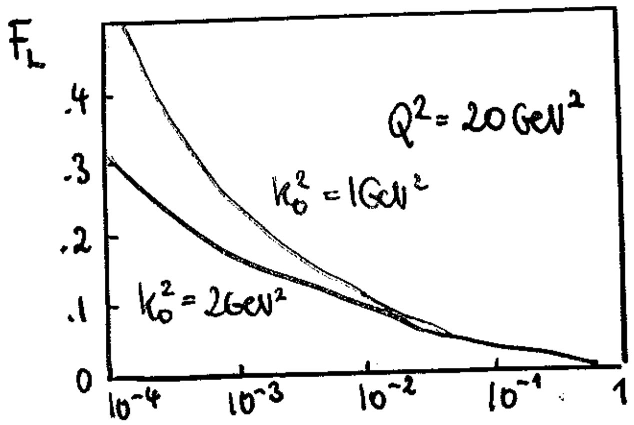
$F_L^g(x,Q^2)$
0.6
0.5
0.4
0.3
0.2
0.1
0
$10^{-4}$
$10^{-3}$
$10^{-2}$
$10^{-1}$
$x$
$Q^2 = 10^4$
$Q_0^2 = 3$ GeV$^2$
CTEQLO
mass factorization, $\mu^2 = Q^2$
mass factorization, $\mu^2 = Q^2(1-z)/z$
$k_T$ factorization




$O(2\%)$ !

$$F_L^g(x,Q^2) = \int_x^1 \frac{dz}{z} f_L^{g,0}(z)\, \frac{x}{z}\, G(z,Q_0^2) + \int_x^1 \frac{dz}{z} \int_{Q_0^2}^{k^2_{max}(z,Q^2)} dk^2\, f_L^g\left(z,\frac{k^2}{Q^2}\right) \frac{x}{z}\, \frac{\partial G(x/z,k^2)}{\partial k^2}\, \theta(k^2_{max}-Q_0^2)$$

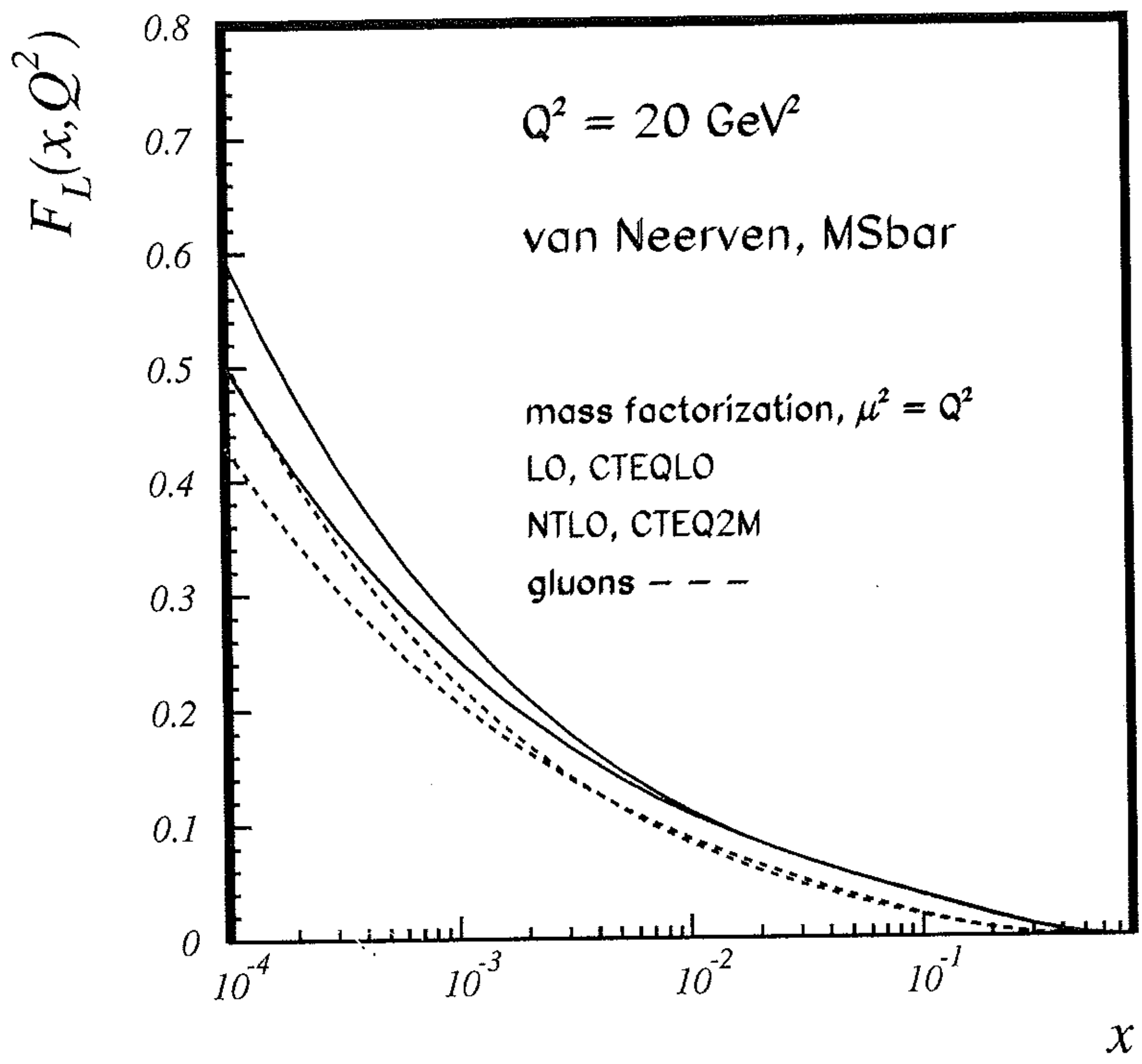
STABLE RESULT UNDER $Q_0^2$ VARIATION !




AKMS '93

DEPENDENCE ON THE INFRARED CUTOFF $k_0$ :

- JOINT EFFEKT OF THE SPECIAL TREATMENT OF THE BFKL EVOL. & FACT. FORMULA.




E. ZIJLSTRA, W. VAN NEERVEN

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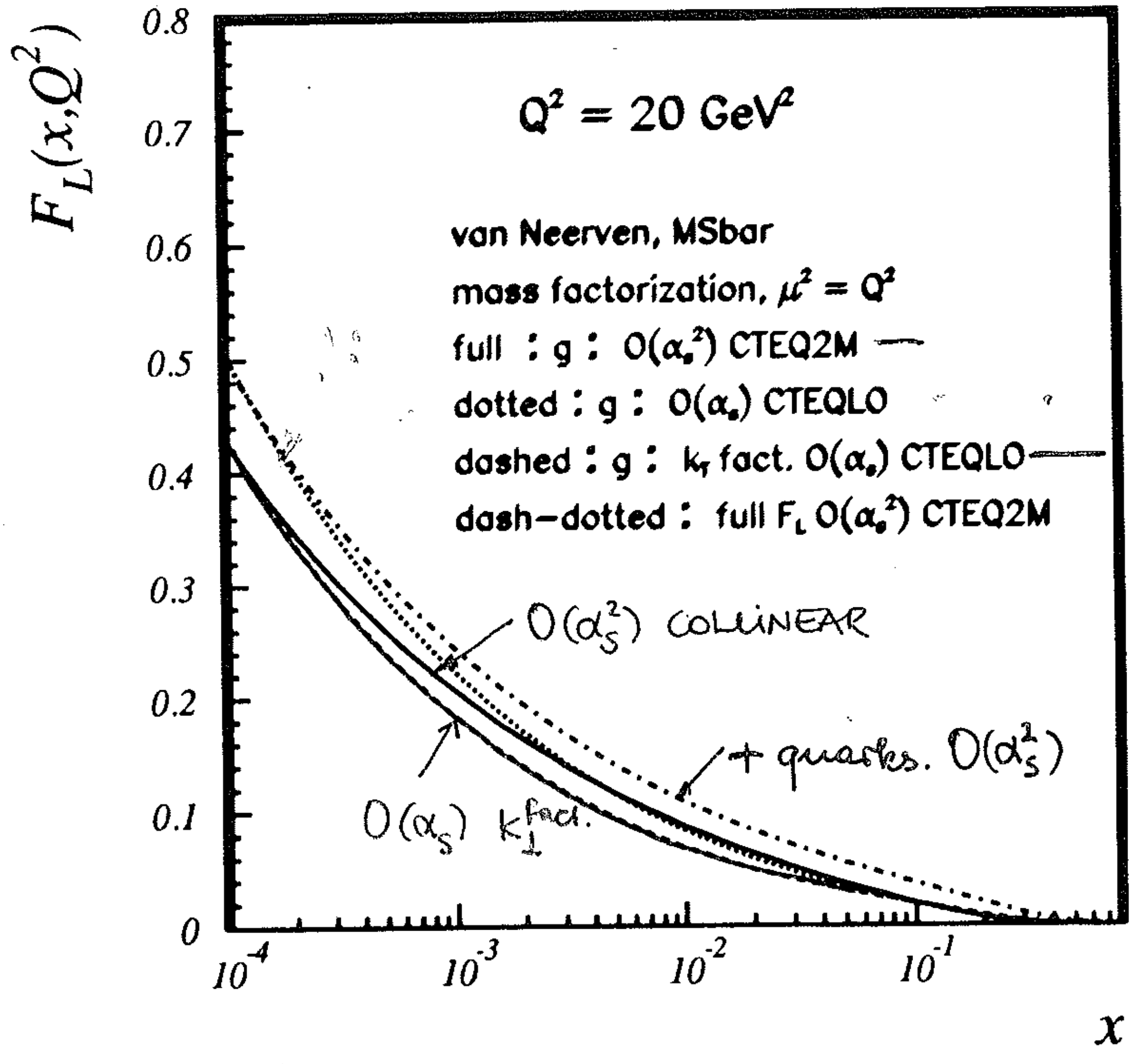
(+ NUMERICAL UPDATE!)



$F_L(x,Q^2)$
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$Q^2 = 20\ GeV^2$
van Neerven, MSbar
mass factorization, $\mu^2 = Q^2$
full : g : $O(\alpha_s^2)$ CTEQ2M
dotted : g : $O(\alpha_s)$ CTEQLO
dashed : g : $k_T$ fact. $O(\alpha_s)$ CTEQLO
dash-dotted : full $F_L$ $O(\alpha_s^2)$ CTEQ2M
$O(\alpha_s^2)$ COLLINEAR
+ quarks. $O(\alpha_s^2)$
$O(\alpha_s)$ $k_\perp^{fact.}$
$10^{-4}$
$10^{-3}$
$10^{-2}$
$10^{-1}$
$x$

# 6 Conclusions

1. A derivation of $k_\perp$ factorization which is consistent with perturbative QCD has been given.
2. The gluon contribution to the structure functions has been calculated using $k_\perp$ factorization *without* approximations of the Mellin convolution and the $x$ dependence of the coefficient functions contrasting earlier investigations. The obtained contributions to the structure functions are *positive* in the whole $x$ range.
3. The derived coefficient functions approach those found using mass factorization in the limit $K^2 \to 0$.
4. The numerical result obtained for $F_i(x, Q^2)$ in $k_\perp$ factorization for suitably 'large' values of $x$ approaches the result obtained ignoring the $k_\perp$ dependence of the coefficient functions. (This is an expectation in the parton model.)
5. The numerical results behave *very* stable against the choice of the scale $Q_0^2$. There is essentially no change for *all* $x$, as long as the condition $Q^2 \gg Q_0^2$ is met. (As we do not provide a thorough inclusion of higher twist effects this is a natural choice.)
6. There is *no fixed onset* (e.g. $x \sim 10^{-2}$) of small $x$ effects observed. Deviations from the $k_\perp \to 0$ results become smaller with rising $Q^2$ at $x = const.$
7. The $k_\perp$ dependence of the coefficient functions and gluon distribution results into SMALLER VALUES of the structure functions in the small $x$ range. A similar behaviour has been observed recently by VAN NEERVEN ET AL. in a higher order calculation using mass factorization.