$\mathcal{O}(\alpha^2 L^2)$ Radiative Corrections to Deep Inelastic ep Scattering for Different Kinematical Variables

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DESY – Zeuthen, FRG

1. The Different Variables
2. The Corrections up to $\mathcal{O}(\alpha^2 L^2)$
3. Numerical Results
4. Conclusions

1. The Different Variables

Goal:

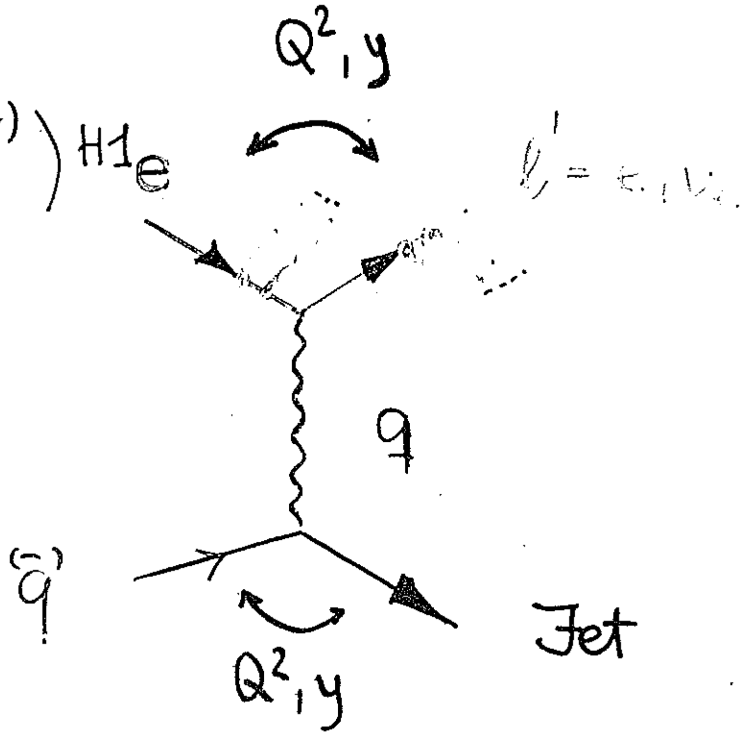
Measurement of a Born Cross section: $2 \rightarrow 2$ Reaction

↪ Integrating over the DOF of the radiated Photon(s).

↪ Different Correction Functions for Different Variables are obtained !

$\delta^{NC,CC}(x,y)$

- Double Angle Method θ_e, θ_J } ZEUS
- θ_e & y_J
- Jet Measurement: NC
- Jet Measurement: CC
- Mixed Variables (Q_e^2, y_J)) H1e
- (Lepton Measurement)



2. The Corrections up to $\mathcal{O}(\alpha^2)$

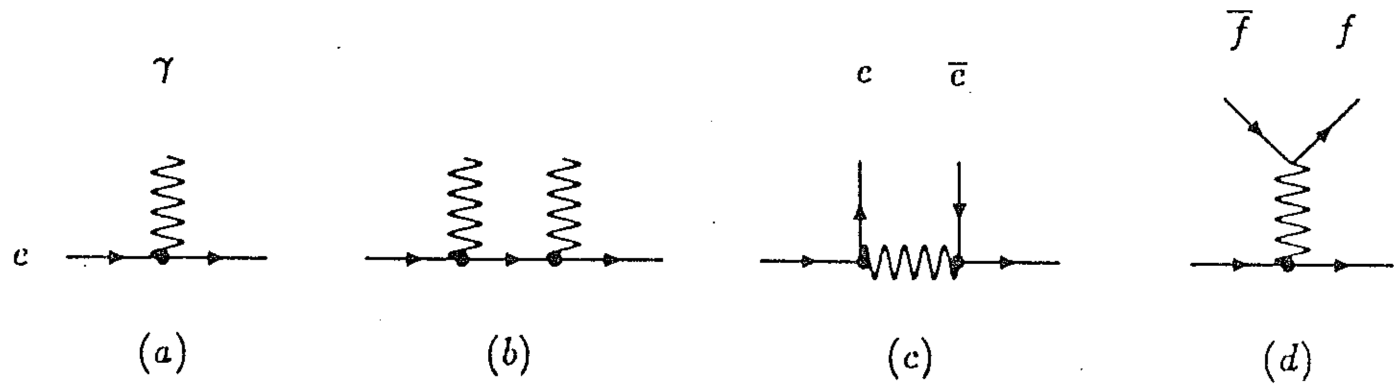
Contributions:

1. Bremsstrahlung: Diagrams a,b
2. Electron Pair Production: Diagram c
3. Fermion Pair Production: Diagram d , $f = e, \mu, \tau, u, d, s, c, b$

The Radiator-Method is applied.

Meaning of the bullet: Collinear Bremsstrahlung contribution
including
soft & virtual corrections.

An individual consideration of initial and final state bremsstrahlung is possible.



$$\frac{d^2\sigma^{(2)}}{dxdy} = \frac{d^2\sigma^{(0)}}{dxdy} + \frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right)\int_0^1 P_{ee}^{(1)}(z)\left\{\theta(z-z_0)\mathcal{J}(x,y,z)\frac{d^2\sigma^{(0)}}{dxdy}\Big|_{x=\hat{x},y=\hat{y},s=\hat{s}} - \frac{d^2\sigma^{(0)}}{dxdy}\right\}$$
$$+ \frac{1}{2}\left[\frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right)\right]^2\int_0^1 P_{ee}^{(2,1)}(z)\left\{\theta(z-z_0)\mathcal{J}(x,y,z)\frac{d^2\sigma^{(0)}}{dxdy}\Big|_{x=\hat{x},y=\hat{y},s=\hat{s}} - \frac{d^2\sigma^{(0)}}{dxdy}\right\}$$
$$+\left(\frac{\alpha}{2\pi}\right)^2\int_{z_0}^1\left\{\ln^2\left(\frac{Q^2}{m_e^2}\right)P_{ee}^{(2,2)}(z) + \sum_{f=l,q}\ln^2\left(\frac{Q^2}{m_f^2}\right)P_{ee,f}^{(2,3)}(z)\right\}\mathcal{J}(x,y,z)\frac{d^2\sigma^{(0)}}{dxdy}\Big|_{x=\hat{x},y=\hat{y},s=\hat{s}}$$

$$\mathcal{J}(x,y,z) = \begin{vmatrix} \partial\hat{x}/\partial x & \partial\hat{y}/\partial x \\ \partial\hat{x}/\partial y & \partial\hat{y}/\partial y \end{vmatrix} \quad (2)$$

$\mathcal{O}(\alpha)$

$$P_{ee}^{(1)}(z) = \frac{1+z^2}{1-z} \quad (4)$$

$\mathcal{O}(\alpha^2)$: Bremsstrahlung

$$P_{ee}^{(2,1)}(z) = \frac{1}{2}\left[P_{ee}^{(1)}\otimes P_{ee}^{(1)}\right](z)$$
$$= \frac{1+z^2}{1-z}\left[2\ln(1-z) - \ln z + \frac{3}{2}\right] + \frac{1}{2}(1+z)\ln z - (1-z) \quad (5)$$

: e^+e^-

$$P_{ee}^{(2,2)}(z) = \frac{1}{2}\left[P_{e\gamma}^{(1)}\otimes P_{\gamma e}^{(1)}\right](z)$$
$$\equiv (1+z)\ln z + \frac{1}{2}(1-z) + \frac{2}{3}\frac{1}{z}(1-z^3) \quad (6)$$

: $f\bar{f}$

$$P_{ee,f}^{(2,3)}(z) = N_c(f)e_f^2\frac{1}{3}P_{ee}^{(1)}(z)\theta\left(1 - z - \frac{2m_f}{E_e}\right) \quad (7)$$

→ DESCRIBE $F_i(x, Q^2)$ FOR $Q^2 \to 0$:

$$\times\left[1-\exp(-A^2\hat{Q}^2)\right] \quad \text{with} \quad A^2 = 3.37\,\text{GeV}^{-2}. \quad (12)$$

SOFT EXPONENTIATION :

SOLVE : LO - GRIBOV LIPATOV eq. (NS) FOR $z \to 1$

$$D_{NS}(z,Q^2) = \zeta(1-z)^{\zeta-1}\frac{\exp\left[\frac{1}{2}\zeta\left(\frac{3}{2}-2\gamma_E\right)\right]}{\Gamma(1+\zeta)} \tag{8}$$

with

$$\zeta = -3\ln\left[1-(\alpha/3\pi)\ln(Q^2/m_e^2)\right] \tag{9}$$

(RUNNING α_{QED} !)

↓ THESE TERMS WERE TAKEN INTO ACC. ALREADY !

$$P_{ee}^{>2,\ soft}(z,Q^2) = D_{NS}(z,Q^2) - \frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right)\frac{2}{1-z}\left\{1+\frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right)\left[\frac{11}{6}+2\ln(1-z)\right]\right\} \tag{10}$$

and[6]

$$\frac{d^2\sigma^{(>2,soft)}}{dxdy} = \int_0^1 dz P_{ee}^{(>2)}(z)\left\{\theta(z-z_0)\mathcal{J}(x,y,z)\frac{d^2\sigma^{(0)}}{dxdy}\Big|_{x=\hat{x},y=\hat{y},s=\hat{s}} - \frac{d^2\sigma^{(0)}}{dxdy}\right\} \tag{11}$$

⟶ NOTE: NO 'UNIQUE' EXPONENTIATION EXISTS !

	$\hat{s}$	$\hat{Q}^2$	$\hat{y}$	$\mathcal{J}(x,y,z)$
lepton measurement	zs	Q^2z	$(z+y-1)/z$	$y/(z+y-1)$
jet measurement	zs	$Q^2(1-y)/(1-y/z)$	y/z	$(1-y)/(z-y)$
mixed variables	zs	Q^2z	y/z	1
double angle method	zs	Q^2z^2	y	z
y_{JB} and θ_e	zs	$Q^2z(z-y)/(1-y)$	y/z	$(z-y)/(1-y)$

Table 1: The shifted variables for different types of cross section measurement

• z_0 :

LEPTON MEASUREMENT
JET MEASUREMENT — $\hat{x}(z_0) = 1$

MIXED VARIABLES — $z_0 = y$
θ_e , y_J
DOUBLE ANGEL : $z_0 \equiv 0$.

} $\hat{\mathcal{J}}(\hat{x} \to 0, \hat{Q}^2 \to 0)$! for $z \to z_0$!

BUT:

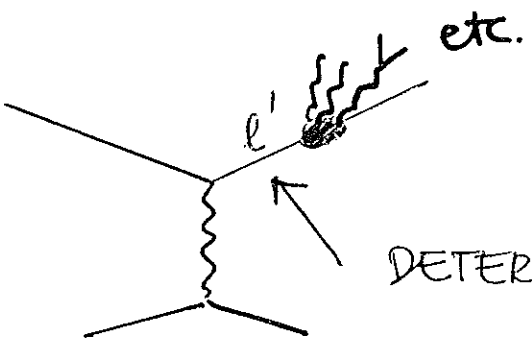
$$2E_e = E'_e(1-\cos\theta_e) + E_J(1-\cos\theta_J) \geq A \quad ! \qquad (3)$$

FORTUNATELY: $z_0 = \frac{A}{2E_e}$.

⟹ ZEUS: THIS HELPS ONLY IN THE CASE OF THE DOUBLE ANGEL METHOD !

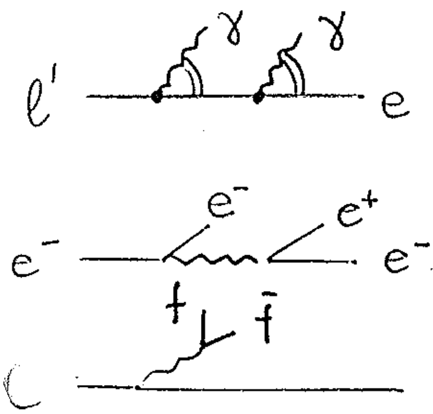
REMARK :

FOR : $\mathcal{O}(\alpha^2)$ FROM LEPTONS.



} CALORIMETRIC MEASUREMENT

DETERMINE ℓ' KINEMATICS!



$$E_{\ell'} = E_e + E_{\gamma_i} + E_{e^+e^-} + \sum_i E_{f_i\bar{f}_i}$$

COLLECT <u>ALL</u> RADIATED ENERGY IN THE ANGULAR VICINITY OF <u>e'</u>!

3. Numerical Results

- UPDATE : $\mathcal{O}(\alpha)$
- STATUS : COMPARISON LLA & FULL CALCULATION IN $\mathcal{O}(\alpha)$
- $\mathcal{O}(\alpha^2 L^2)$ RESULTS IN ALL VARIABLES

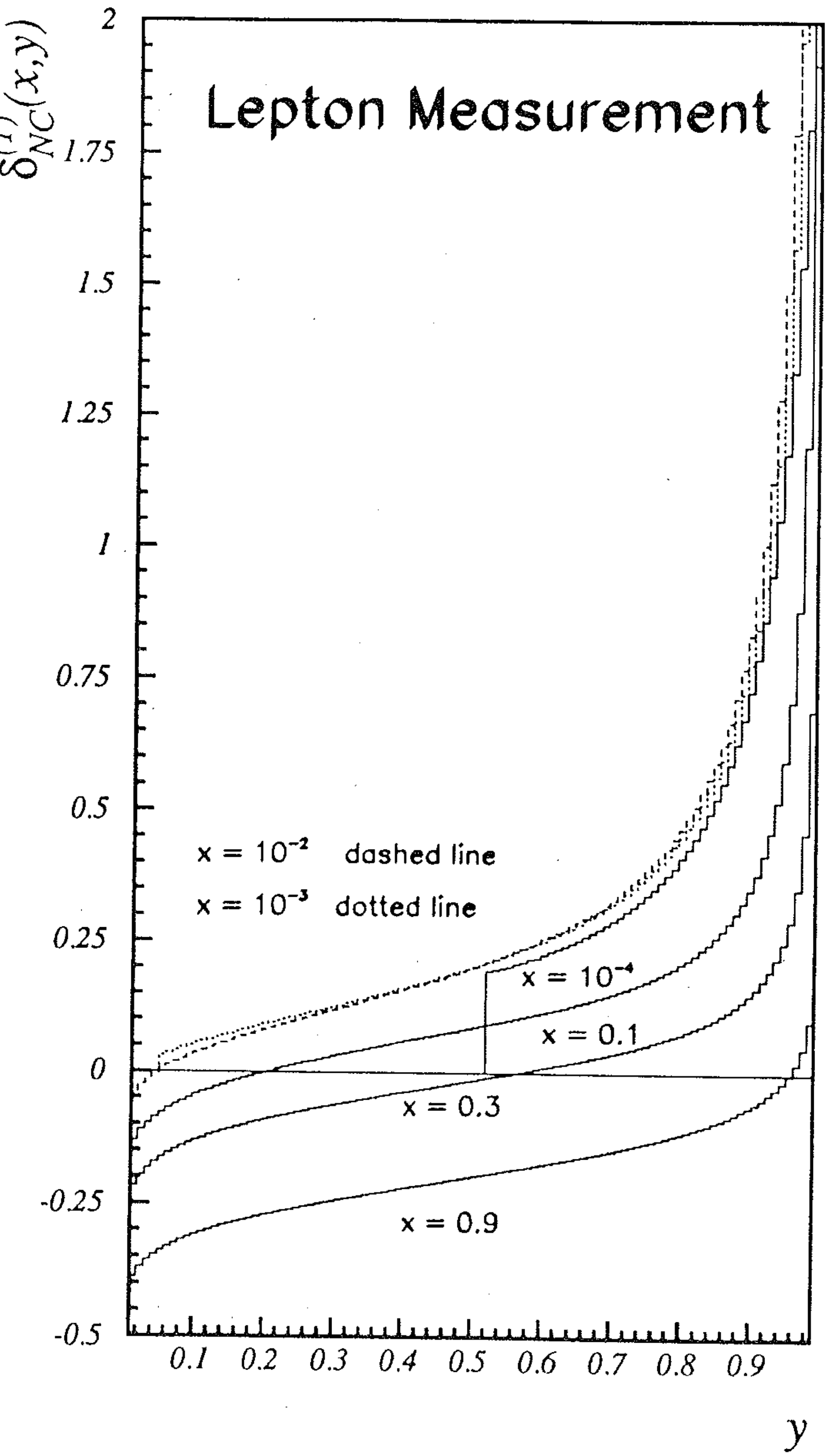
→ ISR LEPTON

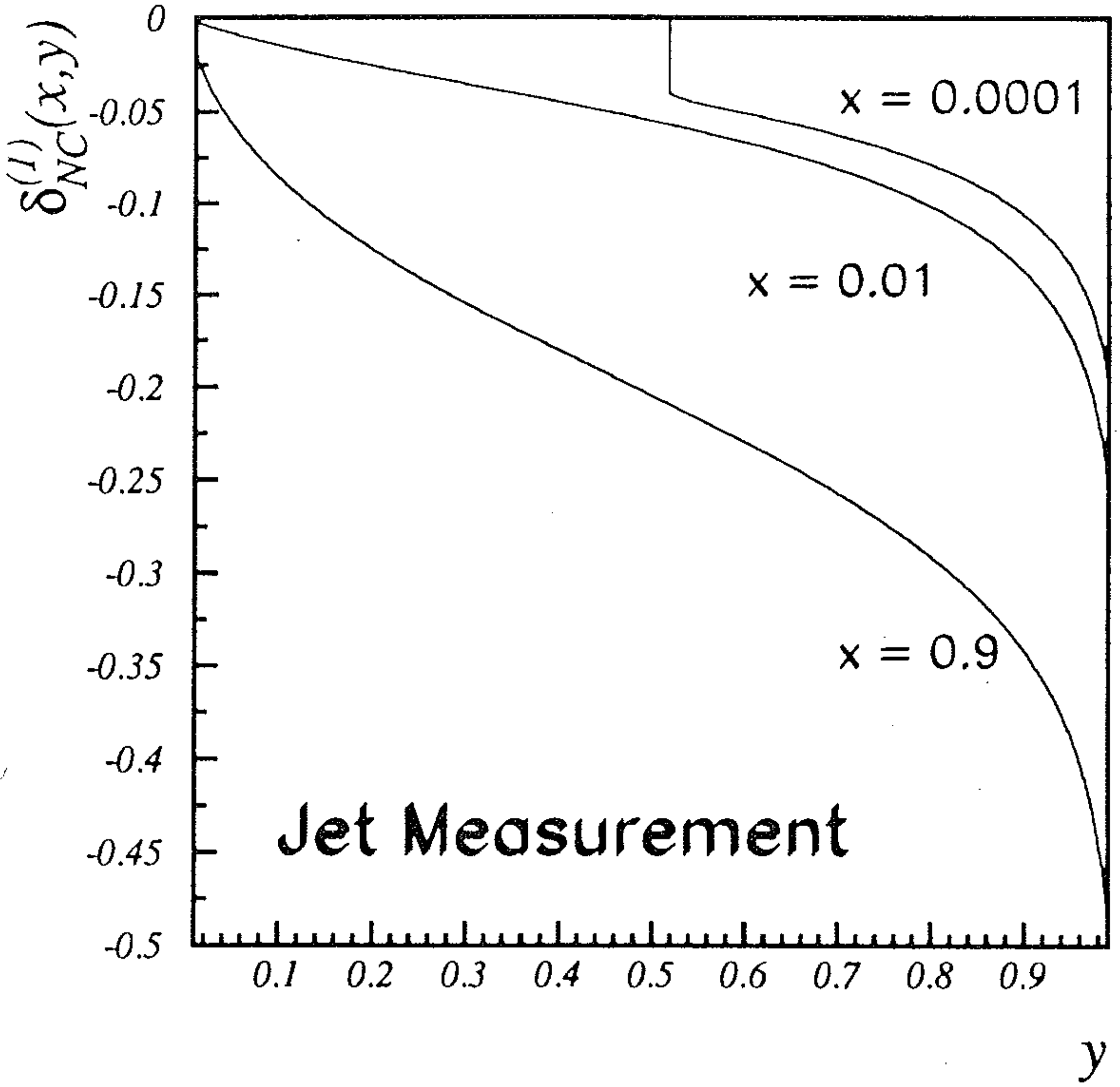
- FSR LEPTON → KLN
- ISR / FSR QUARK → SCAL. VIOL ~ 1%
- COMPTON → EXCL. SIGNATURE

#'s NOT COUNTED TO THE DIS SAMPLE.

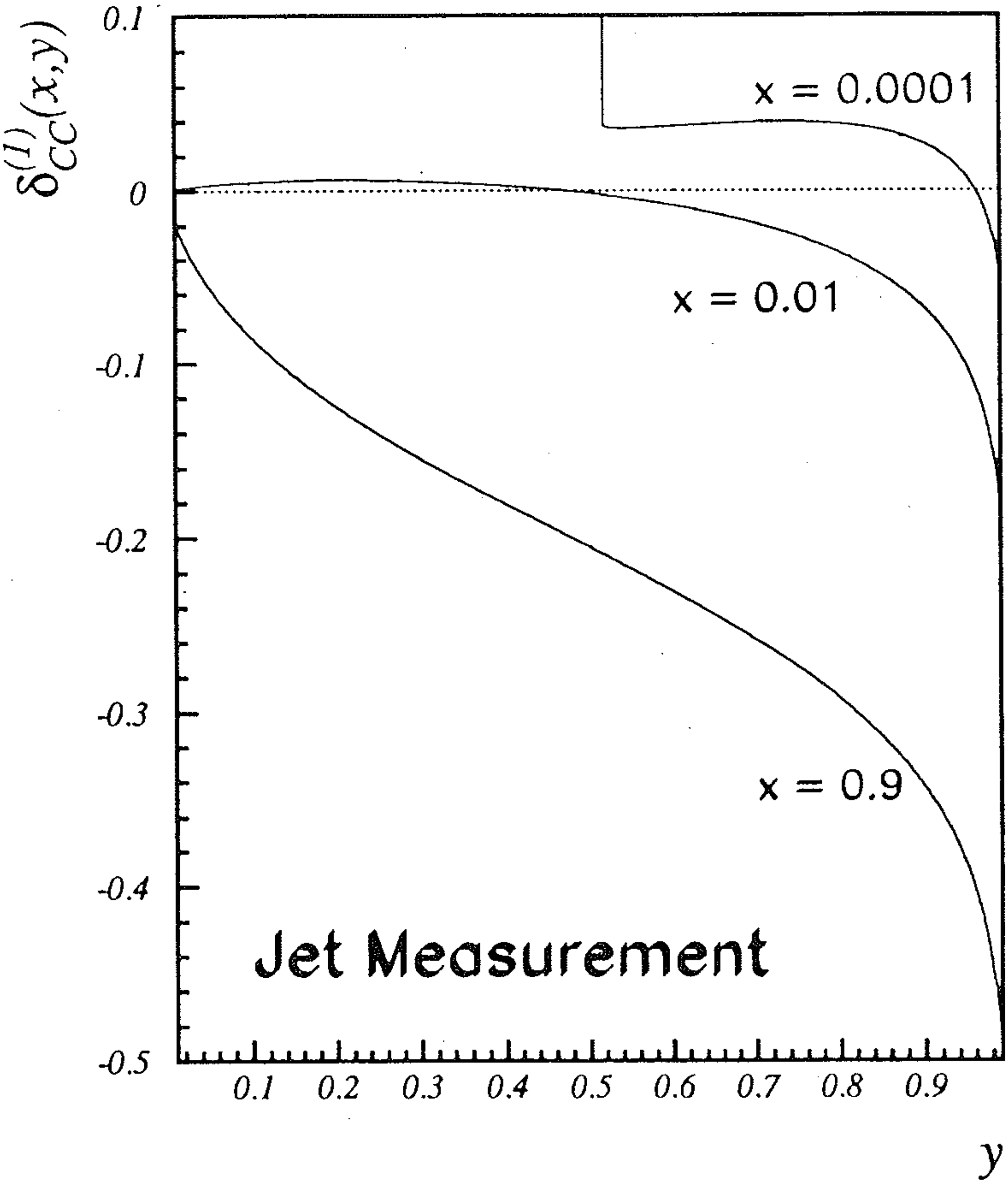
⇨ MRS D'_- , SIMILAR RES. HMRS CETQ 2

$\mathcal{O}(\alpha)$:

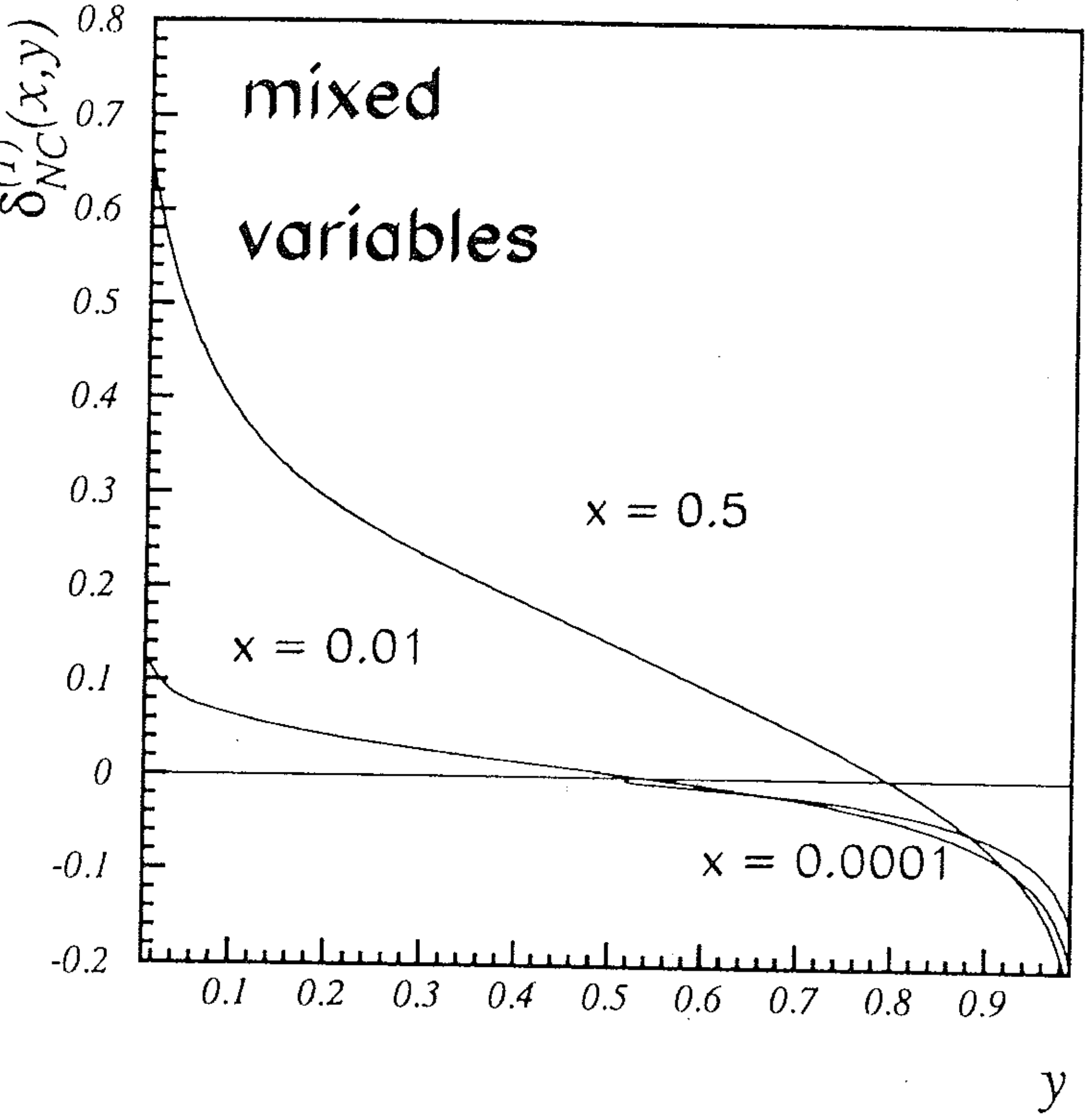




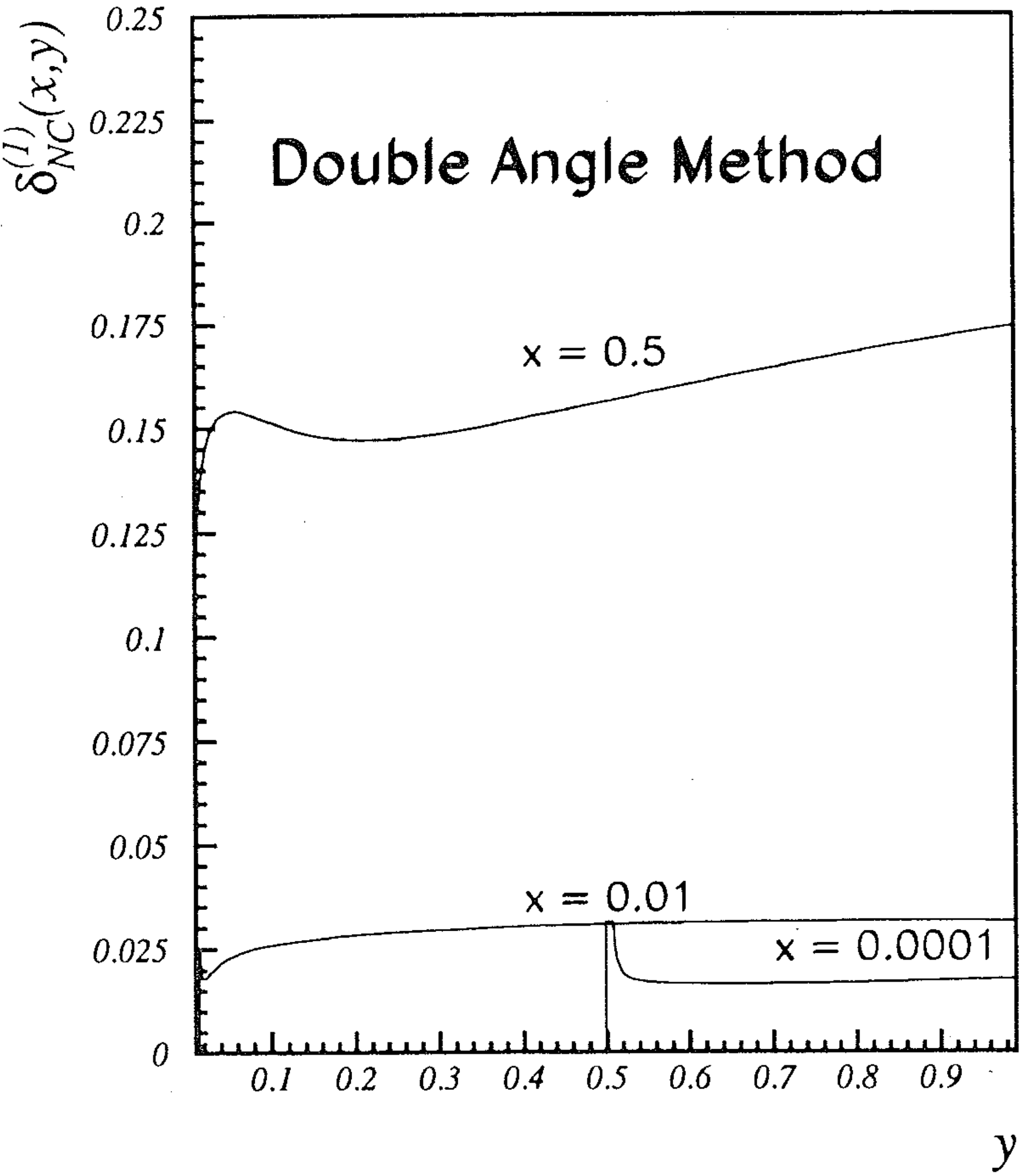
$\delta^{(1)}_{NC}(x,y)$
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
-0.4
-0.45
-0.5
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
y
x = 0.0001
x = 0.01
x = 0.9
Jet Measurement



$\delta_{CC}^{(1)}(x,y)$
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
x = 0.0001
x = 0.01
x = 0.9
Jet Measurement
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
y



mixed
variables
x = 0.5
x = 0.01
x = 0.0001
$\delta_{NC}^{(1)}(x,y)$
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
y



Double Angle Method
$\delta_{NC}^{(1)}(x,y)$
0.25
0.225
0.2
0.175
0.15
0.125
0.1
0.075
0.05
0.025
0
x = 0.5
x = 0.01
x = 0.0001
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
y

A DANGEROUS CASE:

θ_e & y_J

RESCALING: ISR

$$\hat{Q}^2 = Q^2 \, z \, \frac{z-y}{1-y}$$

$$\hat{x} = x \, \frac{z\,(z-y)}{1-y}$$

$$z_0 \equiv y$$

ZEUS: $$z_0 = \max\left\{ \frac{35\ \text{GeV}}{2E_e}, y \right\}$$

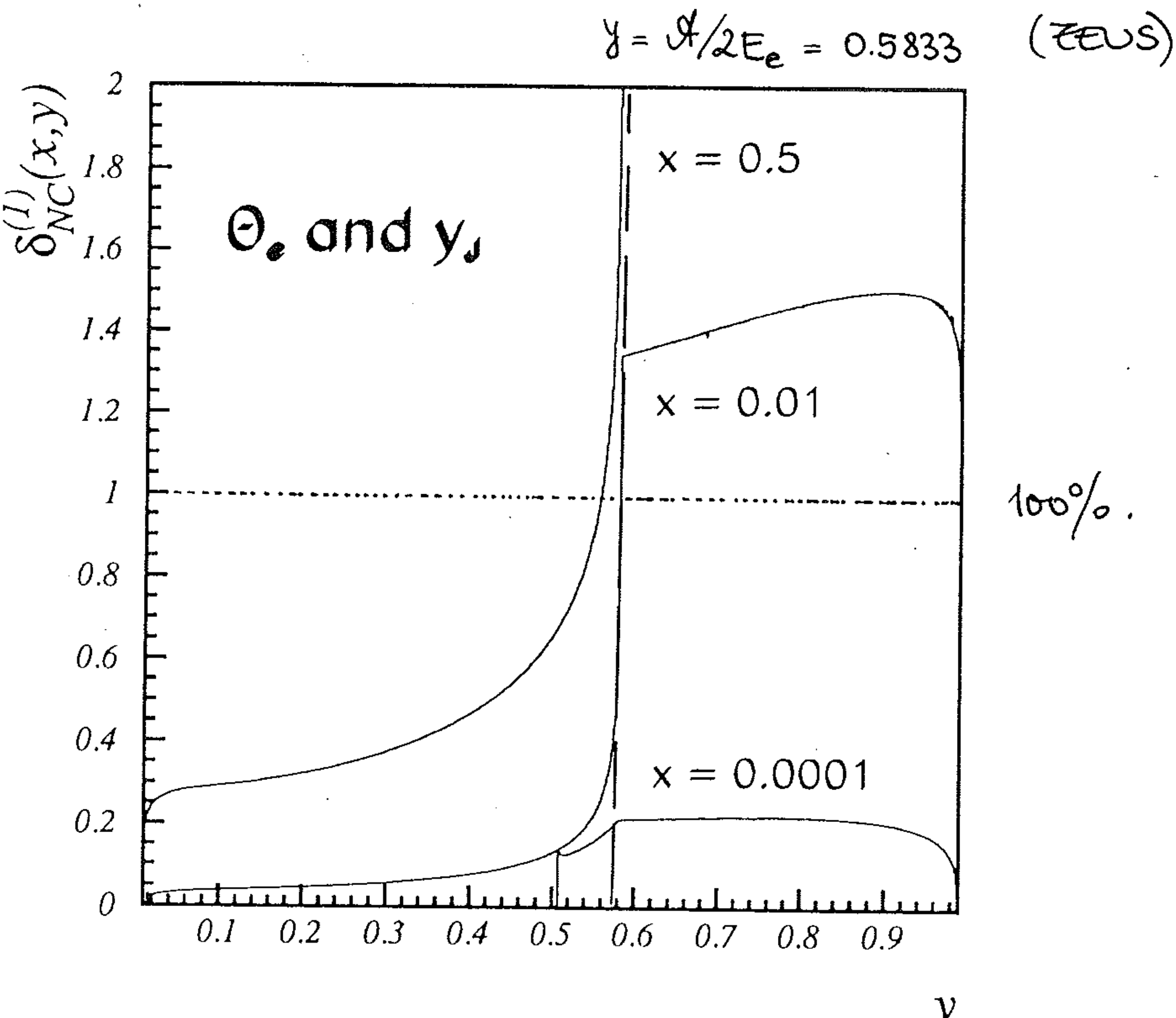
$\delta_{NC}(x,y)$ JUMPS AT $y \gtrsim \frac{\mathcal{A}}{2E_e}$, $\mathcal{A} = 35$ GeV.

$$\frac{\sigma(Q^2, x \to 0)}{\sigma(Q^2, x)} \quad !$$

NO CONTROL ON INPUT AT ALL !

BLIND USE OF 'GENERAL' RC MONTE CARLOS MAY BE DANGEROUS !

⟶ UNFORTUNATE CHOICE OF VARIABLES.



y = ϑ/2E_e = 0.5833
(ZEUS)
δ_NC^(1)(x,y)
Θ_e and y_J
x = 0.5
x = 0.01
x = 0.0001
100%.
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
y

CAN ONE MAKE USE OF THAT ?

→ PARACELSUS' THEOREM: (REVERSED)

- EVERY KNOWN POISON, IF USED IN CORRECT DOSIS, MAY BE APPLIED AS A MEDICINE.

$\mathcal{O}(\alpha)$: (SIM IN HO)

$$\frac{d^2\sigma^{(1)}}{dx\,dy} = \int_0^1 d\tau \; \frac{1+y^2+2\tau y(1-y)+\tau^2(1-y)^2}{(1-y)\qquad(1-\tau)} \frac{\alpha}{2\pi}\log\left(\frac{Q^2}{m_e^2}\right)$$

$$\cdot \frac{2\pi\alpha^2}{xy}\left\{\hat{Y}_+ \underline{\frac{F_2(\hat{x},\hat{Q}^2)}{\hat{Q}^2}} - Y_+ \frac{F_2(x,Q^2)}{Q^2}\right\}$$

$$-\left(\int_0^y dz\, \frac{1+z^2}{1-z}\right) \frac{2\pi\alpha^2}{Q^4}\, S\, Y_+ F_2(x,Q^2)$$

$$\hat{Q}^2 = \tau Q^2 \qquad\qquad Y_+ = 1+(1-y)^2$$

$$\hat{x} = \tau x\,(y+\tau(1-y)) \qquad \hat{y} = y/(y+\tau(1-y))$$

KNOWING: $F_2(x,Q^2)$ ONE MAY DETERMINE

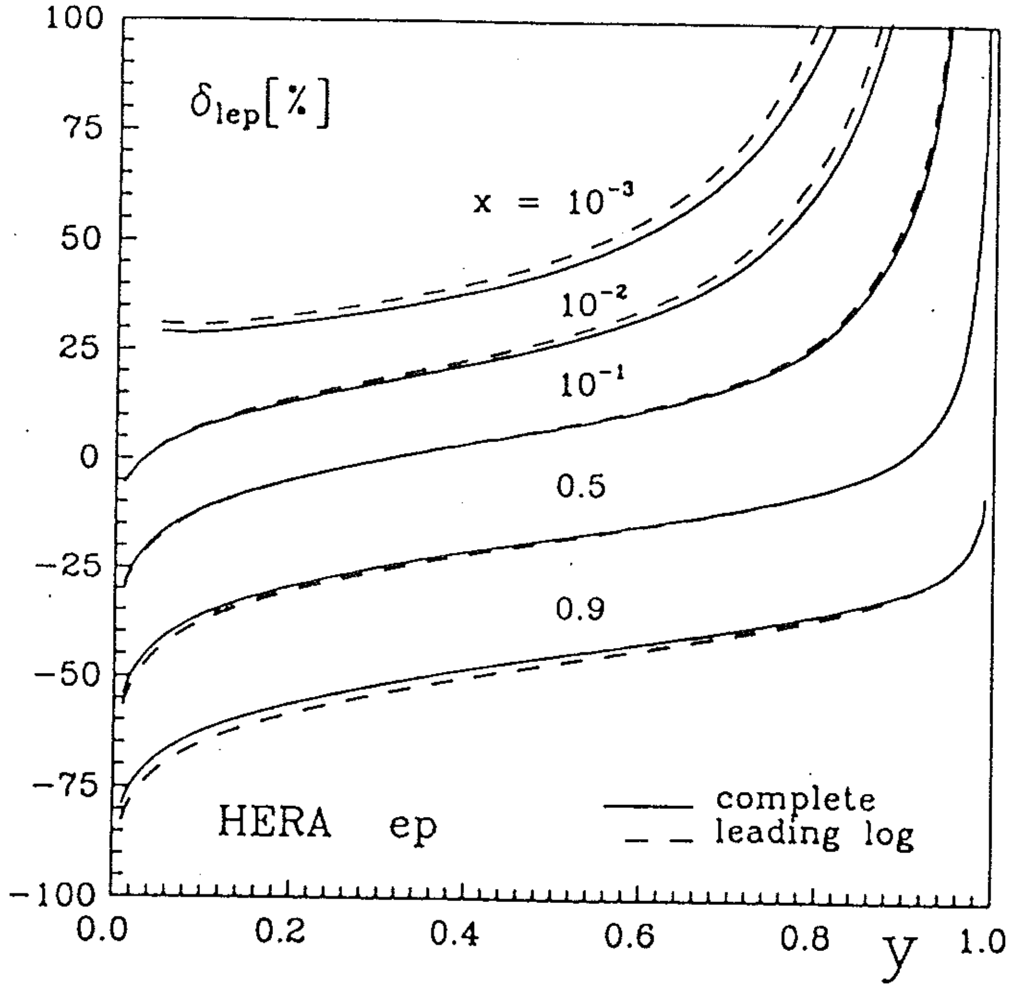
$$F_2(\hat{x},\hat{Q}^2)$$

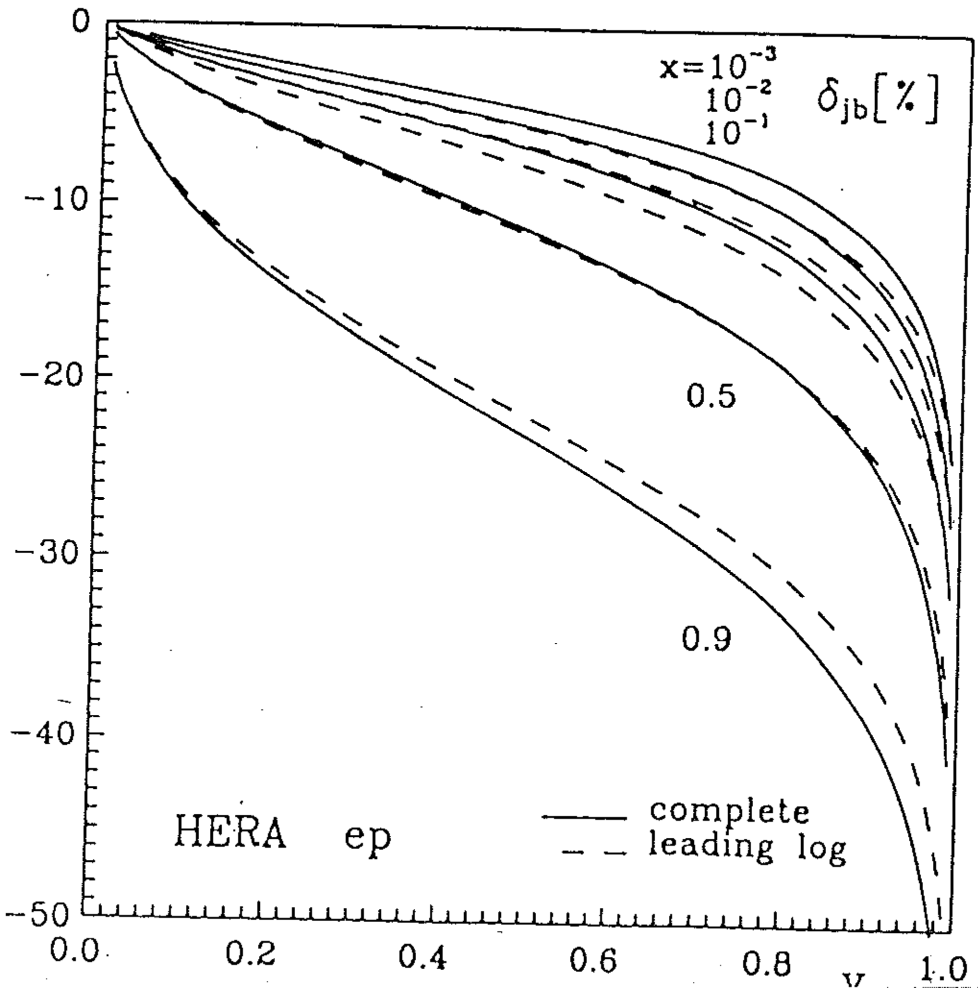
FOR: $\hat{x} \ll x$, $\hat{Q}^2 \ll Q^2$!

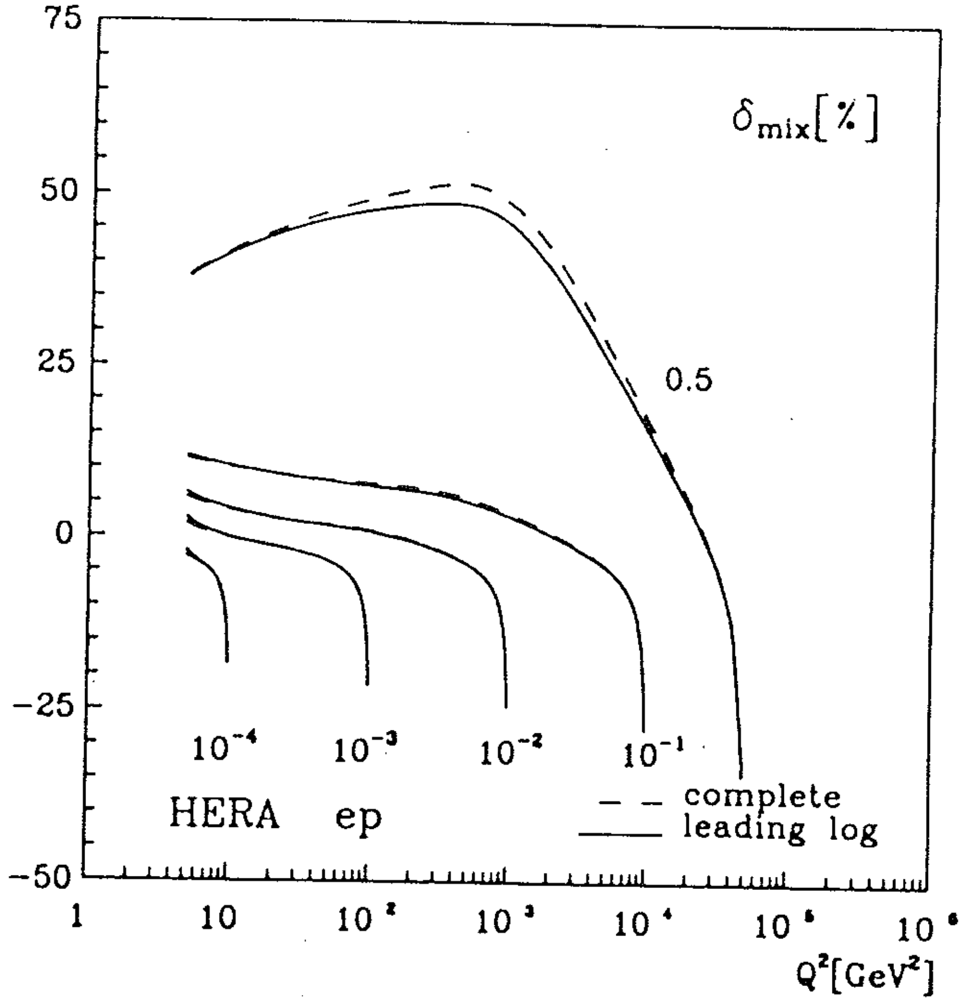
A WINDOW TO THE NON-PERTURBATIVE !

Comparison with a Full $\mathcal{O}(\alpha)$ Calculation

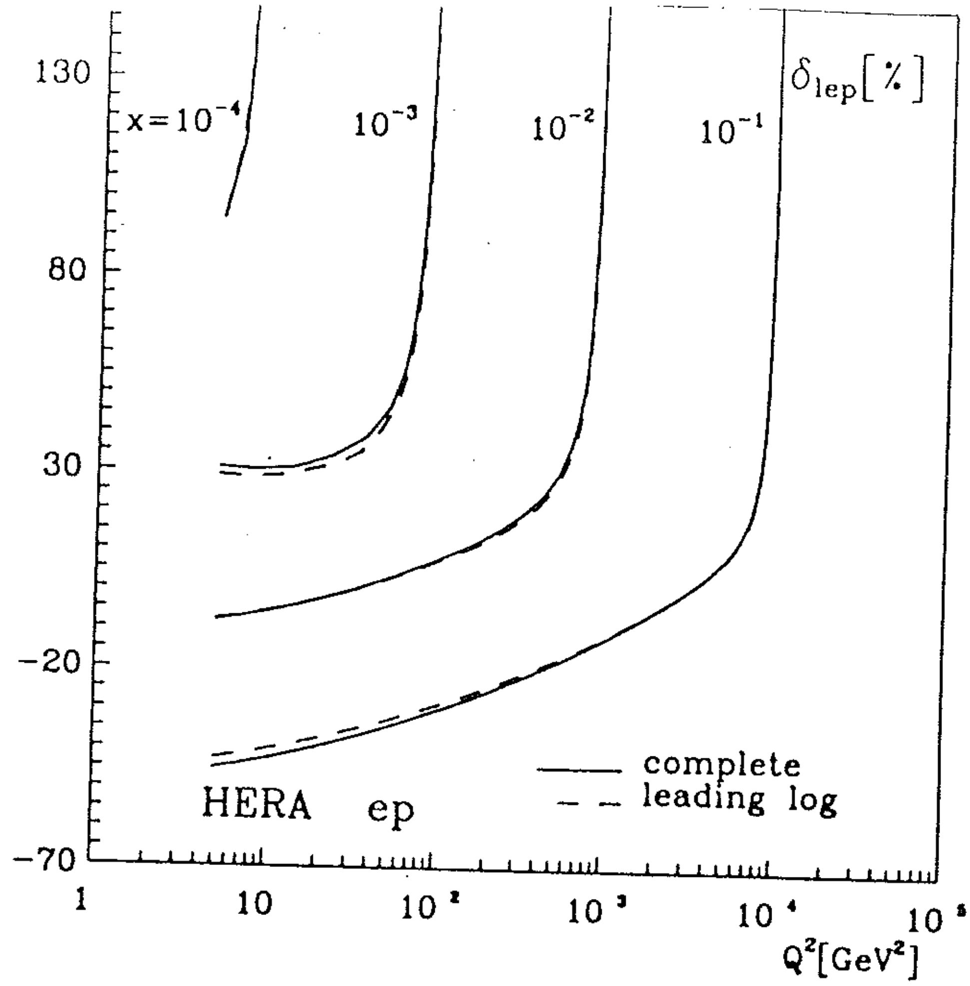
TERAD, D.Y. Bardin et al.



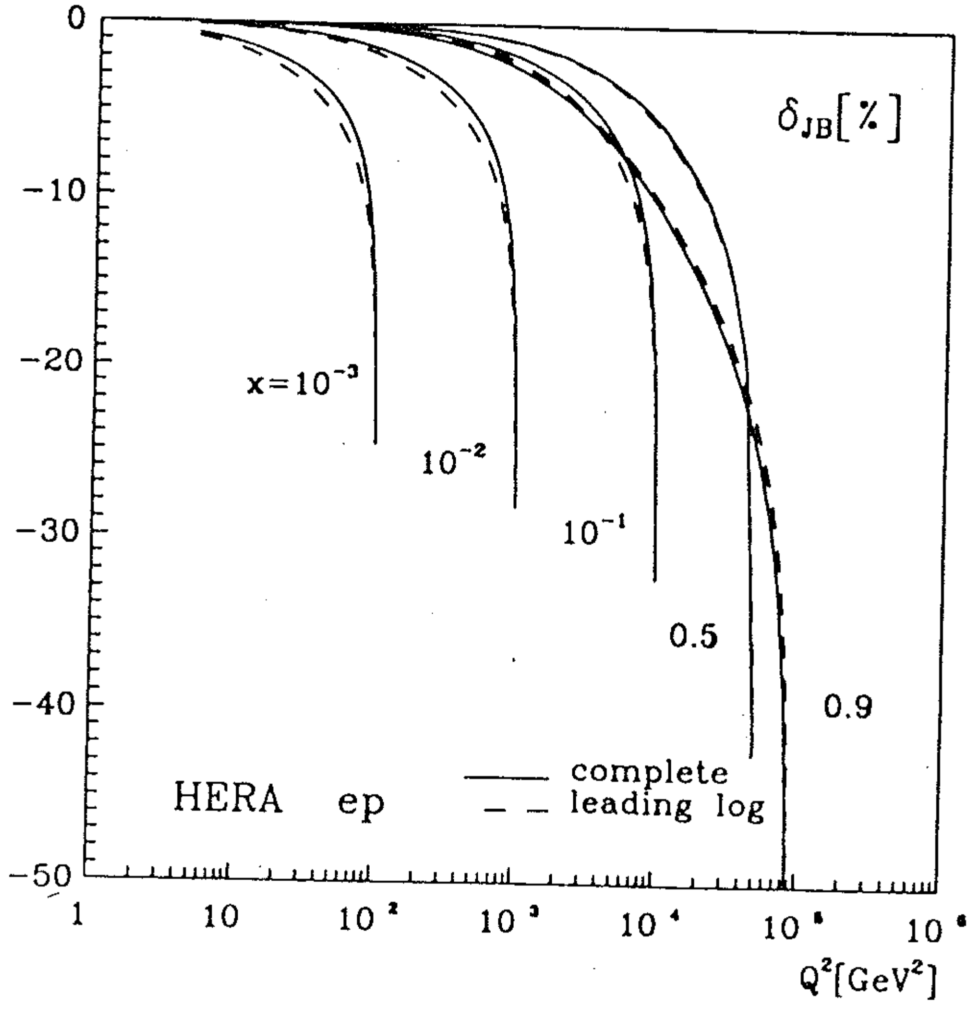




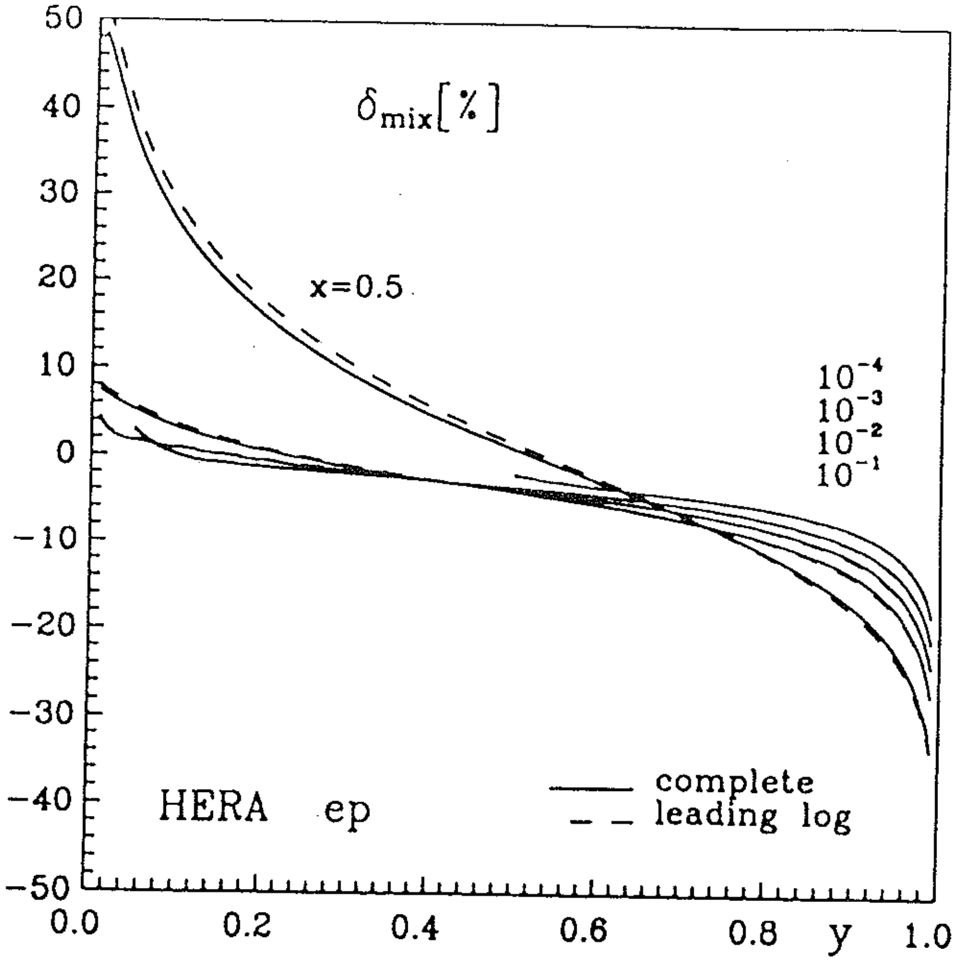
75
50
25
0
−25
−50
δ_{mix}[%]
0.5
10^{-4}
10^{-3}
10^{-2}
10^{-1}
HERA ep
complete
leading log
1
10
10^2
10^3
10^4
10^5
10^6
Q^2[GeV^2]



130
80
30
−20
−70
x=10^−4
10^−3
10^−2
10^−1
δlep[%]
complete
leading log
HERA ep
1
10
10 ²
10 ³
10 ⁴
10 ⁵
Q²[GeV²]

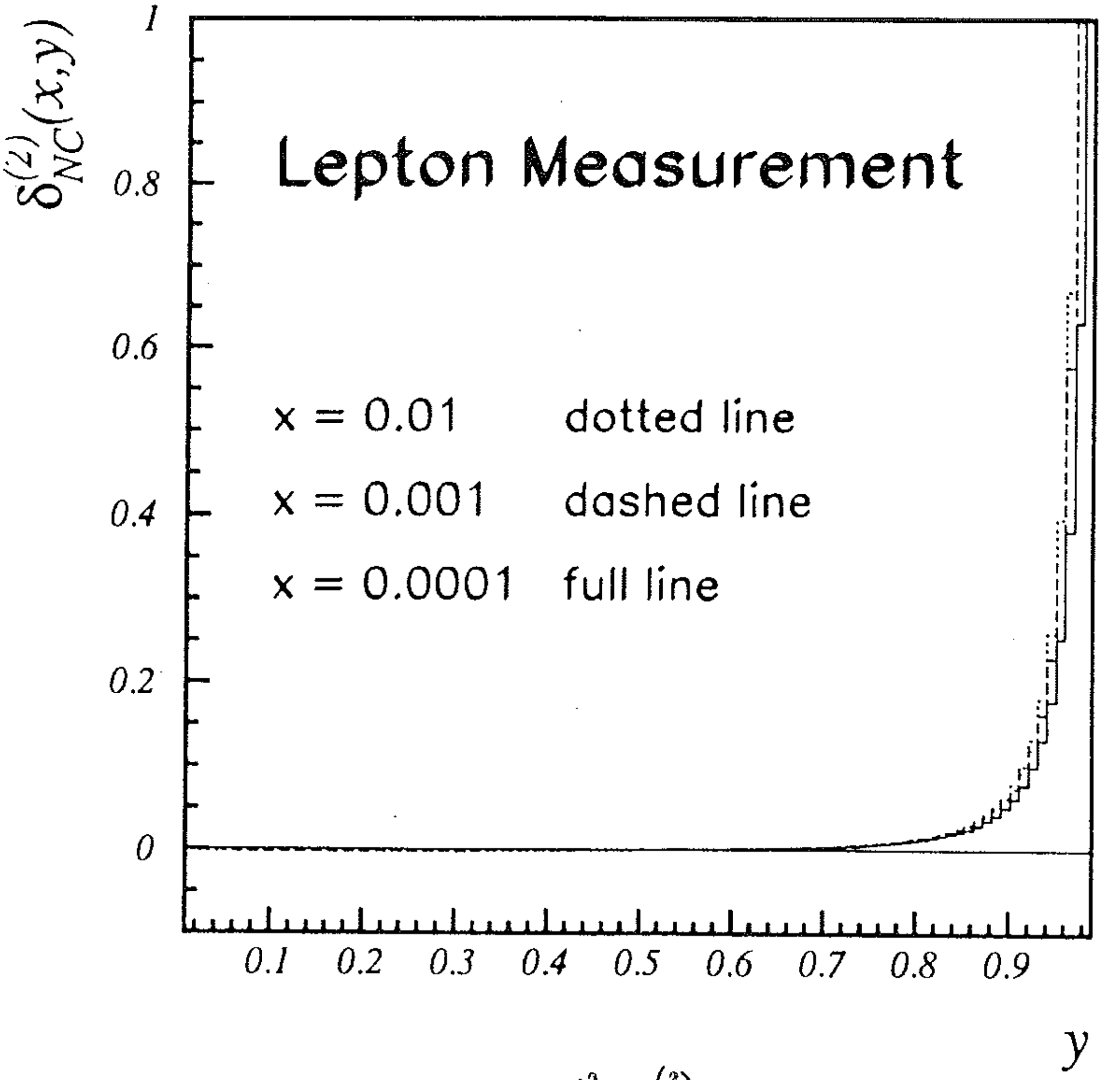


0
−10
−20
−30
−40
−50
δJB[%]
x=10^−3
10^−2
10^−1
0.5
0.9
complete
leading log
HERA ep
1
10
10 ²
10 ³
10 ⁴
10 ⁵
10 ⁶
Q²[GeV²]

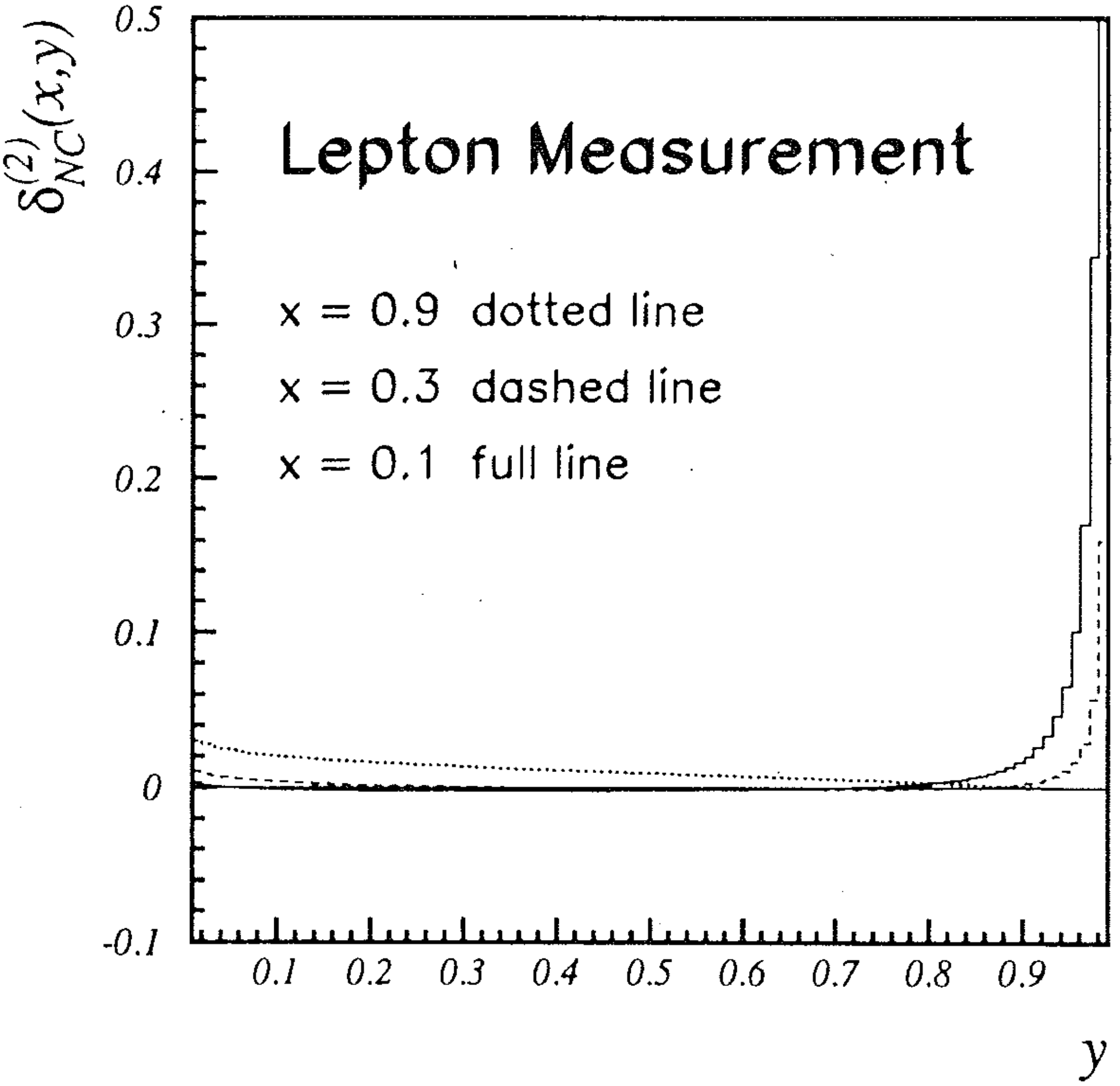


$\delta_{mix}[\%]$
x=0.5
10^{-4}
10^{-3}
10^{-2}
10^{-1}
HERA ep
complete
leading log
50
40
30
20
10
0
−10
−20
−30
−40
−50
0.0
0.2
0.4
0.6
0.8
y
1.0

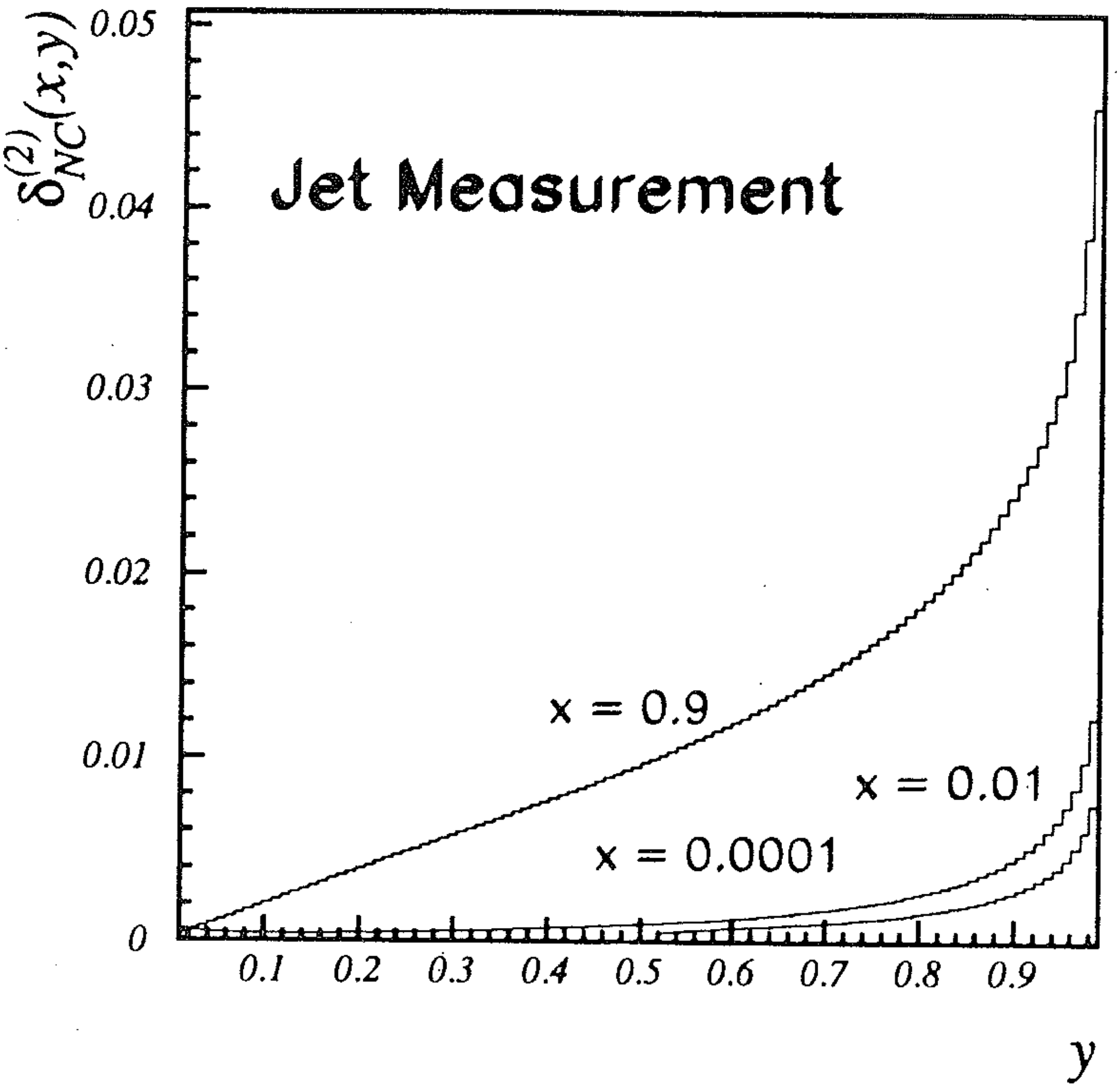
$\mathcal{O}(\alpha^2)$



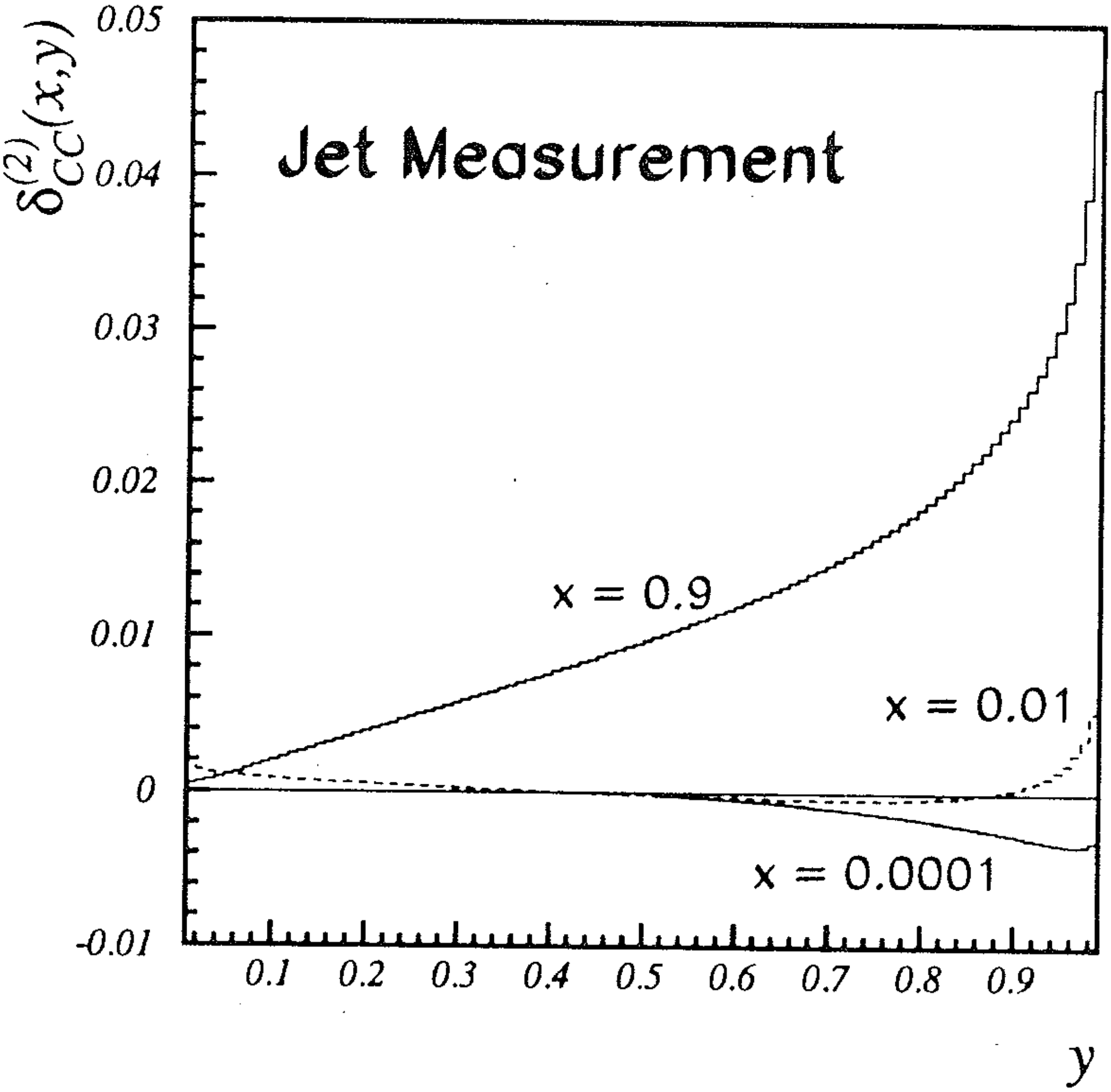
$$\delta_i^{(2)}(x,y) = \frac{\frac{d^2\sigma^{(2)}}{dx\,dy}}{\frac{d^2\sigma^{(0)}}{dx\,dy}}$$

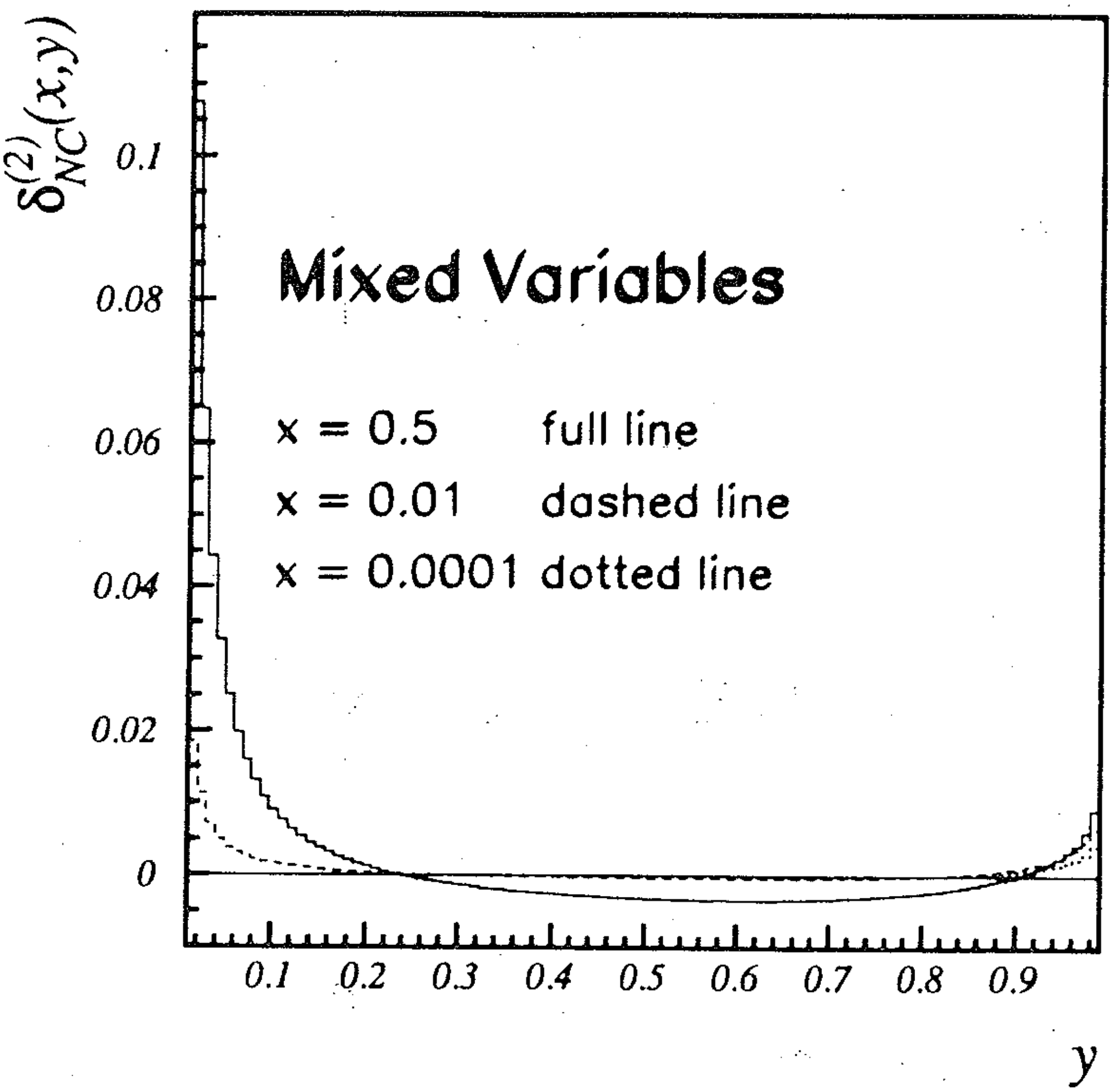


$\delta^{(2)}_{NC}(x,y)$
Lepton Measurement
x = 0.9 dotted line
x = 0.3 dashed line
x = 0.1 full line
0.5
0.4
0.3
0.2
0.1
0
-0.1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
y

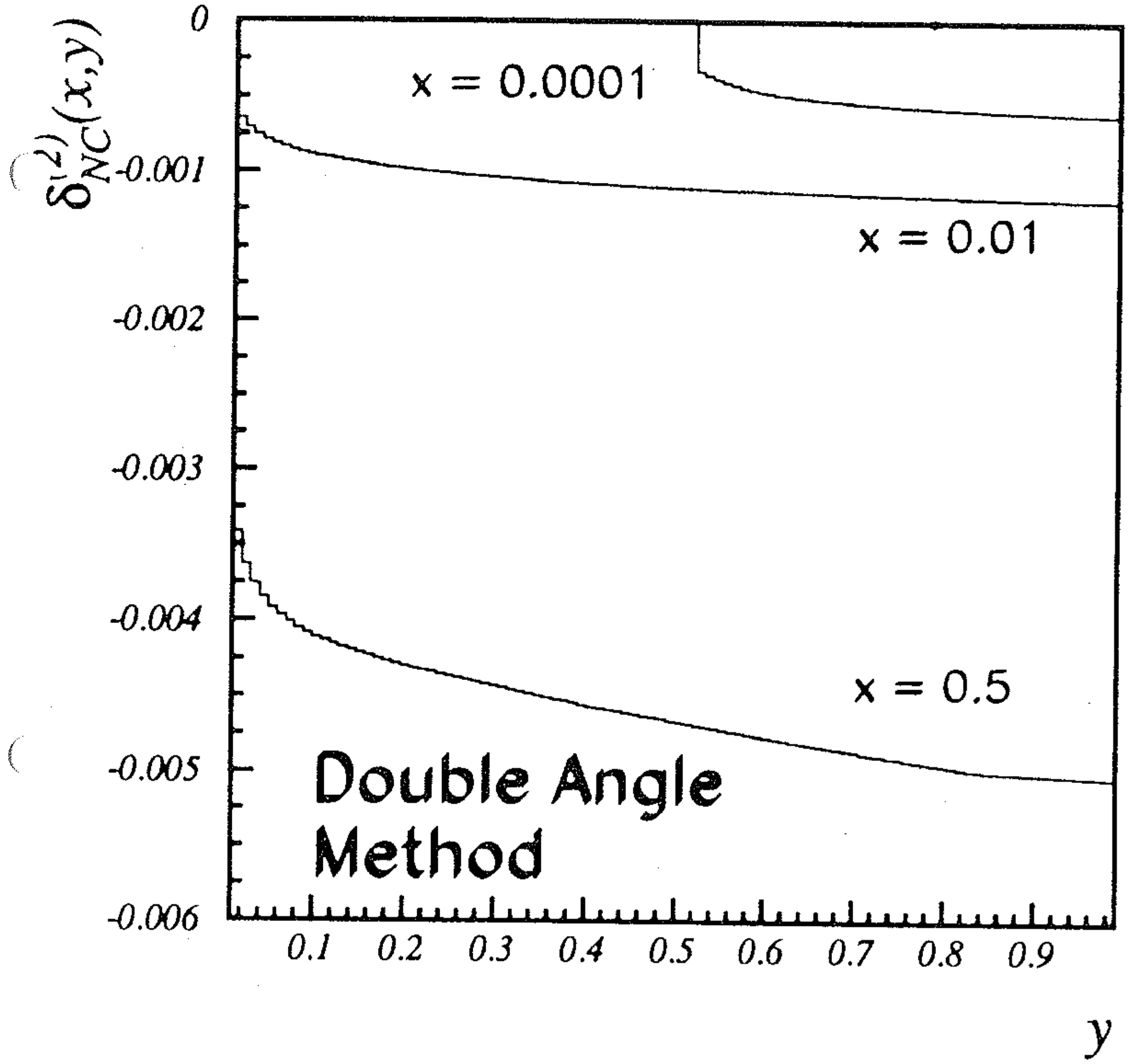


$\delta^{(2)}_{NC}(x,y)$
0.05
0.04
0.03
0.02
0.01
0
Jet Measurement
x = 0.9
x = 0.01
x = 0.0001
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
y

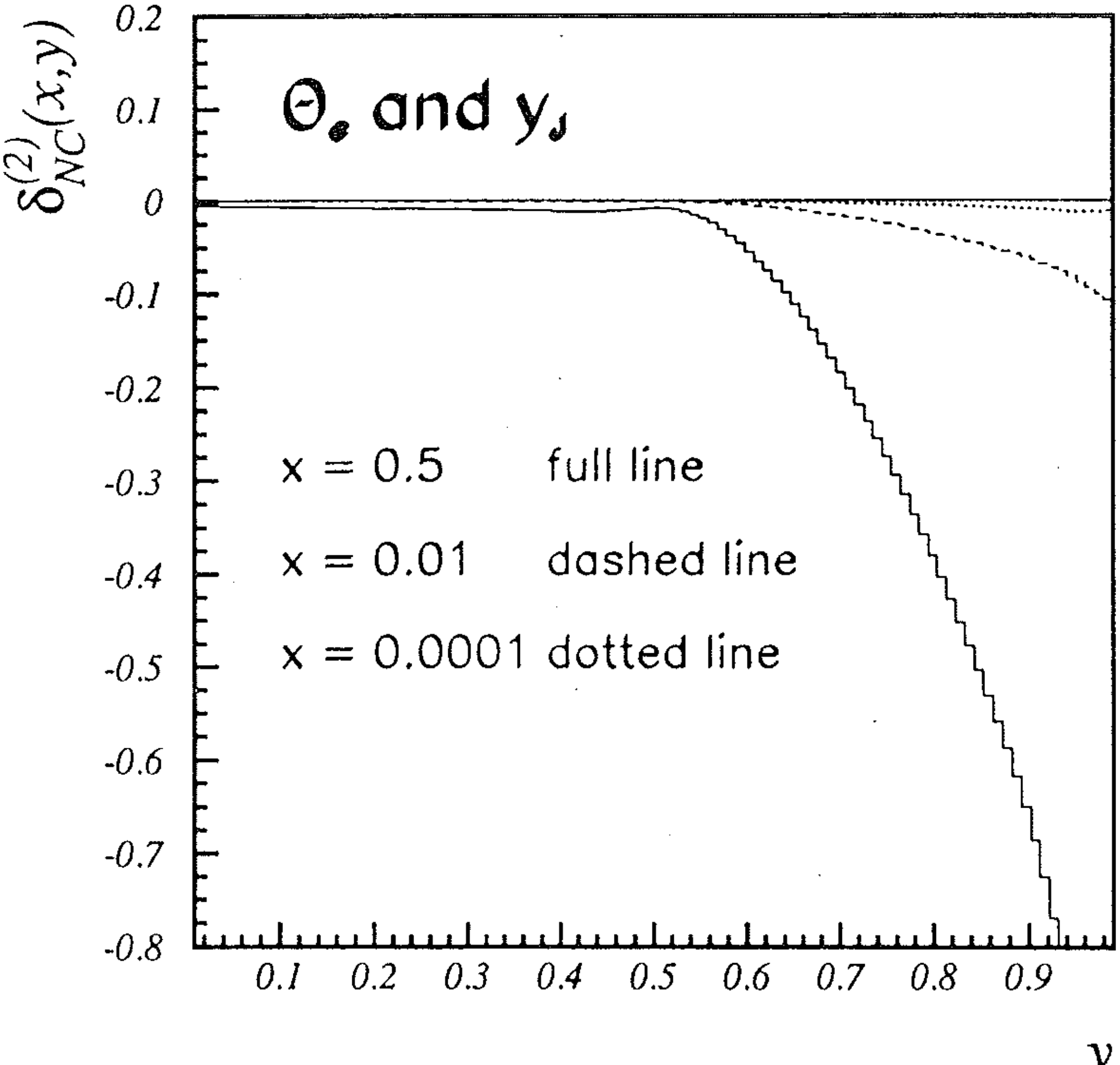

Jet Measurement
$\delta^{(2)}_{CC}(x,y)$
0.05
0.04
0.03
0.02
0.01
0
-0.01
x = 0.9
x = 0.01
x = 0.0001
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
y



$\delta^{(2)}_{NC}(x,y)$
0.1
0.08
0.06
0.04
0.02
0
Mixed Variables
x = 0.5 full line
x = 0.01 dashed line
x = 0.0001 dotted line
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
y



$\delta^{(2)}_{NC}(x,y)$
0
-0.001
-0.002
-0.003
-0.004
-0.005
-0.006
x = 0.0001
x = 0.01
x = 0.5
Double Angle
Method
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
y



$\delta_{NC}^{(2)}(x,y)$
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
Θ_e and y_J
x = 0.5 full line
x = 0.01 dashed line
x = 0.0001 dotted line
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
y

4. Conclusions

1. THE $\mathcal{O}(\alpha L)$ & $\mathcal{O}(\alpha^2 L^2)$ RC'S TO DIS HAVE BEEN CALCULATED FOR:

H1:
- LEPTONIC VARIABLES
- JET MEASUREMENT NC & CC
- MIXED VARIABLES

ZEUS:
- THE DOUBLE ANGLE METHOD
- VARIABLES BASED ON θ_e & y_J.

(1ST CALC.)

2. IN $\mathcal{O}(\alpha)$ THE DOMINANCE OF LLA IS ESTABLISHED, GOOD TO EXCELLENT AGREEMENT WITH FULL $\mathcal{O}(\alpha)$ RESULTS IS FOUND.

3. $\delta_{NS}^{(1)}$: DOUBLE ANGLE METHOD $\lesssim^{+} 18\%$ $x \lesssim 0.5$

   $\sim^{+} 2\%$ $x \sim 10^{-4}$.

   $\delta_{NS}^{(2)}$ $-5‰$ $x \sim .5$

   $-1‰$ $x \sim 10^{-4}$

   FLAT BEHAVIOUR! IN Y, $x = \text{CONST}$.

4. PROBLEMATIC CASE: θ_e & y_J.
   UNSTABLE RC'S FOR $y > \frac{\mathcal{J}}{2E_e}$!
   ONE SHOULD NOT USE IT FOR THE F_2 MEASUREMENT IN THE DIS RANGE!

5. PERHAPS POSSIBLE : $\frac{F_2(x \to 0, Q^2 \to 0)}{}$!
UNFOLDABLE FROM $\delta_{NC}^{(x,y)}$ & $F_2(x, Q^2)$
↑ MEAS. ↑ 'KNOWN' BY A DIFFERENT MEASUREMENT

6. JET MEASUREMENT:

$\delta^{(2)}_{NC,CC}$: $x = 10^{-4}$ $< 5‰$
$x = .9$ $< 5\%$

7. MIXED VARIABLES

$\delta^{(2)}_{NC}$: $x = 0.5$ $\sim 10\%$ $y \to 0$
$\sim 1\%$ $y \to 1$

8. LEPTON MEASUREMENT : $\delta^{(2)}_{NC} > 10\%$
$y > 0.9$

SLIGHT BULK $\sim 5\%$ $x \sim 0.9$ $y \lesssim 0.6$.

9. $\mathcal{O}(\alpha^2)$ CORRECTIONS ARE NEEDED FOR A PRECISION ANALYSIS IN ALL VARIABLES.

$(\delta_{LIM.} < 1\%\ ,\ 0.5\%)$