Status of Polarized and Unpolarized Parton Distributions

Johannes Blümlein

1. Introduction
2. Unpolarized Parton Distributions
3. Polarized Parton Distributions
4. $\alpha_s$ and $\Lambda_{QCD}$
5. Future Avenues
1. Introduction

When is a Parton?

S. DRELL: Infinite Momentum Frame: $P$ - large

$$\tau_{\text{int}} \ll \tau_{\text{life}}$$

$$\tau_{\text{int}} \sim \frac{1}{q_0} = \frac{4Px}{Q^2(1-x)}$$

$$\tau_{\text{life}} \sim \frac{1}{\sum_i E_i - E} = \frac{2P}{\sum_i (k_{1i}^2 + M_i^2)/x_i - M^2} \approx \frac{2Px(1-x)}{k_{1}^2}$$

$$\frac{\tau_{\text{int}}}{\tau_{\text{life}}} = \frac{2k_{1}^2}{Q^2(1-x)^2}$$

Stay away from $x \to 0$, since $xP$ becomes too small.

Stay away from $x \to 1$.

$$Q^2 \gg k_{1}^2.$$
**Main Research Objectives:**

- Precise Measurement of $\alpha_s(M_Z^2)$
- Reveal polarized and unpolarized parton densities at highest precision
- Precision tests of QCD
- Find novel sub-structures

→ **Perturbative QCD:**

**NNLO calculations using new technologies**

→ **Lattice QCD:**

**Calculation of certain non-perturbative quantities a priori**
3 Loop Splitting Functions

\[ \lambda = \frac{1}{N} N^2 \cdot 0.2 = 0.0 \]

\[ (N)^{gg} \lambda \]

\[ (N)^{bb} \lambda \]

\[ \text{NNLO} \]

\[ \text{NLO} \]

\[ \text{LO} \]
3 Loop Coefficient Functions
Unpolarized Parton Distributions

Kinematic Domain

![Graph showing kinematic domain with various datasets and limits](image-url)
H1, ZEUS + fixed target data

\[ F_2 \cdot 2^i \]

\[ x = 0.000050, i = 21 \]
\[ x = 0.000080, i = 20 \]
\[ x = 0.00013, i = 19 \]
\[ x = 0.000020, i = 18 \]
\[ x = 0.00032, i = 17 \]
\[ x = 0.00050, i = 16 \]
\[ x = 0.00080, i = 15 \]
\[ x = 0.0013, i = 14 \]
\[ x = 0.0020, i = 13 \]
\[ x = 0.0032, i = 12 \]
\[ x = 0.0050, i = 11 \]
\[ x = 0.0080, i = 10 \]
\[ x = 0.013, i = 9 \]
\[ x = 0.020, i = 8 \]
\[ x = 0.032, i = 7 \]
\[ x = 0.050, i = 6 \]
\[ x = 0.080, i = 5 \]
\[ x = 0.13, i = 4 \]
\[ x = 0.18, i = 3 \]
\[ x = 0.25, i = 2 \]
\[ x = 0.40, i = 1 \]
\[ x = 0.55, i = 0 \]

- H1 $e^+p$
- ZEUS $e^+p$
- BCDMS
- NMC

H1 PDF 2000
extrapolation

\[ Q^2 / \text{GeV}^2 \]

J. Blümlein
Duality 05, Frascati, June 2005
Scaling violations of $F_2(x, Q^2)$. 

J. Blümlein  
Duality 05, Frascati, June 2005
QCD NS-Analysis to 3 Loops

\[ W^2 > 12.5 \text{ GeV}^2, \quad Q^2 > 4 \text{ GeV}^2 \]

**NNLO:**

\[ \alpha_s(M_Z^2) = 0.1139^{+0.0026}_{-0.0028} \]

J.B., H. Böttcher, A. Guffanti, 2004

J. Blümlein

Duality 05, Frascati, June 2005
# The World Data on $F_2$

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$x$</th>
<th>$Q^2, \text{GeV}^2$</th>
<th>$F_2$</th>
<th>Norm</th>
</tr>
</thead>
<tbody>
<tr>
<td>BCDMS (100)</td>
<td>0.35 - 0.75</td>
<td>11.75 - 75.00</td>
<td>51</td>
<td>1.018</td>
</tr>
<tr>
<td>BCDMS (120)</td>
<td>0.35 - 0.75</td>
<td>13.25 - 75.00</td>
<td>59</td>
<td>1.011</td>
</tr>
<tr>
<td>BCDMS (200)</td>
<td>0.35 - 0.75</td>
<td>32.50 - 137.50</td>
<td>50</td>
<td>1.017</td>
</tr>
<tr>
<td>BCDMS (280)</td>
<td>0.35 - 0.75</td>
<td>43.00 - 230.00</td>
<td>49</td>
<td>1.018</td>
</tr>
<tr>
<td>NMC (comb)</td>
<td>0.35 - 0.50</td>
<td>7.00 - 65.00</td>
<td>15</td>
<td>1.003</td>
</tr>
<tr>
<td>SLAC (comb)</td>
<td>0.30 - 0.62</td>
<td>7.30 - 21.39</td>
<td>57</td>
<td>1.003</td>
</tr>
<tr>
<td>H1 (hQ2)</td>
<td>0.40 - 0.65</td>
<td>200 - 30000</td>
<td>26</td>
<td>1.018</td>
</tr>
<tr>
<td>ZEUS (hQ2)</td>
<td>0.40 - 0.65</td>
<td>650 - 30000</td>
<td>15</td>
<td>1.001</td>
</tr>
<tr>
<td><strong>proton</strong></td>
<td></td>
<td></td>
<td></td>
<td>322</td>
</tr>
<tr>
<td>BCDMS (120)</td>
<td>0.35 - 0.75</td>
<td>13.25 - 99.00</td>
<td>59</td>
<td>0.992</td>
</tr>
<tr>
<td>BCDMS (200)</td>
<td>0.35 - 0.75</td>
<td>32.50 - 137.50</td>
<td>50</td>
<td>0.993</td>
</tr>
<tr>
<td>BCDMS (280)</td>
<td>0.35 - 0.75</td>
<td>43.00 - 230.00</td>
<td>49</td>
<td>0.993</td>
</tr>
<tr>
<td>NMC (comb)</td>
<td>0.35 - 0.50</td>
<td>7.00 - 65.00</td>
<td>15</td>
<td>0.980</td>
</tr>
<tr>
<td>SLAC (comb)</td>
<td>0.30 - 0.62</td>
<td>10.00 - 21.40</td>
<td>59</td>
<td>0.980</td>
</tr>
<tr>
<td><strong>deuteron</strong></td>
<td></td>
<td></td>
<td></td>
<td>232</td>
</tr>
<tr>
<td>BCDMS (120)</td>
<td>0.070 - 0.275</td>
<td>8.75 - 43.00</td>
<td>36</td>
<td>1.000</td>
</tr>
<tr>
<td>BCDMS (200)</td>
<td>0.070 - 0.275</td>
<td>17.00 - 75.00</td>
<td>29</td>
<td>1.000</td>
</tr>
<tr>
<td>BCDMS (280)</td>
<td>0.100 - 0.275</td>
<td>32.50 - 115.50</td>
<td>27</td>
<td>1.000</td>
</tr>
<tr>
<td>NMC (comb)</td>
<td>0.013 - 0.275</td>
<td>4.50 - 65.00</td>
<td>88</td>
<td>1.000</td>
</tr>
<tr>
<td>SLAC (comb)</td>
<td>0.153 - 0.293</td>
<td>4.18 - 5.50</td>
<td>28</td>
<td>1.000</td>
</tr>
<tr>
<td><strong>non-singlet</strong></td>
<td></td>
<td></td>
<td></td>
<td>208</td>
</tr>
<tr>
<td><strong>total</strong></td>
<td></td>
<td></td>
<td></td>
<td>762</td>
</tr>
</tbody>
</table>

- **CUTS**: $0.3 < x < 1.0$ for $F_2^p$ and $F_2^d$
  
- $0.0 < x < 0.3$ for $F_2^{\text{singlet}} = 2(F_2^p - F_2^d)$

- $4.0 < Q^2 < 30000 \text{GeV}^2$, $W^2 > 12.5 \text{GeV}^2$

---

*J. Blümlein*  
*Duality 05, Frascati, June 2005*
Fully Correlated Error Calculation

- The fully correlated $1\sigma$ error for the parton density $f_q$ as given by Gaussian error propagation is

$$
\sigma(f_q(x))^2 = \sum_{i,j=1}^{n_p} \left( \frac{\partial f_q}{\partial p_i} \frac{\partial f_q}{\partial p_j} \right) \text{cov}(p_i, p_j) ,
$$

(1)

where the $\partial f_q/\partial p_i$ are the derivatives of $f_q$ w.r.t. the parameters $p_i$ and the $\text{cov}(p_i, p_j)$ are the elements of the covariance matrix as determined in the fit.

- The derivatives $\partial f_q/\partial p_i$ at the input scale $Q_0^2$ can be calculated analytically. Their values at $Q^2$ are given by evolution.

- The derivatives evolved in MELLIN-N space are transformed back to $x$-space and can then be used according to the error propagation formula above.

$\Rightarrow$ As an example the derivative of $f(x, a, b)$ w.r.t. parameter $a$ in MELLIN–N space reads:
Fit Results

- Parameter values and Covariance Matrix at the input scale $Q_0^2 = 4.0 \, GeV^2$

$$x q_i(x, Q_0^2) = A_i x^{a_i} (1 - x)^{b_i} (1 + \rho_i x^{1/2} + \gamma_i x)$$

<table>
<thead>
<tr>
<th>$u_v$</th>
<th>$a$</th>
<th>0.299 ± 0.007</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$b$</td>
<td>4.157 ± 0.031</td>
</tr>
<tr>
<td></td>
<td>$\rho$</td>
<td>0.751</td>
</tr>
<tr>
<td></td>
<td>$\gamma$</td>
<td>28.833</td>
</tr>
<tr>
<td>$d_v$</td>
<td>$a$</td>
<td>0.488 ± 0.048</td>
</tr>
<tr>
<td></td>
<td>$b$</td>
<td>6.609 ± 0.332</td>
</tr>
<tr>
<td></td>
<td>$\rho$</td>
<td>-1.690</td>
</tr>
<tr>
<td></td>
<td>$\gamma$</td>
<td>17.247</td>
</tr>
</tbody>
</table>

$\Lambda_{QCD}^{(4)} = 233 ± 34 \, MeV$

$\chi^2/ndf = 630/757 = 0.83$

- Covariance Matrix at the input scale $Q_0^2 = 4.0 \, GeV^2$

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_{QCD}^{(4)}$</th>
<th>$a_{u_v}$</th>
<th>$b_{u_v}$</th>
<th>$a_{d_v}$</th>
<th>$b_{d_v}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda_{QCD}^{(4)}$</td>
<td>1.15E-3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_{u_v}$</td>
<td>1.03E-4</td>
<td>5.40E-5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_{u_v}$</td>
<td>-8.45E-5</td>
<td>1.71E-4</td>
<td>9.59E-4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_{d_v}$</td>
<td>4.17E-4</td>
<td>8.84E-6</td>
<td>-4.35E-4</td>
<td>2.32E-3</td>
<td></td>
</tr>
<tr>
<td>$b_{d_v}$</td>
<td>2.32E-3</td>
<td>4.21E-4</td>
<td>-2.28E-3</td>
<td>1.48E-2</td>
<td>1.10E-1</td>
</tr>
</tbody>
</table>
Heavy Flavor NS-contributions

\[
\frac{F_{NS,cc}^{FS}}{(F_{NS}^{FS} + F_{NS,cc}^{FS})}
\]

- \( Q^2 = 10 \text{ GeV}^2 \)
- \( Q^2 = 100 \text{ GeV}^2 \)
- \( Q^2 = 1000 \text{ GeV}^2 \)
- \( Q^2 = 10000 \text{ GeV}^2 \)

\( Q^2 \) range of data: 4 GeV\(^2 < Q^2 < 116 \text{ GeV}^2 \)
Non-Singlet 3-Loop QCD Analysis

$F^\text{NR}_2(x, Q^2) + C$

- BCDMS
- NMC
- SLAC

$x > 0.3$

$x = 0.013, c = 11$
$x = 0.016, c = 10$
$x = 0.025, c = 9$
$x = 0.030, c = 8$
$x = 0.050, c = 7$
$x = 0.070, c = 6$
$x = 0.100, c = 5$
$x = 0.140, c = 4$
$x = 0.180, c = 3$
$x = 0.225, c = 2$
$x = 0.275, c = 1$

$x > 0.3$

$F^p_2(x, Q^2) * 2^i$

- HI
- ZEUS
- BCDMS
- NMC
- SLAC

$x = 0.30, i = 3$
$x = 0.40, i = 2$
$x = 0.50, i = 1$
$x = 0.60, i = 0$

$W^2 > 12.5 \text{ GeV}^2$

$q^p$ and $q^d$

$F^d_2(x, Q^2) * 2^i$

- BCDMS
- NMC

$x = 0.350, i = 4$
$x = 0.450, i = 3$
$x = 0.550, i = 2$
$x = 0.650, i = 1$

$W^2 > 12.5 \text{ GeV}^2$

$q^d$

J. Blümlein  Duality 05, Frascati, June 2005
Higher Twist Contributions:

$$4 < W^2 < 12.5 \text{ GeV}^2, \, Q^2 > 4 \text{ GeV}^2$$

$C_{HT}(x)$ [GeV$^2$]

$4.0 \text{ GeV}^2 < W^2 < 12.5 \text{ GeV}^2$

- Proton
- Deuteron
### Moments and Lattice Results

<table>
<thead>
<tr>
<th>$f$</th>
<th>$n$</th>
<th>This Fit</th>
<th>MRST04</th>
<th>A02</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_v$</td>
<td>2</td>
<td>0.288 ± 0.003</td>
<td>0.285</td>
<td>0.304</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.084 ± 0.001</td>
<td>0.082</td>
<td>0.087</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.0319 ± 0.0004</td>
<td>0.032</td>
<td>0.033</td>
</tr>
<tr>
<td>$d_v$</td>
<td>2</td>
<td>0.113 ± 0.004</td>
<td>0.115</td>
<td>0.120</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.026 ± 0.001</td>
<td>0.028</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.0078 ± 0.0004</td>
<td>0.009</td>
<td>0.010</td>
</tr>
<tr>
<td>$u_v - d_v$</td>
<td>2</td>
<td>0.175 ± 0.004</td>
<td>0.171</td>
<td>0.184</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.058 ± 0.001</td>
<td>0.055</td>
<td>0.059</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.0241 ± 0.0005</td>
<td>0.022</td>
<td>0.024</td>
</tr>
</tbody>
</table>

First lattice results on $u_v - d_v$, $N = 2$ yield promising values using overlap-fermions (QCDSF).

More results also are upcoming.
The Singlet Sector

Parton Densities: Relative Size

![Graph showing parton densities for MRST2001 with $Q^2 = 10 \text{ GeV}^2$.](image)

- up
- down
- antiup
- antidown
- strange
- charm
- gluon

$x f(x, Q^2)$ vs. $x$
Ratios of Unpolarized PDFs

\[ f_{PDF}(x, Q^2) \]
\[ Q^2 = (100 \text{ GeV})^2 \]
MRST/CTEQ

Parton Distributions

H1 PDF 2000: $Q^2 = 4 \text{ GeV}^2$

- Fit to H1 data
- Experimental errors
- Model uncertainties
- Fit to H1 + BCDMS data
- Parton distribution

J. Blümlein

DIS: achievements and needs

DIS05, April 2005
Slope of $F_2$ at low $x$

Very likely, that the $\overline{MS}$-gluon is remains positive!

J. Blümlein

DIS: achievements and needs

DIS05, April 2005
PILE–UP EFFECTS:
Iterative vs Exact Solution of Evolution Equations

Blümlein, Riemersma, van Neerven, Vogt, 1996
Gluon Density

QCD Fits
- (H1 + BCDMS) total uncertainty
- (H1 + BCDMS) exp. + $\alpha_s$ uncert.
- (H1 + BCDMS) exp. uncertainty
- (H1)

$x g(x, Q^2)$

$Q^2 = 20$ GeV$^2$

$Q^2 = 5$ GeV$^2$

$Q^2 = 200$ GeV$^2$

H1 Collaboration

$x$

$10^{-4}$ $10^{-3}$ $10^{-2}$ $10^{-1}$

$x g(x, Q^2)$

$3$

$2$

$1$

$0$

$10^{-6}$ $10^{-4}$ $10^{-2}$ $10^{-1}$
M. Klein, 2004: Projection for a possible measurement at HERA

of central importance to study the small $x$ behaviour of the gluon distribution
\bar{d} - \bar{u}

\begin{equation}
\frac{1}{d(x)} - \frac{1}{u(x)} = 1.195x^{1.24}(1 - x)^{9.10}(1 + 14.05x - 45.52x^2)
\end{equation}

Q^2 = 54 \text{ GeV}^2
Strange quark distribution

- CCFR: iron target, EMC effect. How large?

Can HERMES measure $s(x, Q^2)$?
$c\bar{c}$ Structure Function $F_2$

HERA $F_2^c/F_2$

- $Q^2 = 2 \text{ GeV}^2$
- H1 96-97
- ZEUS 98-00
- ZEUS 96-97
- ZEUS NLO
- QCD

$x$
Mellin-space representation:

$F_{2,q}^{\text{NLO}}(x,Q^2)$

S. Alekhin and J.B., 2004

- necessary for scheme-invariant evolution.
- fast and accurate access to heavy flavor Wilson coefficients.
Polarized Nucleons

**How is the nucleon spin distributed over the partons?**

\[ S_n = \frac{1}{2} \left[ \Delta(u + \bar{u}) + \Delta(d + \bar{d}) + \Delta(s + \bar{s}) \right] + \Delta G + L_q + L_g \]

\[ S_n = \frac{1}{2} \]

\[ \Delta \Sigma = 0.138 \pm 0.082, \quad (0.150 \pm 0.061) \]

\[ \Delta G = 1.026 \pm 0.554, \quad (0.931 \pm 0.679) \]

**EMC, 1987: the nucleon spin is not the sum of the light quark spins.**

**Measure:**

**Polarized parton densities: \( \Delta q_i, \Delta G \)**

**How can one access the parton angular momentum?**

**Polarized heavy flavor contributions.**

- **Polarized structure functions contain also twist 3 contributions.**

**How to unfold these terms?**
Polarized Parton Densities:


Analysis by other groups:
AAC (Japan), 2000, 2004
J.B., H. Böttcher, 2002
Leader et al., 2002
Altarelli et al., 1997

\[ NLO: \quad \alpha_s(M_Z^2) = 0.113^{+0.10}_{-0.08} \]

J.B., H. Böttcher, 2002

---

J. Blümlein

Duality 05, Frascati, June 2005
Currently slight move towards lower values.
Figure 12: Model fit to potential power corrections in $g_1(x, Q^2)$ as extracted from the world polarization asymmetry data in the present analysis (see text). Dashed line: model I, Eq. (70); dotted line: model II, Eq. (71). The full lines correspond to the parameterization (ISET=4) in the present analysis, to which the corresponding power correction model induces a perturbation. The shaded area corresponds to the 1σ correlated error.
**Heavy Flavor:**

$g_1$ : Watson, 1982; Vogelsang, 1990

$g_2$ : J.B., Ravindran, van Neerven, 2003
**Sum Rules and Integral Relations:**

**Twist 2:**

\[ g_2(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{dy}{y} g_1(y, Q^2) \]

Wandzura, Wilczek, 1977;

\[ g_3(x, Q^2) = 2x \int_x^1 \frac{dy}{y^2} g_4(y, Q^2) \]


**Twist 3:**

**Include Nucleon Mass Effects.**

J.B., A. Tkabladze, 1998

\[ g_1(x, Q^2) = \frac{4M^2 x^2}{Q^2} \left[ g_2(x, Q^2) - 2 \int_x^1 \frac{dy}{y} g_2(y, Q^2) \right] \]

\[ \frac{4M^2 x^2}{Q^2} g_3(x, Q^2) = g_4(x, Q^2) \left( 1 + \frac{4M^2 x^2}{Q^2} \right) + 3 \int_x^1 \frac{dy}{y} g_4(y, Q^2) \]

\[ 2xg_5(x, Q^2) = -\int_x^1 \frac{dy}{y} g_4(y, Q^2) \]
Quark Helicity Distributions

HERMES Experiment, hep-ex/0407032
(→ A. Miller's talk)

\[ Q^2 = 2.5 \text{ GeV}^2 \]


H. Böttcher

Phenomenology of PDF's...
allows very precise measurements

Example: Flavor Separation of polarized PDF's

From EIC White Paper 2002 @ 10^{31} luminosity
(J. Stoeszel and E. Kinney)
Comparison with $\Delta q$ from Semi-Incl. Data

$\Delta(u+\bar{u})(x)$

HERMES

BB-LO distribution (1)

$\Rightarrow$ z-range in the Semi-Incl. Analysis: $0.2 < z < 0.7$
**Inclusive + Semi-inclusive Analysis**


Parton densities at $Q^2 = 10 GeV^2$; error bands: $\Delta \chi^2 = 1; 2\%$. 

---

**J. Blümlein**

Duality 05, Frascati, June 2005

52
**Comparison with Lattice Moments:**

<table>
<thead>
<tr>
<th></th>
<th>Moment</th>
<th>BB, NLO</th>
<th>QCDSF</th>
<th>LHPC/SESAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta u_0$</td>
<td>0</td>
<td>0.926</td>
<td>0.889 ± 0.029</td>
<td>0.860 ± 0.069</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.163 ± 0.014</td>
<td>0.198 ± 0.008</td>
<td>0.242 ± 0.022</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.055 ± 0.006</td>
<td>0.041 ± 0.009</td>
<td>0.116 ± 0.042</td>
</tr>
<tr>
<td>$\Delta d_0$</td>
<td>0</td>
<td>-0.341</td>
<td>-0.236 ± 0.027</td>
<td>-0.171 ± 0.043</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-0.047 ± 0.021</td>
<td>-0.048 ± 0.003</td>
<td>-0.029 ± 0.013</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-0.015 ± 0.009</td>
<td>-0.028 ± 0.002</td>
<td>0.001 ± 0.025</td>
</tr>
<tr>
<td>$\Delta u_0 - \Delta d_0$</td>
<td>0</td>
<td>1.267</td>
<td>1.14 ± 0.03</td>
<td>1.031 ± 0.081</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.210 ± 0.025</td>
<td>0.245 ± 0.009</td>
<td>0.271 ± 0.025</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.070 ± 0.011</td>
<td>0.069 ± 0.009</td>
<td>0.115 ± 0.049</td>
</tr>
</tbody>
</table>

1st moments: Still problematic.
\[ \Lambda_{\text{QCD}} \text{ and } \alpha_s(M_Z^2) \]

<table>
<thead>
<tr>
<th>NLO</th>
<th>( \alpha_s(M_Z^2) )</th>
<th>expt</th>
<th>theory</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTEQ6</td>
<td>0.1165</td>
<td>±0.0065</td>
<td></td>
<td>[1]</td>
</tr>
<tr>
<td>MRST03</td>
<td>0.1165</td>
<td>±0.0020</td>
<td>±0.0030</td>
<td>[2]</td>
</tr>
<tr>
<td>A02</td>
<td>0.1171</td>
<td>±0.0015</td>
<td>±0.0033</td>
<td>[3]</td>
</tr>
<tr>
<td>ZEUS</td>
<td>0.1166</td>
<td>±0.0049</td>
<td></td>
<td>[4]</td>
</tr>
<tr>
<td>H1</td>
<td>0.1150</td>
<td>±0.0017</td>
<td>±0.0050</td>
<td>[5]</td>
</tr>
<tr>
<td>BCDMS</td>
<td>0.110</td>
<td>±0.006</td>
<td></td>
<td>[6]</td>
</tr>
<tr>
<td>BB (pol)</td>
<td>0.113</td>
<td>±0.004</td>
<td>±0.009</td>
<td>[7]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NNLO</th>
<th>( \alpha_s(M_Z^2) )</th>
<th>expt</th>
<th>theory</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRST03</td>
<td>0.1153</td>
<td>±0.0020</td>
<td>±0.0030</td>
<td>[2]</td>
</tr>
<tr>
<td>A02</td>
<td>0.1143</td>
<td>±0.0014</td>
<td>±0.0009</td>
<td>[3]</td>
</tr>
<tr>
<td>SY01(ep)</td>
<td>0.1166</td>
<td>±0.0013</td>
<td></td>
<td>[8]</td>
</tr>
<tr>
<td>SY01(\nuN)</td>
<td>0.1153</td>
<td>±0.0063</td>
<td></td>
<td>[8]</td>
</tr>
<tr>
<td>BBG</td>
<td>0.1139</td>
<td>+0.0026/ -0.0028</td>
<td></td>
<td>[9]</td>
</tr>
</tbody>
</table>

**BBG:** \( N_f = 4 \): non-singlet data-analysis at \( O(\alpha_s^3) \):
\[ \Lambda = 233 \pm 30 \text{ MeV} \]

**Alpha Collab:** \( N_f = 2 \) Lattice; non-pert. renormalization
\[ \Lambda = 245 \pm 16 \pm 16 \text{ MeV} \]

**QCDSF Collab:** \( N_f = 2 \) Lattice, pert. reno.
\[ \Lambda = 249 + 13 + 13/ -8 - 17 \text{ MeV} \] also other collab., (cf. PDG).
DIS: $\alpha_s(M_Z^2)$
Future Avenues

HERA:

- Collect high luminosity for $F_2(x, Q^2)$, $F_2^{ee}(x, Q^2)$, $g_2^{ee}(x, Q^2)$, and measure $h_1(x, Q^2)$.

- Measure: $F_L(x, Q^2)$. This is a key-question for HERA.

RHIC & LHC:

- Improve constraints on gluon and sea–quarks: polarized and unpolarized.

JLAB:

- High precision measurements in the large $x$ domain at unpolarized and polarized targets; supplements HERA’s high precision measurements at small $x$. 
ELIC:

- High precision measurements in the medium $x$ domain; both unpolarized and polarized

THE QUEST FOR LARGE LUMINOSITY!
• What is the correct value of $\alpha_s(M_Z^2)$? $\bar{\text{MS}}$-analysis vs. scheme-invariant evolution helps. Compare non-singlet and singlet analysis; careful treatment of heavy flavor. [Theory & Experiment]

• **Flavor Structure of Sea-Quarks:** More studies needed. [All Experiments]

• **Revisit polarized data upon arrival of the 3-loop anomalous dimensions; NLO heavy flavor contributions needed.** [Theory]

• **QCD at Twist 3:** $g_2(x, Q^2)$, semi-exclusive Reactions [High Precision polarized experiments, JLAB, EIC]

• **Comparison with Lattice Results:** $\alpha_s$, Moments of Parton Distributions, Angular Momentum.

• **Calculation of more hard scattering reactions at the 3-loop level:** ILC, LHC

• **Further perfection of the mathematical tools:**
  $\implies$ Algorithmic simplification of Perturbation theory in higher orders.

• **Even higher order corrections needed?**