DIS '95
Paris
April 1995

On the $k_\perp$ dependent gluon density in the proton

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DESY

1. Introduction
2. $k_\perp$ factorization & $k_\perp$ dependent gluon densities
3. An analytical expression for $\mathcal{G}(x, k^2, \mu)$
4. Numerical results
5. Conclusions

1. Introduction

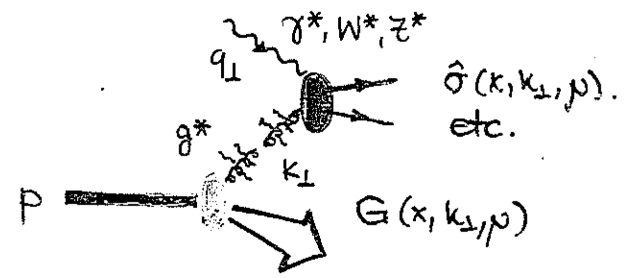
- NEW DYNAMICAL EFFECTS AT SMALL x
  - NO STRONG $k_\perp$ ORDERING
  - EVENTUALLY SCREENING

- 'PARTONDISTRIBUTIONS':

$\phi(x, k_\perp^2, \mu)$ RATHER THAN $\int_0^{\mu^2} dk_\perp^2\, \phi(x, k_\perp^2, \mu) = G(x, \mu)$.

(DRELL 1971)

- AS WELL: $k_\perp$ DEPENDENT COEFFICIENT FUNCTIONS



- $k_\perp$ FACTORIZATION (COVERS THE COLINEAR CASE FOR $\sigma(k_\perp \to 0)$ & $G(k_\perp \to 0)$).

$\longrightarrow$ • ONE MAY TRY TO MEASURE $\phi(x, k_\perp^2, \mu)$ IN DIFFERENT PROCESSES & QUANTITIES:
$F_2, F_L, \sigma(2\,\text{jet}), \sigma(Q\bar{Q}), \sigma(ep \to J/\psi\, \gamma)$ ....

- & FOR DIFFERENT TARGETS: $p, \gamma, \pi, \ldots$

VARIOUS 'PHENOMENOLOGICAL' DESCRIPTIONS

$$\phi(x,k^2) \sim \frac{1}{2\pi}\exp[-\lambda \ln x]\,(k^2)^{-\frac{1}{2}}$$

$\lambda = \bar{\alpha}_s\, 4\ln 2$ — LIPATOV et al.

$$\phi(x,k^2) = N\,\frac{x^{-\lambda}}{\sqrt{k^2}\left(\sqrt{\ln\frac{1}{x}}\right)}\exp\left[-\frac{\ln^2(k^2/k_0^2)}{\frac{14\zeta_3}{\ln 2}\lambda\ln\frac{1}{x}}\right]$$

LEVIN, RYSKIN
FORSHAW et al.
N 'SOME' NORMALIZATION. FORSHAW & ROBERTS

$$\phi(x,k^2) = \phi_0\,(1-x)^3\, f_1(x,k^2)\,\frac{0.05}{x+0.05}$$

$$f_1(x,k^2) = \begin{cases} 1 & k^2 \le q_0^2 \\ q_0^2(x)/k^2 & \text{else} \end{cases}$$

$$q_0^2(x) = Q_0^2 + \Lambda^2\exp\left(3.56\sqrt{\ln(x_0/x)}\right)$$

$Q_0^2 = 2\,\mathrm{GeV}^2$, $\Lambda = 56\,\mathrm{MeV}$, $x_0 = 1/3$. — LEVIN, RYSKIN, SALEEV et al.

$$\phi(x,k^2) \sim \left.\frac{\partial G(x,Q^2)}{\partial Q^2}\right|_{k^2=Q^2}$$

⋮

⟶ CONSISTENT DESCRIPTIONS ARE NEEDED, REFLECTING DYNAMICS BEHIND

⟶ QUANTITATIVE ANALYSIS

2. $k_\perp$ factorization & $k_\perp$ dependent gluon distributions

Factorization relation:

$$O_i(x,\mu) = \int d^2k_\perp f^{O_i}\left(x,\frac{k^2}{\mu^2}\right)\otimes\Phi(x,k^2,\mu) \quad (*)$$

$k_\perp$ distribution of the gluons (Mellin moment): LIPATOV eq.; $\bar{\alpha}_s(\mu)$

$$\widetilde{F}(j,k^2,\mu) = \gamma_c(j,\overline{\alpha}_s)\frac{1}{k^2}\left(\frac{k^2}{\mu^2}\right)^{\gamma_c(j,\overline{\alpha}_s)}\widetilde{f}(j,\mu)$$

SOL. OF THE HOMOG. LIPATOV equ.!

$$\overline{\alpha}_s = \frac{N_c}{\pi}\alpha_s(\mu^2)$$

We shall see later that $F(x,k^2,\mu)$ is *not* enforced to be positive definite through this definition alone (in the infrared).
Here,

$$\widetilde{G}(j) \equiv \mathcal{M}[G](j) = \int_0^1 dx x^{j-1}G(x)$$
$$G(x) \equiv \mathcal{M}^{-1}[\widetilde{G}](x) = \frac{1}{2\pi i}\int_C dz x^{-z}\widetilde{G}(z)$$

where $G = F, f$.

$$F(x,k^2,\mu) = \mathcal{G}(x,k^2,\mu)\otimes f(x,\mu)$$

$$\int_0^{\mu^2} dk^2\,\widetilde{F}(j,k^2,\mu) = \widetilde{f}(j,\mu)$$

The $k_\perp$ spectrum extends beyond μ. Momentum conservation in the conventional sense is only obtained if the contribution from $k_\perp > \mu$ is small.*

$$\int_0^{\mu^2} dk^2\,\mathcal{G}(x,k^2,\mu) = \delta(1-x)$$

* FURTHERMORE IT IS VIOLATED IN THE BFKL EQU.
- EITHER CONFORMAL INVARIANCE & ANALYTIC RESULTS OR E-M CONSERV. & NUM. CALC. ONLY
- 'OTHER WAY OF RESUMMATION.

$$O_i(x,\mu) = \hat{\sigma}^0_{iG}(x,\mu) \otimes G(x,\mu) + \sum_{q\in q\bar{q}} \hat{\sigma}^0_{iq}(x,\mu) \otimes q(x,\mu)$$

$$+ \int_0^\infty dk^2 \left[\hat{\sigma}_{iG}(x,k^2,\mu) - \hat{\sigma}^0_i(x,\mu)\,\theta(\mu^2-k^2)\right] \phi_G(x,k^2,\mu)$$

$$+ \sum_{q\in q\bar{q}} \int_0^\infty dk^2 \left[\hat{\sigma}_{iq}(x,k^2,\mu) - \hat{\sigma}^0_i(x,\mu)\,\theta(\mu^2-k^2)\right] \phi_q(x,k^2,\mu)$$

COLLINS, ELLIS; JB

- $\hat{\sigma}_{i\,q(G)}(x,k^2,\mu^2)$ may contain a 2nd θ function for large k^2.

- $F(x,k^2,\mu) := \phi_G(x,k^2,\mu)$

$\longrightarrow$ 2 pieces in $O_i(x,\mu)$:

a) collinear fact. scheme term:

$\sigma^{(0)}_{iG}(x,\mu) \otimes G(x,\mu)$, (etc. for quarks)

b) term containing $F(x,k^2,\mu)$ (corresp. for quarks)

$F(x,k^2,\mu)$ STARTS WITH: $\propto \bar{\alpha}_S$.

$\longrightarrow$ F is <u>not</u> a probability density, it even can become negative; F is a QCD correction!

$\longrightarrow$ The LHS of the AP equ. $\frac{\partial G}{\partial Q^2}$ is ALSO <u>NOT</u> A PROBABILITY DENSITY (& STARTS WITH $\propto \alpha_S$)

The exponent $\gamma_c(j,\overline{\alpha}_s)$ is determined by the Lipatov (BFKL) equation:

$$\boxed{j-1 = \overline{\alpha}_s \chi\left(\gamma_c(j,\overline{\alpha}_s)\right)}$$

with

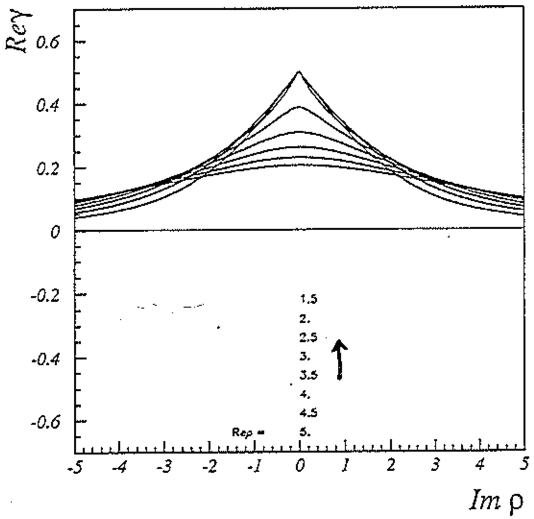
$$g^{-1} := \frac{\bar{\alpha}_s}{j-1}$$

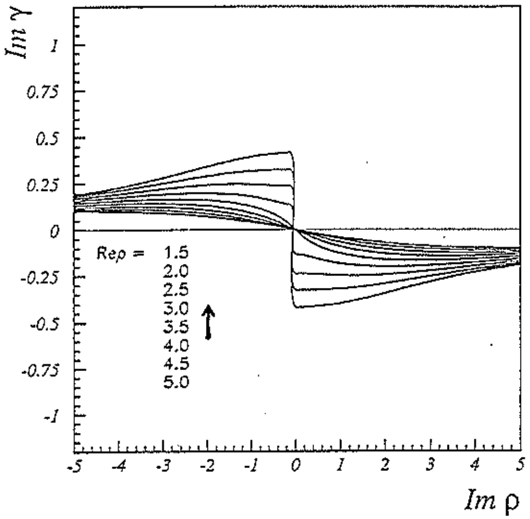
$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$$

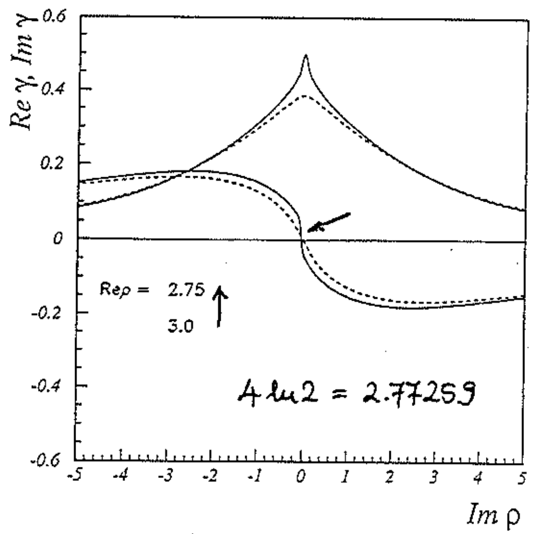
- multivalued function $\chi(\gamma) \to$ define the 'perturbative' sheet for $\gamma_c(z,\overline{\alpha}_s), z \in \mathcal{C}$, i.e. $\gamma_c(z,\overline{\alpha}_s) \sim \overline{\alpha}_s/(z-1)$ for $|z| \to \infty$
- study the structure of $\gamma_c(z,\overline{\alpha}_s)$ in the complex plane
- derive analytic expansions for $\gamma \to 0$ and $Re\gamma \to 1/2$ (may be useful for numerical solutions & the understanding of the qualitative behaviour (to some extent))

The behaviour of $\gamma_c(\rho)$ for $\rho \in C$

$Re\rho \geq 1.5$

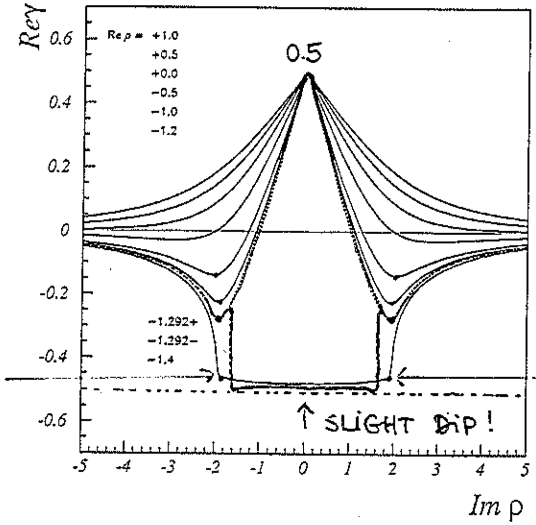






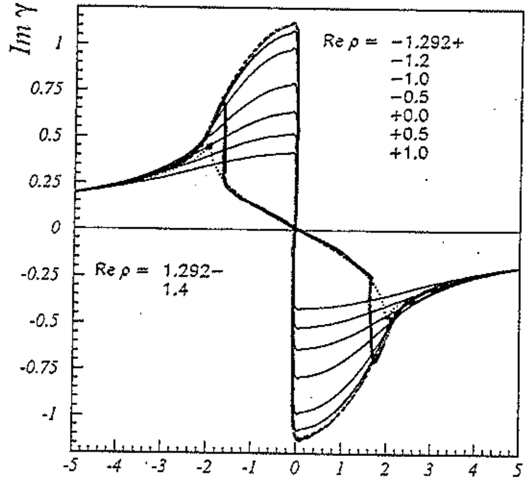
(USE: ADAPTIVE NEWTON ALGORITHM).

$1.5 > Re\rho > -1.5$



RIDGE
↓
BATH TUB

$(-1.41, \pm 1.91)$
BRANCH POINTS
√ ELLIS/HAUTMANN WEBBER.

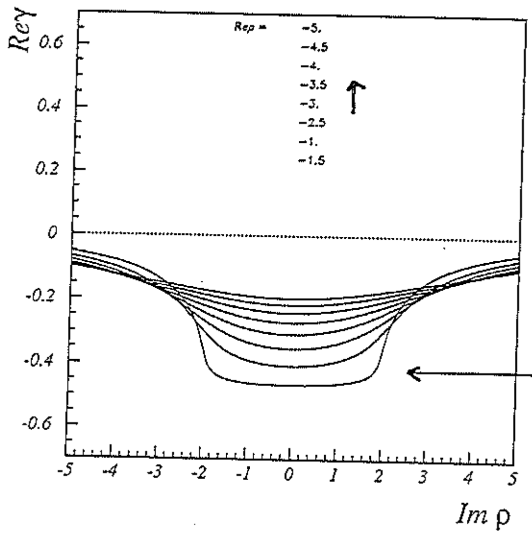


POSITION OF THE 'TRANSITION POINT': $Im\,\rho$ EXPAND AROUND $\gamma_c \sim -\frac{1}{2}$:

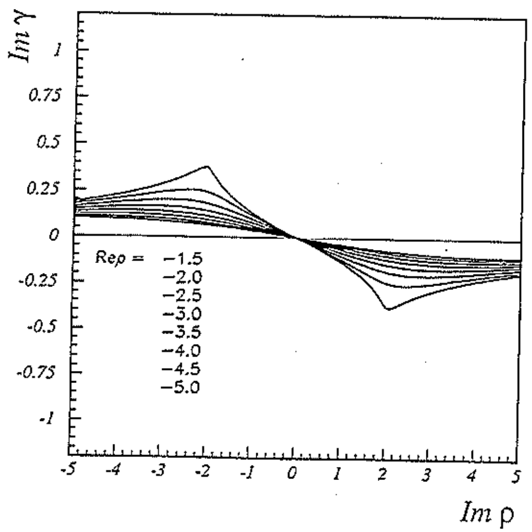
$$\rho = \underbrace{4(\log 2 - 1)}_{-1.22741} - \frac{8\alpha}{1-2\alpha} + \sum_{k=0}^{\infty} b_{2k+1}\left(2^{2(k+1)} - 2\right)\alpha^{2k}$$

$Im\,\alpha = 0$, $Re\,\alpha = 0.0082$ $\rightarrow$ $Re\,\rho \approx -1.292$.

$Re\rho \leq -1.5$



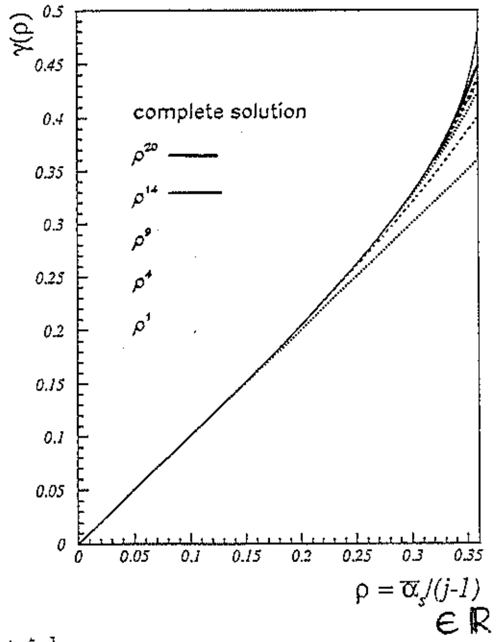
both tub



Solution for $\gamma \to 0$:

$$\gamma_c(j,\overline{\alpha}_s) = \frac{\overline{\alpha}_s}{j-1}\left\{1 + 2\sum_{k=1}^{\infty} \zeta_{2k+1}\gamma_c^{2k+1}(j,\overline{\alpha}_s)\right\}$$

$$\gamma_c(j,\overline{\alpha}_s) \equiv \gamma_c(A) = \sum_{l=1}^{\infty} g_l A^l; \quad A = \frac{\overline{\alpha}_s}{j-1}$$



$$
\begin{aligned}
g_1 &= 1\\
g_2 &= 0\\
g_3 &= 0\\
g_4 &= 2\zeta_3\\
g_5 &= 0\\
g_6 &= 2\zeta_5\\
g_7 &= 12\zeta_3^2\\
g_8 &= 2\zeta_7\\
g_9 &= 32\zeta_3\zeta_5\\
g_{10} &= 2[48\zeta_3^3 + \zeta_9]\\
g_{11} &= 2[20\zeta_3\zeta_7 + 10\zeta_5^2]\\
g_{12} &= 2[220\zeta_3^2\zeta_5 + \zeta_{11}]\\
g_{13} &= 2[440\zeta_3^4 + 24\zeta_3\zeta_9 + 24\zeta_5\zeta_7]\\
g_{14} &= 2[312\zeta_3^2\zeta_7 + 312\zeta_5^2\zeta_3 + \zeta_{13}]\\
g_{15} &= 2[2912\zeta_3^3\zeta_5 + 28\zeta_3\zeta_{11} + 28\zeta_5\zeta_9 + 14\zeta_7^2]\\
g_{16} &= 2[4368\zeta_3^5 + 420\zeta_3^2\zeta_9 + 840\zeta_3\zeta_5\zeta_7 + 140\zeta_5^3 + \zeta_{15}]\\
g_{17} &= 2[6720\zeta_3^2\zeta_5^2 + 4480\zeta_3^3\zeta_7 + 32\zeta_3\zeta_{13} + 32\zeta_5\zeta_{11} + 32\zeta_7\zeta_9]\\
g_{18} &= 2[1088\zeta_3\zeta_5\zeta_9 + 544\zeta_3\zeta_7^2 + 544\zeta_3^2\zeta_{11} + 38080\zeta_3^4\zeta_5 + 544\zeta_5^2\zeta_7 + \zeta_{17}]\\
g_{19} &= 2[6528\zeta_3\zeta_5^3 + 36\zeta_3\zeta_{15} + 19584\zeta_3^2\zeta_5\zeta_7 + 6528\zeta_3^3\zeta_9 + 45696\zeta_3^6 + 36\zeta_5\zeta_{13} + 36\zeta_7\zeta_{11} + 18\zeta_9^2]\\
g_{20} &= 2[1368\zeta_3\zeta_5\zeta_{11} + 1368\zeta_3\zeta_7\zeta_9 + 684\zeta_3^2\zeta_{13} + 124032\zeta_3^3\zeta_5^2 + 62016\zeta_3^4\zeta_7 + 684\zeta_5^2\zeta_9 + 684\zeta_5\zeta_7^2 + \zeta_{19}]
\end{aligned}
$$

MARCHESINI, CATANI et al.

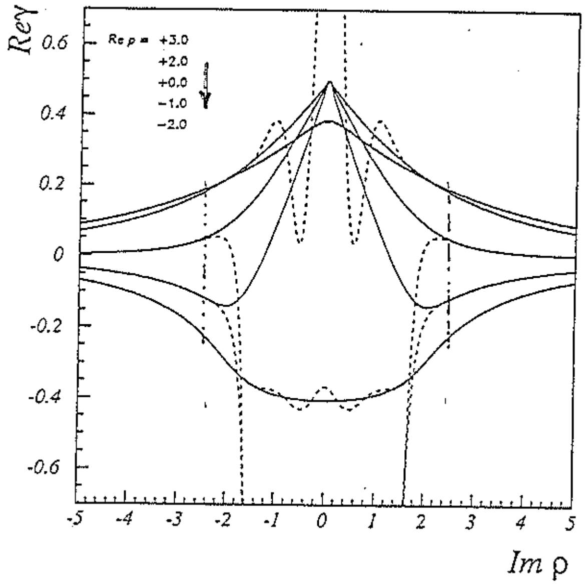
⋮

One has:

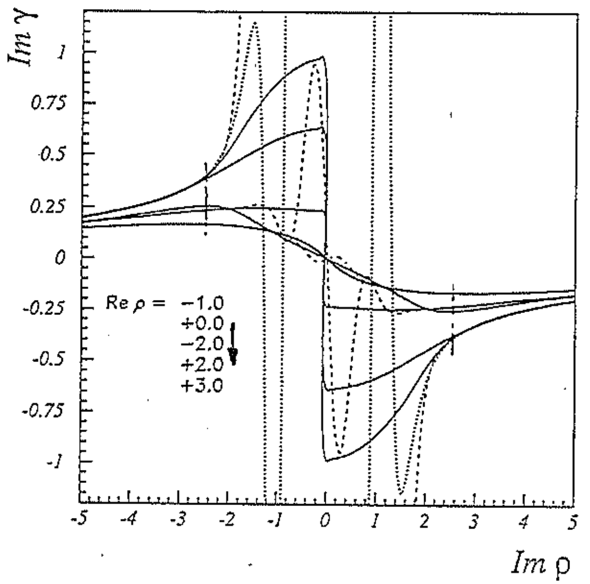
$$g_n \sim \sum_\sigma a_\sigma \left(\prod_i \zeta_{\nu_i}^{\mu_i}\right)_\sigma \Longrightarrow \sum_i \mu_i\nu_i = n-1$$

$\gamma_c(\rho) \approx \sum_{l=1} g_l \rho^l$, $N = 14$ -----

vs. complete sol.



POLE STRUCT. $\leftrightarrow$ POWER SERIES



$$\boxed{\text{Solution for } \gamma \rightarrow 1/2:}$$

$$\frac{1}{\rho} := \frac{j-1}{\overline{\alpha}_s} \qquad \gamma := \frac{1}{2} - \alpha$$

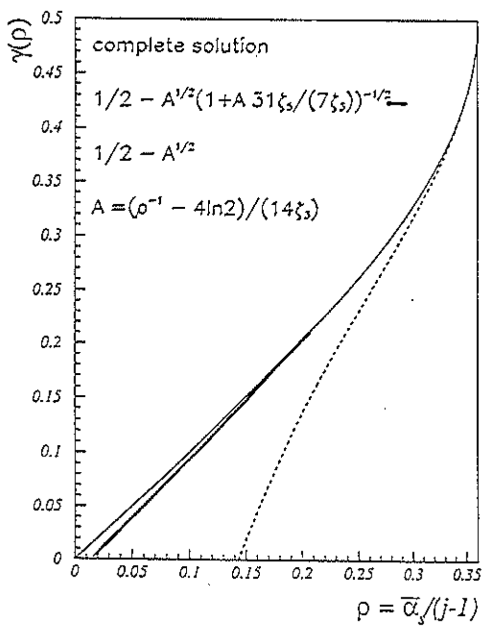
$$\frac{1}{\rho} = 2\psi(1) - \psi\left(\frac{1}{2} - \alpha\right) - \psi\left(\frac{1}{2} + \alpha\right)$$

$$\frac{1}{\rho} = 4\log 2 + \sum_{n=1}^{\infty} \zeta_{2n+1}\left(2^{2(n+1)} - 2\right)\alpha^{2n}$$

$$\alpha_{(0)} \approx 0 \qquad \gamma_c^{(0)} \approx \frac{1}{2}$$

$$\gamma_c^{(1)} \approx \frac{1}{2} - \sqrt{\left(\frac{1}{\rho} - 4\log 2\right)\frac{1}{14\zeta_3}} = \frac{1}{2} - \alpha_{(1)}$$

$$\gamma_c^{(2)} \approx \frac{1}{2} - \frac{\alpha_{(1)}(\rho)}{\sqrt{1 + (31\zeta_5/7\zeta_3)\alpha_{(1)}^2(\rho)}}$$

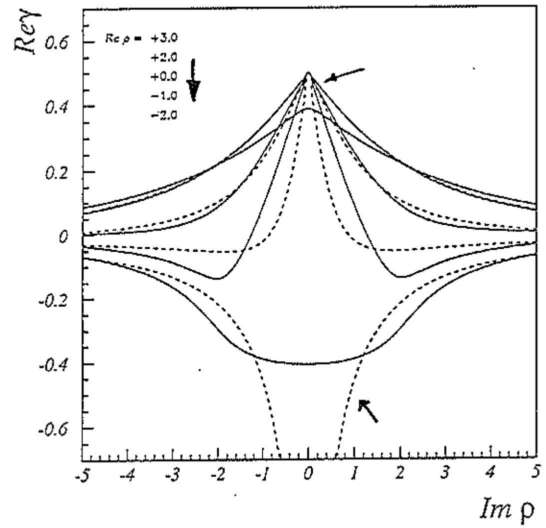


(ONLY UP TO ζ_5 ...)

$\in \mathbb{R}$

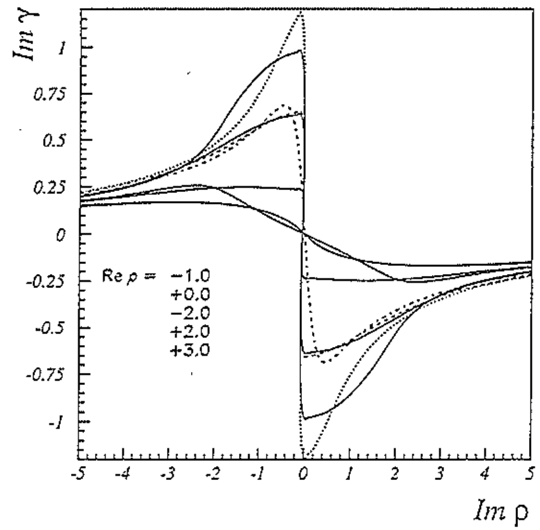
EXPANSION IN $\mathbb{C}$ AROUND $\frac{1}{2}$

$$\gamma_c(\rho) \approx \frac{1}{2} - \sqrt{\frac{\frac{1}{14\zeta_3}(\rho - 4\log 2)}{1 + \frac{31\zeta_5}{98\zeta_3^2}(\rho - 4\log 2)}} \quad ----$$



vs. complete sol.

MORE 'SHAPE PRESERVING' FOR $\mathrm{Re}\,\rho \gtrsim -1$.



3. An analytical solution for $\mathcal{G}(x, k^2, \mu)$ using

$$\gamma_c(\rho) = \sum_{k=1}^{\infty} g_k \rho^k$$

$$k^2 \overline{\mathcal{G}}(z, k^2, \mu) = \gamma_c(z, \overline{\alpha}_s) \exp\left[\gamma_c(z, \overline{\alpha}_s) L\right]$$

with

$$\gamma_c(z, \overline{\alpha}_s) = \sum_{k=1}^{\infty} g_k \left(\frac{\overline{\alpha}_s}{z-1}\right)^k$$

$$L := \log\left(\frac{k^2}{\mu^2}\right)$$

The x dependent function is uniquely determined by the moments due to CARLSON's theorem.

$$\mathcal{G}(x, k^2, \mu) = \mathcal{M}^{-1}\left\{\overline{\mathcal{G}}(z, k^2, \mu)\right\}(x) \tag{1}$$

Particularly one has:

$$\frac{\overline{\alpha}_s^l}{(z-1)^l} = \int_0^1 dx x^{z-1} \left[\otimes_{k=1}^{l} \frac{1}{x}\right] \overline{\alpha}_s^l$$

$$\overline{\alpha}_s^l \otimes_{k=1}^{l} \frac{1}{x} = \frac{1}{x} \frac{\overline{\alpha}_s^l}{(l-1)!} \log^{l-1}\left(\frac{1}{x}\right), l \geq 1 \qquad l=1 \quad \mathrm{LO}$$

$$\left.\begin{aligned} \mathcal{M}^{-1}\{\rho \exp[\rho L]\}(x) &= \frac{\overline{\alpha}_s}{x} I_0\left(2\sqrt{\overline{\alpha}_s \log(1/x) L}\right) & L > 0 \\ &= \frac{\overline{\alpha}_s}{x} J_0\left(2\sqrt{\overline{\alpha}_s \log(1/x)|L|}\right) & L < 0 \end{aligned}\right\} \mathrm{DLA}$$

$$\lim_{k^2 \to 0} \mathcal{M}^{-1}\{\rho \exp[\rho L]\}(x) = \lim_{|L| \to \infty} \frac{\cos(2\sqrt{\overline{\alpha}_s |L| \log(1/x)})}{\sqrt{\pi \sqrt{\overline{\alpha}_s |L| \log(1/x)}}} = 0$$

$$\mathcal{M}^{-1}\{\rho^\sigma \exp[\rho L]\}(x) = \frac{\overline{\alpha}_s}{x} \left(\frac{\overline{\alpha}_s \log(1/x)}{L}\right)^{(\sigma-1)/2} I_{\sigma-1}(2\sqrt{\overline{\alpha}_s L \log(1/x)})$$

~ HIGHER TERMS DUE TO BFKL (NON DLA

DLA

$$
\begin{aligned}
k^2\mathcal{G}(x,k^2,\mu) &= \frac{\overline{\alpha}_s}{x} I_0\left(2\sqrt{\overline{\alpha}_s \log(1/x) L}\right) \\
&+ \frac{\overline{\alpha}_s}{x}\sum_{\nu=4}^{\infty} d_\nu(L)\left(\frac{\overline{\alpha}_s \log(1/x)}{L}\right)^{(\nu-1)/2} I_{\nu-1}\left(2\sqrt{\overline{\alpha}_s \log(1/x) L}\right), \qquad L>0
\end{aligned}
$$

where

$$
\begin{aligned}
d_4 &= g_4 \\
d_5 &= g_4 L \\
d_6 &= g_6 \\
d_7 &= g_7 + g_6 L \\
d_8 &= g_8 + +16\zeta_3^2 L \\
d_9 &= g_9 + g_8 L + 2\zeta_3^2 L^2 \\
d_{10} &= g_{10} + 40\zeta_3\zeta_5 L \\
d_{11} &= g_{11} + (144\zeta_3^3 + 2\zeta_9)L + 4\zeta_3\zeta_5 L^2 \\
d_{12} &= g_{12} + 24(2\zeta_3\zeta_7 + \zeta_5^2)L + 28\zeta_3^3 L^2 \\
d_{13} &= g_{13} + 2(\zeta_{11} + 308\zeta_3^2\zeta_5)L + 2(2\zeta_3\zeta_7 + \zeta_5^2)L^2 + \frac{4}{3}\zeta_3^3 L^3 \\
d_{14} &= g_{14} + 8(7\zeta_3\zeta_9 + 176\zeta_3^4 + 7\zeta_5\zeta_7)L + 100\zeta_3^2\zeta_5 L^2 \\
d_{15} &= g_{15} + [832(\zeta_3\zeta_5^2 + \zeta_3^2\zeta_7) + 2\zeta_{13}]L + (4\zeta_3\zeta_9 + 336\zeta_3^4 + 4\zeta_5\zeta_7)L^2 + 4\zeta_3^2\zeta_5 L^3 \\
d_{16} &= g_{16} + (64\zeta_3\zeta_{11} + 8736\zeta_3^3\zeta_5 + 64\zeta_5\zeta_9 + 32\zeta_7^2)L + 116(\zeta_3\zeta_5^2 + \zeta_3^2\zeta_7)L^2 + \frac{80}{3}\zeta_3^4 L^3 \\
d_{17} &= g_{17} + (2160\zeta_3\zeta_5\zeta_7 + 1080\zeta_3^2\zeta_9 + 14560\zeta_3^5 + 360\zeta_5^3 + 2\zeta_{15})L \\
&+ (4\zeta_3\zeta_{11} + 1792\zeta_3^3\zeta_5 + 4\zeta_5\zeta_9 + 2\zeta_7^2)L^2 + 4(\zeta_3\zeta_5^2 + \zeta_3^2\zeta_7)L^3 + \frac{2}{3}\zeta_3^4 L^4 \\
d_{18} &= g_{18} + [72(\zeta_3\zeta_{13} + \zeta_5\zeta_{11} + \zeta_7\zeta_9) + 19200\zeta_3^2\zeta_5^2 + 12800\zeta_3^3\zeta_7]L \\
&+ (264\zeta_3\zeta_5\zeta_7 + 132\zeta_3^2\zeta_9 + 3920\zeta_3^5 + 44\zeta_5^3)L^2 + \frac{368}{3}\zeta_3^3\zeta_5 L^3 \\
d_{19} &= g_{19} + [2720\zeta_3\zeta_5\zeta_9 + 1360(\zeta_3\zeta_7^2 + \zeta_3^2\zeta_{11} + \zeta_5^2\zeta_7) + 119680\zeta_3^4\zeta_5 + 2\zeta_{17}]L \\
&+ [3456\zeta_3^2\zeta_5^2 + 2304\zeta_3^3\zeta_7 + 4(\zeta_3\zeta_{13} + \zeta_5\zeta_{11} + \zeta_7\zeta_9)]L^2 \\
&+ (8\zeta_3\zeta_5\zeta_7 + 4\zeta_3^2\zeta_9 + 400\zeta_3^5 + \frac{4}{3}\zeta_5^3)L^3 + \frac{8}{3}\zeta_3^3\zeta_5 L^4 \\
d_{20} &= g_{20} + [17952(\zeta_3\zeta_5^3 + \zeta_3^3\zeta_9) + 80(\zeta_3\zeta_{15} + \zeta_5\zeta_{13} + \zeta_7\zeta_{11}) + 53856\zeta_3^2\zeta_5\zeta_7 + 156672\zeta_3^6 + 40\zeta_9^2]L \\
&+ [296\zeta_3\zeta_5\zeta_9 + 148(\zeta_3\zeta_7^2 + +\zeta_3^2\zeta_{11} + \zeta_5^2\zeta_7) + 28288\zeta_3^4\zeta_5]L^2 + (208\zeta_3^2\zeta_5^2 + \frac{416}{3}\zeta_3^3\zeta_7)L^3 + \frac{52}{3}\zeta_3^5 L^4 \\
&\vdots
\end{aligned}
$$

For $L \to 0$ one obtains:

$$
\mu^2\mathcal{G}(x,\mu^2,\mu) = \frac{\overline{\alpha}_s}{x}\sum_{l=1}^{\infty}\frac{g_l}{(l-1)!}\left[\overline{\alpha}_s \log\left(\frac{1}{x}\right)\right]^{l-1}
$$

For $L < 0$ one obtains:

$$
\begin{aligned}
k^2\mathcal{G}(x,k^2,\mu) &= \frac{\overline{\alpha}_s}{x} J_0\left(2\sqrt{\overline{\alpha}_s \log(1/x)|L|}\right) \\
&+ \frac{\overline{\alpha}_s}{x}\sum_{\nu=4}^{\infty} d_\nu(L)\left(\frac{\overline{\alpha}_s \log(1/x)}{|L|}\right)^{(\nu-1)/2} J_{\nu-1}\left(2\sqrt{\overline{\alpha}_s \log(1/x)|L|}\right),
\end{aligned}
$$

i.e. in the infrared range $k^2 < \mu^2$ the Green's function is represented by oscillating functions which are damped as $|L| \to \infty$ since

$$
\begin{aligned}
&\mathcal{M}^{-1}\left\{A^{\rho+4\kappa}L^\kappa \exp[AL]\right\}(x) = \\
&\frac{\overline{\alpha}_s}{x}\left(\overline{\alpha}_s \log(1/x)\right)^{(\rho+4\kappa-1)/2} J_{\rho+4\kappa-1}\left(2\sqrt{\overline{\alpha}_s |L| \log(1/x)}\right)\left(\frac{1}{|L|}\right)^{(\rho+2\kappa-1)/2} \\
&\propto \left(\frac{1}{|L|}\right)^{\rho/2+\kappa-1/4}, \qquad \rho \geq 1, \kappa \geq 0
\end{aligned}
$$

4. Numerical results for

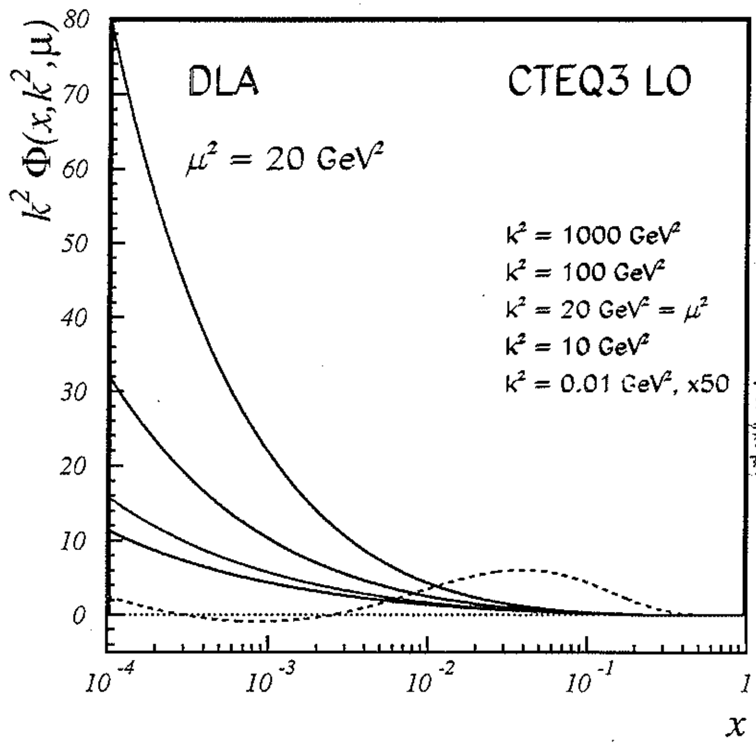
$$k^2\Phi(x,k^2,\mu) = xk^2\{\mathcal{G}(x,k^2,\mu)\otimes G(x,\mu)\}$$

- $k^2\Phi(x,k^2,\mu)^{LO}, k^2\Phi(x,k^2,\mu)^{DLA}$ vs x, k^2
- $k^2\Phi(x,k^2,\mu)^{Lip}$ & $k^2\Phi(x,k^2,\mu)^{DLA}$
- $k^2\Phi(x,k^2,\mu)^{Appr}_{N=20}$ & $k^2\Phi(x,k^2,\mu)^{Lip}$

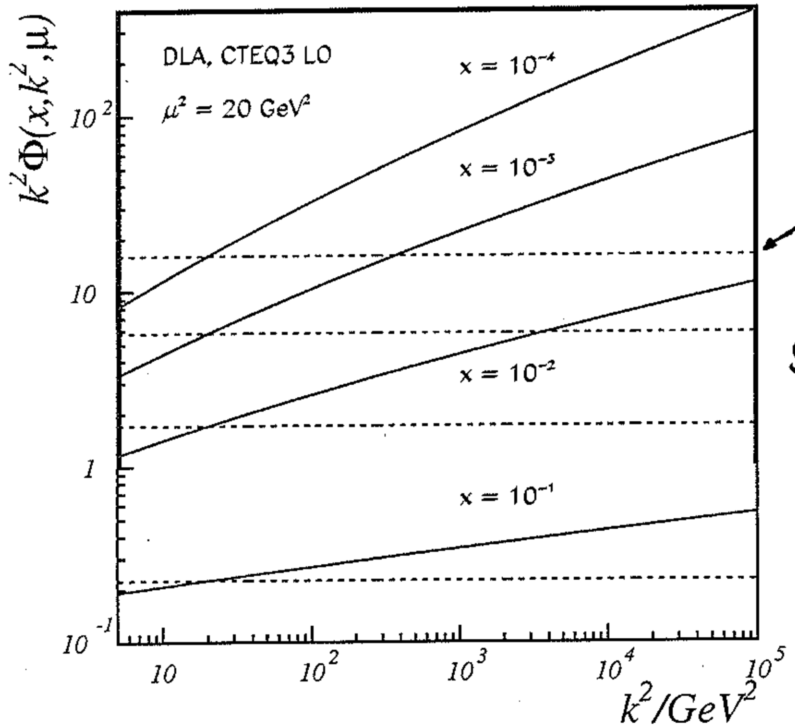
THE LOWEST ORDER TERM :

$$\cdot\ k^2\phi(x,k^2,\mu) = x\,k^2\,\mathcal{G}_0(x,k^2,\mu)\otimes G(x,\mu)$$

$$\mathcal{G}_0(x,k^2,\mu) = \frac{1}{k^2}\frac{\bar{\alpha}_s}{x}\,I_0\left(2\sqrt{\bar{\alpha}_s\log\frac{1}{x}\log\frac{k^2}{\mu^2}}\right)$$

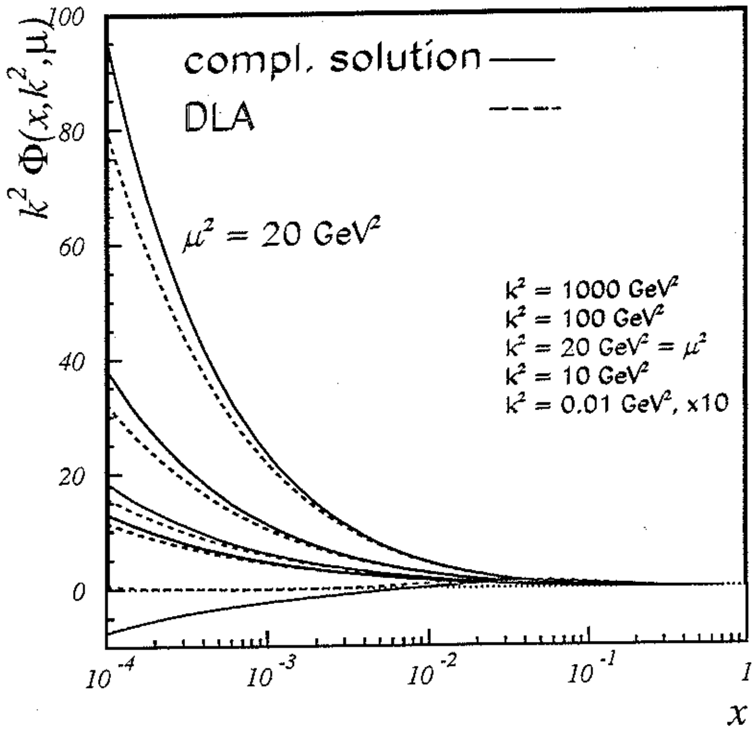


EFFECT OF I_0 AFTER CONVOLUTION



LOWEST ORDER IN $\overline{\alpha}_s$.

$$\mathcal{G}_{00} = \frac{\overline{\alpha}_s}{x k^2}.$$

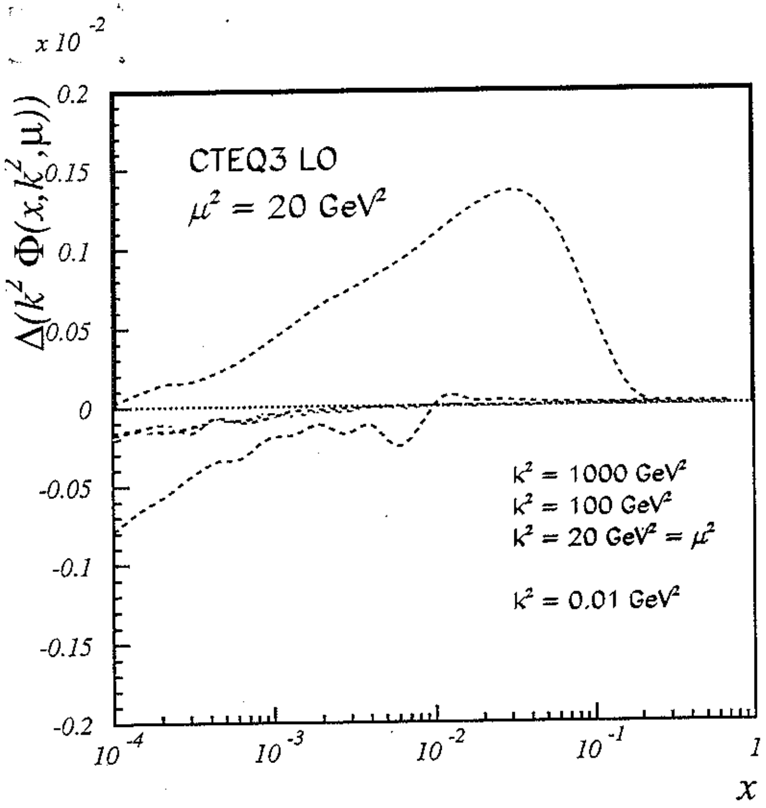


LIPATOV eq. ($\overline{\alpha}_s(\mu)$). MELLIN INVERSION.

USED:

$$\mathcal{G}^{N=20}_{APPR}(x, k^2, \mu) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dz\, \gamma_c^{APPR}(\bar{\alpha}_s, z)\left(\frac{k^2}{\mu^2}\right)^{\gamma_c^{APPR}(\bar{\alpha}_s, z)} \cdot x^{-z}$$

$$\gamma_c^{APPR}(\bar{\alpha}_s, z) = \sum_{k=1}^{N} g_k\left(\frac{\bar{\alpha}_s}{z-1}\right)^k .$$



$\sim \pm 2‰$

@ $N = 20$.

5. Conclusions

1. THE $k_\perp$ DEPENDENT GLUON DISTRIBUTION WAS CALCULATED IN LO (BFKL eq.).

2. A CONSISTENT TREATMENT OF OBSERVABLES IS POSSIBLE IN THE CE-SCHEME.

3. BOTH AN ANALYTICAL $(\mathcal{O}(\bar{\alpha}_s^{20}))$ AND A NUMERICAL SOLUTION (MELLIN INVERSION) FOR $\phi(x, k^2, \mu) = \mathcal{G}(x, k^2, \mu) \otimes G(x, \mu)$ WAS DERIVED.
   THESE REPRESENTATIONS AGREE BETTER THAN 2‰.

4. THE EFFECT OF THE NON-DLA TERMS IN $\phi(x, k^2, \mu)$ IS OF OF $\mathcal{O}(10 \ldots 15\%)$

$\longrightarrow$ THIS WILL BE ABOUT THE LEVEL IN ALL OBSERVABLES DUE TO GLUONS ONLY.

$\Longrightarrow$ CHALLANGE FOR THE EXPERIMENTS !

$\longrightarrow$ BEST OBSERVABLES FOR THIS NEED STILL TO BE FOUND OUT !
($\hat{\sigma}$ MAY BE COUNTERPRODUCTIVE.)

FINAL REMARK: WHERE WE ALLOWED TO RESUM?

$\rightarrow$ ORDER BY ORDER THE TERMS TO BE RESUMMED HAVE TO BE THE DOMINATING ONES !

! OBLIGATORY: TO CHECK THIS. (WOULD BE GOOD TO HAVE EVEN A THEOREM / MORE GENERAL CRITERIA.