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TWIST-2 HEAVY FLAVOR CONTRIBUTIONS TO THE STRUCTURE FUNCTION $g_2(x,Q^2)$

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DESY

1. INTRODUCTION
2. LONGITUDINAL GLUON POLARIZATION
3. THE GENERAL CASE
4. RESULTS
5. CONCLUSIONS

WORK IN COMMON WITH V. RAVINDRAN AND
W.L. VAN NEERVEN

1. INTRODUCTION

- $Q\bar{Q}$ CONTRIBUTIONS TO SF'S ARE LARGE IN THE UNPOLARIZED CASE $O(30-40\%)$ @ SMALL x $\longrightarrow$ LARGE GLUON

- THE POLARIZED GLUON DENSITY IS MORE MODEST BUT NOT SMALL.

- HEAVY FLAVOR CONTRIBUTIONS HAVE DIFFERENT SCALING VIOLATIONS AS THOSE DUE TO LIGHT FLAVORS.

- WHAT ARE THE <u>LO</u> CONTRIBUTIONS TO $g_{1,2}(x, Q^2)$ FOR $Q\bar{Q}$ FINAL STATES ?

NLO : NOT YET KNOWN; $g_{1,LO}^{Q\bar{Q}}$ KNOWN.
WATSON, VOGELSANG

IN THE PAST THE TWIST-2 CONTRIBUTIONS TO g_2 COULD BE RELATED TO THOSE OF g_1 FOR FERMIONIC MATRIX ELEMENTS.

JACKSON, ROBERTS, ROSS, JB, KOCHELEV, TKABLADZE, GEYER, ROBASCHIK

WHAT HAPPENS FOR GLUONIC MATRIX ELEMENTS ?

2. LONGITUDINAL GLUON POLARIZATION

$$s_\mu = \xi_1 S_\mu^L - \xi_2 k_\mu = \xi_3 p_\mu$$

$$W_{\mu\nu}^{(A)} = \frac{i}{q\cdot k}\,\varepsilon_{\mu\nu\rho\sigma}\left\{q^\rho s^\sigma g_1^{PART}(z,Q^2) + \underbrace{\left(q^\rho s^\sigma - \frac{s q}{q k}\, q^\rho k^\sigma\right) g_2^{PART}}_{\equiv\, 0.}\right]$$

TRANSVERSE EFFECTS CANNOT BE DESCRIBED.

SOMETIMES PEOPLE WRITE:

$$\text{"}\,g_2 = -g_1\,\text{"}.$$

→ THIS WILL NOT BE THE SOLUTION.

THE ABOVE REPRESENTATION YIELDS THE CORRECT CONTR. TO $g_1(x,Q^2)$:

$$g_1^{Q\bar{Q}}(x,Q^2) = 2e_Q^2\,\frac{\alpha}{2\pi}\int_{ax}^{1}\frac{dy}{y}\,C_{g_1}^{Q\bar{Q}}\left(\frac{x}{y},\frac{M_Q^2}{Q^2}\right)\Delta G(y,Q^2)$$

$$C_{g_1}^{Q\bar{Q}}(z,Q^2) = \frac{1}{2}\left[\beta(3-4z) - (1-2z)\ln\left|\frac{1+\beta}{1-\beta}\right|\right]$$

$\uparrow$ $z^2-(1-z)^2 = P_{qG}$

$$\beta = \sqrt{1-\frac{4M_Q^2}{Q^2}\,\frac{z}{1-z}}$$

A. WATSON 1982

W. VOGELSANG 1991.

3. THE GENERAL CASE

$$S_\mu = S_\mu^{\parallel} + S_\mu^{\perp} \quad , \quad S_\mu^{\parallel} = (0; 0, 0, M) \qquad \text{RF}$$

$$S_\mu^{\perp} = M(0; \cos\alpha, \sin\alpha, 0)$$

$$W_{\mu\nu}^{(A)}(p, q, S) = \int d^4k \left[f_+(p, k, S) - f_-(p, k, S) \right] \times W_{\mu\nu}^{(A)}(p, q, k, s)$$

$$s_\mu = \frac{p.k}{\sqrt{(pk)^2 k^2 - M^2 k^4}} \left[k_\mu - \frac{k^2}{pk} p_\mu \right].$$

$$s.k = 0$$
$$s.s = -k^2.$$

$$\Delta f = \frac{M\, p.k}{\sqrt{(pk)^2 k^2 - M^2 k^4}} (f_+ - f_-) \equiv \frac{S.k}{M^2} \tilde{f}(p^2; p.k, k^2).$$

$$W_{\mu\nu}^{(A)} = \frac{i}{M^2} \varepsilon_{\mu\nu\alpha\beta} \int d^4k\, \tilde{f}\, \frac{S.k}{q.k} \left[q^\alpha k^\beta \left(\hat{g}_1 + \frac{k^2\, p.q}{qk\, pk} \hat{g}_2 \right) - \frac{q^\alpha p^\beta k^2}{p.k} (\hat{g}_1 + \hat{g}_2) - \frac{k^\alpha p^\beta k^2}{p.k} \hat{v} \right]$$

$$W_{\mu\nu}^{A} = \frac{i}{M^2} \varepsilon_{\mu\nu\alpha\beta} \int d^4k\, (S.K) \left\{ q^\alpha k^\beta \left[\frac{\partial a_1}{\partial pk} \frac{\partial b_1}{\partial qk} + pq \frac{\partial a_2}{\partial pk} \frac{\partial b_2}{\partial q \ldots} \right. \right.$$

$$+ q^\alpha p^\beta \left[\frac{\partial a_3}{\partial pk} \frac{\partial b_3}{\partial qk} + \frac{\partial a_4}{\partial pk} \frac{\partial b_4}{\partial qk} \right]$$

$$\left. + k^\alpha p^\beta \frac{\partial a_5}{\partial pk} \frac{\partial b_5}{qk} \right\}.$$

TENSORIAL DECOMPOSITION:

$$W^{(A)}_{\mu\nu} = \frac{i}{M^2}\,\varepsilon_{\mu\nu\alpha\beta}\,q^\alpha S_\tau \left[I_1^{\beta\tau} + I_2^{\beta\tau} + p^\beta J_1^\tau + p^\beta J_2^\tau \right]$$

$$+ \frac{i}{M^2}\,\varepsilon_{\mu\nu\alpha\beta}\,S_\tau\, p^\beta \left[K^{\alpha\tau} \right]$$

$$I_{1,2}^{\beta\tau} = \int d^4k\, \frac{k^\beta k^\tau}{qk}\, \tilde{f}\, \hat{g}_{1,2} = A_{1,2}\, g^{\beta\tau} + B_{1,2}\, p^\beta p^\tau + C_{1,2}\, q^\beta q^\tau$$

$$+ D_{1,2}\left(p^\beta q^\tau + p^\tau q^\beta\right)$$

$$J_{1,2}^\tau = -\int d^4k\, \frac{k^2}{pk}\, \frac{k^\tau}{qk}\, \tilde{f}\, \hat{g}_{1,2} = E_{1,2}\, p^\tau + H_{1,2}\, q^\tau$$

$$K^{\alpha\tau} = -\int d^4k\, \frac{k^2 k^\alpha k^\tau}{(p\cdot k)(qk)}\, \tilde{f}\, \hat{v} = A_v\, g^{\alpha\tau} + B_v\, p^\alpha p^\tau$$

$$+ C_v\, q^\alpha q^\tau + D_v\left(p^\alpha q^\tau + p^\tau q^\alpha\right)$$

$$B_{1,2} = C_{1,2} = E_{1,2} = B_v = D_v = 0.$$

A_v VANISHES DUE TO GAUGE INVARIANCE.

$$W^{(A)}_{\mu\nu} = i\,\varepsilon_{\mu\nu\alpha\beta}\, q^\alpha S^\beta \left[A_1 + A_2 \right]$$

$$+ i\,\varepsilon_{\mu\nu\alpha\beta}\, q^\alpha p^\beta (S\cdot q)\left[D_1 + D_2 + H_1 + H_2 + C_v \right].$$

ONE OBTAINS:

$$g_1(x,Q^2) + g_2(x,Q^2) = \int d^4k \, \frac{pq}{M^2} \left\{ \left[\frac{k^2}{2pk} + \frac{q^2(pk)^2}{2\,qk\,(pq)^2} - \frac{pk}{pq} \right] * \tilde{f} \cdot \hat{g}_1 \right.$$

$$\left. + \left[\frac{k^4\, pq}{2(qk)^2\, pk} + \frac{q^2 k^2\, pk}{2pq\,(qk)^2} - \frac{k^2}{qk} \right] \tilde{f} \cdot \hat{g}_2 \right\}$$

$$= \int d^4k \left[\frac{q^2\,(pk)^2}{2(qk)(pq)} - (pk) \right] \frac{\tilde{f} \cdot \hat{g}_1}{M^2}$$

$$+ \int d^4k \left(\frac{k^2}{Q^2} \right) \phi_1(k,p,q)$$

$$g_1(x,Q^2) = \int d^4k \, \frac{pq}{M^2} \left\{ \left[-\frac{k^2}{pq} - \frac{q^2(pk)^2}{qk\,(pq)^2} + \frac{pk}{pq} \right] \tilde{f} \hat{g}_1 \right.$$

$$\left. - \left[\frac{q^2 k^2\, pk}{(qk)^2\, pq} \right] \tilde{f} \cdot g_2 - \left[\frac{pk\; k^2}{qk\; pq} \right] \tilde{f} \cdot \hat{v} \right\}$$

$$= \int d^4k \left[-\frac{q^2(pk)^2}{qk\; pq} + pk \right] \frac{\tilde{f}\,\hat{g}_1}{M^2}$$

$$+ \int d^4k \left(\frac{k^2}{Q^2} \right) \phi_2(k,p,q)$$

$$\phi_{1,2}(k,p,q) \xrightarrow[k^2 \to 0]{} \text{FINITE}$$

RELATION TO A GENERATING FUNCTIONAL:

★ $F_i(p^2, pq, q^2) = \int d^4k \, a_i(p^2, pk, k^2) \cdot b_i(q^2, qk, k^2)$

$$\frac{\partial F}{\partial q^\sigma} = p_\sigma \frac{\partial F}{\partial pq} + 2 q_\sigma \frac{\partial F}{\partial q^2}$$

$$\frac{\partial^2 F}{\partial p^\lambda \partial q^\sigma} = g_{\lambda\sigma} \frac{\partial F}{\partial pq} + 2 p_\sigma p_\lambda \frac{\partial^2 F}{\partial p^2 \partial pq} + 2 q_\sigma q_\lambda \frac{\partial^2 F}{\partial q^2 \partial pq}$$
$$+ 2 p_\sigma q_\lambda \frac{\partial^2 F}{\partial (pq)^2} + 4 p_\sigma p_\lambda \frac{\partial^2 F}{\partial q^2 \partial p^2}$$
$$\equiv \int d^4k \Big[k_\lambda k_\sigma \frac{\partial a}{\partial pk} \frac{\partial b}{\partial qk} + 2 k_\lambda q_\sigma \frac{\partial a}{\partial pk} \frac{\partial b}{\partial q^2}$$
$$+ 2 p_\lambda k_\sigma \frac{\partial a}{\partial p^2} \frac{\partial b}{\partial qk} + 4 p_\lambda q_\sigma \frac{\partial a}{\partial p^2} \frac{\partial b}{\partial q^2} \Big].$$

THE FOLLOWING STRUCTURES CONTRIBUTE:

$$\varepsilon_{\mu\nu\alpha\sigma} q^\alpha s_\lambda \int d^4k \, k^\lambda k^\sigma \frac{\partial a}{\partial pk} \frac{\partial b}{\partial qk} = \varepsilon_{\mu\nu\alpha\beta} q^\alpha \Big[s^\beta \frac{\partial F}{\partial pq}$$
$$+ (s \cdot q) p^\beta \frac{\partial^2 F}{\partial (pq)^2} \Big]$$

$$\varepsilon_{\mu\nu\alpha\sigma} p^\alpha s_\lambda \int d^4k \, k^\lambda k^\sigma \frac{\partial a}{\partial pk} \frac{\partial b}{\partial qk} = \varepsilon_{\mu\nu\alpha\sigma} p^\alpha \Big[s^\sigma \frac{\partial F}{\partial p \cdot q}$$
$$+ 2 q.s \, q^\sigma \frac{\partial^2 F}{\partial (pq)^2} \Big]$$

$$\varepsilon_{\mu\nu\alpha\sigma} q^\alpha p^\sigma s_\lambda \int d^4k \, k^\lambda \frac{\partial a}{\partial pk} \cdot b$$
$$= \varepsilon_{\mu\nu\alpha\beta} q^\alpha p^\beta \, q.s \frac{\partial F}{\partial p.q}.$$

THE HADRONIC TENSOR READS:

$$W_{\mu\nu}^{(A)} = \frac{i}{M^2} \varepsilon_{\mu\nu\alpha\beta}\, q^\alpha \Big\{ S^\beta \Big[\frac{\partial F_1}{\partial p\cdot q} + (p\cdot q) \frac{\partial F_2}{\partial pq} \Big]$$

$$+ S\cdot q\, p^\beta \Big[\frac{\partial^2 F_1}{\partial (pq)^2} + (p\cdot q) \frac{\partial^2 F_2}{\partial (pq)^2} \Big]$$

$$+ S\cdot q\, p^\beta \frac{\partial^2 (F_3 + F_4)}{\partial (pq)^2}$$

$$+ 2\, S\cdot q\, p^\beta \Big[\frac{\partial^2 F_5}{\partial pq\, \partial q^2} - \int d^4k \frac{Sk}{Sq} \frac{\partial a_5}{\partial pq} \frac{\partial b_5}{\partial q^2} \Big] \Big\}$$

THE STRUCTURE FUNCTIONS ARE:

$$g_1(x, Q^2) + g_2(x, Q^2) = \frac{p\cdot q}{M^2} \Big[\frac{\partial F_1}{\partial pq} + (pq) \frac{\partial F_2}{\partial pq} \Big]$$

$$g_2(x, Q^2) = -\frac{(pq)^2}{M^2} \Big[\frac{\partial^2 F_1}{\partial (pq)^2} + (pq) \frac{\partial^2 F_2}{\partial (pq)^2} + \frac{\partial^2 (F_3 + F_4)}{\partial (pq)^2}$$

$$+ 2 \frac{\partial^2 F_5}{\partial pq\, \partial q^2} - 2 \int d^4k \frac{Sk}{Sq} \frac{\partial a_5}{\partial pq} \frac{\partial b_5}{\partial q^2} \Big]$$

4. RESULTS

EXISTS A RELATION BETWEEN $g_1^{Q\bar{Q}}$ & $g_2^{Q\bar{Q}}$ AT TWIST-2 ?

REWRITE THE STRUCTURE FUNCTIONS CONSIDERING:

$$\frac{d}{dx}\left[x\left(g_1(x,Q^2)+g_2(x,Q^2)\right)\right]$$

$$= -\frac{(pq)^2}{M^2}\left[\frac{\partial^2 F_1}{\partial(pq)^2}+(pq)\frac{\partial^2 F_2}{\partial(pq)^2}+\frac{\partial F_2}{\partial pq}\right].$$

$$\Longrightarrow\ -x\frac{d}{dx}\left[g_1(x,Q^2)+g_2(x,Q^2)\right] = g_1(x) + \phi(x,Q^2)$$

$$\phi(x,Q^2) = \frac{(pq)^2}{M^2}\left[\frac{\partial(F_3+F_4-F_2)}{\partial p\cdot q}+\frac{\partial^2 F_5}{\partial pq\,\partial q^2} - 2\int d^4k\,\frac{s\cdot k}{q\cdot k}\,\frac{\partial a_5}{\partial pk}\,\frac{\partial b_5}{\partial q^2}\right]$$

$$\equiv \int d^4k\left(\frac{k^2}{Q^2}\right)\phi(p,k,q).$$

TWIST 2 CONTRIBUTION:

$$k^2 \ll Q^2$$

$k^2 \to 0$

$\tau_{int} \ll \tau_{life}$. (DRELL & YAN '70)

EXPAND IN A COLLINEAR BASIS (ELLIS, FURMANSKI, PETRONZIO '83)

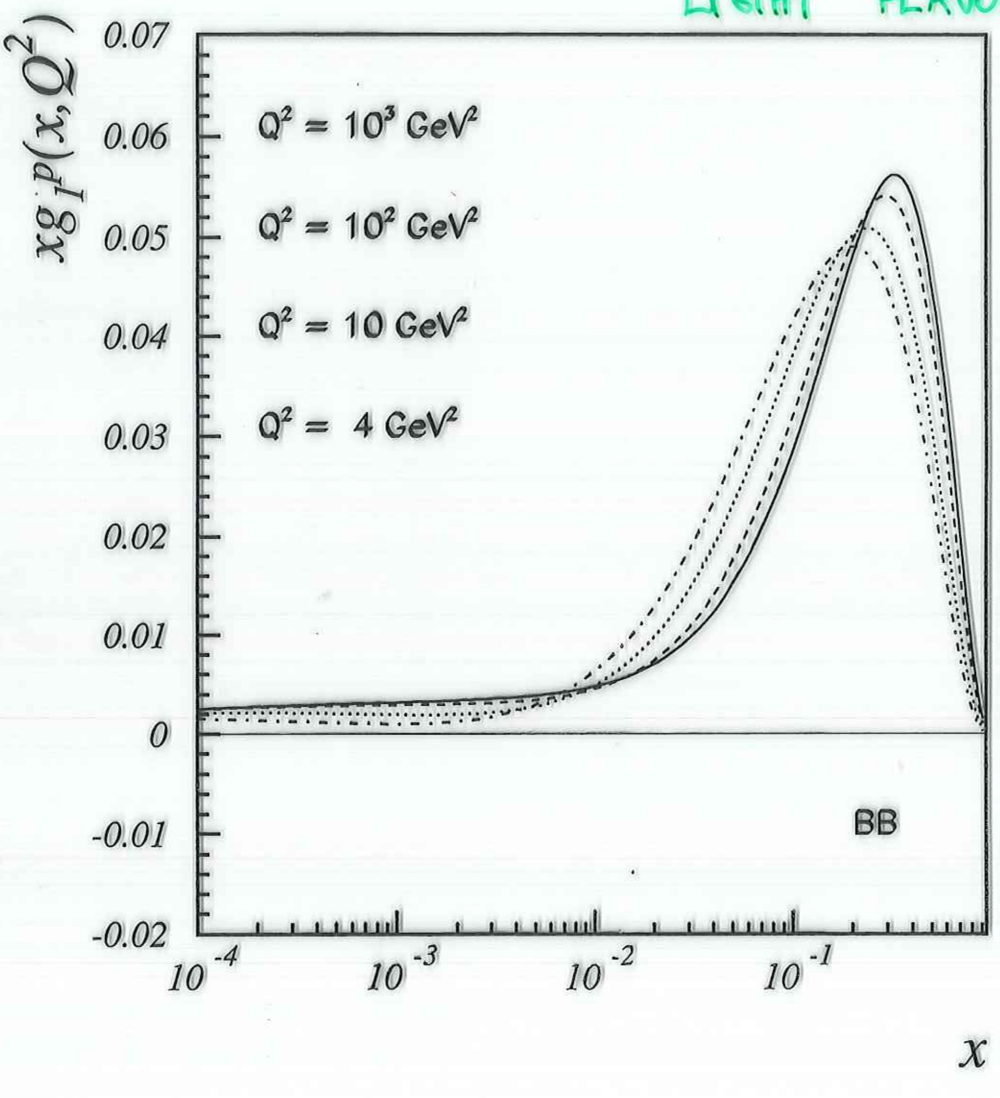
$k_\mu \to z p_\mu$; $p \cdot p = 0$.

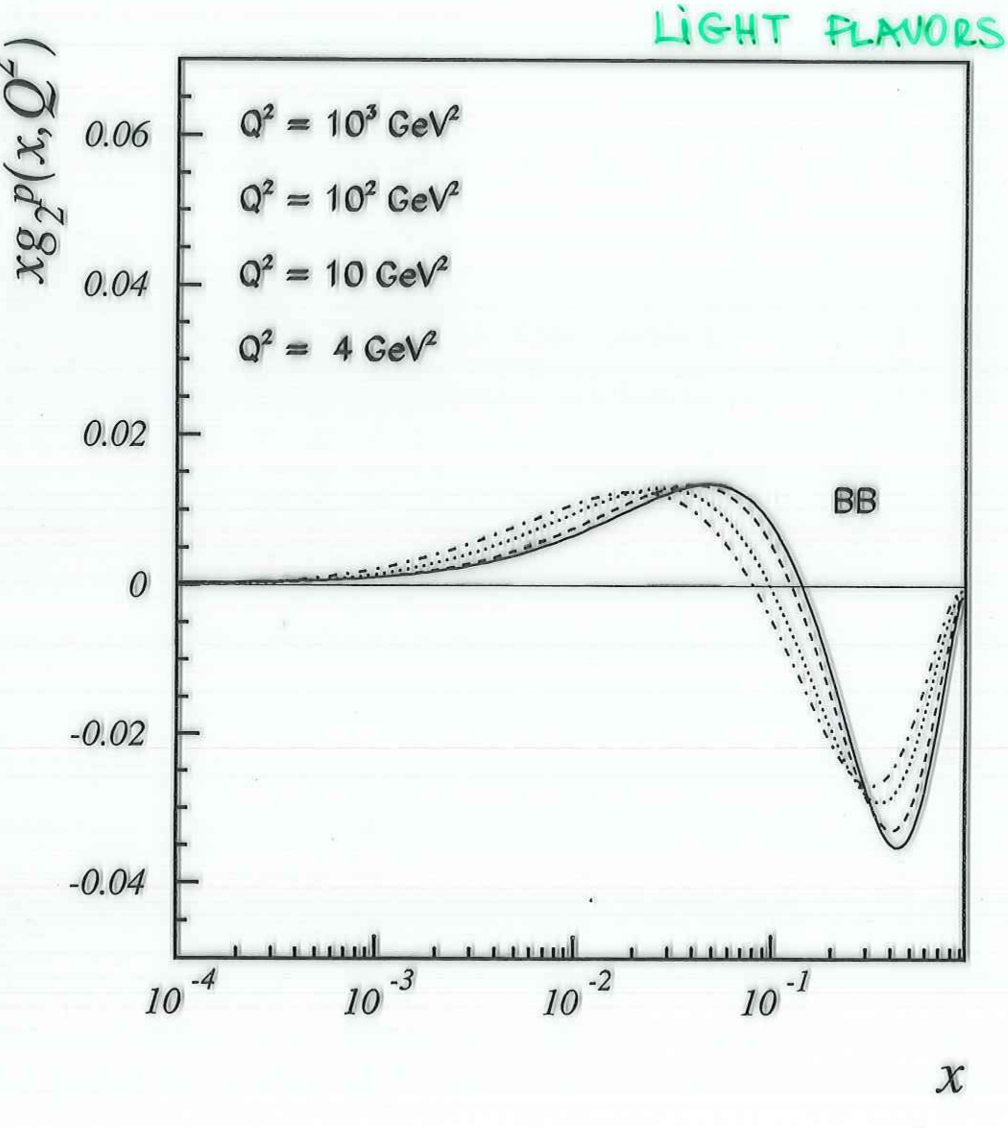
DERIVATIVE TERMS $\partial_{k_\mu} \big|_{k_\mu = z p_\mu}$ CONTRIBUTE AT HIGHER TWIST.

$$\boxed{\begin{gathered} -x \frac{d}{dx}\left[g_1^{\shortparallel} + g_2^{\shortparallel}\right] = g_1^{\shortparallel} \\ g_2^{\shortparallel}(x, Q^2) = -g_1^{\shortparallel}(x, Q^2) + \int_x^1 \frac{dy}{y}\, g_1^{\shortparallel}(y, Q^2) \end{gathered}}$$

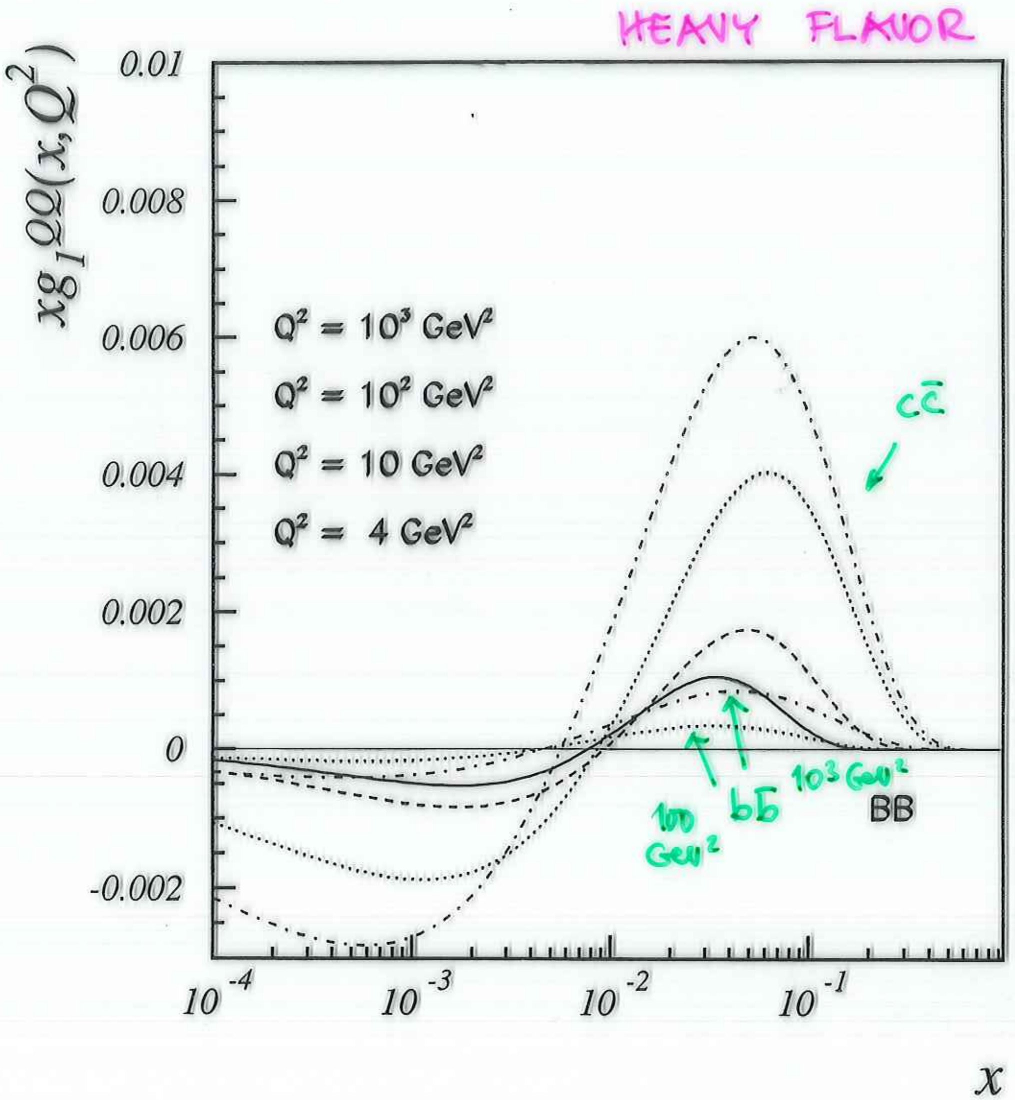
THE WANDZURA - WILCZEK RELATION DESCRIBES THE HEAVY FLAVOR CONTRIBUTIONS TO $g_2(x, Q^2)$ @ TWIST 2.

LIGHT FLAVORS



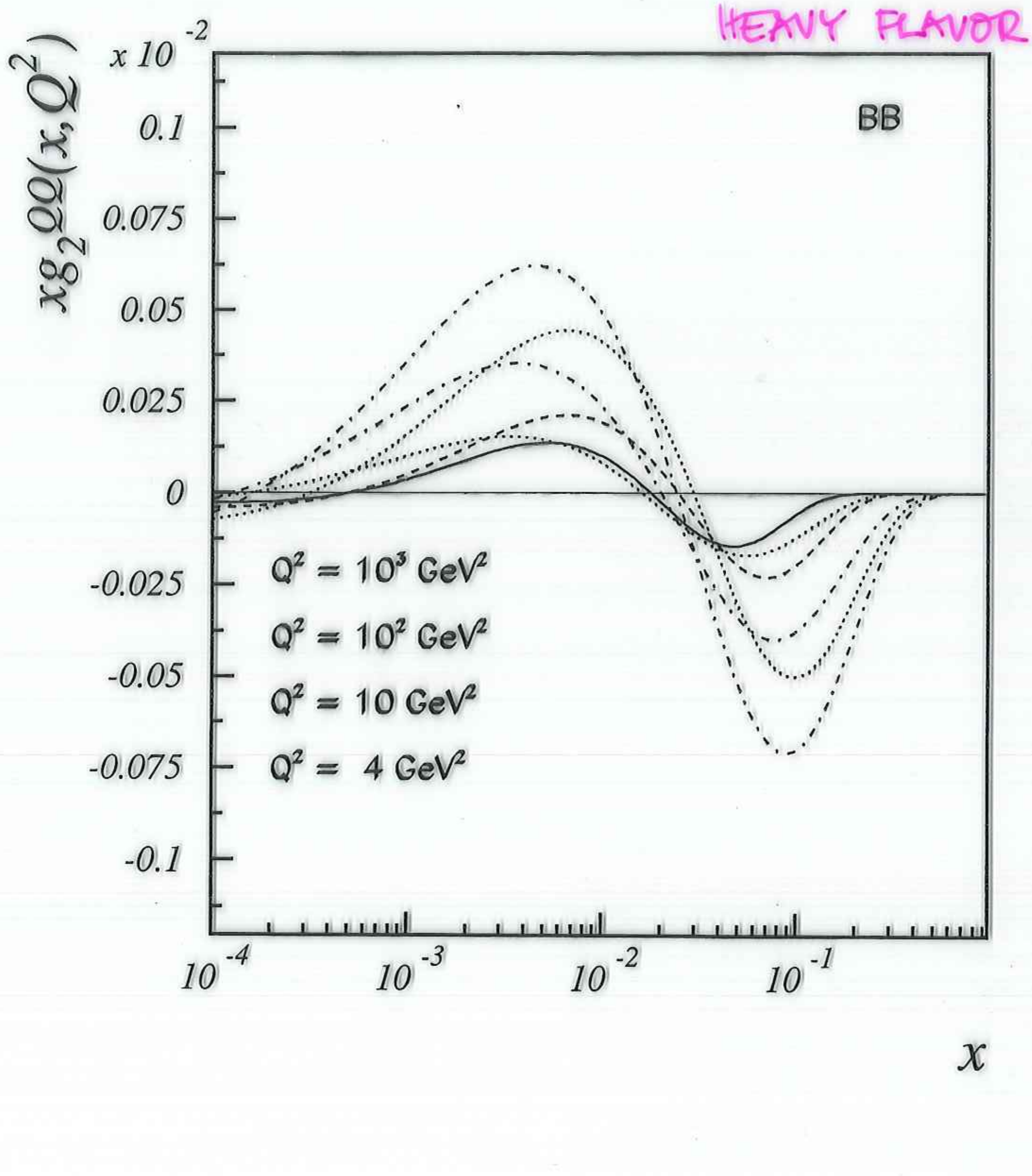


LIGHT FLAVORS
$xg_2^p(x,Q^2)$
0.06
0.04
0.02
0
-0.02
-0.04
$Q^2 = 10^3$ GeV^2
$Q^2 = 10^2$ GeV^2
$Q^2 = 10$ GeV^2
$Q^2 = 4$ GeV^2
BB
10^{-4}
10^{-3}
10^{-2}
10^{-1}
x



HEAVY FLAVOR
$xg_1^{QQ}(x,Q^2)$
0.01
0.008
0.006
0.004
0.002
0
-0.002
$Q^2 = 10^3$ GeV^2
$Q^2 = 10^2$ GeV^2
$Q^2 = 10$ GeV^2
$Q^2 = 4$ GeV^2
$c\bar{c}$
100 GeV^2
$b\bar{b}$
10^3 GeV^2
BB
10^{-4}
10^{-3}
10^{-2}
10^{-1}
x

HEAVY FLAVOR



5. CONCLUSIONS

- WE DERIVED GENERAL REPRESENTATIONS FOR THE POLARIZED STRUCTURE FUNCTIONS IN THE COVARIANT PARTON MODEL.

- PROJECTING ONTO THE TWIST-2 CONTRIBUTIONS $g_1^{Q\bar{Q}}$ AND $g_2^{Q\bar{Q}}$ OBEY THE WANDZURA-WILCZEK RELATION

- THE HEAVY FLAVOR CONTRIBUTIONS SHOULD BE TAKEN INTO ACCOUNT IN FUTURE REFINED QCD ANALYSES.