DESY Academic Training Program

QCD Evolution of Structure Functions

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Further Reading and References

1 Reports and Textbooks

1. G. Altarelli, *Partons in Quantum Chromodynamics*, Phys. Rep. 81, 1 (1982).
2. G. Altarelli, *Experimental tests of perturbative QCD*, Ann. Rev. Nucl. Part. Sci. 39, 357 (1989).
3. V. I. Andreev, *Chromodynamics and hard processes at high energies*, Nauka, Moscow, 1981 (in Russian).
4. A. Bassetto, M. Ciafaloni and G. Marchesini, *Jet structure and infrared sensitive quantities in perturbative QCD*, Phys. Rep. 100, 201 (1983).
5. P. Becher, M. Böhm and H. Joos, *Eichtheorien der starken und electromagnetischen Wechselwirkung*, Teubner Studienbücherei, B. G. Teubner, Stuttgart, 1983, 2nd edition, (in German).
6. A.J. Buras, *Asymptotic freedom in deep inelastic processes in the leading order and beyond*, Rev. Mod. Phys. 52, 199 (1980).
7. Ta-Pei Cheng and Ling Fong, Li, *Gauge Theory of Elementary Particle Physics*, Calendron Press, Oxford, 1984.
8. F.E. Close, *An Introduction to Quarks and Partons*, Academic Press, London, New York, 1979.
9. M. Diemoz, F. Ferroni and E. Longo, *Nucleon structure functions from neutrino scattering*, Phys. Rep. 130, 293 (1986).
10. Yu.L. Dokshitser, R. I. Dyakonov and S. I. Troyan, *Hard processes in Quantum Chromodynamics*, Phys. Rep. 58, 269 (1980).
11. D.W. Duke and R.G. Roberts, *Determination of the QCD strong coupling α_s*, Phys. Rep., 120, 275 (1985).
12. F. Eisele, *High energy neutrino interactions*, Rep. Progr. Phys. 49, 233 (1986).
13. R. P. Feynman, *Photon-Hadron Interactions*, Benjamin Press, Reading, MA, 1972.
14. J.I. Friedman, *Deep inelastic scattering: comparisons with the quark model*, Rev. Mod. Phys. 63, 573 (1991).
15. B. Geyer, D. Robaschik and E. Wieczorek, *Theory of deep inelastic lepton-hadron scattering*, Fortschr. Physik 27, 75 (1979).
16. L.V. Gribov, E.M. Levin and M.G. Ryskin, *Semihard processes in QCD*, Phys. Rep. 100, 1 (1983).
17. H.W. Kendall, *Deep inelastic scattering: experiments on the proton and observation of scaling*, Rev. Mod. Phys. 63, 597 (1991).
18. E.M. Levin and M.G. Ryskin, *High energy hadronic collisions in QCD*, Phys. Rep. 189, 267 (1990).
19. W. Marciano and H. Pagels, *Quantum Chromodynamics*, Phys. Rep. 36, 137 (1978).
20. A.H. Mueller, *Perturbative QCD at high energies*, Phys. Rep. 73, 283 (1981).
21. T. Muta, *Foundations of Quantum Cromodynamics: an introduction to perturbative methods in gauge theories*, World Scientific, Singapore, 1987.
22. O. Nachtmann, *Elementary Particle Physics: Concepts and Phenomena*, Texts and Monographs in Physics, (Springer, Berlin, 1990).

23. H. D. Politzer, *Asymptotic freedom: an approach to strong interactions*, Phys. Rep. 14, 129 (1974).

24. E. Reya, *Perturbative QCD*, Phys. Rep. 69, 3 (1980).

25. R. G. Roberts, *The structure of the proton*, Cambridge Monographs on Mathematical Physics, (Calendron Press, Cambridge ,1990).

26. T. Sloan, G. Smadja and R. Voss, *The quark structure of the nucleon from the CERN muon experiments*, Phys. Rep. 162, 45 (1988).

27. R.E. Taylor, *Deep inelastic scattering: the early years*, Rev. Mod. Phys. 63, 573 (1991).

28. Y. F. Ynduráin, *Quantum Chromodynamics: an introduction to the theory of quarks and gluons*, Texts and Monographs in Physics, Springer, Berlin, New York, 1983.

LECTURE 1

1. INTRODUCTION

2. BASIC RELATIONS IN QCD

3. $\alpha_s\ (Q^2)$

4. STRUCTURE FUNCTIONS FOR DEEP INELASTIC SCATTERING

5. EVOLUTION OF STRUCTURE FUNCTIONS PART 1: OPE

INTRODUCTION

BASIC QUESTIONS: • HOW IS THE PROTON STRUCTURED ?

• IS QCD THE VALID DESCRIPTION ?

AFTER 20 YEARS OF DIS EXPERIMENTS

AND

20 YEARS QCD

HERA STARTS ITS OPERATION.

→ GOALS:

1) EXPLORE THE PROTON STRUCTURE
AT $x \gtrsim 10^{-2}$ AT HIGH Q^2
: 2 MORE ORDERS → EVOLUTION

2) LOWER Q^2 : $Q^2 \lesssim 500\ GeV^2$
: 2 MORE ORDERS IN $x \sim 10^{-4}$
→ NEW EFFECTS

→ AIM ON THEIR ISOLATION !

→ HAVE TO KNOW ALL THE DETAILS OF THE 'STANDARD' BEHAVIOUR.

THESE LECTURES TRY TO GIVE AN OVERVIEW ON THE TOPIC FROM A THEORETICAL POINT OF VIEW.

$\longrightarrow$ TRY TO EXPLAIN THE USEFULNESS OF QCD TESTS WITH DIS

$\longrightarrow$ REQUIRES TO UNDERSTAND:

WHAT STATES <u>QCD</u> ABOUT <u>DIS</u>?

$\longrightarrow$ NEW EVOLUTION EFFECTS AT SMALL x

$\longrightarrow$ REQUIRES TO ISOLATE THE 'OLD' EFFECTS AND STILL TO SEE:

DO WE PROBE NEW DYNAMICAL DEGREES OF FREEDOM?

$\longrightarrow$ FROM FORMULAE TO CODES $\overset{}{\longleftrightarrow}$ DATA

??

BASIC RELATIONS IN QCD

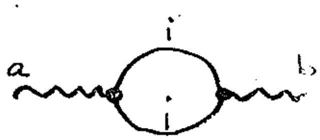
$$\mathcal{L}_{QCD} = -\frac{1}{2} G^a_{\mu\nu} G_a^{\mu\nu} + \sum_{k=1}^{N_f} \bar{q}_k \left(i\gamma^\mu D_\mu - m_k \right) q_k$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, f^a{}_{bc} A^b_\mu A^c_\nu$$

$$D_\mu \equiv \partial_\mu - i g T^a A^a_\mu \quad ; \quad [T_a, T_b] = i f_{abc} T^c$$

$$SU_3^c \qquad\qquad T_a \Rightarrow \frac{\lambda_a}{2}$$

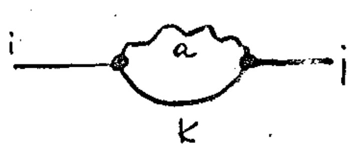
COLOR FACTORS:

$T_2(F)$ 

$\mathrm{tr}\left[T_a T_b\right] = T_2(F)\, \delta_{ab} = \frac{1}{2} \delta_{ab}$

$C_2(A)$ 

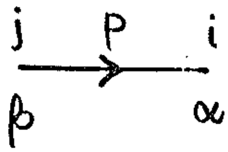
$\sum_{cd}^{8} f_{acd} f_{bcd} = C_2(A)\, \delta_{ab} = \underline{3}\, \delta_{ab}$

$C_2(F)$ 

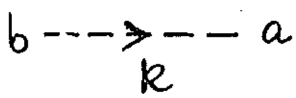
$\sum_{a}^{8} \sum_{k}^{N} T^a_{ik} T^a_{kj} = C_2(F)\, \delta_{ij} = \frac{4}{3} \delta_{ij}$

FRITZSCH, GELL-MANN, LEUTWYLER 1973
GROSS, WILCZEK 1973, 1974
POLITZER 1973
T'HOOFT 1972

PROPAGATORS:



$$= i\,\frac{\delta^{ij}}{\not p - m + i0}\Bigg|_{\alpha\beta}$$ QUARK

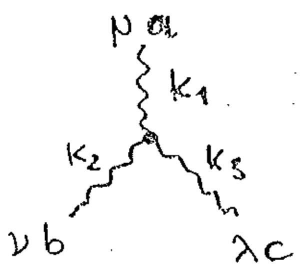


$$= -i\,\frac{\delta^{ab}}{k^2+i0}$$ GHOST

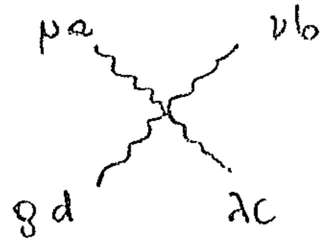
$$= -i\delta^{ab}\frac{1}{k^2+i0}\left[g_{\mu\nu} + (\xi-1)\frac{k_\mu k_\nu}{k^2}\right]$$

GLUON

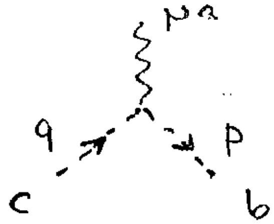
VERTICES:



$$ig\, f^{abc}\Big[(k_1-k_2)_\lambda g_{\mu\nu} + (k_2-k_3)_\mu g_{\nu\lambda} + (k_3-k_1)_\nu g_{\mu\lambda}\Big]$$



$$-ig^2\Big[f^{eab}f^{ecd}(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}) + f^{eac}f^{ebd}(g_{\mu\rho}g_{\lambda\nu} - g_{\mu\nu}g_{\lambda\rho}) + f^{ead}f^{ebc}(g_{\mu\nu}g_{\rho\lambda} - g_{\mu\lambda}g_{\rho\nu})\Big]$$



$$g\, f^{abc}\, p_\mu$$

$$ig(\gamma_\mu)_{\alpha\beta}(T^a)_{ij}$$

THE STRONG COUPLING CONSTANT
$\alpha_s(Q^2)$

$$\Gamma_R^{(N_\psi, N_G)} = \frac{G_R^{(N_\psi, N_G)}}{\prod^{N_f} G_R^{(2,0)} \prod^{N_G} G_R^{(0,2)}}$$

$$G_R^{(N_\psi, N_G)} = \langle 0 | T(\psi_1 \ldots \psi_{N_\psi} G_1 \ldots G_{N_G}) | 0 \rangle$$

N_ψ NUMBER OF EXTERNAL FERMIONS

N_G " " " GLUONS

$$S_R^{-1} \equiv \Gamma_R^{(2,0)}$$

$$\Gamma_R^{(N_\psi, N_G)}(p_j, g, \varepsilon, \mu) = Z_\psi^{N_\psi/2} Z_G^{N_G/2} \Gamma_u^{(N_\psi, N_G)}(p_j, g_0, \varepsilon)$$

$$\Gamma_R^{(N_\psi, N_G)}(p_j, g, \mu) = \lim_{\varepsilon \to 0} \Gamma_R^{(N_\psi, N_G)}(p_j, g, \varepsilon, \mu)$$

RGE: $\boxed{\mu \frac{d}{d\mu} \Gamma_u^{(N_\psi, N_G)}}$

CALLAN
SYMANZIK

$$\left[\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} - N_\psi \gamma_\psi(g) - N_G \gamma_G(g)\right] \Gamma_R^{(N_\psi, N_G)} = 0$$

$$\gamma_\psi(g) = \frac{1}{2} \mu \frac{\partial}{\partial \mu} \ln Z_\psi$$

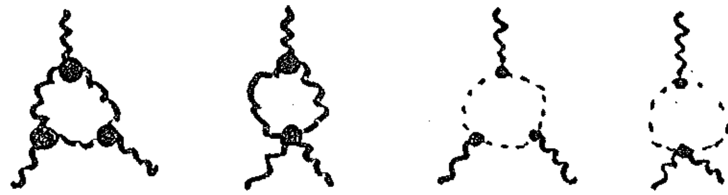
$$\gamma_G(g) = \frac{1}{2} \mu \frac{\partial}{\partial \mu} \ln Z_G$$

$$\beta(g) = \mu \frac{\partial g}{\partial \mu}$$

DETERMINE THE Z_i - FACTORS.

LO:

∴ Γ_{VVV} :



$$\Gamma^{un}_{VVV} = Z_1^{-1} \, \Gamma^{r}_{VVV}$$

$$Z_1 = 1 + \left[\left(\frac{17}{12} - \frac{3}{4}\xi\right) C_G - \frac{4}{6} N_f\right] \frac{g^2}{16\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right)$$

$$G_V^{un} = Z_3 \, G_V$$

$$Z_3 = 1 + \left[\left(\frac{13}{6} - \frac{\xi}{2}\right) C_G - \frac{4}{6} N_f\right] \frac{g^2}{16\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right)$$

$$g = Z_g \, g^{un} = Z_3^{3/2} / Z_1 \; g^{un}$$

$$Z_G = 1 + \Big[\underbrace{\frac{11}{6} C_G - \frac{1}{3} N_f}_{\beta_0/2} \Big] \frac{g^2_{un}}{16\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right)$$

EXPANSION OF $\beta(g)$:

$$\beta(g) = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2} - \beta_2 \frac{g^7}{(16\pi^2)^3} - \dots$$

$$\frac{dg^2}{dt} = g\,\beta(g) \quad , \quad t = \ln\left(\frac{Q^2}{\mu^2}\right)$$

DIFFERENT STEPS OF APPROXIMATION: $N_c = 3$

$\beta_0 = 11 - \frac{2}{3} N_f$ POLITZER, 1973

GROSS, WILCZEK 1973

Asymptotic freedom for $N_f \leq 16$.

$\beta_1 = 102 - \frac{38}{3} N_f$ CASWELL, 1974

JONES, 1974

$\beta_2 = \frac{2857}{2} - \frac{5033}{18} N_f + \frac{324}{54} N_f^2$ TARASOV, VLADIMIROV, ZHARKOV 1980

$N_f = 4: \quad \beta_0 : \beta_1 : \beta_2 = 1 : 6.16 : 48.75$

SOLVING THE DIFFERENTIAL EQU. FOR $\bar{g}$ ONE OBTAINS:

LO: $\boxed{\alpha_s(Q^2) = \frac{4\pi}{\beta_0} \frac{1}{\ln(Q^2/\Lambda^2)}}$

DEFINING $\alpha_s^{-1}(\Lambda^2) = 0$

- THE HIGHER ORDER SOLUTIONS ARE EXPANDED TO THE RESPECTIVE n^{th} ORDER IN $1/\ln(Q^2/\Lambda^2)$
- THERE IS NO CLOSED ANALYTICAL EXACT SOLUTION (→ TRANSC. EQU.!)

1ST ORDER NTLO:

$$\frac{1}{\alpha_s} + \frac{\beta_1}{4\pi\beta_0} \ln \alpha_s \simeq \frac{\beta_0}{4\pi} \ln \frac{Q^2}{\Lambda^2}$$

$$\alpha_s^{(1)}(Q^2) \simeq \frac{4\pi}{\beta_0} \frac{1}{\ln(Q^2/\Lambda^2)} - \frac{4\pi\,\beta_1}{\beta_0^3} \frac{\ln\ln(Q^2/\Lambda^2)}{\ln^2(Q^2/\Lambda^2)}$$

$$\alpha_s^{(1)}(Q^2) = \alpha_s^{(0)}(Q^2) \left[1 - \frac{\beta_1}{\beta_0^2} \frac{\ln\ln(Q^2/\Lambda^2)}{\ln(Q^2/\Lambda^2)} \right]$$

NUMERICAL EXAMPLE:

$Q^2 = 100\ \mathrm{GeV}^2$, $\Lambda = 200\ \mathrm{MeV}$, $N_f = 5$

$$\alpha_s^{(0)}(Q^2) = 0.209$$

$$\beta_1 / \beta_0^2 = 0.658$$

$$\frac{\ln\ln Q^2/\Lambda^2}{\ln Q^2/\Lambda^2} = 0.262$$

$$\alpha_s^{(1)} / \alpha_s^{(0)} = 0.83$$

DIFFERENT RENORMALIZATION SCHEMES:

Z_ψ : $$S_R^{-1}(p) = Z_\psi S^{-1}(p)$$

$$Z_\psi = 1 - C_F \xi \frac{g^2}{16\pi^2}\left[\frac{2}{\varepsilon} - \ln\frac{Q^2}{\Lambda^2} + 1 + \ln 4\pi - \gamma_E\right]$$

$$Z_\psi^{MS} = 1 - C_F \xi \frac{g^2}{16\pi^2} \cdot \left[\frac{2}{\varepsilon}\right]$$

$$Z_\psi^{\overline{MS}} = 1 - C_F \xi \frac{g^2}{16\pi^2} \cdot \left[\frac{2}{\varepsilon} + \ln 4\pi - \gamma_E\right]$$

CONSEQUENCES FOR α_S AND Λ:

$$\Lambda \rightarrow \Lambda' = \Lambda / \kappa$$

$$\alpha_S \rightarrow \alpha_S' - \left(\frac{\beta_0}{4\pi}\ln \kappa\right)\alpha_S^2 + O(\alpha_S^3)$$

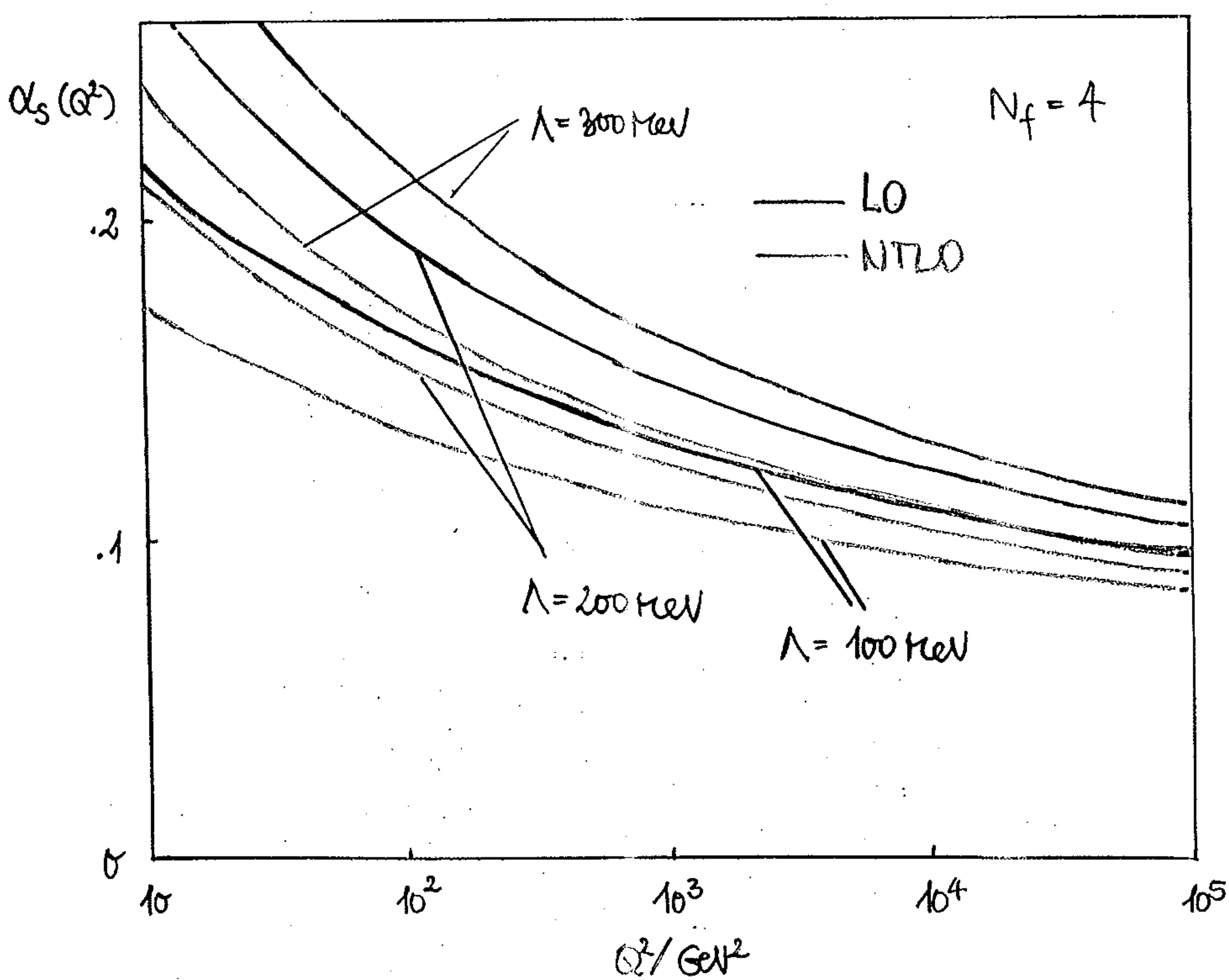
PHYSICAL QUANTITIES HAVE TO BE INVARIANT!

e.g. $d \ln M_n^{NS}(Q^2) / d \ln Q^2$

$$\Lambda_{\overline{MS}} = 2.66\, \Lambda_{MS}$$

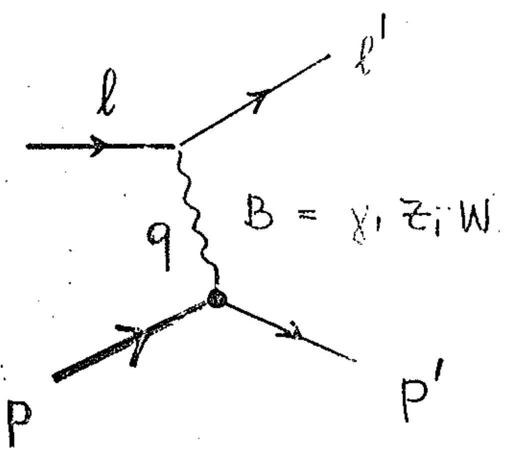
$$\Lambda_{MOM} = 2.55\, \Lambda_{\overline{MS}}$$

FURTHER : N_f DEPENDENCE, cf. SECT. 7.



$\alpha_s(Q^2)$
$N_f = 4$
LO
NTLO
Λ = 300 MeV
Λ = 200 MeV
Λ = 100 MeV
.2
.1
0
10
10^2
10^3
10^4
10^5
Q^2/GeV^2

STRUCTURE FUNCTIONS FOR DEEP INELASTIC SCATTERING



$$\ell_0' \frac{d\sigma^\gamma}{d^3\ell'} = \frac{2M}{S-M^2}\,\frac{\alpha^2}{Q^4}\, L^{\mu\nu} W_{\mu\nu}$$

$$L_{\mu\nu} = 2\left[\ell_\mu \ell'_\nu + \ell'_\mu \ell_\nu - \frac{1}{2} Q^2 g_{\mu\nu}\right]$$

$$W_{\mu\nu} = \frac{1}{4M}\sum_\sigma \int \frac{d^4\xi}{2\pi}\, e^{iq\xi} \langle p,\sigma | [J^{em}_\mu(\xi), J^{em}_\nu(0)] | p,\sigma\rangle$$

$q^\mu W_{\mu\nu} = 0$ P & T invariance

$$\boxed{\begin{aligned} W_{\mu\nu}(p,q) &= -\left(g_{\mu\nu} + \frac{q_\mu q_\nu}{Q^2}\right) W_1 \\ &\quad + \frac{1}{M^2}\left(p_\mu + \frac{p\cdot q}{Q^2} q_\mu\right)\left(p_\nu + \frac{p\cdot q}{Q^2} q_\nu\right) W_2 \end{aligned}}$$

Accordingly, for $\langle \ell | J_{weak} J_{weak} | 0 \rangle$

$$W_{\mu\nu} = -\left(g_{\mu\nu} + \frac{q_\mu q_\nu}{Q^2}\right) W_1$$

$$+ \frac{1}{M^2}\left(p_\mu + \frac{pq}{Q^2} q_\mu\right)\left(p_\nu + \frac{pq}{Q^2} q_\nu\right) W_2$$

$$- \frac{i}{M} \varepsilon_{\mu\nu\rho\sigma} p^\rho p^\sigma W_3$$

$$L_{\mu\nu} = 8\left[l_\mu l'_\nu + l_\nu l'_\mu - g_{\mu\nu} l l' + i \varepsilon_{\mu\nu\rho\sigma} l^\rho l^\sigma\right]$$

$q^\mu W_{\mu\nu} \neq 0$ $\qquad J_{weak}$ NOT CONSERVED.

CP invariance

$q_\mu L^{\mu\nu} \propto m_e, m_\nu$

$q_\mu p_\nu + p_\mu q_\nu \quad \propto m_l / E$

$q_\mu q_\nu \quad \propto (m_e / E)^2$

TRANSFORM NOW

$$
\begin{aligned}
d^3 l' \longrightarrow\; & d\varphi\, dQ^2\, dW^2 \\
& d\varphi\, d\nu\, dQ^2 \\
& d\varphi\, dx\, dQ^2 \\
& d\varphi\, dx\, dy \qquad \text{etc.}
\end{aligned}
$$

$$W^2 = p'^2, \quad Q^2 = -(l - l')^2, \quad \nu = \frac{p \cdot q}{M} = \frac{1}{2M}\left(W^2 + Q^2 - M^2\right)$$

$$x = \frac{Q^2}{yS} = \frac{-(l-l')^2}{2p\,(l-l')}$$

$$y = \frac{p\,(l-l')}{p\,l}$$

GENERAL DECOMPOSITION OF THE $e^{\pm}p$ nc/cc CROSS SECTIONS:

JB, KLEIN, NAUMANN, RIEMANN 1987

NC:

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[Y_+\, \mathbb{F}_2(x, Q^2) + Y_-\, x\mathbb{F}_3(x, Q^2)\right]$$

$$\mathbb{F}_2 = F_2 + K_Z(Q^2)\,(-v \mp \lambda a)\, G_2 + K_Z^2(Q^2)\,(v^2 + a^2 \pm 2\lambda v a)\, H_2$$

$$x\mathbb{F}_3 = K_Z(Q^2)\,(\pm a + \lambda v)\, xG_3 + K_Z^2(Q^2)\,(\mp 2va - \lambda(v^2+a^2))\, xH_3$$

$$K_Z(Q^2) = \frac{Q^2}{\dots}\;\frac{1}{\dots}$$

$$Y_+ = \left(1 + (1-y^2) - y^2 \frac{R}{1+R}\right)$$

$$Y_- = \left(1 - (1-y)^2\right)$$

↑ longitudinal d.f.

5 NC STRUCTURE FUNCTIONS
(+ 3 LONGITUDINAL)

CC :

$$\frac{d^2\sigma^\pm}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4} K_W^2(Q^2)(1\pm\lambda)\cdot\frac{1}{2}\left[Y_+ W_2^\pm \mp Y_- W_3^\pm\right]$$

4 CC STRUCTURE FUNCTIONS

(+ 2 LONGITUDINAL)

DECOMPOSITION OF $W_i^\pm$, F_i, G_i, H_i & F_{iL} INTO PARTONIC DENSITIES

FEYNMAN 1970

BJORKEN 1969 : SCALING
& SCALING VIOLATIONS

⟹ THE STRUCTURE FUNCTIONS MAY BE DECOMPOSED IN TERMS OF PARTON DISTRIBUTIONS

NO $\gamma^*(B^*)\,p$ - BUT $\gamma^*(B^*)\,\overset{(-)}{q_i}$ SCATTERING!

NC

$$F_2(x,Q^2) = x\sum_q e_q^2 \left[q(x,Q^2) + \bar{q}(x,Q^2)\right]$$

$$G_2(x,Q^2) = x\sum_q 2e_q v_q \left[q(x,Q^2) + \bar{q}(x,Q^2)\right]$$

$$H_2(x,Q^2) = x\sum_q (v_q^2 + a_q^2)\left[q(x,Q^2) + \bar{q}(x,Q^2)\right]$$

S+NS

$$xG_3(x,Q^2) = x\sum_q Q_q a_q \left[q(x,Q^2) - \bar{q}(x,Q^2)\right]$$

$$xH_3(x,Q^2) = x\sum_q v_q a_q \left[q(x,Q^2) - \bar{q}(x,Q^2)\right]$$

NS

CC

$$W_2^{\pm}(x,Q^2) = 2x\sum_i \left[q_{d(u)}(x,Q^2) + \bar{q}_{u(d)}(x,Q^2)\right] \quad , \text{S+NS}$$

$$xW_3^{\pm}(x,Q^2) = 2x\sum_i \left[q_{u(d)} - \bar{q}_{d(u)}\right] \quad , \text{NS}$$

LONGITUDINAL: $O(\alpha_s)$ CF. SECT. 5

DIRECT GLUON DRIVEN PART.

CURRENT SETS OF PARTON DISTRIBUTION PARAMETRIZATIONS:

IDIS =1	Duke, Owens	set 1	(1984)
IDIS =2	Duke, Owens	set 2	(1984)
IDIS =3	Owens		(1991)
IDIS =4	Eichten et al.	set 1	(1984)
IDIS =5	Eichten et al.	set 2	(1984)
IDIS =6	Diemoz et al.	LO	(1988)
IDIS =7	Diemoz et al.	NTLO	(1988)
IDIS =8	Harriman et.al.	EMC	(1990)
IDIS =9	Harriman et.al.	BCDMS	(1990)
IDIS =10	Morfin,Tung	LO BCDMS+EMC SU(3) symm.sea	(1991)
IDIS =11	Morfin,Tung	DIS,BCDMS+EMC SU(3) symm.sea	(1991)
IDIS =12	Morfin,Tung	DIS,BCDMS+EMC SU(3) non-symm.sea	(1991)
IDIS =13	Morfin,Tung	DIS,BCDMS1,SU(3) symm.sea	(1991)
IDIS =14	Morfin,Tung	DIS,BCDMS2,SU(3) symm.sea	(1991)
IDIS =15	Morfin,Tung	DIS,EMC,SU(3) symm.sea	(1991)
IDIS =16	Morfin,Tung	MS,BCDMS+EMC SU(3) symm.sea	(1991)
IDIS =17	Morfin,Tung	MS,BCDMS+EMC SU(3) non-symm.sea	(1991)
IDIS =18	Morfin,Tung	MS,BCDMS1,SU(3) symm.sea	(1991)
IDIS =19	Morfin,Tung	MS,BCDMS2,SU(3) symm.sea	(1991)
IDIS =20	Morfin,Tung	MS,EMC,SU(3) symm.sea	(1991)
IDIS =21	Kwiecinski et al.	set B0	(1990)
IDIS =22	Kwiecinski et al.	set B−	(1990)
IDIS =23	Kwiecinski et al.	set B−, weak shadowing	(1990)
IDIS =24	Kwiecinski et al.	set B−, strong shadowing	(1990)
IDIS =25	Glück et al.	LO	(1991)
IDIS =26	Glück et al.	NTLO	(1991)

$$\left\{\begin{array}{l}(u, d, s, c, b, t) \\ (\bar{u}, \bar{d}, \bar{s}, \bar{c}, \bar{b}, \bar{t}) \\ \qquad\qquad xG\end{array}\right.$$

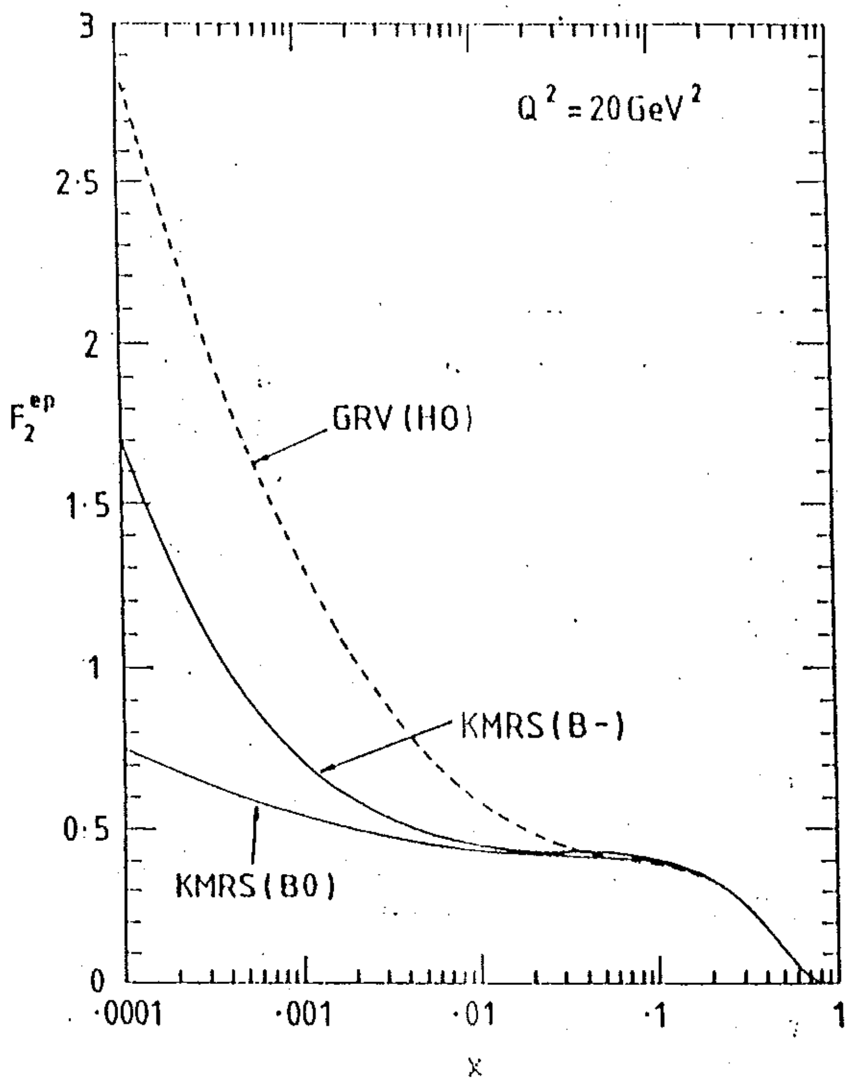


Figure 3: F_2^{ep} structure function predictions.

SUMMARY OF LECTURE 1

QCD

|

NONABELIAN COUPLINGS OF GAUGE FIELDS

|

ASYMPTOTIC FREEDOM $N_f \leq 16$

|

OPE OF $W_{\mu\nu}$ & W_1, W_2, W_3, W_L

|

STUDY TWIST 2 CONTRIBUTION:

$\longrightarrow$ NEGLECT TERMS $\sim \frac{1}{Q^2}, \frac{1}{Q^4} \cdots$

CALCULATE MOST SINGULAR PARTS $\sim \frac{1}{\varepsilon}$

$\longrightarrow$ ANOMALOUS DIMENSIONS $\gamma_{NS}, \gamma_S^{ij}$

EVOLUTION OF STRUCTURE FUNCTIONS: LEADING TWIST

i) OPERATOR PRODUCT EXPANSION

ii) ALTARELLI-PARISI EQUATIONS

IN LEADING & NEXT TO LEADING ORDER

i) OPERATOR PRODUCT EXPANSION

WILSON 1969; GROSS, WILCZEK 1974, GEORGI, POLITZER 1974

$$J(z)\,J(0) = \sum_{i,n} \tilde{C}_n^i(z^2)\, z_{\mu_1} \ldots z_{\mu_n}\, O_i^{\mu_1 \ldots \mu_n}$$

$$\langle p | O_i^{\mu_1 \ldots \mu_n} | p \rangle = A_n^i\, p_{\mu_1} \cdots p_{\mu_n}$$

STRUCTURE FUNCTIONS: CHRIST, HASSLACHER, MUELLER, 1972

$$\int_0^1 dx\, x^{n-2}\, F_k(x, Q^2) = \sum_i A_n^i(\mu^2)\, C_{k,n}^i(Q^2/\mu^2, g^2)$$

$k = 1, 2, 3, L$

A) NON-SINGLET STATES:

$$J(z)\,J(0)\big|_{NS} = \sum_n C_n^{NS}\, O_{NS}^n$$

$$\langle NS | JJ | NS \rangle = \sum_n C_n^{NS}\, O_{NS,NS}^n$$

$$O_{NS,NS}^n = \langle NS | O_{NS}^n | NS \rangle$$

$$\left[\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} - 2\gamma_\psi(g)\right] \langle NS | JJ | NS \rangle = 0$$

$$\left[\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} + \gamma_{NS}^n(g) - 2\gamma_\psi(g)\right] O_{NS,NS}^n = 0$$

$$\left[\mu \frac{\partial}{\partial \mu} \ldots\right] C^{NS}(Q^2 \ldots g^2) \ldots$$

$$C^{NS}_{k,n}\left(\frac{Q^2}{\mu^2}, g^2\right) = C^{NS}_{k,n}(1,\bar{g}^2)\exp\left[-\int_{\bar{g}(\mu^2)}^{\bar{g}(Q^2)} dg' \, \frac{\gamma^n_{NS}(g')}{\beta(g')}\right]$$

$$C^{NS}_{k,n}(1,\bar{g}^2) = \begin{cases} \delta^k_{NS}\left(1+\dfrac{\bar{g}^2}{16\pi^2}B^{NS}_{k,n}\right) & k=1,2,3 \\ \delta^L_{NS}\left(0+\dfrac{\bar{g}^2}{16\pi^2}B^{NS}_{L,n}\right) & k=L \end{cases}$$

$$\gamma^n_{NS}(g) = \gamma^{(0),n}_{NS}\frac{g^2}{16\pi^2} + \gamma^{(1),n}_{NS}\frac{g^4}{(16\pi^2)^2}+\dots$$

δ^j_{NS} - FLAVOUR FACTORS

MOMENTS OF STRUCTURE FUNCTIONS:

1) LEADING ORDER $\qquad M^{k,n}_{NS}(Q^2) = \int_0^1 dx\, F^k(x,Q^2)\, x^{n-2}$

$k = 1,2,3$

$$M^{k,n}_{NS}(Q^2) = \delta^k_{NS}A^n_{NS}(Q_0^2)\left[\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right]^{-d^n_{NS}} = \delta^k_{NS}A^n_{NS}(Q_0^2)\cdot\exp\left[-d^n_{NS}\tilde{s}\right]$$

$$\tilde{s} = \ln\left[\ln Q^2/\Lambda^2 / \ln Q_0^2/\Lambda^2\right]$$

$$M^{L,n}_{NS}(Q^2) = 0$$

$$d^n_{NS} = \frac{\gamma^{(0),n}_{NS}}{2\beta_0} \qquad \gamma^{(0),n}_{NS} = \frac{8}{3}\left[1-\frac{2}{n(n+1)}+4\sum_{j=2}^{n}\frac{1}{j}\right]$$

2) NTLO :

$k = 1, 2, 3$

$$M_{NS}^{k,n}(Q^2) = \delta_{NS}^{k} A_{NS}^{n}(Q_0^2)\left[1 + \frac{R_{k,n}^{NS}(Q^2)}{\beta_0 \ln(Q^2/\Lambda^2)} - \frac{R_{k,n}^{NS}(Q_0^2)}{\beta_0 \ln(Q^2/\Lambda^2)}\right]$$

$$\times \left[\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right]^{-d_{NS}^{n}}$$

$$R_{k,n}^{NS}(Q^2) = R_{k,n}^{NS} - \frac{\beta_1}{\beta_0} d_{NS}^{n} \ln\ln\left(\frac{Q^2}{\Lambda^2}\right)$$

$$M_{NS}^{L,n}(Q^2) = A_{NS}^{n}(Q_0^2)\,\delta_{NS}^{L} \frac{B_{L,n}^{NS}}{\beta_0 \ln(Q^2/\Lambda^2)} \left[\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right]^{-d_{NS}^{n}}$$

with : $B_{L,n}^{NS} = \frac{16}{3} \cdot \frac{1}{n+1}$

$$\frac{1}{n+1} = \int_0^1 dx\, x^{n-2}\ \ \underline{\underline{x^2}}$$

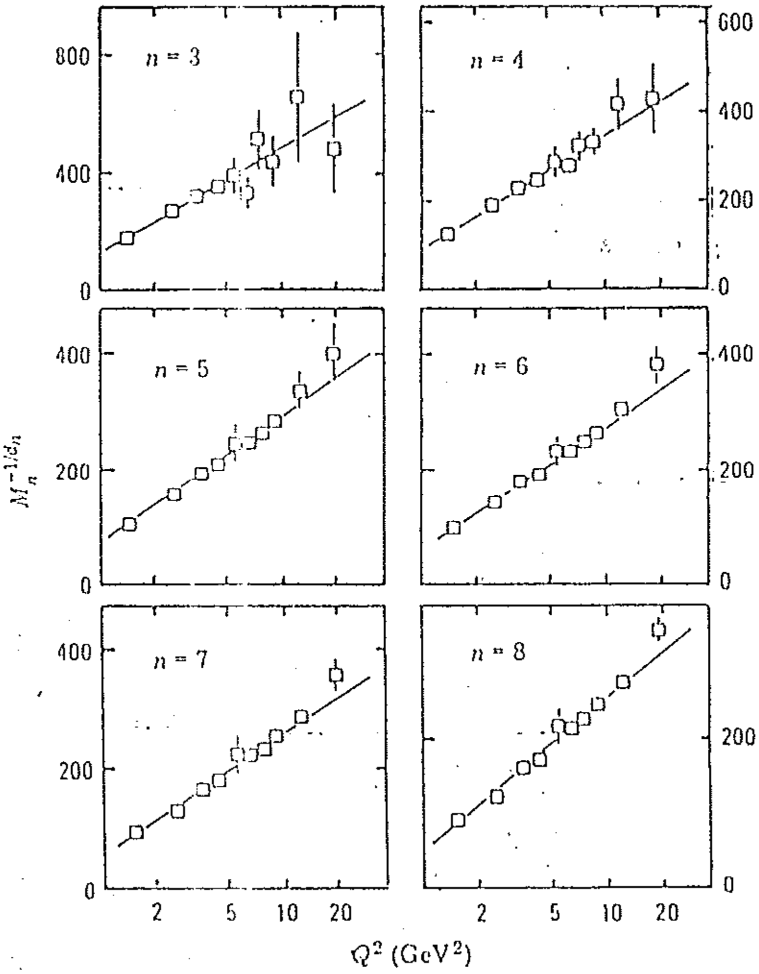


Fig. 5.1 Non-singlet moments $M_n^{NS}(Q^2)$ computed from muon and neutrino data, to the power $-1/d_n^{NS}$ against $\ln Q^2$. Taken from Pennington (1983).

$$M_n^{-\frac{1}{d_N}}\Big|_{NS} = \left[A_n^{-\frac{1}{d_N}} \frac{1}{\ln Q_0^2/\Lambda^2}\right] \cdot \ln(Q^2/\Lambda^2)$$

$\boxed{\text{B) SINGLET STATES}}$

$$\left[\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g}\right] C^i_{k,n}\left(\frac{Q^2}{\mu^2}, g^2\right) = \sum_j \gamma^n_{ji}(g^2)\, C^i_{k,n}\left(\frac{Q^2}{\mu^2}, g^2\right)$$

$$j = \psi, G$$

$$\gamma^n_{ji}(g^2) = \gamma^{(0),n}_{ji} \frac{g^2}{16\pi^2} + \gamma^{(1),n}_{ji} \frac{g^4}{(16\pi^2)^2} + \dots$$

$$C^{\psi}_{k,n}(1, \bar{g}^2) = \begin{cases} \delta^{\psi}_{k}\left[1 + \frac{\bar{g}^2}{16\pi^2} B^{\psi}_{k,n}\right] & k = 1,2 \\ \delta^{\psi}_{L}\left[0 + \frac{\bar{g}^2}{16\pi^2} B^{\psi}_{L,n}\right] & k = L \end{cases}$$

$$C^{G}_{k,n}(1, \bar{g}^2) = \begin{cases} \delta^{\psi}_{k}\left[0 + \frac{\bar{g}^2}{16\pi^2} B^{G}_{k,n}\right] & k = 1,2 \\ \delta^{\psi}_{L}\left[0 + \frac{\bar{g}^2}{16\pi^2} B^{G}_{L,n}\right] & k = L \end{cases}$$

MOMENTS :

1) LEADING ORDER :

$$M^{k,n}_S(Q^2) = \int_0^1 dx\, x^{n-2} F_S(x, Q^2)$$

$$= \delta^k_{\psi} A^+_n \left[\ln \frac{Q^2}{\Lambda^2}\right]^{-d^n_+} + \delta^k_{\psi} A^-_n \left[\ln \frac{Q^2}{\Lambda^2}\right]^{-d^n_-}$$

$$d^n_{\pm} = \frac{\lambda^n_{\pm}}{2\beta_0}, \quad \lambda^n_{\pm} = \frac{\gamma^{0,n}_{\psi\psi} + \gamma^{0,n}_{GG} \pm \sqrt{D^{0,n}}}{2}$$

ANOMALOUS DIMENSIONS:

$$\gamma^{0,n}_{\psi\psi} = \gamma^{0,n}_{NS} = \frac{8}{3}\left[1 - \frac{2}{n(n+1)} + 4\sum_{j=2}^{n}\frac{1}{j}\right]$$

$$\gamma^{0,n}_{\psi G} = -4N_f\,\frac{n^2+n+2}{n(n+1)(n+2)}$$

$$\gamma^{0,n}_{G,\psi} = -\frac{16}{3}\,\frac{n^2+n+2}{n(n^2-1)}$$

$$\gamma^{0,n}_{GG} = 6\left[\frac{1}{3} - \frac{4}{n(n+1)} - \frac{4}{(n+1)(n+2)} + 4\sum_{j=2}^{n}\frac{1}{j}\right] + \frac{4}{3}f$$

(GROSS, WILCZEK CONVENTION)

2) NTLO: FLORATOS, ROSS, SACHRAJDA, 1979
BARDEEN, BURAS 1979 cf. ALSO: BURAS 1980

REPRESENTATION OF STRUCTURE FUNCTIONS:

CF. SECT. 4.

e.g.: $F_2(x,Q^2)$ (γ-EXCHANGE)

$$F_2(x,Q^2) = \frac{5}{18}\Sigma(x,Q^2) + \frac{1}{6}\Delta^{ep}(x,Q^2).$$

$$\Delta_{ep}(x,Q^2) = \sum_i (u_i + \bar{u}_i - d_i - \bar{d}_i)(x,Q^2)$$

$$\Sigma(x,Q^2) = \sum_i (u_i + \bar{u}_i + d_i + \bar{d}_i)(x,Q^2)$$

$$\langle \phi(Q^2) \rangle_n \equiv \int_0^1 dx\, x^{n-1} \phi(x,Q^2)$$

Df: $$\tilde{s} = \ln\left[\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right]$$

$$\langle \Delta(Q^2) \rangle_n = \langle \Delta(Q_0^2) \rangle_n \, e^{-d_{NS}^n \tilde{s}}$$

$$\begin{pmatrix} \langle \Sigma(Q^2) \rangle_n \\ \langle G(Q^2) \rangle_n \end{pmatrix} = \begin{pmatrix} A^n_{\Sigma\Sigma} & A^n_{\Sigma G} \\ A^n_{G\Sigma} & A^n_{GG} \end{pmatrix} \begin{pmatrix} \langle \Sigma(Q_0^2) \rangle_n \\ \langle G(Q_0^2) \rangle_n \end{pmatrix}$$

$$A^n_{\Sigma\Sigma} = (1-\alpha_n)\, e^{-d_+^n \tilde{s}} + \alpha_n\, e^{-d_-^n \tilde{s}}$$

$$A^n_{\Sigma G} = -\tilde{\alpha}_n\, e^{-d_+^n \tilde{s}} + \tilde{\alpha}_n\, e^{-d_-^n \tilde{s}}$$

$$A^n_{G\Sigma} = -\varepsilon_n\, e^{-d_+^n \tilde{s}} + \varepsilon_n\, e^{-d_-^n \tilde{s}}$$

HERE,

$$\alpha_n = \frac{\gamma^{0,n}_{\psi\psi} - \lambda^n_+}{\lambda^n_- - \lambda^n_+} \, , \quad \tilde{\alpha}_n = \frac{\gamma^{0,n}_{\psi G}}{\lambda^n_- - \lambda^n_+} \, , \quad \varepsilon_n = \frac{\gamma^{0,n}_{G\psi}}{\lambda^n_- - \lambda^n_+}$$

HOW TO GET BACK TO THE X-SPACE ?

⟶ INVERSE MELLIN TRANSFORM.

$$F(x,Q^2) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dn \; x^{-n+1} \langle F(Q^2) \rangle_n$$

$$= \frac{1}{\pi} \int_0^\infty dz \, \mathrm{Re} \left\{ x^{1-iz-c} \langle F(Q^2) \rangle_{n=c+iz} \right\}$$

e.g. GROSS 1977

ii) ALTARELLI - PARISI EQUATIONS

EXAMPLE : NON-SINGLET EVOLUTION

MOMENTS $\longrightarrow$ SPLITTING FUNCTIONS

$$\langle \Delta(Q^2) \rangle_n = \langle \Delta(Q_0^2) \rangle_n \exp\left[- d_{NS}^n \tilde{s} \right]$$

$$\frac{d}{d \ln(Q^2/\Lambda^2)} \langle \Delta(Q^2) \rangle_n = - d_{NS}^n \frac{d\tilde{s}}{d \ln(Q^2/\Lambda^2)} \langle \Delta(Q^2) \rangle_n$$

$$d\tilde{s} / d \ln(Q^2/\Lambda^2) = \frac{1}{\ln(Q^2/\Lambda^2)}$$

$$\frac{d \langle \Delta(Q^2) \rangle_n}{d \ln(Q^2/\Lambda^2)} = - \frac{\gamma_{\psi\psi}^{0,n}}{4} \frac{\alpha_s(Q^2)}{2\pi} \langle \Delta(Q^2) \rangle_n$$

MELLIN - TRANSFORM OF A CONVOLUTION :

$$H(x) = \int_0^1 dx_1 \int_0^1 dx_2 \, \delta(x - x_1 x_2) \, H_1(x_1) H_2(x_2)$$

$$M_n[H] = M_n[H_1] \cdot M_n[H_2]$$

i.e. :

$$\frac{d \Delta(x, Q^2)}{d \ln(Q^2/\Lambda^2)} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 dy \frac{1}{y} P_{NS}\left(\frac{x}{y}\right) \Delta(y, Q^2)$$

WITH:

$$- \frac{\gamma_{\psi\psi}^{0,n}}{4} = \int_0^1 dx \, x^{n-1} P_{NS}(x)$$

$$-\frac{1}{4}\gamma^{0,n}_{\psi\psi} = +\frac{2}{3}\left[-1+\frac{2}{n(n+1)}-4\sum_{j=2}^{n}\frac{1}{j}\right]$$

$$= \frac{2}{3}\left\{-\sum_{j=1}^{n-1}\frac{1}{j}-\sum_{j=3}^{n+1}\frac{1}{j}\right\}$$

$$= \frac{2}{3}\int_0^1 dx\left[\frac{x^{n-1}-1}{1-x}+\frac{x^{n+1}-1}{1-x}\right]$$

$$= \frac{4}{3}\int_0^1 dx\, x^{n-1}\,\frac{1+x^2}{(1-x)_+}$$

$\forall n$

$$P_{NS}(x) = \frac{4}{3}\left[\frac{1+x^2}{1-x}+\frac{3}{2}\,\delta(1-x)\right]$$

EXERCISE: SINGLET CASE:

$$\boxed{\int_0^1 dx\, x^{n-1}\,P^S_{ij}(x) = -\frac{1}{4}\gamma_{ij}}$$

LEADING ORDER SPLITTING FUNCTIONS

$$P_{qq}(x) = P_{NS}(x) = \frac{4}{3}\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\,\delta(1-x)\right] = \frac{4}{3}\left(\frac{1+x^2}{1-x}\right)_+$$

$$P_{qG}(x) = 2T_R N_f\left[x^2 + (1-x)^2\right], \quad T_R = \frac{1}{2}, \quad C_F = \frac{4}{3}$$

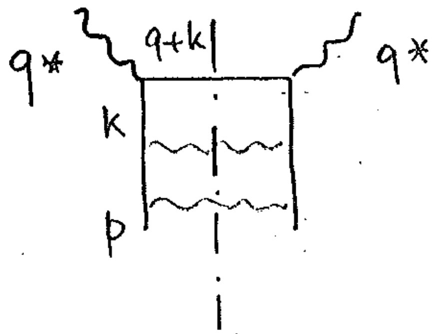
$$C_G = 3$$

$$P_{Gq}(x) = \frac{4}{3}\,\frac{1+(1-x)^2}{x}$$

$$P_{GG}(x) = 2C_G\left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right] + \left(\frac{11}{6}C_G - \frac{2T_R}{3}\right)\cdot\delta(1-x)$$

THE OPE IS NOT THE ONLY WAY TO DERIVE THE SPLITTING FUNCTIONS.

CONSIDER:



- USE THE AXIAL GAUGE:

$$d_{\mu\nu}(k) = g_{\mu\nu} - (k_\mu \eta_\nu - \eta_\mu k_\nu)\frac{1}{k\eta} + \eta^2 \frac{k_\mu k_\nu}{(k.\eta)^2}$$

NO FADDEEV - POPOV GHOSTS : PHYSICAL GLUONS

- SUDAKOV VARIABLES

$$k_\mu = \xi p_\mu + \beta q'_\mu + k_{\perp\mu}$$

$$q'_\mu = q_\mu + x p_\mu \quad , \quad k_\perp p = k_\perp q' = 0.$$

$$d^4k = 2\pi (pq)\, d\beta\, d\xi\, dk_\perp^2$$

FEW KINEMATICAL STEPS ...

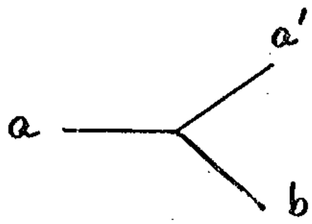
$$W_{\mu\nu}(p,q) = \frac{g^2}{6\pi^2}\int \frac{dk^2}{k^2}\left(\frac{1+x^2}{1-x}\right)\left[\frac{x}{\nu}\frac{p_\mu p_\nu}{p^2} + \dots\right]$$

↑ AP: NS

first rank:

ITERATE : $p^2 \ll k_1^2 \ll k_2^2 \dots k_N^2 \ll Q^2$.

ONE CALCULATES THE CONTRIBUTIONS OF



$a \to b$ IN THE COLLINEAR APPROXIMATION

$m_a = m_b = 0 \quad , \quad p_b = z\, p_a$

$\longrightarrow$ CONTRIBUTION $\sim$ MASS SINGULARITY

$$\sim \ln\left(\frac{P_\perp^{max^2}}{m^2}\right).$$

$$P_{qq}(z) = \frac{1+z^2}{1-z}\Bigg|_{z<1}$$

$$P_{gq}(z) = \frac{1+(1-z)^2}{z}$$

$z \to (1-z)$

FURTHER :

$$P_{qG}(z) = P_{qG}(1-z) \quad , \quad P_{GG}(z) = P_{GG}(1-z)$$

MOMENTUM CONSERVATION:

$$\int_0^1 dz \; z \left[P_{qq}(z) + P_{Gq}(z) \right] = 0$$

$$\int_0^1 dz \; z \left[2N_f \, P_{qG}(z) + P_{GG}(z) \right] = 0$$

CALCULATING SPLITTING FUNCTIONS

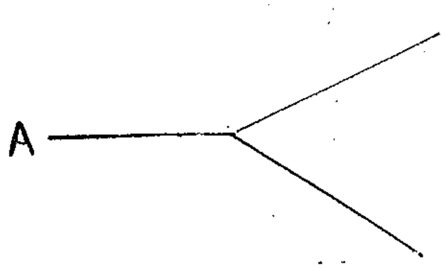
à la ALTARELLI - PARISI :

$\longrightarrow$ EQUIVALENT PHOTON METHOD OF QED

WEIZSÄCKER-WILLIAMS APPROXIMATION.

$$P_{BA}(x) = \frac{x(1-x)}{2} \overline{\sum} \frac{|V_{A\to BC}|^2}{k_\perp^2} \qquad x<1$$

PARTON SPLITTING:



$$k_B = \left(xp + \frac{k_\perp^2}{2xp}, xp, \vec{k}_\perp\right)$$

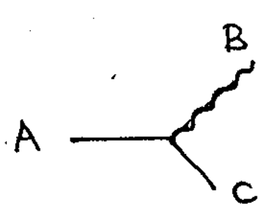
$$k_C = \left((1-x)p + \frac{k_\perp^2}{2(1-x)p}; (1-x)p, -\vec{k}_\perp\right)$$

$$K_A = (p; p, \vec{0})$$

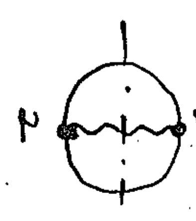
$$P_{BA}(x) = P_{CA}(1-x) \quad , \quad x<1$$

e.g.: $P_{Gq}(x)$: $\qquad \overline{\sum}|V_{q\to Gq}|^2 = \frac{1}{2} C_F \, \mathrm{tr}[k\!\!\!/_C \gamma_\mu k\!\!\!/_A \gamma_\nu]$ (Spin)

A, C quarks
B gluon



$\equiv$



Color

$$\times \sum_{pol} \epsilon_B^{*\mu} \epsilon_B^\nu$$

$$\delta_{ab} C_F = \sum_A (T^A T_A)_{ab}$$

$$\sum_{pol} \epsilon_B^{*\mu} \epsilon_B^\nu = \begin{cases} \delta^{ij} - k_B^i k_B^j / \vec{k}_B^{\,2} & \mu,\nu = 1,2,3 \\ 0 & \mu \text{ and/or } \nu = 0 \end{cases}$$

- ONE HAS NOW TO EXPAND ALL THE PRODUCTS :

i.e. $\quad tr\left[\not{k}_C \gamma_\mu \not{k}_A \gamma_\nu \right] = 4\left[k_C^\mu k_A^\nu + k_C^\nu k_A^\mu - g_{\mu\nu}\, k_C \cdot k_A \right]$

- CONTRACT WITH $\sum_{pol} \epsilon^{*\mu} \epsilon^\nu$

USE : $\quad p^2 = 0$

- $k_\perp^2 \approx 0 \qquad \left(\text{i.e. } \frac{1}{k_\perp^2} \; k_\perp^4 \; ! \right)$

NUMERATOR TERMS $\propto k_\perp^2$

THESE ARE PURELY x - DEPENDENT FUNCTIONS !

(FOR $k_\perp^2 \to 0$: WEIZSÄCKER / WILLIAMS !)

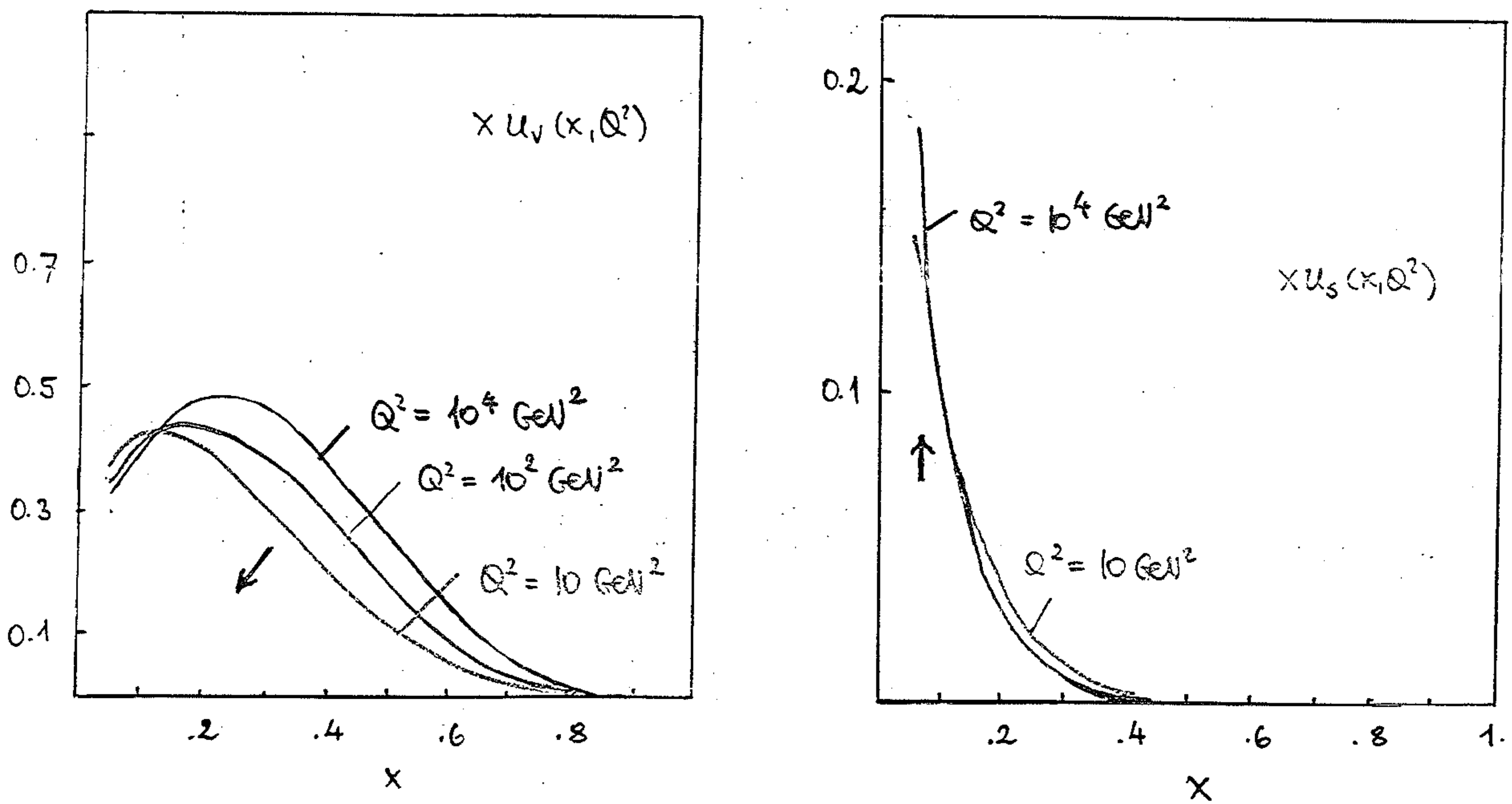
$$\boxed{P_{Gq}(x) = C_F \, \frac{1 + (1-x)^2}{x}}$$

LO Probability of a gluon in a quark.

Question : Prob. : quark in a quark: $\quad x \to (1-x)$
cf. above.

$$\boxed{P_{qq}(x) = C_F \, \frac{1+x^2}{1-x} \qquad x < 1}$$

DO 1



$x u_v(x,Q^2)$
$Q^2 = 10^4\ GeV^2$
$Q^2 = 10^2\ GeV^2$
$Q^2 = 10\ GeV^2$
0.7
0.5
0.3
0.1
.2
.4
.6
.8
x
0.2
0.1
$Q^2 = 10^4\ GeV^2$
$x u_s(x,Q^2)$
$Q^2 = 10\ GeV^2$
.2
.4
.6
.8
1.
x

NEXT TO LEADING ORDER

NS :

LO

$$\hat{P}_{qq}(x,\alpha) = \underline{\left(\frac{\alpha}{2\pi}\right) C_F \left(\frac{1+x^2}{1-x}\right)}$$
$$+ \left(\frac{\alpha}{2\pi}\right)^2 \left[C_F^2 P_F(x) + \tfrac{1}{2} C_F C_G P_G(x) + C_F N_F T_F P_{N_F}(x) \right], \tag{4.50}$$

$$\underline{\hat{P}_{q\bar{q}}(x,\alpha)} = \left(\frac{\alpha}{2\pi}\right)^2 \left(C_F^2 - \tfrac{1}{2} C_F C_G \right) P_A(x), \tag{4.51}$$

$\uparrow$

where

$$P_F(x) = -2\frac{1+x^2}{1-x} \ln x \ln(1-x) - \left(\frac{3}{1-x} + 2x\right) \ln x - \tfrac{1}{2}(1+x)\ln^2 x - 5(1-x), \tag{4.52}$$

$$P_G(x) = \frac{1+x^2}{1-x}\left[\ln^2 x + \tfrac{11}{3}\ln x + \tfrac{67}{9} - \tfrac{1}{3}\pi^2\right] + 2(1+x)\ln x + \tfrac{40}{3}(1-x), \tag{4.53}$$

$$P_{N_F}(x) = \tfrac{2}{3}\left[\frac{1+x^2}{1-x}\left(-\ln x - \tfrac{5}{3}\right) - 2(1-x)\right], \tag{4.54}$$

$$P_A(x) = 2\frac{1+x^2}{1+x}\int_{x/(1+x)}^{1/(1+x)} \frac{dz}{z} \ln\frac{1-z}{z} + 2(1+x)\ln x + 4(1-x). \tag{4.55}$$

CURCI et al.
1980

NTLO : NS

TABLE I

Detailed contribution of various diagrams to $\Gamma_{qq}(x, \alpha, 1/\varepsilon)$

DIAGRAMS :

$\Gamma_{qq}(x,\alpha,1/\varepsilon)$	C_F^2					SUM	$\frac{1}{2}C_F C_G$				SUM	$\frac{1}{2}N_F C_F$
$1/\varepsilon$ A: $\frac{1+x^2}{1-x}$	$7-\frac{2}{3}\pi^2$	$-7+\frac{2}{3}\pi^2$	0	0	0	0	$7-\frac{2}{3}\pi^2$	0	$-11+\pi^2$	$\frac{103}{9}-\frac{2}{3}\pi^2$	$\frac{67}{9}-\frac{1}{3}\pi^2$	$-\frac{10}{9}$
$\frac{1+x^2}{1-x}\ln^2 x$	-2	1	-1	2	0	0	-1	1	1	0	1	0
$(1+x)\ln x$	0	$-\frac{7}{2}$	2	-1	0	$-\frac{5}{2}$	$\frac{7}{2}$	-2	$\frac{1}{2}$	0	2	0
$1-x$	3	-11	0	3	0	-5	11	0	-1	$\frac{10}{3}$	$\frac{40}{3}$	$-\frac{4}{3}$
B: $(1+x)\ln^2 x$	0	0	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	0	0	0	0	0
$\frac{1+x^2}{1-x}\ln^2(1-x)$	0	0	0	0	0	0	0	0	2	-2	0	0
$\frac{1+x^2}{1-x}\ln x\ln(1-x)$	-4	2	0	0	0	-2	-2	0	6	-4	0	0
$\frac{1+x^2}{1-x}\ln x$	0	$-\frac{3}{2}$	0	0	0	$-\frac{3}{2}$	$\frac{3}{2}$	0	$-\frac{3}{2}$	$\frac{11}{3}$	$\frac{11}{3}$	$-\frac{2}{3}$
$\frac{1+x^2}{1-x}\ln(1-x)$	3	-3	4	-4	0	0	3	-4	5	-4	0	0
$(1-x)\ln x$	-4	2	0	3	0	1	-2	0	2	0	0	0
$(1-x)\ln(1-x)$	0	0	0	0	0	0	0	0	4	-4	0	0
$I_1\frac{1+x^2}{1-x}$	4	-4	0	0	0	0	4	0	-8	4	0	0
$I_0\frac{1+x^2}{1-x}$	0	0	4	-4	0	0	0	-4	8	-4	0	0
$I_0\frac{1+x^2}{1-x}(\ln x+\ln(1-x))$	-4	4	0	0	0	0	-4	0	8	-4	0	0
$I_0(1-x)$	-4	4	0	0	0	0	-4	0	8	-4	0	0

$I_0 = \int_0^1 du \frac{u}{u^2+\delta^2}$

$I_1 = \int_0^1 du \frac{u}{u^2+\delta^2} \ln u$

Appropriate colour factors are shown in the first line. Terms of type A satisfy the Gribov-Lipatov relation while those of type B break it.

THE NTLO SINGLET SPLITTING FUNCTIONS:

Our results for the two-loop probabilities $\hat{P}(x)$ are [13]:

$$
\begin{aligned}
\hat{P}^{(1,S)}_{FF} &= C_F^2[-1+x+(\tfrac{1}{2}-\tfrac{3}{2}x)\ln x-\tfrac{1}{2}(1+x)\ln^2 x-(\tfrac{3}{2}\ln x+2\ln x\ln(1-x))p_{FF}(x)+2p_{FF}(-x)S_2(x)] \\
&+ C_F C_G[\tfrac{14}{3}(1-x)+(\tfrac{11}{6}\ln x+\tfrac{1}{2}\ln^2 x+\tfrac{67}{18}-\tfrac{1}{6}\pi^2)p_{FF}(x)-p_{FF}(-x)S_2(x)] \\
&+ C_F T_R N_F[-\tfrac{16}{3}+\tfrac{40}{3}x+(10x+\tfrac{16}{3}x^2+2)\ln x-\tfrac{112}{9}x^2+\tfrac{40}{9}x^{-1}-2(1+x)\ln^2 x-(\tfrac{10}{9}+\tfrac{2}{3}\ln x)p_{FF}(x)]\,, \\
\hat{P}^{(1,S)}_{FG} &= C_F^2[-\tfrac{5}{2}-\tfrac{7}{2}x+(2+\tfrac{7}{2}x)\ln x+(-1+\tfrac{1}{2}x)\ln^2 x-2x\ln(1-x)+(-3\ln(1-x)-\ln^2(1-x))p_{FG}(x)] \\
&+ C_F C_G[\tfrac{28}{9}+\tfrac{65}{18}x+\tfrac{44}{9}x^2+(-12-5x-\tfrac{8}{3}x^2)\ln x+(4+x)\ln^2 x+2x\ln(1-x)+(-2\ln x\ln(1-x) \\
&+ \tfrac{1}{2}\ln^2 x+\tfrac{11}{3}\ln(1-x)+\ln^2(1-x)-\tfrac{1}{6}\pi^2+\tfrac{1}{2})p_{FG}(x)+p_{FG}(-x)S_2(x)] \\
&+ C_F T_R N_F[-\tfrac{4}{3}x-(\tfrac{20}{9}+\tfrac{4}{3}\ln(1-x))p_{FG}(x)]\,, \\
\hat{P}^{(1,S)}_{GF} &= C_F T_R N_F[4-9x+(-1+4x)\ln x+(-1+2x)\ln^2 x+4\ln(1-x) \\
&+ (-4\ln x\ln(1-x)+4\ln x+2\ln^2 x-4\ln(1-x)+2\ln^2(1-x)-\tfrac{2}{3}\pi^2+10)p_{GF}(x)] \\
&+ C_G T_R N_F[\tfrac{182}{9}+\tfrac{14}{9}x+\tfrac{40}{9}x^{-1}+(\tfrac{136}{3}x-\tfrac{38}{3})\ln x-4\ln(1-x)-(2+8x)\ln^2 x+(-\ln^2 x \\
&+ \tfrac{44}{3}\ln x-2\ln^2(1-x)+4\ln(1-x)+\tfrac{1}{3}\pi^2-\tfrac{218}{9})p_{GF}(x)+2p_{GF}(-x)S_2(x)]\,, \\
\hat{P}^{(1,S)}_{GG} &= C_F T_R N_F[-16+8x+\tfrac{20}{3}x^2+\tfrac{4}{3}x^{-1}+(-6-10x)\ln x+(-2-2x)\ln^2 x] \\
&+ C_G T_R N_F[2-2x+\tfrac{26}{9}x^2-\tfrac{26}{9}x^{-1}-\tfrac{4}{3}(1+x)\ln x-\tfrac{20}{9}p_{GG}(x)]\,, \\
&+ C_G^2[\tfrac{27}{2}(1-x)+\tfrac{67}{9}(x^2-x^{-1})+(-\tfrac{25}{3}+\tfrac{11}{3}x-\tfrac{44}{3}x^2)\ln x+4(1+x)\ln^2 x+(\tfrac{67}{9}-4\ln x\ln(1-x) \\
&+ \ln^2 x-\tfrac{1}{3}\pi^2)p_{GG}(x)+2p_{GG}(-x)S_2(x)]\,.
\end{aligned}
\qquad (11)
$$

Here and in the following, $p_{AB}(x)$ are defined as in eqs. (8) and

$$S_2(x) \equiv \int_{(1+x)/x}^{1/(1+x)} \frac{dz}{z}\ln\left(\frac{1-z}{z}\right);\quad S_1(x) \equiv \int_0^{1-x} \frac{dz}{z}\ln(1-z)\,.$$

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PETRONZIO '80