Parton Densities and Structure Functions Beyond the Next-To-Leading Order

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Topics

NNLO evolution of parton distributions

towards NNLO calculations for Tevatron/LHC

Structure functions at NNLO and beyond (N^3LO, soft-gluon resummation)

towards $\Delta_{\rm th}\, \alpha_s(M_Z^2) \simeq 1\%$ from DIS (NLO: $\simeq 5\%$)

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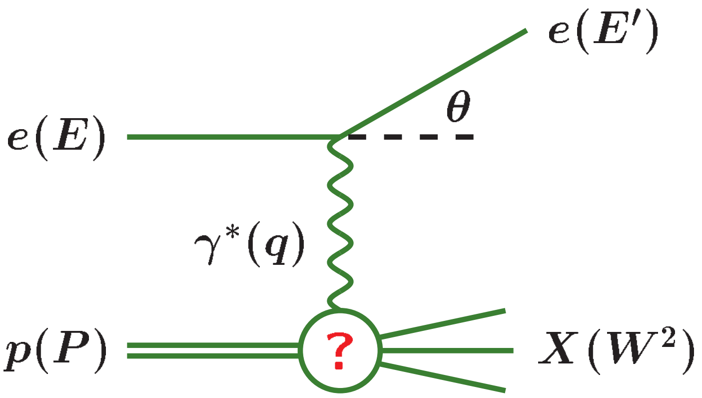
hep-ph/9907472, 9910545, 0006154, 0007362, 0010146, ...

Inelastic electron-proton scattering



Inclusive scattering: X unspecified $(W^2 > m_p^2)$

Resolution, Bjorken variable (inelasticity)

$$Q^2 \equiv -q^2 = 4EE' \sin^2 \frac{\theta}{2}$$

$$x = \frac{Q^2}{Q^2 + W^2 - m_p^2} = \frac{Q^2}{2P \cdot q}$$

Deep-inelastic scattering

$$Q^2 \gg m_p^2 \,, \qquad W^2 \gg m_p^2$$

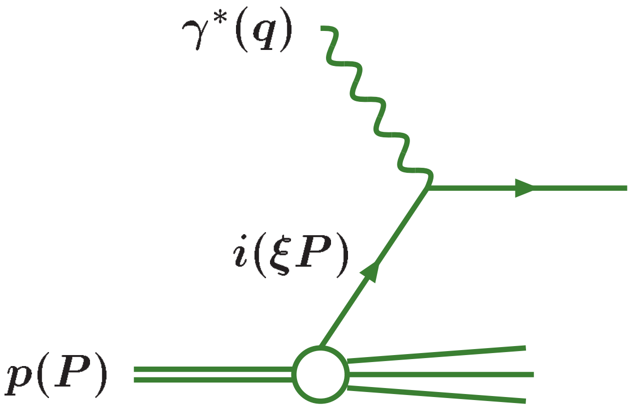
Cross section (symmetry constraints)

$$\frac{d\sigma}{d\Omega \, dE'} \stackrel{\text{lab}}{=} \underbrace{\frac{\alpha^2 \cos^2 \frac{\theta}{2}}{4E^2 \sin^4 \frac{\theta}{2}}}_{\text{Mott result (pointlike)}} \left\{ W_2(x, Q^2) + \tan^2 \frac{\theta}{2} \, W_1(x, Q^2) \right\}$$

$W_i(x, Q^2)$: structure functions, $(E - E')W_2 \equiv F_2$

The parton model

DIS = elastic, incoherent scattering off quasi-free pointlike constituents ('partons')



Elasticity, quasi-freedom

$$(\xi P + q)^2 \simeq 0 \qquad \rightarrow \qquad \xi \simeq x$$

Pointlike coupling, incoherence: $F_i(x, \cancel{Q}^2)$, scaling

$$F_2(x) = \sum_i e_i^2 \, x f_i(x)$$

$x f_i(x)$: momentum probability distributions

Quantum numbers

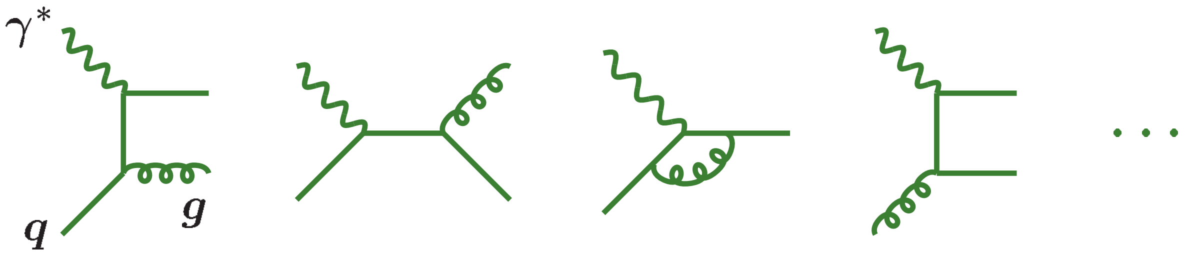
$$F_2^n / F_2^p \ , \ \nu p \rightarrow e X \ , \qquad F_1 / F_2$$

$\rightarrow$ charged: $|e_i| = 2/3, \ 1/3,$ spin 1/2 (quarks)

$$\int_0^1 dx \, F_2(x)$$

$\rightarrow$ neutrals: $\approx 50\%$ of momentum P (gluons)

QCD corrections in DIS



After renormalization (scale μ_r)

$$F_2 = \hat{F}_{2,k}(\alpha_s(\mu_r^2), \frac{Q^2}{\mu_r^2}, \varepsilon) \otimes \hat{f}_k$$

Initial-state mass singularities $\sim \epsilon^{-l}$, $\epsilon = \mathrm{dim.} - 4$

$$(p-k)^2 = -2|\vec{p}\,||\vec{k}|(1-\cos\vartheta) \xrightarrow{\vartheta \to 0} -|\vec{p}\,||\vec{k}|\vartheta^2$$

Mass factorization (scale μ_f)

$$F_2(Q^2) = \mathcal{C}_{2,i}(\alpha_s(\mu_r^2), \frac{Q^2}{\mu_f^2}, \frac{\mu_f^2}{\mu_r^2}) \otimes \underbrace{\Gamma_{ik}(\alpha_s(\mu_r^2), \frac{\mu_f^2}{\mu_r^2}, \varepsilon) \otimes \hat{f}_k}_{f_i(\mu_f^2,\, \mu_r^2)}$$

Γ_{ik} : includes (universal) pole part of $\hat{F}_{2,k}$

$\mathcal{C}_{2,i}$: coefficient functions (scheme dependent)

f_i : renormalized parton densities

$$\frac{\partial}{\partial \ln \mu_f^2} f_i = \frac{\partial}{\partial \ln \mu_f^2} \Gamma_{ik} \otimes \hat{f}_k = \mathcal{P}_{ij} \otimes f_j$$

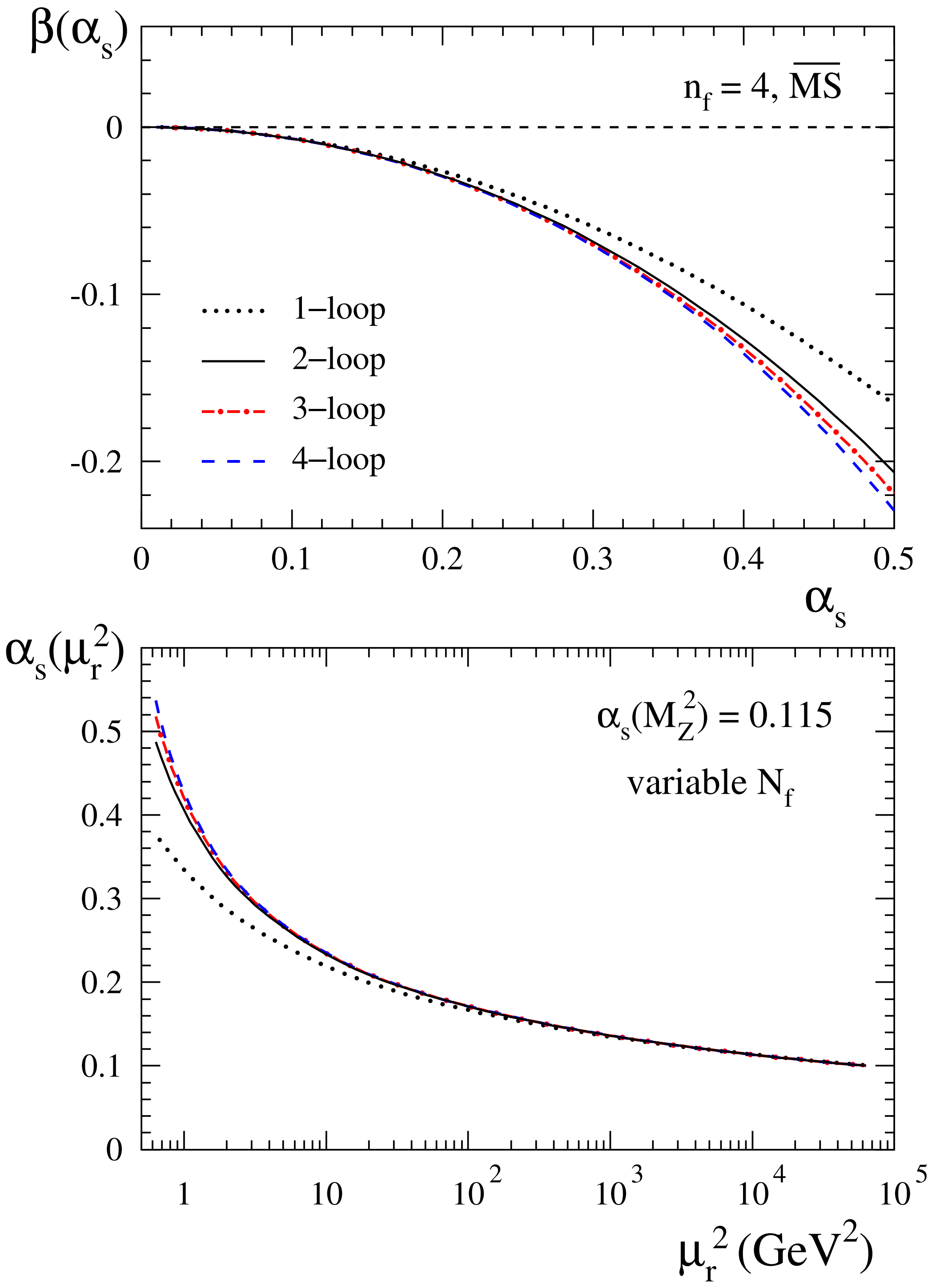
$\mathcal{P}_{ij}$: splitting functions

The running coupling constant

Renormalization group (μ_r independence)

$$da_s/d\ln\mu_r^2 = -\beta_0\, a_s^2 - \beta_1\, a_s^3 - \beta_2\, a_s^4 - \beta_3\, a_s^5 - \ldots$$

β_3 : van Ritbergen, Vermaseren, Larin (97)



Evolution of parton distributions

Non-singlet/singlet distributions $q^{\pm}$, q^V/q^S

$$\begin{aligned} q_{ik}^{\pm} &= q_i \pm \bar{q}_i - (q_k \pm \bar{q}_k) \\ q^V &= \sum_{r=1}^{N_f} (q_r - \bar{q}_r) \\ q^S &= \begin{pmatrix} \Sigma \\ g \end{pmatrix} \quad , \quad \Sigma = \sum_{r=1}^{N_f} (q_r + \bar{q}_r) \end{aligned}$$

Evolution equations ($\otimes$: Mellin convolution)

$$\frac{d}{d \ln \mu_f^2} q(x, \mu_f^2, \mu_r^2) = \left[\mathcal{P}(\alpha_s(\mu_r^2), \frac{\mu_f^2}{\mu_r^2}) \otimes q(\mu_f^2, \mu_r^2) \right](x)$$

$2N_f - 1$ scalar (NS) equations + 2×2 singlet case

Splitting functions, $\mu_f = \mu_r$

$$\mathcal{P} = a_s\, P^{(0)}(x) + a_s^2\, P^{(1)}(x) + a_s^3\, P^{(2)}(x) + \ldots$$

$\mu_f \neq \mu_r$

$$\begin{aligned} \mathcal{P}^{(0)} &= P^{(0)} \\ \mathcal{P}^{(1)} &= P^{(1)} - L\, \beta_0 P^{(0)} \\ \mathcal{P}^{(2)} &= P^{(2)} + L^2\, \beta_0^2 P^{(0)} - L\left(\beta_1 P^{(0)} + 2\beta_0 P^{(1)}\right) \end{aligned}$$

$L = \ln \frac{\mu_f^2}{\mu_r^2}$. $\mu_f = \mathcal{O}(\mu_r)$: no large logarithms

Non-singlet splitting functions

Results on $P_{\mathrm{NS}}^{(2)}(x)$

- $P^{(2)+}(N)$, $N = 2, \ldots, 14$ (even)

  Larin et al. (94, 96)

  Retey, Vermaseren (00)
- $P^{(2)-,V}(N)$, $N = 1, \ldots, 13$ (odd) RV (00)
- N_f^2 contribution Gracey (94)
- small-x terms ($\propto \ln^4 x$) Blümlein, A.V. (95)

General form in $\overline{\mathrm{MS}}$

$$P_{\mathrm{NS}}^{(2)}(x) \;=\; P_0^{(2)}(x) + N_f P_1^{(2)}(x) + N_f^2 P_2^{(2)}(x)$$

$$P_i^{(2)}(x) \;=\; A_i\,(1-x)_+^{-1} + B_i\,\delta(1-x) + f_{i,\mathrm{smooth}}(x) + \sum_{l=1} C_{i,l}\,\ln^l(1-x) + \sum_{l=1} D_{i,l}\,\ln^l x$$

no $[\ln^n(1-x)]/(1-x)$ terms Korchemsky (89)

Approximate reconstructions

A_i, B_i, parameter(s) of $f_{i,\mathrm{smooth}}$, some of $C_{i,l}$, $D_{i,l}$: fixed by moments. Variation: uncertainty estimate

Singlet splitting functions

Results on $P_{ab}^{(2)}(x)$

- $P_{ab}^{(2)}(N)$ for even $N = 2, \ldots, 12$ — Larin et al. (94, 96); Retey, Vermaseren (00)
- $N_f^2 C_A$ term of $P_{gg}^{(2)}$ — Bennett, Gracey (97)
- small terms, $\frac{1}{x} \ln x$ (except gq) — Catani, Hautmann (94); Fadin, Lipatov (98); Camici, Ciafaloni (98)

Assumption

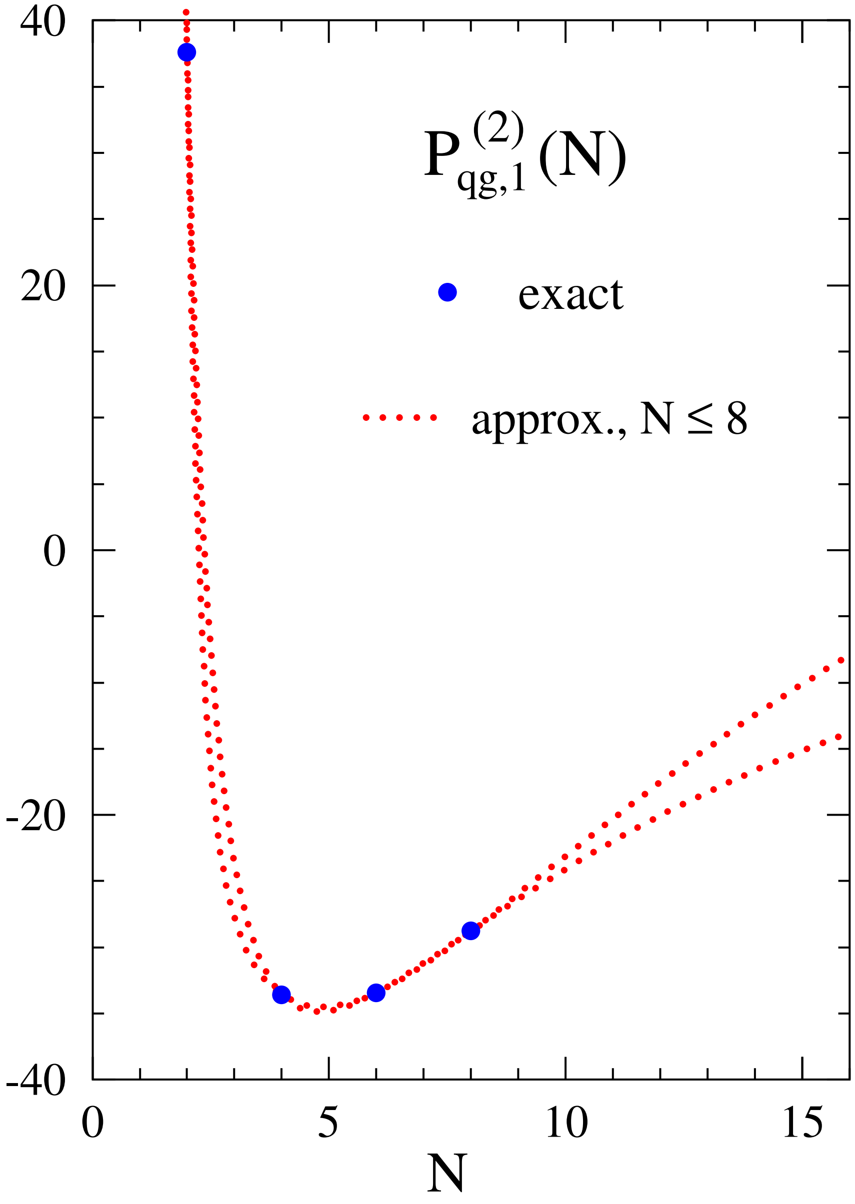
- $1/(1-x)_+$ terms of P_{gg}, P_{qq} : C_A/C_F

Approximate reconstructions

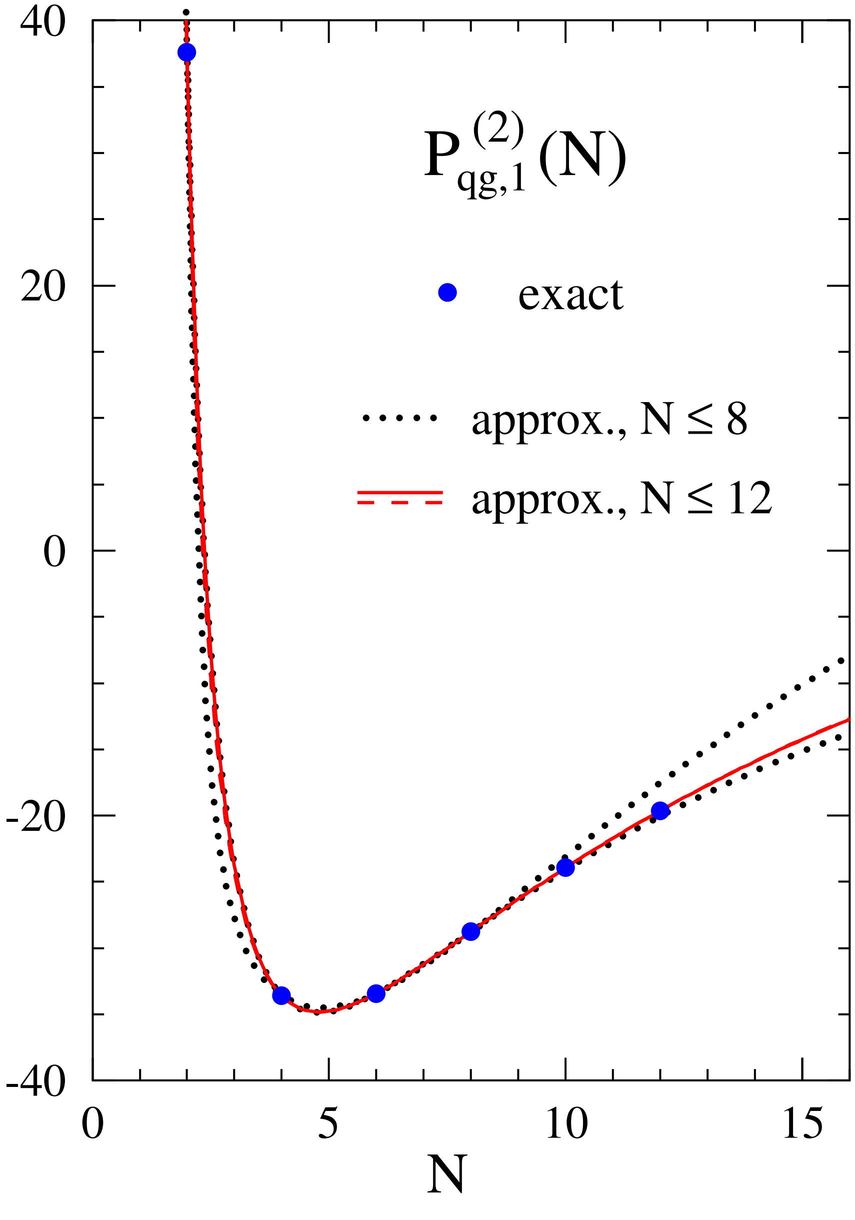
$P_{ab,i}^{(2)}(x)$: 10 functions, analogous to non-singlet

Illustrations ($\mu_f^2 \approx 30$ GeV^2)

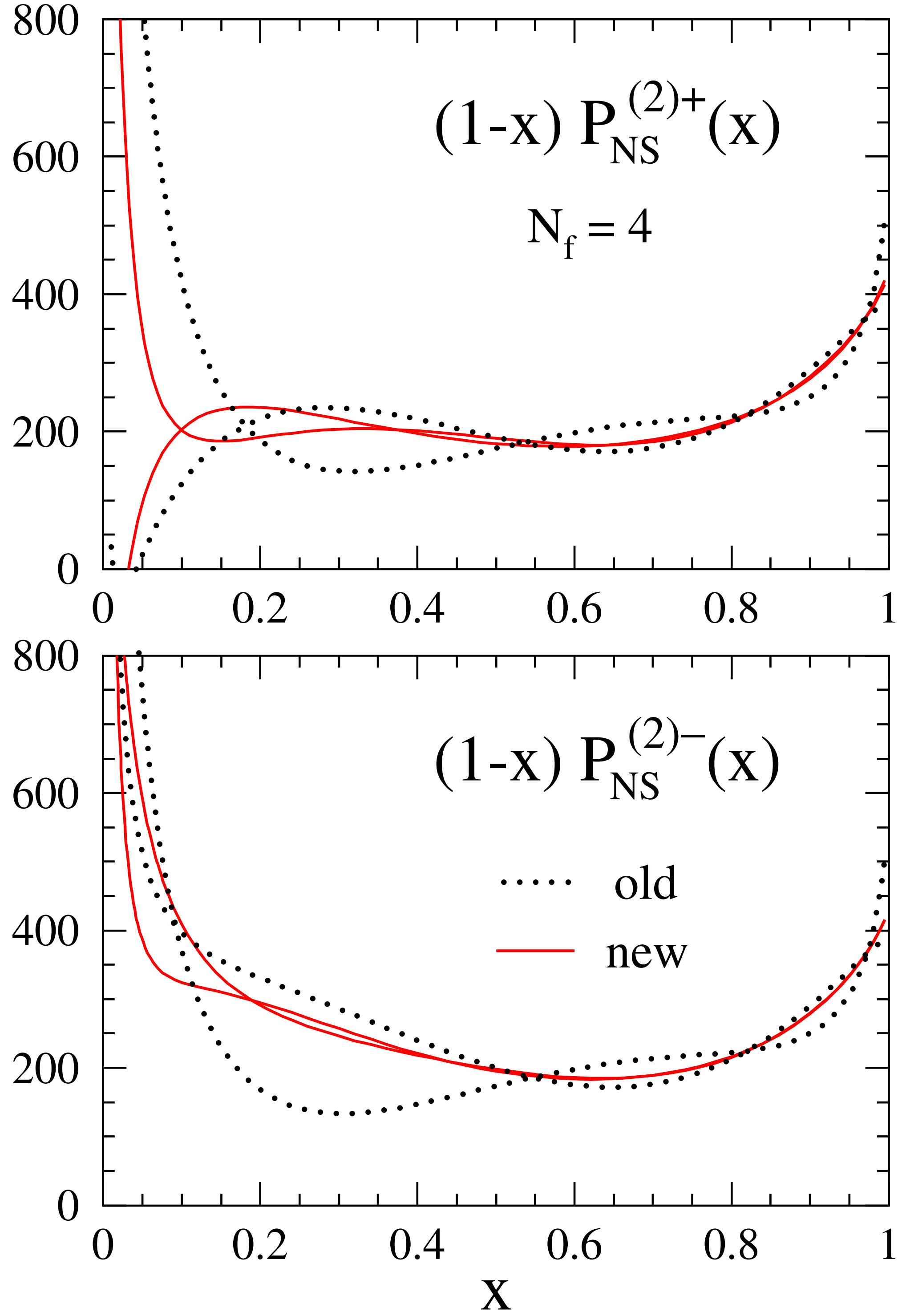
$$
\begin{aligned}
x\Sigma &= 0.6\, x^{-0.3}\, (1-x)^{3.5}\, (1 + 5\, x^{0.8}) \\
xg &= 1.0\, x^{-0.37} (1-x)^5 \\
N_f &= 4 \;, \quad \alpha_s(\mu_r^2 = \mu_f^2) \;=\; 0.2
\end{aligned}
$$



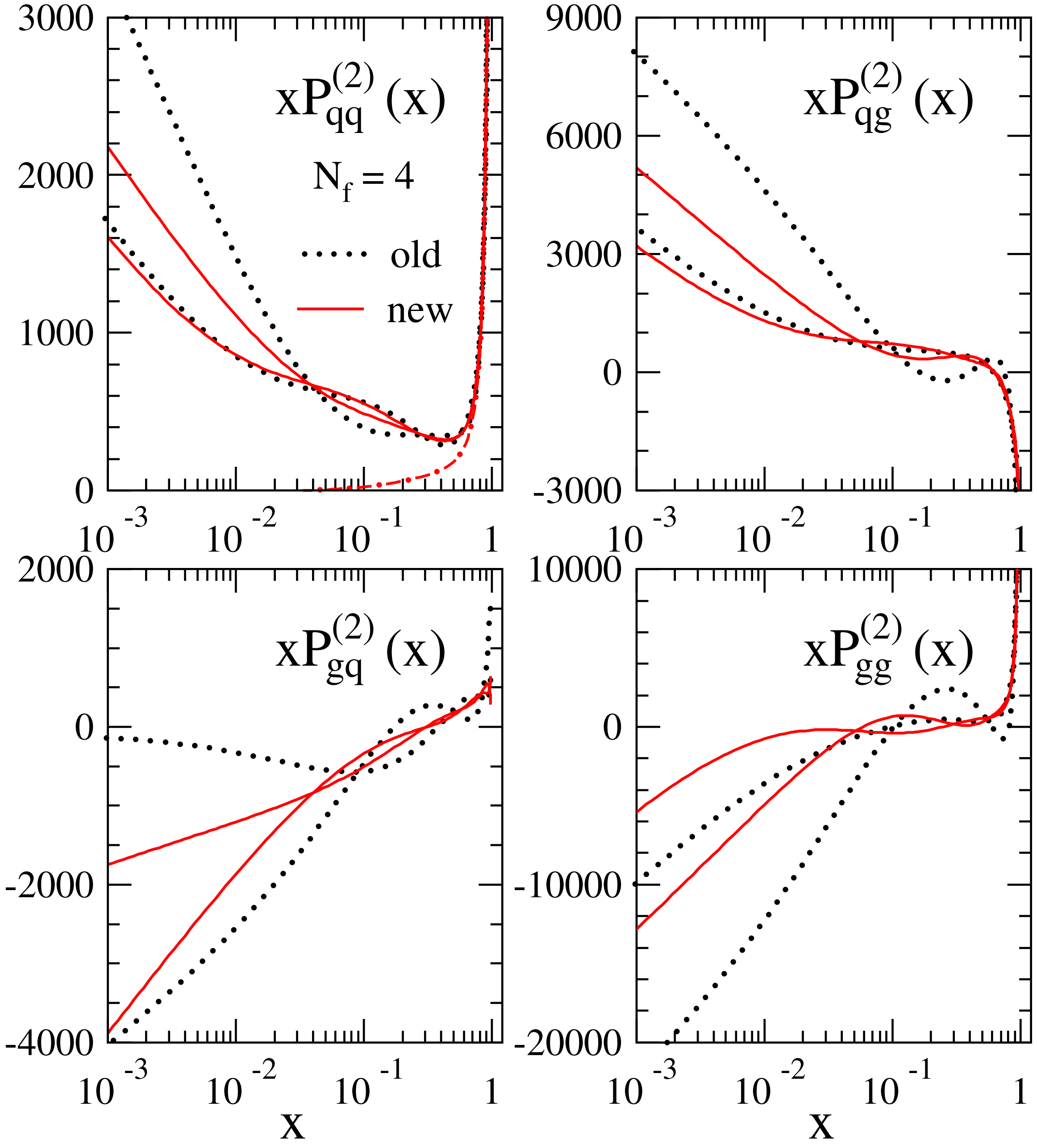
$P^{(2)}_{qg,1}(N)$
exact
approx., $N \leq 8$
40
20
0
-20
-40
0
5
10
15
N



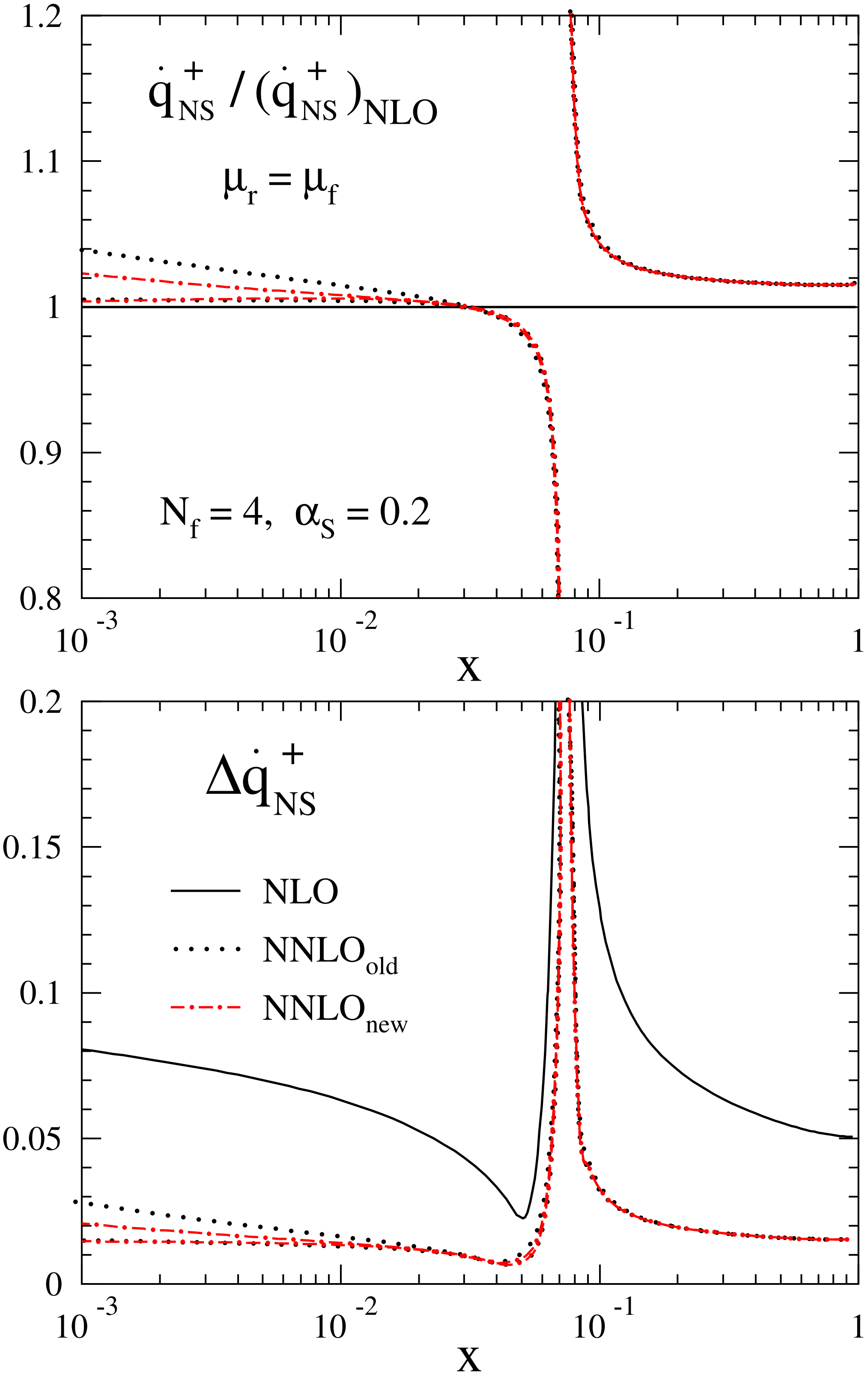
$P^{(2)}_{qg,1}(N)$
exact
approx., $N \leq 8$
approx., $N \leq 12$
40
20
0
-20
-40
0
5
10
15
N



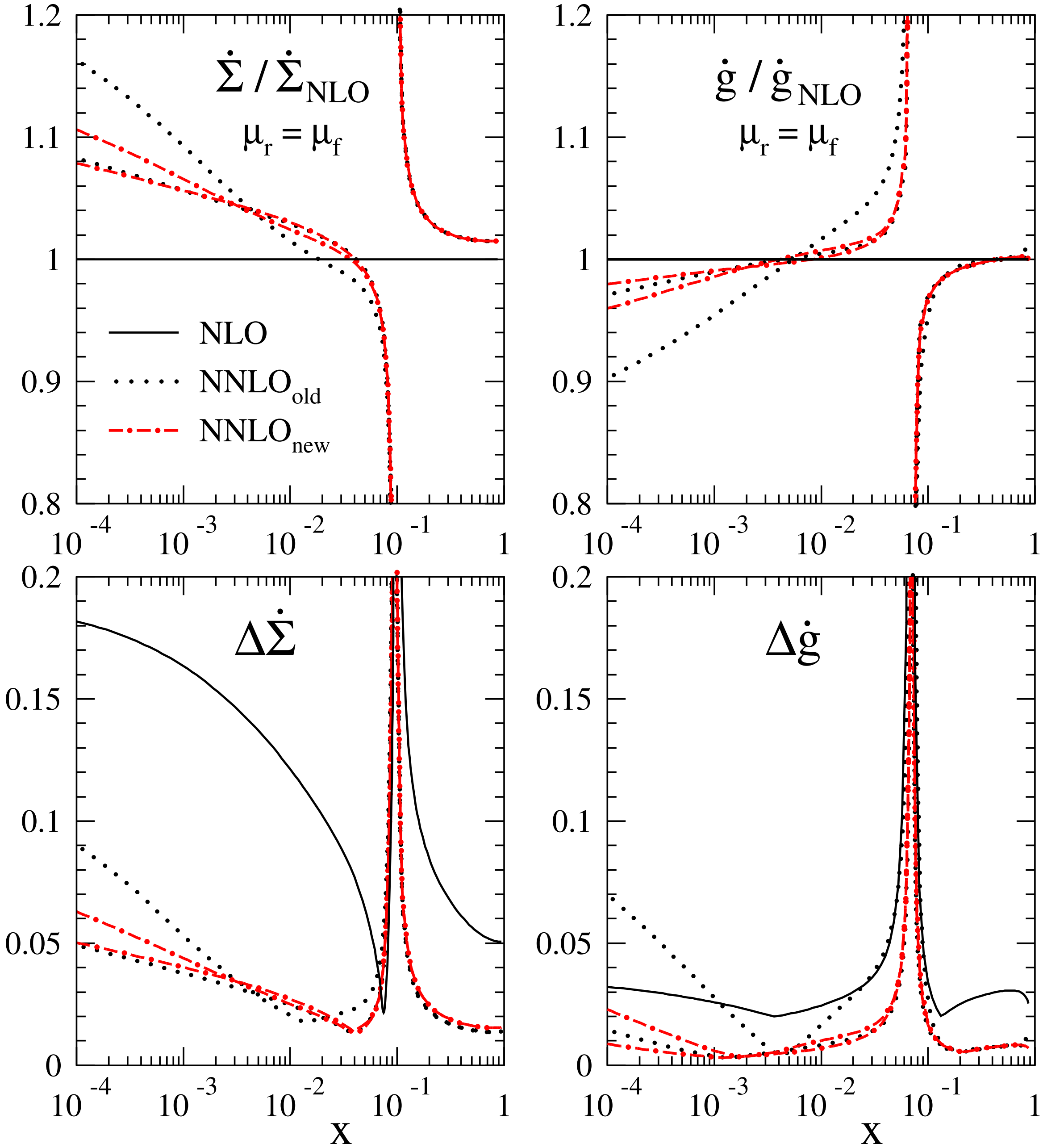
$(1-x)\, P_{NS}^{(2)+}(x)$
$N_f = 4$
$(1-x)\, P_{NS}^{(2)-}(x)$
old
new
x
0
200
400
600
800
0
0.2
0.4
0.6
0.8
1



$xP^{(2)}_{qq}(x)$
$N_f = 4$
old
new
$xP^{(2)}_{qg}(x)$
$xP^{(2)}_{gq}(x)$
$xP^{(2)}_{gg}(x)$
x
x



$\dot{q}^+_{NS} / (\dot{q}^+_{NS})_{NLO}$
$\mu_r = \mu_f$
$N_f = 4, \ \alpha_S = 0.2$
1.2
1.1
1
0.9
0.8
10^{-3}
10^{-2}
10^{-1}
1
x
$\Delta\dot{q}^+_{NS}$
NLO
$NNLO_{old}$
$NNLO_{new}$
0.2
0.15
0.1
0.05
0
10^{-3}
10^{-2}
10^{-1}
1
x



$$\dot{q} \;\equiv\; d \ln q / d \ln \mu_f^2$$

$$\Delta f \;=\; \frac{\max\ f - \min\ f}{2\,|\,\text{average}\ f|}\ ,\ \ \mu_r^2 = \frac{1}{4}\,\mu_f^2 \ldots\ 4\,\mu_f^2$$

Evolution of structure functions

Structure functions $\mathcal{F}_{1,2,3}$ $(2F_1,\ F_2/x,\ F_3)$

$$\mathcal{F}_a(x,Q^2) \;=\; \Big[\,\mathcal{C}_a(\alpha_s(\mu_r^2),\frac{Q^2}{\mu_f^2},\frac{\mu_f^2}{\mu_r^2}) \otimes q(\mu_f^2,\mu_r^2)\,\Big](x)$$

Coefficient functions for $\mu_r^2 = \mu_f^2 = Q^2$

$$\mathcal{C}_a = \delta(1-x) + a_s\, c^{(1)}_{a,q}(x) + a_s^2\, c^{(2)}_a(x) + a_s^3\, c^{(3)}_a(x) + \ldots$$

Non-singlet: Physical evolution kernels

$$\frac{d}{d\ln Q^2}\mathcal{F}_a^{\pm}(x,Q^2) \;=\; \Big[\,\mathcal{K}_a^{\pm}(\alpha_s(\mu_r^2),\frac{Q^2}{\mu_r^2}) \otimes \mathcal{F}_a^{\pm}(Q^2)\,\Big](x)$$

$\mu_r^2 = Q^2$:

$$\begin{aligned}
\mathcal{K}_a^{\pm} \;=\;\; & a_s\, P^{(0)}_{qq} \\
& + a_s^2 \left(P^{(1)\pm} - \beta_0 c^{(1)}_{a,q}\right) \\
& + a_s^3 \left(P^{(2)\pm} - \beta_1 c^{(1)}_{a,q} - \beta_0\{2c_a^{(2)\pm} - c^{(1)}_{a,q}\otimes c^{(1)}_{a,q}\}\right) \\
& + a_s^4 \Big(P^{(3)\pm} - \beta_2 c^{(1)}_{a,q} - \beta_1\{2c_a^{(2)\pm} - c^{(1)}_{a,q}\otimes c^{(1)}_{a,q}\} \\
& \qquad - \beta_0\{3c_a^{(3)\pm} - 3c_a^{(2)\pm}\otimes c^{(1)}_{a,q} + c^{(1)}_{a,q}\otimes c^{(1)}_{a,q}\otimes c^{(1)}_{a,q}\}\Big) \\
& + \ldots
\end{aligned}$$

$\mu_r^2 \neq Q^2$: analogous to parton evolution

Independent of factorization scheme/scale

Evolution kernels for $F_{2,\mathrm{NS}}$ and F_3

$c^{(2)}_{a,i}(x)$: known, checked — Zijlstra, van Neerven (91); Moch, Vermaseren (99)

Results on $c^{(3)}_{2,\mathrm{NS}}(x)$ and $c^{(3)}_3(x)$

- $c^{(3)+}_{2,\mathrm{NS}}(N)$, $N = 2, \ldots, 14$

  Larin et al. (94, 96)
  Retey, Vermaseren (00)

- $c^{(3)-}_3(N)$, $N = 1, \ldots, 13$ — RV (00)

- large-x terms, $\dfrac{\ln^k(1-x)}{1-x}$ — soft-gluon resumm.

  $k = 5, 4$: Larin ...
  $k = 3, 2$: A.V. (99)

Approximate reconstructions

for $N_f = 3, \ldots 5$, analogous to $P^{(2)}(x)$

$P^{(3)}_{\mathrm{NS}}$: expect small effect. Assume

$$P^{(3)}_{\mathrm{NS}}(x) = (15.7 \pm 15.7) \cdot P^{(2)}_{\mathrm{NS}}(x)$$

$\left(\frac{1}{4} \pm \frac{1}{4}\right.$ of $P^{(2)}_{\mathrm{NS}}$ effect on scale derivative for $\left.\alpha_s = 0.2 \right)$

Soft-gluon resummation in DIS/DY

Large-x terms of $c_{i,q}^{(k)}$, $l = 1, \ldots, 2k$

$$\left[\frac{\ln^{l-1}(1-x)}{1-x} \right]_+ \quad \longleftrightarrow \quad \frac{(-1)^l}{l} \underbrace{\ln^l N}_{L^l} + \ldots$$

Resummation

$$C_q^N \overset{N \to \infty}{=} (1 + a_s g_{01} + a_s^2 g_{02} + \ldots) \exp(G^N)$$
$$G^N = L g_1(a_s L) + g_2(a_s L) + a_s g_3(a_s L) + \ldots$$

g_1, g_2 : LL, NLL known — Sterman (87), Catani, Trentadue (89)

$g_3 = g_{32} a_s L + \ldots$ — Matsuura, van Neerven (88)

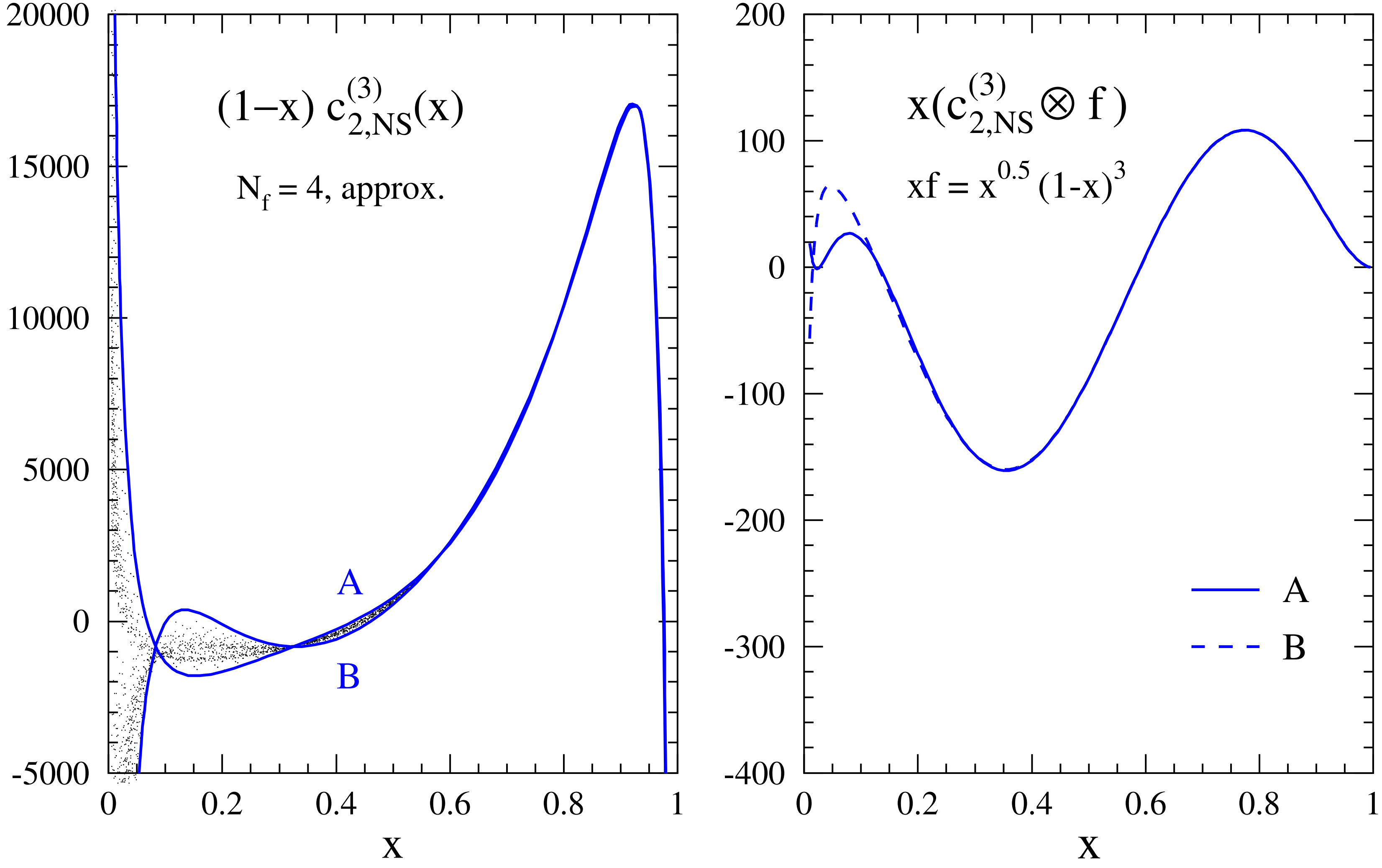
g_3 , NNLL: DY known
DIS almost known — A.V. (00)

Landau poles in $g_{2,3}$ at $(2) a_s \beta_0 L = 1$: prescriptions

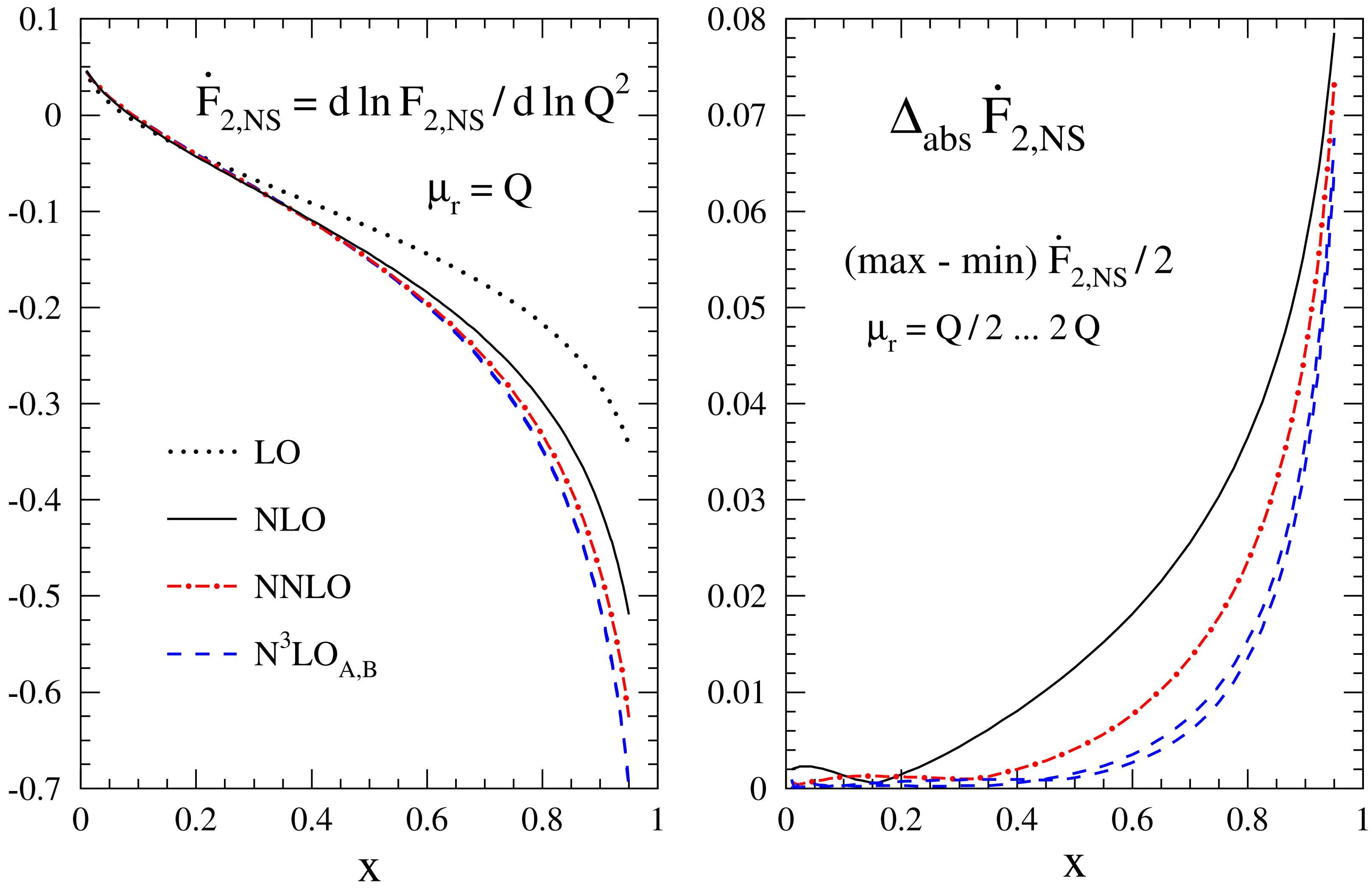
Expansion in α_s (no poles)

$$C_{i,q}^N = 1 + \sum_{k=1}^{\infty} a_s^k \sum_{m=1}^{2k} c_{km} L^{2k+1-m} + \ldots$$

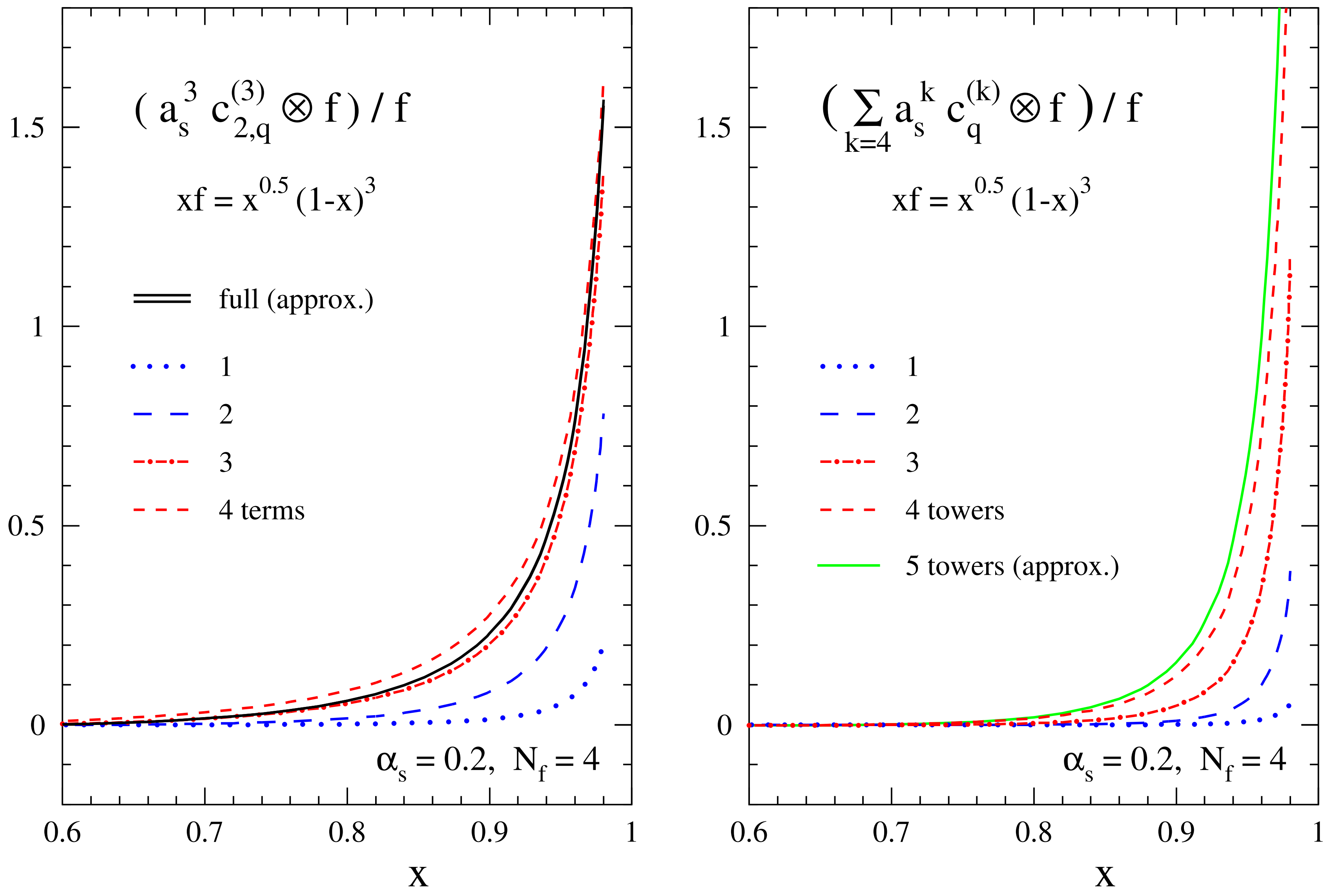
Coefficients $c_{k1} \ldots c_{k4}$, c_{k5}^{DY} fixed. — A.V. (99,00)



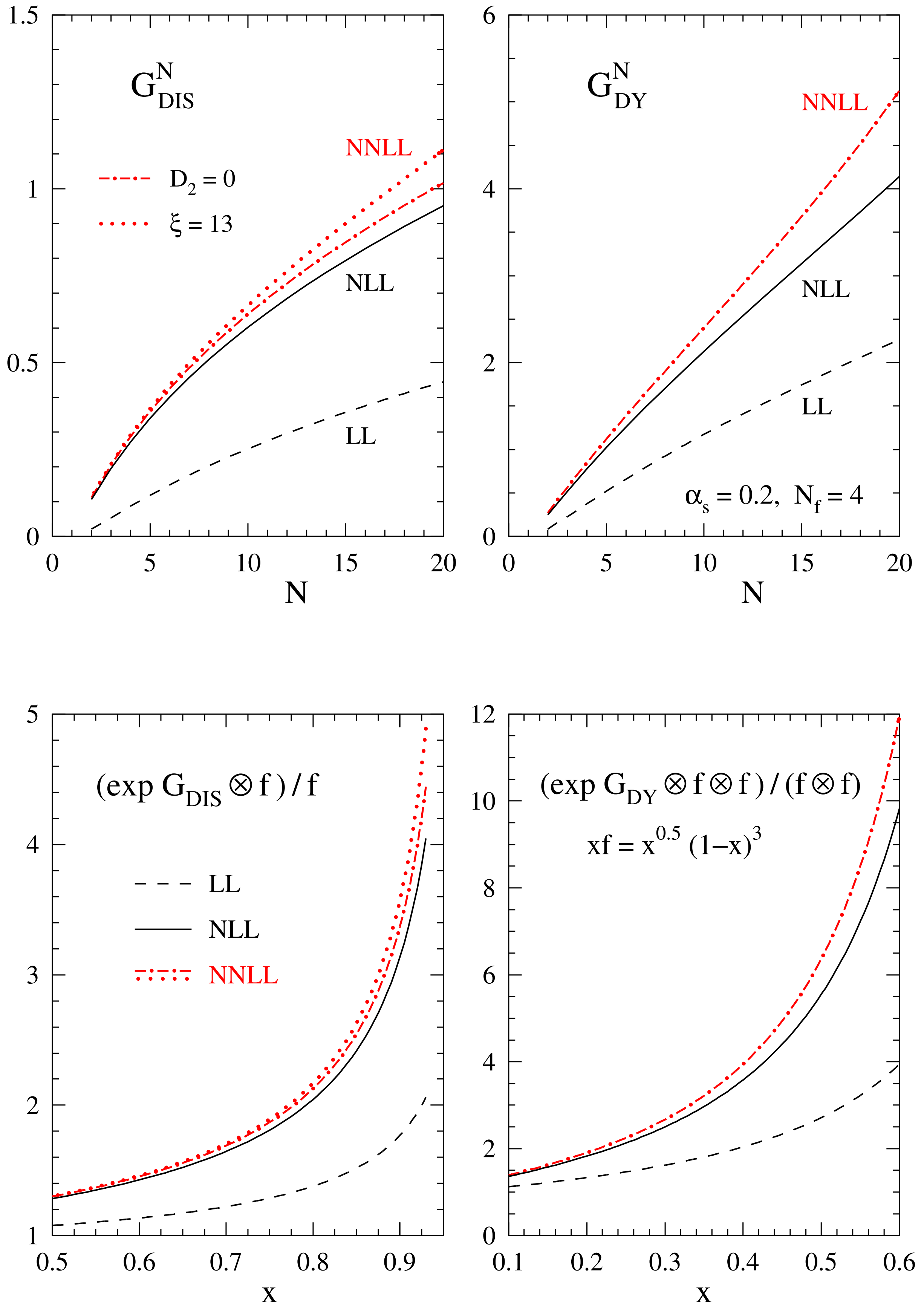
$(1-x)\, c^{(3)}_{2,\mathrm{NS}}(x)$
$N_f = 4$, approx.
A
B
20000
15000
10000
5000
0
-5000
0
0.2
0.4
0.6
0.8
1
x
$x(c^{(3)}_{2,\mathrm{NS}} \otimes f)$
$xf = x^{0.5}\,(1-x)^3$
A
B
200
100
0
-100
-200
-300
-400
0
0.2
0.4
0.6
0.8
1
x



$\dot{F}_{2,NS} = d \ln F_{2,NS} / d \ln Q^2$
$\mu_r = Q$
LO
NLO
NNLO
$N^3LO_{A,B}$
x
$\Delta_{abs} \dot{F}_{2,NS}$
(max - min) $\dot{F}_{2,NS} / 2$
$\mu_r = Q/2 \ldots 2Q$
x



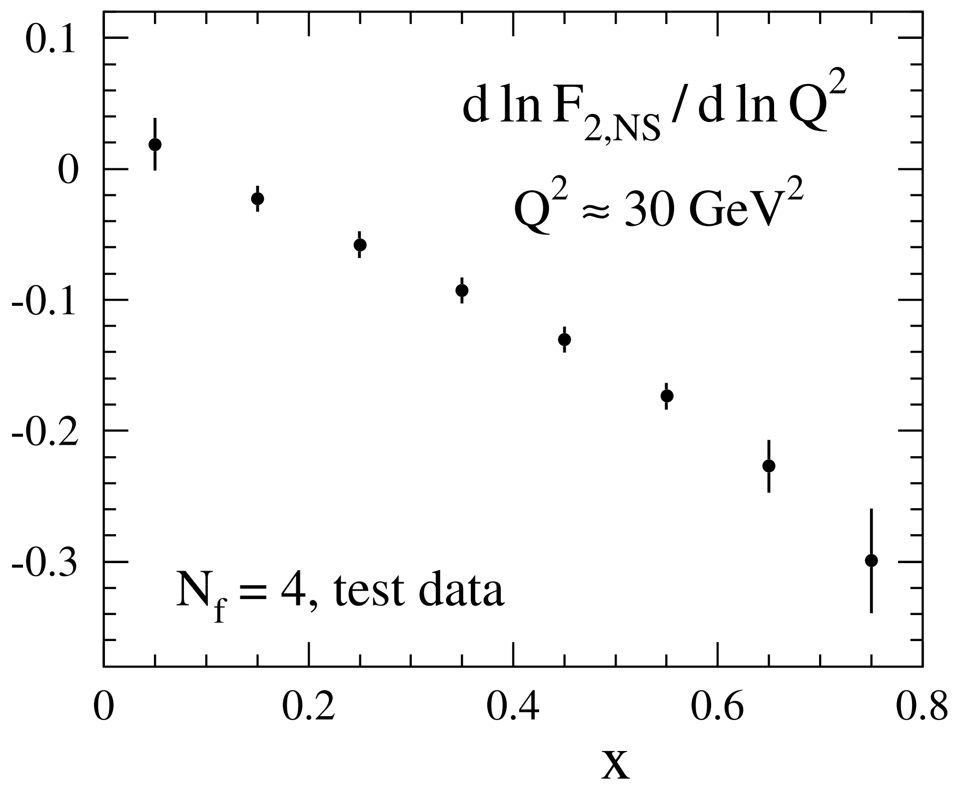
$(a_s^3 \, c_{2,q}^{(3)} \otimes f) / f$
$xf = x^{0.5} (1-x)^3$
full (approx.)
1
2
3
4 terms
$\alpha_s = 0.2, \; N_f = 4$
$\left(\sum_{k=4} a_s^k \, c_q^{(k)} \otimes f \right) / f$
$xf = x^{0.5} (1-x)^3$
1
2
3
4 towers
5 towers (approx.)
$\alpha_s = 0.2, \; N_f = 4$
x
x



G^N_{DIS}
NNLL
$D_2 = 0$
$\xi = 13$
NLL
LL
G^N_{DY}
NNLL
NLL
LL
$\alpha_s = 0.2,\ N_f = 4$
N
N
$(\exp G_{DIS} \otimes f\,)\,/\,f$
LL
NLL
NNLL
$(\exp G_{DY} \otimes f \otimes f\,)\,/\,(f \otimes f)$
$xf = x^{0.5}\,(1-x)^3$
x
x

α_s from scaling violations of $F_{2,\mathrm{NS}}$

Simplifying assumption: $F_{2,\mathrm{NS}}$ known at Q_0^2
one-parameter fit to 'data' on $dF_{2,\mathrm{NS}}/d\ln Q^2$



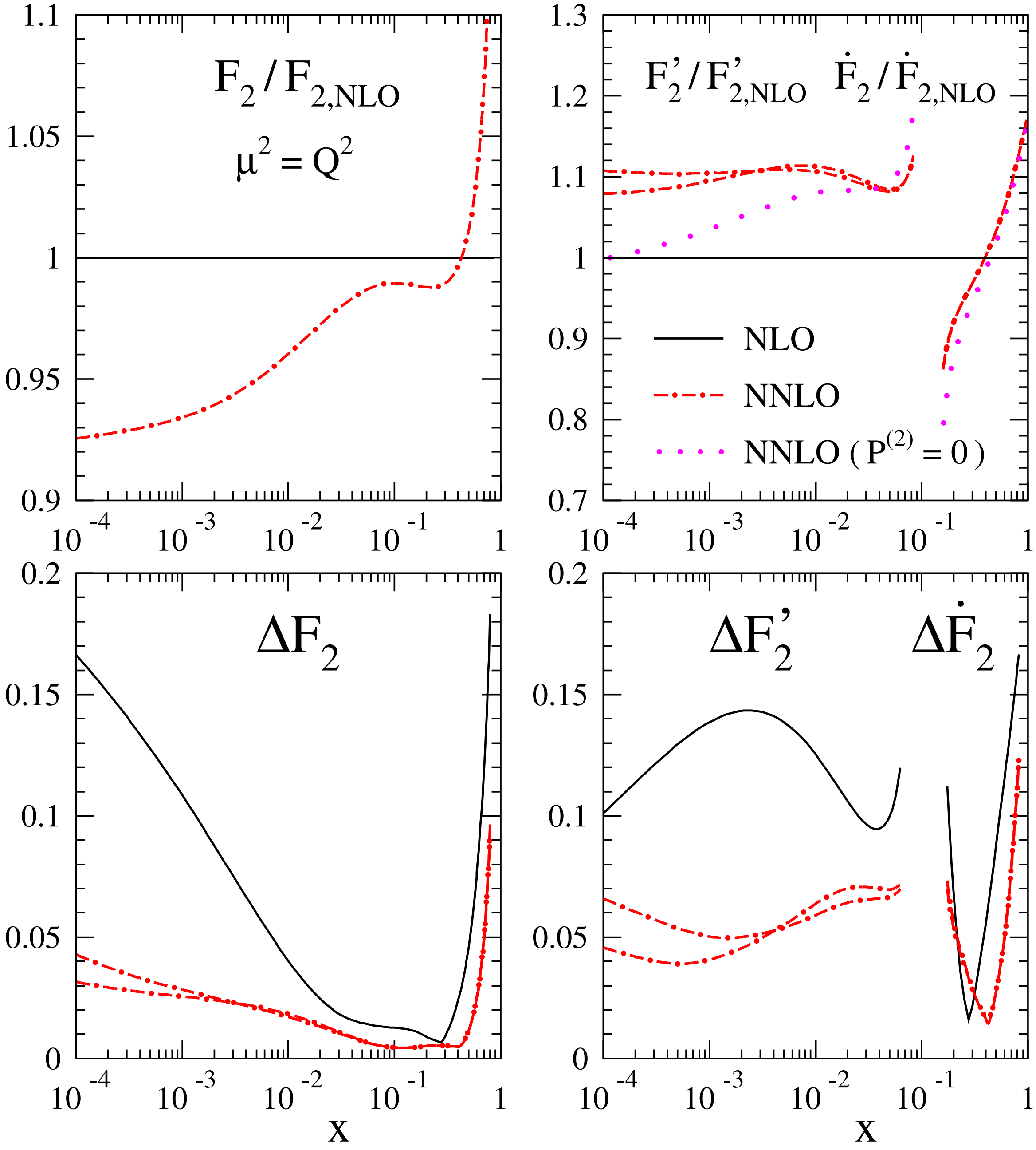
Central values and scale uncertainties

$$\alpha_s(Q_0^2)_{\mathrm{NLO}} = 0.2085 \begin{array}{l} +\ 0.021 \\ -\ 0.013 \end{array}$$

$$\alpha_s(Q_0^2)_{\mathrm{NNLO}} = 0.2020 \begin{array}{l} +\ 0.008 \\ -\ 0.0025 \end{array}$$

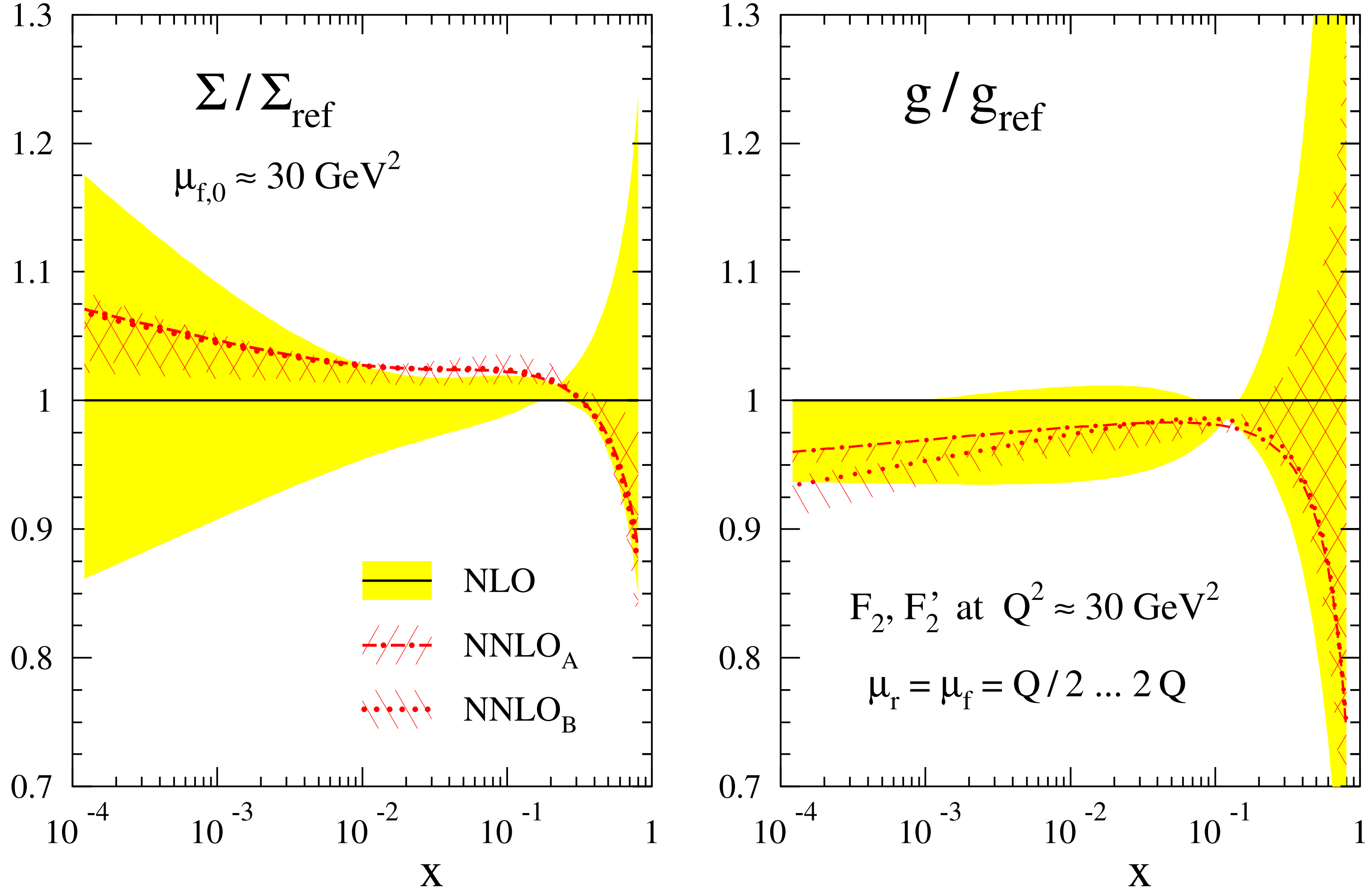
$$\alpha_s(Q_0^2)_{\mathrm{N^3LO}} = 0.2005 \begin{array}{l} +\ 0.0035 \\ -\ 0.0015 \end{array}$$

M_Z^2: Differences / errors smaller by factor of 3



$$\dot{F}_2 \;\equiv\; d \ln F_{2,S} / d \ln Q^2 \quad , \quad F_2' \;\equiv\; dF_{2,S} / d \ln Q^2$$

$$\Delta f \;=\; \frac{\max\ f - \min\ f}{2\,|\,\text{average}\ f|} \;\; , \;\; \mu_r^2 = \mu_f^2 = \frac{1}{4}\, Q^2 \ldots\, 4\, Q^2$$



Σ / Σ_{ref}
$\mu_{f,0} \approx 30\ GeV^2$
NLO
$NNLO_A$
$NNLO_B$
g / g_{ref}
F_2, F_2' at $Q^2 \approx 30\ GeV^2$
$\mu_r = \mu_f = Q/2 \ldots 2Q$
1.3
1.2
1.1
1
0.9
0.8
0.7
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x

Summary

Evolution of parton densities at NNLO

- Residual uncertainties of 3-loop splitting fct's
  irrelevant for $x \gtrsim 10^{-2}$, tolerable down to $x \approx 10^{-4}$
- Fast convergence of the series, e.g. at large x
  NNLO effects, μ_r variation: $< 2\%$ for $\alpha_s = 0.2$

Structure functions at NNLO and beyond

- Information on 3-loop coefficient fct's for $F_{2,3}$
  $\mathrm{N^3LO}$ analyses possible for $x > 10^{-2}$
- Sizeable higher-order effects at large x
  11% / 5% for NNLO / $\mathrm{N^3LO}$, resp., at $x = 0.8$
- Theoretical error on α_s from scaling violations
  Gain of factors $\simeq 3$ (NNLO)
  $\simeq 6$ ($\mathrm{N^3LO}$) wrt. NLO
- Soft-gluon resummation up to NNLL (also DY)
  reliable estimate possible, beyond $\mathrm{N^3LO}$ in DIS:
  relevant for $x \gtrsim 0.8$

Compact expressions for all ingredients

http://www.lorentz.leidenuniv.nl/~avogt/