Diffractive structure function at very small β

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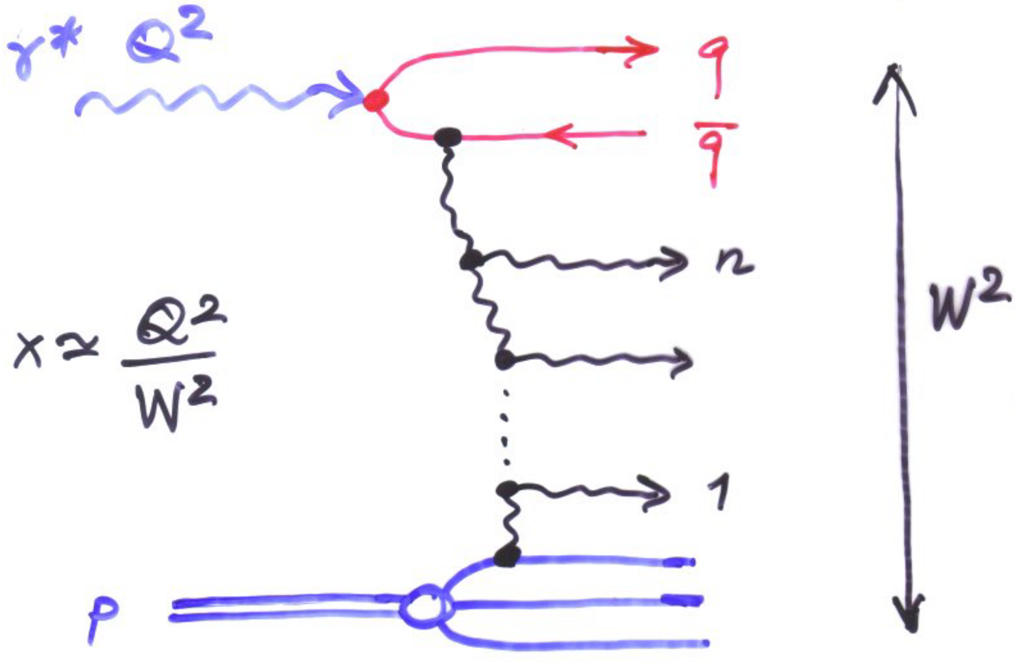
- color dipole approach to small-x physics
  The color dipole Regge expansion.
- Extension of the CD-Regge expansion to diffractive DIS.
  Unitarity corrections.
  Predictions

- Color dipole (CD) formulation of small x DIS



$x \simeq \frac{Q^2}{W^2}$

$\sigma_n \sim \log^n\left(\frac{1}{x}\right)$

- CD-Fock space decomposition:

$$|\gamma^*\rangle = |\gamma^*\rangle_{bare} + \sum \Psi_{q\bar{q}}(z,\vec{r})\,|q\bar{q};\vec{r}\rangle$$

$$+ \sum_n \Psi_{q\bar{q}g_1\cdots g_n}(z,\vec{r},z_i,\vec{\rho}_i)\,|q\bar{q}g_1\cdots g_n;\vec{r},\vec{\rho}_i\rangle$$

→ Exact conservation of $\vec{r},\vec{\rho}_i$ in a scattering process ≡ angular momentum conservation

→ The effect of multigluon Fock states is reabsorbed into $\log(1/x)$ evolution of the interaction of the $q\bar{q}$-dipole with the target:

$$\sigma^{\gamma^* p}_{T,L}(x,Q^2) = \int dz\, d^2\vec{r}\, |\Psi_{T,L}(Q^2,z,\vec{r})|^2 \cdot \sigma(x,\vec{r})$$

- Inclusive DIS: exact color dipole factorization:

$$\sigma^{\gamma^* p}_{T,L}(x,Q^2) = \int dz\, d^2\vec{r}\, |\Psi_{T,L}(Q^2; z, r)|^2 \cdot \sigma(x, \vec{r})$$

$$|\Psi_T|^2 = \frac{6\alpha_{em}}{(2\pi)^2} \sum_f e_f^2 \left\{ [z^2 + (1-z)^2]\, \varepsilon^2 K_1^2(\varepsilon r) + m_f^2 K_0^2(\varepsilon r) \right\}$$

$$|\Psi_L|^2 = \frac{6\alpha_{em}}{(2\pi)^2} \sum_f e_f^2\, 4Q^2 z^2 (1-z)^2 K_0^2(\varepsilon r)$$

$$\varepsilon^2 \equiv z(1-z)Q^2 + m_f^2$$

Nikolaev, Zakharov '91

- Relationship between $\sigma(x,r)$ and the gluon SF of the target:

$$\sigma(x,r) = \frac{4\pi}{3} \alpha_s(r) \int \frac{d^2\vec{\kappa}}{\kappa^2} \frac{1 - \exp[i \vec{\kappa} \cdot \vec{r}]}{\kappa^2} \cdot \frac{\partial G(x, \vec{\kappa}^2)}{\partial \log \vec{\kappa}^2}$$

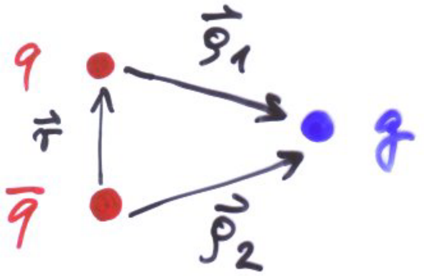
$$\simeq \frac{\pi^2}{3} r^2 \alpha_s(r)\, G\left(x, q^2 \sim \frac{10}{r^2}\right)$$

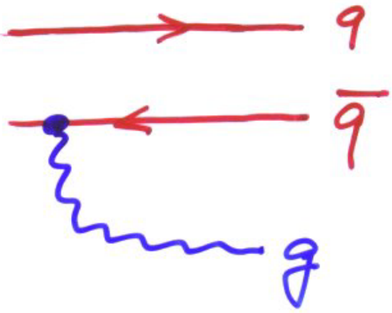
NZ '93

- Color dipole formulation of BFKL:

  Nikolaev, Zakharov, Zoller '94 ; running α_s, $N_c = 3$

  Mueller '94 ; fixed α_s, $N_c \to \infty$





- $$\frac{\partial \sigma(x,r)}{\partial \log\left(\frac{1}{x}\right)} = \mathcal{K} \otimes \sigma(x,r)$$

$$= \frac{3}{8\pi^3} \int d^2\vec{\rho}_1 \, |\vec{E}(\rho_1) - \vec{E}(\rho_2)|^2$$

$$* \left[\sigma(x,\rho_1) + \sigma(x,\rho_2) - \sigma(x,r)\right]$$

$* \; \vec{E}(\vec{\rho}) = g_S(R = \min(r,\rho)) \cdot \frac{\vec{\rho}}{\rho^2} \cdot$ (IR screening)

$* \; \alpha_S(r) = \frac{g_S^2(r)}{4\pi}$

! Asymptotic freedom (AF) enhances the IR sensitivity of BFKL & necessitates IR regularization.

- Finite propagation (screening) radius for perturbative gluons: $R_c \sim 0.2 \div 0.3$ fm

- IR freezing of α_S.

- Solutions to the CD-equation – their **Regge theory properties:**

$$\sigma_i(x,r) = \sigma_i(r)\left(\frac{1}{x}\right)^{\Delta_i} \quad ?$$

- fixed α_s ; ~~AF~~

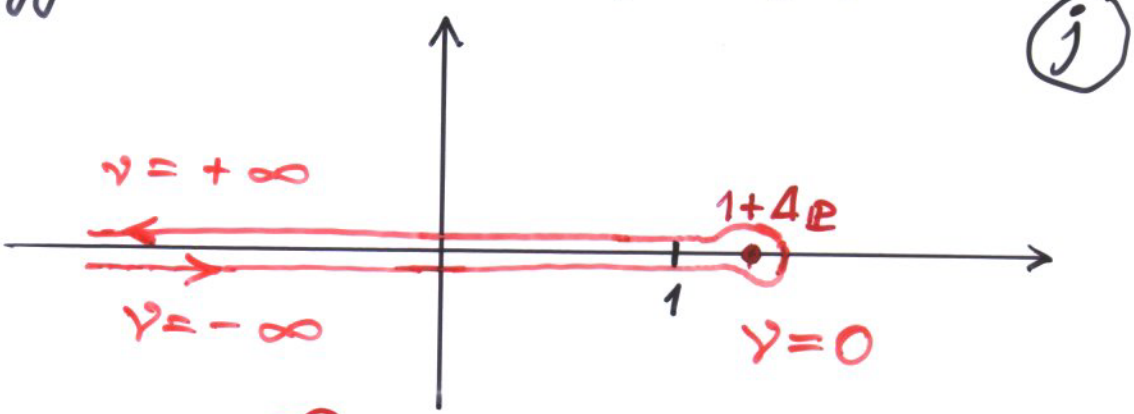
$\rightarrow \mathcal{K} \propto \frac{r^2}{\rho_1^2 \rho_2^2} \Rightarrow$ scale invariance

$$\frac{\sigma_\nu(r)}{r} = \exp[i\nu \log r^2]$$

"plane waves" in $\log r^2$-space with a highly non-linear "kinetic energy":

$$T(\nu) = -\Delta(\nu) = \frac{3\alpha_s}{\pi}\left[2\psi(1) - \psi\left(\tfrac{1}{2} - i\nu\right) - \psi\left(\tfrac{1}{2} + i\nu\right)\right]$$

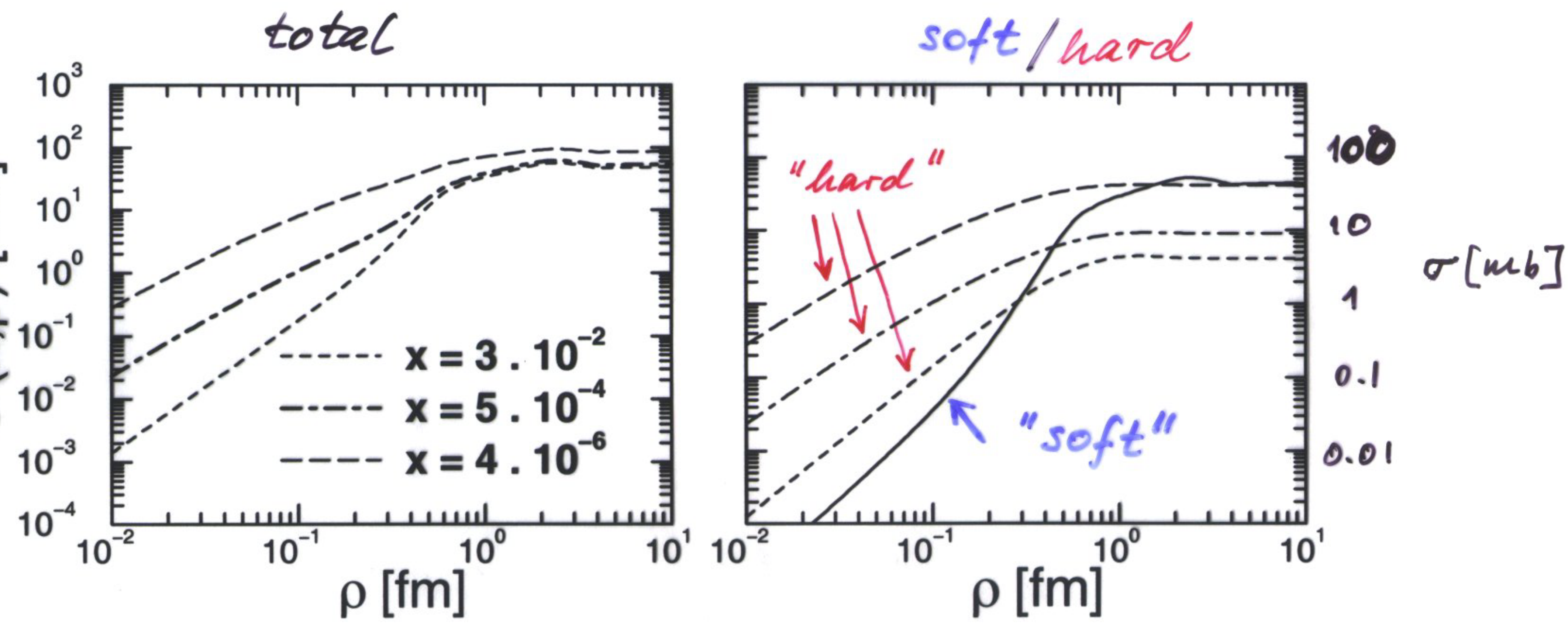
- Regge cut in the complex j-plane:



$$\sigma(x,r) = r \cdot \int_{-\infty}^{\infty} d\nu\, f(\nu) \exp[2i\nu \log r] \left(\frac{1}{x}\right)^{\Delta(\nu)}$$

$$\Delta_{\mathbb{P}} = \frac{12 \log 2}{\pi} \alpha_s$$

Color dipole cross section

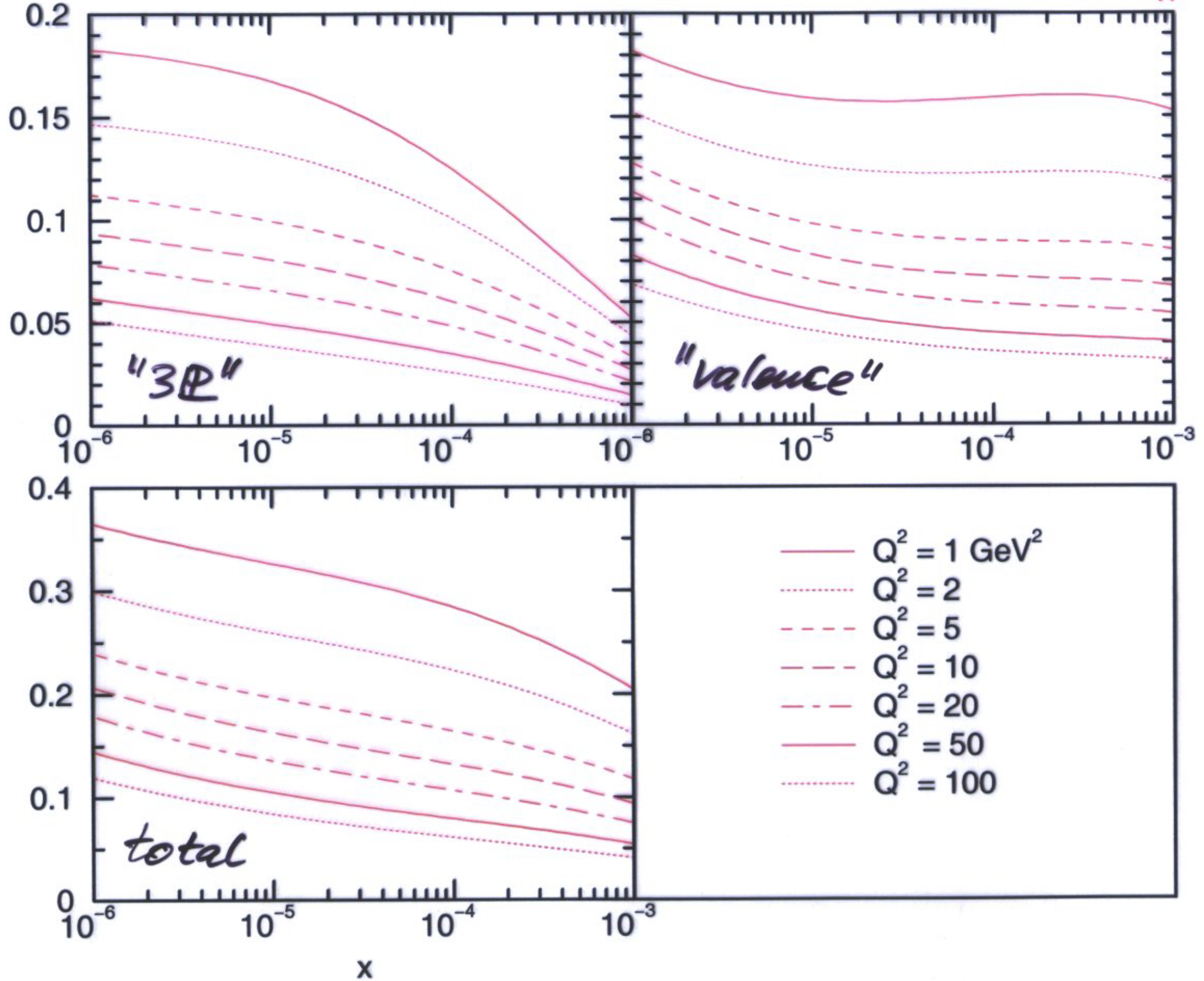


diffractive / total:

$F_2^D(x,Q^2)/F_{2p}(x,Q^2)$

$$F_2^D(x,Q^2) \equiv \int_x^{0.03} \frac{dx_{I\!P}}{x_{I\!P}} F_2^{D(3)}\left(x_{I\!P}, \frac{x}{x_{I\!P}}, Q^2\right)$$

$$\sim \int_x^{0.03} \frac{dx_{I\!P}}{x_{I\!P}} \varphi(x_{I\!P}) \cdot F_{2I\!P}\left(\frac{x}{x_{I\!P}}, Q^2\right)$$



photoproduction at HERA:

$$\frac{\text{diff}}{\text{total}} \sim 22\,\%$$

- AF ⇒ chromoelectric field in a color dipole be calculated with running $\alpha_s(r)$.

⇒ The Regge cut of the approximation $\alpha_s \equiv$ const. is split into a sequence of (infinitely many) Regge poles.

(Fadin, Kuraev, Lipatov '75)

Their intercepts satisfy: (Lipatov '86)

$$\Delta_n \simeq \frac{\Delta_{\mathbb{P}}}{n+1}$$

Such a behavior is indeed found from the numerical solution of the eigenvalue equation:

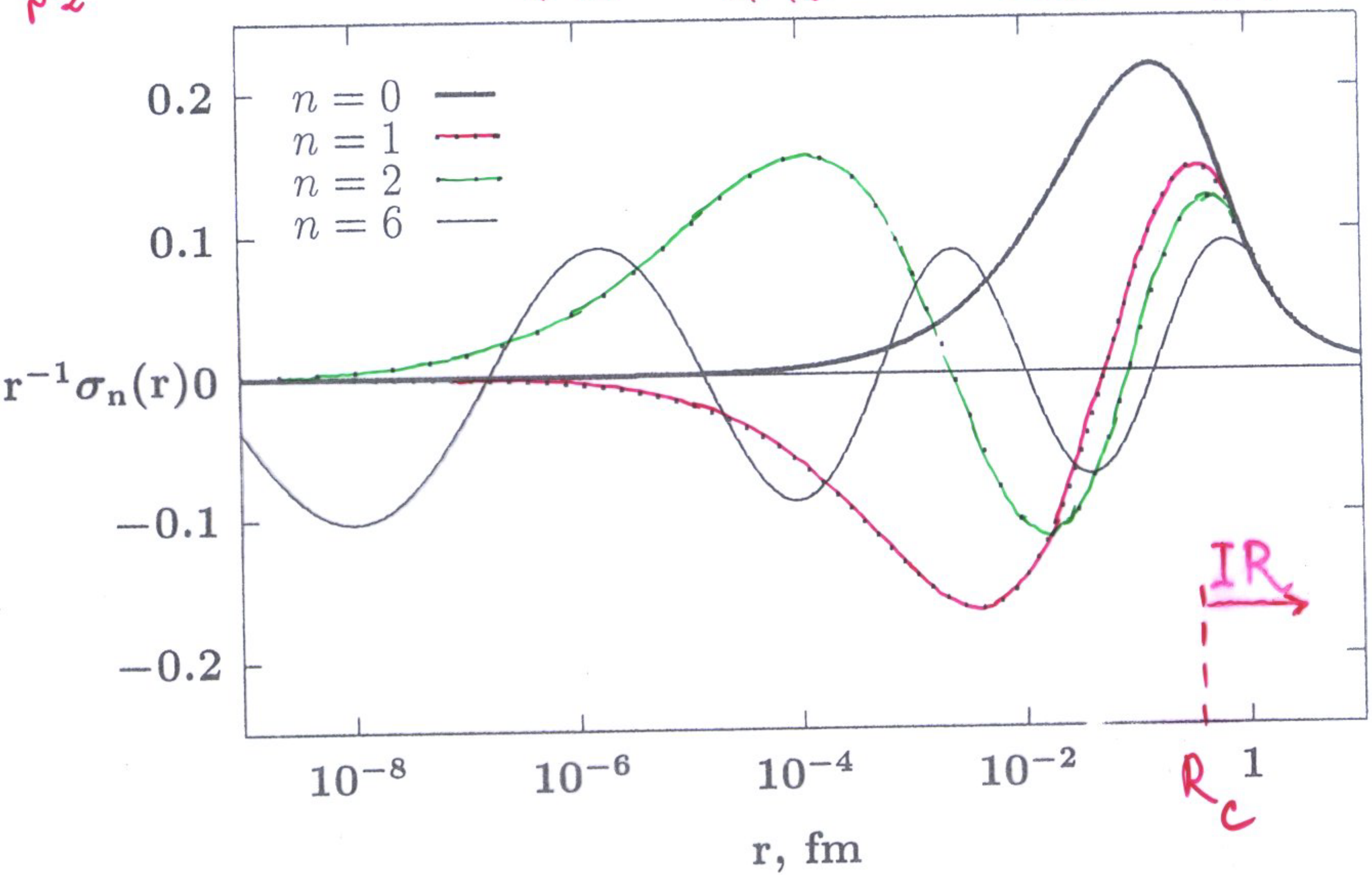
$$\Delta_m \sigma_m(r) = \mathcal{K} \otimes \sigma_m(r)$$

(Nikolaev, Zakharov, Zoller, '97)

The nodal properties of eigen-cross sections of CD BFKL equation

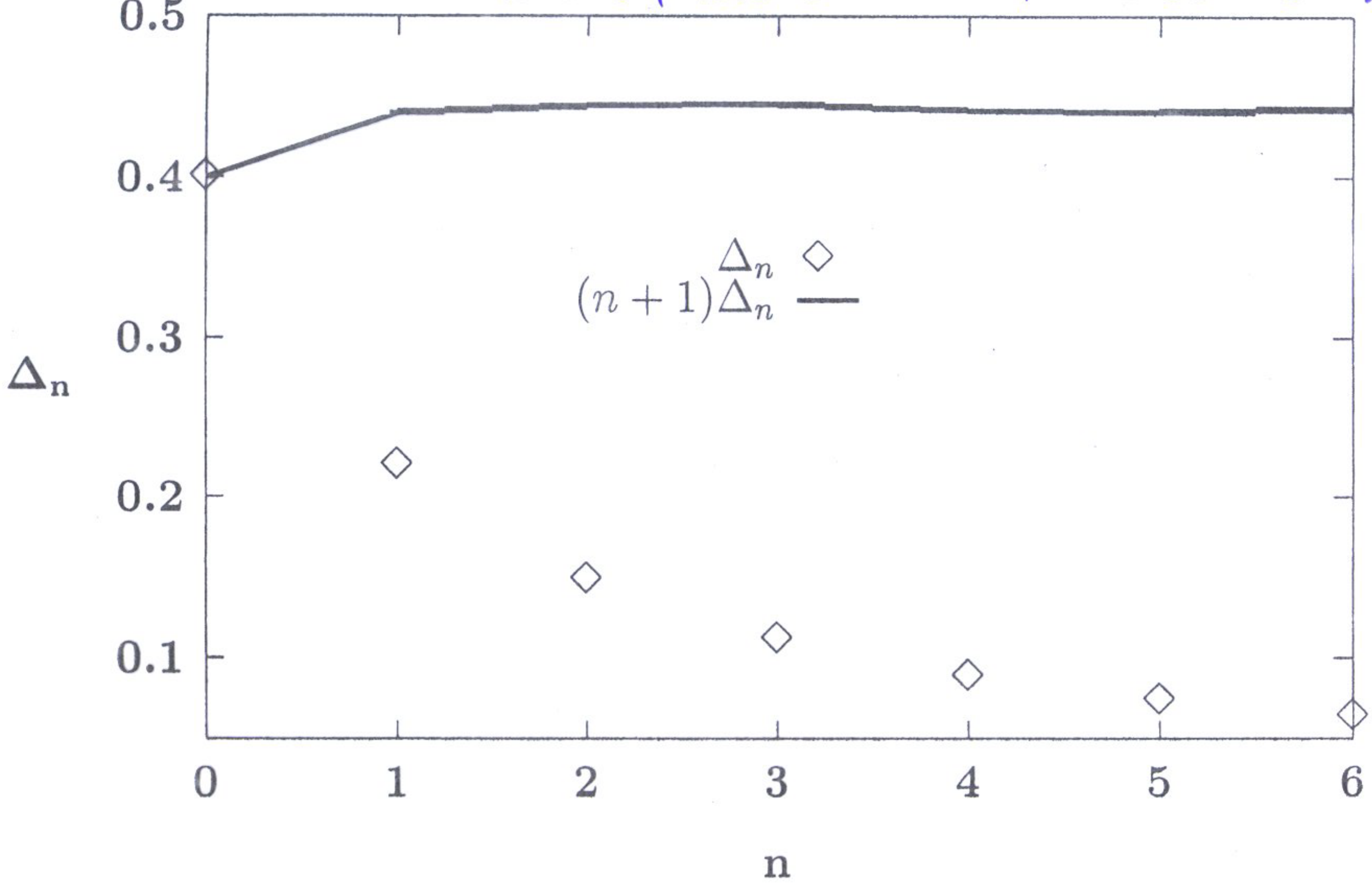
NNN, B.G.Zakharov, V.R.Zoller 97

$Q^2_{eff} \sim \frac{10\ GeV^2}{r^2}$



Intercept of the n-node CD BFKL pole

N.N.N., B.G.Zakharov, V.R.Zoller 97



- The practical BFKL–Regge phenomenology of $F_{2p}(x, Q^2)$

→ Take the IR-regulated 2-G exchange as boundary condition to the $\log(1/x)$-evolution. (might be too restrictive, though)

starting point x_0 is the sole parameter

! The so obtained hard $\sigma(x,r)$ is short of strength at $r > R_c$.
Add a purely phenomenological soft Pomeron for $r > R_c$.
Adjust the strength to $\sigma_{\pi N}$, diffraction etc...

$\Delta_{soft} = 0$.

$$F_{2p}(x,Q^2) = \int_0^1 dz\, d^2\vec{r}\, |\Psi_{\gamma^*}(z,r)|^2 \cdot \sigma(x,r) \cdot \frac{Q^2}{4\pi^2 \alpha_{em}}$$

$$= \int \frac{dr^2}{r^2} \cdot \frac{\sigma(x,r)}{r^2} \cdot \Phi_2(Q^2, r^2)$$

- Plateau-like Φ_2 → onset of the leading $\log Q^2$ regime.
- $r > R_c$ – the infrared sensitive contribution

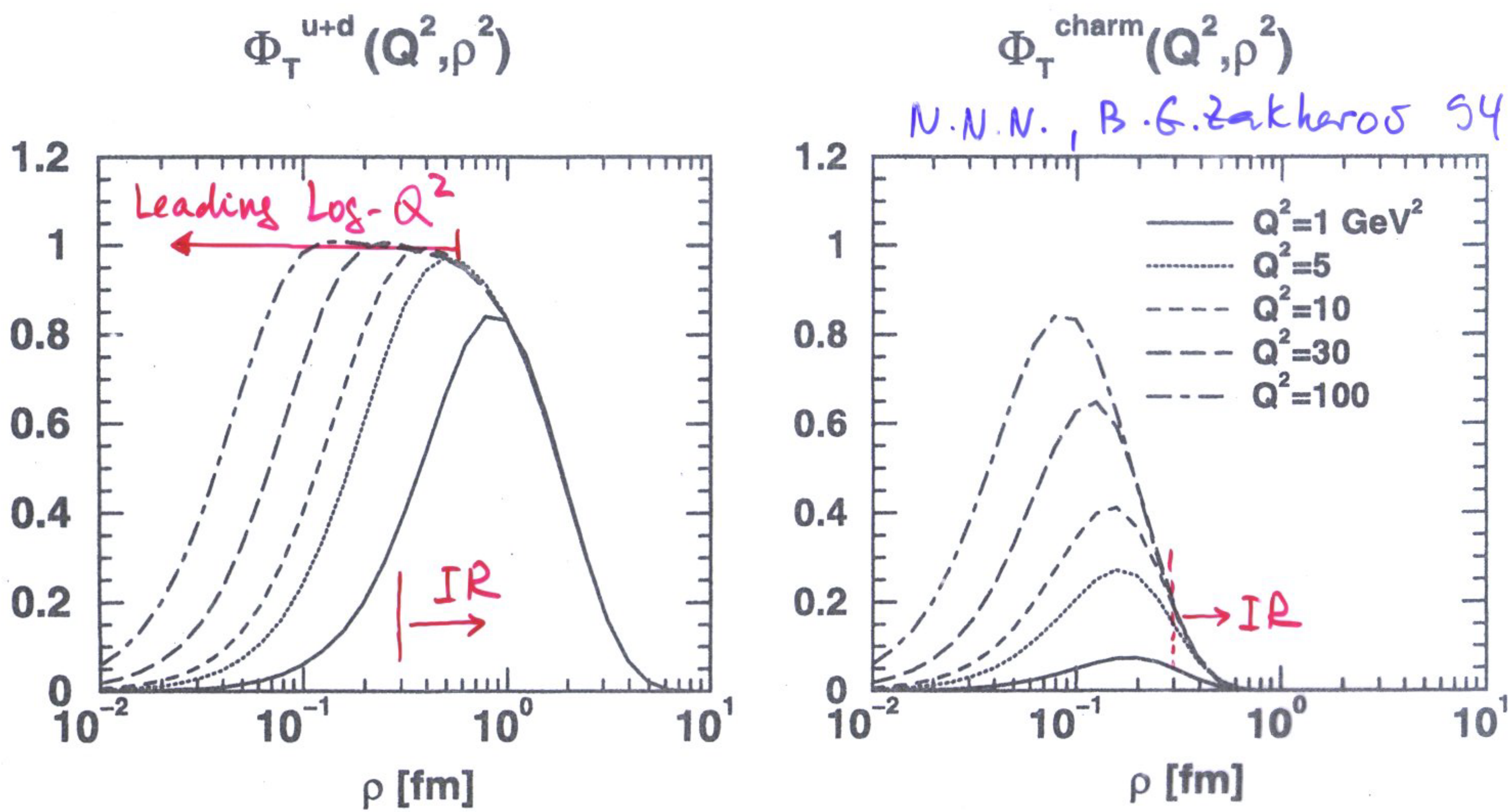


Figure 3.3: *The kernel* $\Phi_T(Q^2, \rho^2)$. *The left panel shows the result for light flavours* $(u\bar{u} + d\bar{d})$, *the right panel is for charm* $c\bar{c}$.

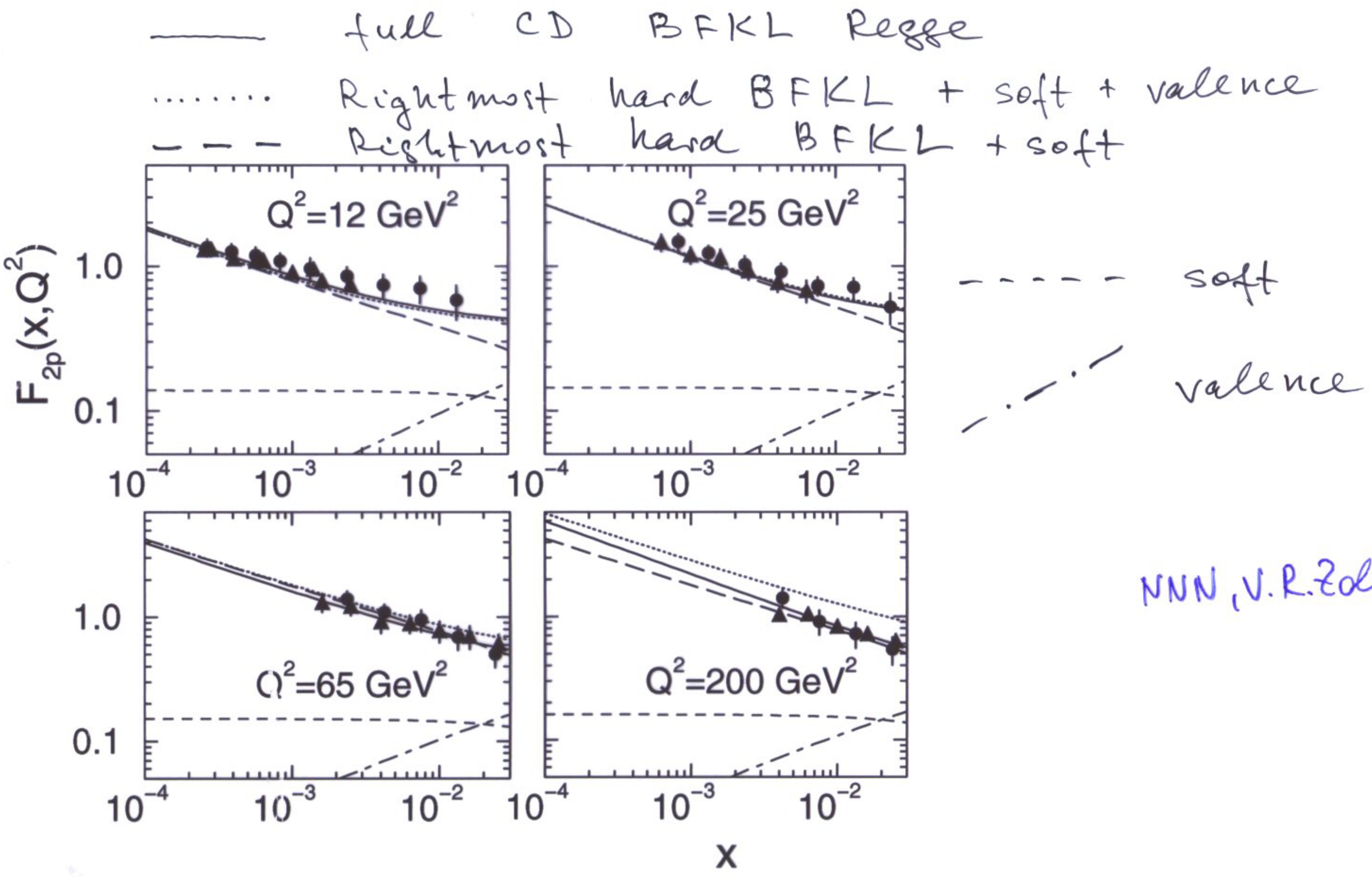
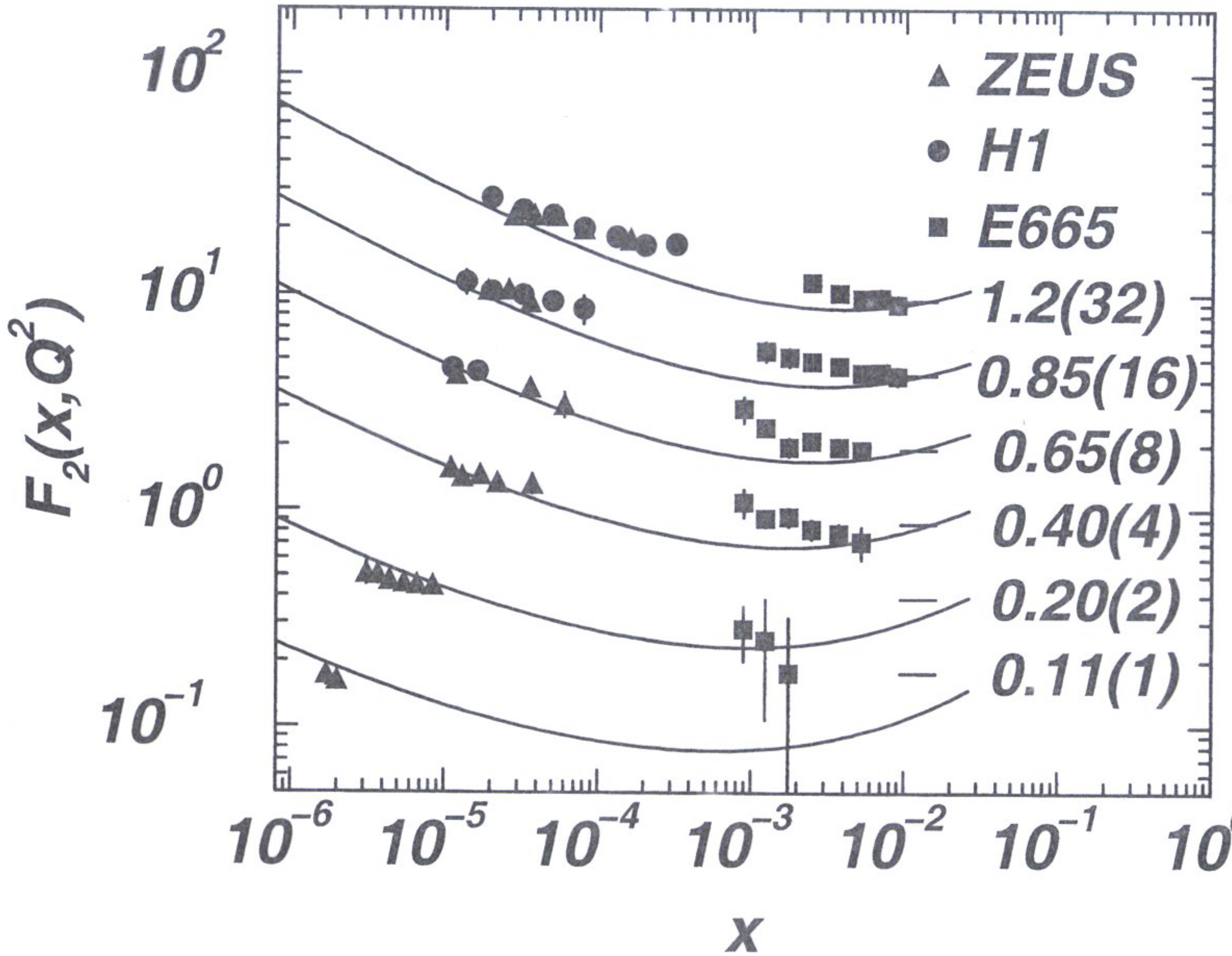


Fig. 2

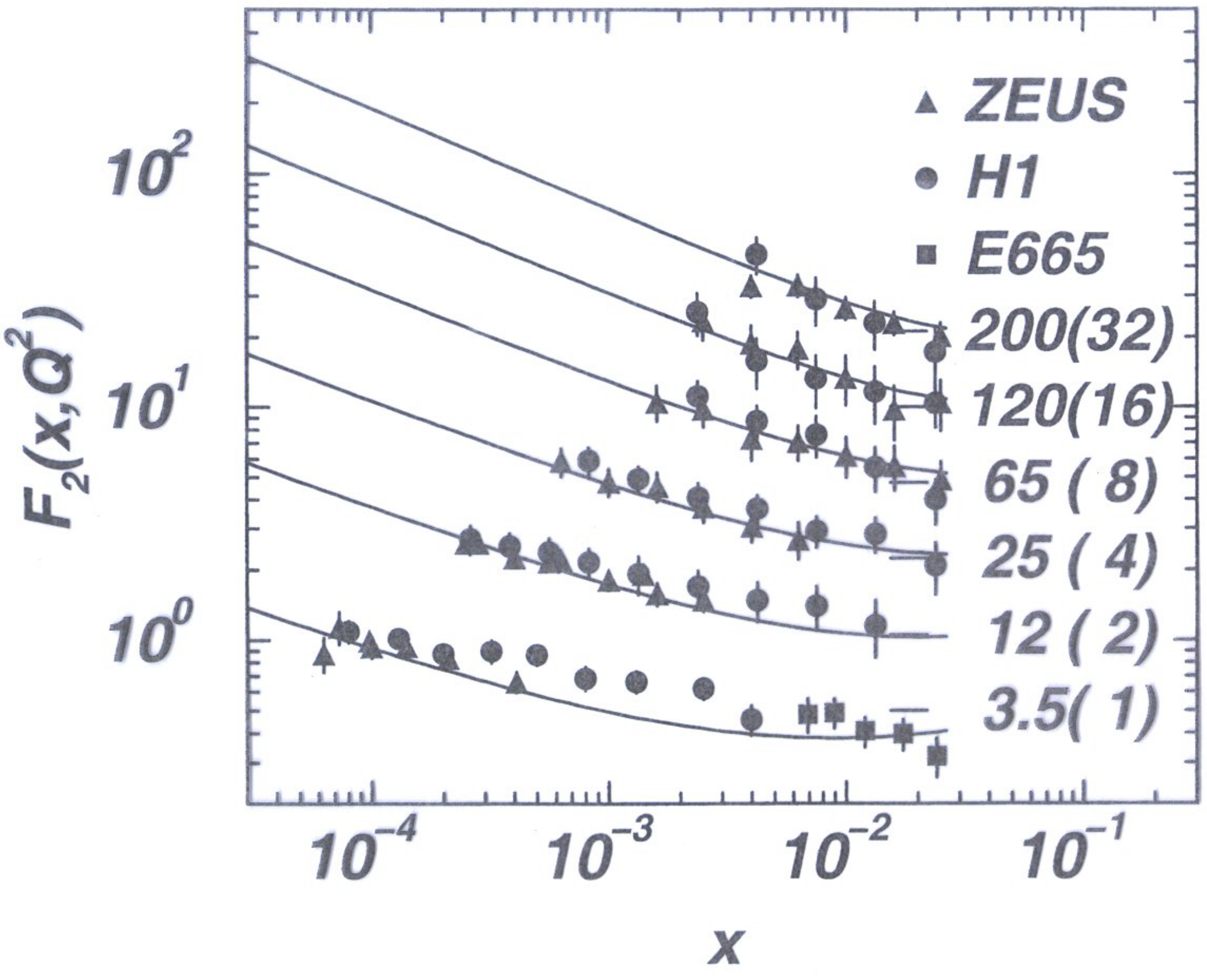
NNN, V.R. Zoller 99

CD BFKL + Soft + Valence (GRV)



CD BFKL + Soft + Valence (GRV)

Natural 2G-exchange boundary condition for CD BFKL



NNN
V.R.Zoller 98

- Extension to other targets:
  beam-target interaction as dipole-dipole interaction.

⇒ Exploit the factorization properties of the BFKL-Regge poles:

$$\sigma(x, r_b, r_t) = \sum_n C_n \, \sigma(r_b) \, \sigma(r_t) \left(\frac{x_0}{x}\right)^{\Delta_n}$$

⇒ $$\sigma_{bt} = \int d^2 r_b \int d^2 r_t \, |\Psi_b(r_b)|^2 \, |\Psi_t(r_t)|^2 * \sigma(x, r_b, r_t)$$

⇒ predictions for $F_{2\pi}$ at small x;
hard contribution to hadronic & real photon interactions.

⇒ There is a need for unitarization corrections.

Hard BFKL contribution to hadronic and real photon interactions

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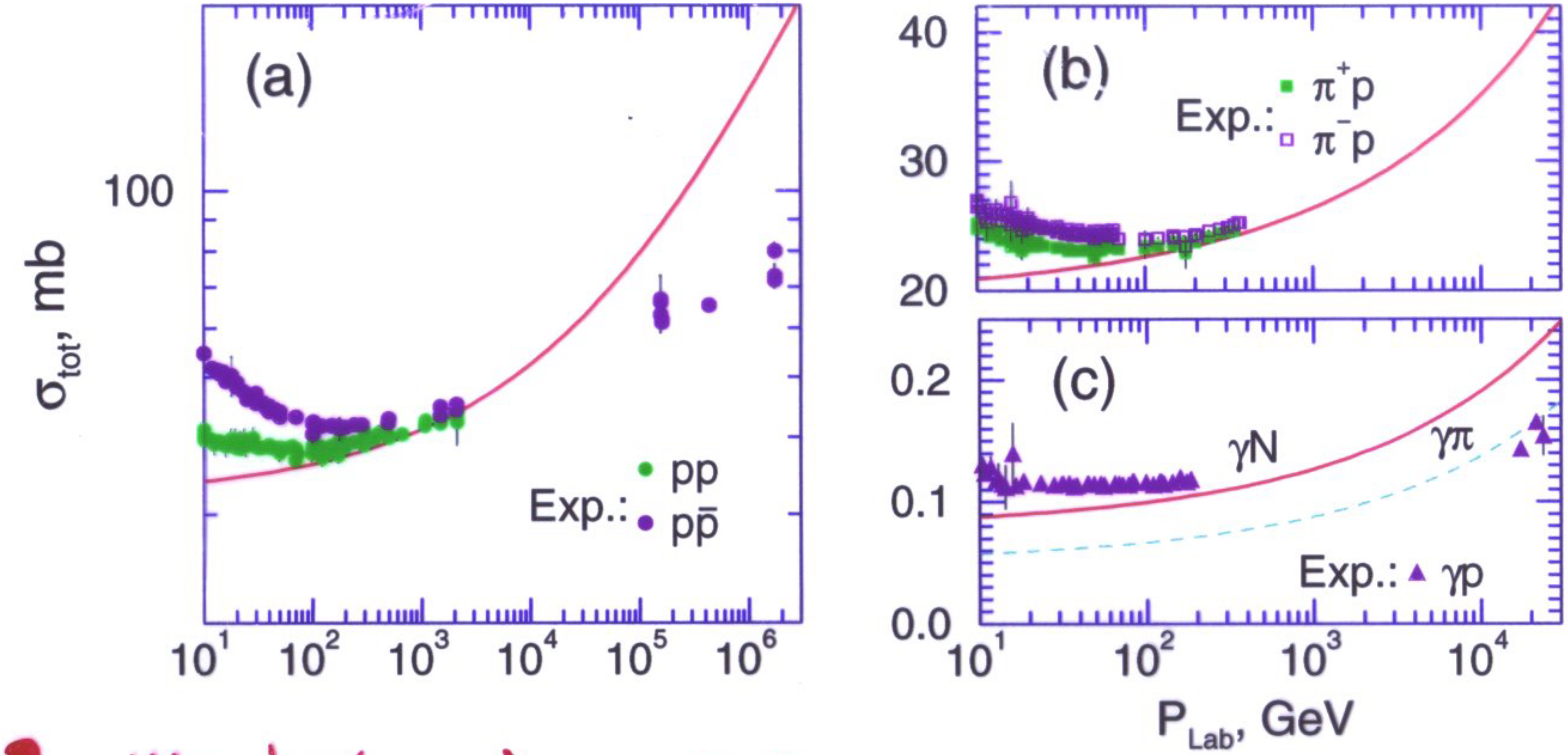
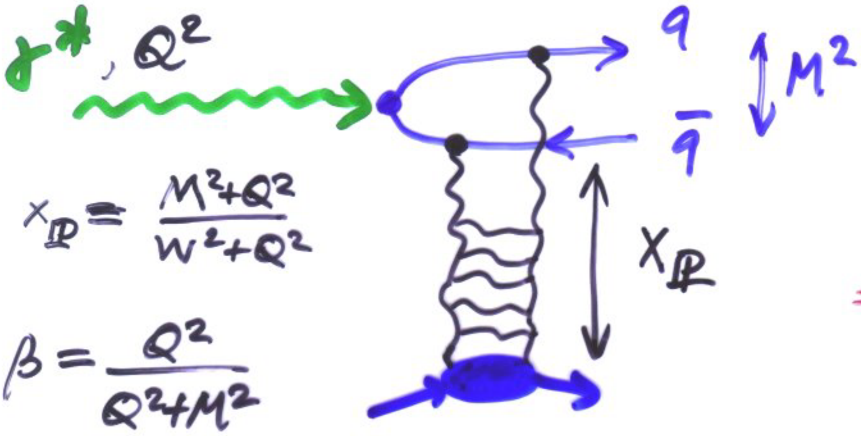


Fig. 3

! We had $\Delta_{soft} = 0$.

- BFKL diffusion from hard to soft scattering too strong at very high energy and invites unitarization by factor $\sim 2 \div 3$ at $x \sim 10^{-6}$. Not unfeasible.

- Diffraction dissociation $\gamma^* p \to X p$ of the virtual photon:



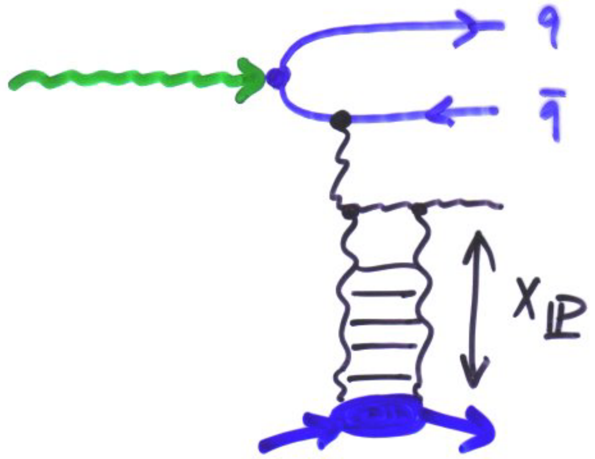
$x_{\mathbb{P}} = \frac{M^2+Q^2}{W^2+Q^2}$

$\beta = \frac{Q^2}{Q^2+M^2}$

small mass excitation, large & moderate β.

⇒ valence quark SF of the $\mathbb{P}$

$F^{\mathbb{P}}_{val}(\beta) \propto \beta(1-\beta)$



higher mass excitation smaller β

⇒ valence gluon SF of the $\mathbb{P}$.

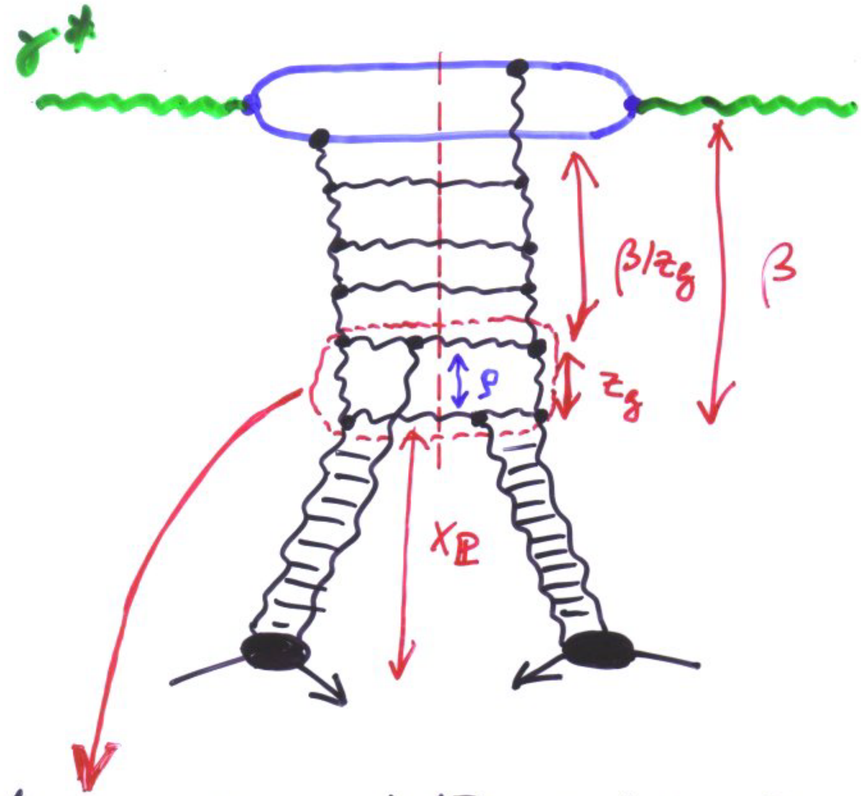
$G_{\mathbb{P}}(\beta) \sim (1-\beta)$

Again, we take the $x_{\mathbb{P}}$-dependence into account through $\sigma(x_{\mathbb{P}}, r)$.

→ Successful description of diffractive DIS at HERA.

Genovese, Nikolaev, Zakharov '94
Bartels, Ellis, Kowalski, Wüsthoff '98
Bartels & Royon '98, M. Bertini et al. '98.

... building up high mass states:



two-gluon WF of the $\mathbb{P}$:

$$|\Psi_{\mathbb{P}}(z_g, \vec{\rho}, x_{\mathbb{P}})|^2 = \frac{1}{\sigma^{pp}_{tot}} \cdot \frac{2}{\pi^4} \cdot \frac{1}{z_g} \cdot \left[\frac{\sigma_{gg}(x_{\mathbb{P}}, \rho)}{\rho^2}\right]^2 \cdot F\left(\rho/R_c\right)$$

$$\sigma_{gg} = \frac{C_A}{C_F} \cdot \sigma = \frac{9}{4} \cdot \sigma$$

- Take the full β-dependence of the CD-Regge expansion into account. $|\Psi_{\mathbb{P}}|^2$ defines the target-WF.
- Remember that the hard, rising part of $\sigma(x_{\mathbb{P}}, \rho)$ was short ranged! Should be subject to unitarization corrections.

$$\sigma(x_{\mathbb{P}}, \rho) \rightarrow \sigma^{u}(x_{\mathbb{P}}, \rho) \quad \text{(e.g. eikonal)}$$

- CD-Regge expansion for the $\gamma^* \mathbb{P}$ cross section:

$$\sigma(\gamma^* \mathbb{P} | \beta, x_{\mathbb{P}}) = \int dz_1 \, d^2\vec{r}_1 \, |\Psi_{\gamma^*}(z_1, r_1)|^2$$

$$\cdot \int dz_g \, d^2\vec{\rho} \, |\Psi_{\mathbb{P}}(z_g, \rho, x_{\mathbb{P}})|^2$$

$$\cdot \; \sigma(r_1, \rho, \beta/z_g)$$

$$\sigma(r_1, \rho, \beta/z_g) = \sum_{m \geq 0} C_m \, \sigma_m(r_1) \, \sigma_m(\rho) \cdot \left[\frac{z_g \beta_0}{\beta}\right]^{\Delta_m}$$

$$\Rightarrow \sigma(\gamma^* \mathbb{P} | \beta, x_{\mathbb{P}}) = \sum_m \sigma_m^{\gamma^*}(Q^2) \cdot C_m$$

$$\cdot \int d^2\vec{\rho} \, |\varphi_{\mathbb{P}}(\rho, x_{\mathbb{P}})|^2 \sigma_m(r) \cdot \left[\frac{\beta_0}{\beta}\right]^{\Delta_m}$$

$$\cdot \int_\beta^1 \frac{dz_g}{z_g} \, z_g^{\Delta_m} (1 - z_g)^2$$

(where $|\Psi_{\mathbb{P}}|^2 = \frac{1}{z_g} \cdot |\varphi_{\mathbb{P}}|^2$)

Recall that $|\varphi_{\mathbb{P}}|^2$ is subject to the unitarization.

$$\Rightarrow \sigma(\gamma^* \mathbb{P}, \beta, x_{\mathbb{P}}) = \sum_m \varphi_m(x_{\mathbb{P}}) \, \sigma_m(\gamma^* \mathbb{P}; \beta)$$

- Recall, that $\sigma(x_{\mathbb{P}}, \rho)$ consisted of two pieces:

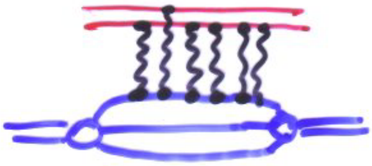
$$\sigma(x_{\mathbb{P}}, \rho) = \sigma_{pt}(x_{\mathbb{P}}, \rho) + \sigma_{np}(\rho)$$

↑ short range, strong $x_{\mathbb{P}}$-dep. ⇒ unitarization.

↑ long range, $x_{\mathbb{P}}$-indep. no unitarization.

The hard, driving piece of the $x_{\mathbb{P}}$-dep. was of short range $\sim R_c$.

⇝ Additive quark counting for scattering on the proton emerges:



$$R_c^2 / R_p^2 \ll 1$$

⇝ The relevant profile function is the one for dipole-quark scattering:

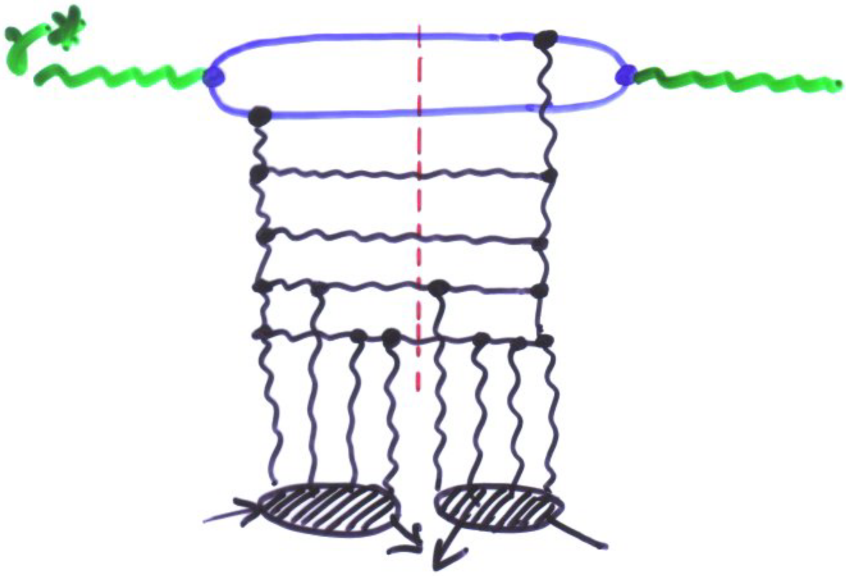
$$\Gamma_0(x_{\mathbb{P}}, \rho, \vec{b}) = \frac{1}{3} \cdot \frac{\sigma_{pt}(x_{\mathbb{P}}, \rho)}{4\pi B(x)} \cdot \exp\left[-\frac{\vec{b}^2}{2B(x)}\right]$$

$$B(x) = \frac{1}{3} R_c^2 + 2\alpha'_{\mathbb{P}} \log\left(\frac{x_0}{x}\right) ; \quad \alpha'_{\mathbb{P}} = 0.07 \text{ GeV}^{-2}, \quad R_c \sim 0.2 \div 0.3 \text{ fm}$$

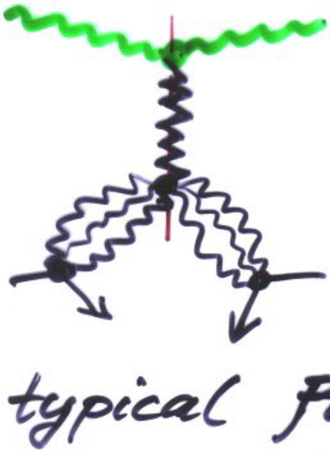
⇝ eikonal:

$$\sigma_{pt}^{u}(x_{\mathbb{P}}, \rho) = 2 \cdot \int d^2\vec{b} \left[1 - \exp[-\Gamma_0(x_{\mathbb{P}}, \rho, \vec{b})]\right]$$

- The dominant unitarization corrections look like:

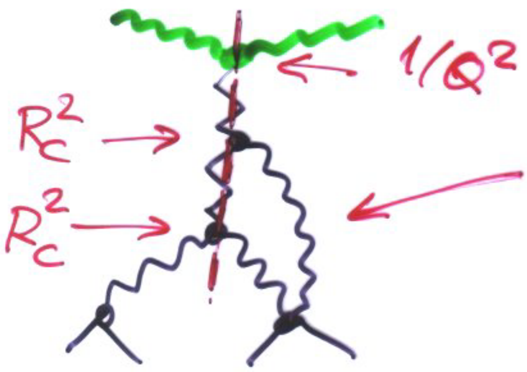


or if reinterpreted in terms of Pomeron diagrams:

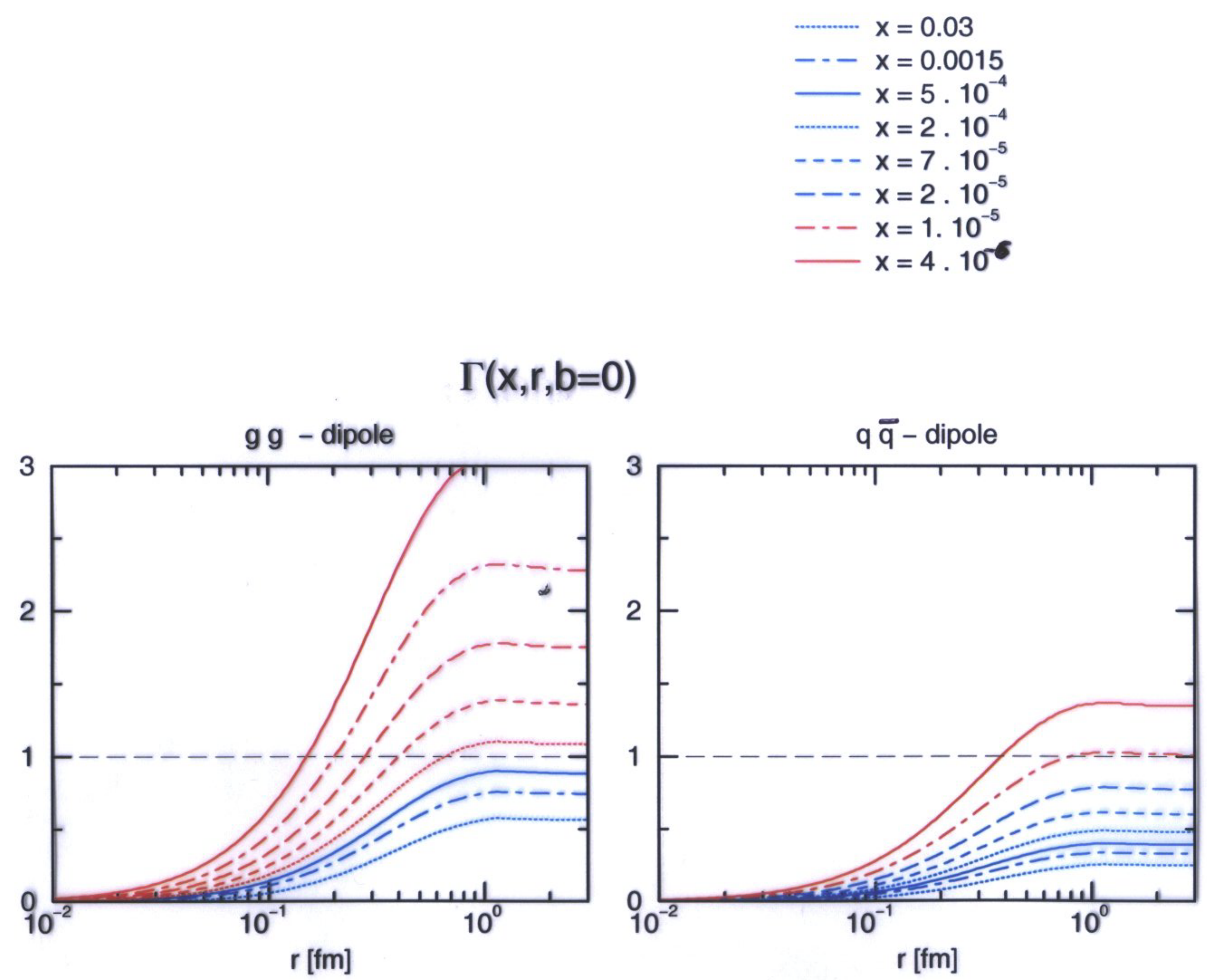


etc...
with many ℙ's coupled into one point.

The typical Fan-diagram shall in comparison lose in the β-dependence:



coupling this ℙ into lower vertex would give the whole β-span evolution from $1/Q^2$ to R_c^2!



x = 0.03
x = 0.0015
x = 5 . 10^-4
x = 2 . 10^-4
x = 7 . 10^-5
x = 2 . 10^-5
x = 1. 10^-5
x = 4 . 10^-6
Γ(x,r,b=0)
g g – dipole
q q̄ – dipole
0
1
2
3
10^-2
10^-1
10^0
r [fm]

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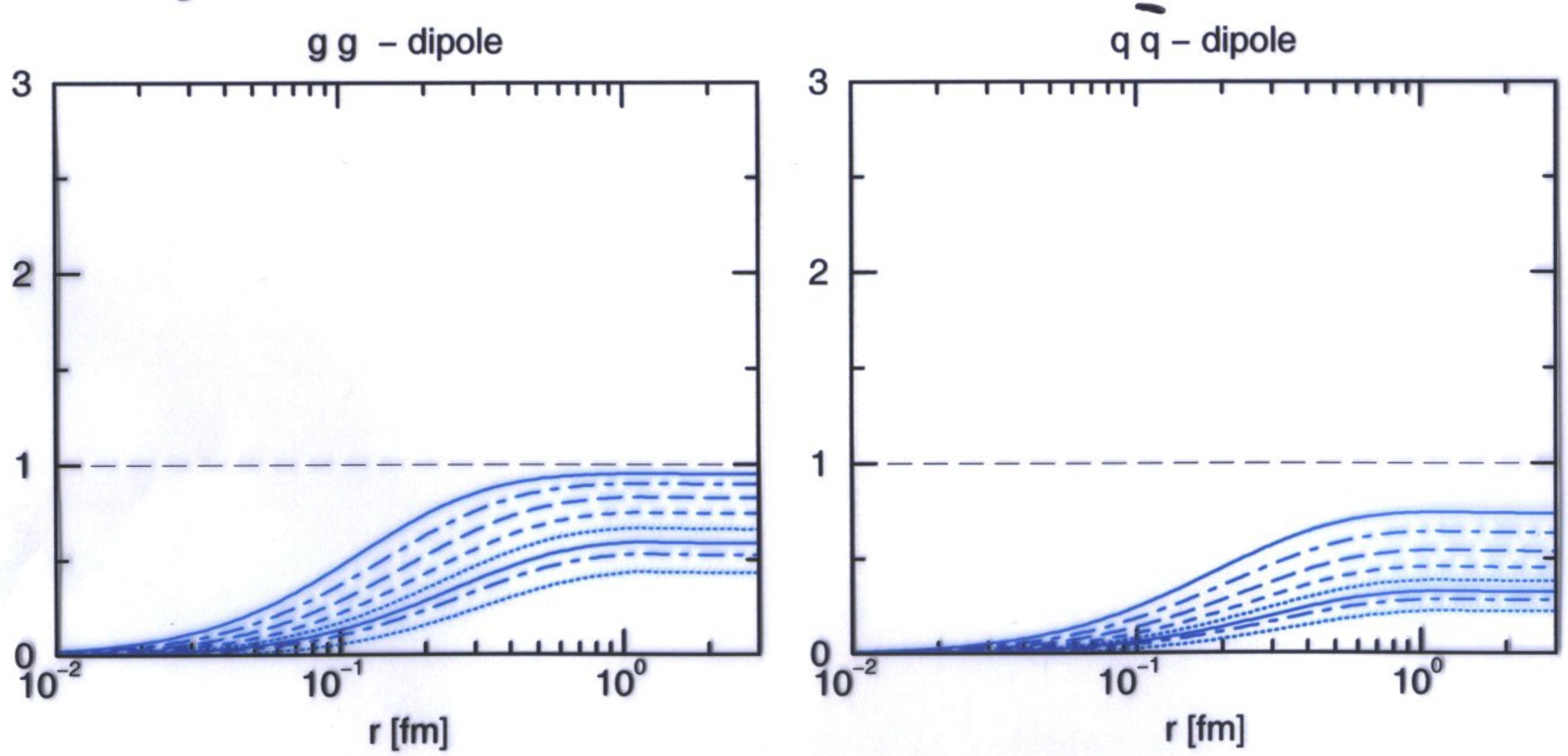
$$\Gamma_U(x,r,b=0) = 1 - \exp(-\Gamma(x,r,b=0))$$



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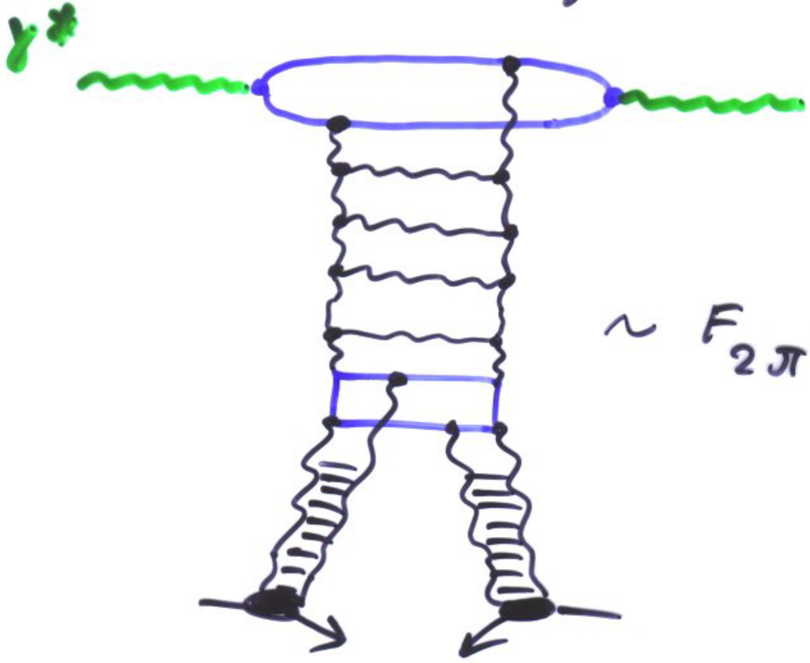
- We also have to account for the sea, that evolved from the input valence quarks:

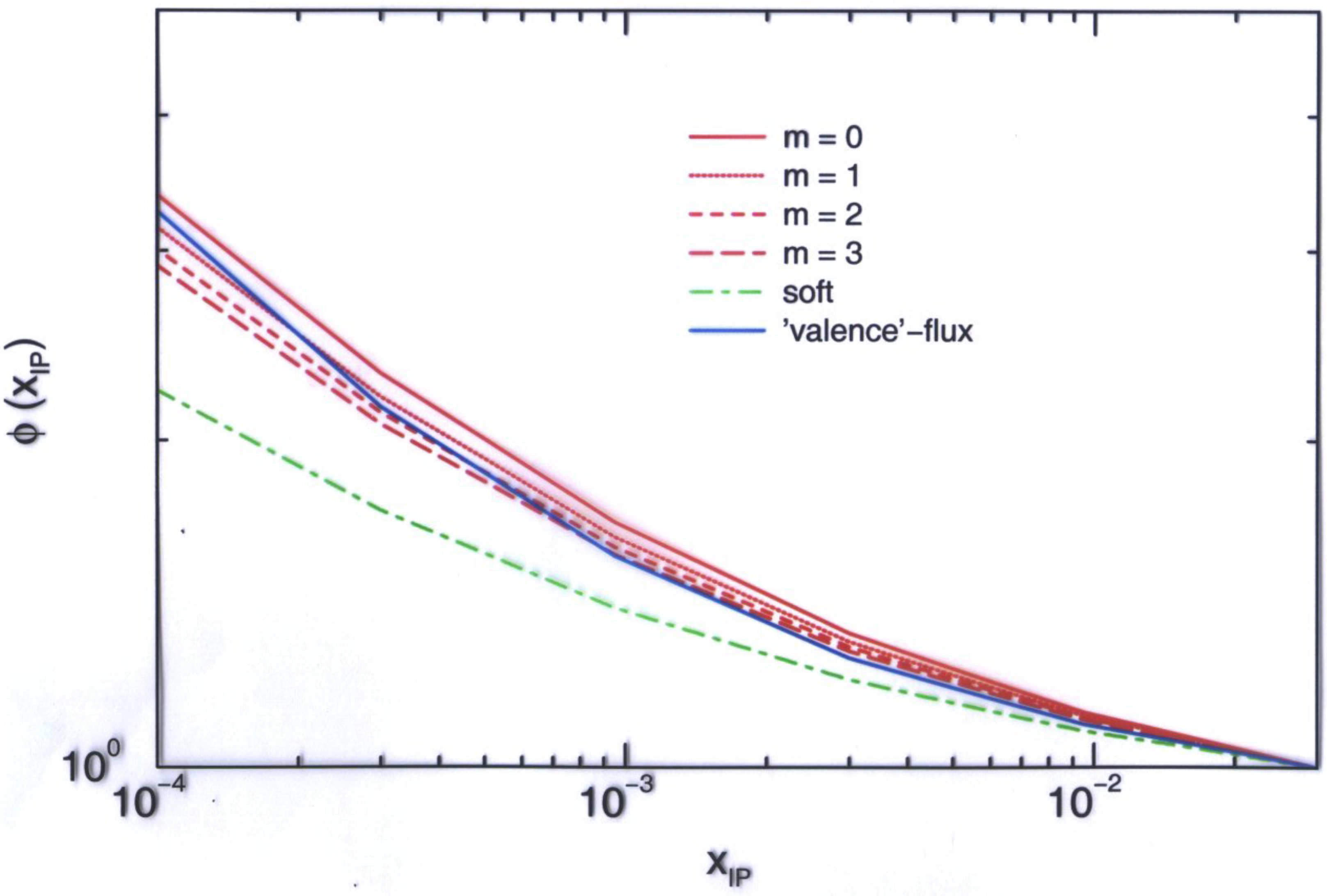


$\sim F_{2\pi}(\beta, Q^2)$

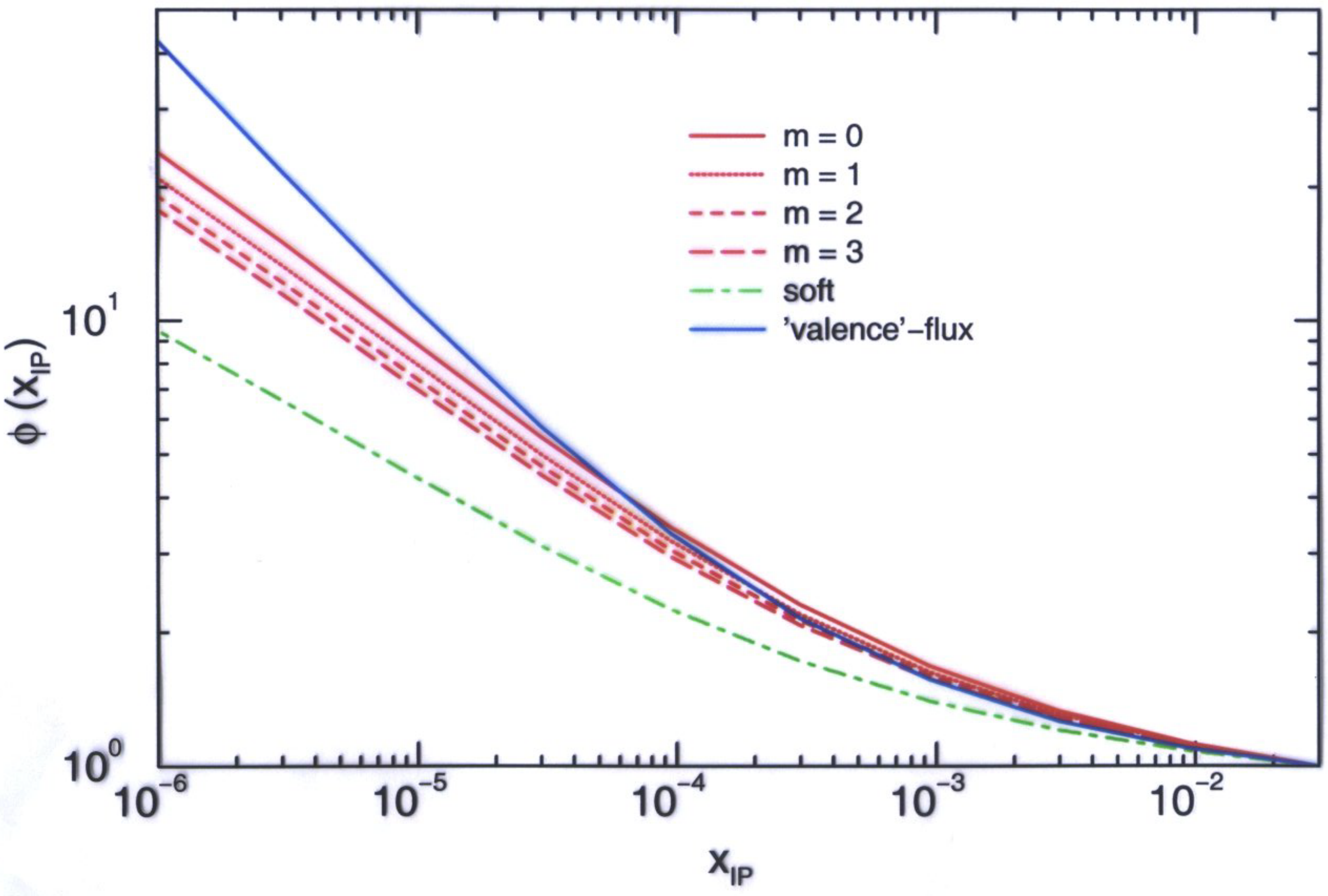
Here the cross section that is to be unitarized, is down by a factor 9/4 compared to the $q\bar{q}$ – dipole!

Unitarized Pomeron fluxes $\varphi(x_{IP})$

... almost universal.



Unitarized $\mathbb{P}$- fluxes in a wider range of $x_{\mathbb{P}}$.

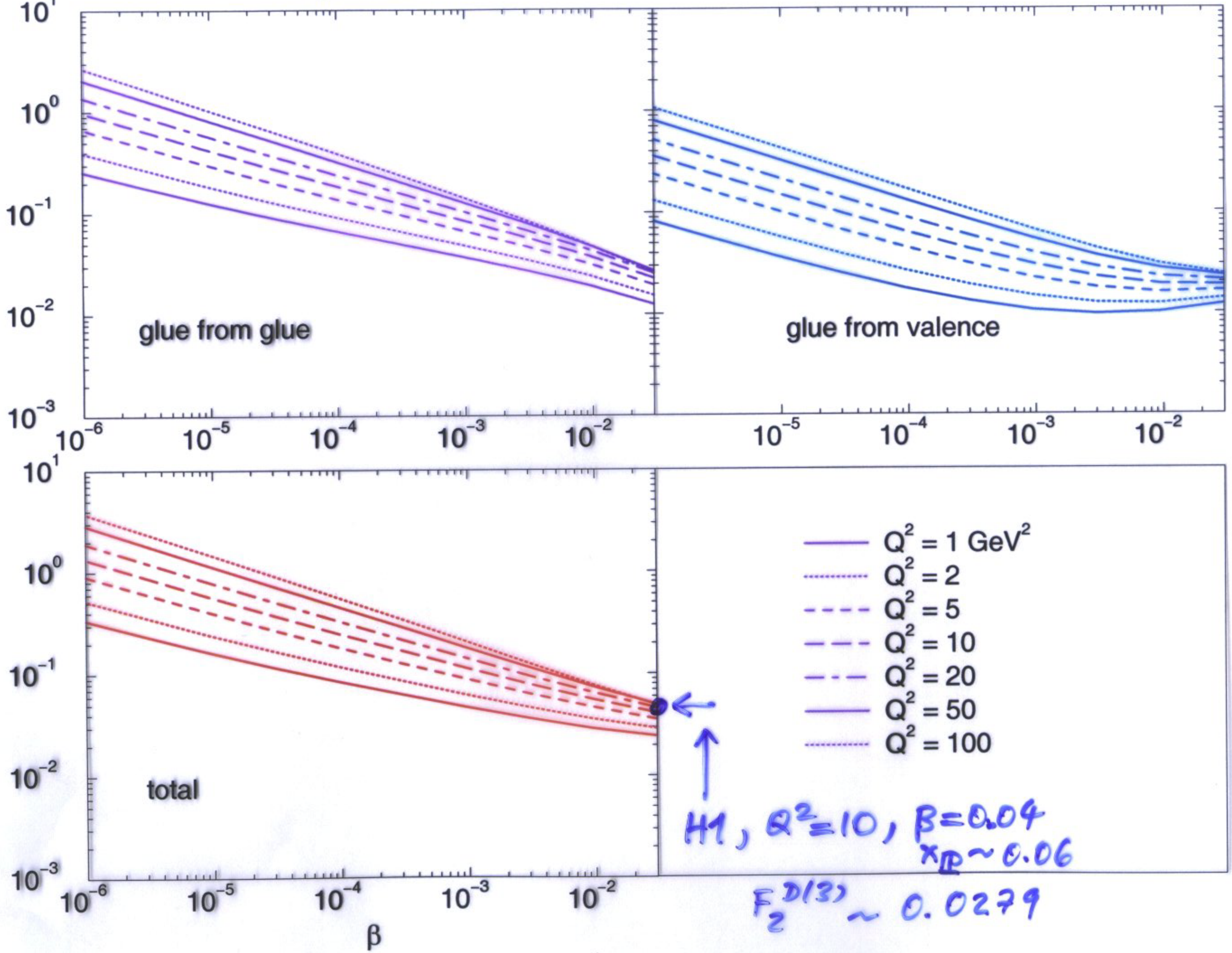


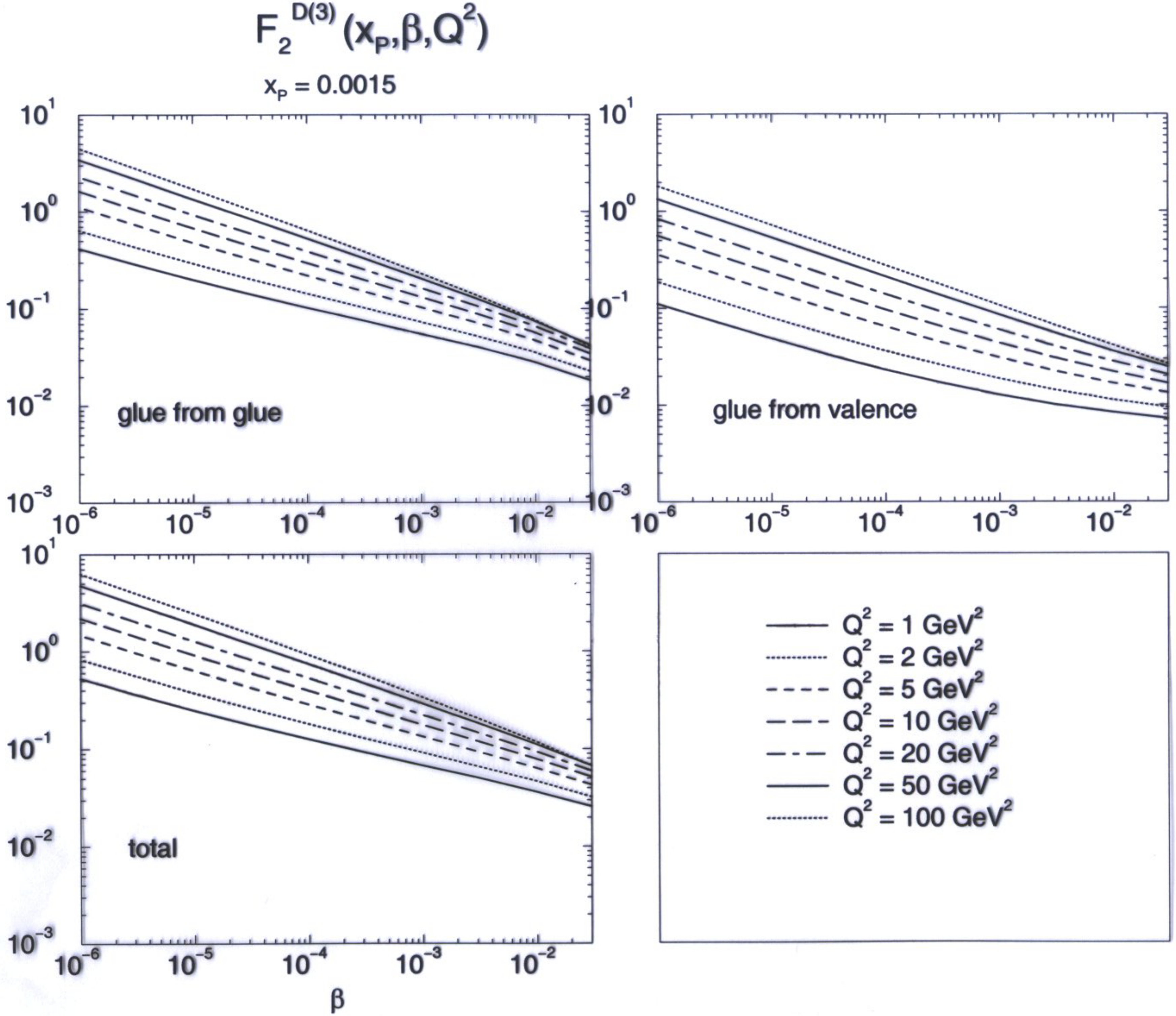
$$F_2^{D(3)}(x_P,\beta,Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}} \cdot \int dp_\perp^2 \, x_{\mathbb{P}} \frac{d\sigma}{dx_{\mathbb{P}}\, dp_\perp^2}$$

$x_P = 0.03$

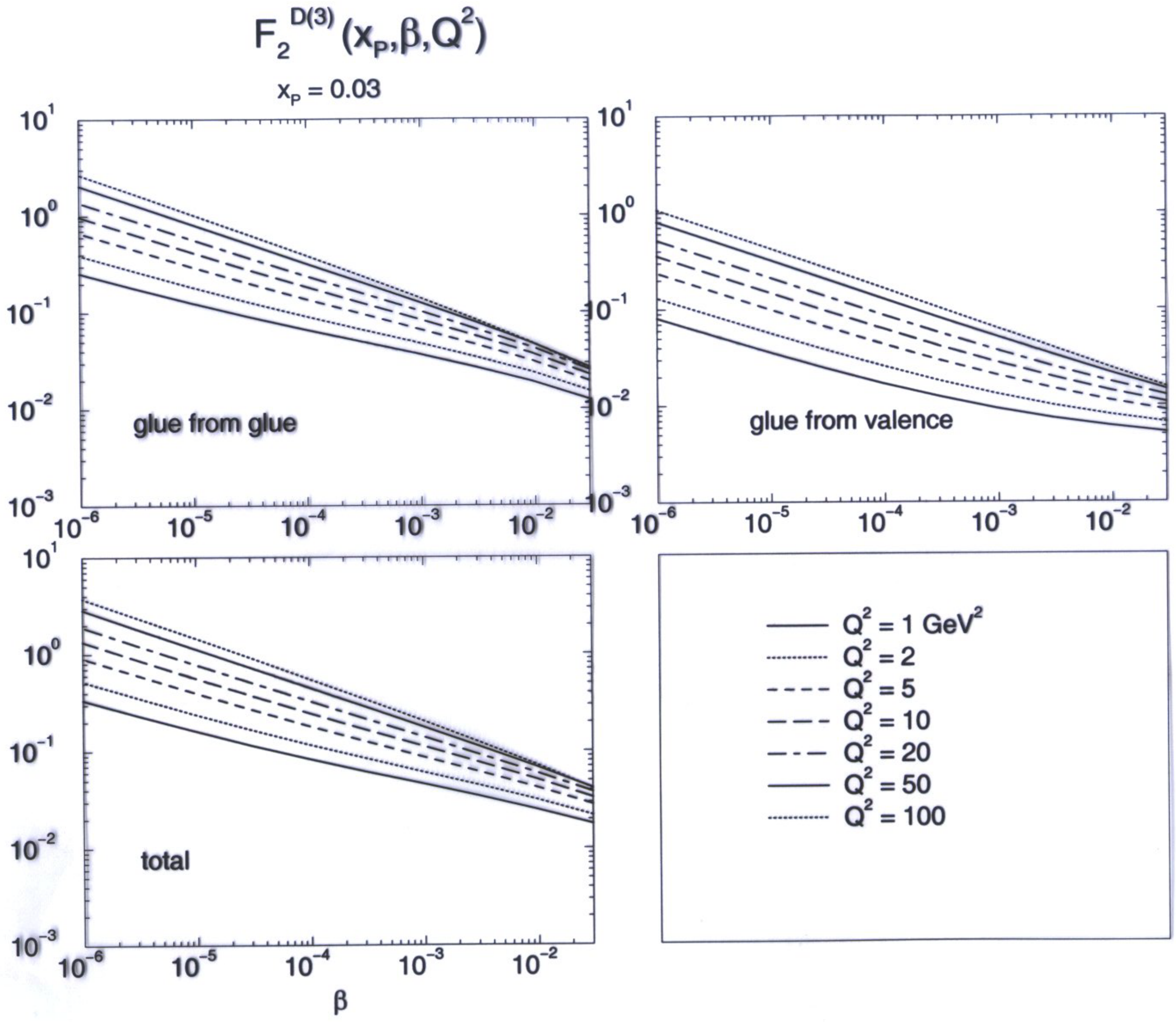
$B_{3\mathbb{P}} \sim 6\ \mathrm{GeV}^{-2}$

$B_{val} \sim 10\ \mathrm{GeV}^{-2}$

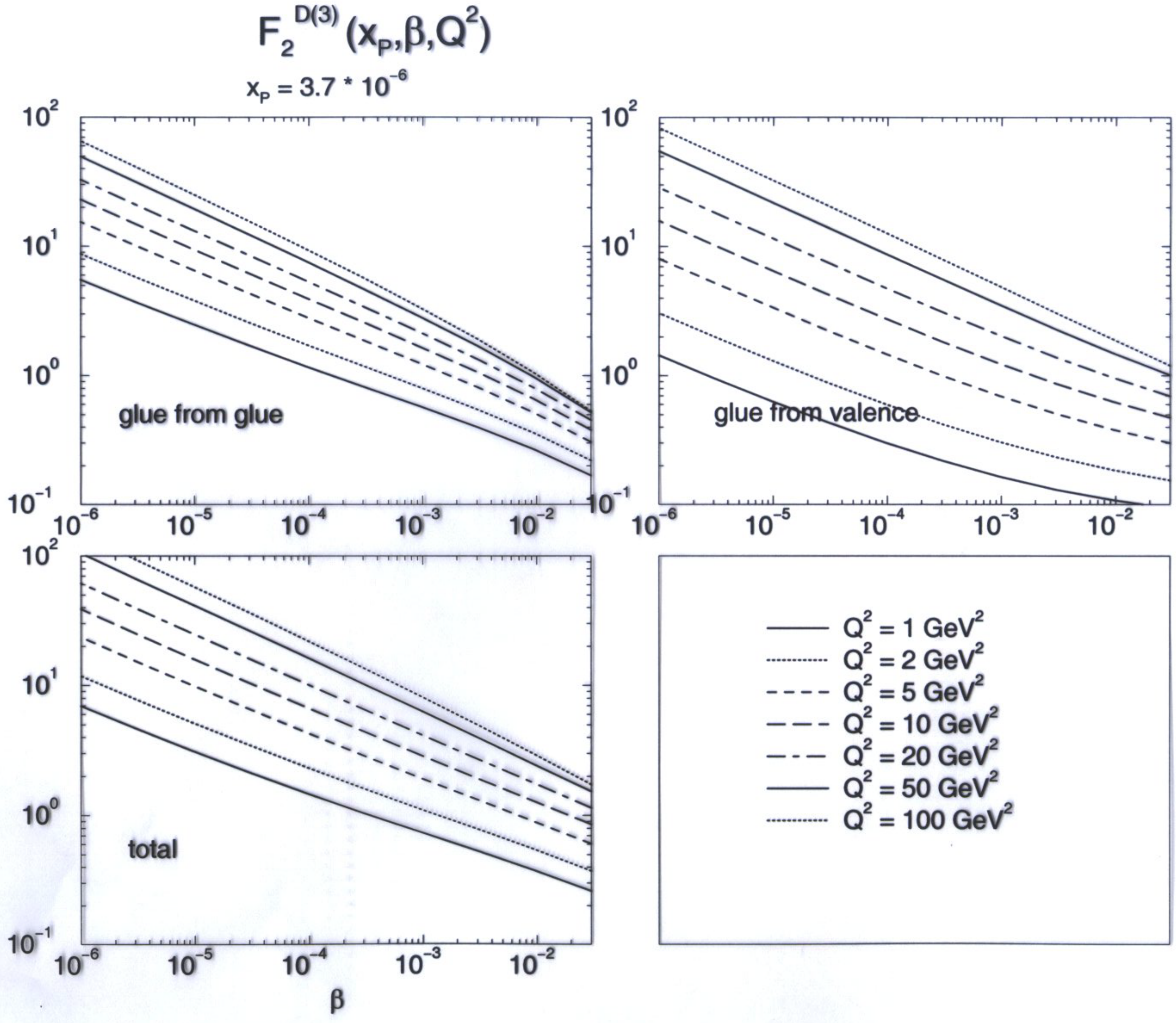




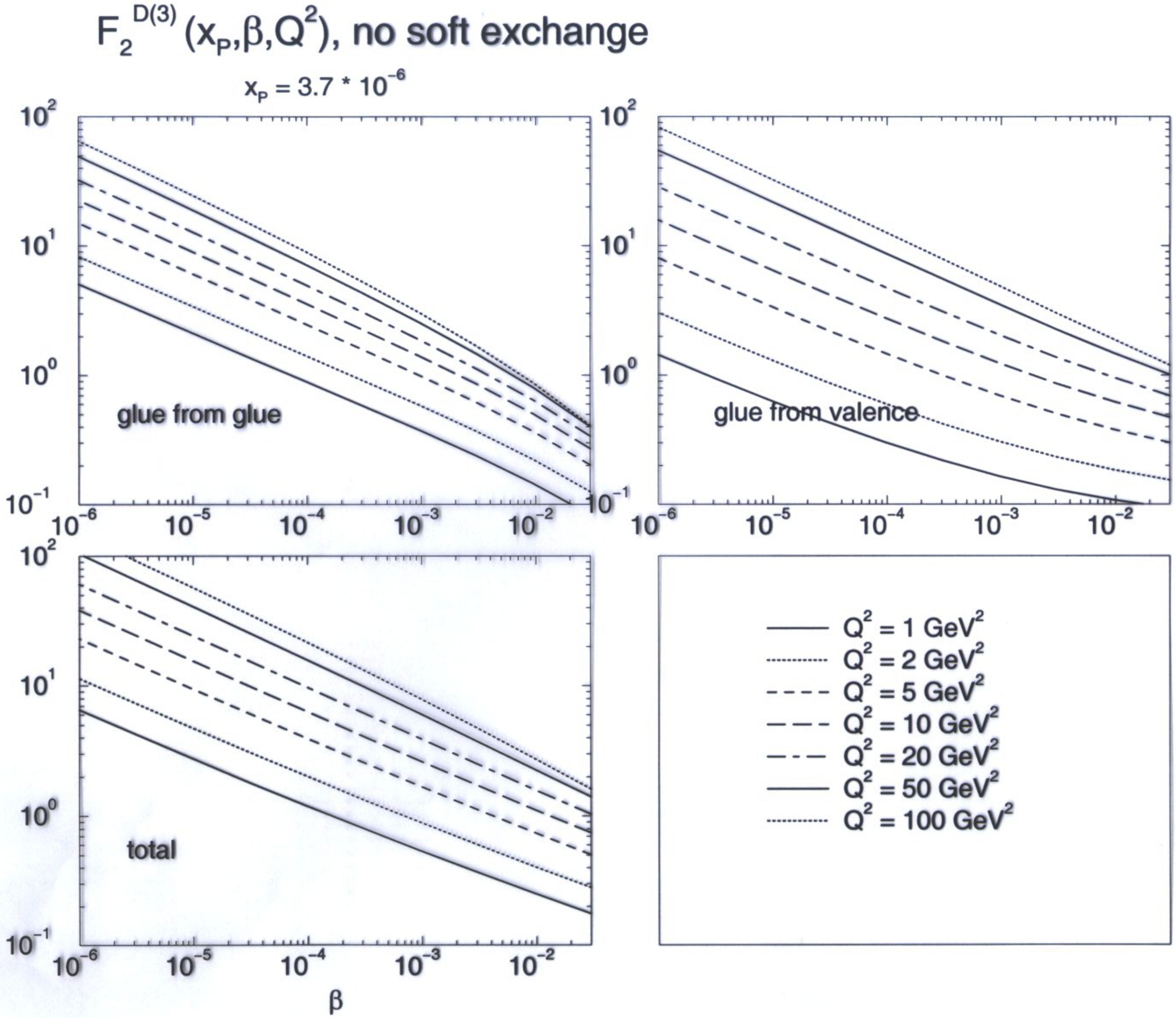
$F_2^{D(3)}(x_P,\beta,Q^2)$
$x_P = 0.0015$
glue from glue
glue from valence
total
β
Q^2 = 1 GeV^2
Q^2 = 2 GeV^2
Q^2 = 5 GeV^2
Q^2 = 10 GeV^2
Q^2 = 20 GeV^2
Q^2 = 50 GeV^2
Q^2 = 100 GeV^2

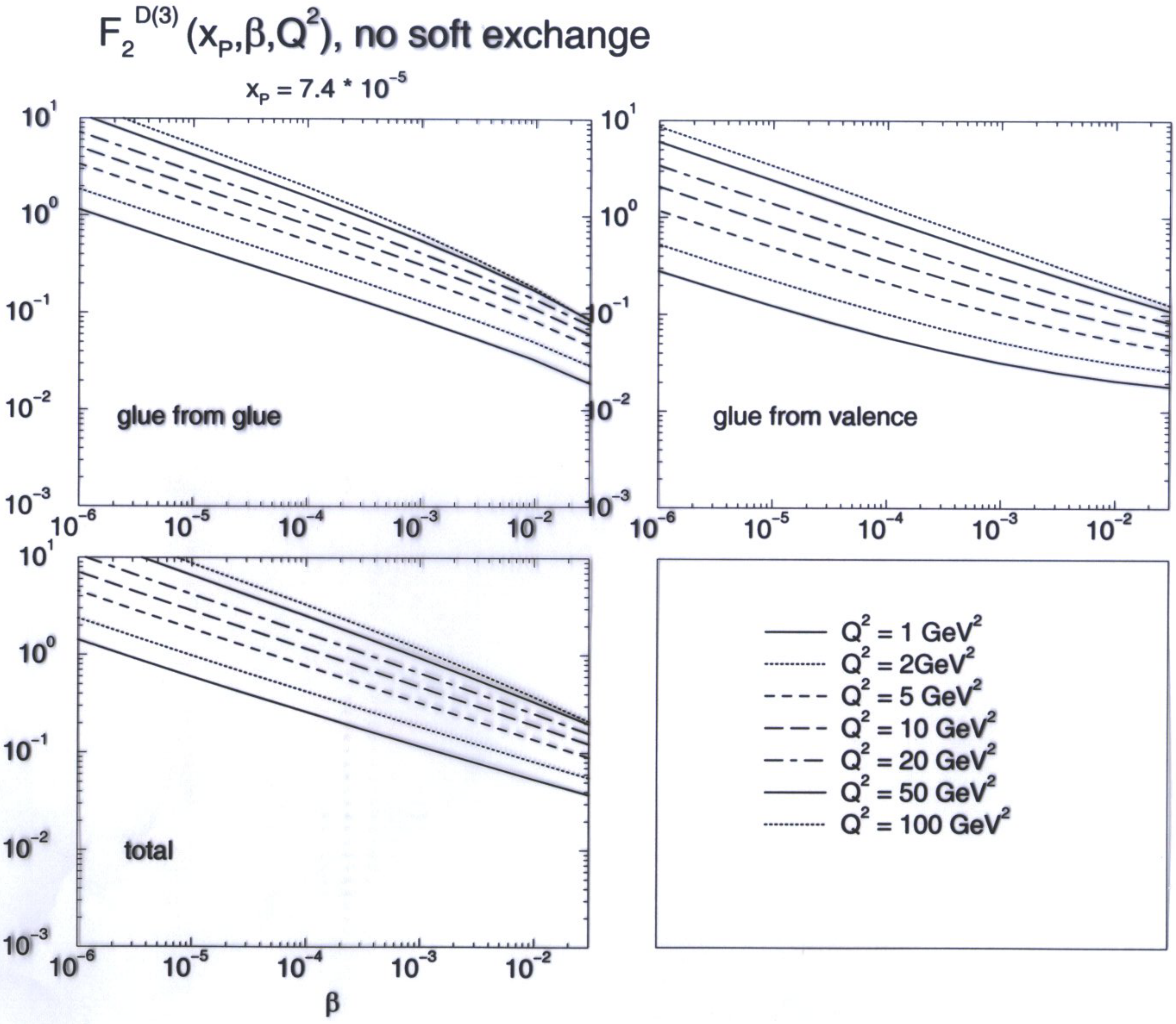


$F_2^{D(3)}(x_P,\beta,Q^2)$
$x_P = 0.03$
glue from glue
glue from valence
total
β
$Q^2 = 1$ GeV^2
$Q^2 = 2$
$Q^2 = 5$
$Q^2 = 10$
$Q^2 = 20$
$Q^2 = 50$
$Q^2 = 100$

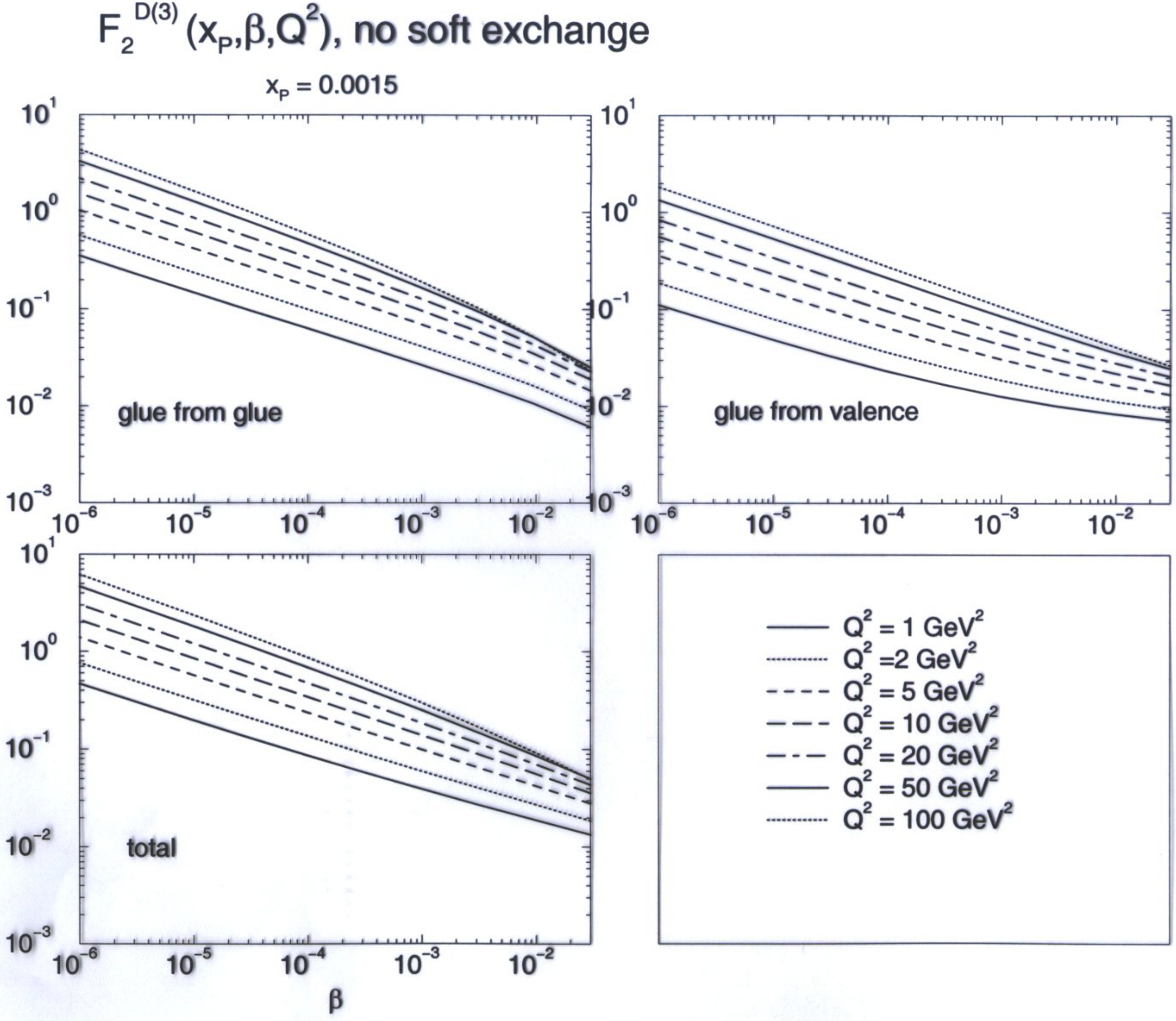


$F_2^{D(3)}(x_P,\beta,Q^2)$
$x_P = 3.7 * 10^{-6}$
glue from glue
glue from valence
total
β
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Q^2 = 1 GeV^2
Q^2 = 2 GeV^2
Q^2 = 5 GeV^2
Q^2 = 10 GeV^2
Q^2 = 20 GeV^2
Q^2 = 50 GeV^2
Q^2 = 100 GeV^2

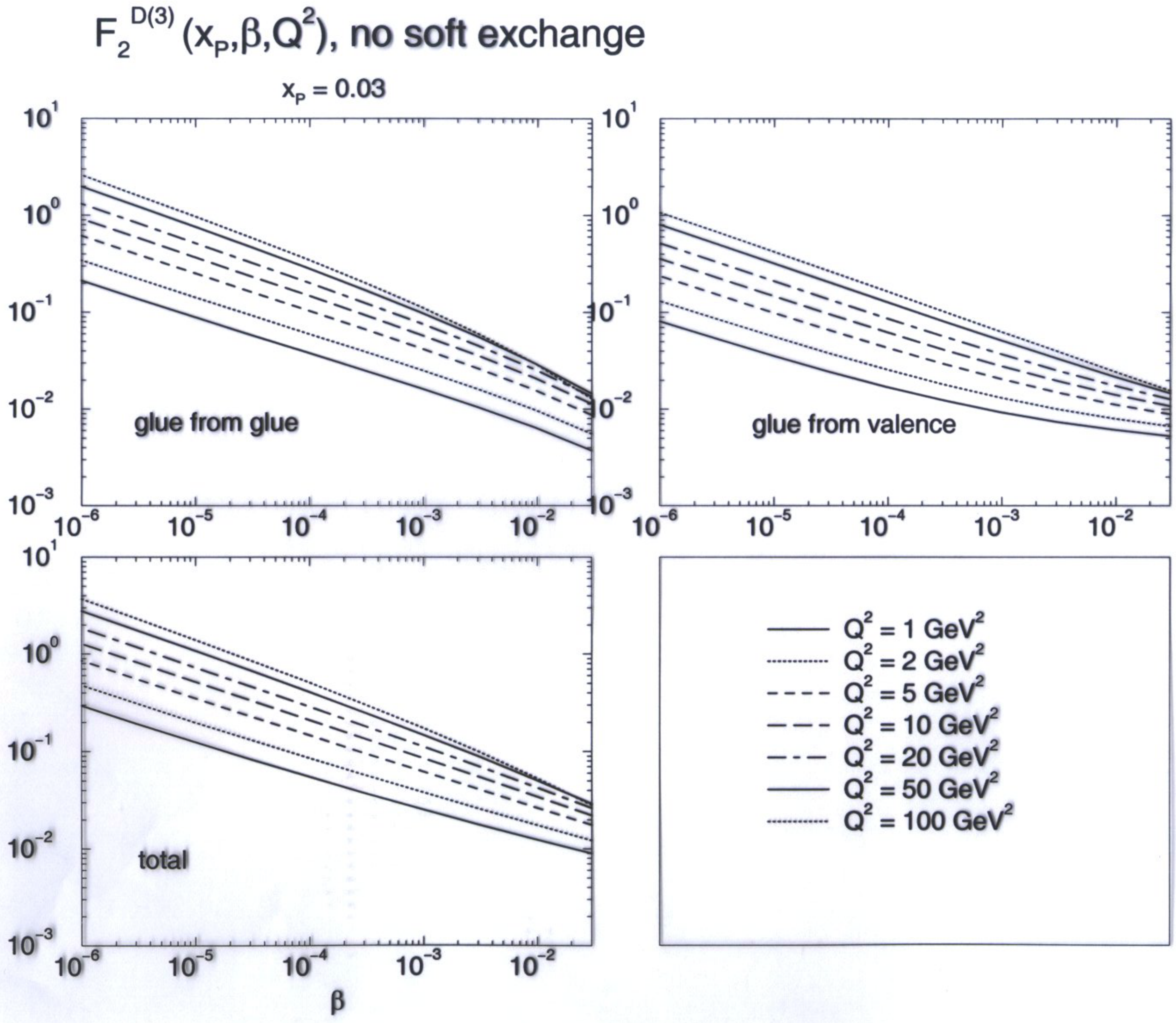

$F_2^{D(3)}(x_P,\beta,Q^2)$, no soft exchange
$x_P = 3.7 * 10^{-6}$
glue from glue
glue from valence
total
β
10^{-1}
10^{0}
10^{1}
10^{2}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
$Q^2 = 1$ GeV^2
$Q^2 = 2$ GeV^2
$Q^2 = 5$ GeV^2
$Q^2 = 10$ GeV^2
$Q^2 = 20$ GeV^2
$Q^2 = 50$ GeV^2
$Q^2 = 100$ GeV^2



F2 D(3) (xP,β,Q2), no soft exchange
xP = 7.4 * 10-5
glue from glue
glue from valence
total
β
Q2 = 1 GeV2
Q2 = 2GeV2
Q2 = 5 GeV2
Q2 = 10 GeV2
Q2 = 20 GeV2
Q2 = 50 GeV2
Q2 = 100 GeV2



$F_2^{D(3)}(x_P,\beta,Q^2)$, no soft exchange
$x_P = 0.0015$
glue from glue
glue from valence
total
β
10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2}
10^{1} 10^{0} 10^{-1} 10^{-2} 10^{-3}
$Q^2 = 1$ GeV^2
$Q^2 =2$ GeV^2
$Q^2 = 5$ GeV^2
$Q^2 = 10$ GeV^2
$Q^2 = 20$ GeV^2
$Q^2 = 50$ GeV^2
$Q^2 = 100$ GeV^2


$F_2^{D(3)}(x_P, \beta, Q^2)$, no soft exchange
$x_P = 0.03$
glue from glue
glue from valence
total
β
$Q^2 = 1$ GeV^2
$Q^2 = 2$ GeV^2
$Q^2 = 5$ GeV^2
$Q^2 = 10$ GeV^2
$Q^2 = 20$ GeV^2
$Q^2 = 50$ GeV^2
$Q^2 = 100$ GeV^2

- Conclusions:

* Starting from the color dipole description of diffractive DIS and the 2G-wave-function of the $I\!P$, we identified the arguably dominant unitarization corrections.
An extension of the CD-Regge expansion allows predictions for the β-dependence of $F_2^{D(3)}$.

* We found a weak dependence on x of the diffractive/total ratio.
(which we maybe overestimate somewhat)
No sign of a black disc limit is found, if we interpret $F_2^D(x,Q^2)$ as the screening correction to the proton structure function, following AGK rules.
Still, the shadowing corrections to the proton SF are quite sizeable!