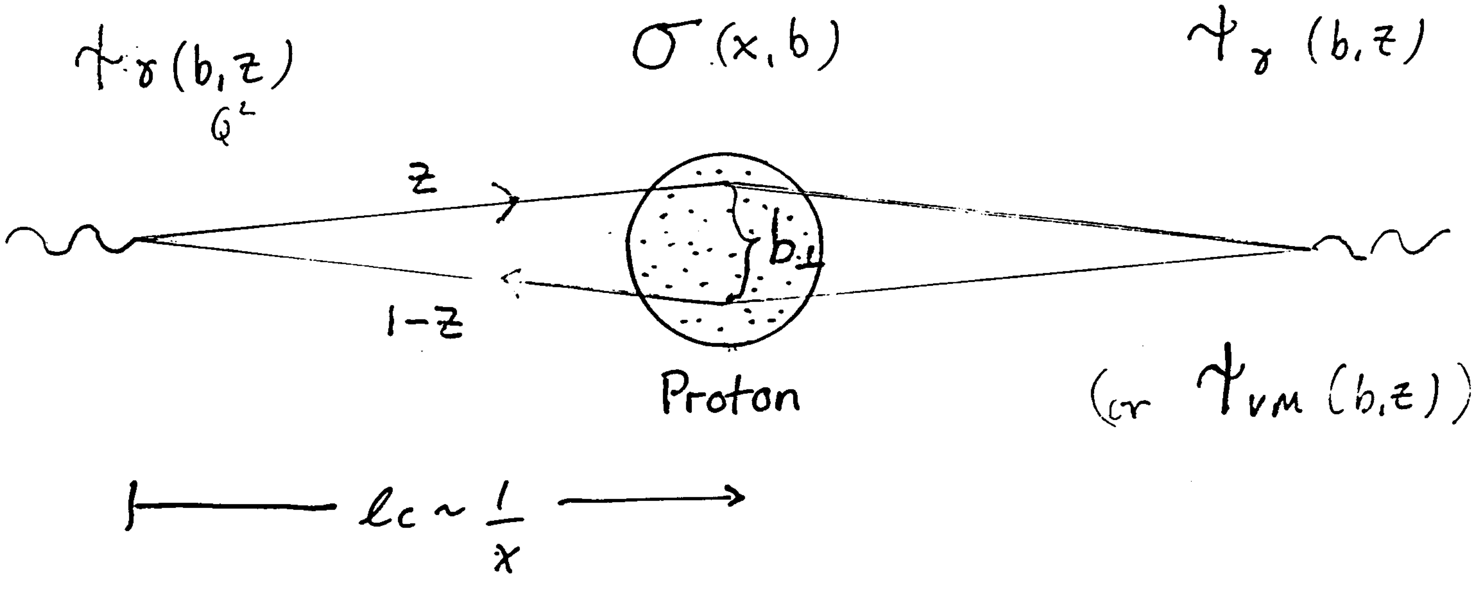
Unitarity and the QCD-improved Dipole Picture.

Martin McDermott (Manchester)



with
L. Frankfurt
V. Guzey
M. Strikman

M. McDermott

Small-x Structure Functions in b-space

mm. Frankfurt, Guzey, Strikman hep-ph/9912547

Total $\gamma^* P$ cross sections

$$\sigma^b_{L,T}(x,Q^2) = \int_0^1 dz \int d^2 b_\perp \; \hat{\sigma}(b^2, x) \, |\psi_{L,T}(z, b^2, Q^2)|^2$$

light-cone wave functions

$$|\psi_L(z,b)|^2 = \frac{6\,\alpha_{em}}{\pi^2} \sum_{q=1}^{nf} e_q^2 \, z^2(1-z)^2 Q^2 K_0^2(\epsilon b)$$

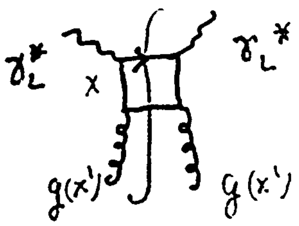
$$|\psi_T(z,b)|^2 = \frac{3\,\alpha_{em}}{2\pi^2} \sum_{q=1}^{nf} e_q^2 \left\{ (z^2+(1-z)^2)\,\epsilon^2 K_1^2(\epsilon b) + m_q^2 K_0^2(\epsilon b) \right\}$$

$$\epsilon^2 = z(1-z)\,Q^2 + m_q^2$$

Structure Functions

$$F^b_{L,T}(x,Q^2) = \frac{Q^2}{4\pi^2 \alpha_{em}} \, \sigma^b_{L,T}(x,Q^2) \quad ; \quad F_2^b = F_L^b + F_T^b$$

c.f. $$F_L^q = F_L^{LO\,pQCD}(x,Q^2) = \frac{4\,\alpha_s(Q^2)\,T_R}{2\pi} \sum_{q=1}^{2nf} e_q^2 \int_x^1 dx'\, x' g(x',Q^2) \frac{x^2}{x'^3}\left(1 - \frac{x}{x'}\right)$$



$\langle x' \rangle \simeq 1.75\,x$

LC wfs weight universal $\hat{\sigma}$ differently in dipole size according to x, Q^2 and process

$$\sigma_{L,T} = \int_0^\infty db \; I_{L,T}(b, x, Q^2)$$

Generic Integral

$\rightarrow$?

QCD-improved ansatz for $\sigma(b^2; x)$

[C.f. Collins
Frankfurt
Strikman
factorization proofs]

For small dipoles we know

$$\hat{\sigma}_{pQCD}(x, b) = \pi b_\perp^2 \cdot \frac{\pi}{3} \cdot \alpha_s(\bar{Q}^2)\, x' g(x', \bar{Q}^2) \Big|_{\bar{Q}^2(b^2)} \qquad (1).$$

We choose

Fr. Kopf. Strikman.

i) $\bar{Q}^2 = \dfrac{\lambda}{b^2}$ $\quad \lambda = 10 \quad (\simeq \langle b\rangle^2_{\sigma_L} Q^2$ from σ_L^b)

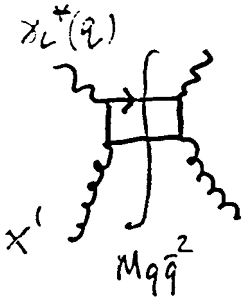
$\hookrightarrow$ see later

ii) $\boxed{x' = x + (\langle x\rangle - x)\dfrac{\langle b\rangle^2}{b^2}}$ $= x \quad b^2 \gg \langle b\rangle^2$

$= \langle x\rangle \quad b^2 = \langle b\rangle^2 \quad \langle x\rangle = 1.75x$ from σ_L

$=$ few $x \quad b^2 \ll \langle b\rangle^2$

recall



$$(x'P + q)^2 = M_{q\bar{q}}^2 = \frac{k_\perp^2 + m_q^2}{z(1-z)} \propto \frac{1}{b_\perp^2}$$

$$\hookrightarrow x' = x_{Bj} + \frac{c}{b^2 W^2}$$

But i) implies eqn (1) not defined for large dipoles

$$b^2 > b_{Q_0}^2 = \frac{\lambda}{Q_0^2} \simeq (0.4\,\text{fm})^2$$

$\hookrightarrow$ input scale of parton densities

Large b model and matching

Large b "dipole" becomes complicated partonic state

"b" refer to typical transverse size of system.

Ansatz

Match to measured experimental $\sigma_{\pi p}$ at $b \simeq b_\pi \simeq 0.65\,\mathrm{fm}$

$$\hat{\sigma}(b_\pi, x) = \sigma_{\pi, N}\left(\frac{x_0}{x}\right)^{0.08} = 24\left(\frac{x_0}{x}\right)^{0.08} \mathrm{mb}.$$

↳ Donnachie-Landshoff

slow energy growth.

Interpolation for moderate $x \simeq 10^{-3}$

$$\hat{\sigma}_I(x, b) = \left[\hat{\sigma}_{\pi p}(x, b_\pi^2) - \hat{\sigma}_{pQCD}(b_{Q_0}^2)\right] H(b^2) + \hat{\sigma}_{pQCD}(b_{Q_0}^2)$$

where

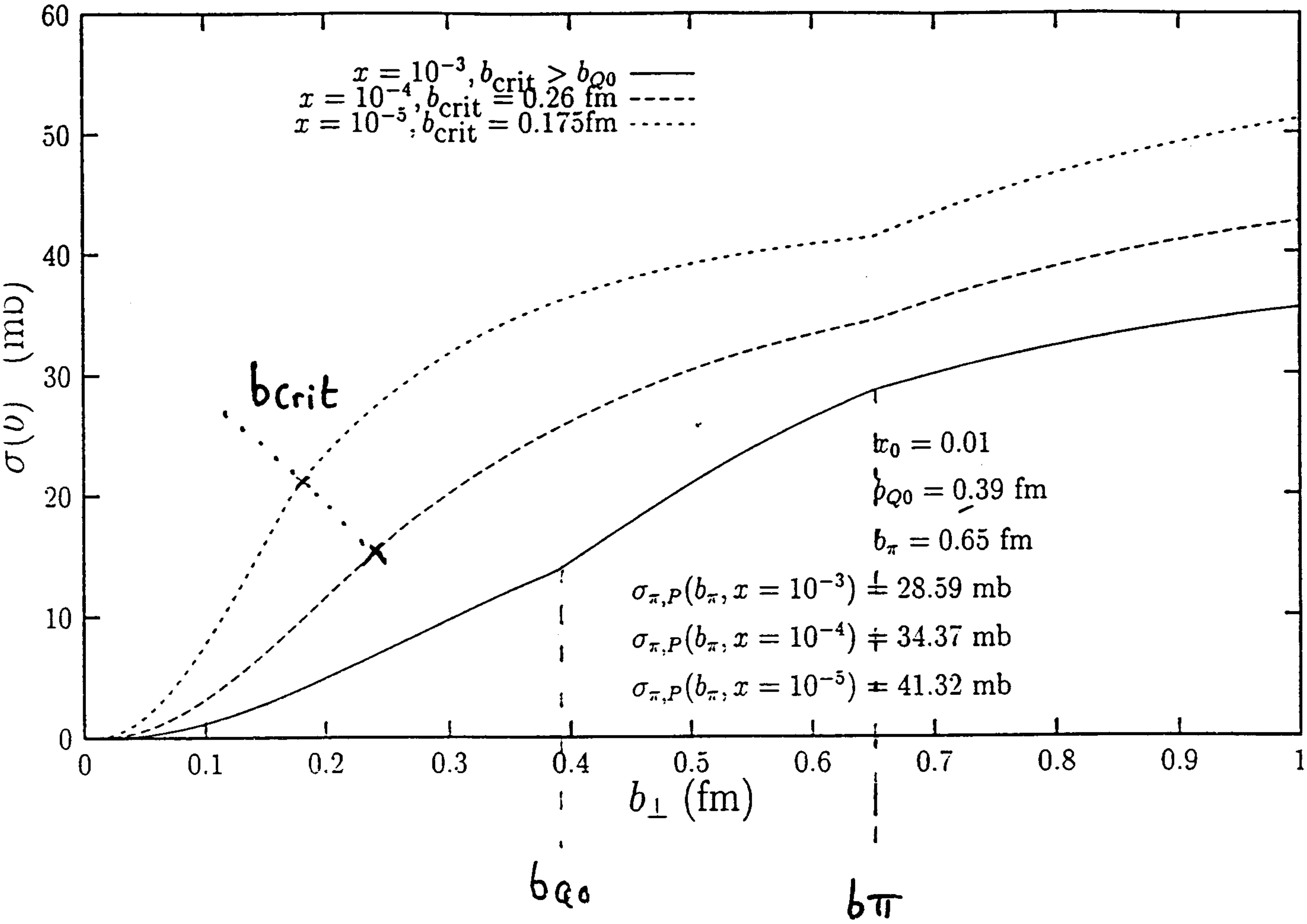
$$H(b^2) = \frac{e}{(e-1)}\left[1 - e^{-(b^2 - b_{Q_0}^2)/(b_\pi^2 - b_{Q_0}^2)}\right] = 0 \quad b = b_{Q_0}$$

$$= 1 \quad b = b_\pi.$$

linear in b^2 around $b = b_{Q_0}$

flatter in b^2 " $b = b_\pi$

Dipole Cross Section as a function of transverse size.



Very Small $b^2 < b^2_{crit}$

$$\hat{\sigma} = \hat{\sigma}_{PQCD}(b^2, x) = \frac{\pi^2 b^2}{3} \alpha_s(\bar{Q}^2)\, x' g(x', \bar{Q}^2) \Big|_{\bar{Q}^2 = \frac{\lambda}{b^2}}$$

Unitarity Corrections at very small x

Problem

Because the gluon density rises steeply (x^{-n} $n > 0.08$) at small before long $\hat{\sigma}_{pQCD} \simeq \hat{\sigma}_{\pi p}$

Infact $\hat{\sigma}_{pQCD} = \hat{\sigma}_{inel}$ (implicit assumed $\hat{\sigma}_{el} \ll \hat{\sigma}_{inel}$)

In black disc limit $\sigma_{el} = \sigma_{inel} = \frac{\sigma_{tot}}{2}$

To prevent disaster we tame this small x growth at

$$\hat{\sigma}_{pQCD}(b^2_{crit}, x) = \frac{\hat{\sigma}_{\pi p}(x, b_\pi^2)}{2}$$

For small enough-x this tamed must occur in the perturbative region (perhaps calculable!)

$$b_{crit}(x) < b_{Q_0}$$

New Interpolation for $b_{crit} < b < b_\pi$

$$\hat{\sigma}_I = \left(\frac{b^2}{b^2+a^2}\right)^n \sigma_0$$

↳ match at b_π.

match at b_{Q_0} (a)
and fit below b_{Q_0} (n fit)

Gribov's Black Limit (1969)

Gribov considered Photon-Proton at very high energies : proton looks "black" for any fluctuation

$$\sigma^{\gamma^* p}(x,Q^2) = \int d^2b_\perp\, dz\ \hat{\sigma}(z, b_\perp^2, x)\,|\psi^{\gamma^*}(Q^2, z, b_\perp)|^2$$

$$= \int d^2b_\perp\, dz\ 2\pi\left(r_N + \tfrac{b}{2}\right)^2 |\psi^{\gamma^*}(Q^2, z, b_\perp^2)|^2$$

$$+ \int_0^{b_0} d^2b_\perp\, dz\ \left[\hat{\sigma}_{pQCD}(z, b, x) - 2\pi\left(r_N + \tfrac{b}{2}\right)^2\right] |\psi^{\gamma^*}|^2$$

Structure functions no longer Scale !

$$F_2(x,Q^2) \simeq \frac{2\pi r_N^2}{12\pi^3} Q^2 \rho \ln(\delta/x) \qquad \propto Q^2 \ !$$

$$= \frac{2\pi r_N^2}{12\pi^3} \cdot Q^2 \int_0^{\delta s} \frac{\rho(M^2)\, dM^2}{(M^2+Q^2)}$$

<u>For</u> $Q^2 = 1\ \mathrm{GeV}^2$, $\rho \simeq 2.5$, $\delta/x \simeq 10$, $x \simeq 10^{-5} - 10^{-6}$ $\Rightarrow$ $F_2(x,Q^2) \simeq 1.5 - 3.0$.

HERA $F_2(10^{-5}, 1) \simeq 0.5 - 0.7$

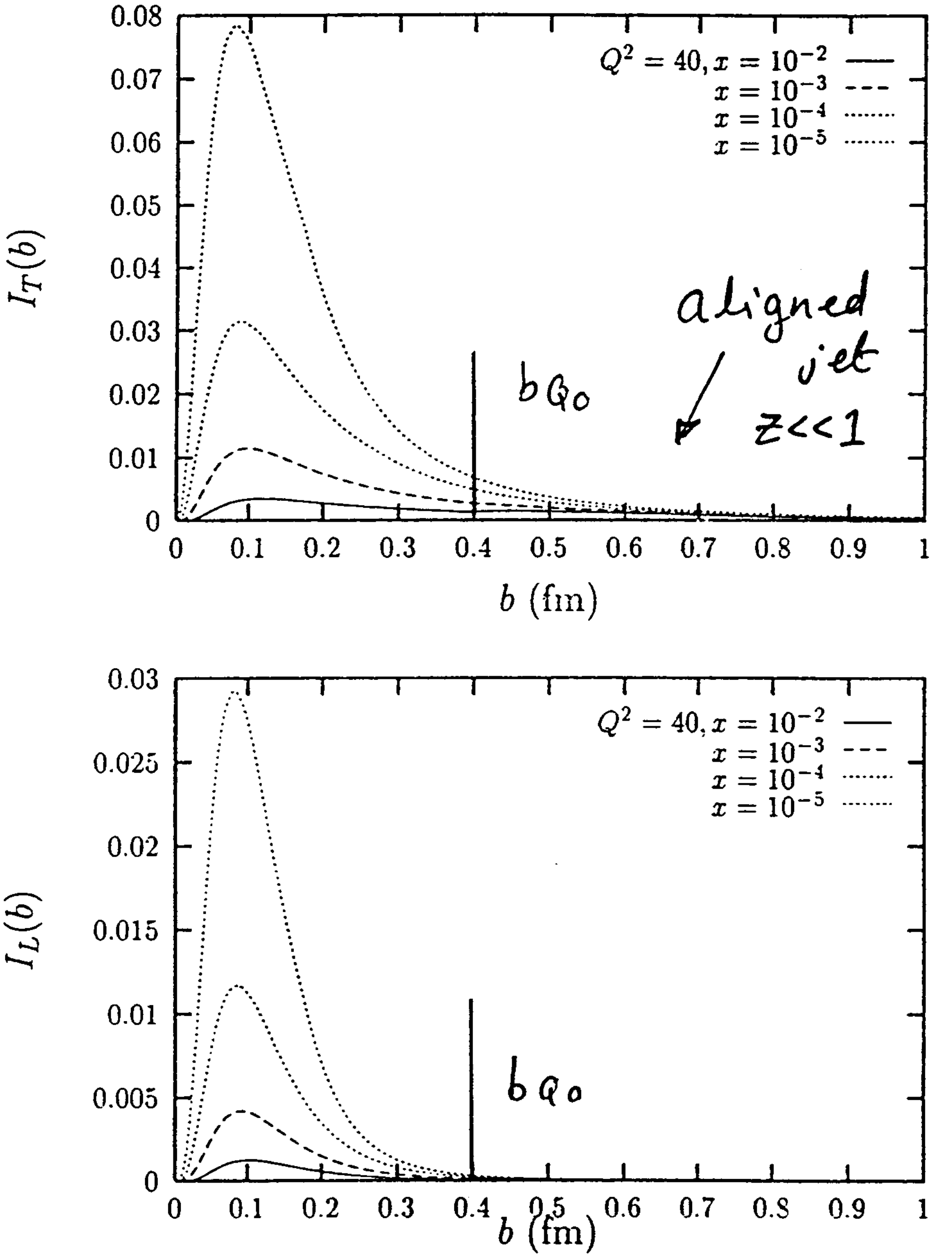
Perhaps the black limit is nearby !

$\hookrightarrow$ look for signals for it at THERA.

b-integrand of σ_T, σ_L for large $Q^2 \simeq 40\,\mathrm{GeV}^2$

$I_T(b)$

broad



$I_L(b)$

narrow.

Figure 12: Integrands in b-space for $Q^2 = 40$ GeV^2.

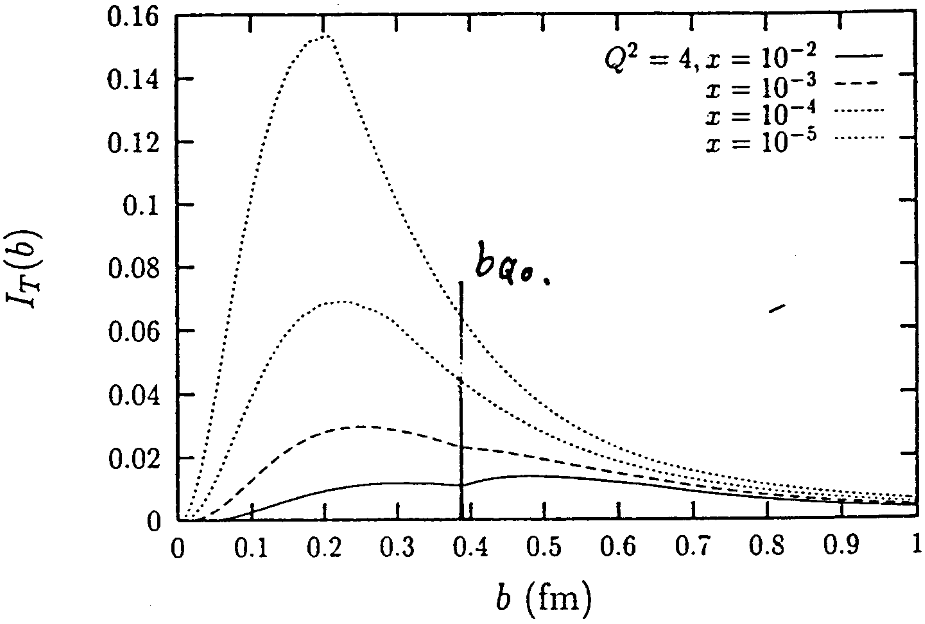
8

b- integrand of σ_T, σ_L for low $Q^2 = 4\ GeV^2$

CTEQ4L gluons $Q_0^2 = 2.56\ GeV^2$

$b_{Q_0} = \frac{1}{Q_0} = 0.39 f.$

I_T



I_L

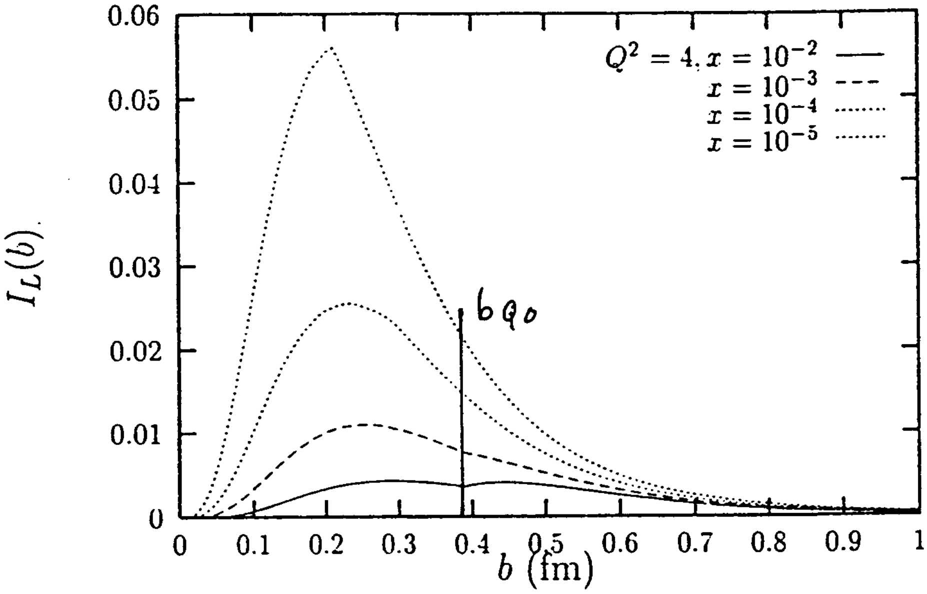


Figure 13: Integrands in b-space for $Q^2 = 4\ \mathrm{GeV}^2$.

Comparison of F_L^q with b-space F_L^b

$b > b_{90}$.

x CTEQ4L	F_L^q	F_L^b	% diff. = 100 × $(F_L^q - F_L^b)/F_L^q$	$R_L(\%)$
$Q^2 = 45$				
10^{-2}	0.0638	0.0716	-12.3	5.1
10^{-3}	0.224	0.225	-0.3	2.4
10^{-4}	0.620	0.594	4.1	1.3
10^{-5}	1.53	1.40	8.2	0.7
$Q^2 = 10$				
10^{-2}	0.0704	0.0750	-6.53	26.2
10^{-3}	0.204	0.187	8.46	15.2
10^{-4}	0.493	0.427	13.4	9.87
10^{-5}	1.10	0.869	21.1	6.38
$Q^2 = 4$				
10^{-2}	0.0714	0.0759	-6.32	54.3
10^{-3}	0.174	0.153	12.0	37.5
10^{-4}	0.378	0.307	18.6	26.9
10^{-5}	0.787	0.558	29.1	19.4

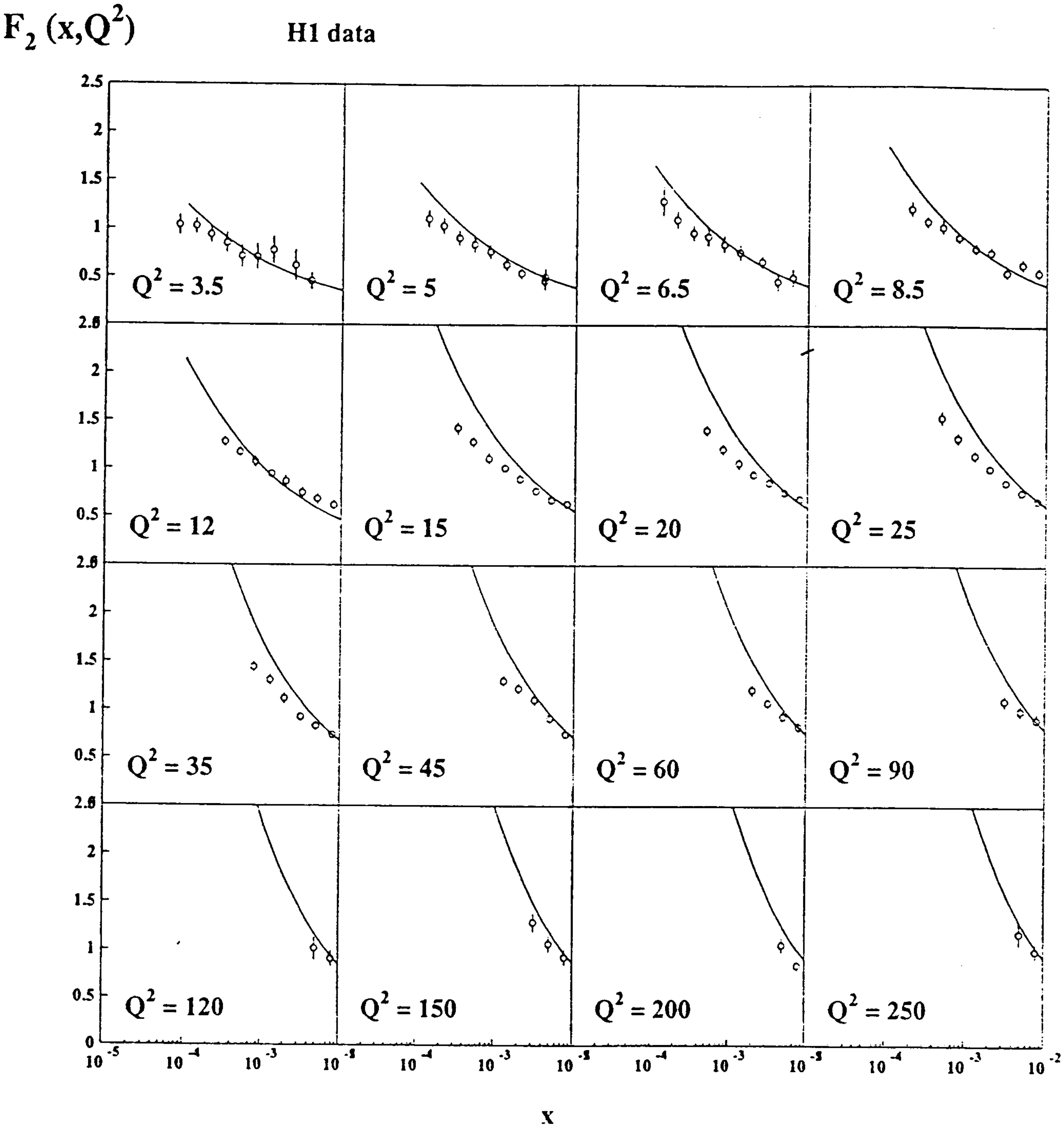
$Q_0^2 = 2.56$ GeV^2.

Table 1: A comparison of the b-space formula, F_L^b, using our ansätze for the DCS, with the standard perturbative QCD result, F_L^q, for $F_L^{n_f=3}(x, Q^2)$ as a function of x for $Q^2 = 4, 10, 45$ GeV^2. We used CTEQ4L gluons.

Good agreement for large Q^2 (recall $\lambda = 10$, $\langle x \rangle = 1.75x$)

Deviation for low Q^2

Comparison with H1 1994 data on $F_2(x, Q^2)$



Also reasonable agreement with low Q^2 data

Comparison to other models for $\hat{\sigma}$

Forshaw, Kerley, Shaw

Phys Rev D 60 (1999) 074012

"Ad hoc", fitted to data.

$$\hat{\sigma}(b^2, W^2) = \frac{a\, P_s^2(b)}{1 + P_s^2(b)} (b^2 W^2)^{\lambda_S} + b^2 P_h^2(b)\, e^{-\gamma_h^2 b} (b^2 W^2)^{\lambda_H}$$

Scaling of F_2 $\to \frac{1}{x}$ $\quad b^2 = \frac{\lambda}{Q^2}$!

but large W^2 not monotonically increasing function of b^2

QCD-info not included (not $\sim b^2$ small b^2!).

Golec-Biernat, Wusthoff.

Phys Rev D59 (1999) 014017.
D60 (1999) 114023.

"Saturating" ansatz

$$\hat{\sigma}(x, b^2) = \sigma_0 \left(1 - e^{-\frac{b^2 Q_0^2}{4 (x_0/x)^\lambda}}\right)$$

but

i) pQCD not explicitly included.

ii) Constant at large b with W^2

iii) Unitarity Corr. ad hoc.

Phys Lett B437 (1998) 408

Donnachie, Landshoff

Some shared features with Regge 2 pomeron model

Gotsman, Levin, Maor, Natali

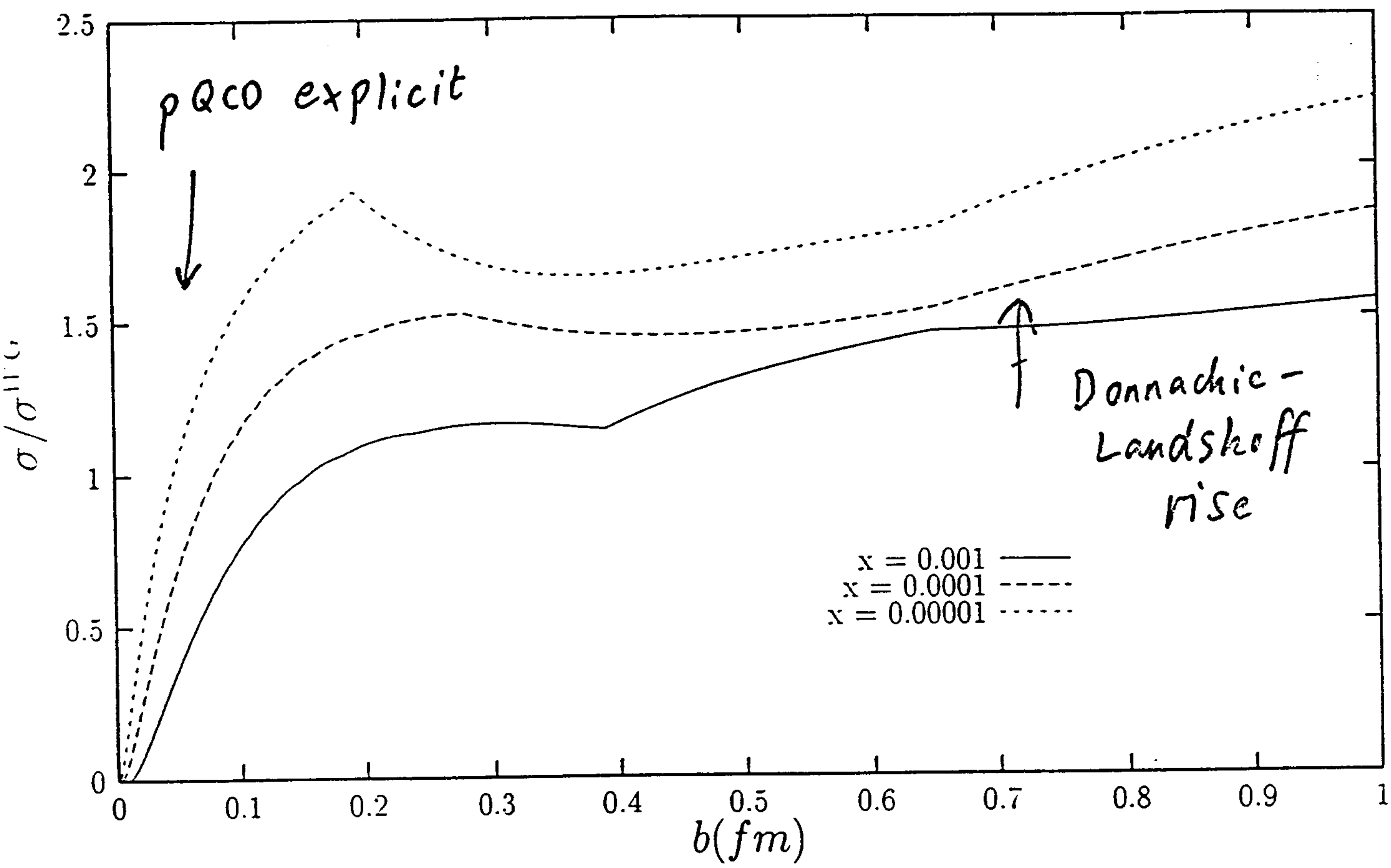
e.g Nucl Phys B539 (1999) 535

"eikonal" model, too naive

$$\sigma \sim (1 - e^{-c b^2 \alpha_s x g})$$

e.g. Eur Phys J C10 (1999) 689

Comparison with Wüsthoff - Golec-Biernat model.



Distinguish in regions where R changes quickly with x.

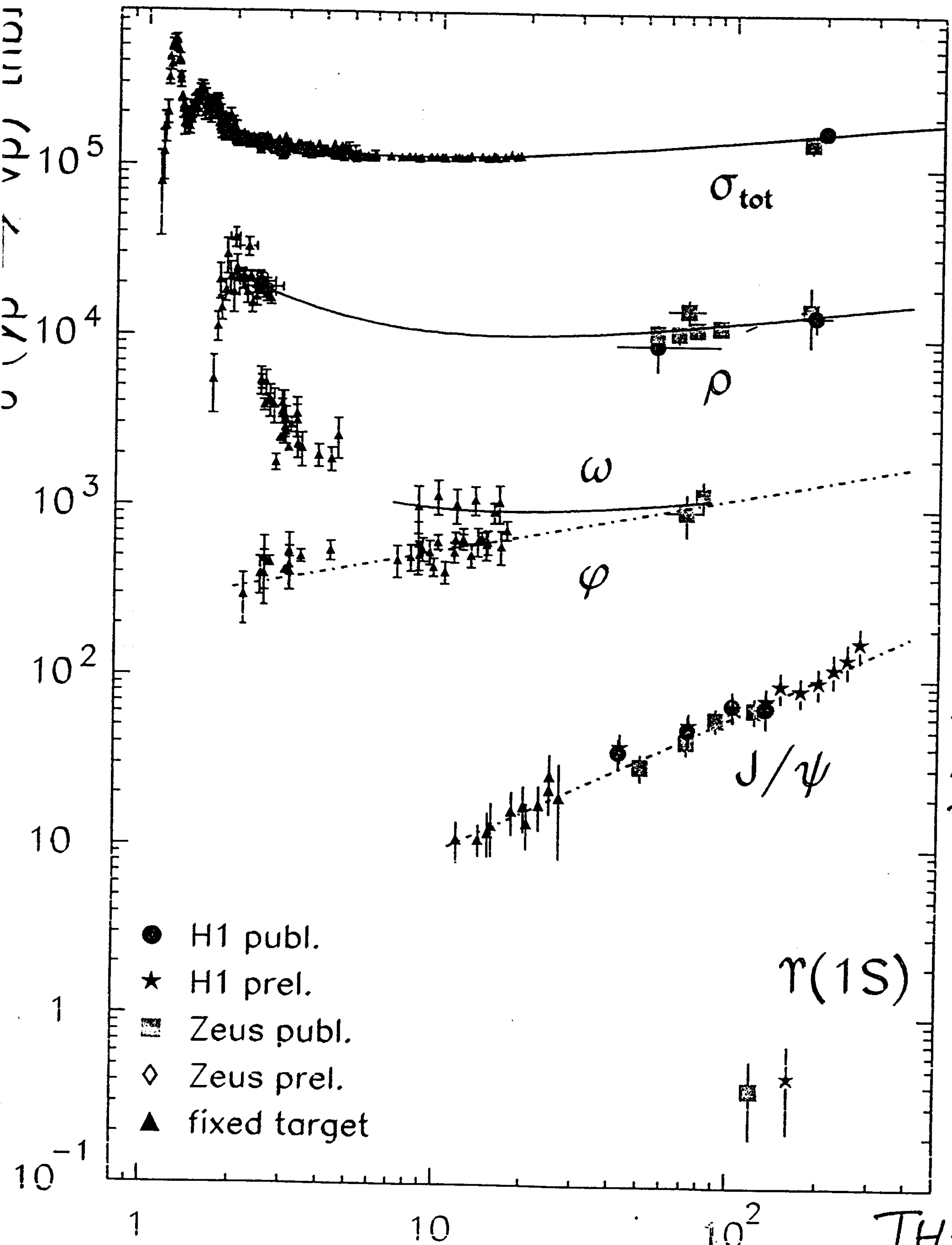
$$\sigma^{WG}(x,b^2) = \sigma_0 \left(1 - e^{-\frac{b^2 Q_0^2}{4(x_0/x)^{\lambda}}}\right)$$

$Q_0^2 \to 1\ \mathrm{GeV}^2$

$x_0 \to 3\times10^{-3}$

$\sigma_0 \to 23\,\mathrm{mb}$

Summary of Vector Meson data in Photoproduction.



$W^{0.32}$

$W^{0.8}$

Also ρ in DIS?

J/ψ in DIS?

THERA?

Photoproduction of J/ψ

Relative light mass $\simeq 10\,GeV^2$
relevant Scale $\simeq 5\,GeV^2$ (Q^2_{eff})
$\langle b \rangle_{J/\psi} \simeq 0.3\,fm$
Can act as precursor to Gribov black limit

$$\sigma(\gamma P \to J/\psi P) = \frac{|A|^2 N^2}{16\pi B_D}$$

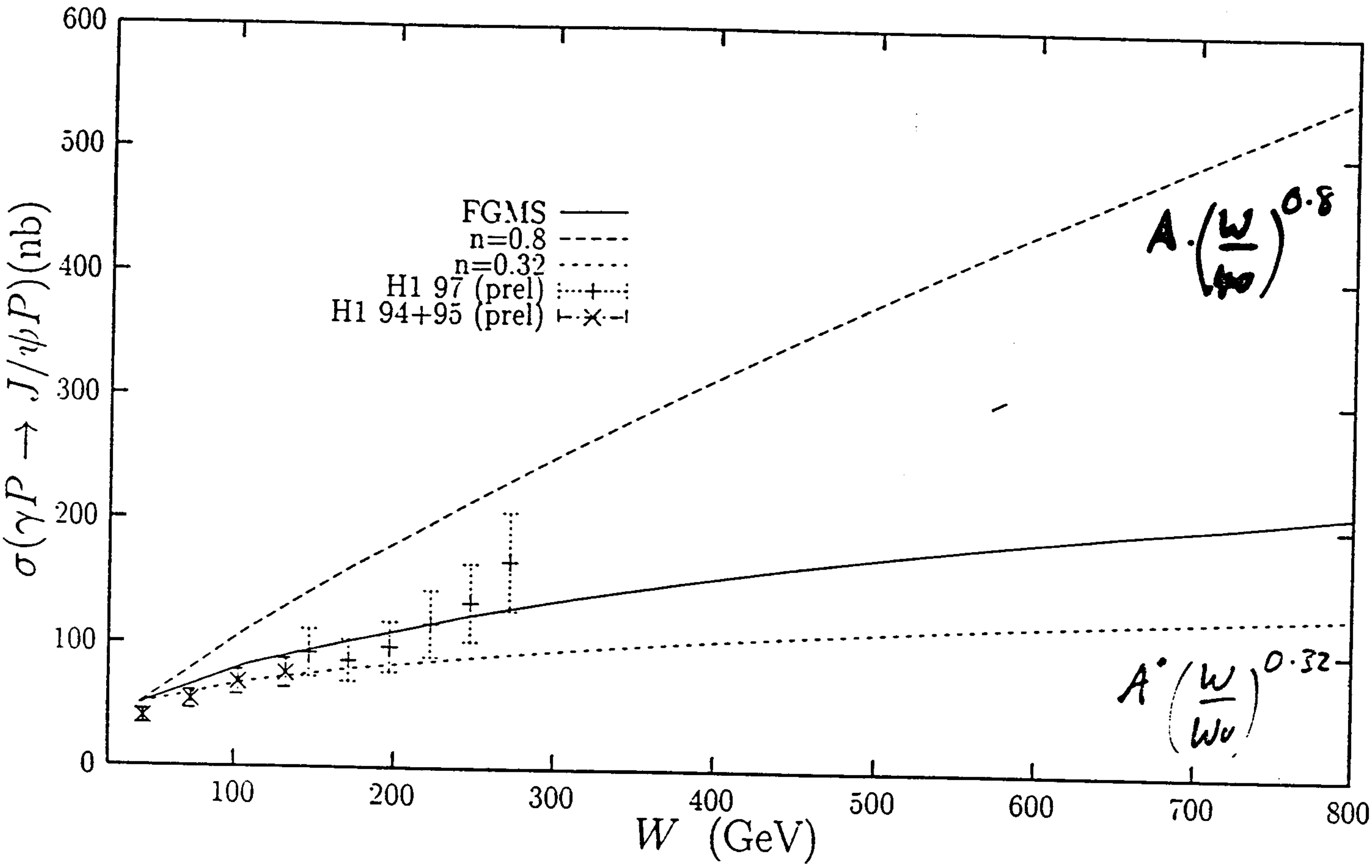
$$A = \int d^2b \int dz \; \underbrace{m_q K_0(bm_q)}_{\gamma^T} \; \hat{\sigma}(b, x') \; \underbrace{\frac{\phi_V(b,z) M_q}{z(1-z)}}_{VM^T}$$

$$N^2 = \frac{2\pi}{3 M_q^2}$$

ϕ_V + hard corrections $\qquad \phi_V \sim a_0\, z(1-z)$ small b
$\hookrightarrow$ norm. to $\Gamma_{e^+e^-}$

$$x'_{BJ} \simeq 1.4\, x_{BJ} = \frac{1.4\, M^2_{J/\psi}}{W^2}$$

Taming of photoproduction of J/ψ. at THERA



↑ HERA. $x_{min} \simeq 10^{-4}$

↑ THERA. Reach. $x_{min} = 10^{-5}$

THERA machine can distinguish between "steep" rise and Donnachie-Landshoff rise.

We predict taming of J/ψ, Υ in γP and γ*P

↳ depends on ⟨b⟩ (Q^2) due to blackening of interactio

Conclusions

- Universal $\hat{\sigma}(x,b)$ small x.

  $\hookrightarrow$ $F_{2,L}$, VM, DVCS, F_2^D

  σ_{tot} . A .

- Small dipole $\hat{\sigma} \propto b^2 \alpha_s xg$

- Match to $\mathbb{P}$ at large b .

- Physical process have contbns. both regions.

- Small enough x $\Rightarrow$ unitarity corrections required.

- Widely applicable and predictive approach.

  eg. $\sigma(\gamma P \to J/\psi P)$ should flatten at large enough W^2. – THERA.

- Small enough x see approach to Gribov black. limit