Higher twists and gluon saturation at THERA

Kirill Tuchin

Tel Aviv University

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Based on paper with : E.Gotsman, E. Levin, U. Maor L.McLerran and K.Tuchin

The general goal:

To find new observables for gluon saturation at THERA

Why higher twists?

1. The first indication of gluon saturation: the leading twist (LT) contribution to the cross section is equal to next-to-leading twist (NLT) contribution:

   LT=NLT

   What is a kinematic region where it happens?

2. Ratios like F_{2L}/F_{2T} and F_{2L}^{D}/F_2 have maximum at $Q^2_{max} \propto Q_s(x)^2$. This is the scale at which all twists give the same contributions.

   How does this maximum depend on x?

   It will be very instructive to study F_{2T}^{D}/F_2 as well.

Outline:

1. Interaction of photon with proton at rest: eikonal approximation;
2. First sign of SC: Extraction of LT and NLT.
3. SC: Maximum $Q^2_{\max}$ of ratios

   $\frac{F_{2L}}{F_{2T}}$, $\frac{F^D_{2L}}{F_2}$ and $\frac{F^D_{2T}}{F_2}$ and their x dependence.
4. Why SC are still small at HERA.
5. **How to measure?**

Our idea and method of higher twist extraction are similar to that of the paper

J. Bartels, K. Golec-Biernat and K. Peters, *"An estimate of higher twist at small x_B and low Q^2 based upon a saturation model."* (BGP)

Unlike BGP we:

- Calculate contributions of longitudinal and transverse photons to the diffraction dissociation cross section
- Our formula match the DGLAP evolution equations at large Q^2
- Take care of impact parameter distribution, which allows a generalization to DIS off nuclei

The Model:

1. Interaction of photon with the proton at rest (Gribov,1970)

   (a) development of the photon dipole wave function $|\Psi(z,r)|^2$;

   (b) interaction of each dipole with proton $\sigma_{\rm dipole}(x,r,b)$;

Cross section:

$$\sigma(x,Q^2) = \int dz\, d^2r\, d^2b |\Psi(z,r)|^2\, \sigma_{\rm dipole}(x,r,b)$$

2. Account of Shadowing Corrections (SC): Mueller-Glauber formula (A. Mueller, 1994)

   - for $\sigma(x,Q^2)_{\rm total}$:

     $$\sigma_{\rm dipole}(x,r,b) = 2\left(1 - e^{-\Omega(x,r,b)/2}\right)$$

   - for $\sigma(x,Q^2)_{\rm diffr.}$:

     $$\sigma_{\rm dipole}(x,r,b) = \left(1 - e^{-\Omega(x,r,b)/2}\right)^2$$

3. $$\Omega(x, r^2, b) = \kappa(x, r^2) \cdot S(b^2)$$

$$\kappa(x, r^2) = \frac{\alpha_S(4/r^2)\pi r^2}{6R^2} xG^{DGLAP}(x, \tfrac{4}{r^2})$$

Assume:

4. **Gaussian profile function:** $S(b^2) = e^{-\frac{b^2}{R^2}}$

5. $$xG(x, \tfrac{4}{r^2}) \approx xG(x, Q^2)$$

Final result of integrations:

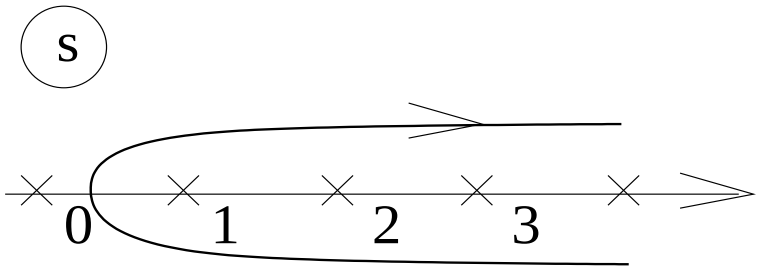
$$F_{2L}(x,Q^2) = \frac{\alpha_S(Q^2)\, xG(x,Q^2)}{24} \cdot G^{32}_{34}\left(\kappa^{-1}(x,Q^2)\,\middle|\,{}^{0,1;\frac{5}{2}}_{1,1,2;-\frac{1}{2}}\right)$$

G is a Meijer function: generalization of the hypergeometric one.

$$G^{32}_{34}\left(\kappa^{-1}(x,Q^2)\,\middle|\,{}^{0,1;\frac{5}{2}}_{1,1,2;-\frac{1}{2}}\right) =$$

$$\frac{1}{2\pi i}\int ds\,\kappa^{-s}\,\frac{\Gamma(s)\Gamma(1+s)\Gamma(1-s)^2\Gamma(2-s)}{\Gamma(\frac{5}{2}-s)\Gamma(\frac{3}{2}+s)}$$

Integration contour:



$$F_{2T}(x,Q^2) = \frac{\alpha_S(Q^2)\, xG(x,Q^2)}{12} \times$$

$$\left[G^{32}_{34}\left(\kappa^{-1}\,\middle|\,{}^{1,1;\frac{5}{2}}_{1,1,2;\frac{1}{2}}\right) - \frac{1}{2}G^{32}_{34}\left(\kappa^{-1}\,\middle|\,{}^{0,1;\frac{5}{2}}_{1,1,2;-\frac{1}{2}}\right) + \right.$$

$$\left.\kappa\left\{\frac{1}{4}G^{32}_{34}\left(\kappa^{-1}\,\middle|\,{}^{1,1;\frac{7}{2}}_{2,2,2;\frac{1}{2}}\right) - \frac{1}{2}G^{32}_{34}\left(\kappa^{-1}\,\middle|\,{}^{1,2;\frac{7}{2}}_{2,2,2;\frac{3}{2}}\right)\right\}\right]$$

$$F^D_{2L/T}(\kappa^{-1}) = F_{2L/T}(\kappa^{-1}) - F_{2L/T}\left(\frac{\kappa^{-1}}{2}\right)$$

Extraction of different twists

- Closing the contour over the left poles gives series in Q^{-2}
  $\Rightarrow$ contribution of any twist

- Closing the contour over the right poles gives series in Q^2
  $\Rightarrow$ behavior at small photon virtualities.

Example:

- **Leading Twist** contribution to F_{2L} is:

$$\frac{\alpha_S(Q^2)\, xG(x,Q^2)}{12} \cdot \frac{8}{3\pi}$$

- **Next-To-Leading Twist** contribution to F_{2L} is:

$$-\frac{\alpha_S(Q^2)\, xG(x,Q^2)}{12} \left(\frac{8(17 + 30\gamma + 30\ln(\kappa^{-1}(x,Q^2))}{225\pi} \right) \kappa(x,Q^2)$$

- **Small Q^2** contribution to F_{2L} is:

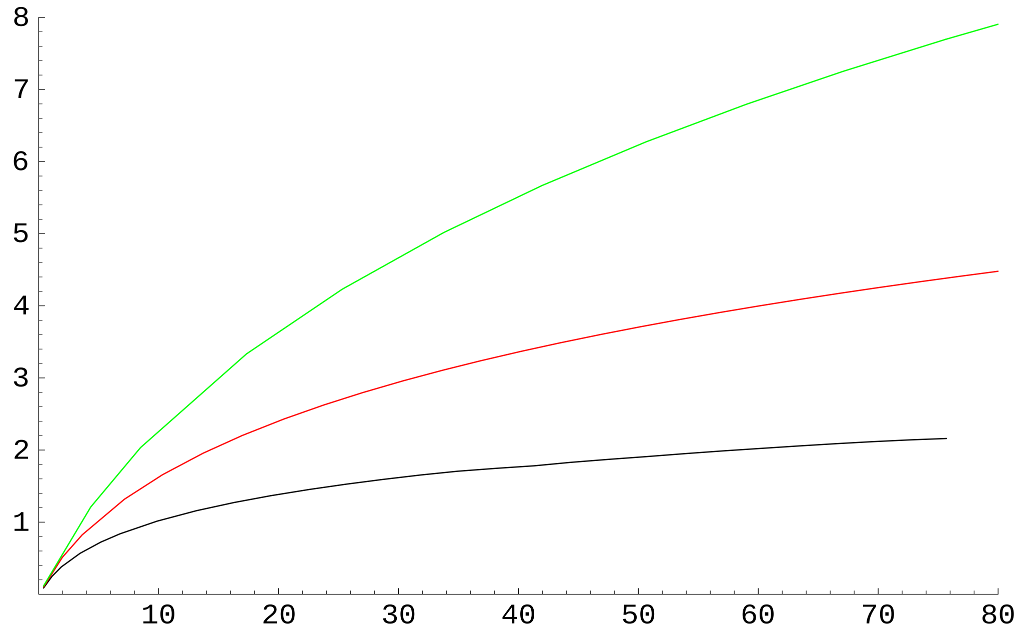
$$\frac{\alpha_S(Q^2)\, xG(x,Q^2)}{12} \left(\frac{8\kappa^{-1}(x,Q^2)(1 + 3\gamma + 3\ln \kappa^{-1}(x,Q^2)}{9\pi} \right)$$

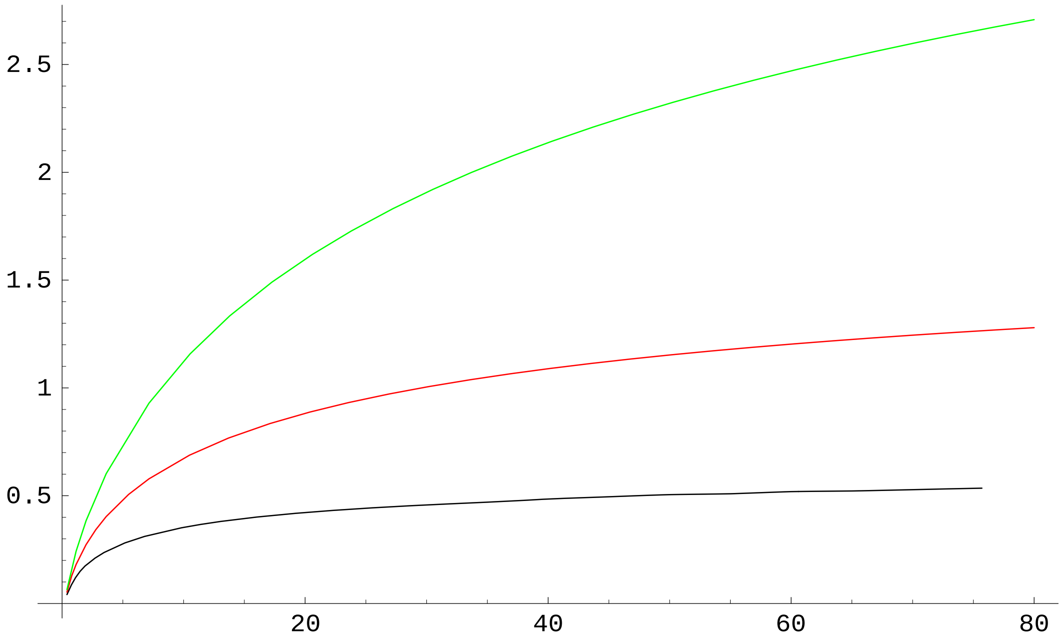
$$\kappa(x,Q^2) = \frac{2\alpha_S(Q^2)\pi}{3Q^2R^2} xG^{DGLAP}(x,Q^2)$$

Numerical results

F_{2T} and F_{2L} for

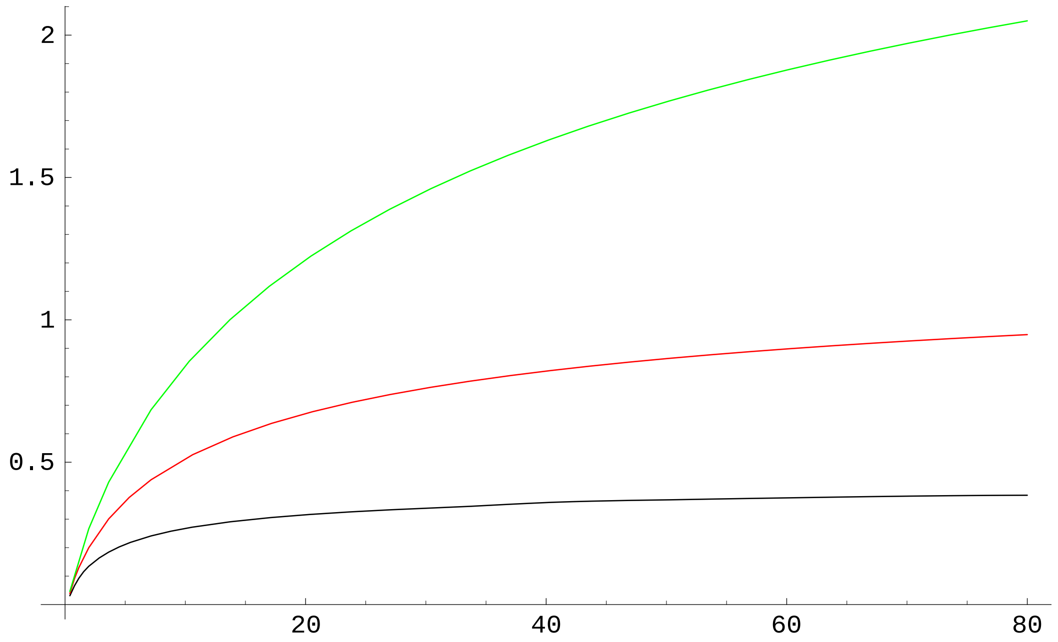
$x = 10^{-4}$ $x = 10^{-5}$ $x = 10^{-6}$

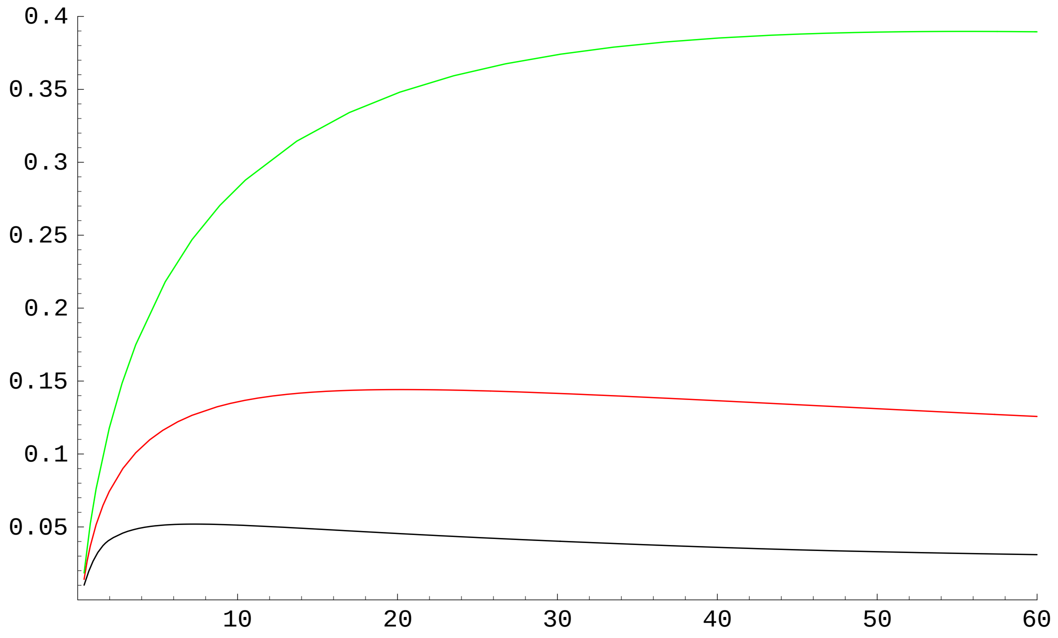




F_{2T}^{D} and F_{2L}^{D} for

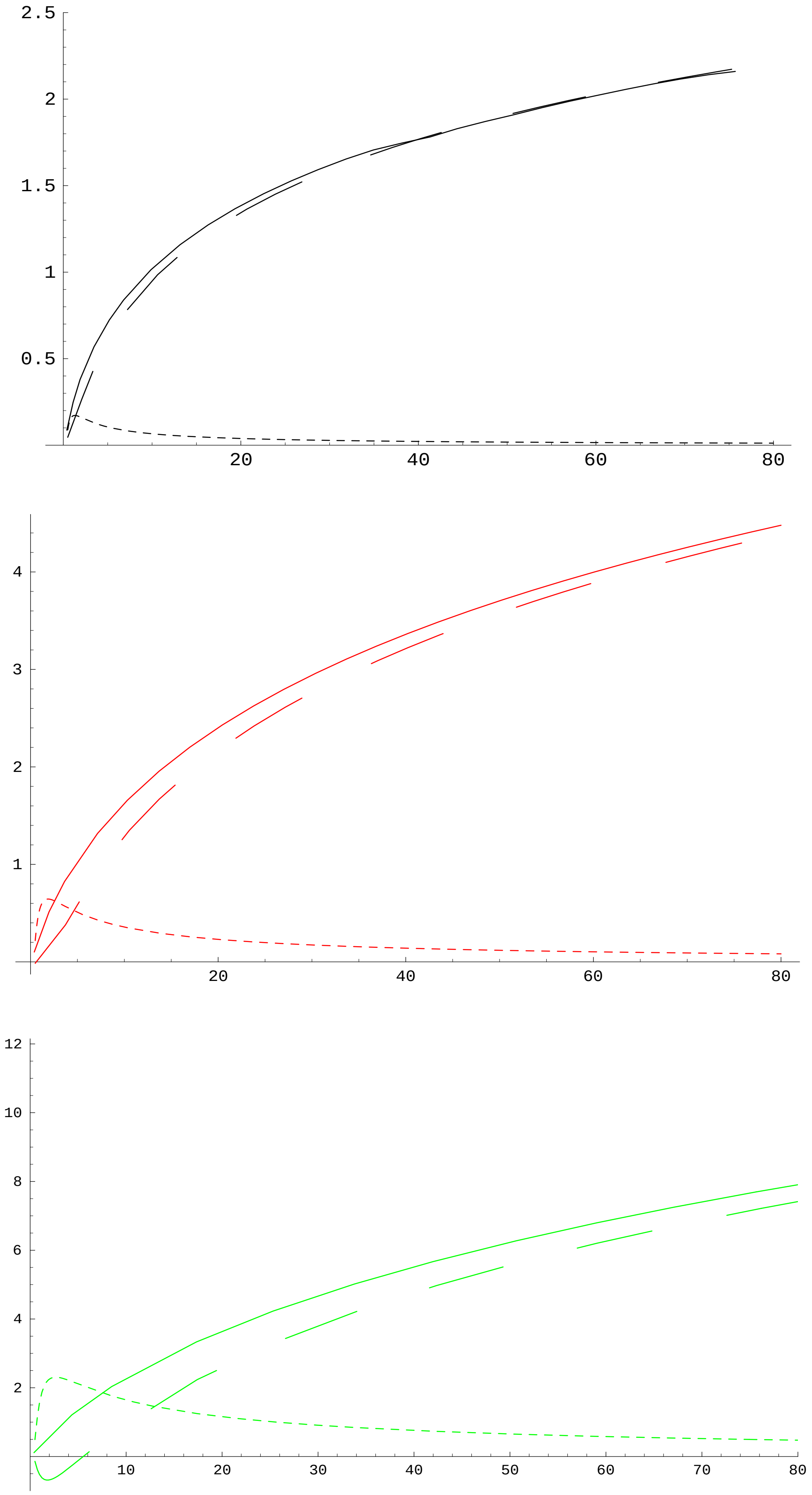
$x = 10^{-4}$ $x = 10^{-5}$ $x = 10^{-6}$





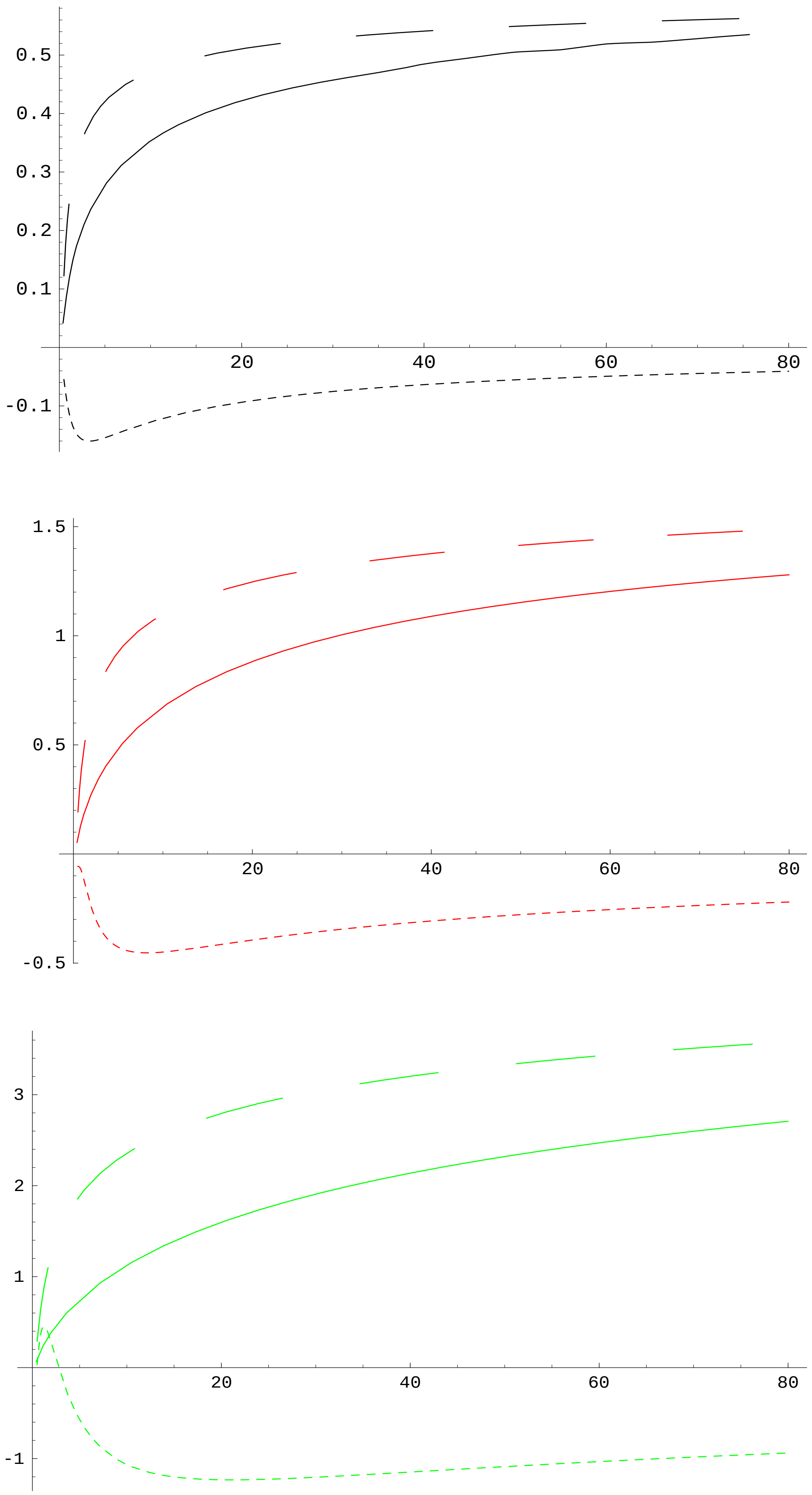
Contributions of different twists to F_{2T} for

$x = 10^{-4}$ $x = 10^{-5}$ $x = 10^{-6}$



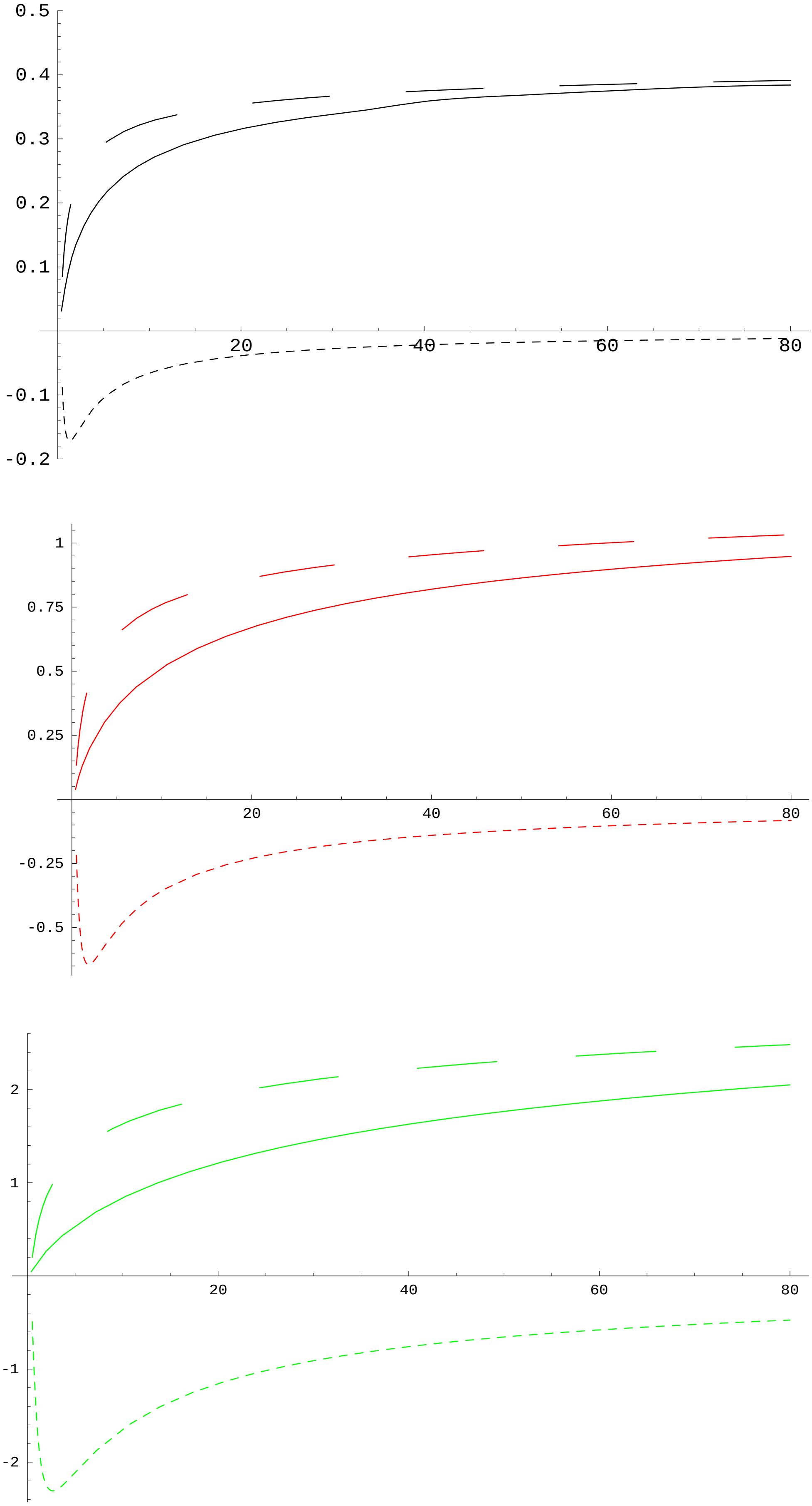
Contributions of different twists to F_{2L} for

$x = 10^{-4}$ $x = 10^{-5}$ $x = 10^{-6}$



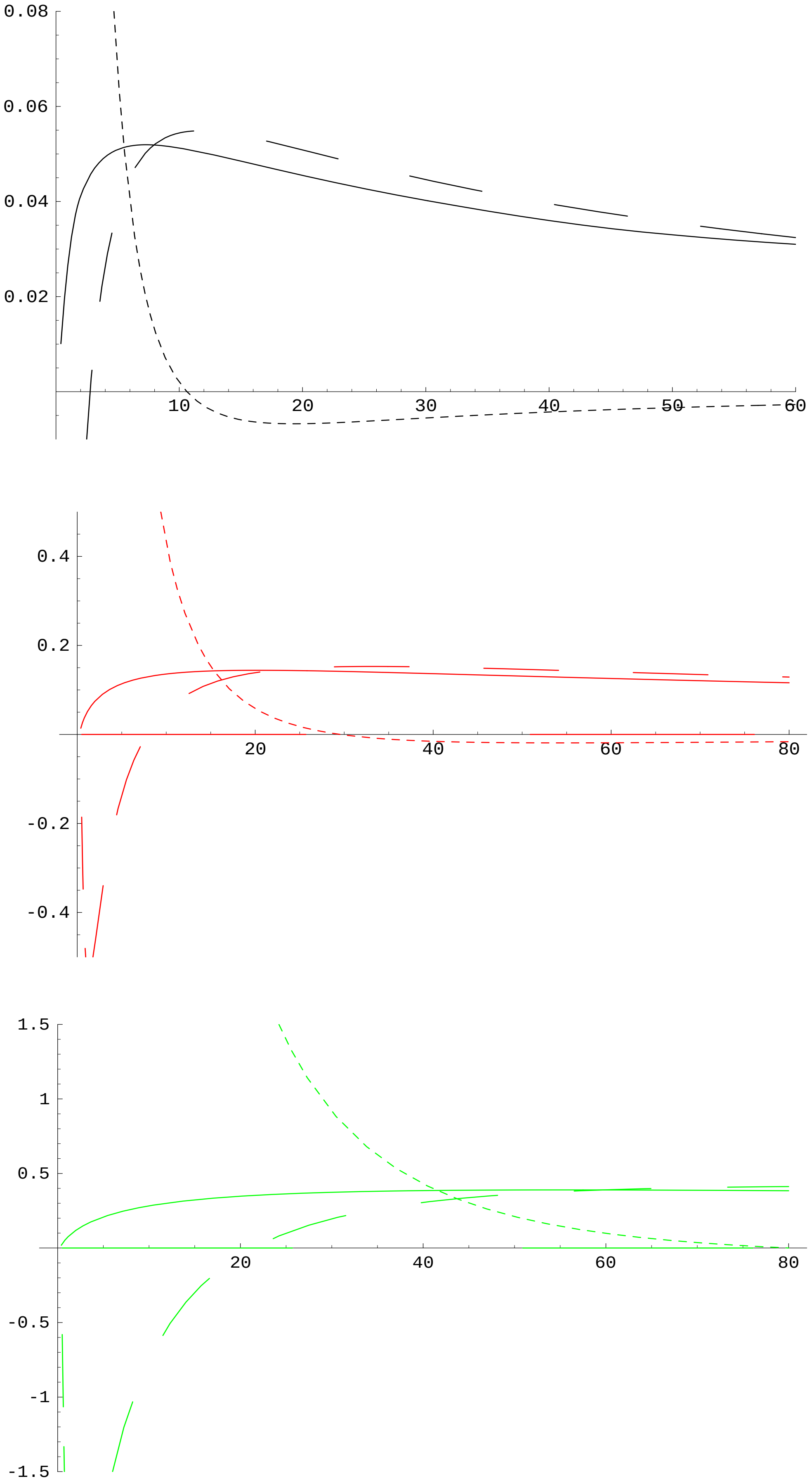
Contributions of different twists to F_{2T}^{D} for

$x = 10^{-4}$ $\quad$ $x = 10^{-5}$ $\quad$ $x = 10^{-6}$



Contributions of different twists to F_{2L}^{D} for

$x = 10^{-4}$ $x = 10^{-5}$ $x = 10^{-6}$



Ratios of structure functions

We expect to see the maximum in such ratios as

$$F_{2L}/F_{2T} \text{ and } F_{2L}^D/F_2$$

Indeed:

- the real photon $(Q^2 = 0)$ is transverse $\Rightarrow F_{2L}(Q^2 = 0) = 0$
- $\frac{F_{2L}}{F_{2T}}$, $\frac{F_{2L}^D}{F_2} \rightarrow 0$ as $Q^2 \rightarrow \infty$;

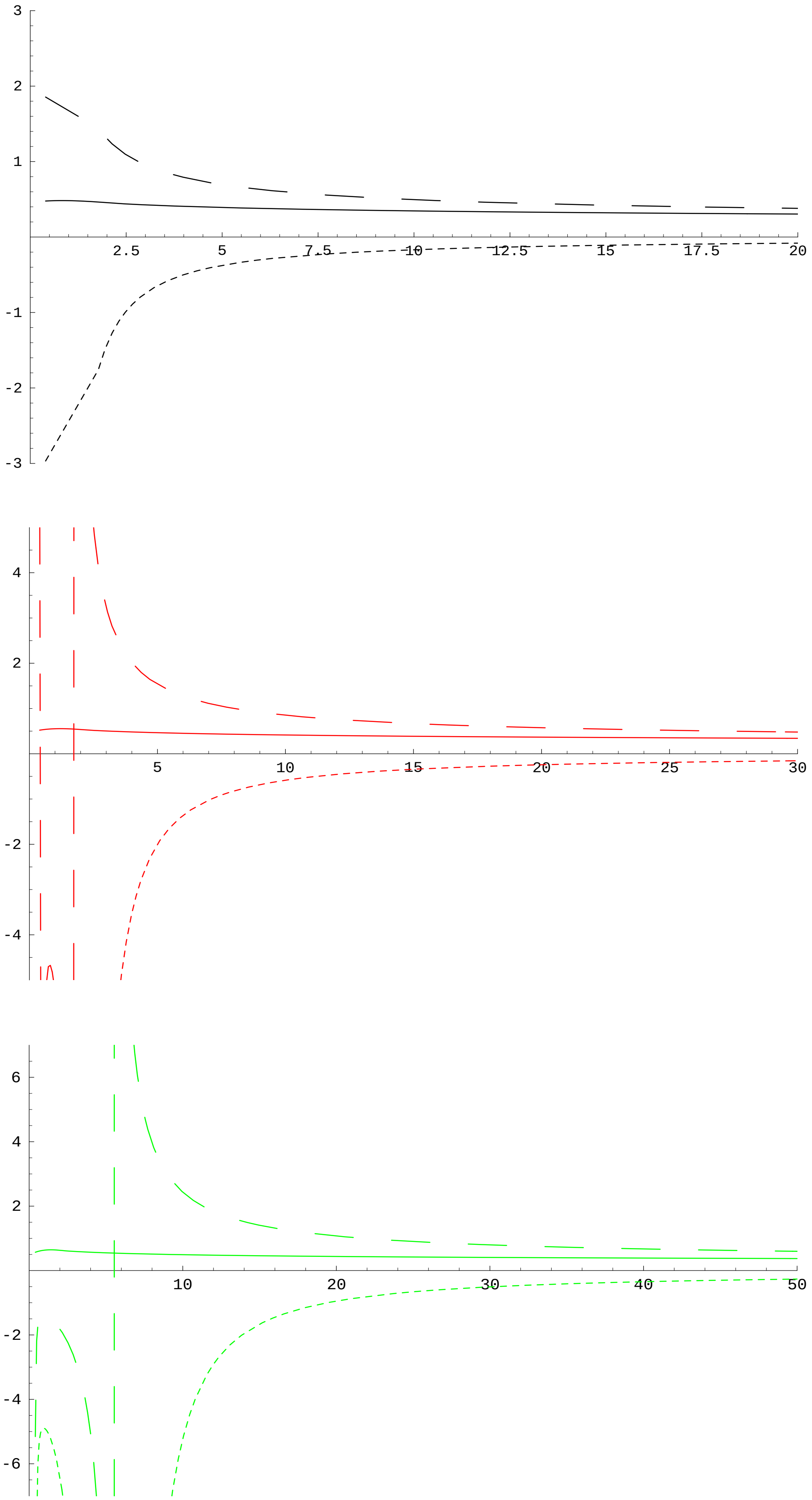
Two scales can be responsible for the maximum:

1. quark mass m_q^2
2. Saturation Scale $Q_s^2(x)$

Important signature of $Q_s^2(x)$: dependence on x

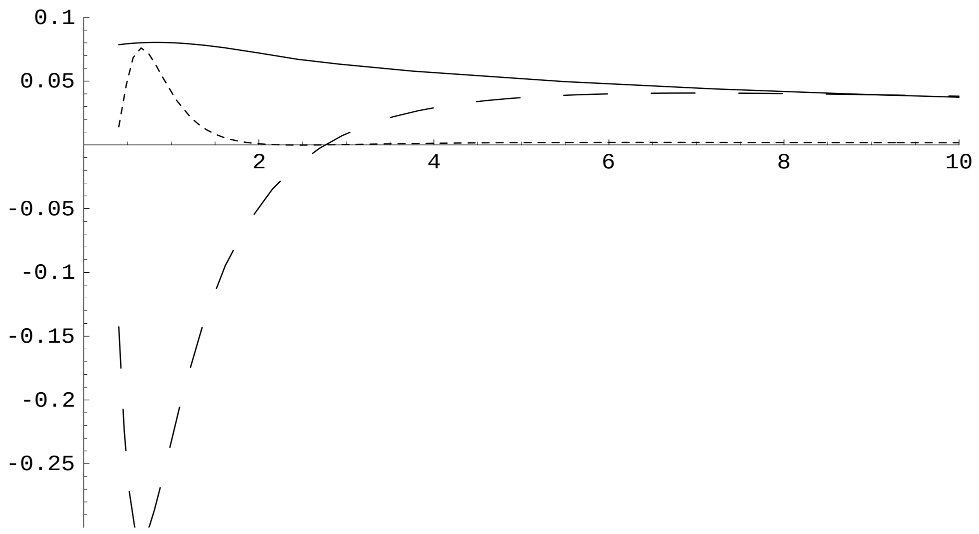
Contributions of different twists to F_{2L}/F_{2T} for

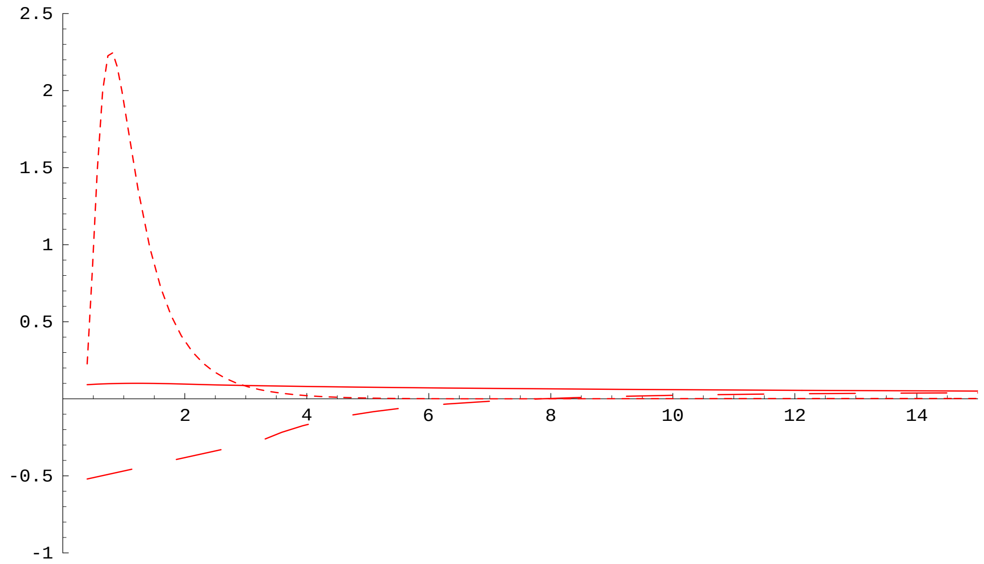
$x = 10^{-4}$ $x = 10^{-5}$ $x = 10^{-6}$

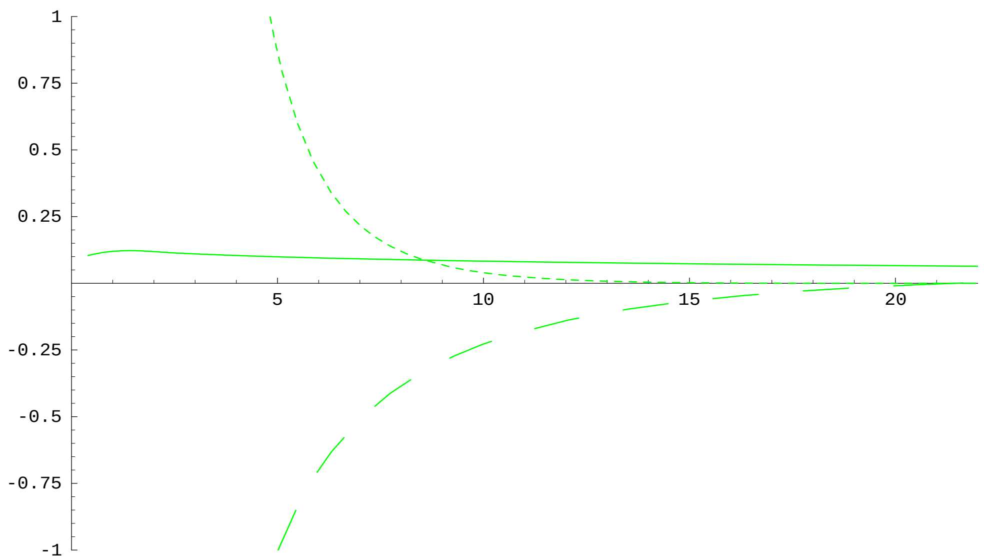


Contributions of different twists to F_{2L}^{D}/F_2 for

$x = 10^{-4}$ $\quad$ $x = 10^{-5}$ $\quad$ $x = 10^{-6}$







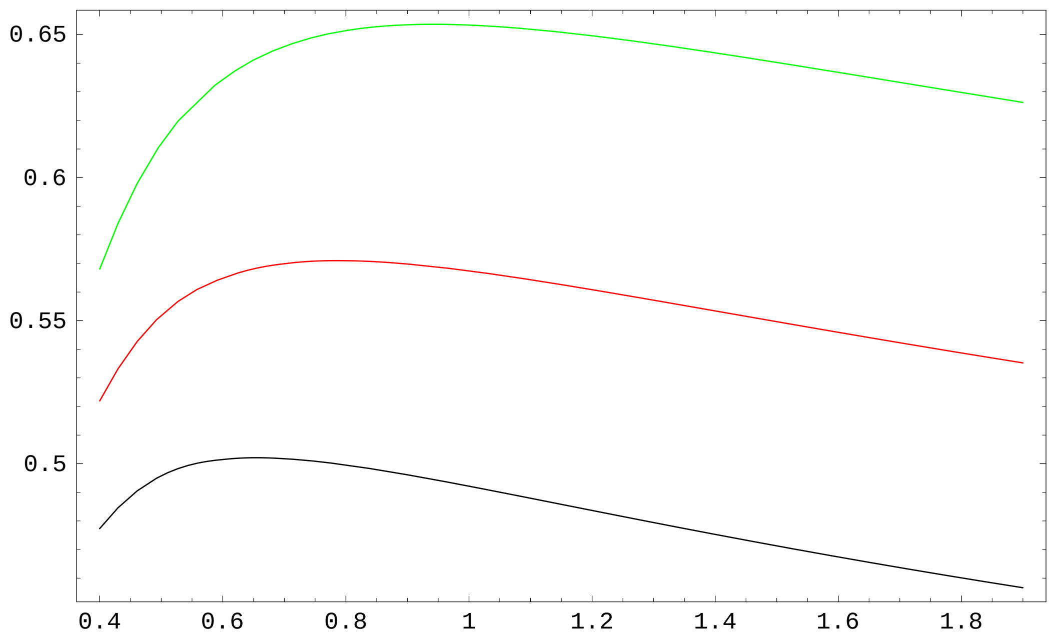
The maximum occurs

1. at $Q^2_{max}(x) \gg m_q^2$

2. $Q^2_{max}(x)$ depends on x

3. $Q^2_{max}(x) \propto Q_s^2(x)$

4. at the same Q^2_{max} for two ratios F_{2L}/F_{2T} and F^D_{2L}/F_2

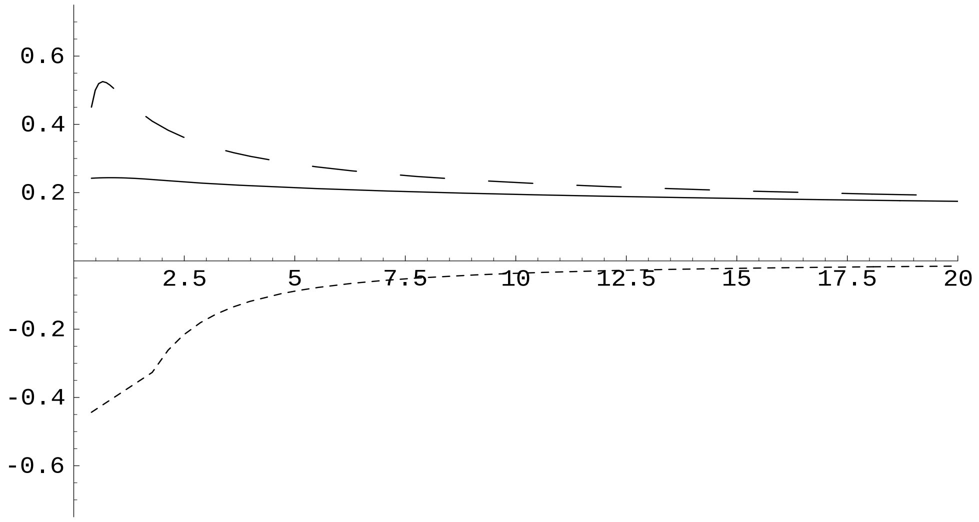


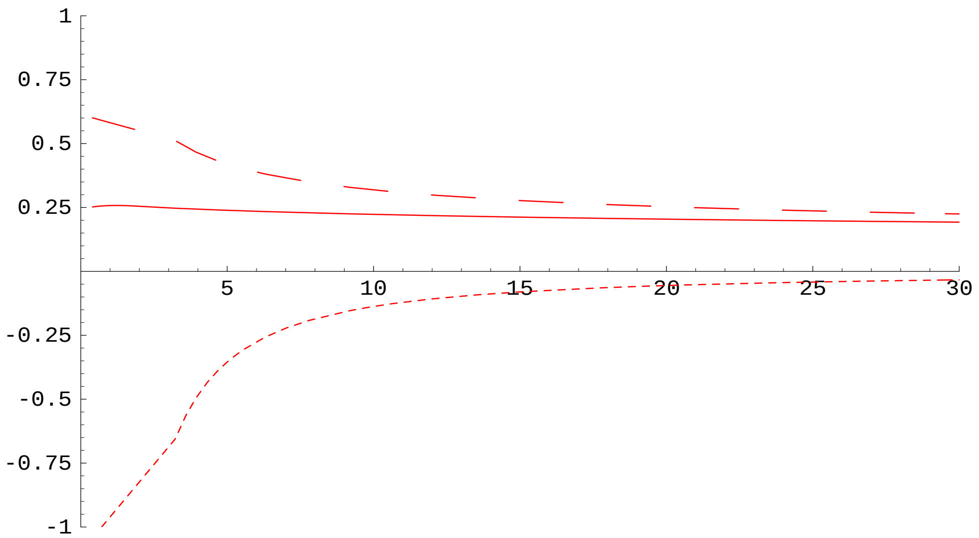
There is also maximum in F^D_{2T}/F_2 at the same $Q^2_{max}(x)$!

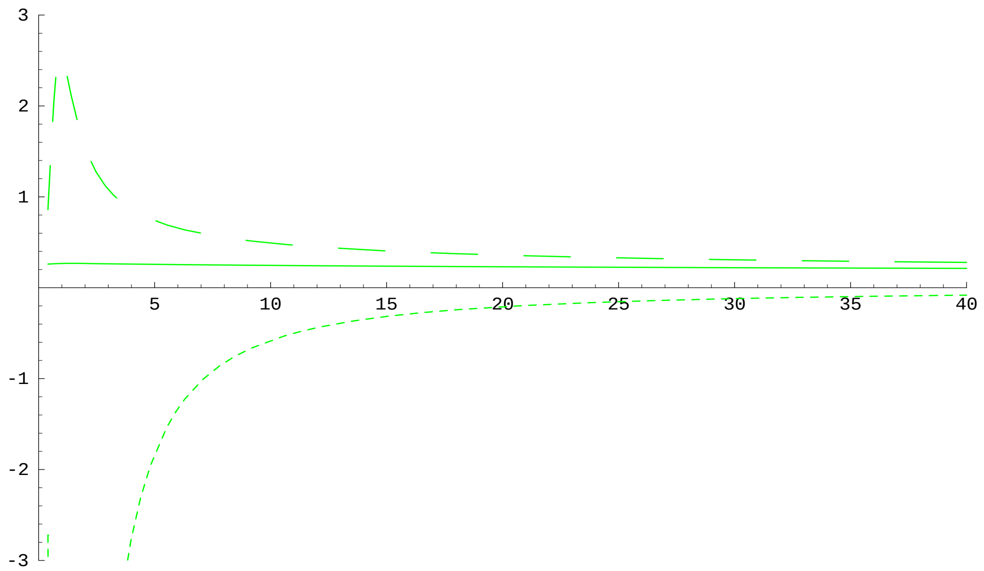
Is this result the artifact of the model?

Contributions of different twists to F_{2T}^{D}/F_2 for

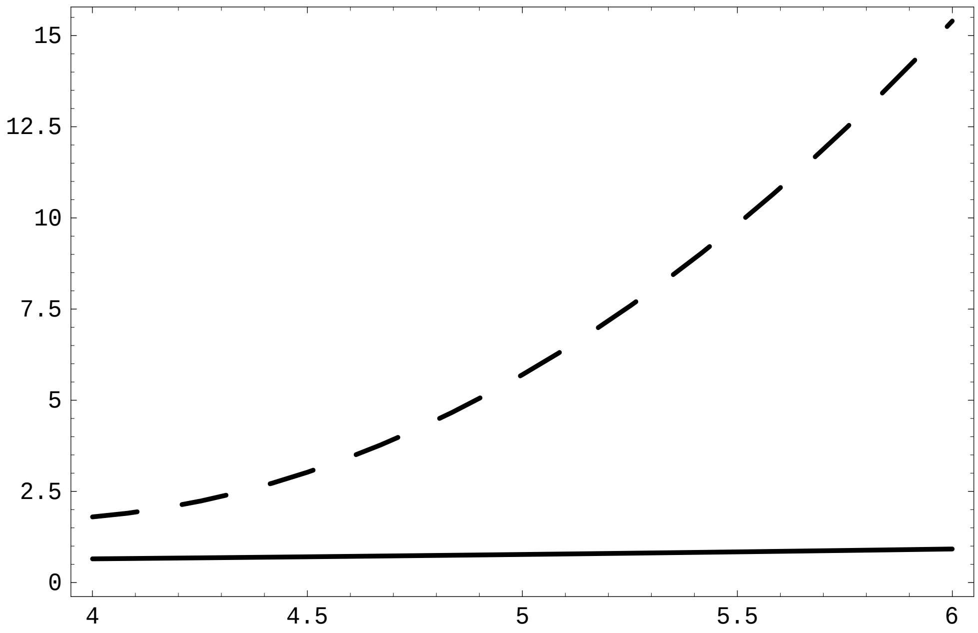
$x = 10^{-4}$ $x = 10^{-5}$ $x = 10^{-6}$







Saturation Region



- dashed line – solution of equation $\kappa(x, Q_s^2(x)) = 1$ for $Q_s^2(x)$
- solid line – $Q_{max}^2(x)$ from figures

Perhaps it is the explanation of HERA puzzle.

To evaluate $Q_s(x)$ analytically with better accuracy one need to solve high density evolution equation.

Conclusions

- Measurements of ratios of structure functions versus photon virtuality Q^2 is an important tool in investigation of Shadowing Corrections. It allows to detect a Saturation Scale.

- At $x = 10^{-6}$ higher twists start to be essential in:

  F_{2L}/F_{2T} and F_{2T}^D/F_2 at $Q^2 \sim 15$ GeV^2;

  F_{2L}^D/F_2 at $Q^2 \sim 20$ GeV^2

  While at $x = 10^{-5}$ this scale is at 5–7 GeV^2

  It gives a hope to detect them.

- We found x dependence of Q^2_{max}.
  For $x = 10^{-6}$ it occurs at $Q^2 \sim 1$ GeV^2 and could be measured at THERA, contrary to $x = 10^{-4}$ where it ~ 0.6 GeV^2.

- For $\gamma^* A$ reactions: we investigate the A-dependence of $Q^2_{max}(x)$ and hope that it will yield us with a tool to distinguish HT from LT

- The question *how to detect higher twist contributions* is still under discussion