Two Loop Electroweak Corrections to Δr

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- intro
- matching and factorization theorem
- calculation
- conclusions

Intro

Precision test of the Standard Model (SM)

Accurately measured quantities

$\alpha^{-1} = 137.03599976(50)$	(fine structure constant)
$M_W = 80.423(39)$ GeV	(W-boson mass)
$M_Z = 91.1876(21)$ GeV	(Z-boson mass)
$m_\mu = 105.658357(5)$ MeV	(muon mass)
$\tau_\mu = 2.19703(4) \cdot 10^{-6}$ s	(muon lifetime)

There is a relation between these parameters in the SM!

$$\boxed{\frac{1}{\tau_\mu} = \frac{\alpha^2 m_\mu^5}{384\pi M_W^4 (1 - M_W^2/M_Z^2)^2}}$$

quantum corrections (?)

in future — increasing experimental accuracy:

$$M_W,\ M_t \longleftarrow \text{LHC, TESLA}$$

$$\tau_\mu \longleftarrow \text{PSI (?), Brookhaven (?)}$$

$\Longrightarrow$ need to compute τ_μ beyond one loop!

Today:

- $O(\alpha^2)$ top-quark correction
- $O(\alpha^2)$ light fermionic correction
- $O(\alpha^2)$ leading large Higgs boson mass correction
- partly $O(\alpha^2)$ bosonic correction
- up to $O(\alpha\alpha_s^2)$ QCD corrections
- two loop bosonic correction is missing!

Computation of process $\mu \longrightarrow e\bar{\nu}_e\nu_\mu$

- too many scales from ~ 1 MeV to > 100 GeV — very bad!

- $\left.\begin{array}{l}\sim 50 \text{ diagrams at 1-loop} \\ \sim 5000 \text{ diagrams at 2-loop}\end{array}\right\}$ today not a problem

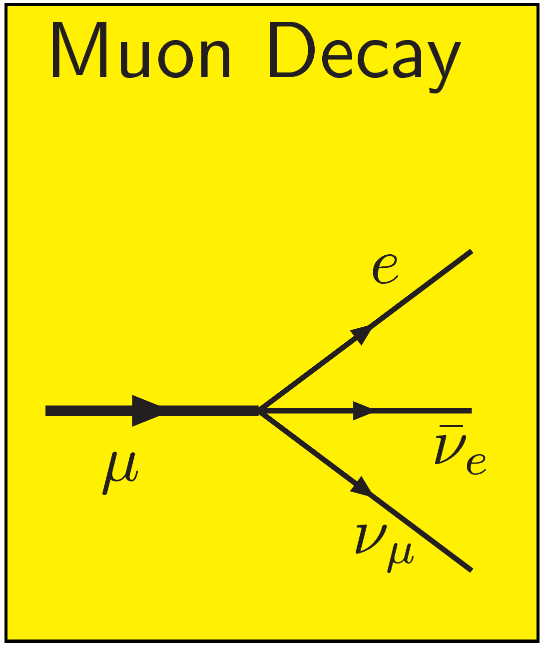
- complicated phase space integrals — too bad!

$\Longrightarrow$ direct computation is not feasible

therefore: low energy effective theory (Fermi theory)

factorization of long and short distance dynamics

Fermi Lagrangian



$$\mathcal{L} = \mathcal{L}_{\mathrm{QED}} + \frac{G_F}{\sqrt{2}}[\bar{\nu}_\mu \gamma^\alpha (1-\gamma_5)\mu][\bar{e}\gamma_\alpha(1-\gamma_5)\nu_e]$$

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}(1+\Delta q)$$

S.M. Berman

T. Kinoshita, A. Sirlin

R. Stuart, T. van Ritbergen

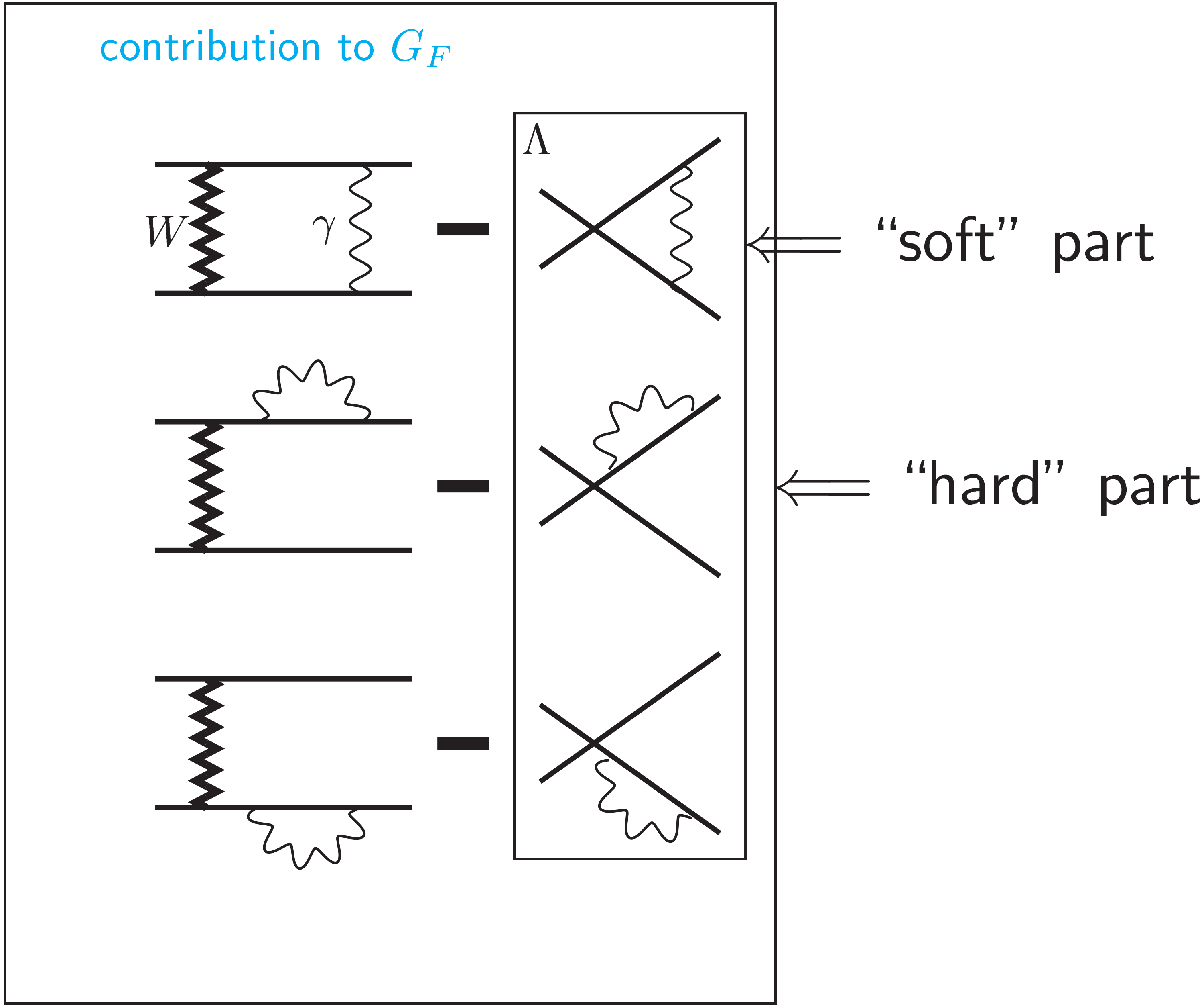
G_F is a Wilson coefficient!

Matching in the Standard Model:

$$\frac{G_F}{\sqrt{2}} = \frac{\alpha\pi}{2(1-M_W^2/M_Z^2)M_W^2}(1+\Delta r)$$

$\Delta r = \Delta r(\alpha, M_W, M_Z, M_H, \ldots)$ appears due to quantum corrections

Subtraction of the soft part with Pauli–Villars regularization



Separation of "soft" and "hard" dynamics

At the level of a single diagramm $\Longrightarrow$ asymptotic expansion

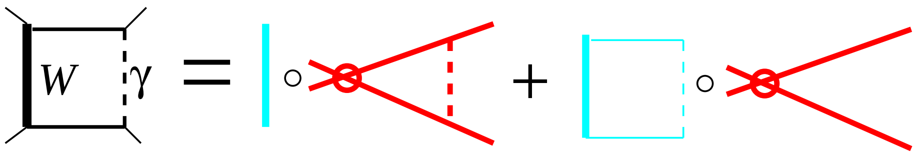
F. Tkachov

K. Chetyrkin

V. Smirnov

$$\boxed{F \overset{\text{as}}{\sim} \sum_{H} H \circ S}$$

example:



At the level of S-matrix $\Longrightarrow$ "integrate out" "hard" modes

1. matching $\longrightarrow$ matching equations
2. factorization theorem based on S.G. Gorishnii

- special structure of asymptotic expansion
- combinatorial properties of R-operation

How it works

Standard Model

$$\left\langle e^{\int \mathcal{L}_{\rm SM}+j\phi} \right\rangle$$

integrating out
heavy modes

Effective Theory

$$\left\langle e^{\int \sum C_i O_i + j\phi_{\rm eff}} \right\rangle$$

Wick's
theorem

R-operation

sum of
Feynman diagrams

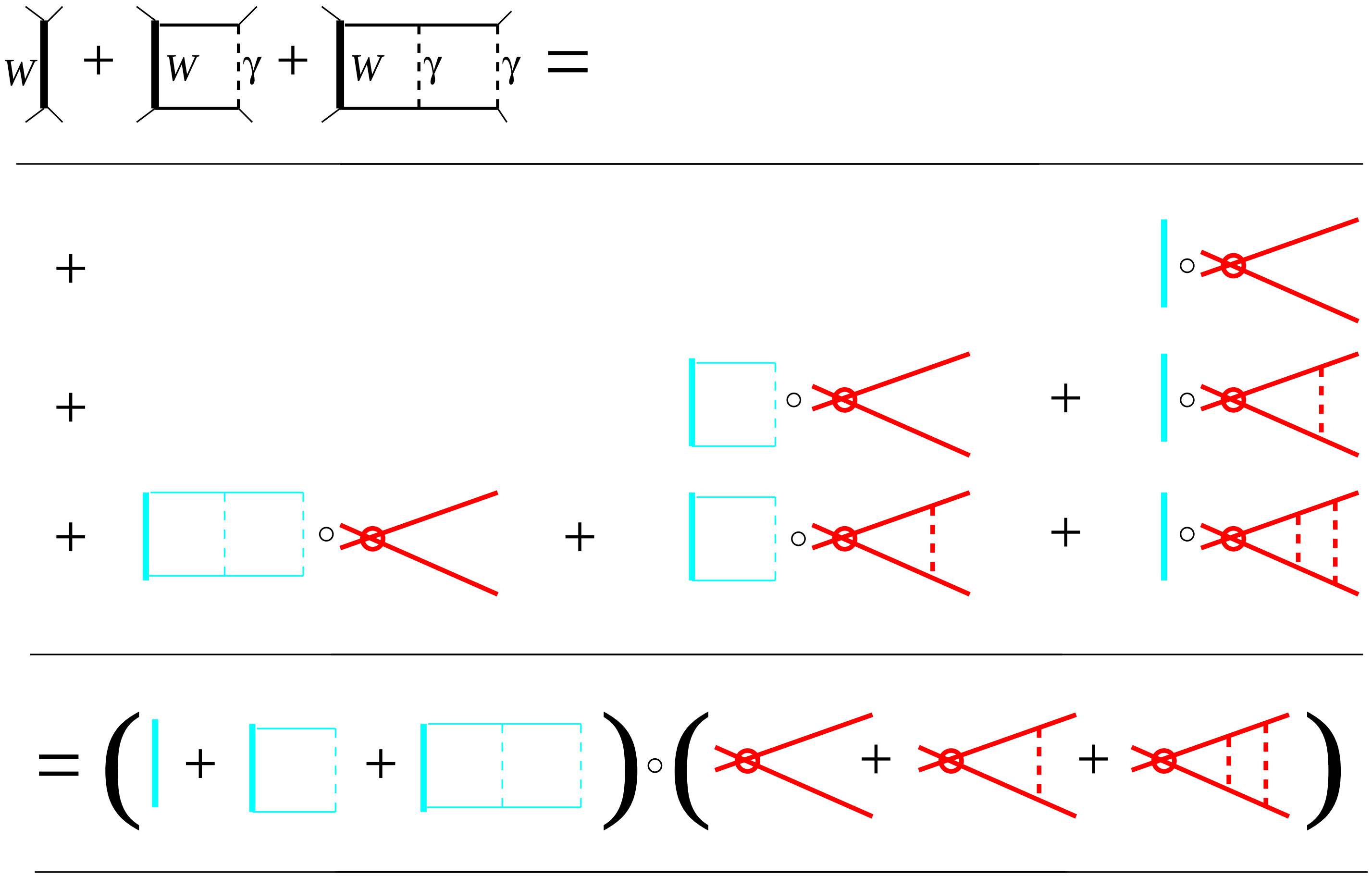
$$\sum_{\rm loops} F$$

asymptotic
expansion

sum of F.D.
sum of subgraphs

$$\sum_{\rm loops} \sum_{H} H \circ S$$

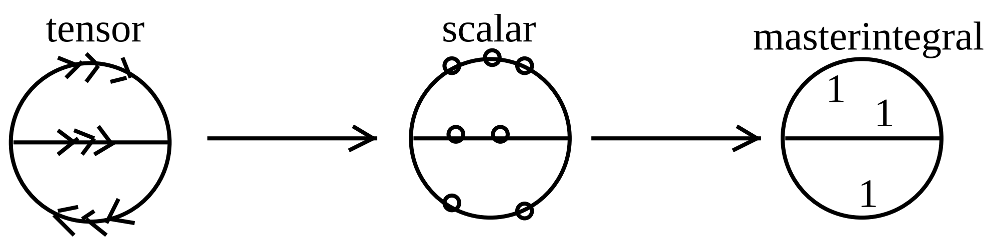
Factorization theorem (illustration)



$$= C(\Lambda) \circ \langle O \rangle_\Lambda + \ldots$$

Calculation

- R_ξ gauge with 3 gauge parameters ξ_W, ξ_Z and ξ_γ
- generation of the input $\Longrightarrow$ DIANA M.Tentyukov, J. Fleischer
- analytical evaluation of bubble diagrams



- $\overline{\mathrm{MS}}$ renormalization
- on-shell integrals for mass counterterms $\Longrightarrow$ expansions
  $\Longrightarrow$ ONSHELL2 M.Kalmykov, J. Fleischer
- transformation to on-shell scheme

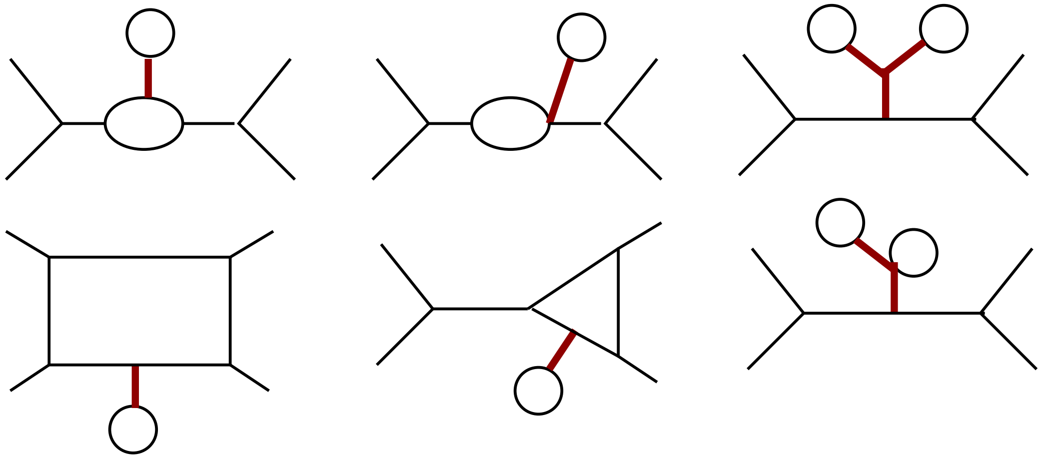
Gauge invariance and tadpoles

Gauge invariant renormalization:

- gauge invariant bare expressions
- $\Longrightarrow$ gauge invariant counterterms

$\Big\}$ in order to have this the tadpoles should be added

Example



Renormalization

- charge $e_0 = \mu^\varepsilon Z_e e(\mu)$
- masses m_Z, m_W, m_H $m_0^2 = Z_m m^2(\mu)$
- no ghost sector or gauge parameter renormaliztion !!!
- no boson field renormalization !!!

$Z_{m_Z}, Z_{m_W}, Z_{m_H} \longleftarrow$ F.Jegerlehner, M.Kalmykov, O.V.

$Z_e \longleftarrow$ renormalization group equation

$$0 = \frac{d}{d\log\mu^2} e_0 = \frac{d}{d\log\mu^2}\Big(\mu^\varepsilon Z_e e(\mu)\Big)$$

$$\frac{d}{d\log\mu^2} e(\mu) = -\frac{\varepsilon}{2} e(\mu) + \beta_e$$

β-function β_e is known to two loops D.R.T.Jones
M.E.Machacek, M.T.Vaughn

$$Z_e = 1 + \frac{e^2}{4\pi}\left(-\frac{7}{2\varepsilon}\right) + \left(\frac{e^2}{4\pi}\right)^2\left(\frac{147}{8\varepsilon^2} + \frac{1}{\varepsilon}\left(-\frac{125}{12\sin^2\theta_W} + \frac{1}{4\cos^2\theta_W}\right)\right)$$

Expansions

Three parameters: $m_Z,\, m_W,\, m_H$ too many to keep them all !

(we can but why???)

1. Small parameter: $\sin^2\theta_W = \frac{m_Z^2 - m_W^2}{m_Z^2} \sim 0.23$ good!

   expansion in $\sin^2\theta_W$ is very simple $\longrightarrow$ Taylor expansion

2. Two regimes of Higgs boson mass. Expand when

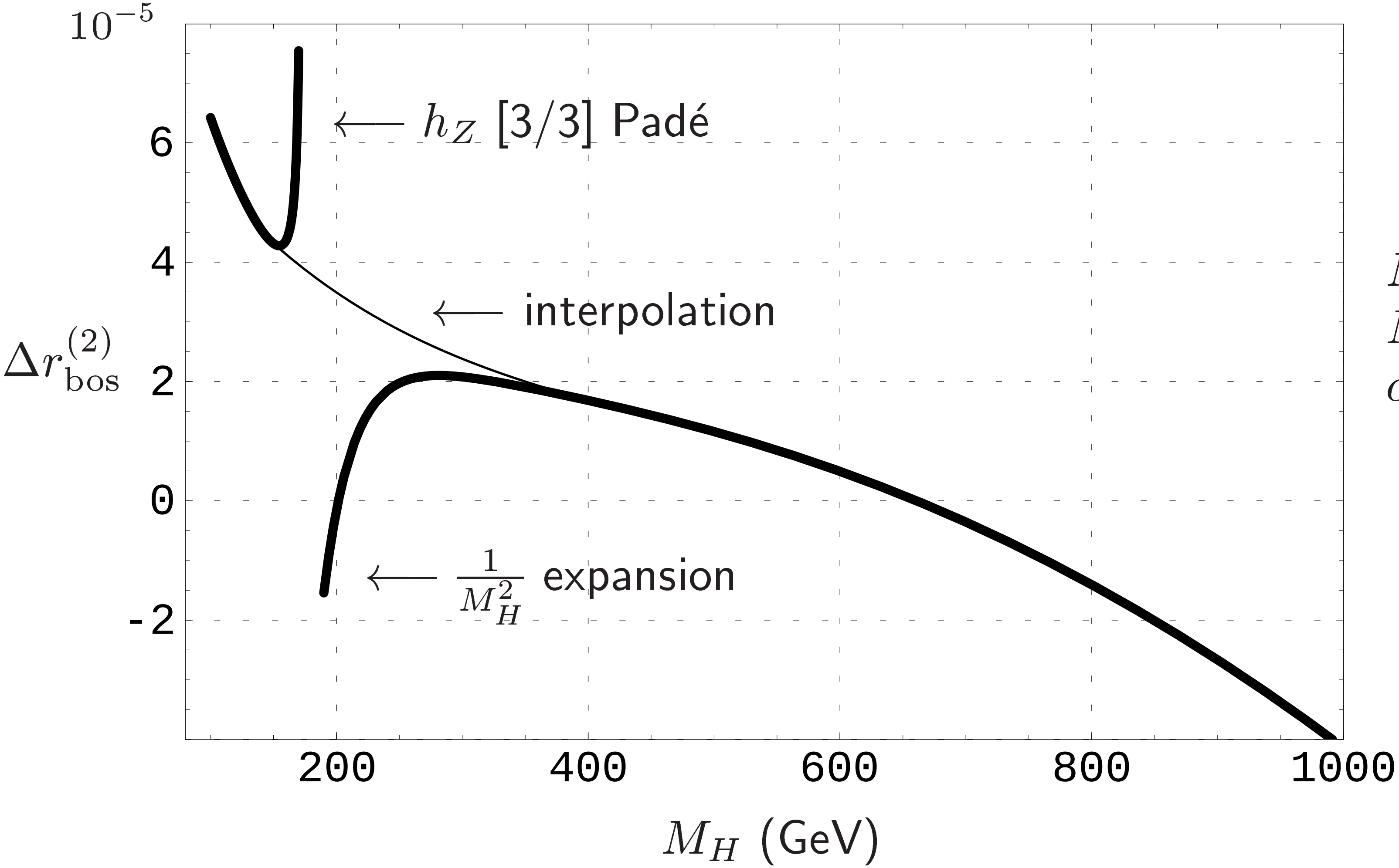
   - m_H is large (z_H-expansion)
   - m_H is close to m_Z (h_Z-expansion)

   single scale problem

   $\Longrightarrow$ analytical result in terms of $\log 2,\ \log 3,\ \zeta_2,\ \zeta_3,\ \frac{\pi}{\sqrt{3}},\ S_2$

3. Improve h_Z-expansion by Padé

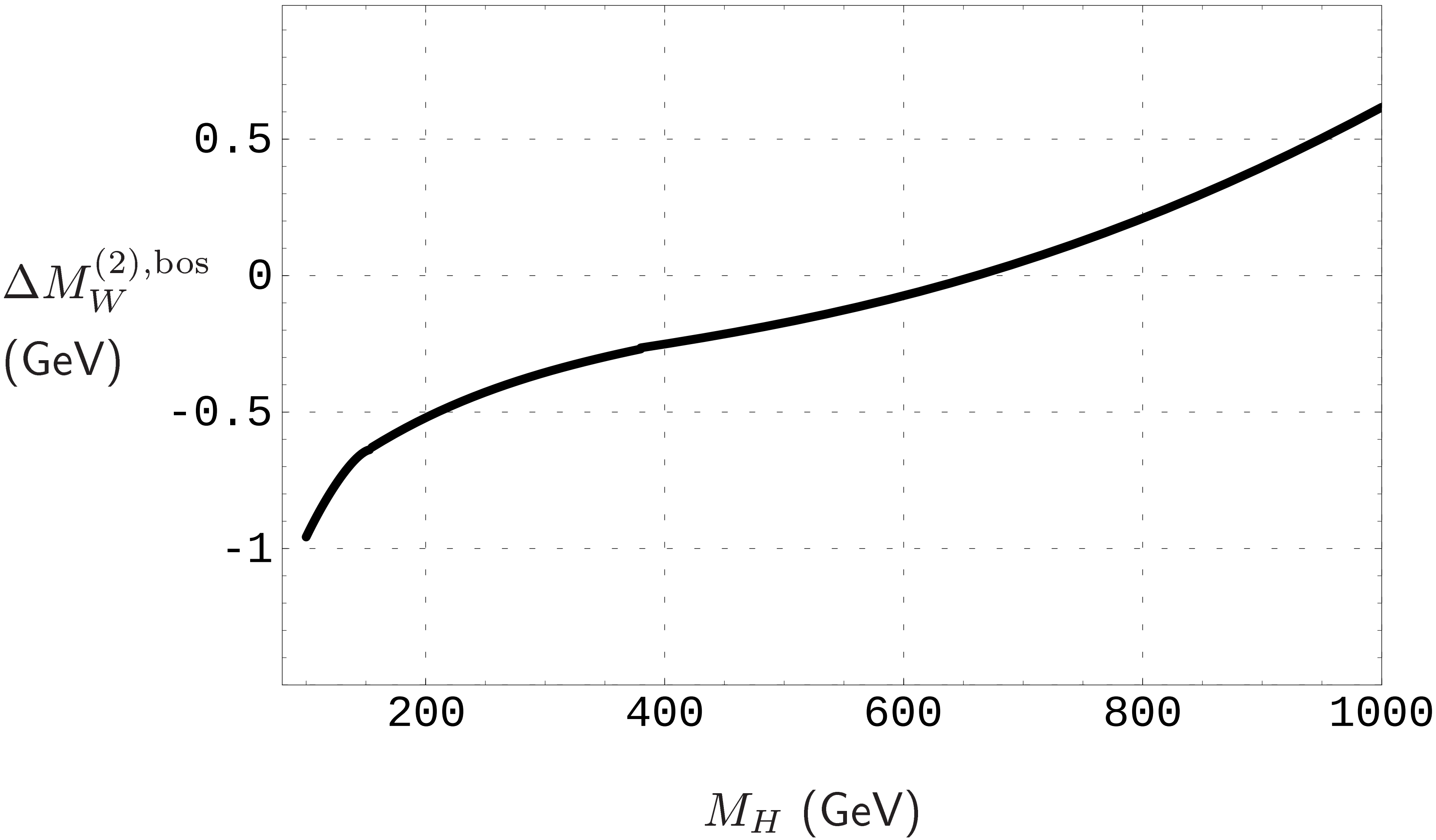
$\Delta r^{(2)}_{\text{bos}}$ as a function of M_H



$M_Z = 91.1876$ GeV
$M_W = 80.423$ GeV
$\alpha^{-1} = 137.036$

<u>There is full agreement with calculation of M. Awramik & M. Czakon!</u>

Shift of $$M_W = M_Z \sqrt{\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi\alpha}{\sqrt{2} G_F M_Z^2}(1 + \Delta r)}}$$



Conclusions

- bosonic two loop electroweak corrections to Δr have been computed using methods of asymptotic expansion

- the framework for the simple and economical calculation of G_F based on the low energy factorization theorem is given

- it is shown that the results of expansions in regimes of Higgs boson mass being large or close to M_Z together with Padé approximations cover the whole region M_H

- the shift of M_W due to two loop bosonic correction does not exceed 1 MeV in the region of Higgs boson mass from 100 to 1000 GeV