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Virtual Photon Structure Functions

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In collaboration with

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1. Introduction and Motivation
2. Virtual Photon Structure Functions
3. $F_2^\gamma(x, Q^2, P^2)$ & $g_1^\gamma(x, Q^2, P^2)$
4. Positivity Constraints
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1. Introduction -Motivation-

- The structure of the virtual photon can be studied from
  - double tagged two-photon process in e^+e^- collisions

    or
  - 2-jet events in resolved photon processes of $e^{-(+)}p$ collisions.

$\Rightarrow$ Effective photon structure functions

$$F_{\rm eff}^{\gamma} \simeq F_{TT}^{\gamma} + F_{TL}^{\gamma} + F_{LT}^{\gamma} + F_{LL}^{\gamma}$$
$$\simeq \frac{1}{x}F_2^{\gamma} + \frac{3}{2}F_L^{\gamma}$$

$\Rightarrow$ Photon structure functions

$$F_2^{\gamma}(x, Q^2), \quad F_L^{\gamma}(x, Q^2)$$

$\Rightarrow$ Quark and gluon distributions inside the photon q^{γ}, G^{γ} extracted

- **Recent review articles**

M. Krawczyk, Talk at Photon 2000, "Structure Functions for the Virtual and Real Photons"
AIP Conf. Proc. hep-ph/0012179

R. Nisius, Talk at Photon 2001, "Summary of the Structure Session at Photon 2001"
To appear in the Proceedings, hep-ph/0110078

M. Klasen, "Theory of Hard Photoproduction"
Rev. Mod. Phys. to appear, hep-ph/0206169

I. Schienbein, "Two-Photon Prosesses and Photon Structure"
hep-ph/0205301

Recent interests in

Photon's spin stuture functions

$\Rightarrow$ Photon structure functions

$g_1^\gamma(x, Q^2, P^2)$, $g_2^\gamma(x, Q^2, P^2)$

- Theor. prediction for the 1st moment of $g_1^\gamma(x, Q^2, P^2) \Rightarrow$ Sum Rule

- Pol. ep colliders will provide the information on spin structure of the photon

- Pol. e^+e^- collision in the future linear colliders

  $\Rightarrow$

  spin-dependent photon structure function

- The $g_2^\gamma(x, Q^2, P^2)$ would provide the information on twist-3 effects.

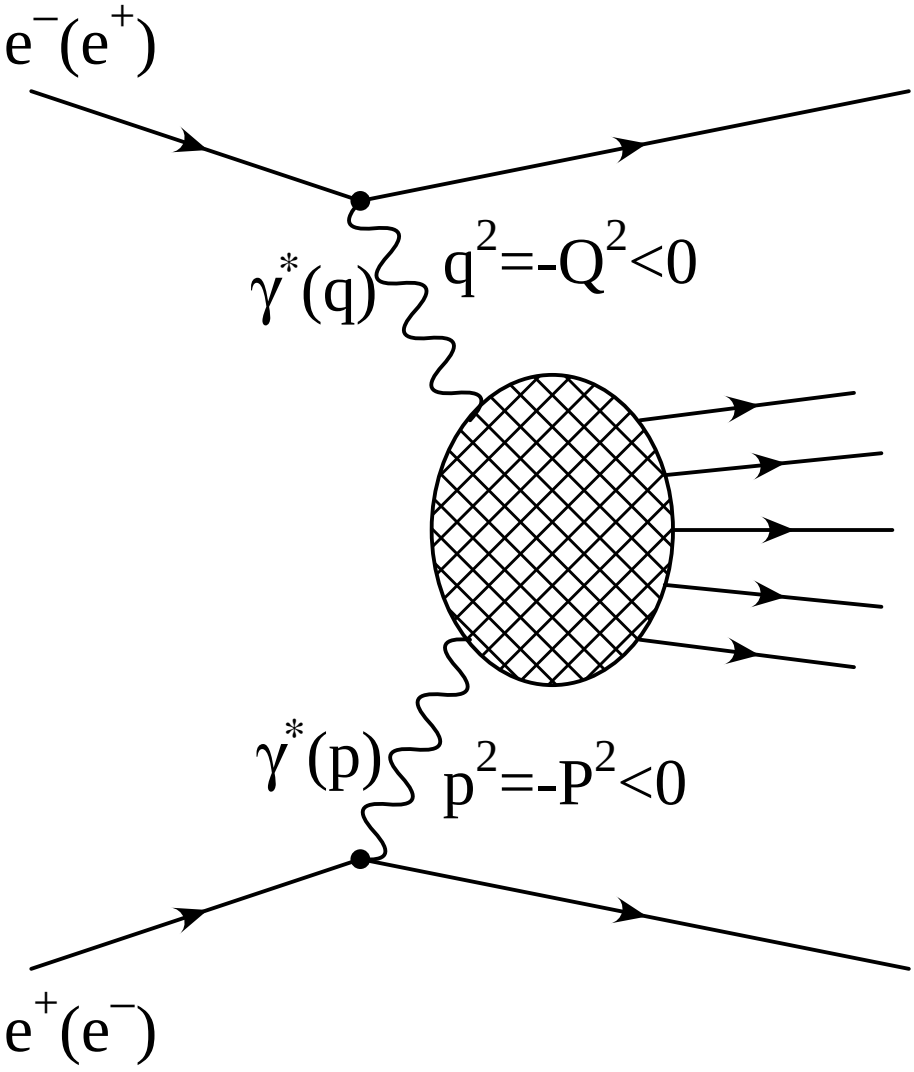
  $\Rightarrow$ Ken Sasaki's Talk

Two-photon process in e^+e^- collision

$\Rightarrow$ **Photon structure structure fns.**

$$F_2^\gamma(x, Q^2, P^2),\ F_L^\gamma(x, Q^2, P^2)$$

$$g_1^\gamma(x, Q^2, P^2),\ \ g_2^\gamma(x, Q^2, P^2)$$



$$\Lambda^2 \ll P^2 \ll Q^2$$

• Virtual photon target $(\Lambda^2 \ll P^2 \ll Q^2)$
Good place to study the structure of the current product:

$$J_\mu(x)J_\nu(0) \sim \sum_n C_n(x)O_n(0)$$

$$W_{\mu\nu}(p,q,s) =$$
$$\int d^4x e^{iq\cdot x}\langle\gamma(p,s)|J_\mu(x)J_\nu(0)|\gamma(p,s)\rangle$$
$$= W^s_{\mu\nu}(p.q) + iW^A_{\mu\nu}(p,q,s)$$

For the virtual photon target, we can calculate $\langle\gamma(p,s)|O_n(0)|\gamma(p,s)\rangle$.

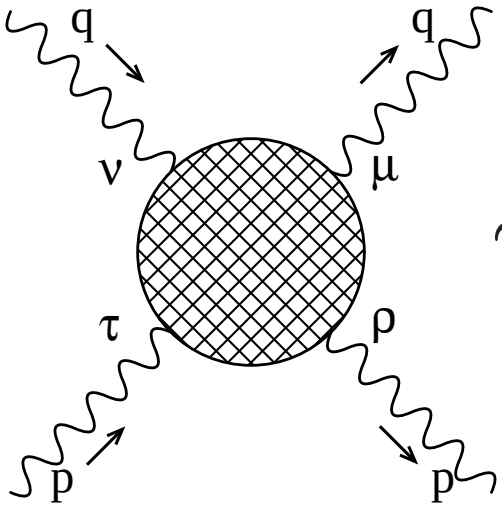
• PQCD gives the whole information
up to NLO on

- $F_2^\gamma(x,Q^2,P^2)$ & $F_L^\gamma(x,Q^2,P^2)$
unpolarized structure functions

- $g_1^\gamma(x,Q^2,P^2)$
polarized structure function

2. Virtual Photon Structure Functions

There are 8 independent structure functions for the virtual photon-photon scattering



$\sim W_{\mu\nu\rho\tau}(p,q)$

Structure Tensor

$$W_{\mu\nu\rho\tau}(p,q) = \frac{1}{\pi}\text{Im}\, T_{\mu\nu\rho\tau}(p,q)$$

$$T_{\mu\nu\rho\tau}(p,q) = i\int d^4x d^4y d^4z e^{iq\cdot x} e^{ip\cdot(y-z)} \times \langle 0|T\left[J_\rho(y)J_\mu(x)J_\nu(0)J_\tau(z)\right]|0\rangle$$

For $\Lambda^2 \ll P^2 \ll Q^2$, or $x^2 \ll (y-z)^2$ we have the Operator Product Expansion (OPE):

$$J_\mu(x)J_\nu(0) \sim \sum_n C_n(x)O_n(0)$$

• Decomposition of Structure Tensor

Budnev-Chernyak-Ginzburg, Nucl. Phys. **B34** (1971) 470

$$
\begin{aligned}
W^{\mu\nu\rho\tau}(p,q) = & \\
& (P_{TT})^{\mu\nu\rho\tau} W_{TT} + (P^a_{TT})^{\mu\nu\rho\tau} W^a_{TT} \\
& +(P^\tau_{TT})^{\mu\nu\rho\tau} W^\tau_{TT} + (P_{ST})^{\mu\nu\rho\tau} W_{ST} \\
& +(P_{TS})^{\mu\nu\rho\tau} W_{TS} + (P_{SS})^{\mu\nu\rho\tau} W_{SS} \\
& -(P^\tau_{TS})^{\mu\nu\rho\tau} W^\tau_{TS} - (P^{\tau a}_{TS})^{\mu\nu\rho\tau} W^{\tau a}_{TS}
\end{aligned}
$$

$(P_{TT})^{\mu\nu\rho\tau}$ etc. : Projectors

$$
(P_{TT})^{\mu\nu\rho\tau} = R^{\mu\nu} R^{\rho\tau}
$$

$$
R^{\mu\nu} = -g^{\mu\nu} + \frac{1}{X}\left[w(q^\mu p^\nu + p^\mu q^\nu) - q^2 p^\mu p^\nu - p^2 q^\mu q^\nu \right]
$$

$w = p \cdot q, \quad X = (p \cdot q)^2 - p^2 q^2$

$(P_{TT}) \cdot (P_{TT}) = 4$ etc.

$(P_i)^{\mu\nu\rho\tau} (P_j)_{\mu\nu\rho\tau} = 0$ for $i \neq j$

T: Transverse Polarization

S: Longitudinal polarization

Helicity amplitudes for $\gamma - \gamma$ scattering

S-channel helicity amplitudes for virtual photon-photon forward scattering:

$$\gamma(q) + \gamma(p) \to \gamma(q) + \gamma(p)$$

$$W(ab|a'b') = \epsilon^{*\mu}(a)\epsilon^{*\rho}(b)W_{\mu\nu\rho\tau}\epsilon^{\nu}(a')\epsilon^{\tau}(b')$$

where $a, a' = 0, \pm 1$ and $b, b' = 0, \pm 1$.

From parity conservation, hermiticity, and angular momentum conservation, the number of independent amplitudes is 8.

helicity non-flip: 5

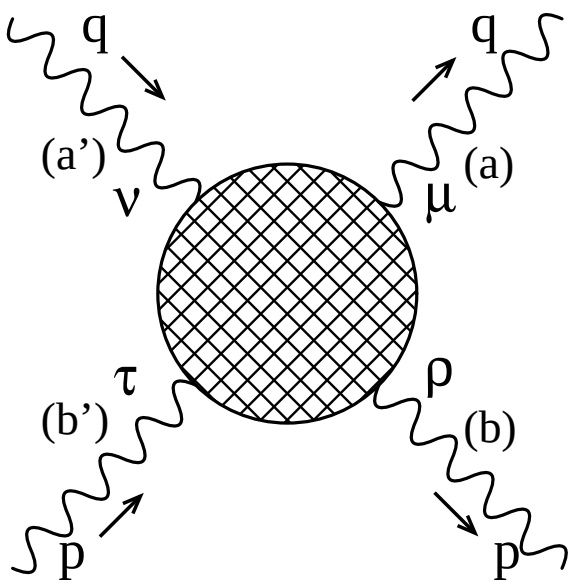
$W(1,1|1,1)$

$W(1,-1|1,-1)$

$W(1,0|1,0)$

$W(0,1|0,1)$

$W(0,0|0,0)$



helicity flip: 3

$W(1,1|-1,-1)$

$W(1,1|0,0)$

$W(1,0|0,-1) = W(0,1|-1,0)$

- Relation between the 8 structure functions and the helicicty amplitudes

$$
\begin{aligned}
W_{TT} &= \frac{1}{2}\left[W(1,1|1,1) + W(1,-1|1,-1)\right] \\
W_{ST} &= W(0,1|0,1) \\
W_{TS} &= W(1,0|1,0), \quad W_{SS} = W(0,0|0,0) \\
W^a_{TT} &= \frac{1}{2}\left[W(1,1|1,1) - W(1,-1|1,-1)\right] \\
W^\tau_{TT} &= W(1,1|-1,-1) \\
W^\tau_{TS} &= \frac{1}{2}\left[W(1,1|0,0) - W(1,0|0,-1)\right] \\
W^{\tau a}_{TS} &= \frac{1}{2}\left[W(1,1|0,0) + W(1,0|0,-1)\right]
\end{aligned}
$$

- The helicity-nonflip amps. are non-negative

  $\Rightarrow$

  The first four structure fns. are postive-definite and the last four are not.

- Symmetry: All structure functions are symmetric under intechange of $p \leftrightarrow q$ except W_{ST} and W_{TS}:

$$W_{ST}(w, q^2, p^2) = W_{TS}(w, p^2, q^2)$$

3. $F_2^\gamma(x, Q^2, P^2)$ & $g_1^\gamma(x, Q^2, P^2)$

Basic frameworks for NLO QCD calculation

- Operator Product Expansion (OPE) + Renormalization Group (RG)

  or

- DGLAP type Parton Evolution Eq.

For $g_1^\gamma(x, Q^2, P^2)$

Two-loop anomalous dimensions (or two-loop parton splitting functions) were calculated by

- Mertig and van Neerven, Z. Phys. C70 (1996) 637
- Vogelsang, Phys. Rev. D54 (1996) 2023; Nucl. Phys. B475 (1996) 47.

$\Rightarrow$ Next-to-leading order (NLO) calculation possible.

OPE

$$J_\mu(x)J_\nu(0) \sim \sum_{n,i} O_n^i(0) C_n^i(x)$$

twist-2 ops. coefficient fns.

The n-th moment of $F_2^\gamma(x, Q^2, P^2)$ & $g_1^\gamma(x, Q^2, P^2)$ for the kinematical region:

$$\Lambda^2 \ll P^2 \ll Q^2$$

$$\begin{aligned}
&\int_0^1 dx x^{n-1} g_1^\gamma(x, Q^2, P^2) \\
&= \frac{\alpha}{4\pi} \frac{1}{2\beta_0} \Bigg[\sum_{i=+,-,NS} L_i^n \frac{4\pi}{\alpha_s(Q^2)} \left\{ 1 - \left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{\lambda_i^n/2\beta_0+1} \right\} \\
&\quad + \sum_{i=+,-,NS} \mathcal{A}_i^n \left\{ 1 - \left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{\lambda_i^n/2\beta_0} \right\} \\
&\quad + \sum_{i=+,-,NS} \mathcal{B}_i^n \left\{ 1 - \left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{\lambda_i^n/2\beta_0+1} \right\} \\
&\quad + \mathcal{C}^n + \mathcal{O}(\alpha_s) \Bigg]
\end{aligned} \tag{1}$$

$\cdots$ Master equation

where L_i^n, $\mathcal{A}_i^n$, $\mathcal{B}_i^n$ and $\mathcal{C}^n$ are computed from the NLO QCD perturbation theory.

λ_i^n $(i = +, -, NS)$: eigenvalues of one-loop anomalous dim.

NLO QCD calculations of g_1^γ:

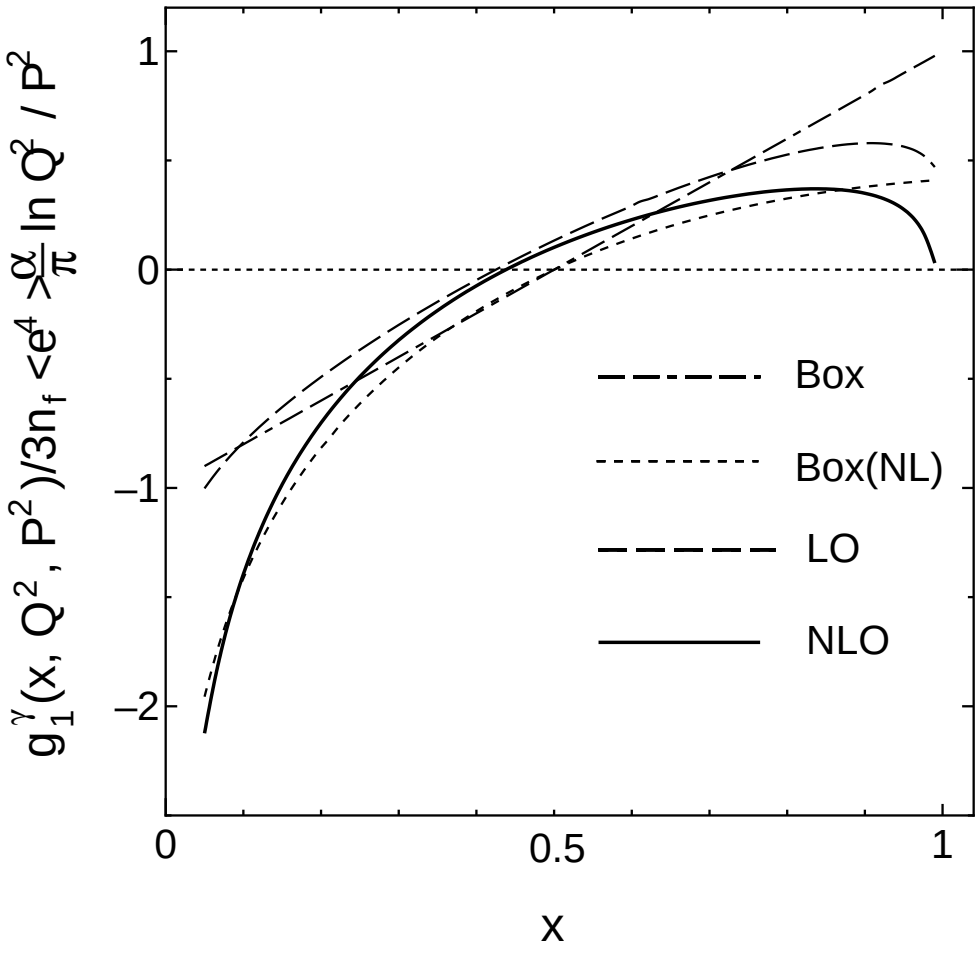
- M. Stratmann and W. Vogelsang
  Phys. Lett. B386 (1996) 370
- K. Sasaki and T.U.
  Phys. Rev. D59 (1999) 11401
  Phys. Lett. B473 (2000) 309
- K. Sasaki and T.U.
  Eur. Phys. J. C20 (2001) 283
- M. Glück, E. Reya and C. Sieg
  Phys. Lett. B503 (2001) 285
- M. Glück, E. Reya and C. Sieg
  Eur. Phys. J. C20 (2001) 271

$$|g_1^\gamma| \leq F_1^\gamma$$

Polarized Virtual Photon Structure Function g_1^γ

$Q^2 = 30\ \mathrm{GeV}^2$

$P^2 = 1\ \mathrm{GeV}^2$



For real photon ($P^2 = 0$) with QCD effects

Bass-Brodsky-Schmidt, Phys. Lett. B437 (1998) 424

$$\int_0^1 dx g_1^\gamma(x, Q^2) = 0$$

The $n = 1$ moment

$$\int_0^1 dx g_1^\gamma(x, Q^2, P^2) \qquad ?$$

Taking $n \to 1$ limit of (1)

Note that the eigenvalues of 1-loop anomalous dim. matrix $\hat{\gamma}^{(0)}_{n=1}$:

$$\lambda_+^{n=1} = 0, \lambda_-^{n=1} = -2\beta_0$$

The first 3 terms vanish

$$L_i^n \to 0, \sum_i \mathcal{A}_i^n\{\ \} \to 0, \sum_i \mathcal{B}_i^n\{\ \} \to 0$$

$$\int_0^1 dx g_1^\gamma(x, Q^2, P^2) = \frac{\alpha}{4\pi} \frac{1}{2\beta_0} \mathcal{C}^{n=1} + \mathcal{O}(\alpha_s)$$

$$\boxed{\int_0^1 dx g_1^\gamma(x, Q^2, P^2) = -\frac{3\alpha}{\pi} \sum_{i=1}^{n_f} {e_i}^4 + \mathcal{O}(\alpha_s)}$$

We can go further to

$\mathcal{O}(\alpha_s)$ QCD Corrections

$$\int_0^1 dx g_1^{\gamma}(x, Q^2, P^2)$$
$$= -\frac{3\alpha}{\pi}\left[\sum_{i=1}^{n_f} e_i^4\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)\right.$$
$$\left. - \frac{2}{\beta_0}(\sum_{i=1}^{n_f} e_i^2)^2\left(\frac{\alpha_s(P^2)}{\pi} - \frac{\alpha_s(Q^2)}{\pi}\right)\right]$$
$$+\mathcal{O}(\alpha_s^2)$$

for

$\Lambda^2 \ll P^2 \ll Q^2$ $\quad n_f$:# of active flavors

Sasaki & T.U.

Phys. Rev. D59 (1999) 11401

coincides with the result of

Narison-Shore-Veneziano

Nucl. Phys. B391 (1993) 69

obtained from a different approach.

4. Positivity Constraints

K. Sasaki, J. Soffer and T. Uematsu, Phys. Lett. B522 (2001) 22.

K. Sasaki, J. Soffer and T. Uematsu, Phys. Rev. D66 (2002) 034014.

Cauchy-Schwarz inequality

$$|W(a,b|a',b')| \leq \sqrt{W(a,b|a,b)W(a',b'|a',b')}$$

$\Rightarrow$ **Three positivity constraints**

$$|W(1,1|-1,-1)| \leq W(1,1|1,1)$$

$$|W(1,1|0,0)| \leq \sqrt{W(1,1|1,1)W(0,0|0,0)}$$

$$|W(1,0|0,-1)| \leq \sqrt{W(1,0|1,0)W(0,1|0,1)}$$

In terms of structure functions

$$|W^{\tau}_{TT}| \leq (W_{TT} + W^{a}_{TT})$$

$$|W^{\tau}_{TS} + W^{\tau a}_{TS}| \leq \sqrt{(W_{TT} + W^{a}_{TT})W_{SS}}$$

$$|W^{\tau}_{TS} - W^{\tau a}_{TS}| \leq \sqrt{W_{TS}W_{ST}}$$

The virtual photon structure functions F_2^γ, F_L^γ, and g_1^γ are related to W_i's as

$$
\begin{aligned}
F_1^\gamma(x,Q^2,P^2) &= 2\left[W_{TT} - \frac{1}{2}W_{TS}\right] \\
F_2^\gamma(x,Q^2,P^2) &= \frac{2x}{\widetilde{\beta}^2}\left[W_{TT} + W_{ST} - \frac{1}{2}W_{SS} - \frac{1}{2}W_{TS}\right] \\
F_L^\gamma(x,Q^2,P^2) &= \widetilde{\beta}^2 F_2^\gamma - xF_1^\gamma, \\
g_1^\gamma(x,Q^2,P^2) &= \frac{2}{\widetilde{\beta}^2}\left[W_{TT}^a - \frac{(P^2Q^2)^{1/2}}{w}W_{TS}^{\tau a}\right] \\
g_2^\gamma(x,Q^2,P^2) &= -\frac{2}{\widetilde{\beta}^2}\left[W_{TT}^a - \frac{w}{(P^2Q^2)^{1/2}}W_{TS}^{\tau a}\right] \\
&\qquad \text{with} \quad \widetilde{\beta} = \left(1 - \frac{P^2Q^2}{w^2}\right)^{1/2}
\end{aligned}
$$

For $\widetilde{\beta} \approx 1$

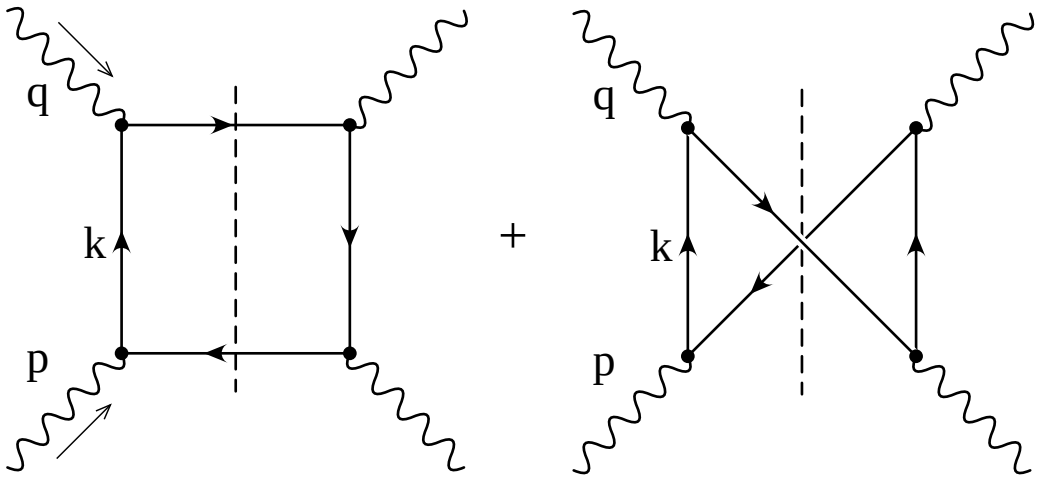
$$
\begin{aligned}
W_{TT}(x,Q^2,P^2) &\approx \frac{1}{2}F_1^\gamma(x,Q^2,P^2) \\
&= \frac{1}{2x}\left\{F_2^\gamma(x,Q^2,P^2) - F_L^\gamma(x,Q^2,P^2)\right\} \\
W_{TT}^a(x,Q^2,P^2) &\approx \frac{1}{2}g_1^\gamma(x,Q^2,P^2)
\end{aligned}
$$

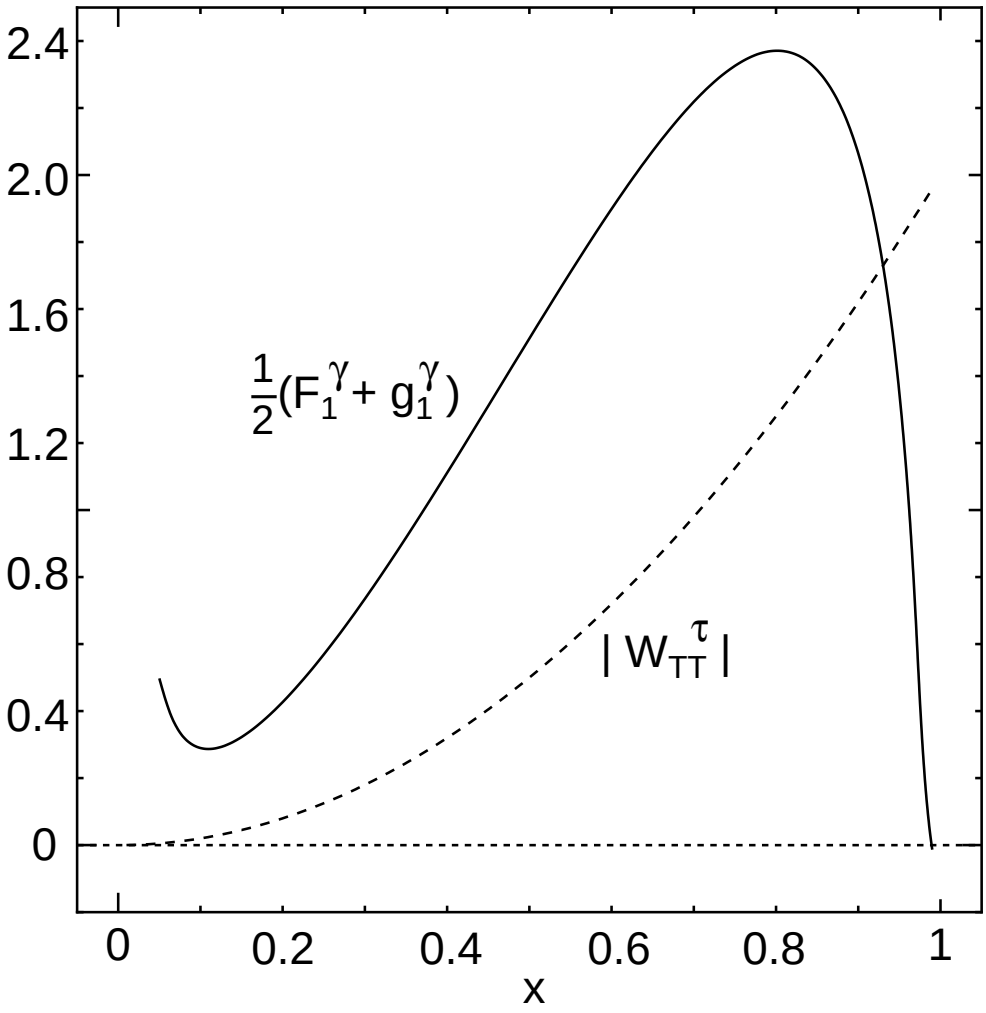
Let's see if the first positivity constriant are satisfied in the pQCD results.

$$|W_{TT}^{\tau}(x,Q^2,P^2)| \lesssim \frac{1}{2}\left[F_1^{\gamma}(x,Q^2,P^2) + g_1^{\gamma}(x,Q^2,P^2)\right]$$

The parton model BOX diagram calculation for $W_{TT}^{\tau}(x,Q^2,P^2) = 2W_3^{\gamma}$ reads

$$W_{TT}^{\tau}(x,Q^2,P^2) = \frac{\alpha}{2\pi}\delta_{\gamma}\left[(-2x^2) + \mathcal{O}(\alpha_s(Q^2))\right]$$




2.4
2.0
1.6
1.2
0.8
0.4
0
0
0.2
0.4
0.6
0.8
1
x
$\frac{1}{2}(F_1^\gamma + g_1^\gamma)$
$|W_{TT}^\tau|$

5. Concluding Remarks

- Virtual photon structure provides

  Good testing ground for the structure of the current product:

$$J_\mu(x)J_\nu(0) \sim \sum_n C_n(x)O_n(0)$$

- Virtual photon's unpolarized and polarized structure fn.:

$$F_2^\gamma(x,Q^2,P^2) \qquad g_1^\gamma(x,Q^2,P^2)$$

$$\Lambda^2 \ll P^2 \ll Q^2$$

  Perturbatively calculable to NLO QCD

  $g_2^\gamma(x,Q^2,P^2)$ computed for large N_c for flavor non-singlet case to LO.

- 1st moment of g_1^γ non-vanishing

  $\Rightarrow$ Sum Rule

$$\int_0^1 dx g_1^\gamma(x,Q^2,P^2) = -\frac{3\alpha}{\pi}\sum_{i=1}^{n_f} e_i^4 + \mathcal{O}(\alpha_s)$$

in contrast to the real photon case:

$$\int_0^1 dx g_1^\gamma(x, Q^2) = 0$$

cf. Drell-Hearn-Gerasimov sum rule

- The NLO results qualitatively consistent with the analysis by
  Stratman & Vogelsang,
  Phys. Lett. B386(1996)370

  Glück, E. Reya & Sieg,
  Phys. Lett. B503 (2001) 285
  Eur. Phys. J. C20 (2001) 271

- Three positivity constraints on virtual photon structure functions derived

Future subjects

- Full QCD analysis of g_2^γ structure function
  twist-3 effects in $g_2^\gamma(x, Q^2, P^2)$ $\Leftarrow$ needs to solve the mixing problem.
- How can we understand the transition from vanishing 1st moment for real photon ($P^2 = 0$) to non-vanishing one for virtual photon ($P^2 \gg \Lambda^2$)?
- Power corrections of the form

$$(P^2/Q^2)^k \quad (k = 1, 2, \cdots)$$

  target mass effects and higher-twist effects
- There remain a number of virtual photon structure functions yet to be studied.