$$\Delta_n = \begin{vmatrix} Y_{11} & Y_{12} & \ldots & Y_{1n} \\ Y_{12} & Y_{22} & \ldots & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{1n} & Y_{2n} & \ldots & Y_{nn} \end{vmatrix}, \qquad Y_{ij} = -(p_i - p_j)^2 + m_i^2 + m_j^2,$$

p_i, p_j are externa momenta and m_j is the mass of the j-th line , $\partial_j \equiv \partial/\partial m_j^2$

$$G_{n-1} = -2^n \begin{vmatrix} p_1p_1 & p_1p_2 & \ldots & p_1p_{n-1} \\ p_1p_2 & p_2p_2 & \ldots & p_2p_{n-1} \\ \vdots & \vdots & \ddots & \vdots \\ p_1p_{n-1} & p_2p_{n-1} & \ldots & p_{n-1}p_{n-1} \end{vmatrix}.$$

Integrals with zero kinematic determinants

When one or both of the determinants G_{n-1} and Δ_n are equal to zero it is possible to express integrals with n lines as a combination of integrals with $n-1$ lines.

$$I_n^{(d)} = -\sum_{k=1}^{n} \frac{(\partial_j \partial_k \Delta_n)}{(\partial_j \Delta_n)} \, \mathbf{k}^- I_n^{(d)}.$$

If $\Delta_n = 0$ use relation

$$(d - \sum_{i=1}^{n} \nu_i - 1) G_{n-1} I_n^{(d)} = \sum_{k=1}^{n} (\partial_k \Delta_n) \mathbf{k}^- I_n^{(d-2)},$$

and also

$$\sum_{j=1}^{n} \nu_j \mathbf{j}^+ I_n^{(d+2)} = -I_n^{(d)} .$$