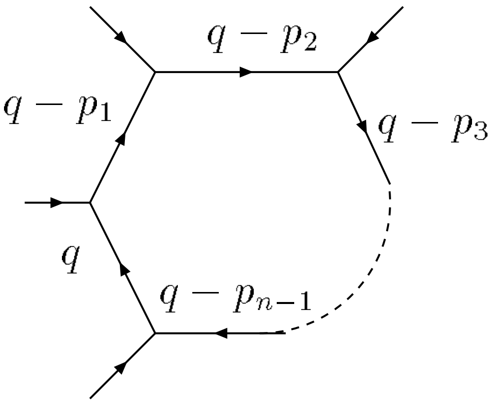

Integrals with non-zero Gram determinants

Relation for removing dots from lines

$$2\Delta_n \nu_j \mathbf{j}^+ I_n^{(d)} = \sum_{k=1}^{n} (1+\delta_{jk}) \left(\frac{\partial \Delta_n}{\partial Y_{jk}}\right) \left[d - \sum_{i=1}^{n} \nu_i (\mathbf{k}^- \mathbf{i}^+ + 1)\right] I_n^{(d)},$$

Reducing dimension and index of the j-th line

$$G_{n-1} \nu_j \mathbf{j}^+ I_n^{(d+2)} = \left[(\partial_j \Delta_n) + \sum_{k=1}^{n} (\partial_j \partial_k \Delta_n) \mathbf{k}^-\right] I_n^{(d)},$$

Reducing the space-time dimension

$$(d - \sum_{i=1}^{n} \nu_i + 1) G_{n-1} I_n^{(d+2)} = \left[2\Delta_n + \sum_{k=1}^{n} (\partial_k \Delta_n) \mathbf{k}^-\right] I_n^{(d)}.$$

$\mathbf{j}^{\pm}$ etc. shift the indices $\nu_j \to \nu_j \pm 1$